

1a

$$\underline{N = 2}$$

$$6 \mid 2^3 - 2 = 6$$

$$\underline{N = 3}$$

$$6 \mid 3^3 - 3 = 24$$

$$\underline{N = k}$$

$$6 \mid (k^3 - k)$$

$$k^3 - k = 6x$$

$$6x = ((k+1)^3 - (k+1))$$

$$6x = (k^3 + 3k^2 + 3k + 1 - (k+1))$$

$$6x = k^3 + 3k^2 + 3k + 1 - k - 1$$

$$6x = k^3 + 3k^2 + 2k$$

$$6x = (k^3 - k) + 3k(k+1)$$

k or k+1 must be even

1b

$$N = 1$$

$$3 \cdot 1 - 1 = 2 = 1(3 \cdot 1 + 1)/2 = 2$$

$$N = 2$$

$$3 \cdot 2 - 1 + 2 = 7 = 2(3 \cdot 2 + 1)/2 = 7$$

$$N = k$$

$$2 + 5 + 8 \dots + (3k-1) = k(3 \cdot k + 1)/2$$

$$\sum 3k-1 = k(3 \cdot k + 1)/2$$

2

$$A_n = 4a_{n-1} - 4a_{n-2} \quad a_n = 2^n(n-2)$$

$$A_3 = 4a_2 - 4a_1 \quad a_3 = 2^3(3-2)$$

$$A_3 = 4 \cdot 0 - 4 \cdot -2 \quad a_3 = 2^3(1)$$

$$A_3 = 8 \quad a_3 = 8$$

$$A_{k+1} = 2^{k+1}(k+1-2)$$

$$A_{k+1} = 2 \cdot 2^k \cdot (k+1-2)$$

$$A_{k+1} = 2(k+1-2) \cdot 2^k$$

$$A_{k+1} = 2^k(2k-2)$$

3

i $\text{Gcd}(a, a^2) = a$

ii 1

iii if a is even: 2

 If a is odd: 1