Maths for Informatics: 4b Geomatry
Revision Notes
Ver 0.1

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Contents

1 Introduction **1**

2 Mapping **2**
2.1 Matrix 2

3 Shapes **2**
3.1 Types 2
3.2 Generalisation 3
3.3 Idetification 3

4 Examples **4**
4.1 Example 1 4

5 Matrix and Vector **5**
5.1 Matrix 5
5.2 Vector 5
5.3 Mix 5

1 Introduction

This document is a set of revision notes for the Geomatry for Informatics course at the Univerisy of Edinbugh.

2 Mapping

2.1 Matrix

If a matrix is arthoginal in $\mathbb{R}^3$ on can show that $MM^T = I$ then:

If $det(m) = 1$

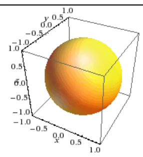
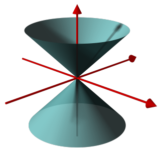
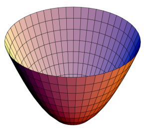
This ia a rotation around some line $L \in \mathbb{R}^3$ that passes through $0(0,0,0)$ at angle α.

If you find the *Eigan Vector* that is a line parralles to L, set it to $(0,0,0)$ and we have the line.

If $det(m) = -1$

This is a reflection in a plane.

3 Shapes

Example	Name	Visual Representation
$x^2 + y^2 + z^2 = 1$	Sphere	 [1]
$x^2 + y^2 = z^2$	Conic	
$x^2 + y^2 = z$	Poraboloid	

3.1 Types

Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Parabola

$$y^2 = 4ax$$

Other

Union of two line		
Line	Shortest distance between two points	$y = Mx + c$
Point	A zero dimensinal object	(x, y)
Empty	Nothing	$\{\}$

3.2 Generalisation

$$Ax^2 + Bxy^2 + Cy^2 + Dx^2 + F = 0$$

or

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} A & \frac{B}{2} \\ \frac{B}{2} & C \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} D \\ E \end{bmatrix} + F = 0$$

3.3 Idetification

$$N = \begin{bmatrix} A & \frac{B}{2} & \frac{D}{2} \\ \frac{B}{2} & C & \frac{E}{2} \\ \frac{D}{2} & \frac{E}{2} & F \end{bmatrix} M = \begin{bmatrix} A & \frac{B}{2} \\ \frac{B}{2} & C \end{bmatrix}$$

Ellipse

$4AC - B^2 > 0$ and $det(N)(A + C) < 0$

Hyperbola

$4AC - B^2 < 0$ and $det(N) \neq 0$

Parabola

$4AC - B^2 = 0$ and $det(N) \neq 0$

4 Examples

4.1 Example 1

Note: Simmilar to assignment

Question

$S : x^2 + y^2 - z^2 = 1$
$S \subset \mathbb{R}^3$
If you are given a point and asked to find all lines that pass through it
Hint: There will allways be two

Answer

$pt(1, 1, 1)$
$P \leq L \leq S \subseteq \mathbb{R}^3$
$L = (1 + a\alpha, 1 + b\alpha, 1 + c\alpha)$

Inserted into questions formula
$(1 + a\alpha)^2 + (1 + b\alpha)^2 + (1 + c\alpha)^2 = 1 \forall \alpha$

Expand to
$1 + ... = 1$

seperate varibles
$\alpha(2a + 2b - 2c) + \alpha^2(a^2 + b^2 - c^2) = 0$

Divide by α
$(2a + 2b - 2c) = 0 or (a^2 + b^2 - c^2) = 0$

let a = 1 (as 1 is modulo all numbers)
$(1, b, c)$
$1 + b^2 - c^2 = 0 and 1 + b - c = 0$
$b = 0 and c = 1$

Form to a equation
(are intial point and are new derived point)
$\underline{(1, 1, 1) + \alpha(1, 0, 1)}$

5 Matrix and Vector

5.1 Matrix

TODO

5.2 Vector

TODO

5.3 Mix

Orthoginal $a \cdot b = 1$

Parrallel $a \cdot b = 0$

Cross Product
$$\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Dot Product $(a_1xb_1) + (a_2xb_2) + (a_2xb_2)$

Angle $cos(\theta) = \frac{a \cdot b}{|a||b|}$

Determinate
$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Determinate
$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{array}{ccccc} a & b & c & \nearrow & \\ d & e & f & \nearrow & Y \\ g & h & i & \nearrow & \\ a & b & c & \searrow & \\ d & e & f & \searrow & X \\ g & h & i & \searrow & \end{array} \quad X - Y$$

References

[1] "Sphere Graphic", Author="Wolfram—Alpha", Publisher"Wolfram Alpha LLC", Accessed="26-April-2011", Year="2011", "http://www.wolframalpha.com/input/?i=x^2%2By^2%2Bz^2%3D1"