1

a

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

B

$$\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}^2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \quad \ldots^3 = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$$

$$\ldots^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So order is 3

C

Let D be a diagonal matrix in G with non-zero entries d1 and d2 on the diagonal, where d1, d2 ∈ {1,2}. We need to show that the order of D is either 1 or 2.

d1 = d2 = d

$$\begin{bmatrix} d & 0 \\ 0 & d \end{bmatrix}$$

$$D^2 = \begin{bmatrix} d & 0 \\ 0 & d \end{bmatrix}^2 = \begin{bmatrix} d^2 & 0 \\ 0 & d^2 \end{bmatrix}$$

Since we are working modulo 3, we have d^2 = d (mod 3) for any d ∈ {1,2}. Therefore, D^2 is equal to D, which means that the order of D is 2.

d1 = 1 and d2 = 2. Then, D is of the form:

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$D^2 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2

a

Z∗20={1,3,7,9,11,13,17,19}

B

X	1	3	7	9
1	1	3	7	9
3	3	9	1	7
7	7	1	9	3
9	9	7	3	1

C

1^1 ≡ 1 (mod 20), 1^2 ≡ 1 (mod 20), 1^3 ≡ 1 (mod 20), ...

3^1 ≡ 3 (mod 20), 3^2 ≡ 9 (mod 20), 3^3 ≡ 7 (mod 20), 3^4 ≡ 1 (mod 20), ...

7^1 ≡ 7 (mod 20), 7^2 ≡ 9 (mod 20), 7^3 ≡ 3 (mod 20), 7^4 ≡ 1 (mod 20), ...

9^1 ≡ 9 (mod 20), 9^2 ≡ 1 (mod 20), 9^3 ≡ 9 (mod 20), ...

1 = 3^0

3 = 3^1

7 = 3^3

9 = 3^2

3