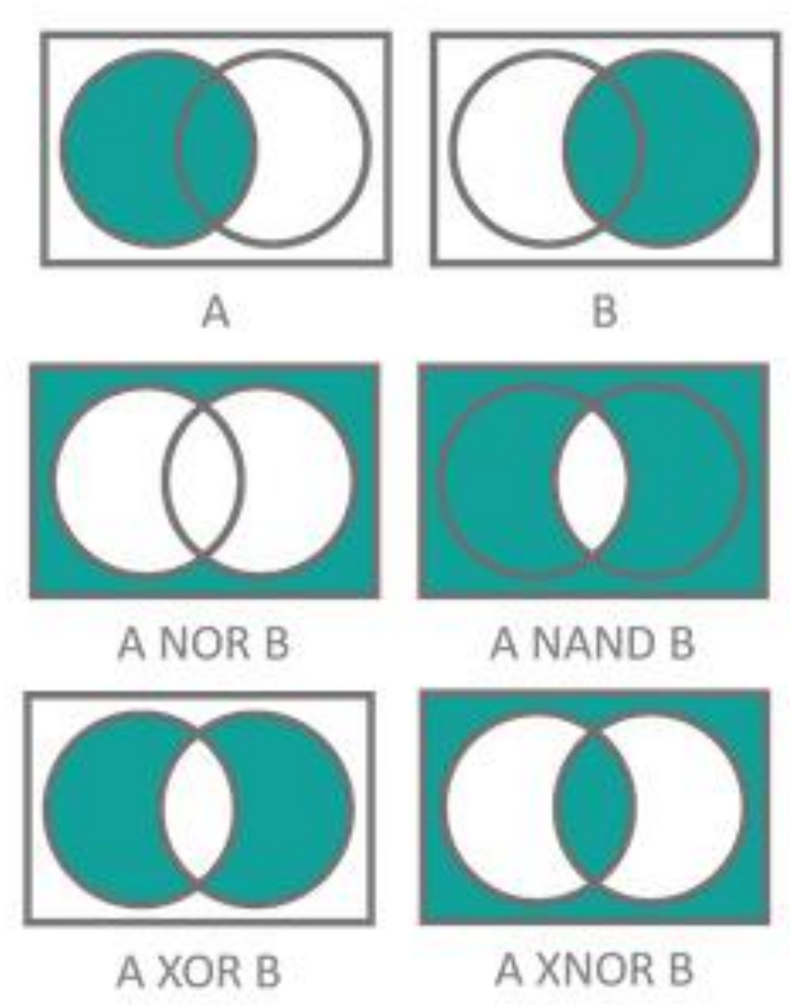
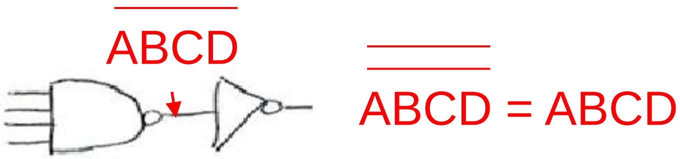
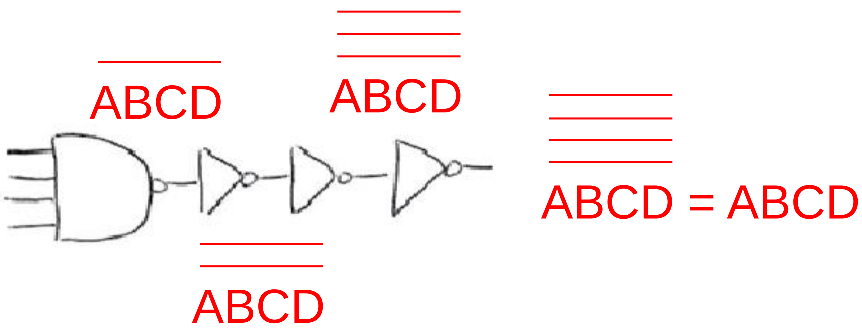
2.5 Boolean Functions

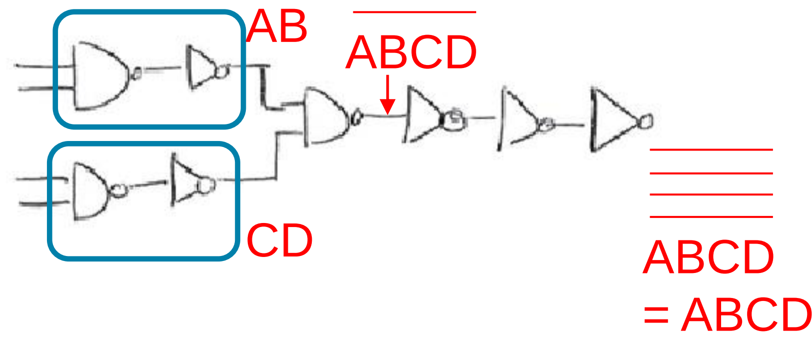
Venn Diagrams of Some Basic Functions

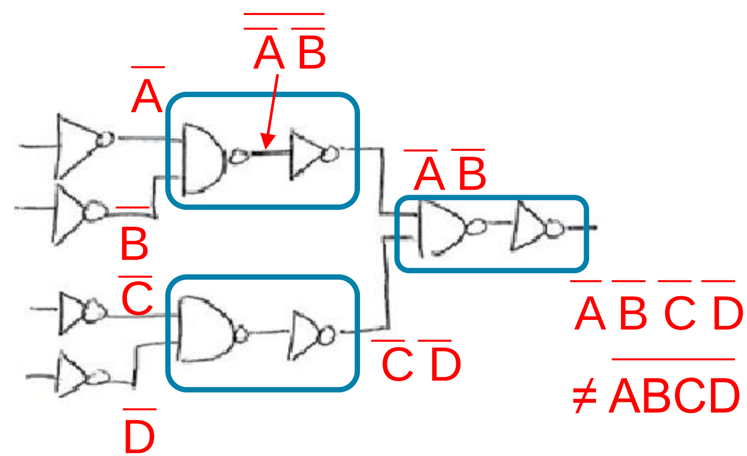


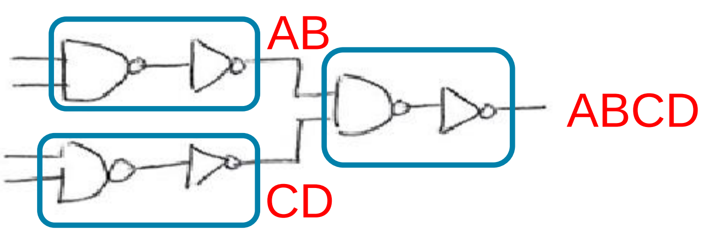
4-Input AND Functions











2.4 (c) Reduce to one literal

$$A'B(D'+C'D)+B(A+A'CD)$$

factor $A'BD'+A'BC'D+BA+BA'CD$

factor out B $B(A'D'+A'C'D+A+A'CD)$

factor out A'D $B(A'D'+A'D(C'+C)+A)$

$$B(A'D'+A'D+A)$$

factor out A' $B(A'(D'+D)+A)$

$$B(A'+A)$$

$$B$$

2.6 Canonical and Standard forms Minterms/maxterms

- A shorthand notation for describing functions
- Any Boolean function can be represented by a truth table with 2^N rows (N variables)
- The literal combinations for 1's are called minterms and are grouped together with OR functions
- The complemented literal combinations for 0's are called maxterms and are grouped together with AND functions

2.6 Minterms/maxterms

Inputs		Output
A	*B*	*X*
0	0	0
0	1	1
1	0	1
1	1	0

The minterms of the XOR gate are:

A'B + AB'
Can be written as $m_1 + m_2$

$X = m_1 + m_2$ (minterm 1 OR minterm 2)
(product of sums)

The maxterms of the XOR gate are:
(Note how maxterms are written: don't sum the zeros)

(A + B) (A' + B')
Can be written as M_0M_3

$X = M_0M_3$ (maxterm 0 AND maxterm 3)
(sum of products)

2.6 Minterms/maxterms

Table 2.3
Minterms and Maxterms for Three Binary Variables

			Minterms		Maxterms	
x	y	z	Term	Designation	Term	Designation
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7

2.6 Minterms/maxterms

Table 2.4
Functions of Three Variables

x	y	z	Function f_1	Function f_2
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$f_1 = m_1 + m_4 + m_7$

$= x'y'z + xy'z' + xyz$

$f_1 = M_0\ M_2\ M_3\ M_5\ M_6$

$=(x+y+z)(x+y'+z)(x+y'+z')(x'+y+'z)(x'+y'+z)$

$f_2 = m_3 + m_5 + m_6 + m_7$

$=x'yz + xy'z + xyz' +xyz$

$f_2 = M_0\ M_1\ M_2\ M_4$

$=(x+y+z)(x+y+z')(x+y'+z)(x'+y+z)$

2.6 Canonical and Standard forms minterms/maxterms

Example of a digital logic function
seat-belt interlock system

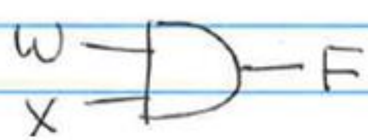
all combinations of two variables

W	X	F
0	0	0
0	1	0
1	0	0
1	1	1

W – foot on brake
X – driver seat belt on

$F = m_3 = WX$

Both a foot must be on the brake and the driver seat belt must be on



2.6 Canonical and Standard forms
Minterms/maxterms

Example of a digital logic function
seat-belt interlock system

w x y z	F
0 0 0 0	0
0 0 0 1	0
0 0 1 0	0
0 0 1 1	0
0 1 0 0	0
0 1 0 1	0
0 1 1 0	0
0 1 1 1	0
1 0 0 0	0
1 0 0 1	0
1 0 1 0	0
1 0 1 1	0
1 1 0 0	0
1 1 0 1	0
1 1 1 0	0
1 1 1 1	1

y - door shut
z - passenger seat belt on
w - foot on brake
x - driver seat belt on

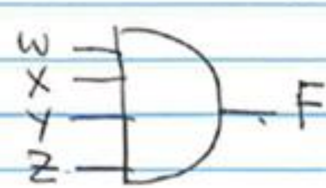
N=4 variables
2^N rows of the truth table

All four conditions must be true for the car to start

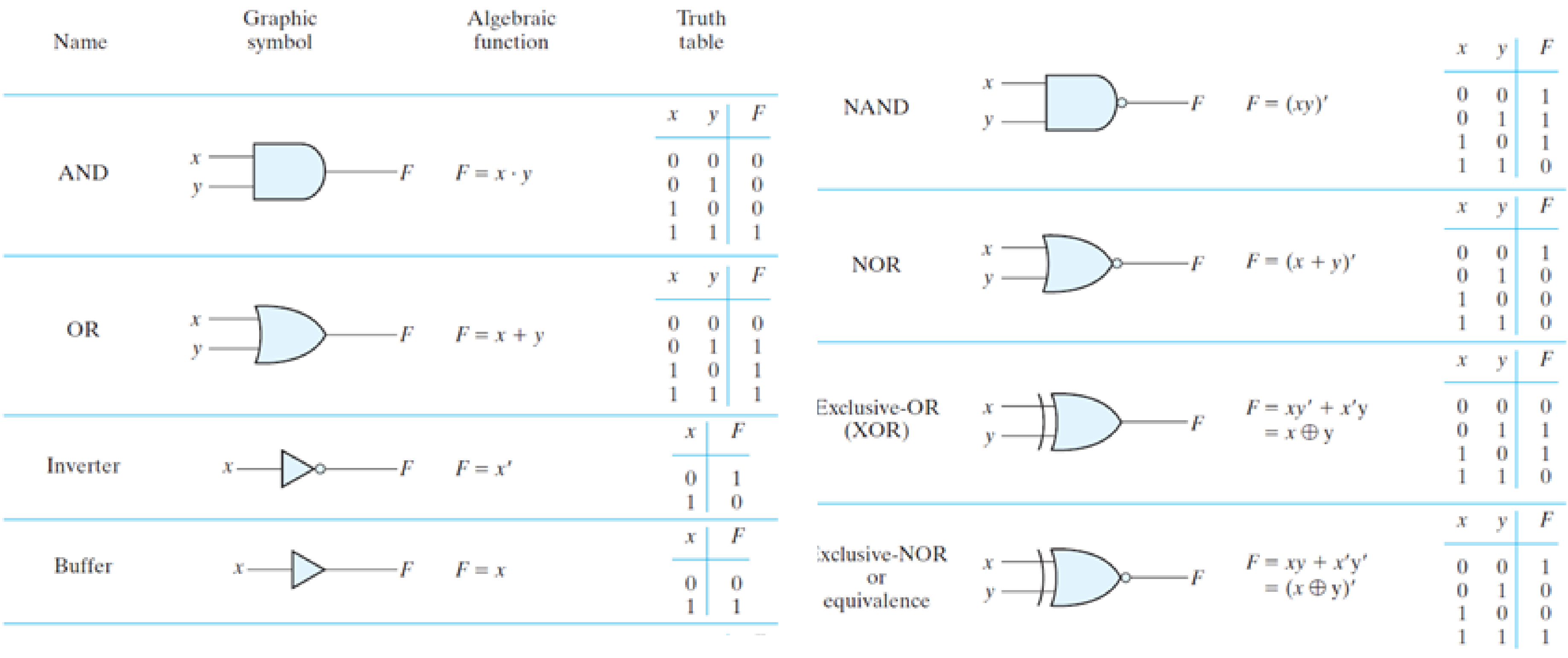
truth table = representation of function

minterms = $F = m_{15} = wxyz$

circuit



2.8 Digital Logic Gates



Name	Graphic symbol	Algebraic function	Truth table
AND		$F = x \cdot y$	x y F 0 0 0 0 1 0 1 0 0 1 1 1
OR		$F = x + y$	x y F 0 0 0 0 1 1 1 0 1 1 1 1
Inverter		$F = x'$	x F 0 1 1 0
Buffer		$F = x$	x F 0 0 1 1
NAND		$F = (xy)'$	x y F 0 0 1 0 1 1 1 0 1 1 1 0
NOR		$F = (x + y)'$	x y F 0 0 1 0 1 0 1 0 0 1 1 0
Exclusive-OR (XOR)		$F = xy' + x'y$ $= x \oplus y$	x y F 0 0 0 0 1 1 1 0 1 1 1 0
Exclusive-NOR or equivalence		$F = xy + x'y'$ $= (x \oplus y)'$	x y F 0 0 1 0 1 0 1 0 0 1 1 1

FIGURE 2.5
Digital logic gates

Truth Table

truth table - defines a function

2^n rows for N variables

	A	B	C	Y
m_0	0	0	0	0
m_1	0	0	1	1
m_2	0	1	0	1
m_3	0	1	1	0
m_4	1	0	0	0
m_5	1	0	1	0
m_6	1	1	0	0
m_7	1	1	1	0

minterms

$$Y = \Sigma(m_1, m_2)$$

$$= m_1 + m_2 \qquad \text{(or)}$$

$$= \bar{A}\bar{B}C + \bar{A}B\bar{C}$$

maxterms

$$Y = \Pi(M_0 M_3 M_4 M_5 M_6 M_7) \qquad \text{(AND)}$$

$$= M_0 M_3 M_4 M_5 M_6 M_7$$

$$= \underset{M_0}{(A+B+C)}\underset{M_3}{(A+\bar{B}+\bar{C})}\underset{M_4}{(\bar{A}+B+C)} \ldots\ldots$$

(change operation & complement variable)

Convert Representations of a Function

truth table ⟷ Boolean expression

circuit

$Y = (A+B)C + B$

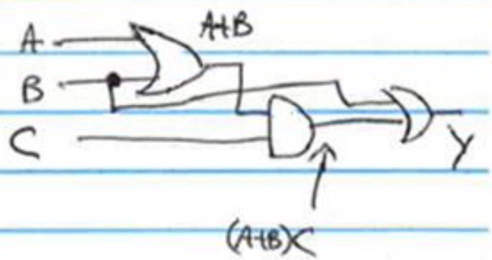
what is the truth table?
what does the circuit look like?
what is the Boolean expression?

3 variables ⇒ $2^3 = 8$ rows   what is the B

A	B	C	(A+B)	(A+B)C	Y=(A+B)C+B
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	1	0	0
1	0	1	1	1	1
1	1	0	1	0	1
1	1	1	1	1	1

AND these columns

$$Y = m_2 + m_3 + m_5 + m_6 + m_7$$
$$= \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$



$Y = (\overline{A+B})\,C$

what is the truth table?

what does the circuit look like?

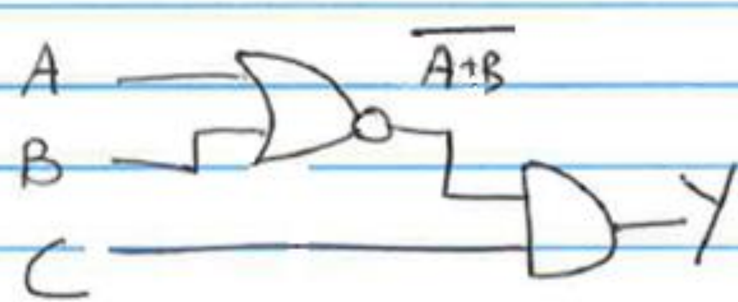
A B C	$\overline{A+B}$	$Y=(\overline{A+B})C$
0 0 0	1	0
0 0 1	1	1
0 1 0	0	0
0 1 1	0	0
1 0 0	0	0
1 0 1	0	0
1 1 0	0	0
1 1 1	0	0

A B	$\overline{A+B}$
0 0	1
0 1	0
1 0	0
1 1	0

NOR

Y can be written as $Y = M_1$

$= \overline{A}\,\overline{B}C$



2.17a Obtain the truth table

$$y=(b+cd)(c+bd)$$

bcd	cd	(b+cd)	bd	(c+bd)	y
000	0	0	0	0	0
001	0	0	0	0	0
010	0	0	0	1	0
011	1	1	0	1	1
100	0	1	0	0	0
101	0	1	1	1	1
110	0	1	0	1	1
111	1	1	1	1	1

AND

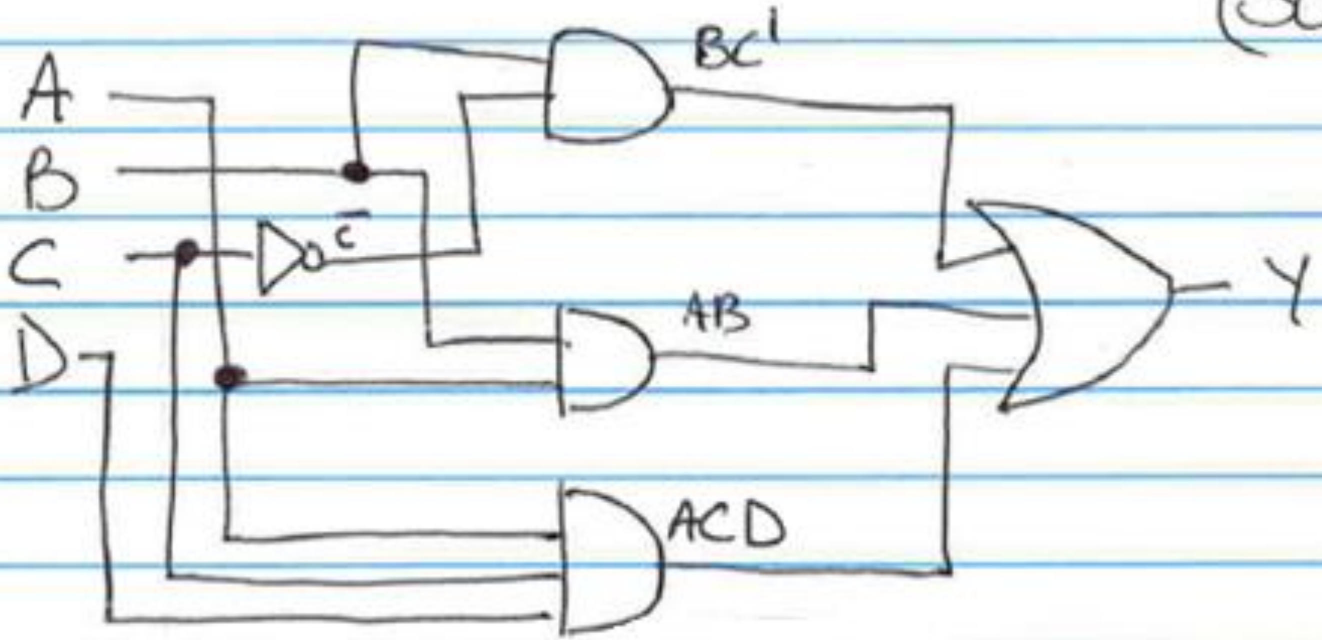
2.17 (b) $Y = (cd + b'c + bd')(b+d)$

bcd	cd	$b'c$	bd'	$(cd + b'c + bd')$	$(b+d)$	Y
000	0	0	0	0	0	0
001	0	0	0	0	1	0
010	0	1	0	1	0	0
011	1	1	0	1	1	1
100	0	0	1	1	1	1
101	0	0	0	0	1	0
110	0	0	1	1	1	1
111	1	0	0	1	1	1

AND

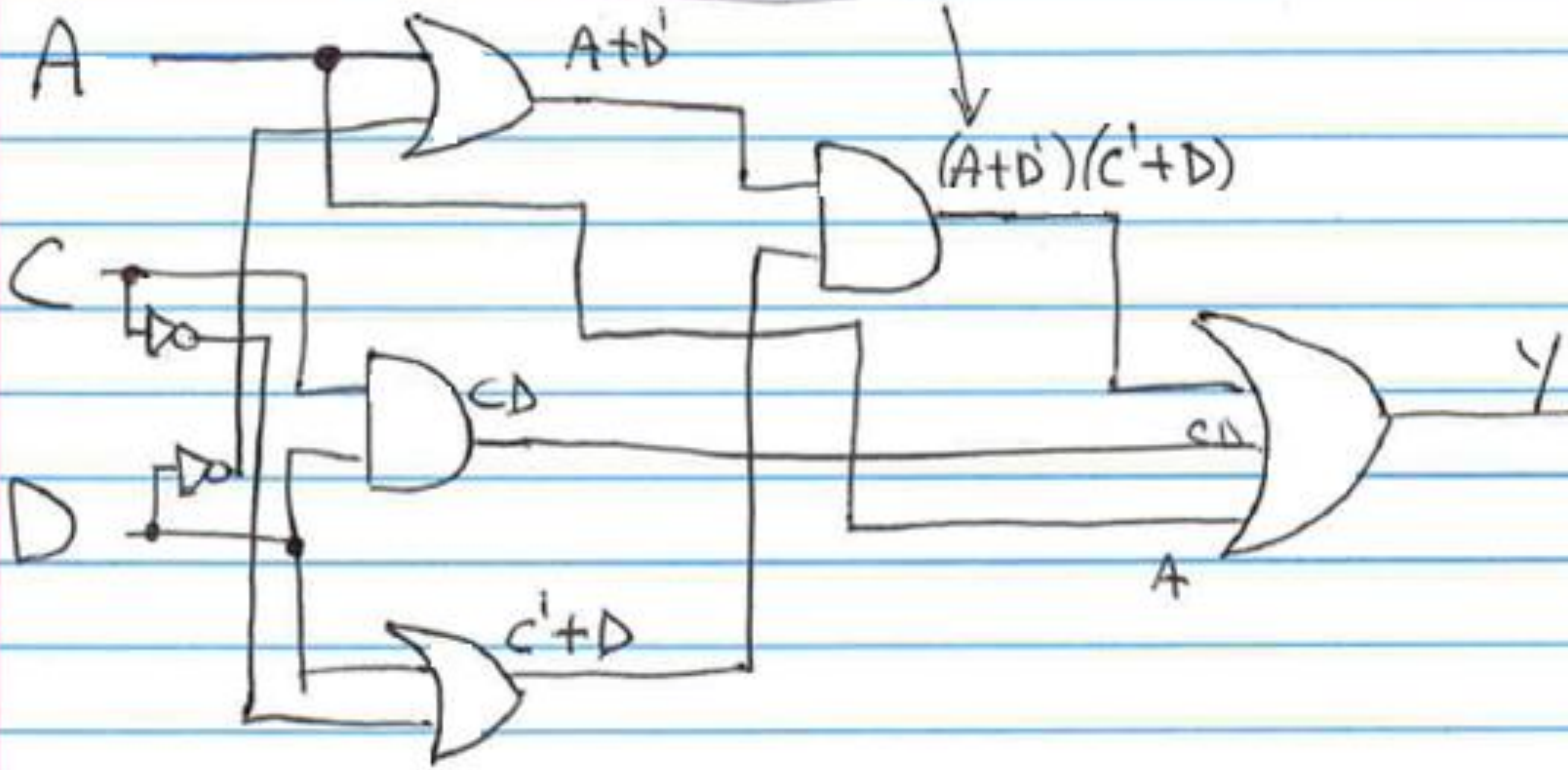
2.23 Draw the logic diagram for the following expression without simplifying

(a) $Y = BC' + AB + ACD$ ⇒ sum-of-products form (SOP)

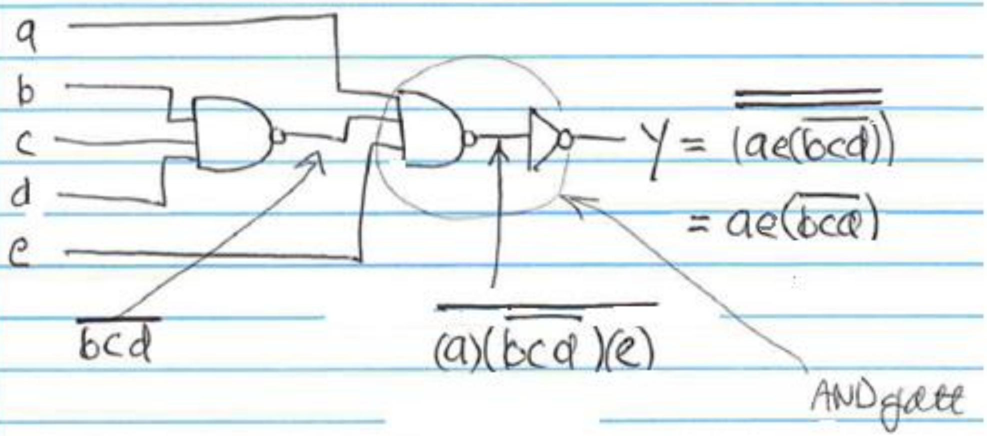


2.23

(d) $Y = A + CD + (A + D')(C' + D)$



2.28 Write the Boolean expression and truth table
(a) describing the outputs of the circuit



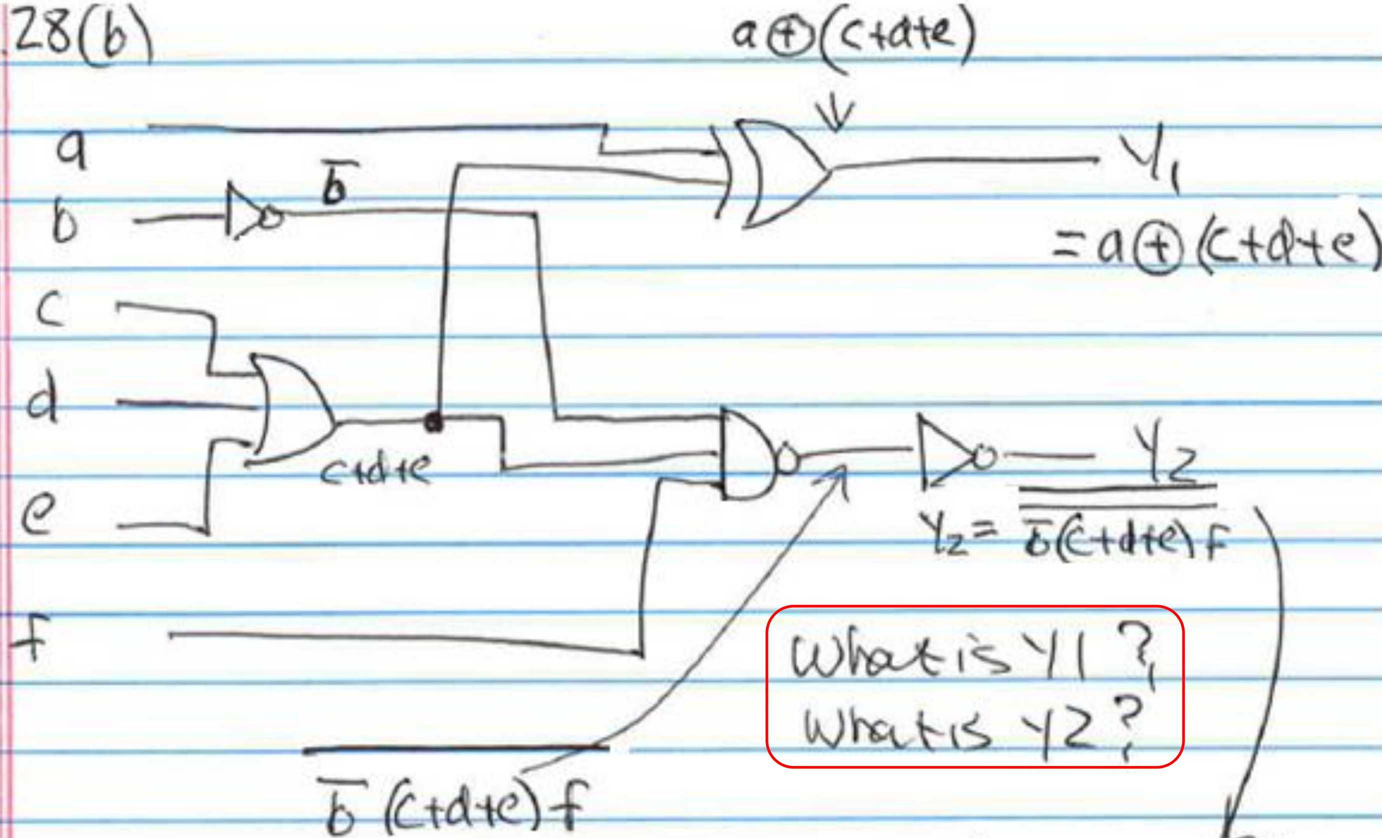
$$Y = \overline{\overline{(ae(\overline{bcd}))}}$$
$$= ae(\overline{bcd})$$

truth table

a	b	c	d	e	$(\overline{bcd})$	ae	$ae(\overline{bcd})$
0	0	0	0	0	1	0	0
0	0	0	0	1	1	0	0
0	0	0	1	0	1	0	0
0	0	0	1	1	1	0	0
0	0	1	0	0	1	0	0
0	0	1	0	1	1	0	0
0	0	1	1	0	1	0	0
0	0	1	1	1	1	0	0
0	1	0	0	0	1	0	0
0	1	0	0	1	1	0	0
0	1	0	1	0	1	0	0
0	1	0	1	1	1	0	0

⋮

2.28(b)



What is Y1?
What is Y2?

What is Y1?
What is Y2?

complements cancel.

$Y_2 = \bar{b}(c+d+e)f$

2.20 Find the complement of the following functions in sum-of-minterms form

(a) $F(A,B,C,D) = \sum(2,4,7,10,12,14)$

A	B	C	D	F	$\bar{F}$
0	0	0	0	0	1
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	0	1
0	1	0	0	1	0
0	1	0	1	0	1
0	1	1	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	0	1	0	1
1	0	1	0	1	0
1	0	1	1	0	1
1	1	0	0	1	0
1	1	0	1	0	1
1	1	1	0	1	0
1	1	1	1	0	1

$\bar{F}(A,B,C,D) = \sum(0,1,3,5,6,8,9,11,13,15)$

(b) $F(X,YZ) = \prod(3,5,7)$

X	Y	Z	F	$\bar{F}$
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	1	0
1	1	1	0	1

$\bar{F} = \sum(3,5,7)$

2.29 True or False

Use Boolean algebra

$$x'y' + x'z + x'z' \overset{?}{=} x'z' + y'z' + x'z$$

$$x'(y' + z + z') = x'z' + y'z' + x'z$$

$$x' : x'(z' + z) + y'z'$$

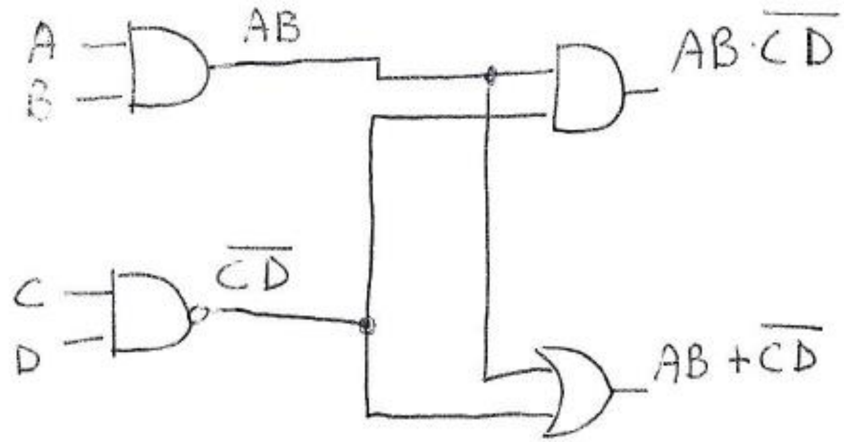
$$x' = x' + y'z'$$ Not equal

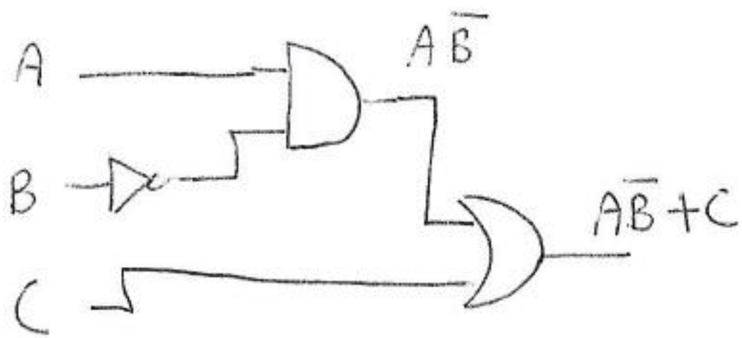
Or a truth table

X Y Z	x'	$y'z'$	$x'+y'z'$
0 0 0	1	1	1
0 0 1	1	0	1
0 1 0	1	0	1
0 1 1	1	0	1
1 0 0	0	1	1
1 0 1	0	0	0
1 1 0	0	0	0
1 1 1	0	0	0

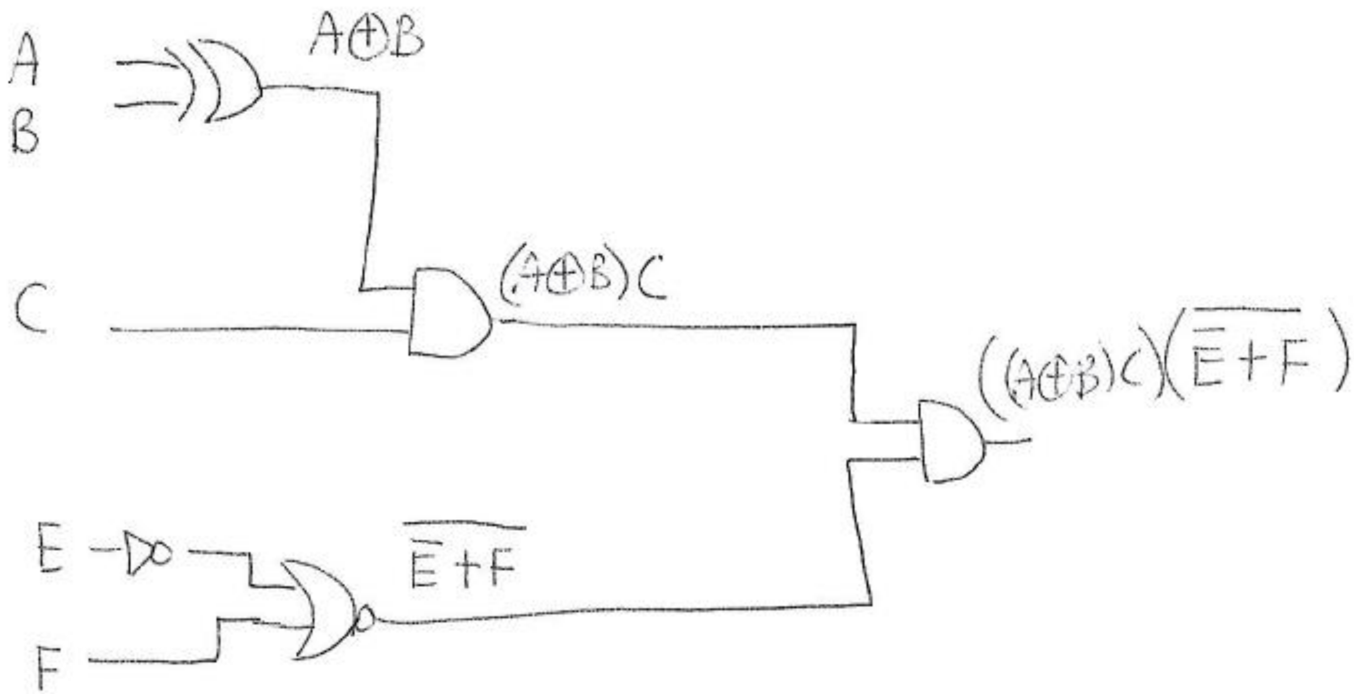
not equal

Boolean Expressions from Circuits





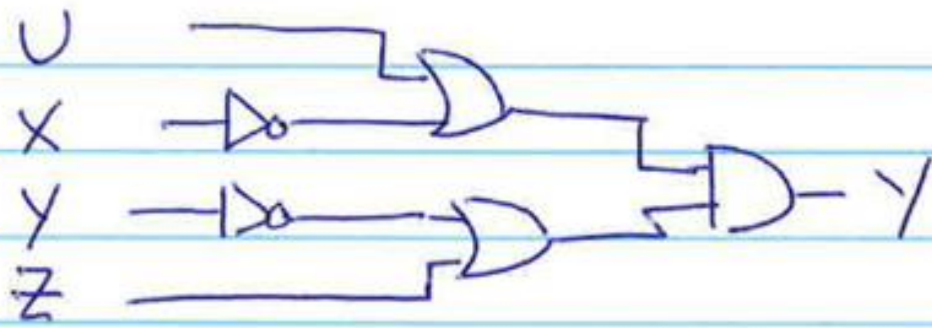
Boolean Expressions from Circuits



2.13 Draw the logic diagram to implement the following Boolean expressions.

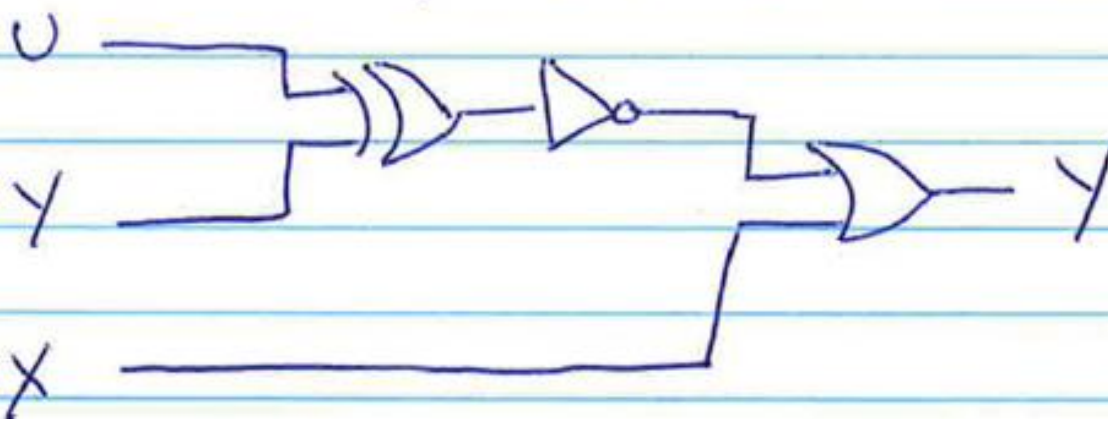
(a) $y = (u + x')(y' + z)$

Bad notation – y should not be used as and input and output

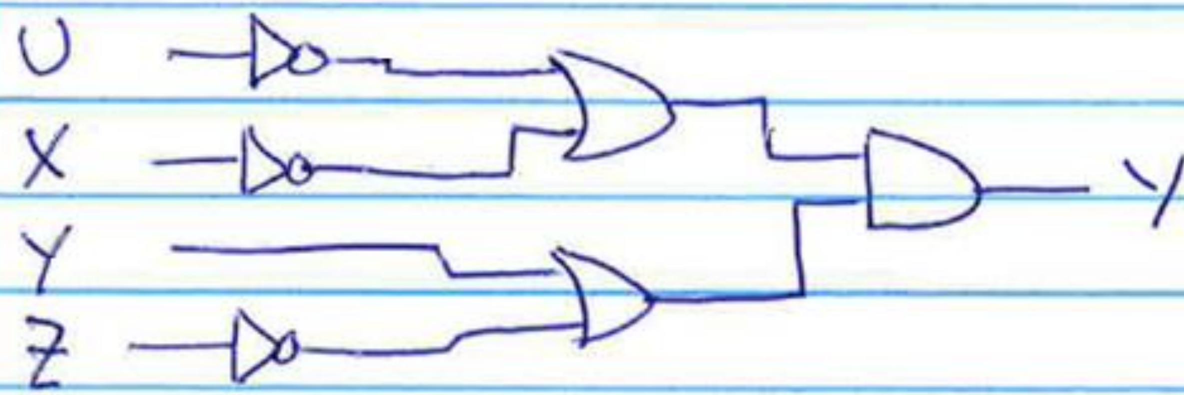


$y = (u \oplus y)' + x$

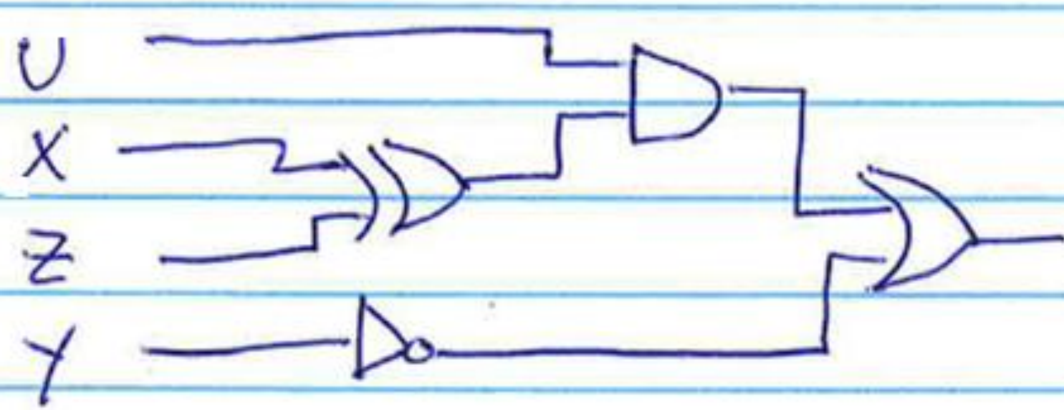
(b)



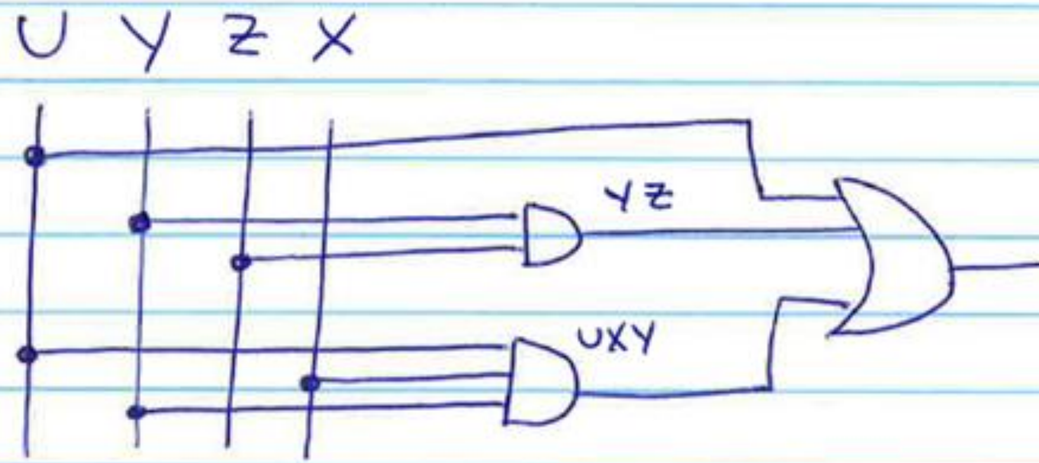
2.13 (c) $Y = (U' + X')(Y + Z')$



2.13 (d) $Y = U(X \oplus Z) + Y'$



2.13(e) $Y = U + YZ + UXY$



2.13(f) $Y = U + X + X'(U+Y')$