Write as a linear combination of the standard basis vectors I, j, k

(-1, 0, 12) = -i + 12k

Unit vector:

= x/||x||

Eg. s = (3, 4)

||s|| = √(32 + 4^2)

=

Parallel vectors if same direction or opposite

If scale multiple of other vector

Write down a vector parametric description of the x, y and z axis

X axis rx = s(1, 0, 0) = si

Y axis ry = sj

Z = rz = sk

For (1, 4, -3)

R = s(1, 4, -3)

Rx = si

Ry = 4sj

Rz = -3sk

Find a vector parametric form of the line y=2x + c final form (x, y) + f()

Introduce a parameter, for example x=f

Then y = 2f+3

= (0, 3) + (f, 2f)

= (0, 3) + f(1, 2)

Line through 2 points

Vector A goes to point A, Vector B foes to point B

B-A = Vector between 2 A and B

A = s(2, 4)

B = s(3, 1)

C = (3, 1) - s(1, -3) where s = any real number

Line through 3 points

A = (4, -5, 2)

B = (3, -1, 6)

B – A = (-1, 4, 4)

= (3, -1, 6) + s(-1, 4, 4) where s is any real number

Distances

Length/magnitude/norm = ||x|| = pythag

Distance between 2 points is d(x, y) == ||x – y||

What is the distance between the points (1, 2, 3) and (5, 6, 0)

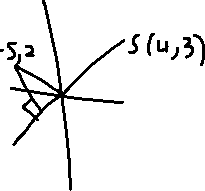
||x – y|| = ||(1, 2, 3) – (5, 6, 0)|| = ||(-4, -4, 3)||

= √(4^2 + 4^2 + 3^2) = √(41)

Dot product

(x1, y1, z1) \* (x2, y2, z2) = (x1\*x2) + (y1\*y2) + (z1\*z2) = scalar

Distance from the line r = s(4, 3) and the point (-5, 2)



Y = projr x = r\*x/||r||2 \* r

= (4,3) \* (-5, 2)/ 25

= -14/25 (4, 3)

Planes

Vector parametric of a plane

R = r0 + sd + te, s, t are real numbers

One point on the plane and 2 vectors parallel to the plane

R = r­0 + sd + te

The vector parametric form of the plane through the point (1, 2, 3) with direction vectors (4, 5, 6) and (7, 8, 9)

X =

Y =

Z =

Find the plane though the points (1, 2, 1), (5, 0, -1) and (3, -1, -1)

R = r0 +sd + te, st is real

R0 = (1, 2, 1)

D = b – a = (5, 0, -1) – (1, 2, 1) = (4, -2, -2)

E = c – a = (5, 0, -1) - (3, -1, -1) = (2, 1, 0)

R = (1, 2, 1) + s(4, -2, -2) + t(2, 1, 0). St are real

One point and one normal direction vector

Perpendicular = dot product = 0

N . (r-r0) = 0 point normal form of a plane

Find point normal equation for the plane though the point (-2, 5, 4) and n = (3, 1, -2)

(r-r0) . n = 0

(r – (-2, 5, 4)) . (3, 1, 2) = 0

Scalar form of a plane

Ax + by + cz = d

Normal = (a, b ,c)

Scalar form from normal form

R0 = (-2, 5, 4)

N = (3, 1, -2)

(r – (-2, 5, 4)) \* (3, 1, 2) = 0

R = (x, y, z)

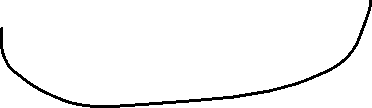
((x, y, z) – (-2, 5, 4)) \* (3, 1, 2) = 0

(x + 2, y – 5, z - 4) \* (3, 1, 2) = 0

3(x + 2) + 1(y - 5) + 2(z - 4) = 0 (r – r0) \* n = 0

3x + 6 + y-5 – 2z + 8 r\*n – r0\*n = 0

3x + y – 2z = -9



# Week 2

Write the plane x + 2y + 3z = 4 in vector parametric form



R = r0 + sd + te one equation with 3 variables, so we have 2 free variables

Choose 2 parameters to represent the free variables

Y = s

Z = t

Then write the other variables in terms of these parameters

X + 2s + 3t = 4 x = 4 – 2s – 3t

R = (x, y, z) = (4-2s-3t, s, t)

= (4 – 2s – 3t, 0+s + 0, 0 + 0 + t)

= (4 ,0, 0) + s(-2, 1, 0) + t(-3, 0, 1)

Vector parametric of the plane

Vector parametric form to point normal and scalar form

R = (1, 2, 3) + s(2, 0, 1) + t(4, 1 ,-2)

Ax + by + dz = d

N = (a, b, c) = normal of the plane

Because n is normal to the plane, it is normal to both directional vectors

So (2, 0, 1) . (a, b, c) = 0

And (4, 1, -2) . (a, b, c) = 0

2a + c = 0

4a + b – 2c = 0

C = -2a

4a + b – 2(-2a) = 0 = 8a + b = 0

-8a = b

A = 1 (we choose)

B = -8 c = -2

N = (1, -8, -2)

# Cross product



X = (1, 2, 3) and y = (4, 5, 6)

(x2\*y3 – x3\*y2) – (x1\*y3 – x3\*y1) + (x1\*y2 – x2\*y1)



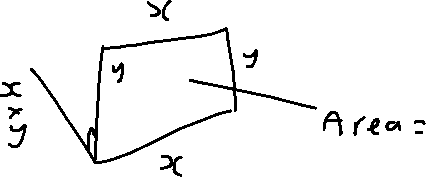
A = (1, 2, -2) b = (3, 0, 1)

Find A x B

1, 2, -2

3, 0, 1

(2\*1 - -2\*0) – (1\*1 - -2\*3) + (1\*0 – 2\*3) = (2) - (7) + (-6) = (2, -7, -6)



X x Y = normal to both X and Y

Area of plane with any angle

Cross product of parallel vectors gives 0

Find a scalar form of the plane via the cross product of r = (1, 2, 3) + s(2, 0, 1) + t(4, 1, -2)

2 0 1

4 1 -2 = (0\*-2 – 1\*1) – (2\*-2 – 1\*4) + (2\*1 – 0\*4) = (-1) – (8) + (2) = (-1, 8, 2)

Final form = (r – r0). N = 0

r.n – r0 . n = 0

r.n = r0 . n

(x, y, z).(-1, 8, 2) = (1, 2, 3).(-1, 8, 2)

-x + 8y + 2z = -1 + 16 + 6

-x + 8y + 2z = 21

Point normal form to vector parametric form

Write the plane (3, 1, 2) . (r – (-2, 5, 4))

In vector parametric form r = r0 +sd + te

**All vectors orthogonal to n are parallel to the plane, therefore if we take the cross product with n with any other vector (non parallel to n), we will obtain a vector parallel to the plane. To find d and e, cross product of n with a vector of our choosing eg. (1, 0, 0)**

d = (3, 1, 2) x (1, 0, 0) = (0, -2, -1)

e = (3, 1, 2) x (0, 1, 0) = (2, 0, 3) OR e = d x n

r = (-2, 5, 4) + s(0, -2, 1) + t(2, 0, 3)

# Area of a parallelogram

Find the area of a parallelogram determines by the vectors u = (1, -1, 2) and v = (0, 3, 1)

A = base \* height

So A = ||u x v||

U x v = 1 -1 2

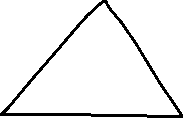
0 3 1 = (-1\*1 – 2\*3) – (1\*1 – 2\*0) + (1\*3 - -1\*0) = (-7) – (1) + (3) = (-7, -1, 3)

||(-7, -1, 3)|| = √(49 + 1 + 9) = √(59)

# Area of a triangle

Find the area of a triangle with the vertices at A(1, -1, 0), B(2, 1, -1) and C(-1, 1, 2)

b-a = (2, 1, -1) – (1, -1, 0) = (1, 2, -1)



c – a = (-1, 1, 2) – (1, -1, 0) = (-2, 2, 2)

Area = 0.5\*||(b-a)x(c-a)||

Find cross multiply

1 2 -1

-2 2 2 (4 - -2) – (2 - 2) + (2 - -4) = (6, 0, 6)

= 0.5 \* √(36 + 36) = √(72)

= 0.5 \* √(9) \* √(4) \* √(2) = 0.5 \* 3 \* 2 \* √(2) = 3\*√(2)

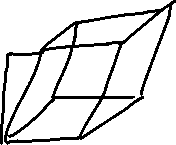
# Volume of a parallelopiped

Show the volume of the parallelopiped determined by the 3 vectors x, y and z is |x . (y x z)|

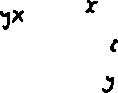
3d parallelogram

V = Barea \* height perpendicular

z = depth



y = horizontal



x = height

perpendicular height direction = y x z

perpendicular height = projyxz x

V = ||yxz|| x ||projyxzx||

= ||yxz|| x ||x.(yxz)/||yxz||2 \* (yxz)||



= ||x . (y x z)||

# Finding Volume

Find the volume of parallelopiped with axis a = (2, -6, 2), b = (0, 4, -2), c = (2, 2, -4)

Vol = |a . (b x c)|

B x c = (-12, -4, -8)

Then |a(b x c)| = |(2, -6, 2) . (-12, -4, -8)| = -24 + 24 – 16 = -16

# Find the intersection of a plane and a line

Where does the line r = (1, 2, 3) + s(3, 2, -1) meet the plane x + y + 2z = 3

R = (1, 2, 3) + s(3, 2, -1)

X = 1 + 3s

Y = 2 + 2s

Z = 3 – s

Substitute into plane

X + y + 2z = 3

(1 + 3s) + (2 + 2s) + 2(3-s) = 3

1 + 3s + 2 + 2s + 6 + 2s = 3

3s = -6

S = -2 this is the value of s at the point of intersection

R = (1, 2, 3) -2(3, 2, -1) = (-5, -2. 5)

Does the line r = (1, 2 ,3) + s(-1, -1, -1) meet the plane x + y + 2z = 3

X = (1 -s)

Y = (2 - s)

Z = (3 + s)

(1 - s) + (2 - s) + 2(3 + s) = 3

1 – s + 2 – s + 6 + 2s = 3

0 = -6 s is undefined

Find normal of plane

N = (1, 1, 2)

N . (-1, -1, 1) = 0

Lines are parallel

# Intersections of lines

R1 = (2, -4, 2) + s(3, 1, -2) and r2 = (3, -1, 0) + t(-2, 2, 1)

X: 2 + 3s = 3 – 2t

Y: -4 + s = 3 + 2t

Z: 2 - 2s = t

2 + 3s = 3 – 2(2 + 2s) = s = 3

2 + 2\*(3) = t = -4

-4 + 3 = 3 + 2\*-4

-1 = -9 Not True

There are no values of s and t which satisfy all the equations therefore the two lines do not intersect

They are skew lines

Show the lines intersect

R1 = (1, 2, -1) + s(3, -1, 2) and r2 = (3, 2, 0) + t(4, -2, 3)

X: 1 + 3s = 3 + 4t

Y: 2 – s = 2 – 2t s = 2t

Z: -1 + 2s = 3t

1 + 3(2t) = 3 + 4t

1 + 6t = 3 + 4t 2 = 2t t=1

S = 2(1) = 2

-1 + 2(2) = 3(1)

-1 + 4 = 3

Intersection point Is r1 = (1, 2, -1) + 2(3, -1, 2) = (7, 0, 3)

R2 = (3, 2, 0) + 1(4, -2, 3) = (7, 0, 3)

# Distance from point to plane

Which point on the plane x – y + z = 0 is closest to the point (5, 6 ,7)

How far is the point

Step 1: n = (1, -1, 1) = d

R = r0 + sd

R = (5, 6, 7) + s(1, -1, 1)

Step 2: find s

X = 5 + s

Y = 6 – s

Z = 7 + s

Sub into plane

(5 + s) – (6 - s) + (7 + s) = 0

Step 3: 5 – 6 + 7 + s + s + s = 0 6+3s = 0 s=-2

Then x is x = 5 – 2 = 3

Y = 6 – -2 = 8

z = 7 – 2 = 5

x = (3, 8, 5)

find distance from x to p

||x-p|| = ||(3, 8, 5) – (5 ,6, 7)||

= √(4 + 4 + 4) = √(12) = √(4)\*√(3) = 2\*√(3)

# Week 3

Linear equations

3x = 2 0x = 2 0x = 0

X = 2/3 no solution x = t

3 cases fpr the solution of ax = b are unique solution, no solution, or infinite solutions

3x + 5y = 15

3x = 15 – 5y

X = 5-5/3y

Y = t

So the solution is x = 5-5/3t = scalar parametric form

Elimination method

Case 1: find a solution

Case 2: 2 = 0, no solutions

Case 3: 0 = 0, infinite solutions, so find general solution

General solution, substitute x or y or a variable

Eg. (t, 3t + 4)

Systems with 3 variables

X + y + z = 2

It represents a plane geometrically

Has 3 free variables

Solve for z : z = 2 – x- y solve for x: x = 2-y-z

General form x=t x = 2-t-s

Y=s y = t

Z = 2 – t-s z = s

(t, s, 2 – t - s) (2 – t – s, t, s)

# Back substitution

X + y + z – w = 2

2z + 2w = 0 z and z are pivot variables

Set free variables to some free parameters

W = s

Y = t

2z + 2s = 0 z = -s

X + t + (-s) – s = 2 x = 2 + 2s -t

The general solution to this system

X = 2 + 2s – t

Y = t

Z = -s

W = s where s, t are real

Solve the following system of equations

X + 2y = 2

3x + 4y = 12

3x + 4y = 12 – 3x + 6y = 6 -2y = 6 y = -3

X + 2(-3) = 2 x + -6 = 2 x = 8

24 – 12 = 12

Write as a matrix

2 -5 : 0



7 2 : 0

2 -1 3 : 0



0 2 -1 : 6



1 2 2

3 4 12

2\*(Row1) – row2 = 2 4 4 0 2 -6 y = -3

3 4 12 = -1 0 -8 X = 8

1 -1 1 0 1 -1 1 0 1 -1 1 0

-1 1 -1 0 0 0 0 0 0 -28 -16 -90

20 10 0 80 0 -30 20 -80 0 -30 20 -80

9 19 25 90 0 -28 -16 -90 0 0 0 0

1 -1 1 0

0 -28 -16 -90

0 -0 -30\*-16-20\*-28 -90\*-30—80\*-28

0 0 0 0

R4: 0=0

R3: 80 = 460 =6

R2:

# Week 5

Set notation

(a, b] = {x: a < x <= b}

[a, b) = {x: a <= x <= b}

-x/3 < 2x + 1

-x < 6x + 3

-x – 3 < 6x

-3 < 7x

X > -3/7

{x: x > -3/7}

b)

6/(x-1) >= 5 where x != 1

6 >= 5(x-1) 6>= 5x – 5

11 >= 5x

11/5 >= x

X <= 11/5

{x: 1 < x <= 11/5}

6-2x >= 4 6-2x <= -4

-2x >= -2 -2x <= -10

X <= 1 x >= 5

(-inf, 1] U [5, inf)