CS3010 #3

1 a) $f(x) = x^3 + 3x - 1$ on $[0, 1]$

$a = 0$ $f(a) = -1$ $c = .5$ $f(a) \cdot f(c) < 0$, so $b = c$

$b = 1$ $f(b) = 3$ $f(c) = .675$

$a = 0$ $f(a) = -1$ $c = .3125$ $f(c) \cdot f(c) > 0$, so $a = c$

messed up this section → $b = .625$ $f(c) = -.03198242$

$a = .3125$ $f(a) = -.03$ $c = .46875$ $f(a) \cdot f(c) < 0$, so $b = c$

$b = .625$ $f(c) = .5492468$

$a = .3125$ $f(a) = -.03$ $c = .390625$ $f(a) \cdot f(c)$ <0, so $b = c$

$b = .46875$ $f(c) = .23148$

$a = .3125$ $f(a) = -.03$ $c = .3515625$ $f(a) \cdot f(c) < 0$, so $b = c$

$b = .390625$ $f(c) = .098139$

$a = .3125$ $f(a) = -.03$ $c = .33203125$ $f(a) \cdot f(c) < 0$, so $b = c$

$b = .3515625$ $f(c) = .032698$

$a = .3125$ $f(a) = -.03$ $c = .32226625$ $f(a) \cdot f(c) < 0$, so $b = c$

$b = .33203125$ $f(c) = .001266$

~~$f(c) < .01$, so the zero is about $x = .32226625$~~

cont. next pg →

b) $g(x) = x^3 - 2\sin x$ on $[.5, 2]$

$a = .5$ $f(a) = -.833851$ $c = 1.25$ $f(a) \cdot f(c) < 0$, so $b = c$

$b = 2$ $f(c) = .055156$

$a = .5$ $f(a) = -.833$ $c = .875$ $f(a) \cdot f(c) > 0$, so $a = c$

$b = 1.25$ $f(c) = -.865165$

$a = .875$ $f(a) = -.87$ $c = 1.0625$ $f(a) \cdot f(c) > 0$, so $a = c$

$b = 1.25$ $f(c) = -.548$

$a = 1.0625$ $f(a) = -.549$ $c = 1.15625$ $f(a) \cdot f(c) > 0$, so $a = c$

$b = 1.25$ $f(c) = -.28474$

$a = 1.15625$ $f(a) = -.285$ $c = 1.203125$ $f(a) \cdot f(c) > 0$, so $a = c$

$b = 1.25$ $f(c) = -.124749$

$a = 1.203125$ $f(a) = -.124$ $c = 1.2265625$ $f(a) \cdot f(c) > 0$, so $a = c$

$b = 1.25$ $f(c) = -.03736$

$a = 1.2265625$ $f(a) = -.03736$ $c = 1.23828125$ $f(a) \cdot f(c) < 0$, so $b = c$

$b = 1.25$ $f(c) = .008258$

cont. next pg →

$a = 1.23242188$ $f(a) = -.0014$ $c = 1.23535156$

$b = 1.23828125$ $f(c) = -.00326$

$a = 1.2265625$ $f(a) = -.0037$ $c = 1.23242187$

$b = 1.23828125$ $f(c) = -.01471$

1a) cont.

a = .7125 f(a) = -.032 c = .31738 2813 f(a)·f(c) > 0, so a = c
b = .32226 5625 f(c) = -.01588 1985
a = .31738 2813 f(a) = -.015 c = .31982 4219 f(a)·f(c) > 0, so a = c
b = .32226 5625 f(c) = -.00 7813 314
a = .31982 4219 f(a) = -.0078 c = .32044 922 f(a)·f(c) > 0, so a = c
b = .32226 5625 f(c) = -.00 3774
a = .32044 922 f(a) = -.00 3774 c = .32165 5274 f(a)·f(c) > 0, so a = c
b = .32226 5625 f(c) = -.00 1755
a = .32165 5274 f(a) = -.00 1755 c = .32196 0449 f(c) < .001, so x ≈ .32211 3037
b = .32226 5625 f(c) = -.00 0745
a = .32196 0449 f(a) = -.0017 c = .32211 3037
b = .32226 5625 f(c) = -.00 0239

1b.) cont.

a = 1.23553 51563 f(a) = -.00 3266 c = 1.23681 6407 f(a)·f(c) < 0, b = c
b = 1.23828 125 f(c) = .00 2486
a = 1.23553 51563 f(a) = -.00 0396 c = 1.23668 3985 f(c)·f(c) > 0, a = c
b = 1.23681 6407 f(c) = -.00 0392
a = 1.23668 3985 f(a) = -.00 0392 c = 1.23644 5096 f(a)·f(c) < 0, b = c
b = 1.23681 6407 f(c) = .00 1046
a = 1.23668 3985 f(a) = -.00 0392 c = 1.23626 7091 f(a)·f(c) < 0, b = c
b = 1.23645 6196 f(c) = .00 0327
a = 1.23668 3985 f(a) = -.00 0392 c = 1.23617 5538 f(a)·f(c) > 0, a = c
b = 1.23626 7091 f(c) = -.00 0033
a = 1.23617 5538 f(a) = -.00 0033 c = 1.23622 1314 f(a)·f(c) < 0, b = c
b = 1.23626 7091 f(c) = .00 0147
a = 1.23617 5538 f(a) = -.00 0033 c = 1.23619 8426 f(a)·f(c) < 0, b = c
b = 1.23622 1314 f(c) = .00 0057
a = 1.23617 5538 f(a) = -.00 0033 c = 1.23618 6982
b = 1.23619 8426 f(c) = .00 0012

f(c) < .00001, so x ≈ 1.23618 6982

c) $h(x) = x + 1\phi - x\cosh(5\phi/x)$ $a=12\phi$, $b=13\phi$

$a=12\phi$ $f(a) = -.56824555$ $c=125$ $f(a)\cdot f(c) > \phi$, so $a=c$

$b=13\phi$ $f(c) = -.1347464$

$a=125$ $f(a) = -.134$ $c=127.5$ $f(a)\cdot f(c) < \phi$, so $b=c$

$b=13\phi$ $f(c) = .\phi 69789\phi 6$

$a=125$ $f(a) = -.134$ $c=126.25$ $f(a)\cdot f(c) > \phi$, so $a=c$

$b=127.5$ $f(c) = -.\phi 31\phi 8\phi 615$

$a=126.25$ $f(a) = -.\phi 31\phi 8\phi 615$ $c=126.875$ $f(a)\cdot f(c) < \phi$, so $b=c$

$b=127.5$ $f(c) = .\phi 1461238$

$a=126.25$ $f(a) = -.\phi 311$ $c=126.5625$ $f(a)\cdot f(c) > \phi$, so $a=c$

$b=126.875$ $f(c) = -.\phi\phi 5669134$

$a=126.5625$ $f(a) = -.\phi\phi 4466$ $c=126.71875$ $f(a)\cdot f(c) < \phi$, so $b=c$

$b=126.875$ $f(c) = .\phi\phi 6487$

$a=126.5625$ $f(a) = -.\phi\phi 567$ $c=126.64\phi 625$ $f(a)\cdot f(c) > \phi$, so $b=c$

$b=126.71875$ $f(c) = .\phi\phi\phi 63786$

$a=126.5625$ $f(a) = -.\phi\phi 567$ $c=126.6\phi 15625$ $f(a)\cdot f(c) > \phi$, so $a=c$

$b=126.64\phi 625$ $f(c) = -.\phi\phi 25\phi 1862$

$a=126.6\phi 15625$ $f(a) = -.\phi\phi 254$ $c=126.621\phi 938$ $f(a)\cdot f(c) > \phi$, so $a=c$

$b=126.64\phi 625$ $f(c) = -.\phi 6\phi 418985$

$a=126.621\phi 938$ $f(a) = -.\phi\phi\phi 92$ $c=126.63\phi 8594$ $f(a)\cdot f(c) > \phi$, so $a=c$

$b=126.64\phi 625$ $f(c) = -.\phi\phi\phi 127$

$a=126.63\phi 8594$ $f(a) = -.\phi\phi\phi 127$ $c=126.6357422$ $f(a)\cdot f(c) < \phi$, so $b=c$

$b=126.64\phi 625$ $f(c) = .\phi\phi\phi 2678\phi 2$

$a=126.63\phi 8594$ $f(a) = -.\phi\phi\phi 127$ $c=126.633\phi\phi 8$ $f(a)\cdot f(c) < \phi$, so $b=c$

$b=126.6357422$ $f(c) = .\phi\phi\phi\phi 7\phi\phi 58$

$a=126.63\phi 8594$ $f(a) = -.\phi\phi\phi 127$ $c=126.632\phi 8\phi 1$ $f(a)\cdot f(c) > \phi$, $a=c$

$b=126.6333\phi 08$ $f(c) = -.\phi\phi\phi 4\phi 28877$

$a=126.632\phi 8\phi 1$ $f(a) = -.\phi\phi\phi 428$ $c=126.6326\phi 25$ $f(a)\cdot f(c) < \phi$, $b=c$

$b=126.6333\phi 08$ $f(c) = .\phi\phi\phi\phi 2\phi 611$

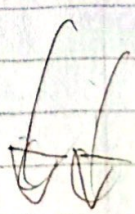
$a=126.632\phi 8\phi 1$ $f(a) = -.\phi\phi\phi 428$ $c=126.6323853$ $f(a)\cdot f(c) > \phi$, $a=c$

$b=126.6326925$ $f(c) = -.\phi\phi\phi\phi\phi 4111$

$a=126.6323853$ $f(a) = -.\phi\phi\phi\phi\phi 4111$ $c=126.6325379$ $f(a)\cdot f(c) < \phi$, $b=c$

$b=126.6326925$ $f(c) = .\phi\phi\phi\phi\phi\phi 8252$

$|f(c)| < .00001$ so $x \approx 126.6325379$



2. $f(x) = x^3 + 2x^2 + 10x - 20 \qquad x_0 = 2$

$f'(x) = 3x^2 + 4x + 10$

$x_1 = 2 - \frac{16}{30} = 1.46667$

$x_2 = 1.46627 - \frac{2.123852}{22.32} = 1.37151207$

$x_3 = 1.37151207 - \frac{.007086}{21.129184} = 1.36881246$

$x_4 = 1.36881246 - \frac{.000045}{21.096164} = 1.36880813$

$x_5 = 1.36880813 - \frac{.000000019}{21.096134} = 1.36880808$

$f(x_4) < .0001$, so $x \approx 1.36880808$

3. $f(x) = x^3 + 2x^2 + 10x - 20$, $x_0 = 2$, $x_1 = 1$

$$x_2 = x_1 - \frac{(x_1 - x_0) f(x_1)}{f(x_1) - f(x_0)} = 1 - \frac{(1-2)(-7)}{-7-16} = 1 - \frac{(-1)(-7)}{-23} = 1.3043478\overline{26}$$

$$x_3 = x_2 - \frac{(x_2 - x_1) f(x_2)}{f(x_2) - f(x_1)} = 1.3043478\overline{26} - \frac{(.3043478\overline{26} - 1)(-1.334758)}{-1.334758 + 7} = 1.14044886$$

$$x_4 = 1.14044886 - \frac{(1.14044886 - 1.3043478\overline{26})(-4.510969)}{(-4.510969) + (-1.334758)} = 1.37322346\overline{1}$$

$$x_5 = 1.37322346\overline{1} - \frac{(1.37322346\overline{1} - 1.14044886)(.093266)}{(.093266) + (-4.510969)} = 1.3685\overline{82}46$$

$$x_6 = 1.3685\overline{82}46 - \frac{(1.3685\overline{82}46 - 1.37322346\overline{1})(-.006325)}{(-.006325) - (.093266)} = 1.3688\overline{77}68$$

$$x_7 = 1.3688\overline{77}68 - \frac{(1.3688\overline{77}68 - 1.3685\overline{82}46)(-.000\overline{68})}{(-.0000\overline{68}) - (-.006325)} = 1.3688\overline{73}29$$

$f(x_6) < .0001$, so zero is $x \approx 1.3688\overline{73}29$