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Optimization Algorithms in Deep Learning

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PhD in Computer Science

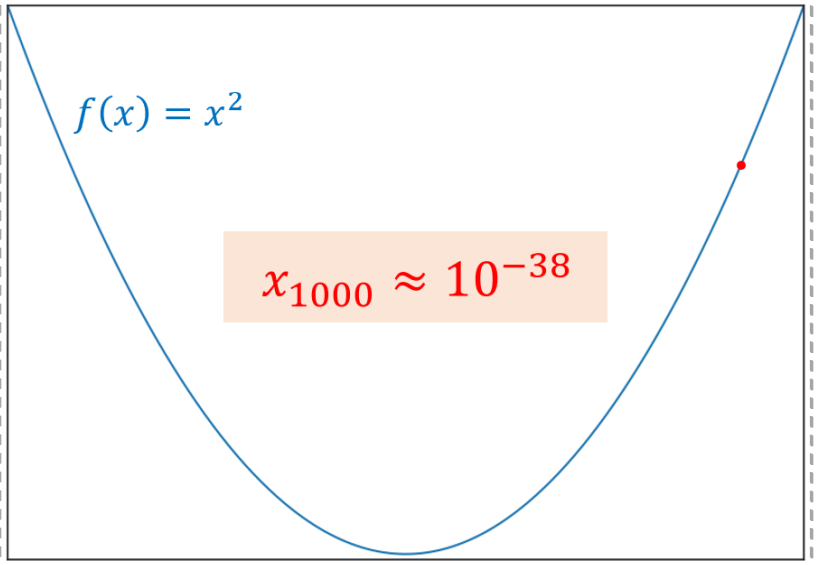
Year 2024

code&data

Objectives

SGD Insight

$$x_t = x_{t-1} - \eta f'(x)$$



Adaptive L. Rate

$$g_t = \nabla_\theta L$$

$$s_t = s_{t-1} + g_t^2$$

$$\theta_t = \theta_{t-1} - \frac{\eta}{\sqrt{s_t + \epsilon}} g_t$$

current summation

expected summation

Momentum&Adam

Simpler version of Adam

$$g_t = \nabla_\theta L$$

$$m_t = \beta_1 m_{t-1} + (1 - \beta_1) g_t$$

$$v_t = \beta_2 v_{t-1} + (1 - \beta_2) g_t^2$$

$$\theta_t = \theta_{t-1} - \frac{\eta}{\sqrt{v_t + \epsilon}} m_t$$

Outline

SECTION 1

SGD Insight

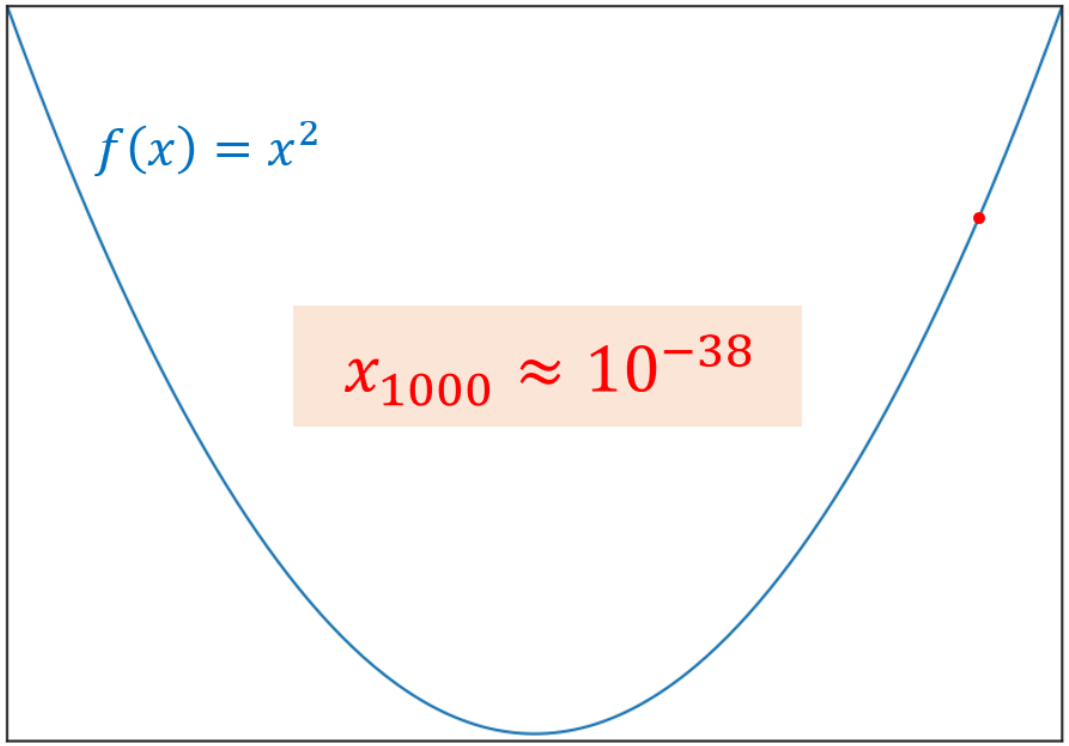
SECTION 2

Adaptive Learning Rate

SECTION 3

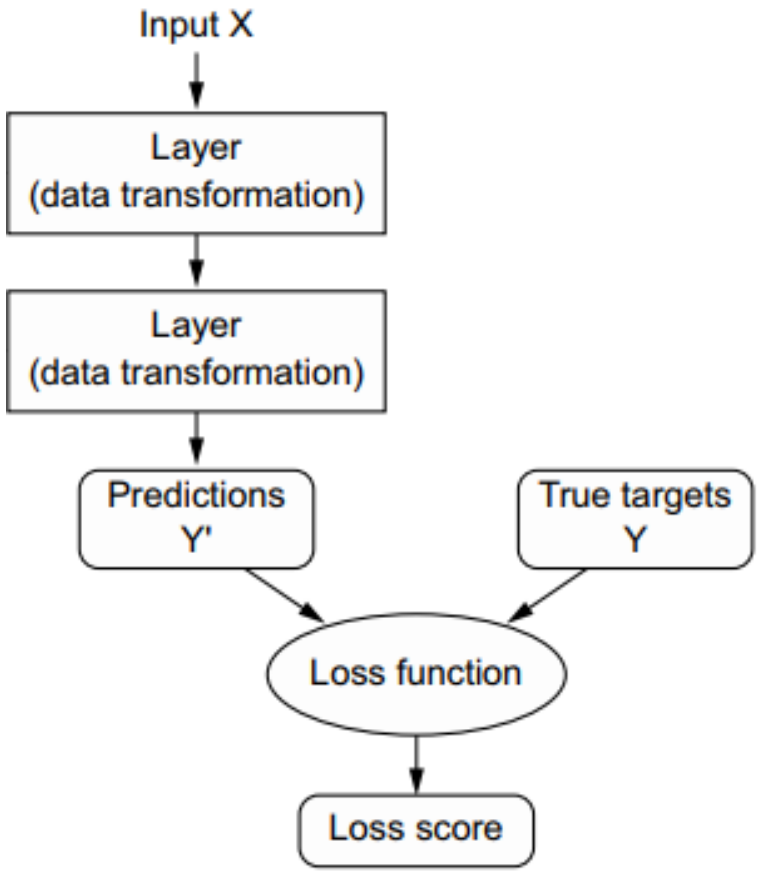
Momentum and Towards Adam

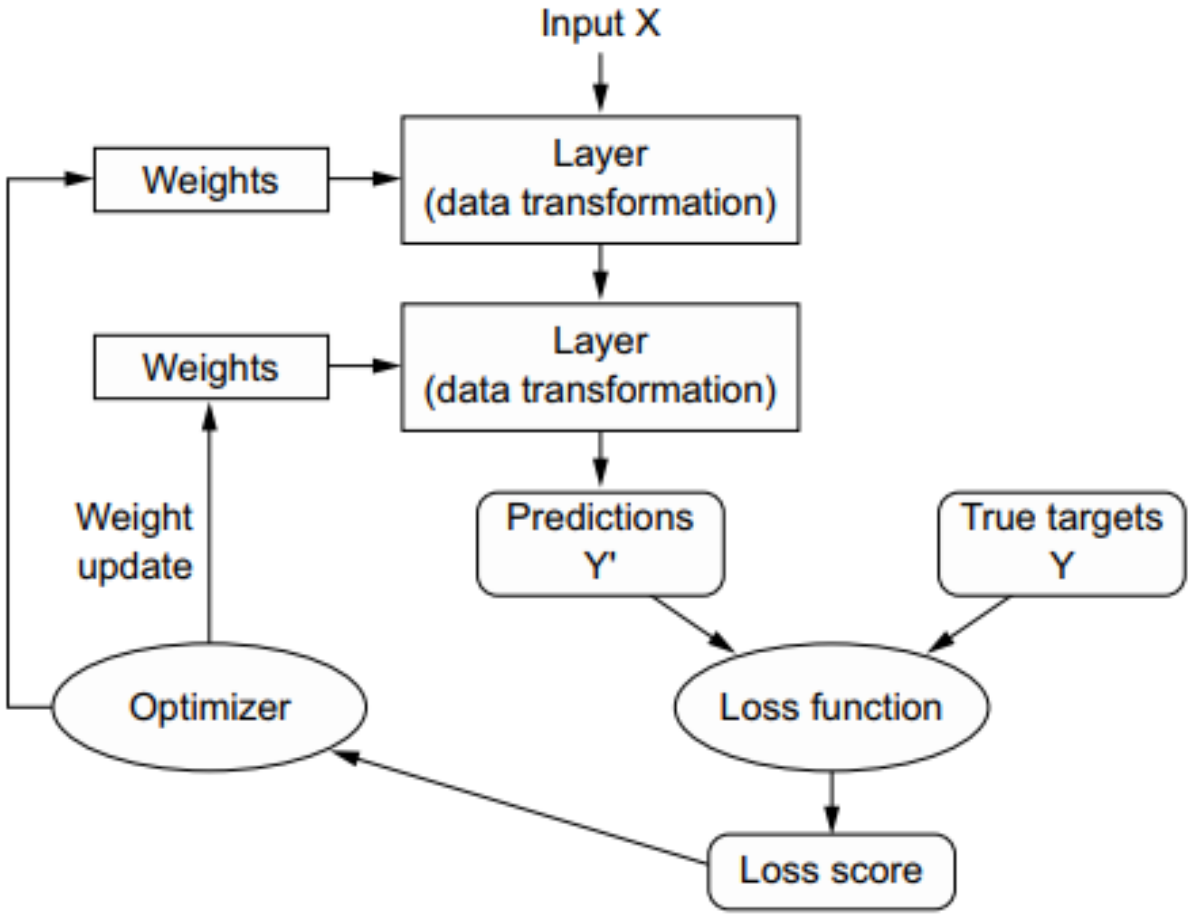
$$x_t = x_{t-1} - \eta f'(x)$$



Optimization Algorithms

❖ Overview

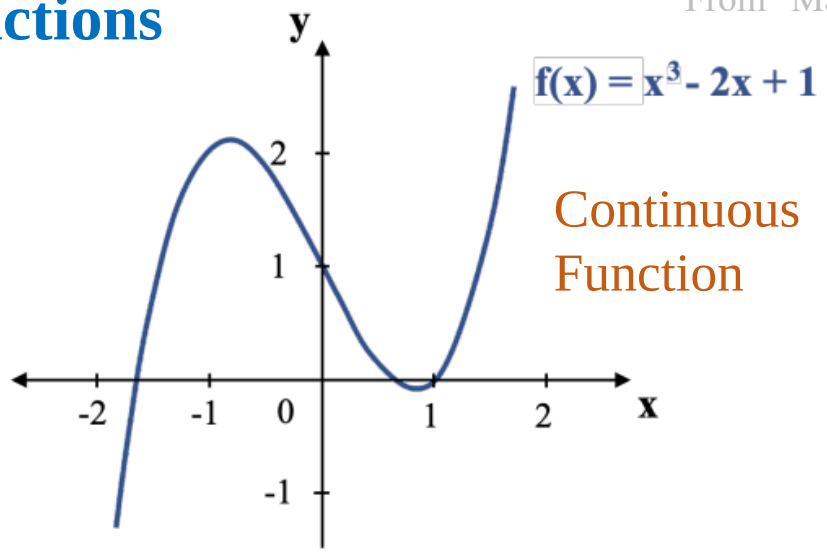




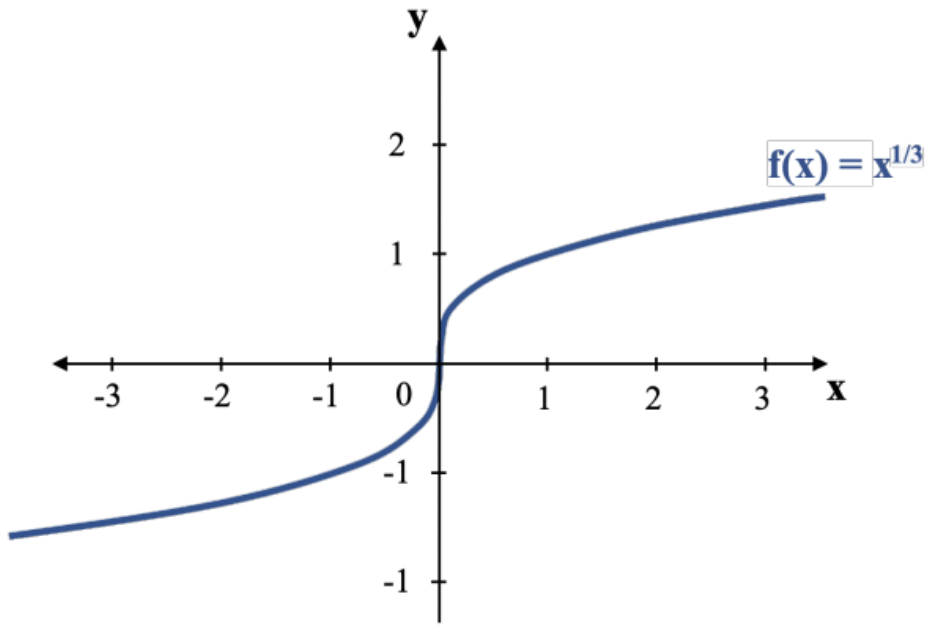
Optimization Algorithms

Loss functions

From "Machine Learning Simplified"

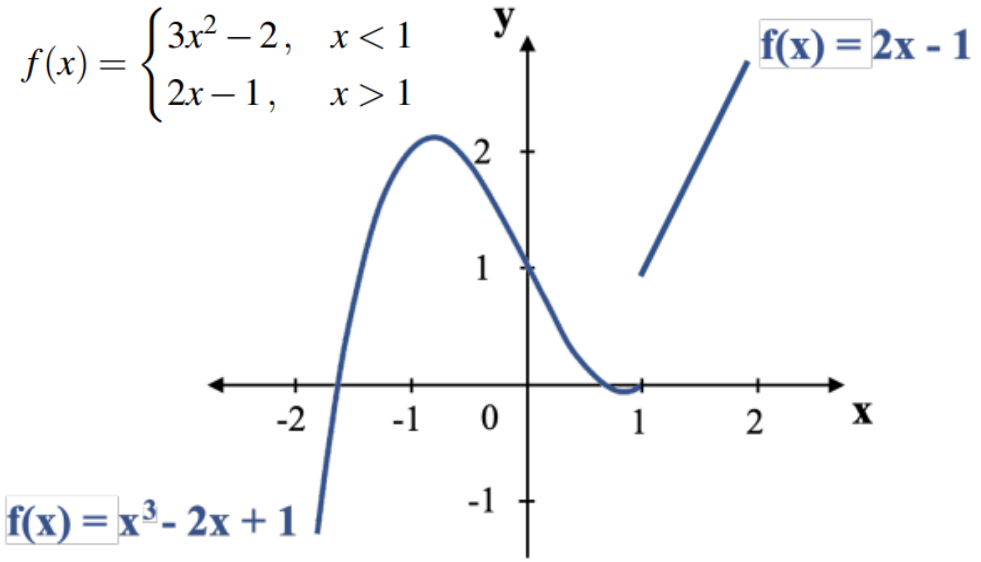


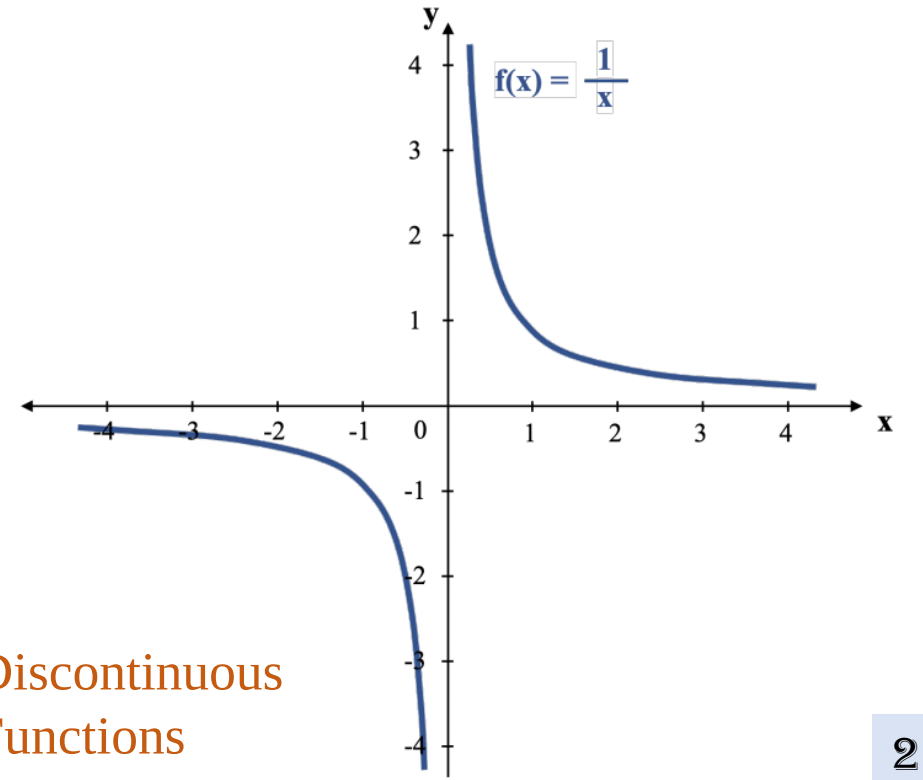
Continuous Function



Continuous non-differentiable functions

$$f(x) = \begin{cases} 3x^2 - 2, & x < 1 \\ 2x - 1, & x > 1 \end{cases}$$



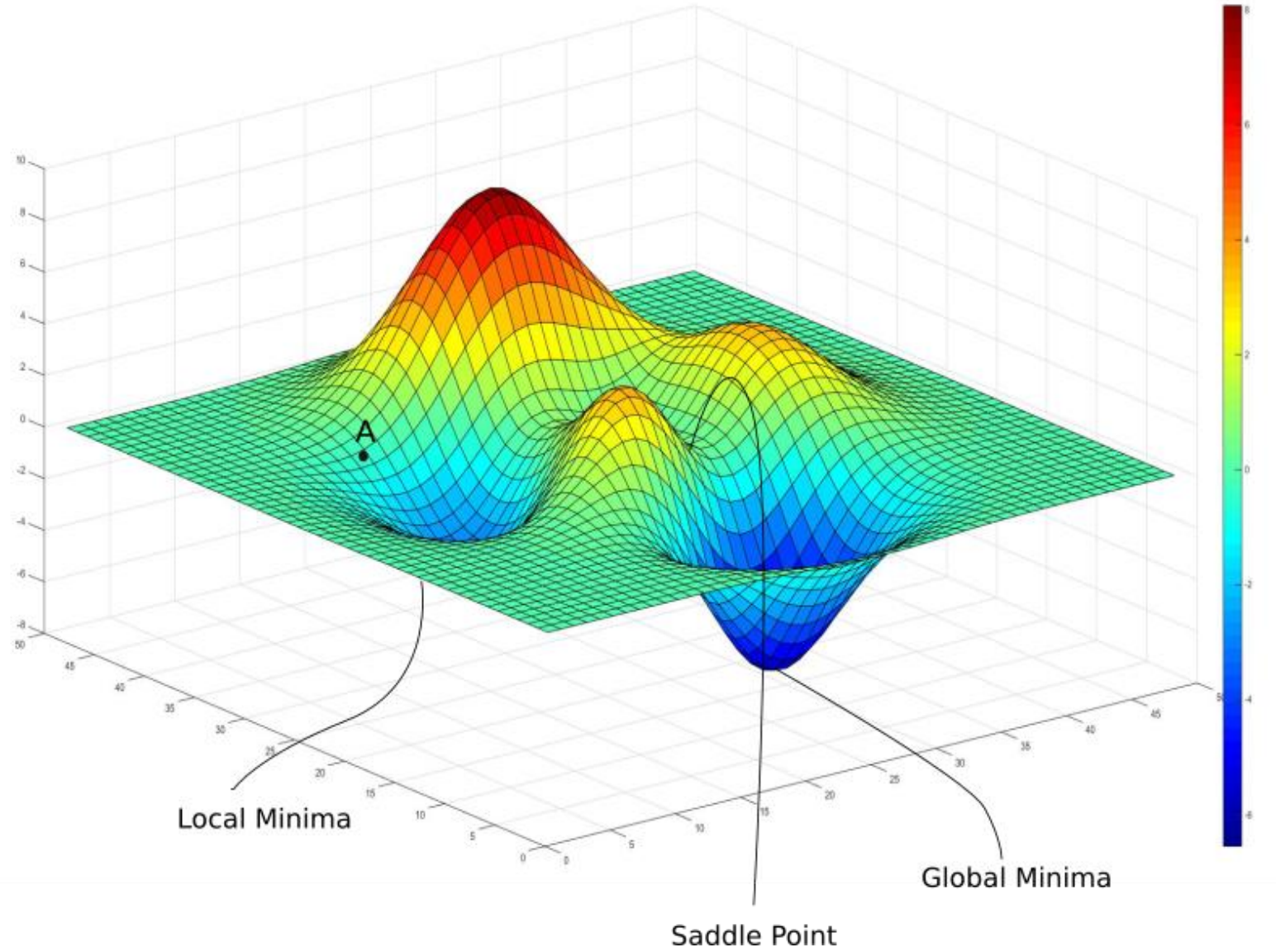


Discontinuous Functions

Optimization Algorithms

❖ Challenges

- Local minima
- Global minima
- Saddle points

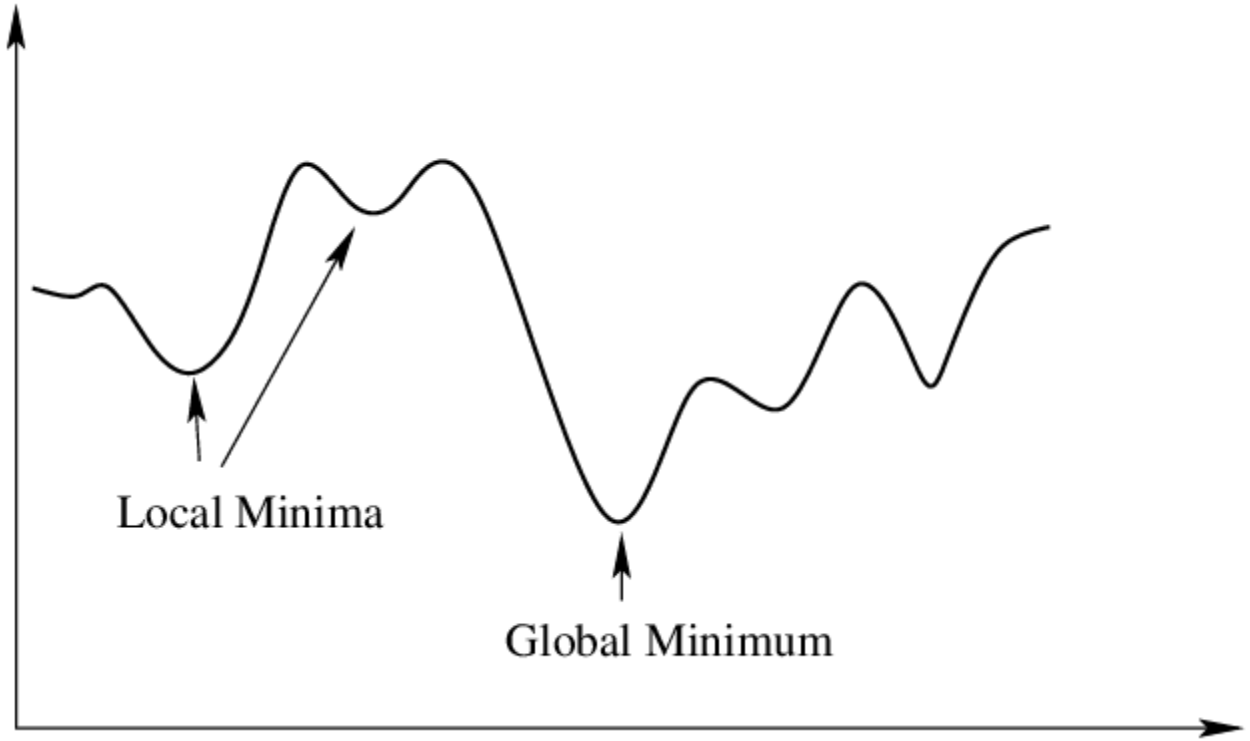


https://blog.paperspace.com/intro-to-optimization-in-deep-learning-gradient-descent/

Optimization Algorithms

❖ Challenges

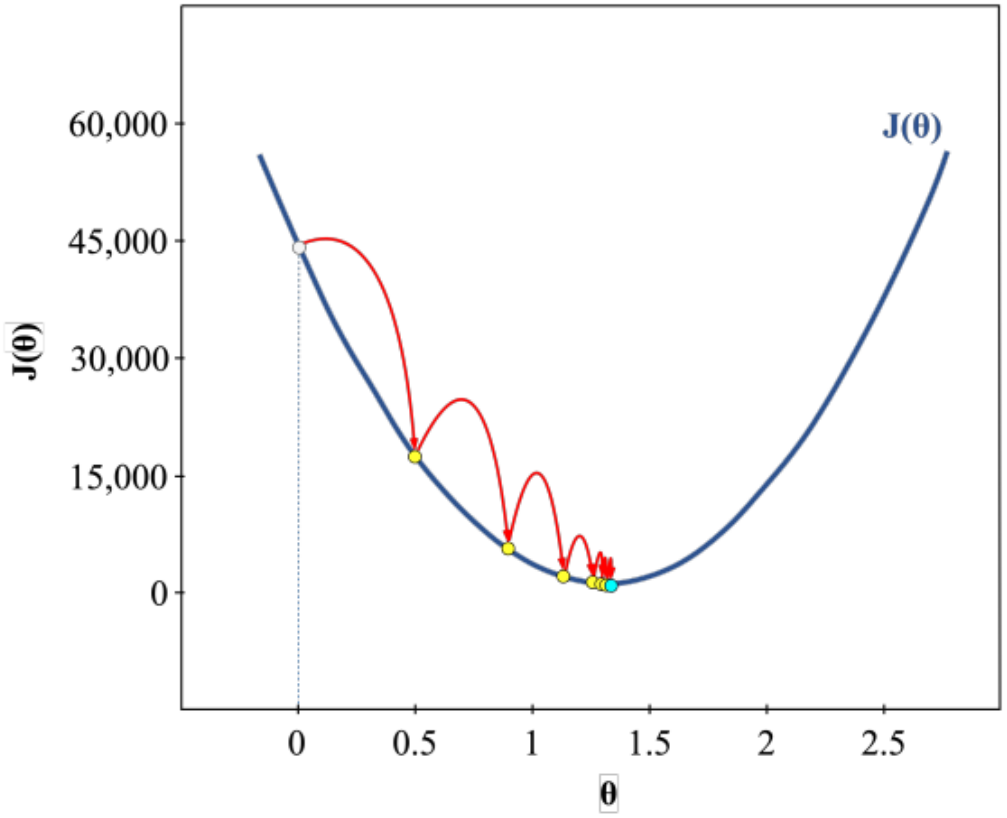
- Local minima
- Global minima
- Saddle points



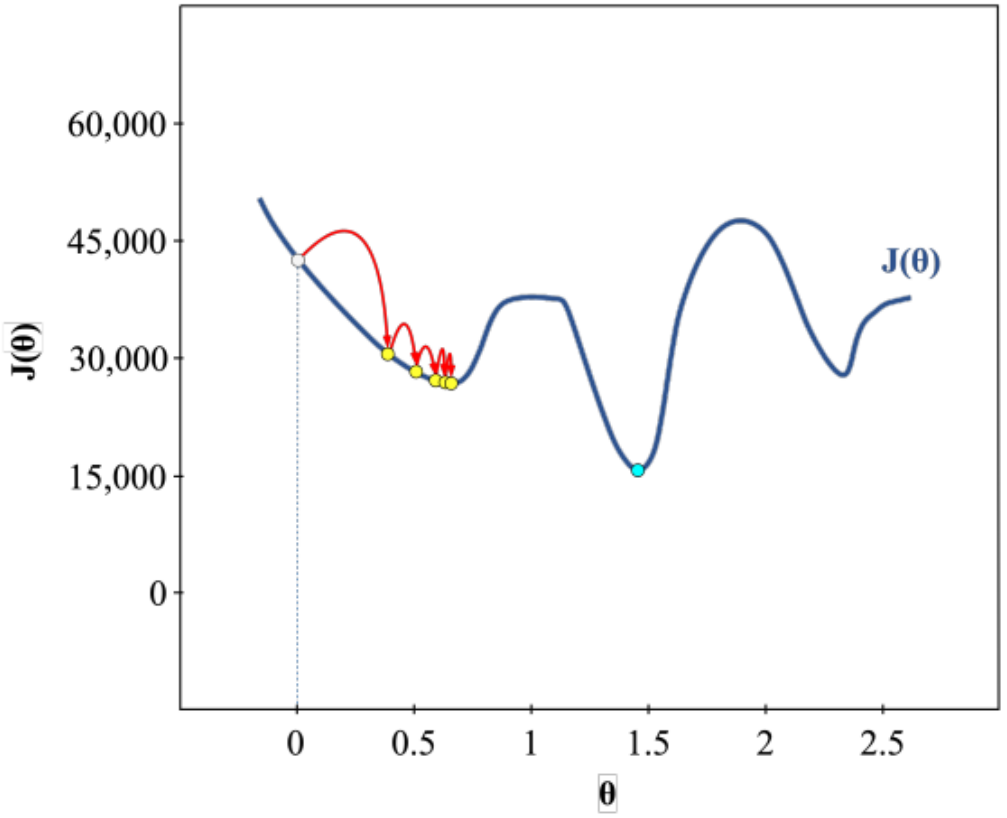
https://vitalflux.com/local-global-maxima-minima-explained-examples/

Optimization Algorithms

❖ Challenges: Local minima



(a) Gradient Descent on a Convex Cost Function.



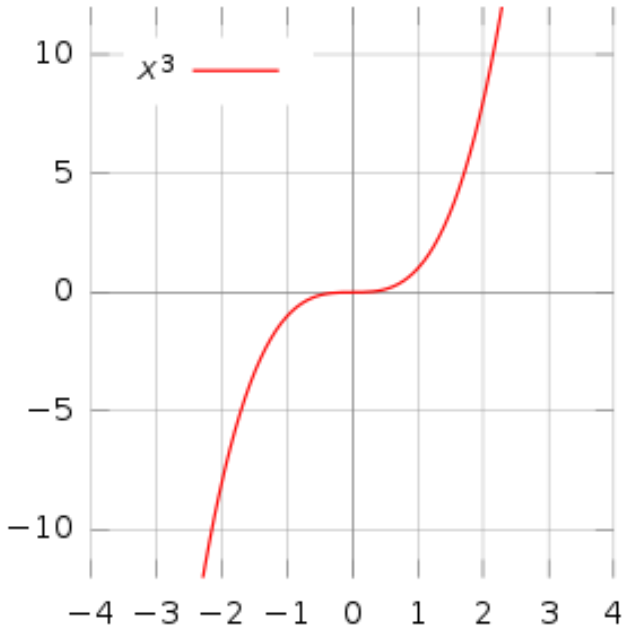
(b) Gradient Descent on a Non-convex Cost Function.

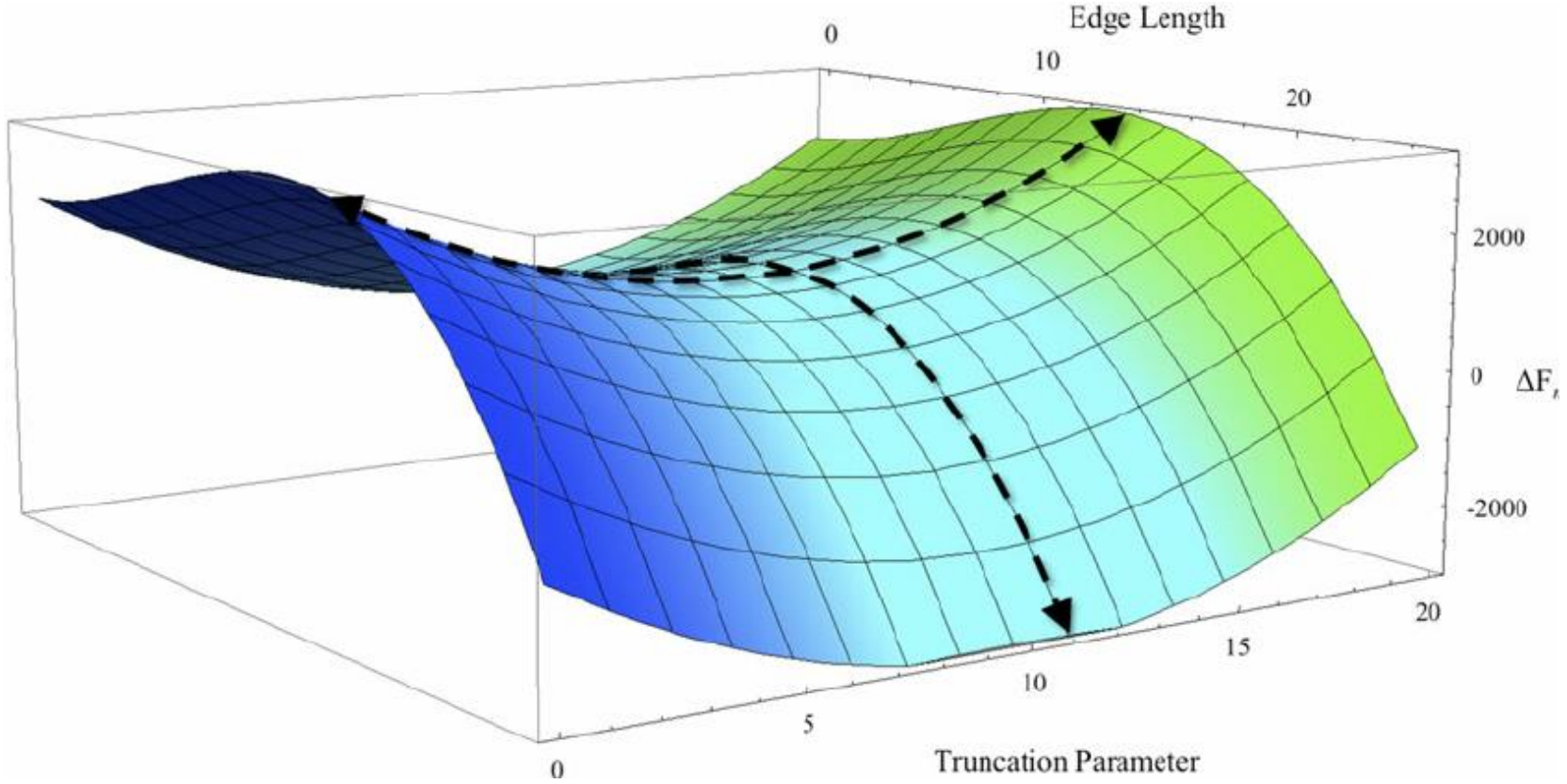
From "Machine Learning Simplified"

Optimization Algorithms

❖ Challenges

Saddle points

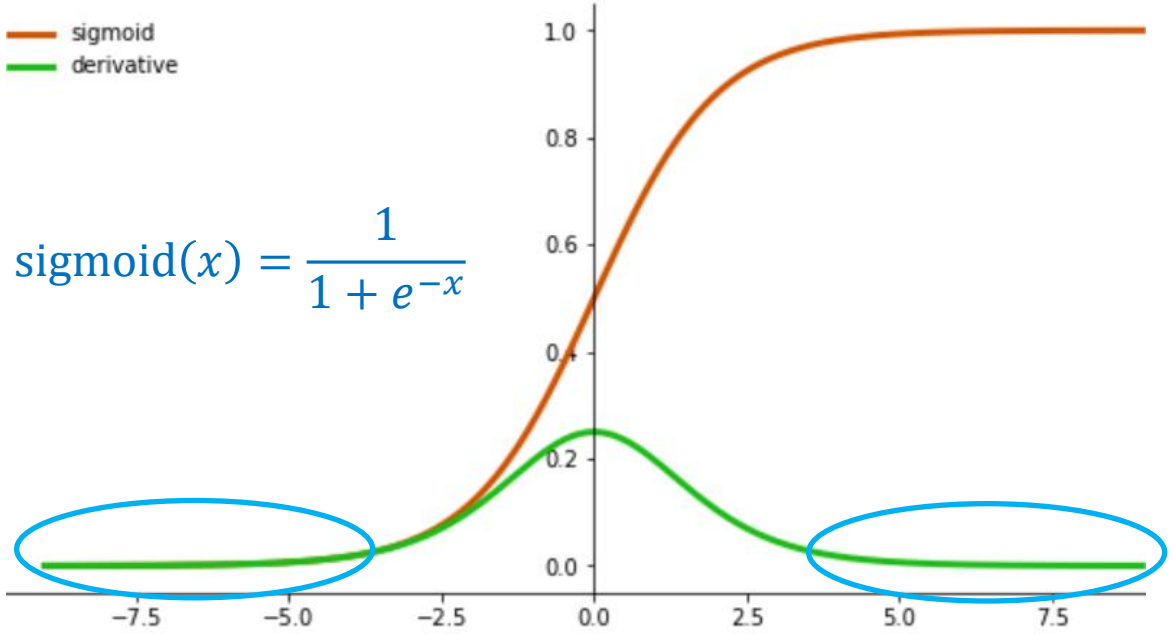




https://www.wikiwand.com/en/Saddle_point

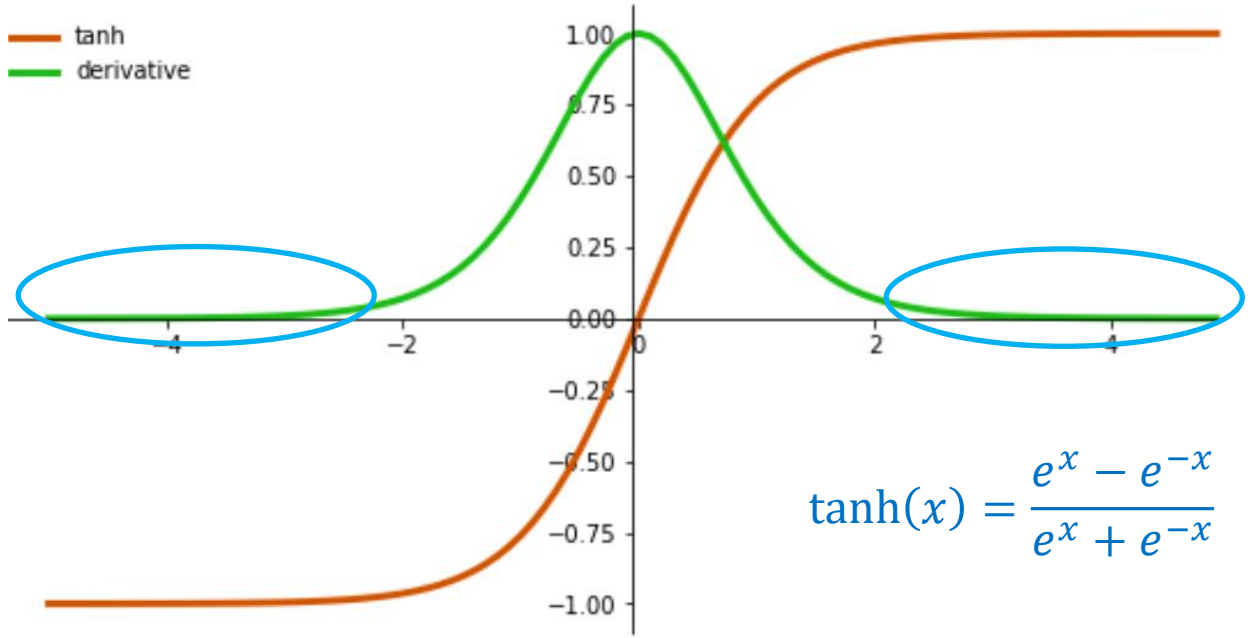
Optimization Algorithms

❖ Challenges: Gradient vanishing



$$\text{sigmoid}(x) = \frac{1}{1 + e^{-x}}$$

$$\text{sigmoid}'(x) = \text{sigmoid}(x)\,(1 - \text{sigmoid}(x))$$

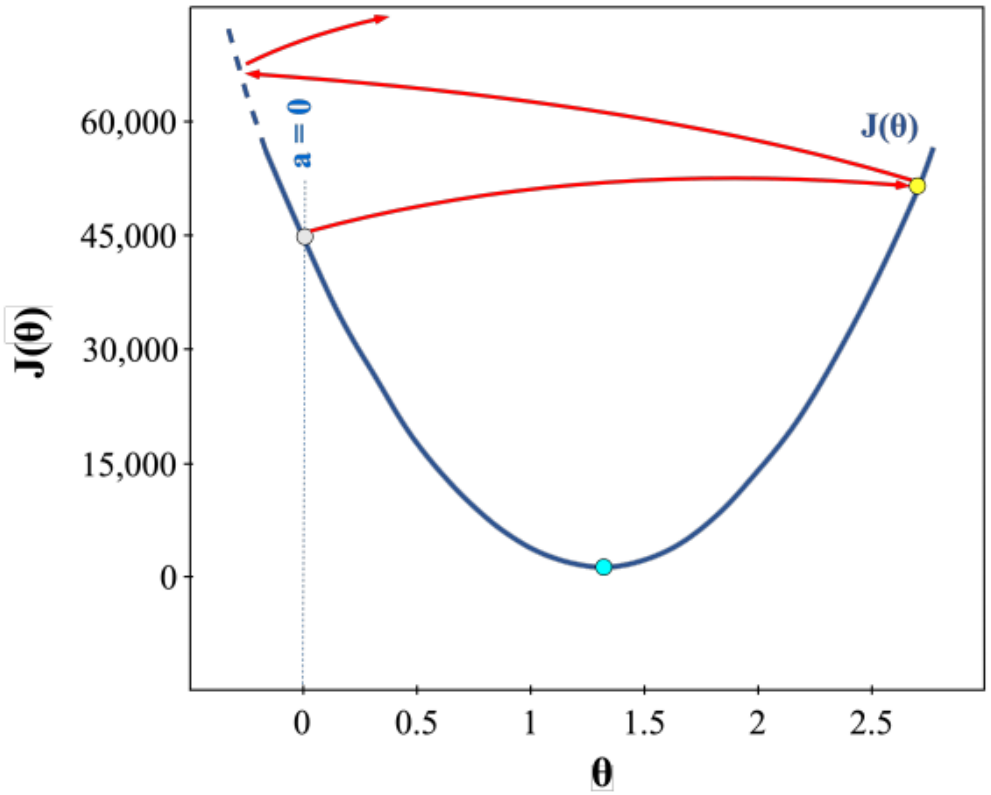


$$\tanh(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

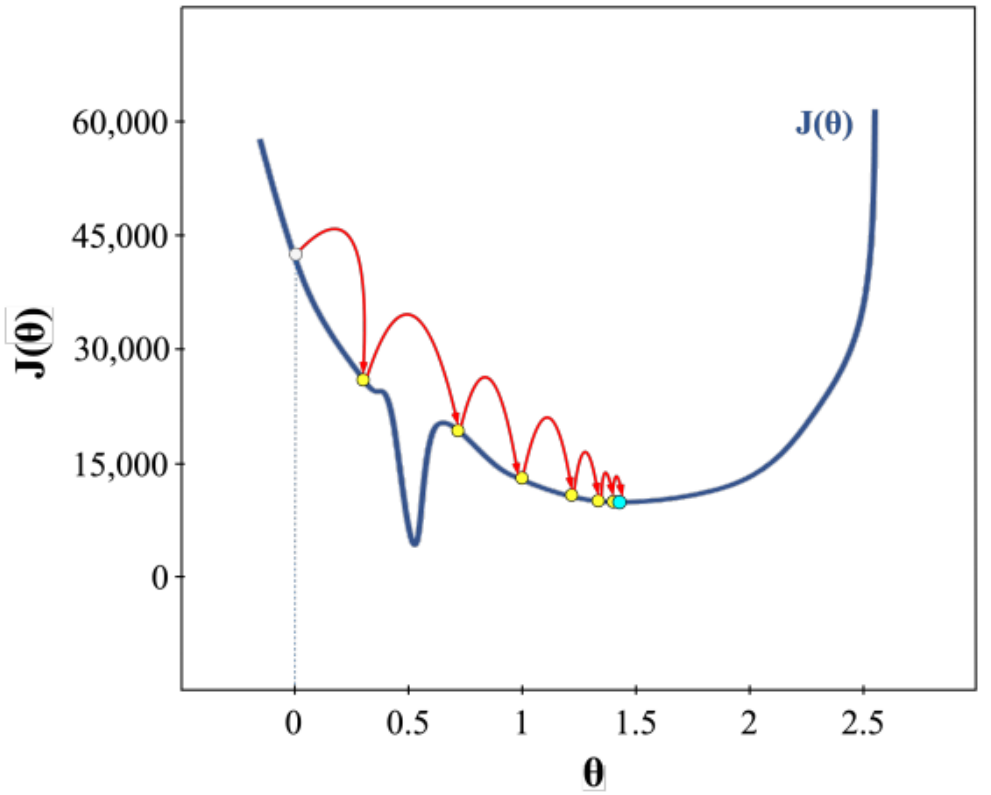
$$\tanh'(x) = 1 - \tanh^{2}(x)$$

Optimization Algorithms

❖ Learning rate



(a) Gradient descent missing global minimum on a convex cost function due to a very large learning rate.



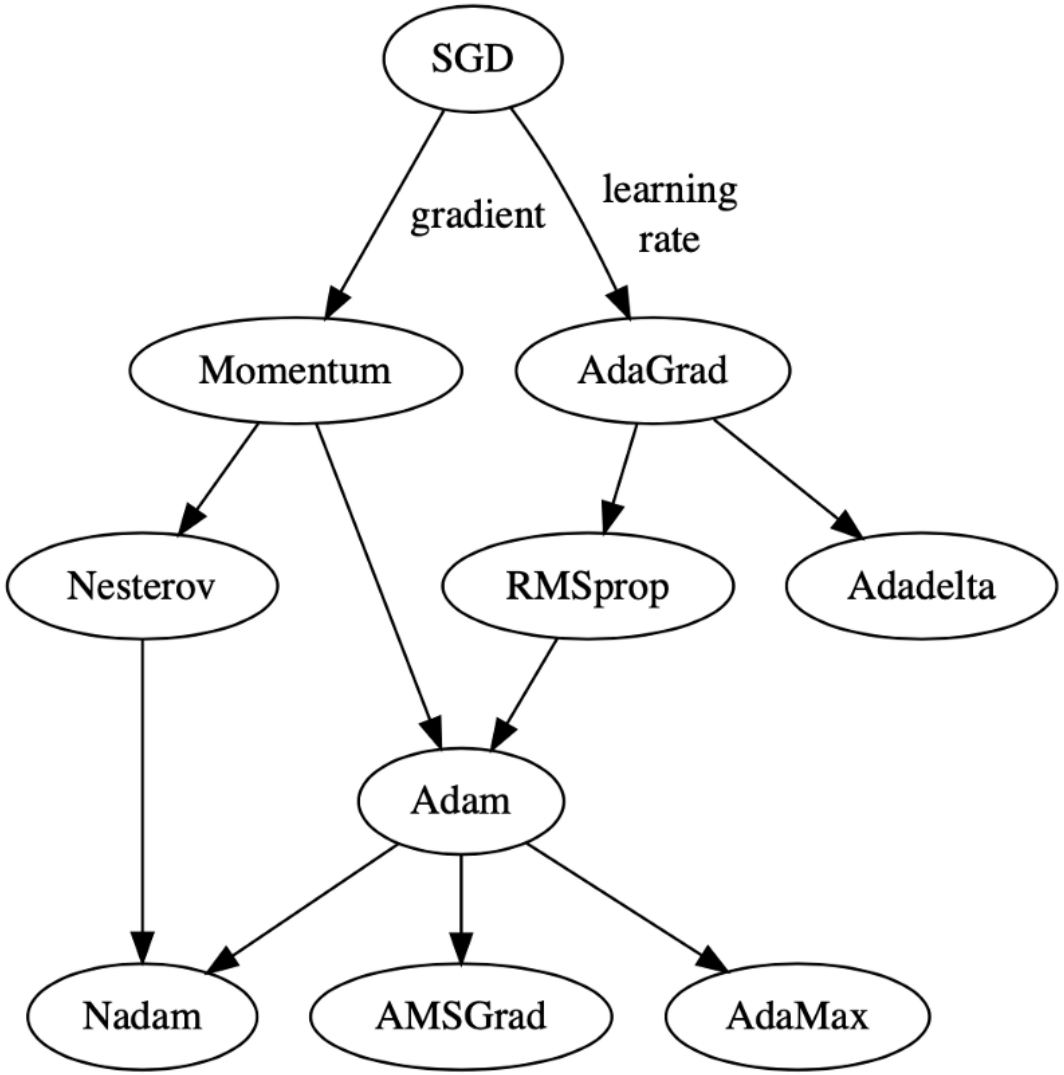
(b) Gradient Descent missing global minimum on a non-convex cost function due to a very large learning rate.

From "Machine Learning Simplified"

Optimizers

❖ Optimizer Selection

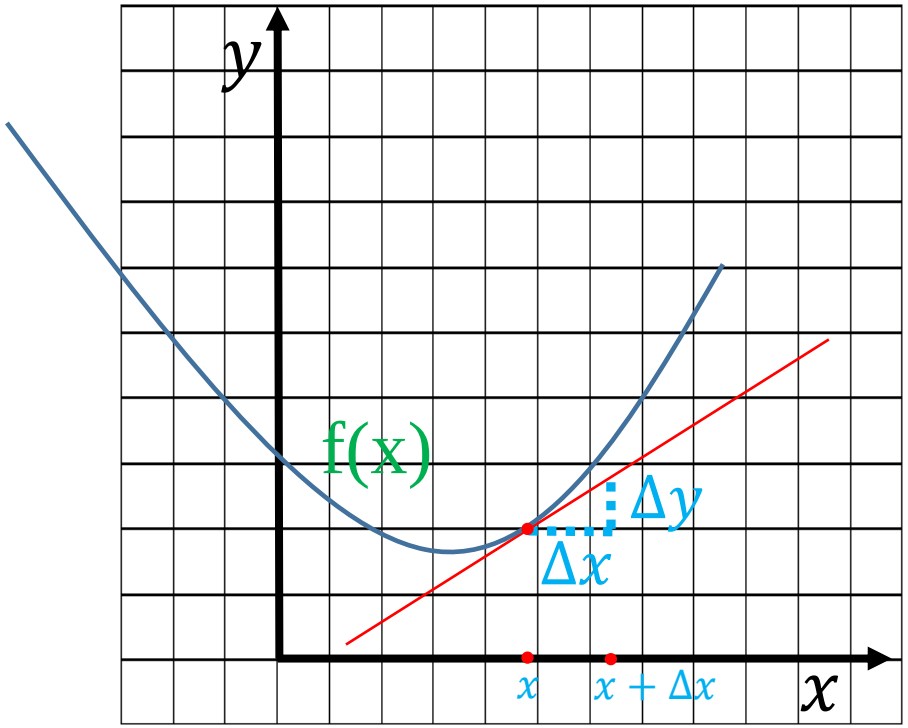
Define a way to update parameters



https://www.kdnuggets.com/2019/06/gradient-descent-algorithms-cheat-sheet.html

Derivative/Gradient

❖ Đạo hàm cho hàm liên tục



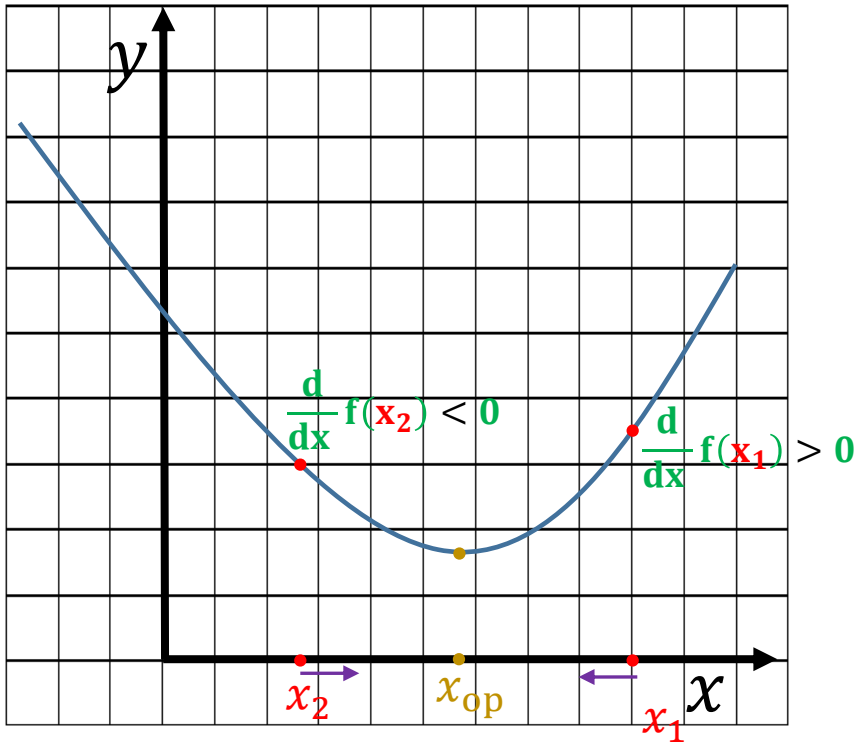
$$\text{Đạo hàm} = \frac{Thay\ đổi\ theo\ y}{Thay\ đổi\ theo\ x} = \frac{\Delta y}{\Delta x}$$

$$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Δx cần tiến về 0 để đường tiếp tuyến tiến về hàm f(x) trong vùng lân cận tại x

Gradient-based Optimization

❖ Tìm giá trị min



Quan sát: x_{op} ở vị trí ngược hướng đạo hàm tại x_1 và x_2

Cách xử lý việc di chuyển ngược hướng đạo hàm cho x_1 và x_2 (để tìm x_{op}) khác nhau hình thành các thuật toán tối ưu hóa khác nhau

Cách cập nhật giá trị x đơn giản

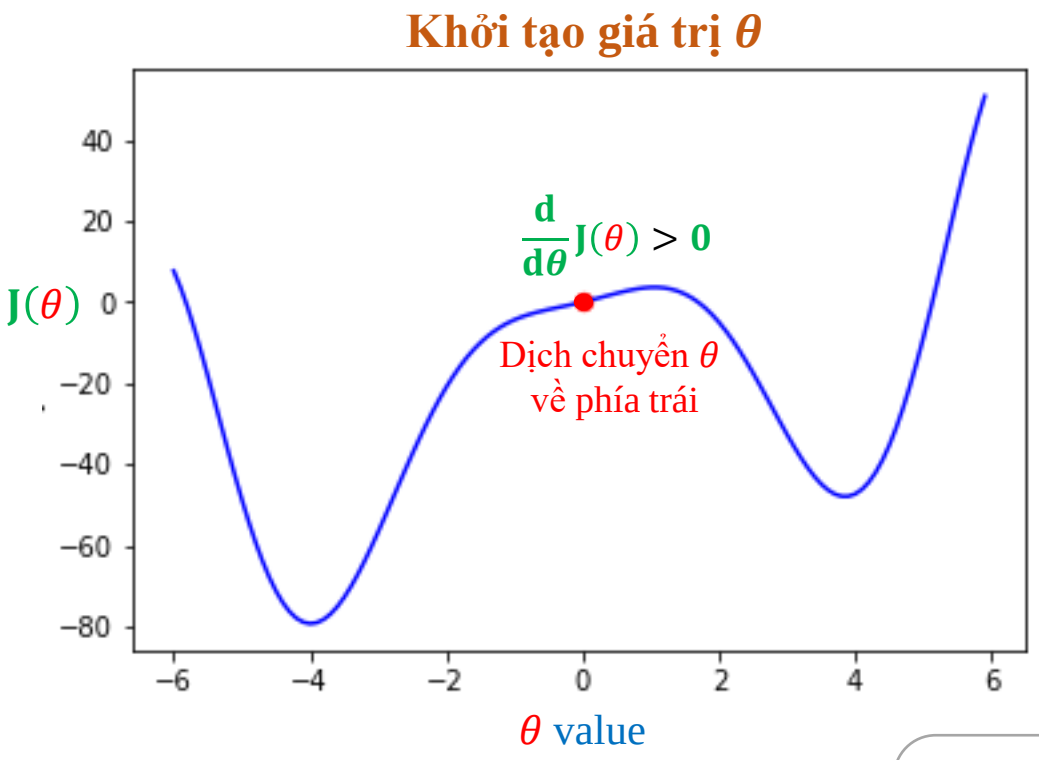
$$x = x - \eta \frac{d}{dx} f(x)$$

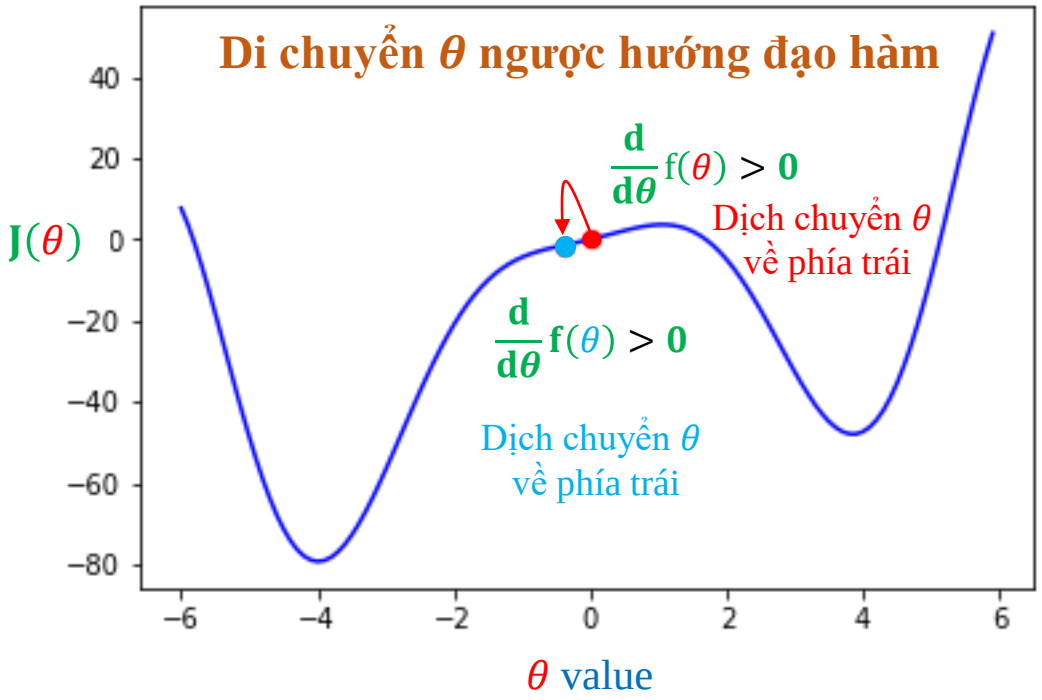
Đạo hàm tại x

Trọng số

Optimization

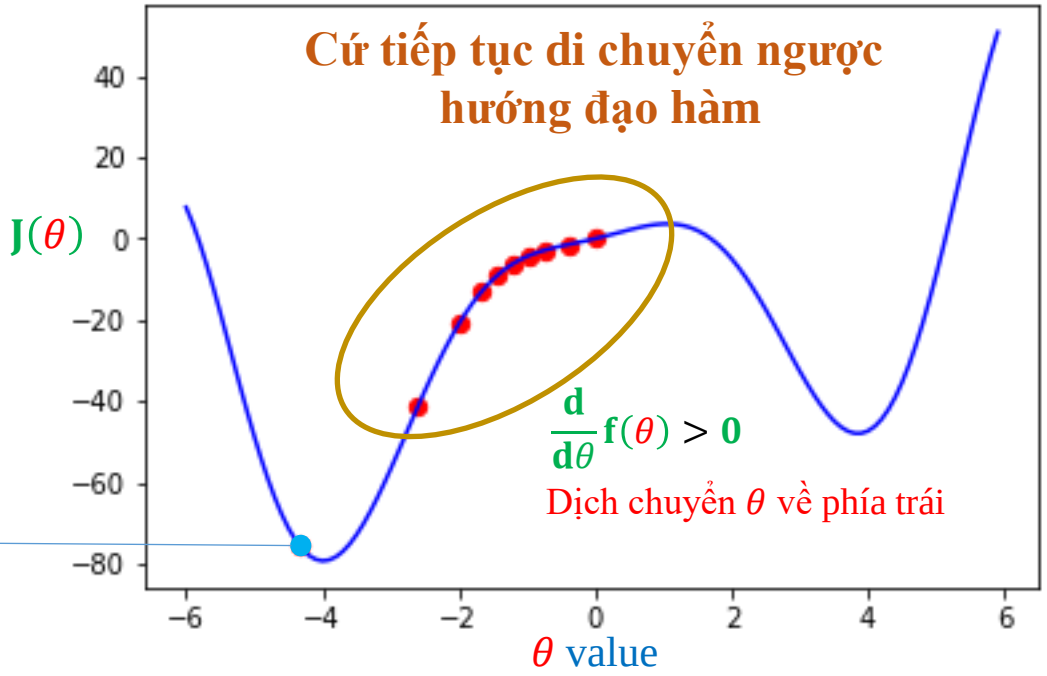
❖ A cue to optimize a function





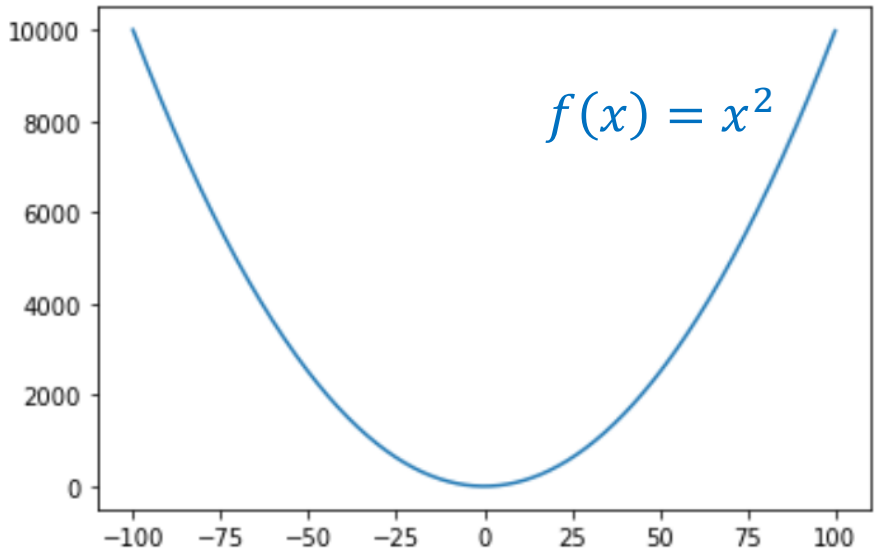
$\frac{d}{d\theta}f(\theta) < 0$

Dịch chuyển θ về phía phải



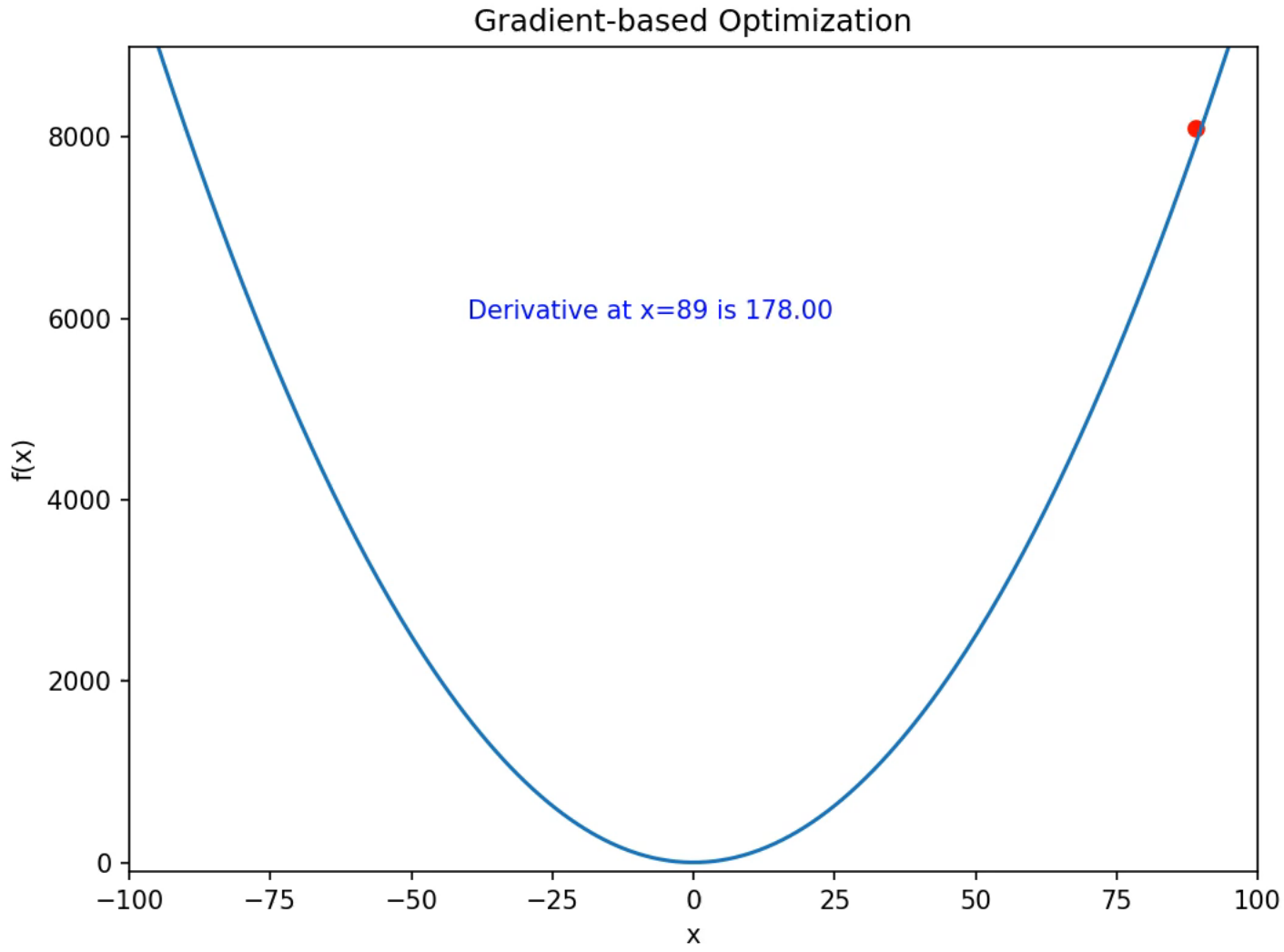
Gradient-based Optimization

❖ Square function



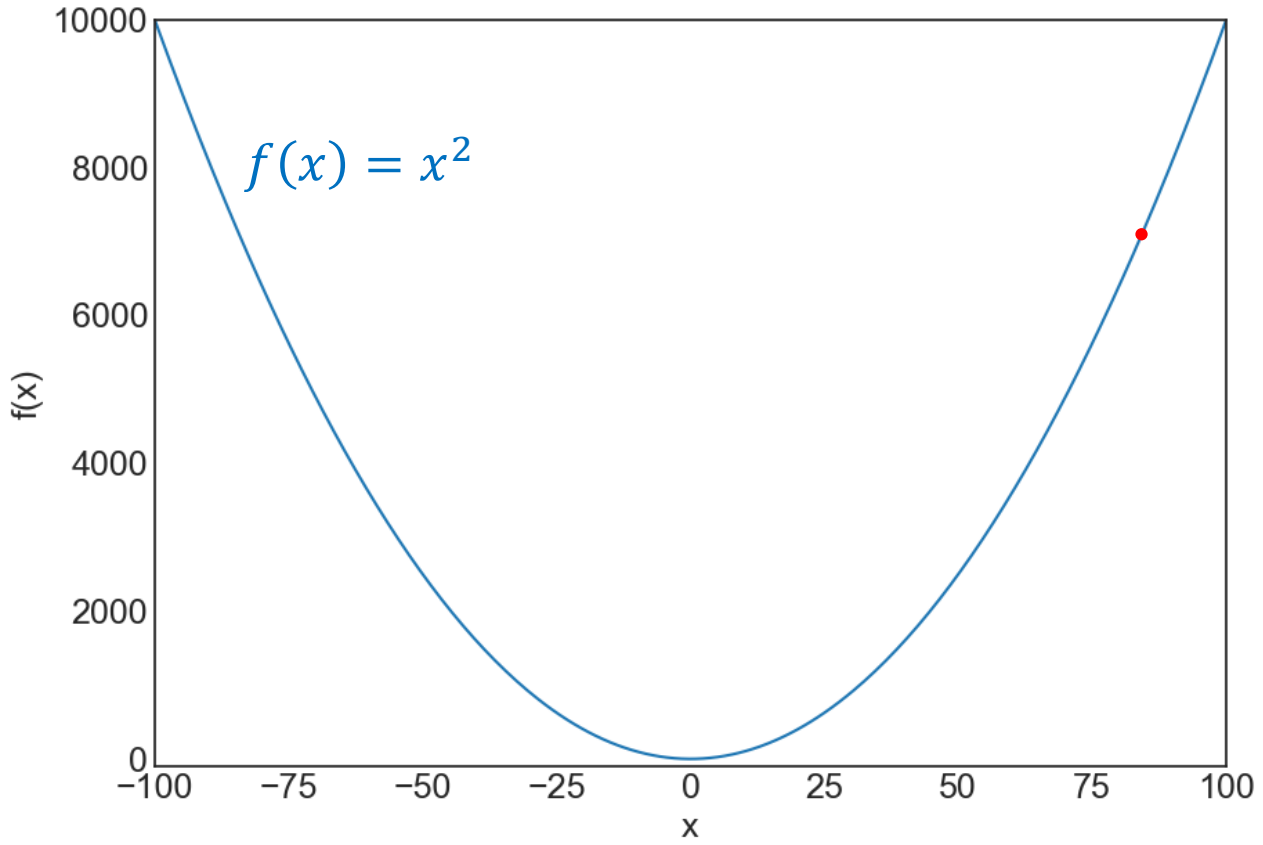
$$-100 \leq x \leq 100$$

$$x \in \mathbb{N}$$



Gradient-based Optimization

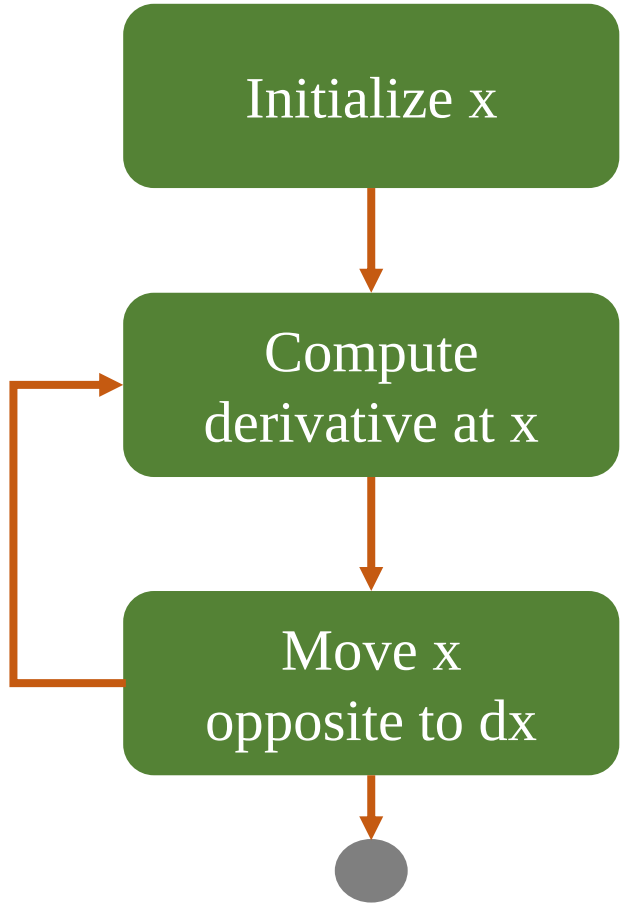
❖ Square function



$$-100 \leq x \leq 100$$

$$x \in \mathbb{N}$$

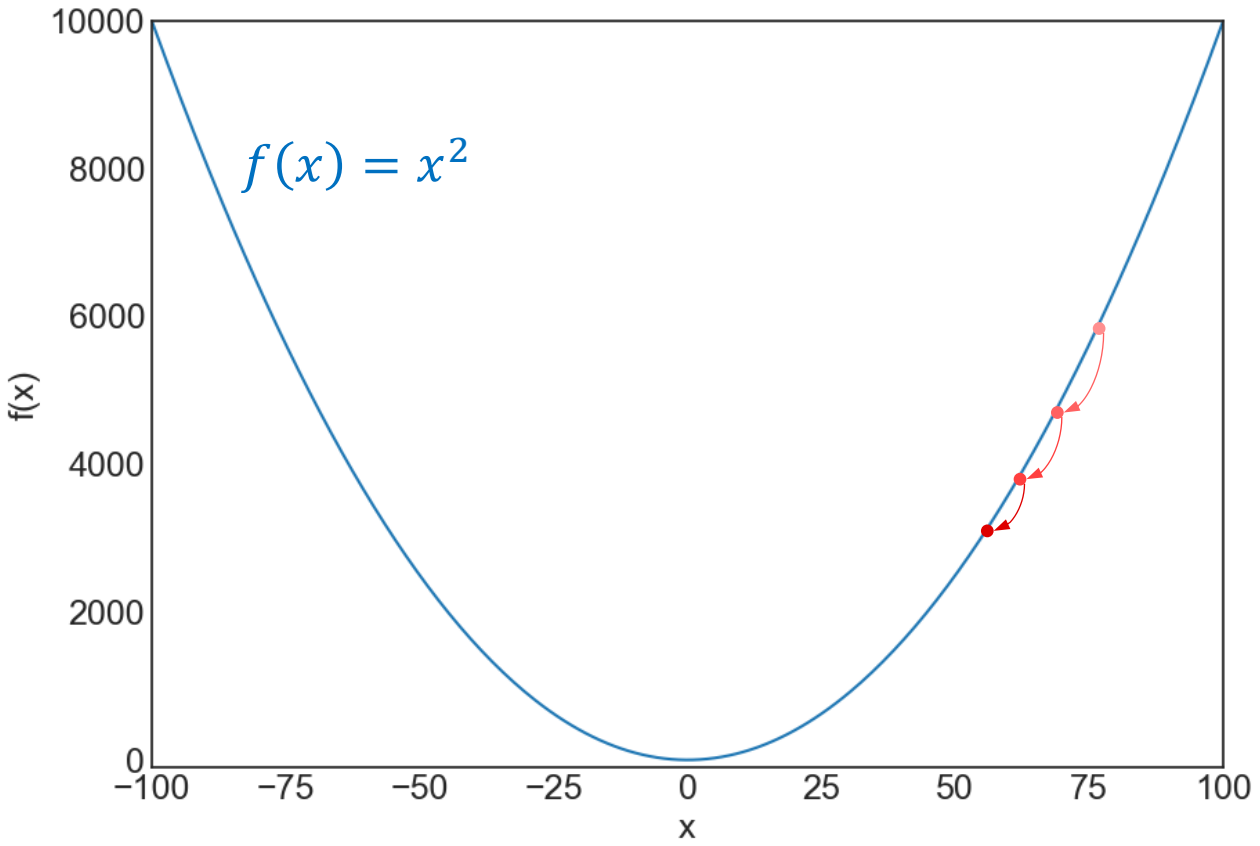
$$x_t = x_{t-1} - \eta f'(x_{t-1})$$



Optimization

$$x_t = x_{t-1} - \eta f'(x_{t-1})$$

❖ Square function



$$-100 \leq x \leq 100$$

$$x \in \mathbb{N}$$

$x_0 = 70.0 \qquad \eta = 0.1$

$f'(x_0) = 140.0$

$x_1 = x_0 - \eta f'(x_0) = 56.0$

$f'(x_1) = 112.0$

$x_2 = x_1 - \eta f'(x_1) = 44.8$

$f'(x_2) = 89.6$

$x_3 = x_2 - \eta f'(x_2) = 35.84$

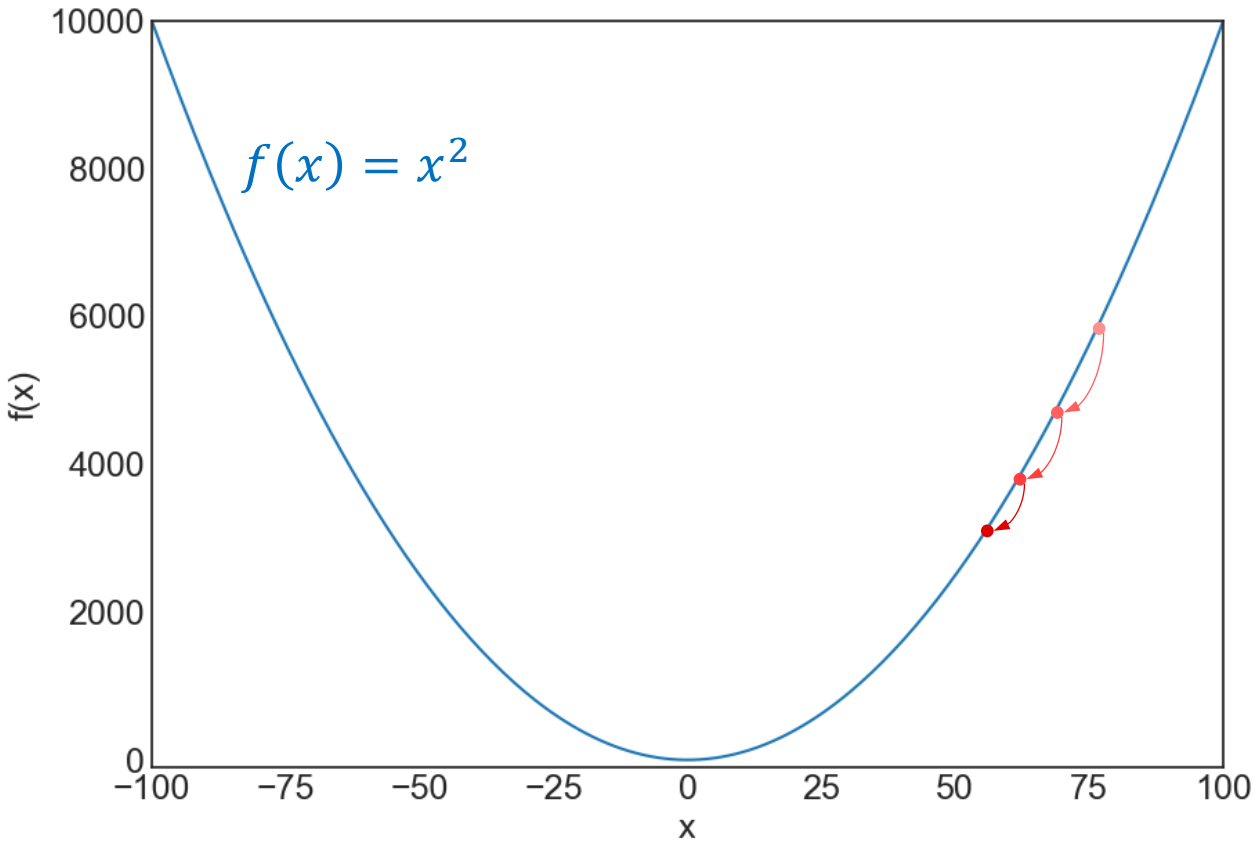
$f'(x_3) = 71.68$

$x_4 = x_3 - \eta f'(x_3) = 28.672$

Optimization

$$x_t = x_{t-1} - \eta f'(x_{t-1})$$

❖ Square function



$x_{10} = 6.012 \qquad \eta = 0.1$

$f'(x_{10}) = 12.02$

$x_{11} = x_{10} - \eta f'(x_{10}) = 4.81$

$f'(x_{11}) = 9.62$

$x_{12} = x_{11} - \eta f'(x_{11}) = 3.84$

$f'(x_{12}) = 7.69$

$x_{13} = x_{12} - \eta f'(x_{12}) = 3.078$

$f'(x_{13}) = 6.15$

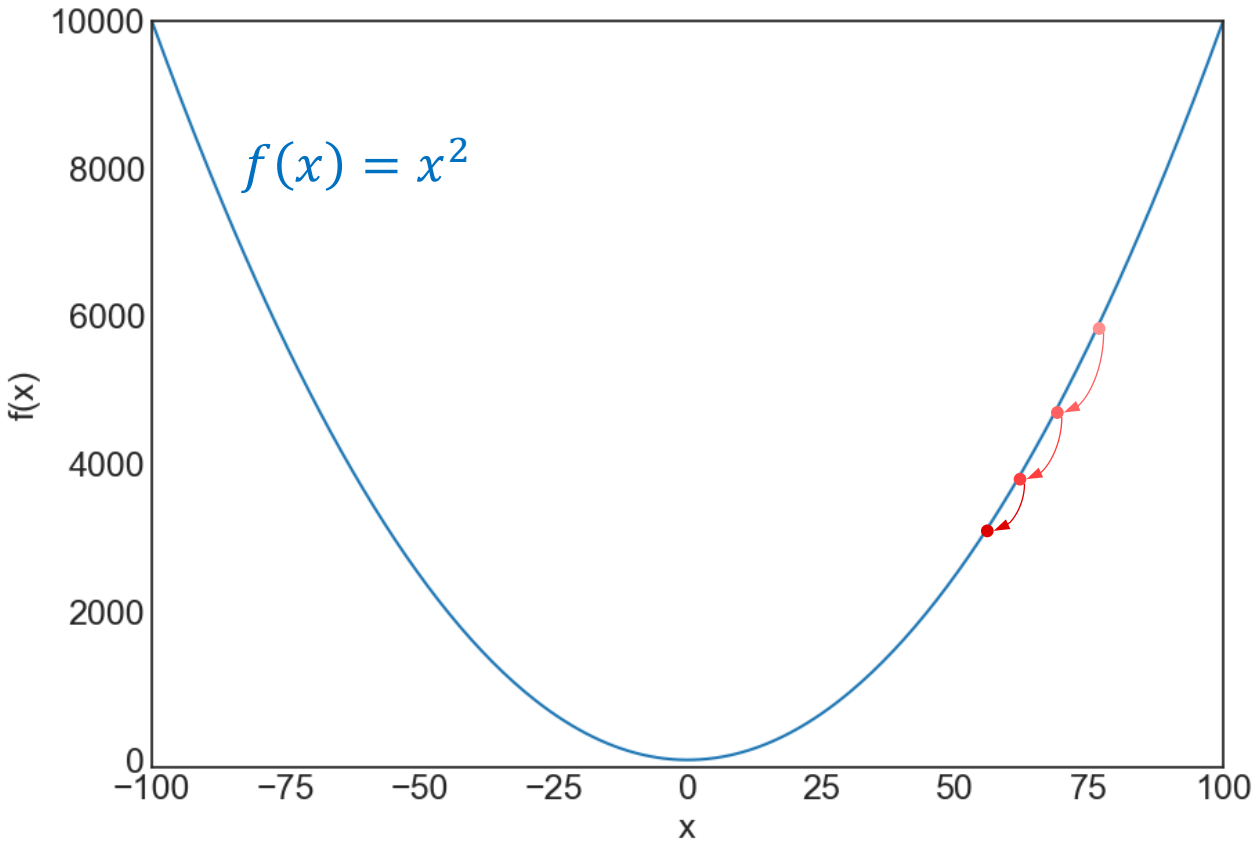
$x_{14} = x_{13} - \eta f'(x_{13}) = 2.46$

Keep doing

Optimization

$$x_t = x_{t-1} - \eta f'(x_{t-1})$$

❖ Square function



$x_{30} = 0.069 \qquad \eta = 0.1$

$f'(x_{30}) = 0.138$

$x_{31} = x_{30} - \eta f'(x_{30}) = 0.055$

$f'(x_{31}) = 0.11$

$x_{32} = x_{31} - \eta f'(x_{31}) = 0.044$

$f'(x_{32}) = 0.88$

$x_{33} = x_{32} - \eta f'(x_{32}) = 0.035$

$f'(x_{34}) = 0.071$

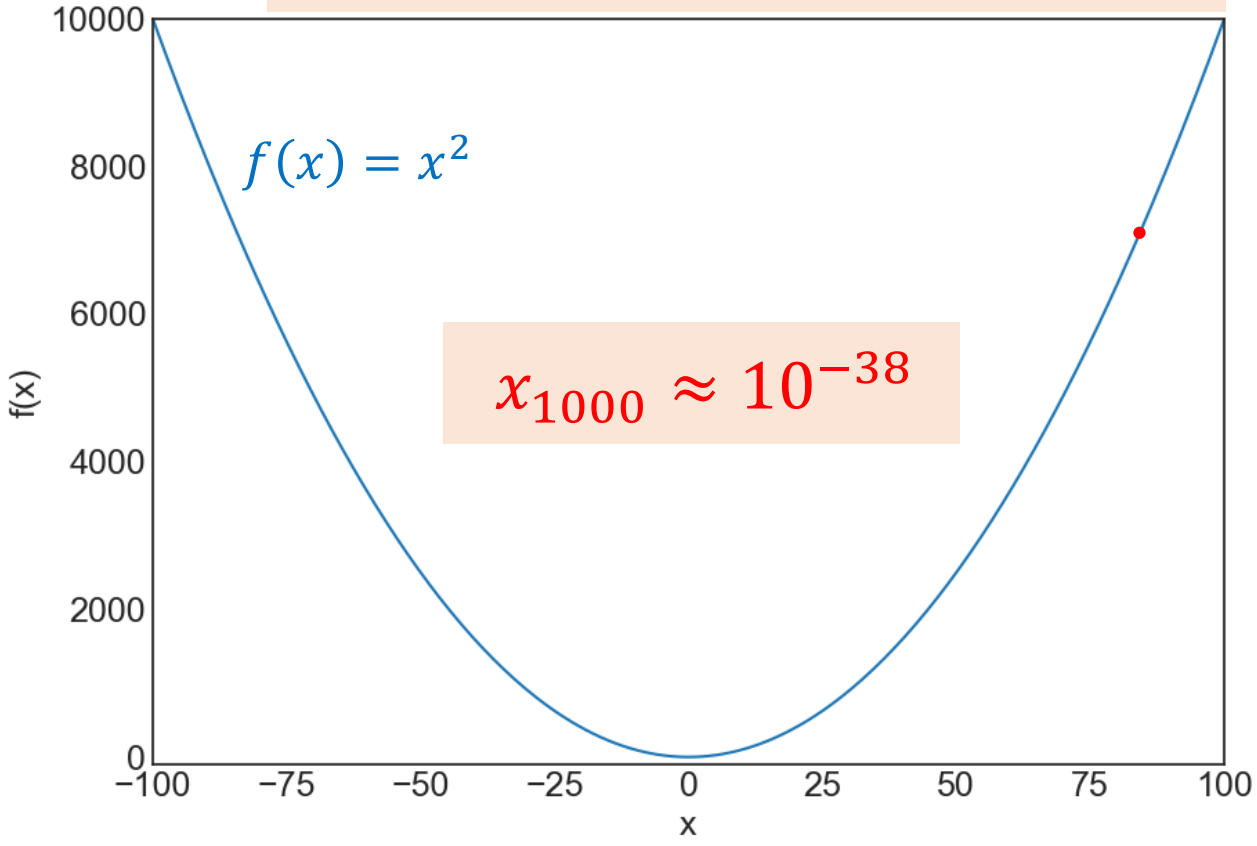
$x_{34} = x_{33} - \eta f'(x_{33}) = 0.028$

Keep doing

Optimization

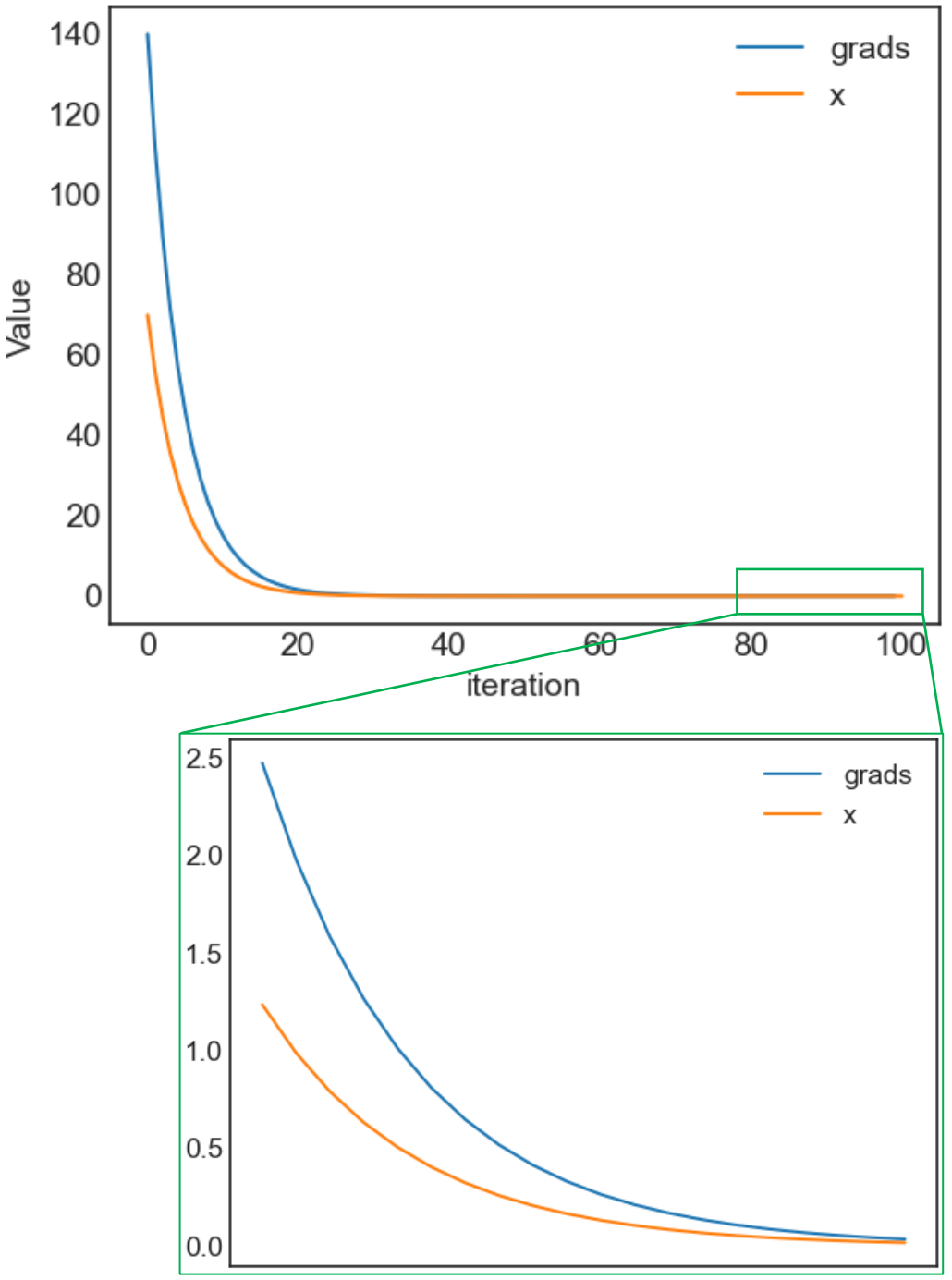
❖ Square function

$$x_t = x_{t-1} - \eta f'(x)$$



$$x_{1000} \approx 10^{-38}$$

Optimized successfully!



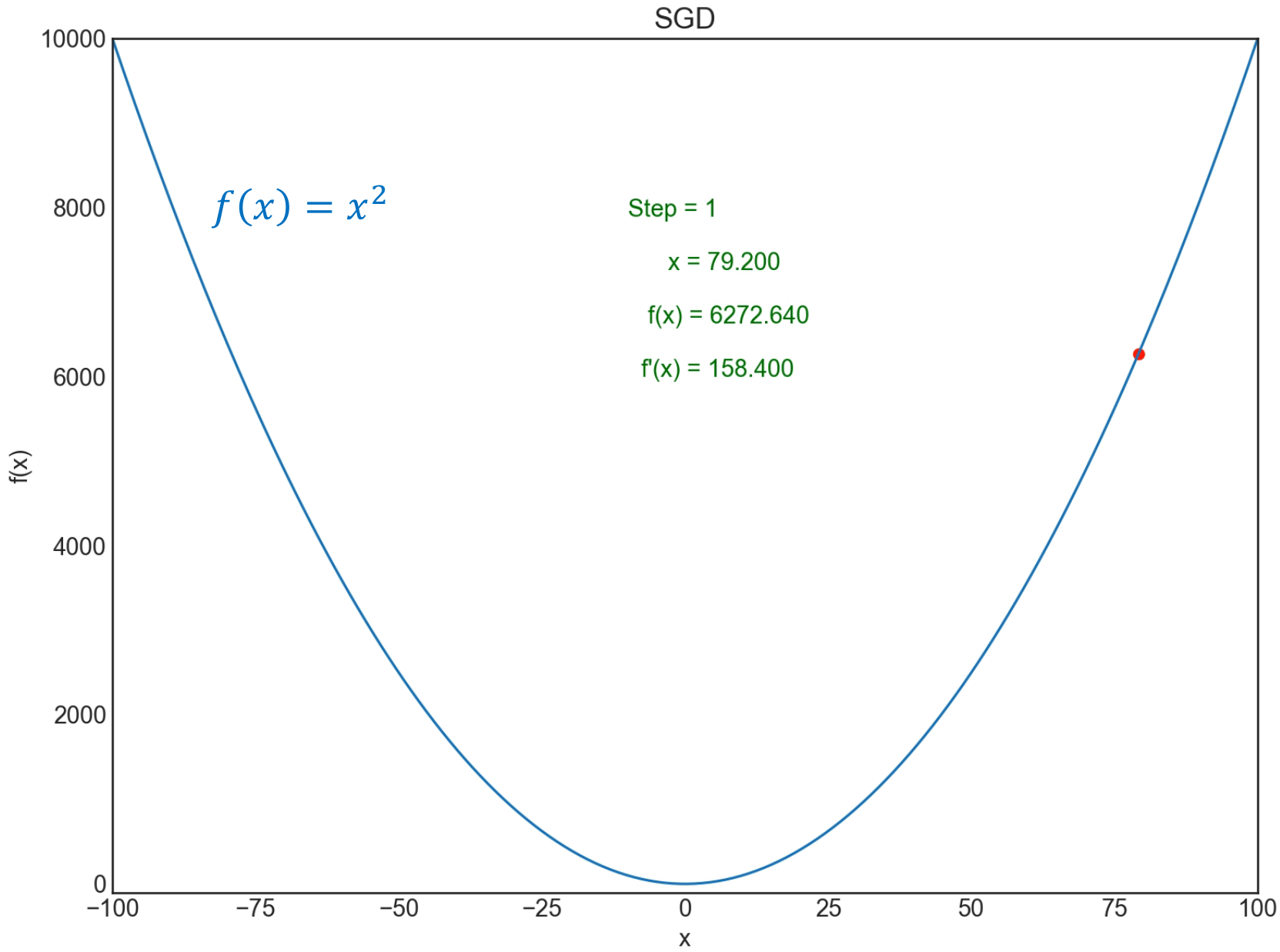
Optimization

❖ **Square function**

$x_0 = 99.0$

$\eta = 0.1$

$x_t = x_{t-1} - \eta f'(x)$



Optimization

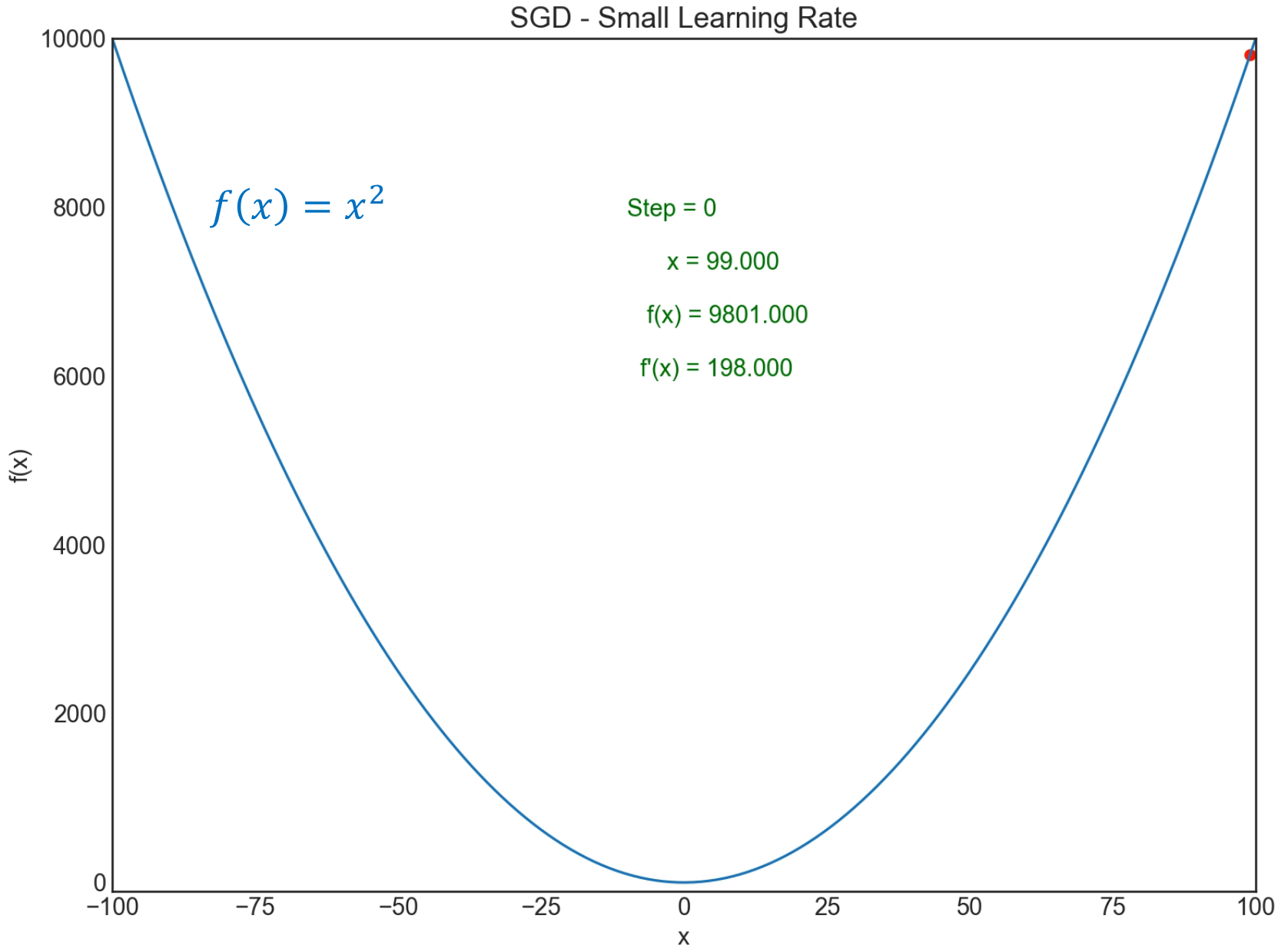
❖ **Square function**

Discussion

$x_0 = 99.0$

$\eta = 0.001$

$x_t = x_{t-1} - \eta f'(x)$



Optimization

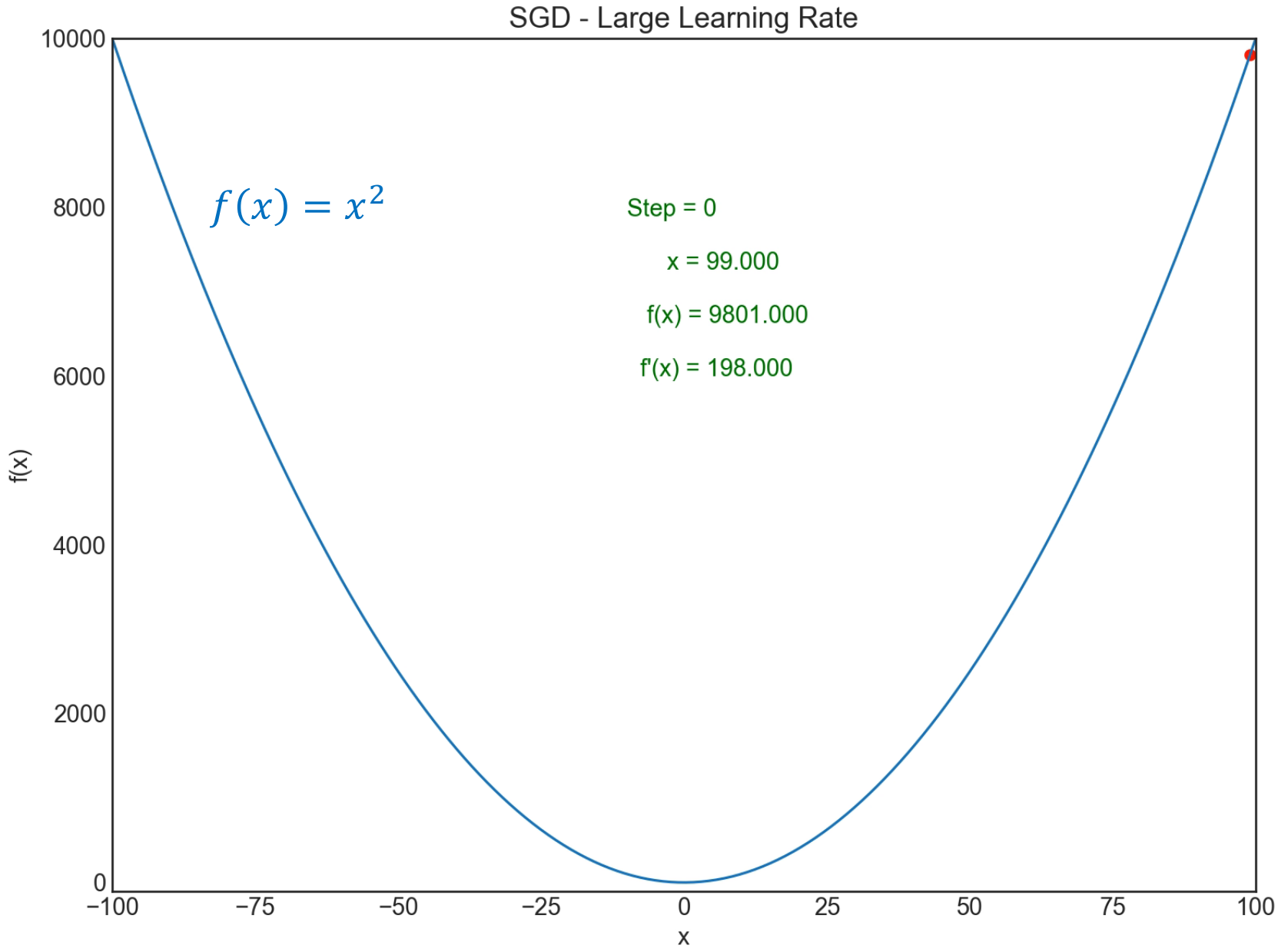
❖ **Square function**

Discussion

$$x_0 = 99.0$$

$$\eta = 0.8$$

$$x_t = x_{t-1} - \eta f'(x)$$



Optimization

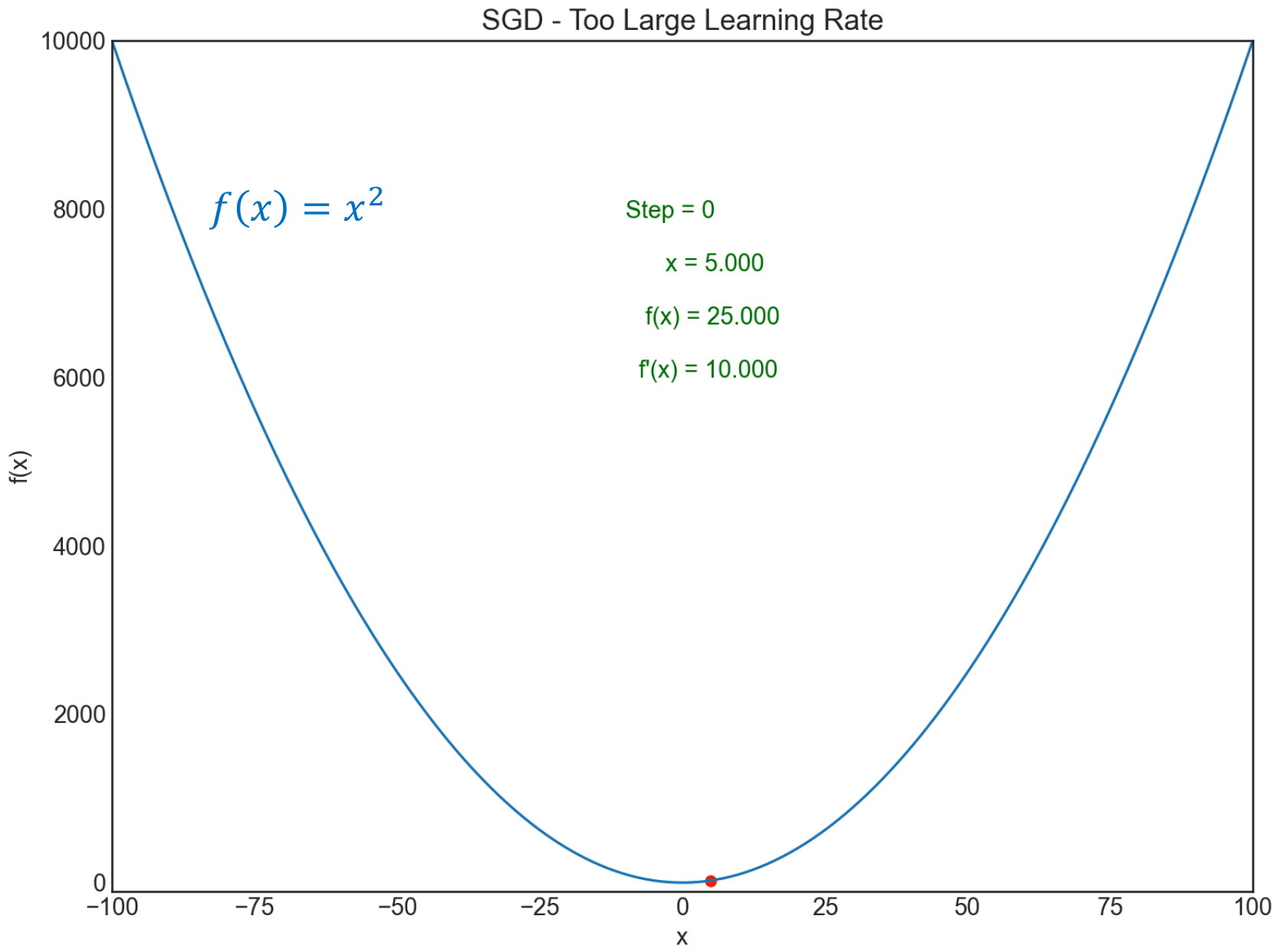
❖ **Square function**

Discussion

$$x_0 = 99.0$$

$$\eta = 1.1$$

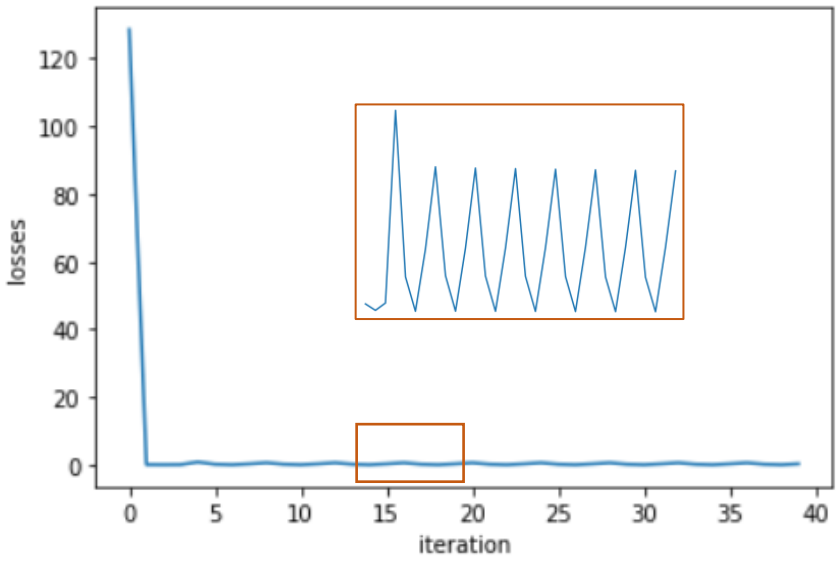
$$x_t = x_{t-1} - \eta f'(x)$$



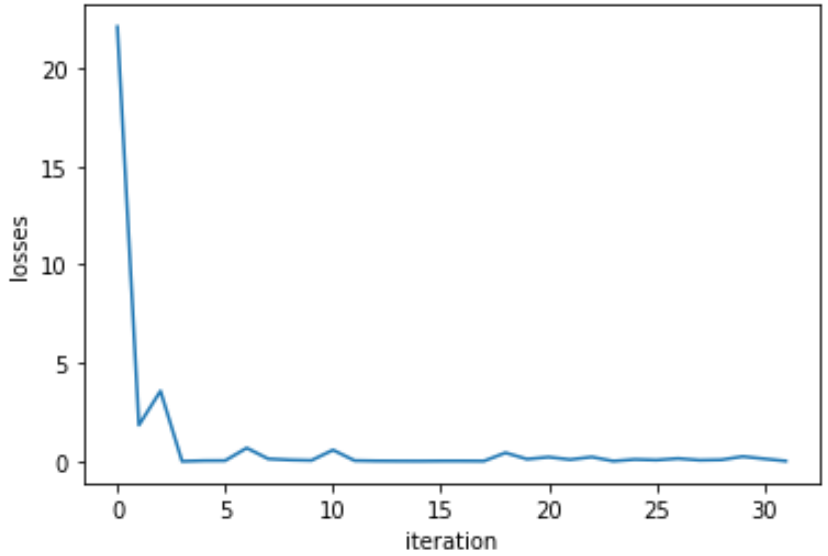
Optimization Algorithms

❖ Stochastic gradient descent

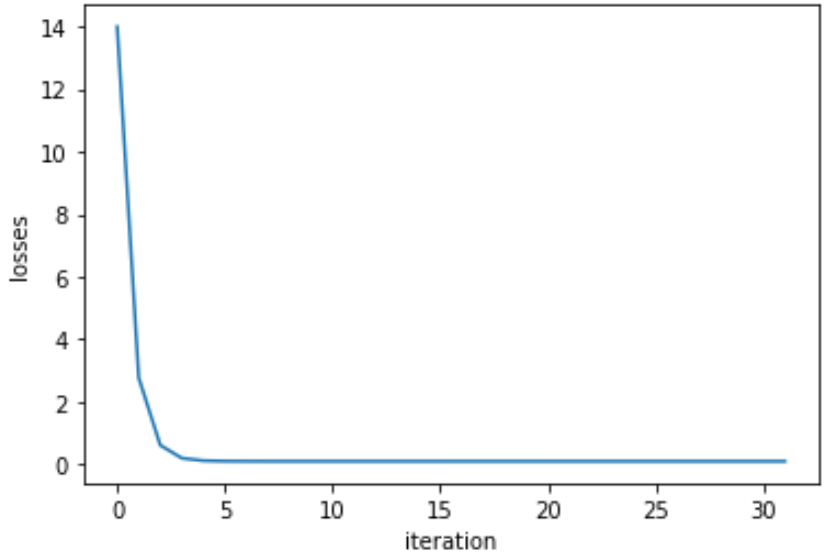
$$\theta_t = \theta_{t-1} - \eta \nabla_\theta L$$



1-sample



m-sample



N-sample

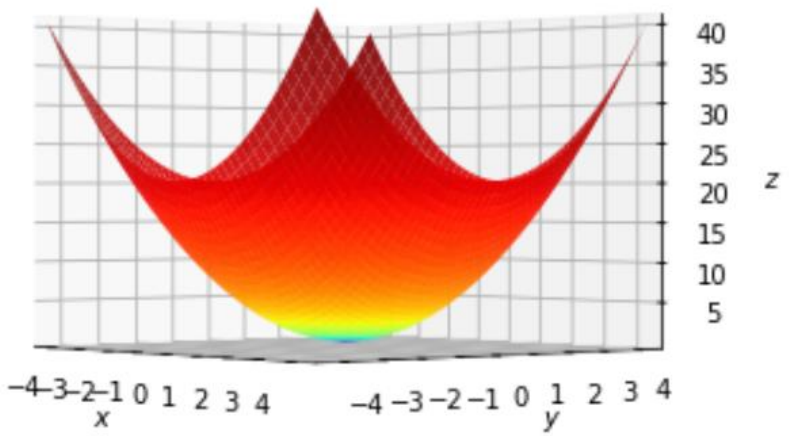
Optimization

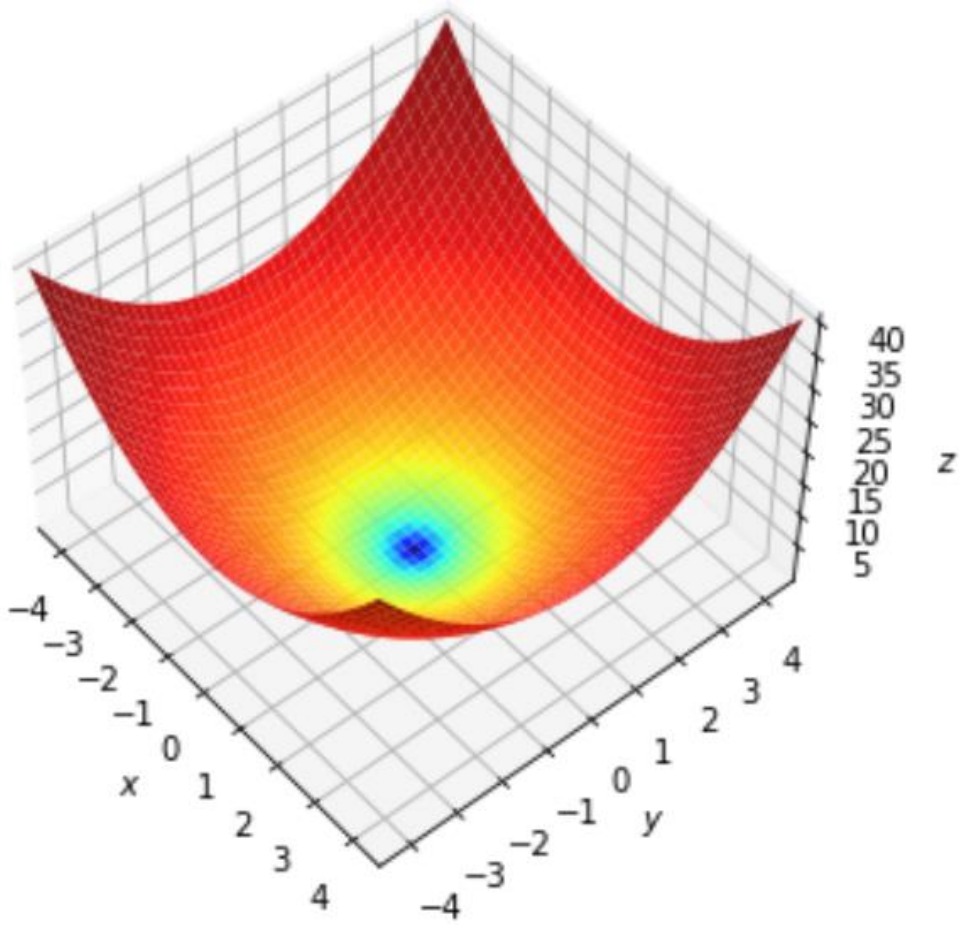
❖ Optimization: 2D function

$$f(x, y) = x^2 + y^2$$

$$-100 \leq x, y \leq 100$$

$$x, y \in \mathbb{N}$$





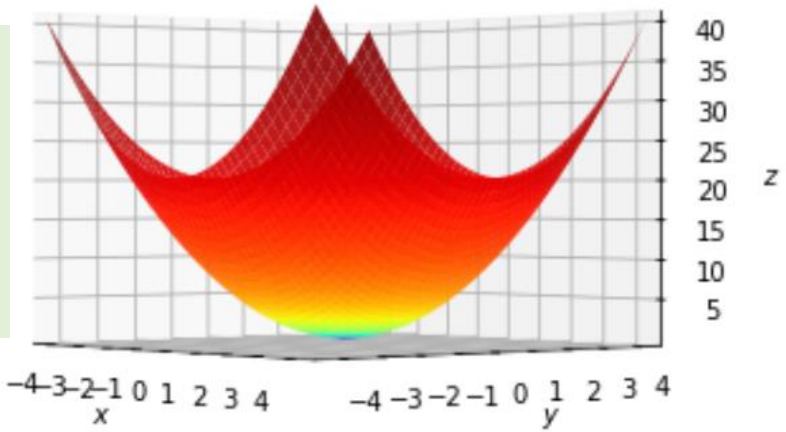
Optimization

❖ Optimization: 2D function

$$f(x,y) = x^2 + y^2$$

$$-100 \le x, y \le 100$$

$$x, y \in \mathbb{N}$$



$$x = x - \eta \frac{\partial f(x,y)}{\partial x}$$

$$y = y - \eta \frac{\partial f(x,y)}{\partial y}$$

$x_0 = 6.0 \qquad y_0 = 9.0 \qquad \eta = 0.1$

$\frac{\partial f(x_0,y_0)}{\partial x} = 12 \qquad \frac{\partial f(x_0,y_0)}{\partial y} = 18$

$x_1 = 4.8 \qquad y_1 = 7.2$

$\frac{\partial f(x_1,y_1)}{\partial x} = 9.6 \qquad \frac{\partial f(x_1,y_1)}{\partial y} = 14.4$

$x_2 = 3.84 \qquad y_2 = 5.75$

$\frac{\partial f(x_2,y_2)}{\partial x} = 7.68 \qquad \frac{\partial f(x_2,y_2)}{\partial y} = 11.51$

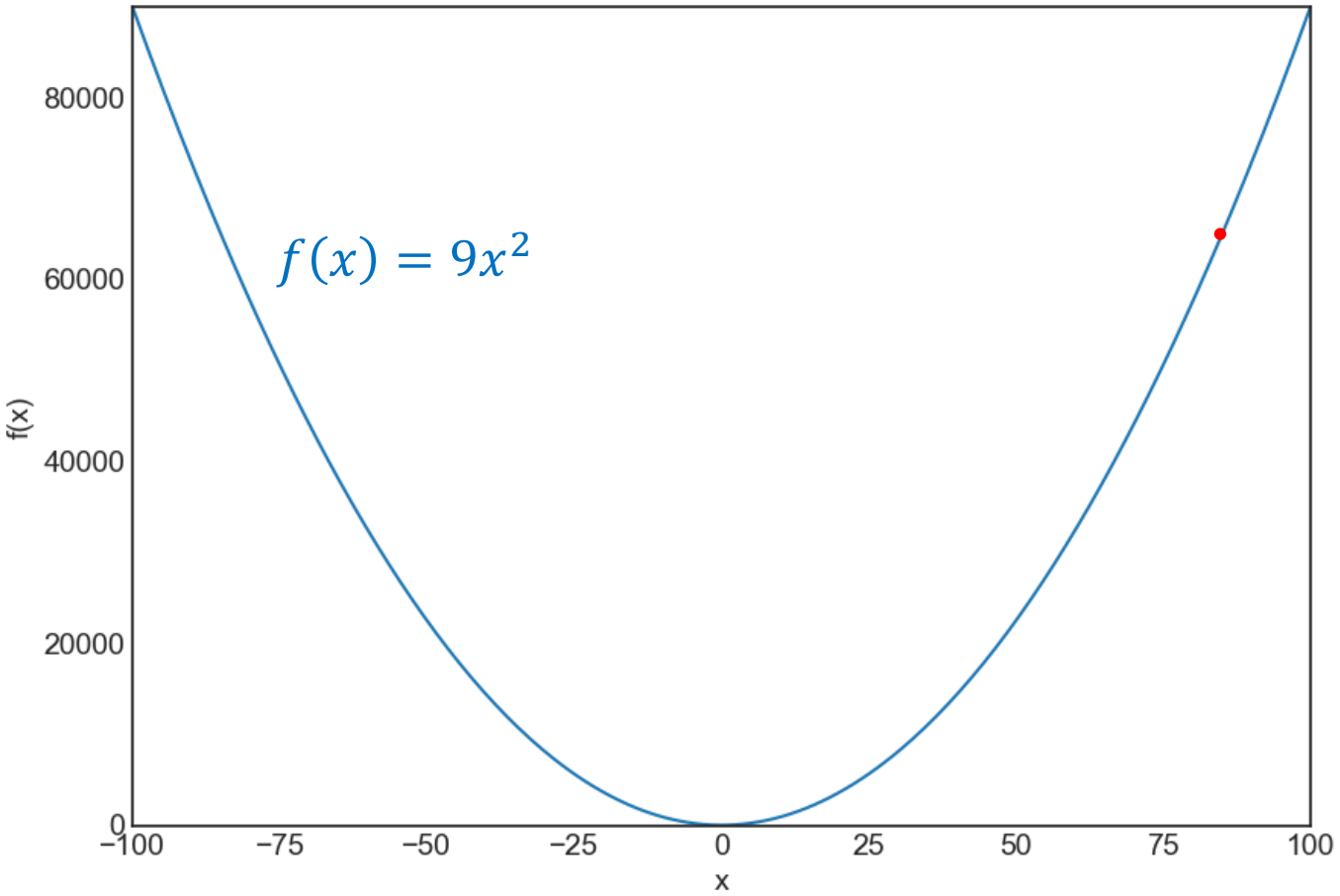
$x_3 = 3.07 \qquad y_3 = 4.608$

$\frac{\partial f(x_3,y_3)}{\partial x} = 6.14 \qquad \frac{\partial f(x_3,y_3)}{\partial y} = 9.21$

$x_4 = 2.45 \qquad y_4 = 3.68$

Gradient-based Optimization

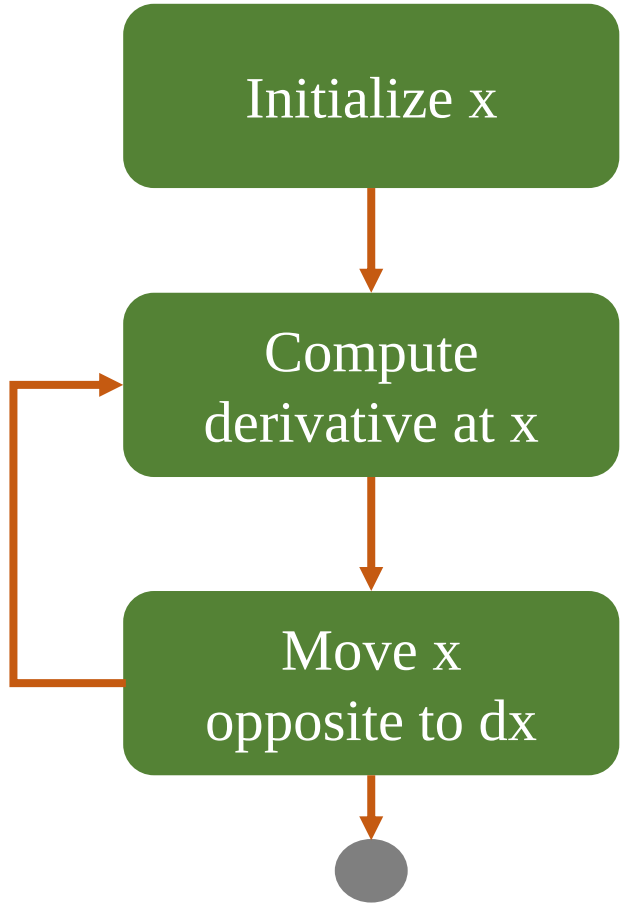
❖ Another Square function



$$-100 \leq x \leq 100$$

$$x \in \mathbb{N}$$

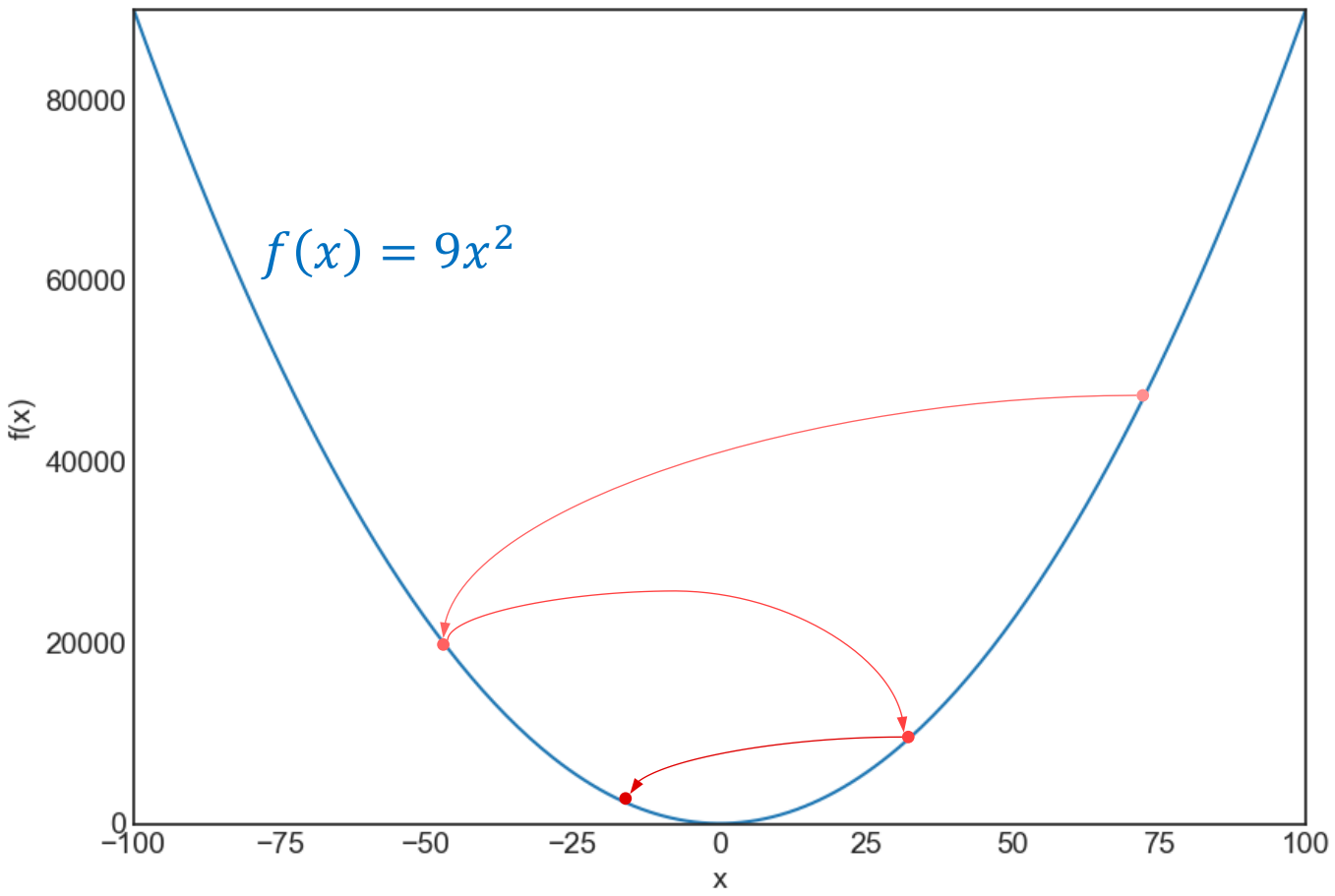
$$x_t = x_{t-1} - \eta f'(x)$$



Optimization

$$x_t = x_{t-1} - \eta f'(x_{t-1})$$

❖ Another Square function



$$-100 \leq x \leq 100$$

$$x \in \mathbb{N}$$

$x_0 = 70.0$ $\eta = 0.1$

$f'(x_0) = 1260.0$

$x_1 = x_0 - \eta f'(x_0) = -56.0$

$f'(x_1) = -1008.0$

$x_2 = x_1 - \eta f'(x_1) = 44.8$

$f'(x_2) = 806.4$

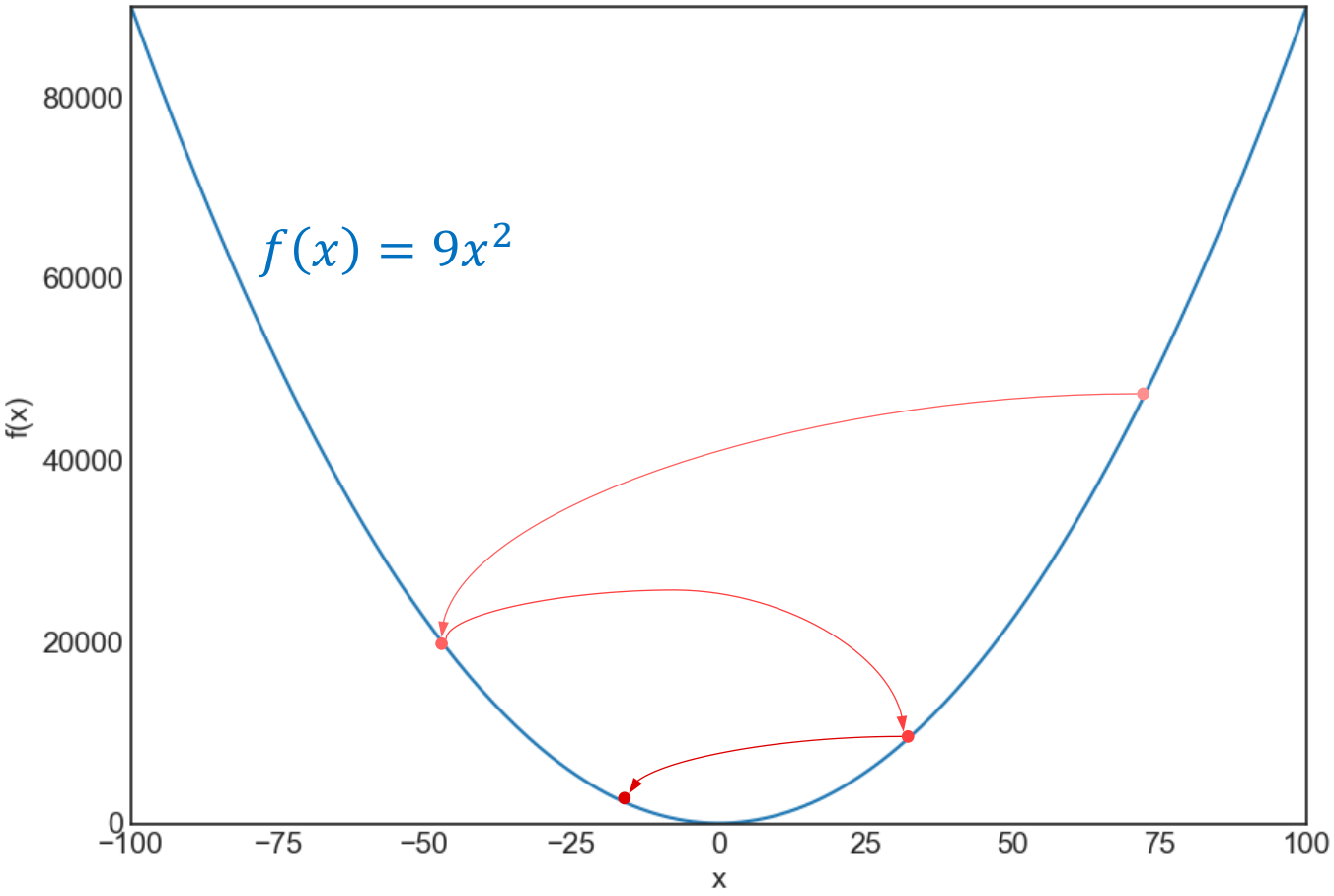
$x_3 = x_2 - \eta f'(x_2) = -35.84$

$f'(x_3) = -645.12$

$x_4 = x_3 - \eta f'(x_3) = 28.672$

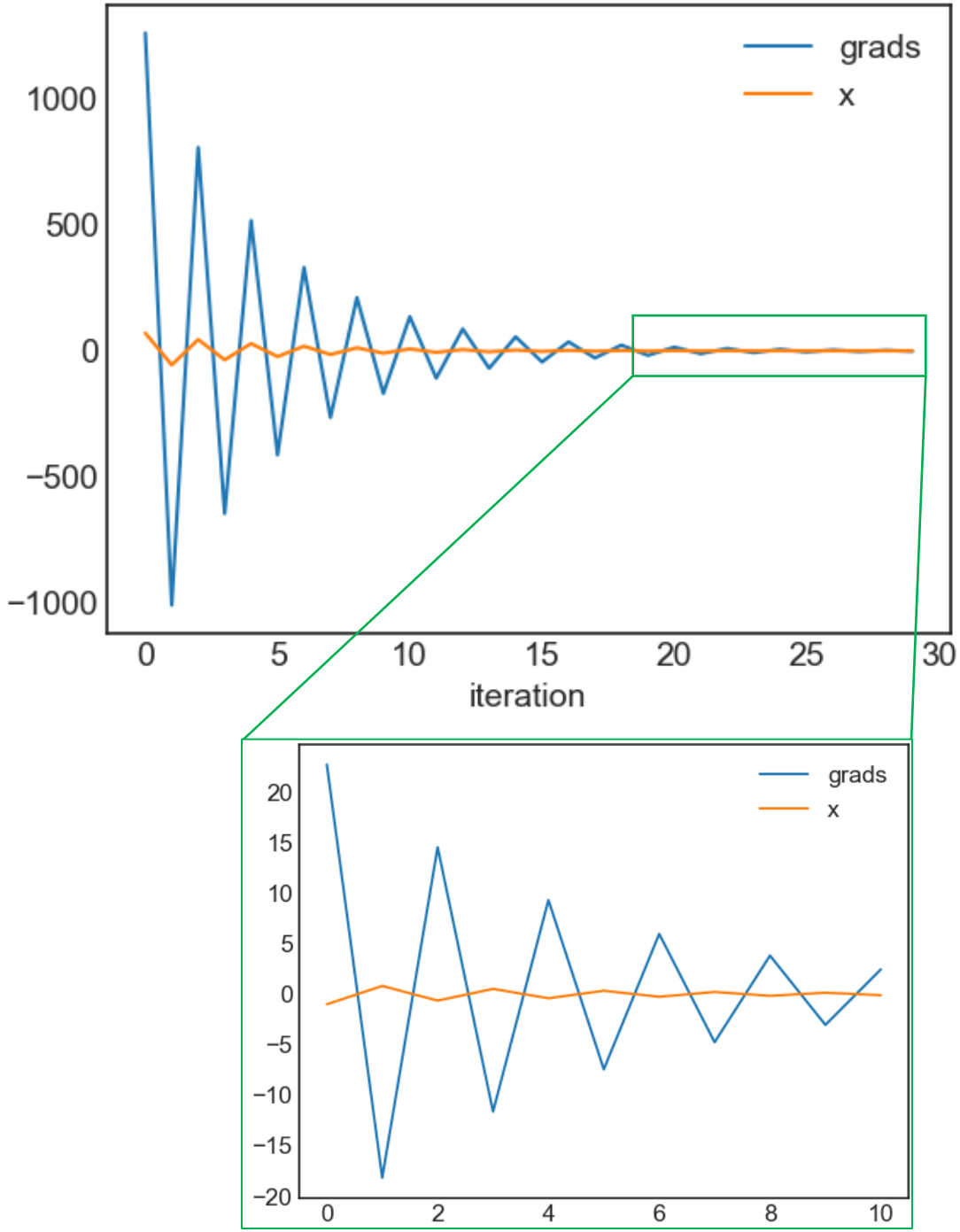
Optimization

❖ Another Square function



$$-100 \leq x \leq 100$$

$$x \in \mathbb{N}$$



Optimization

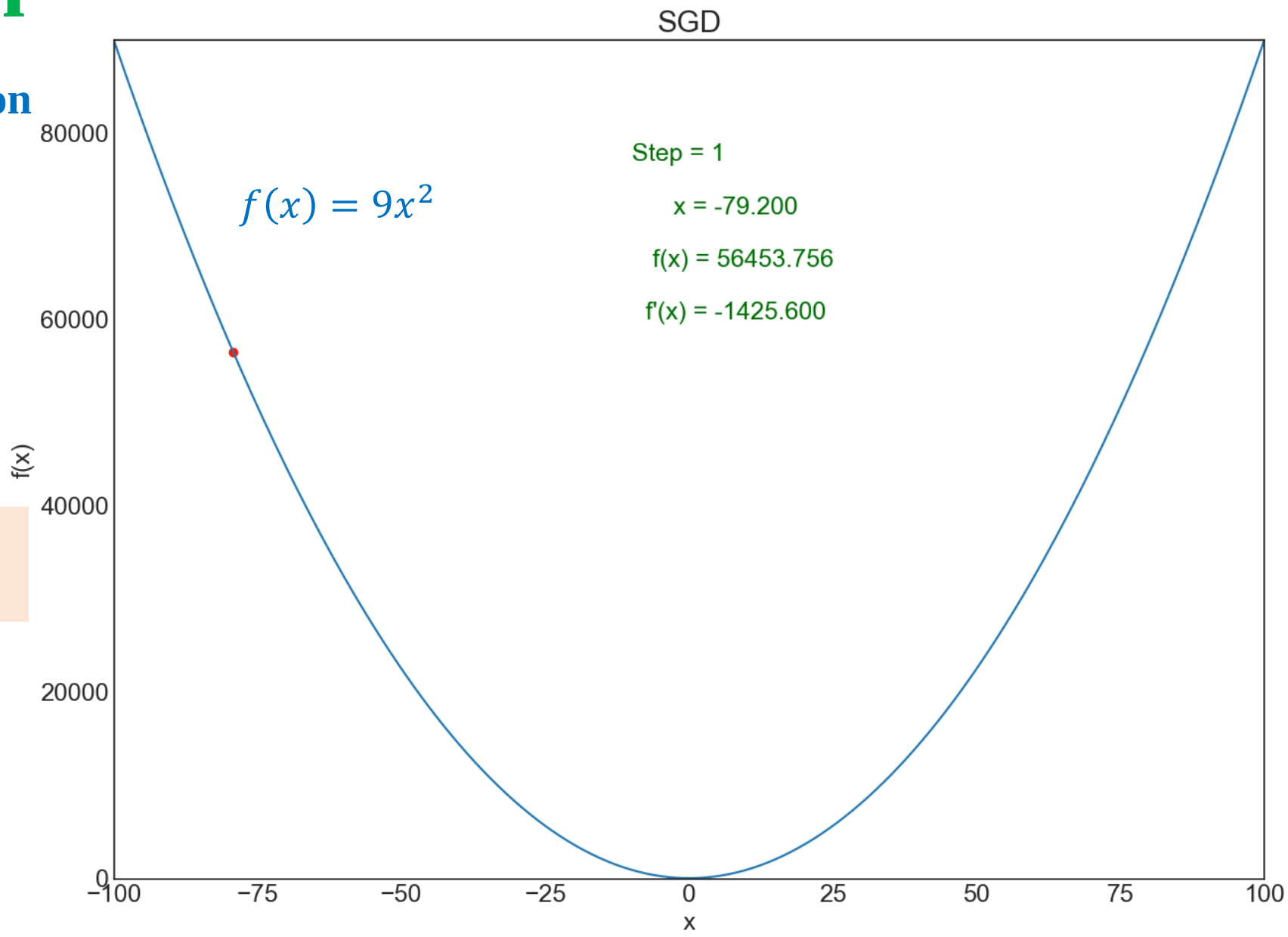
❖ **Another Square function**

$x_0 = 99.0$

$\eta = 0.1$

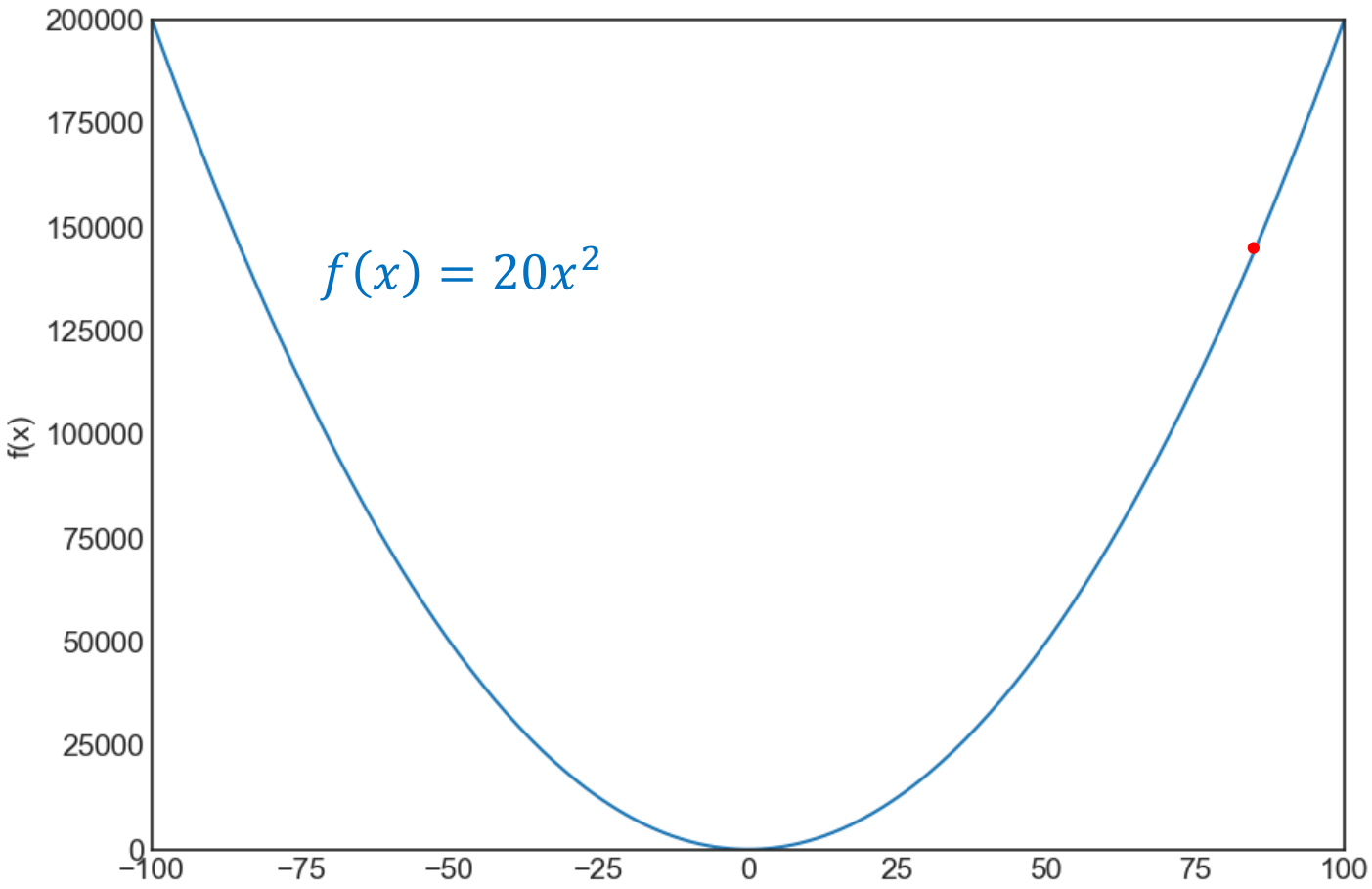
$$x_t = x_{t-1} - \eta f'(x)$$

Observation?



Gradient-based Optimization

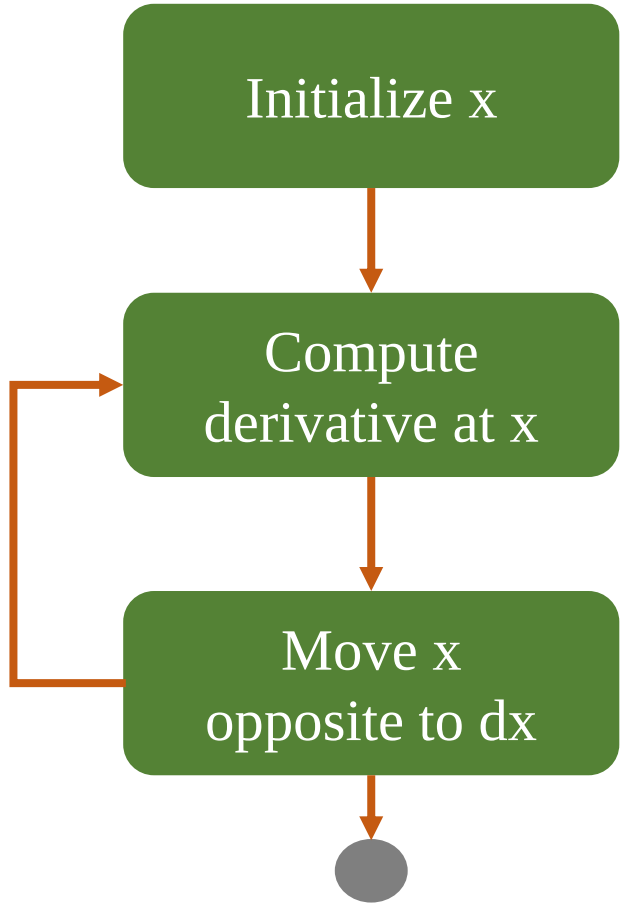
❖ Square function



$$-100 \leq x \leq 100$$

$$x \in \mathbb{N}$$

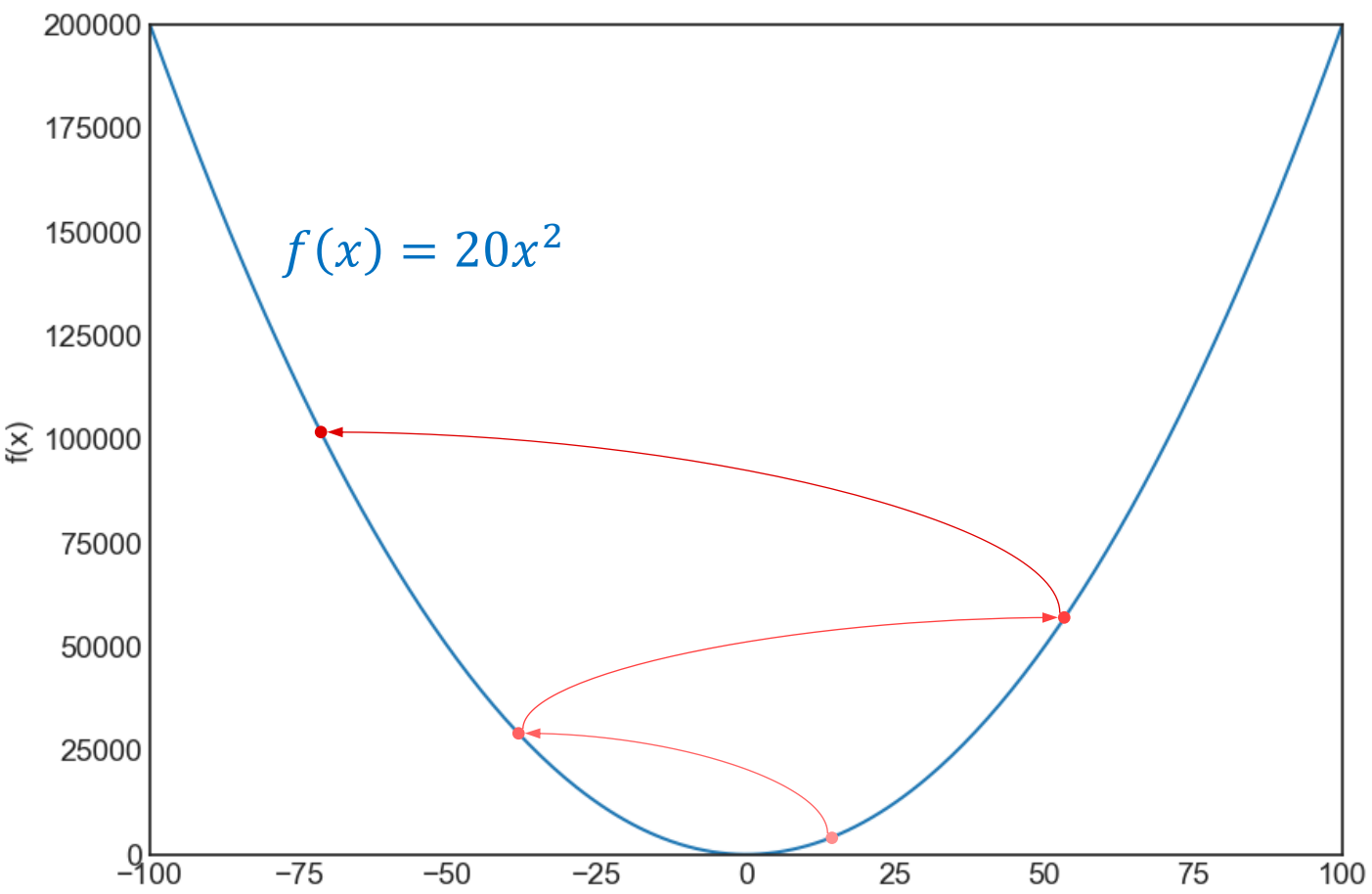
$$x_t = x_{t-1} - \eta f'(x)$$



Optimization

$$x_t = x_{t-1} - \eta f'(x_{t-1})$$

❖ Square function



$x_0 = 2.0$ $\eta = 0.1$

$f'(x_0) = 80.0$

$x_1 = x_0 - \eta f'(x_0) = -6.0$

$f'(x_1) = -240.0$

$x_2 = x_1 - \eta f'(x_1) = 18.0$

$f'(x_2) = 720.0$

$x_3 = x_2 - \eta f'(x_2) = -54.0$

$f'(x_3) = -2160.0$

$x_4 = x_3 - \eta f'(x_3) = 162.0$

$-100 \leq x \leq 100$

$x \in \mathbb{N}$

Optimization

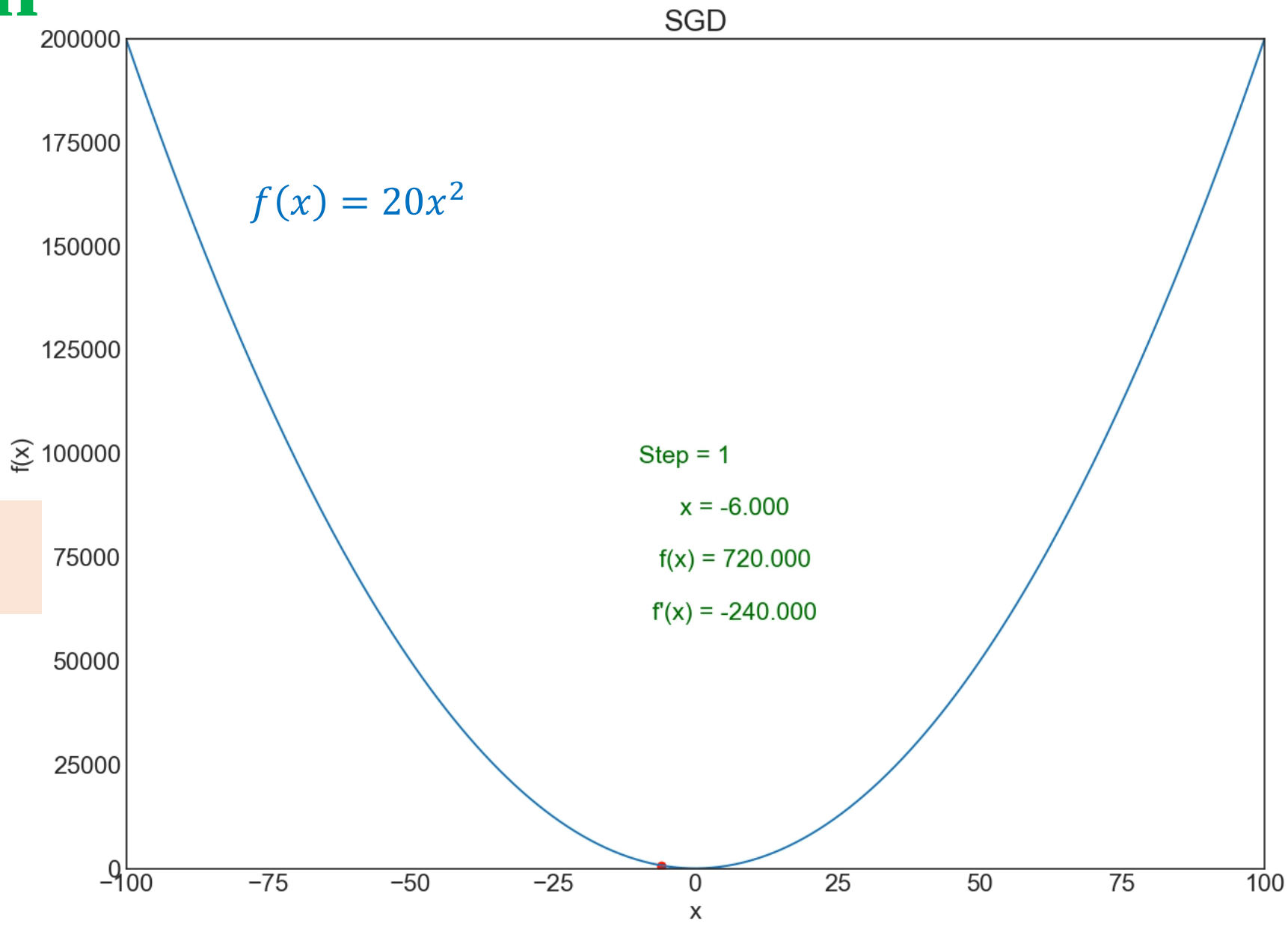
❖ **Square function**

$x_0 = 2.0$

$\eta = 0.1$

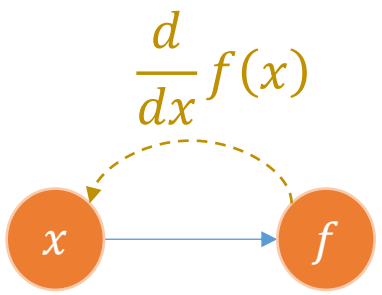
$x_t = x_{t-1} - \eta f'(x)$

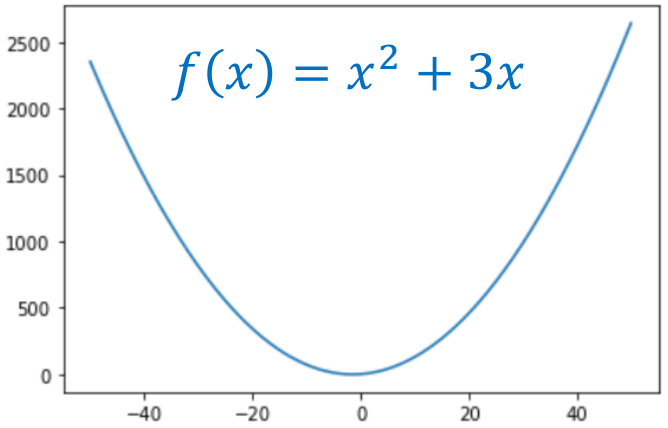
Observation?

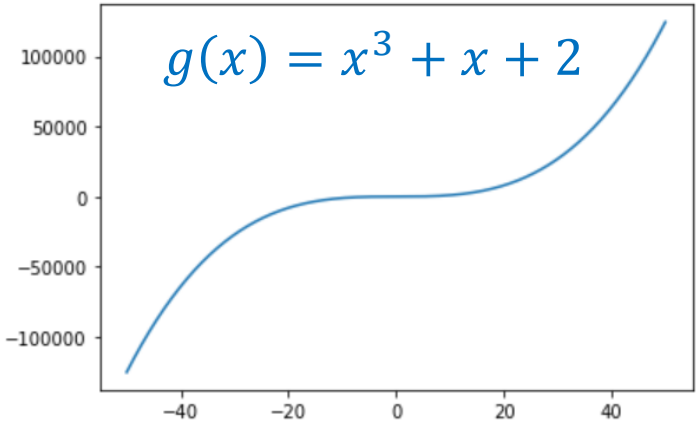


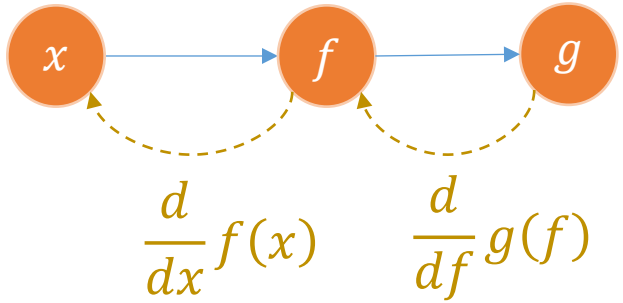
Gradient-based Optimization

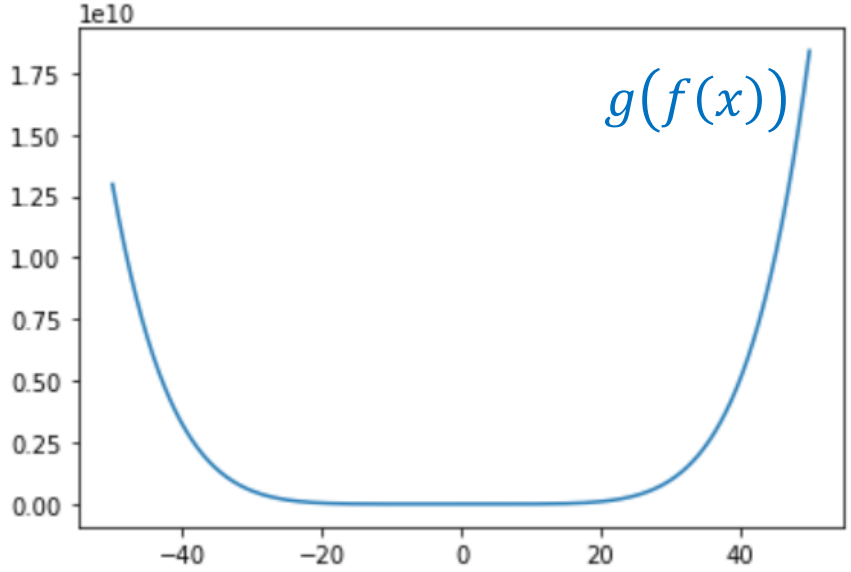
❖ For composite function







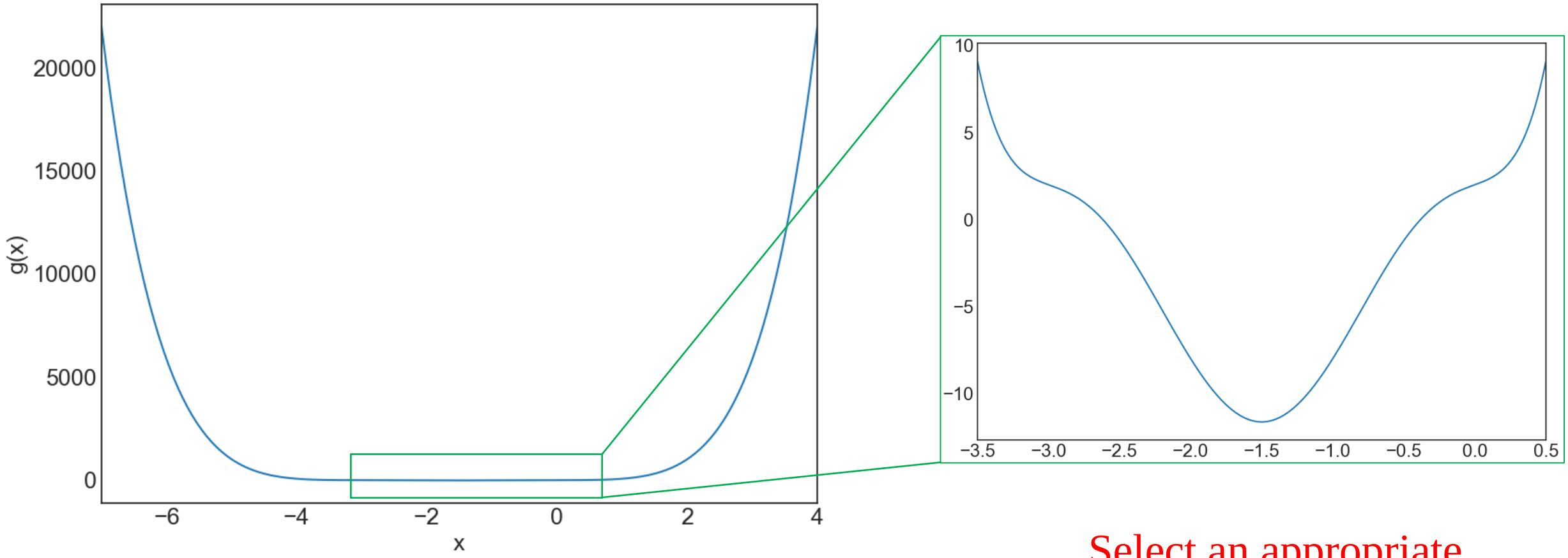




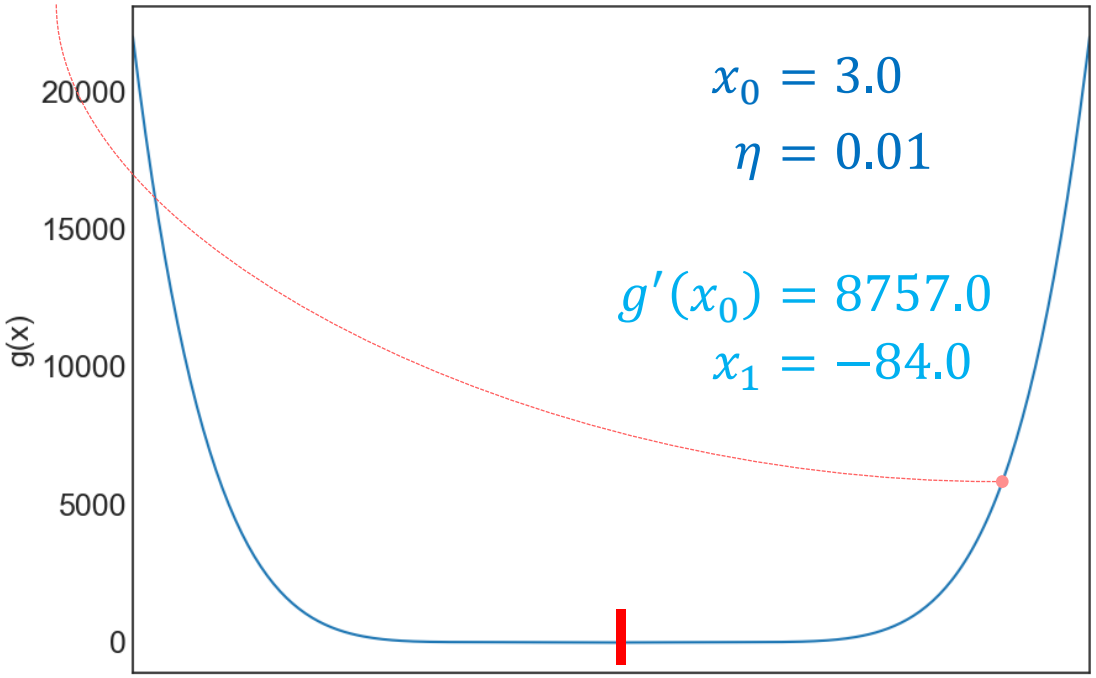
$$\frac{d}{dx}g(f(x)) = \left[\frac{d}{df}g(f)\right] * \left[\frac{d}{dx}f(x)\right]$$

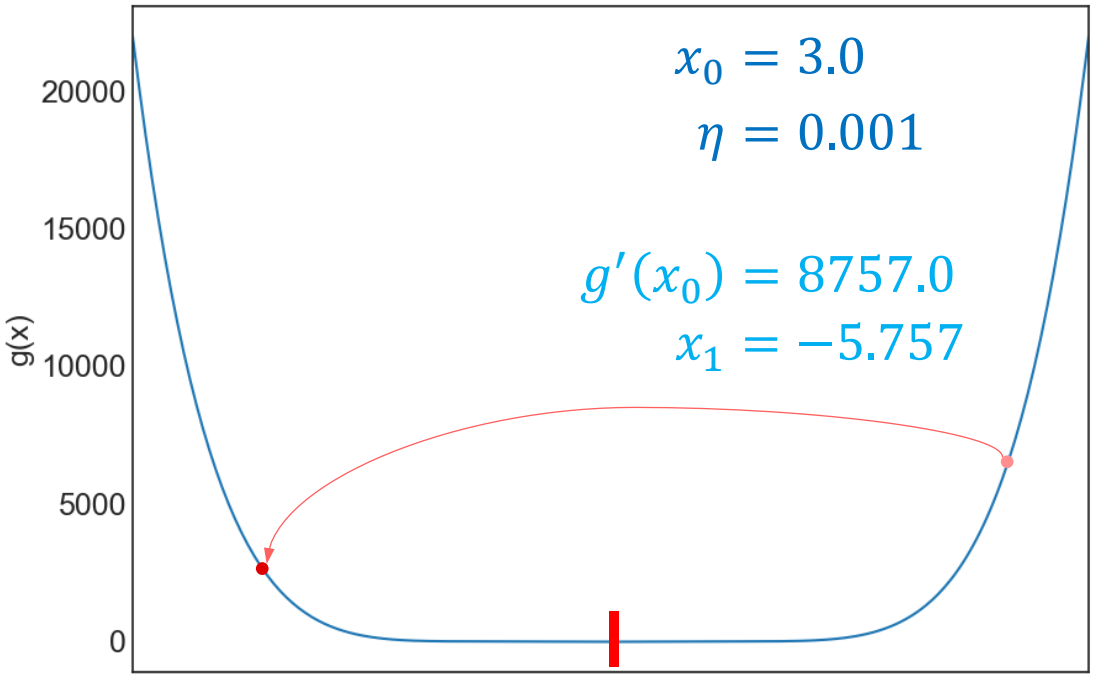
Gradient-based Optimization

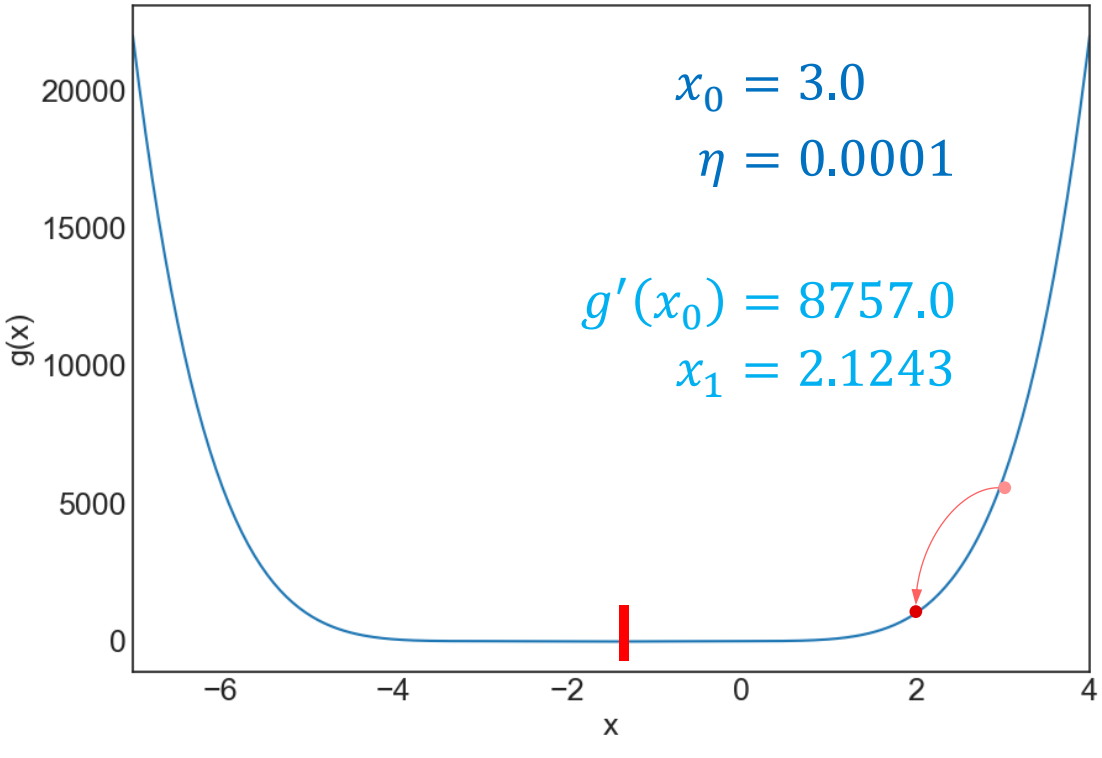
❖ For composite function

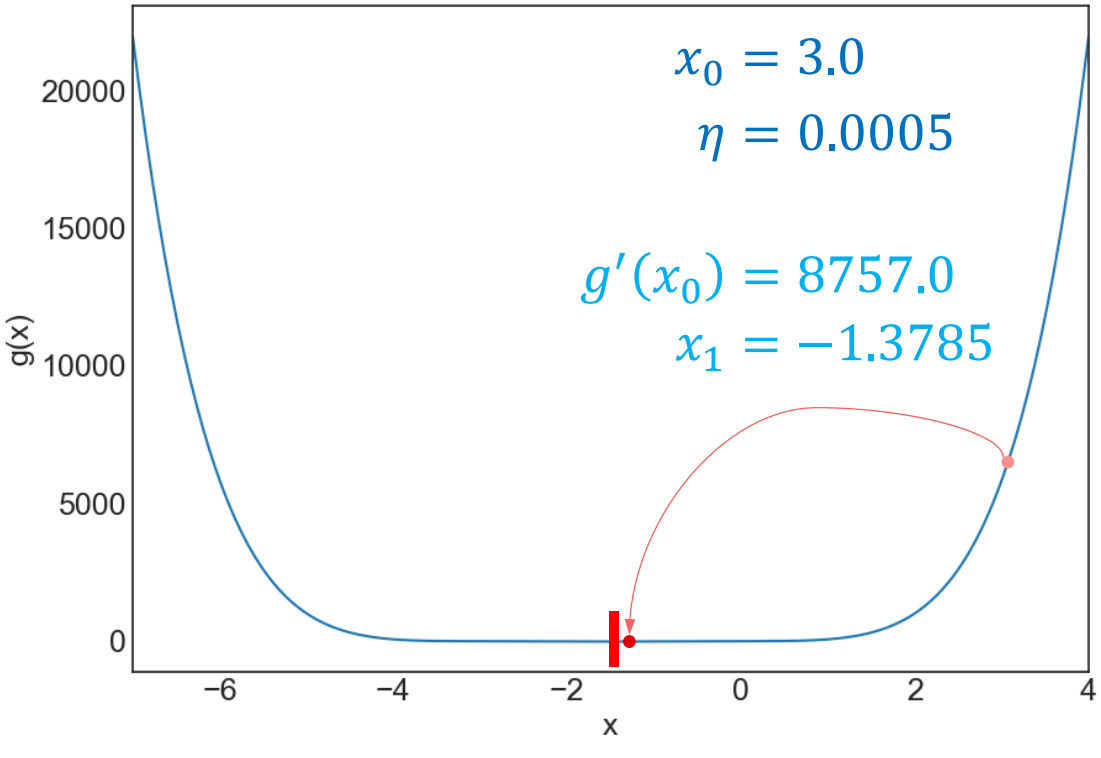


Select an appropriate value for learning rate


$x_0 = 3.0$
$\eta = 0.01$
$g'(x_0) = 8757.0$
$x_1 = -84.0$
g(x)
20000
15000
10000
5000
0


$x_0 = 3.0$
$\eta = 0.001$
$g'(x_0) = 8757.0$
$x_1 = -5.757$
g(x)
20000
15000
10000
5000
0


$x_0 = 3.0$
$\eta = 0.0001$
$g'(x_0) = 8757.0$
$x_1 = 2.1243$
g(x)
20000
15000
10000
5000
0
−6
−4
−2
0
2
4
x


$x_0 = 3.0$
$\eta = 0.0005$
$g'(x_0) = 8757.0$
$x_1 = -1.3785$
g(x)
20000
15000
10000
5000
0
−6
−4
−2
0
2
4
x

Optimization

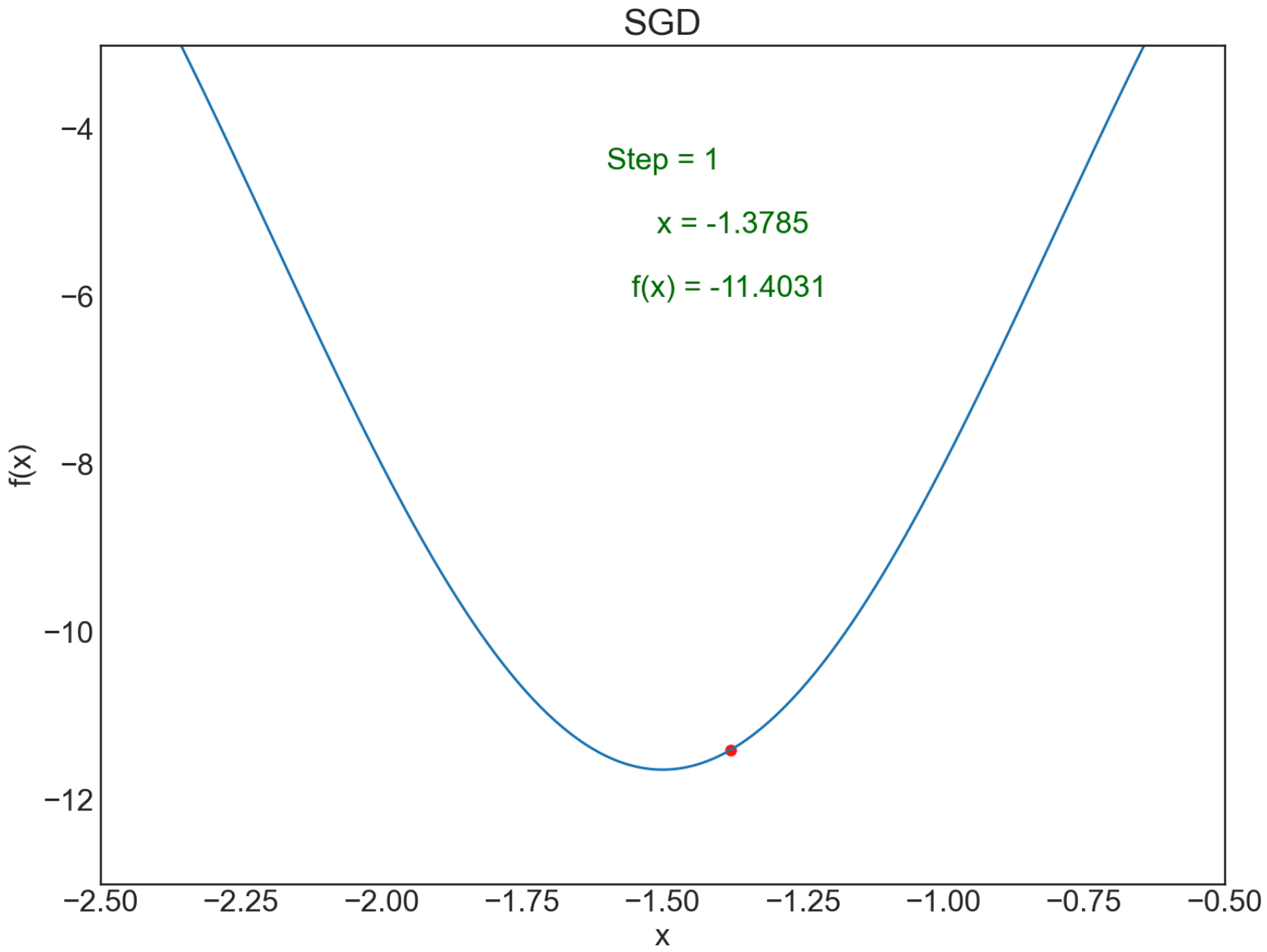
❖ **Composite function**

$x_0 = 3.0$

$\eta = 0.0005$

$$x_t = x_{t-1} - \eta f'(x)$$

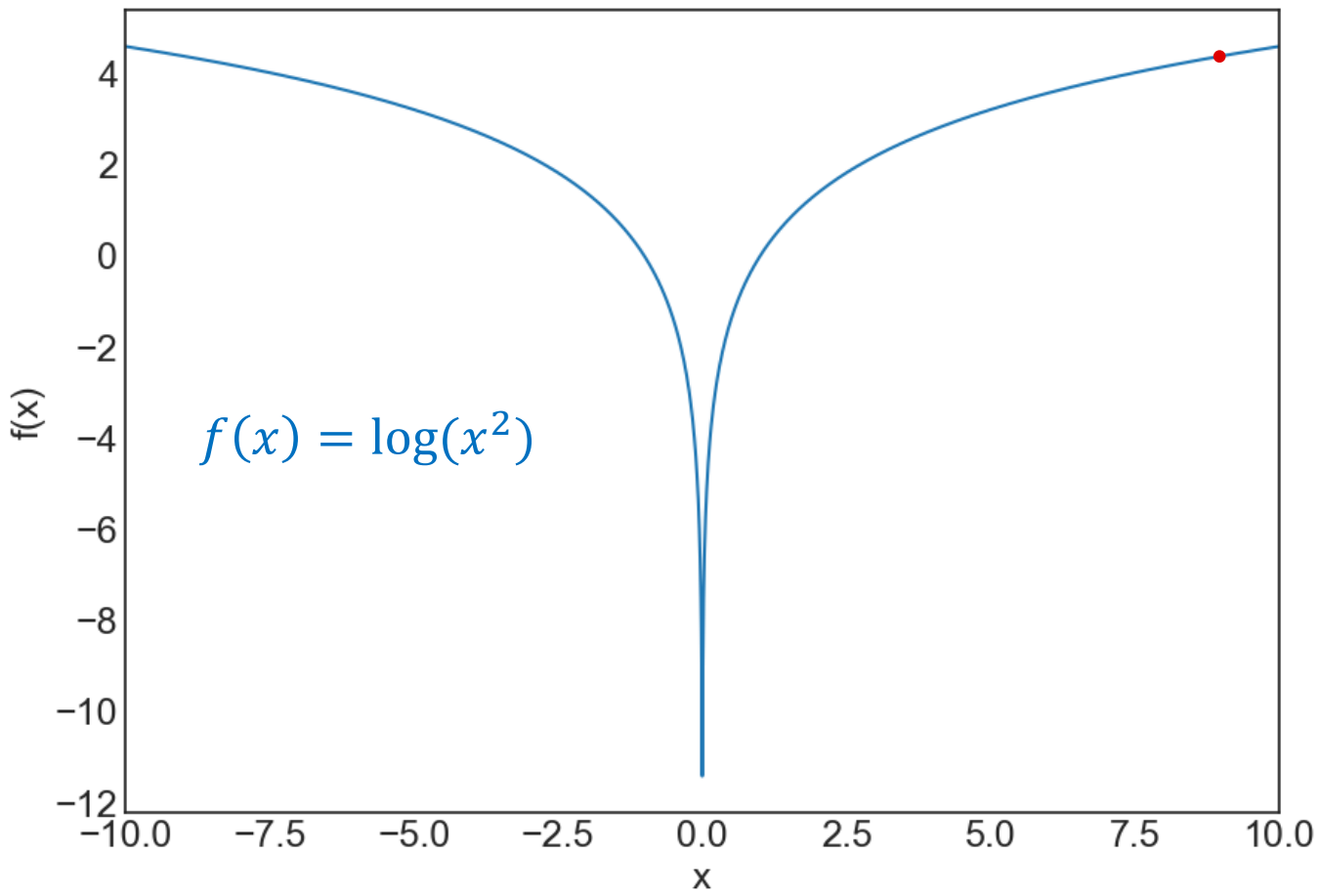
Observation?



Gradient-based Optimization

$$x_t = x_{t-1} - \eta f'(x_{t-1})$$

❖ **Another function**



$x_0 = 9.0$ $\eta = 10.0$

$f'(x_0) = 0.22$

$x_1 = x_0 - \eta f'(x_0) = 6.77$

$f'(x_1) = 0.295$

$x_2 = x_1 - \eta f'(x_1) = 3.82$

$f'(x_2) = 0.52$

$x_3 = x_2 - \eta f'(x_2) = -1.39$

$f'(x_3) = -1.429$

$x_4 = x_3 - \eta f'(x_3) = 12.89$

Optimization

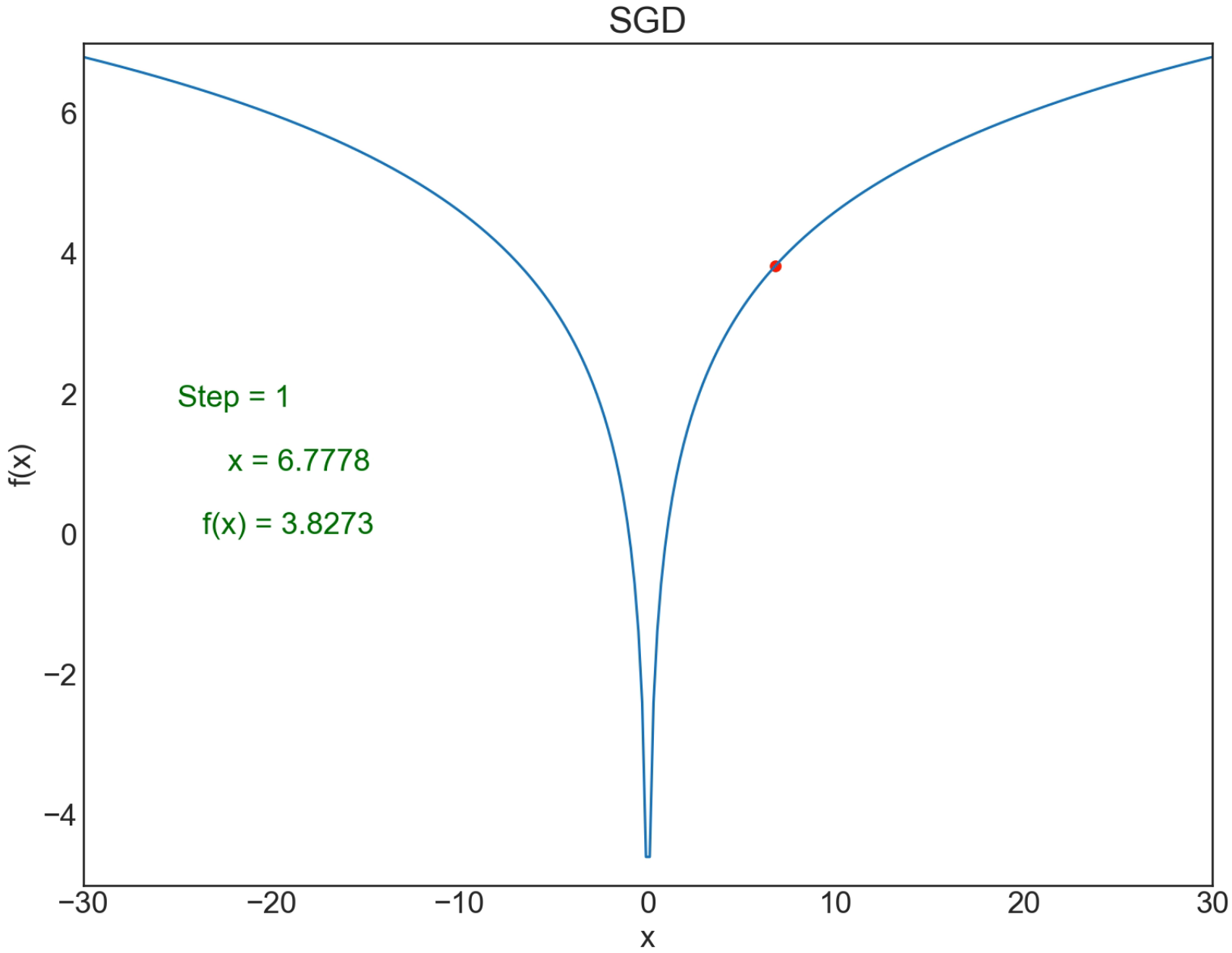
❖ **Another function**

$x_0 = 9.0$

$\eta = 10.0$

$$x_t = x_{t-1} - \eta f'(x)$$

Observation?



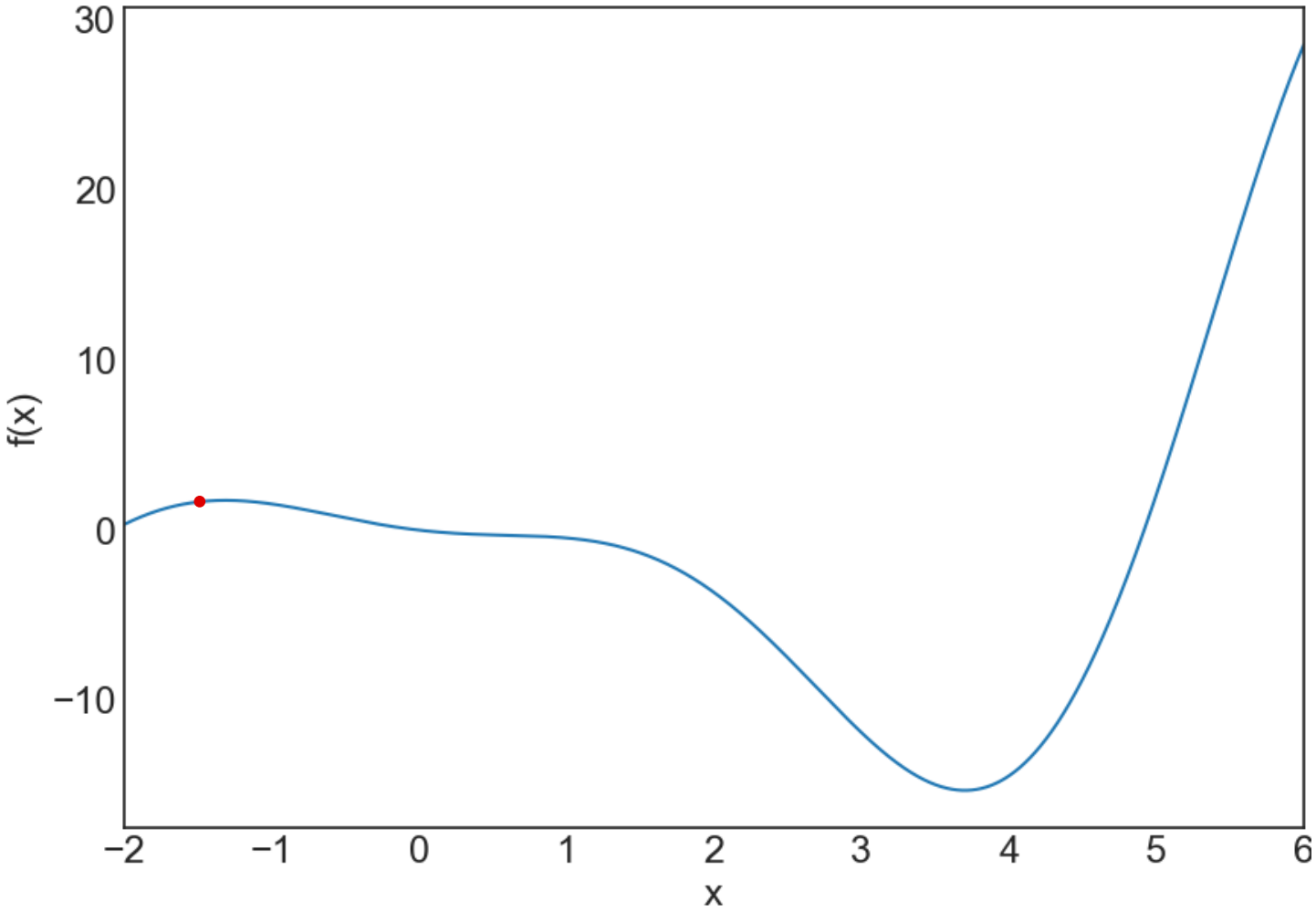
Gradient-based Optimization

❖ Another function

$$x_0 = -1.5$$

$$\eta = 0.2$$

$$x_t = x_{t-1} - \eta f'(x)$$



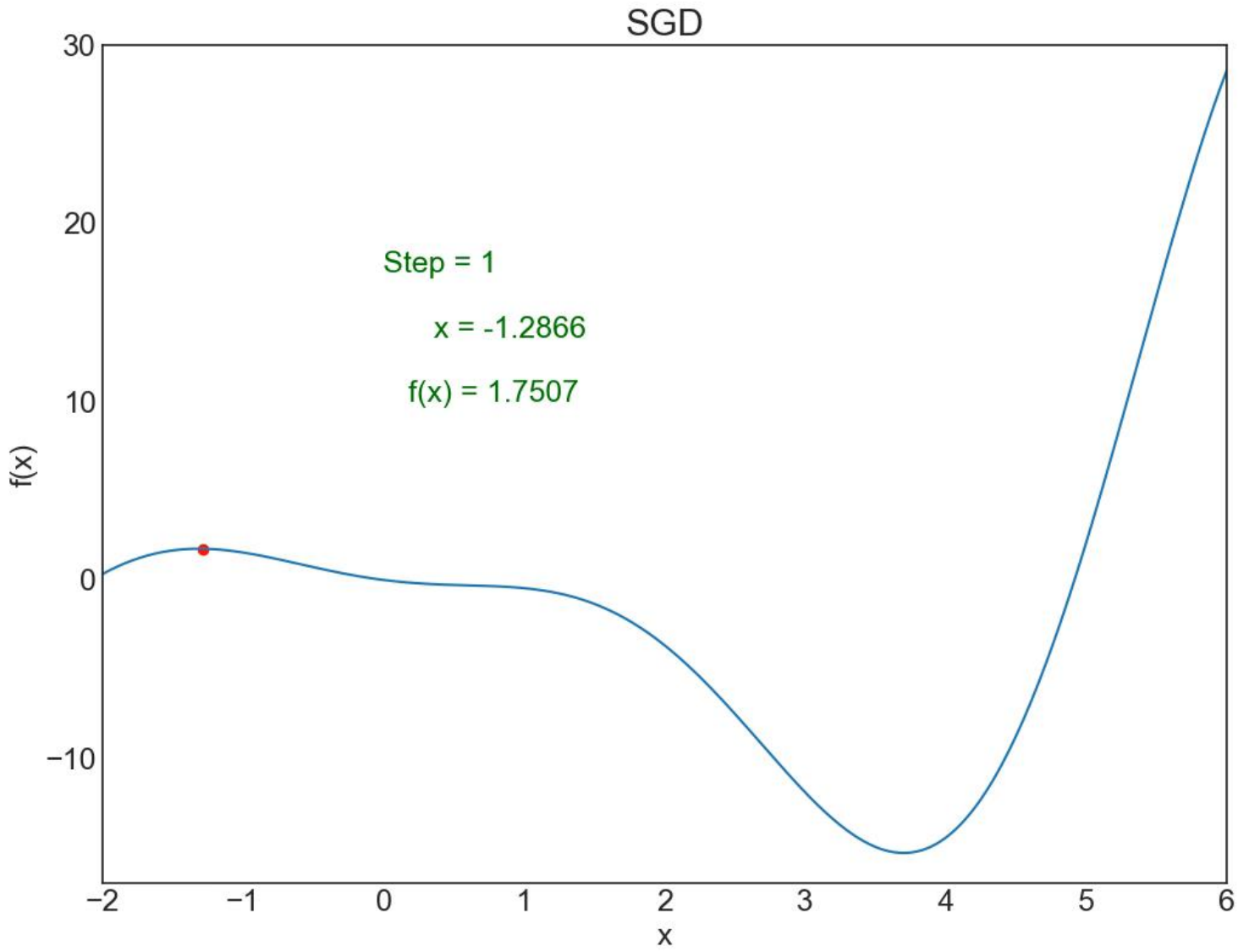
Gradient-based Optimization

❖ Another function

$$x_0 = -1.5$$

$$\eta = 0.2$$

$$x_t = x_{t-1} - \eta f'(x)$$



Outline

SECTION 1

SGD Insight

SECTION 2

Adaptive Learning Rate

SECTION 3

Momentum and Towards Adam

$$g_t = \nabla_\theta L$$

$$s_t = s_{t-1} + g_t^2$$

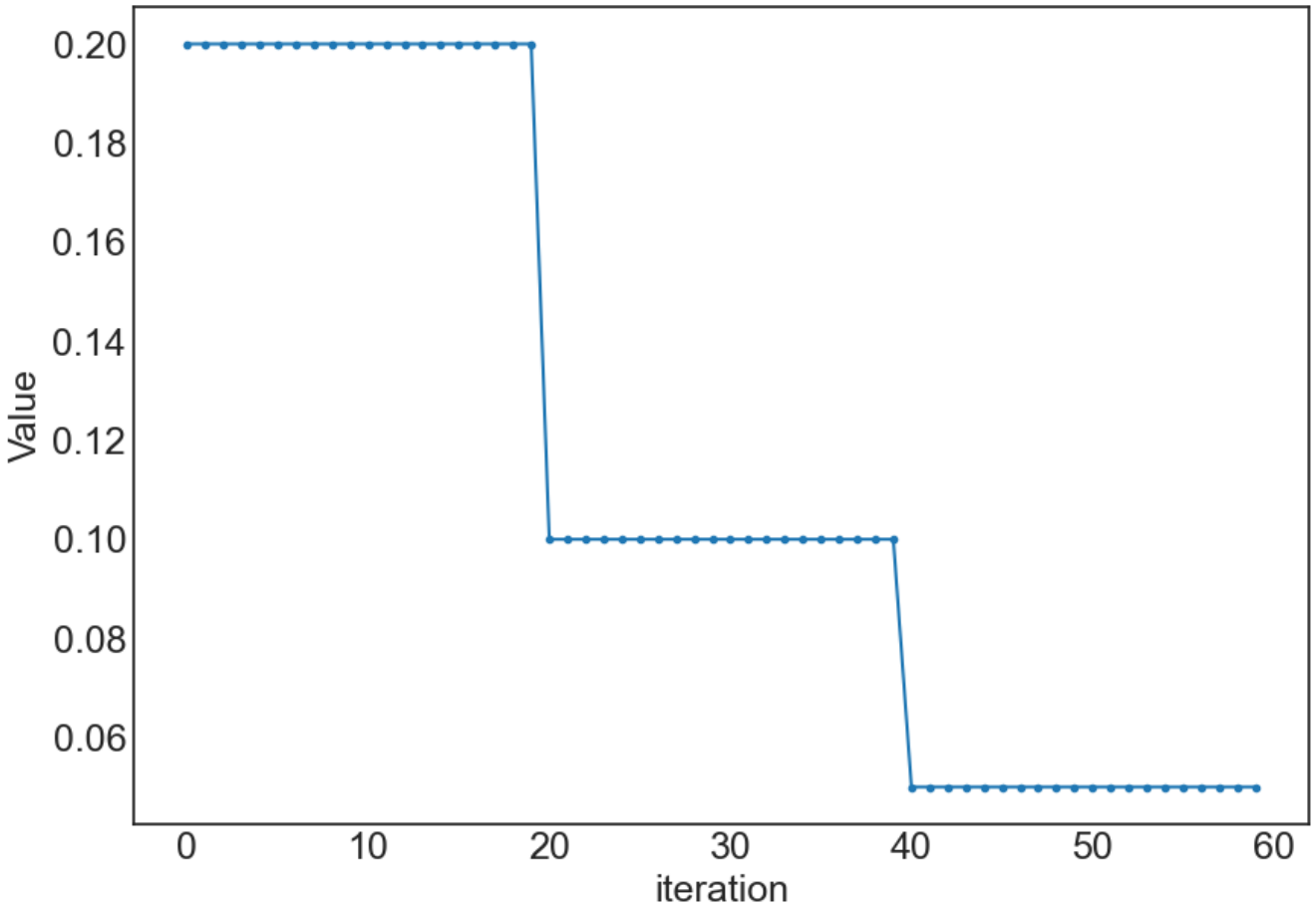
$$\theta_t = \theta_{t-1} - \frac{\eta}{\sqrt{s_t + \epsilon}} g_t$$

current summation

expectation

Adaptive Learning Rate

❖ Learning rate decay



Iteration i

Period p

Reduction rate k

$\eta = 0.2$

$if\ i\%p == 0$:

$\eta = \eta/k$

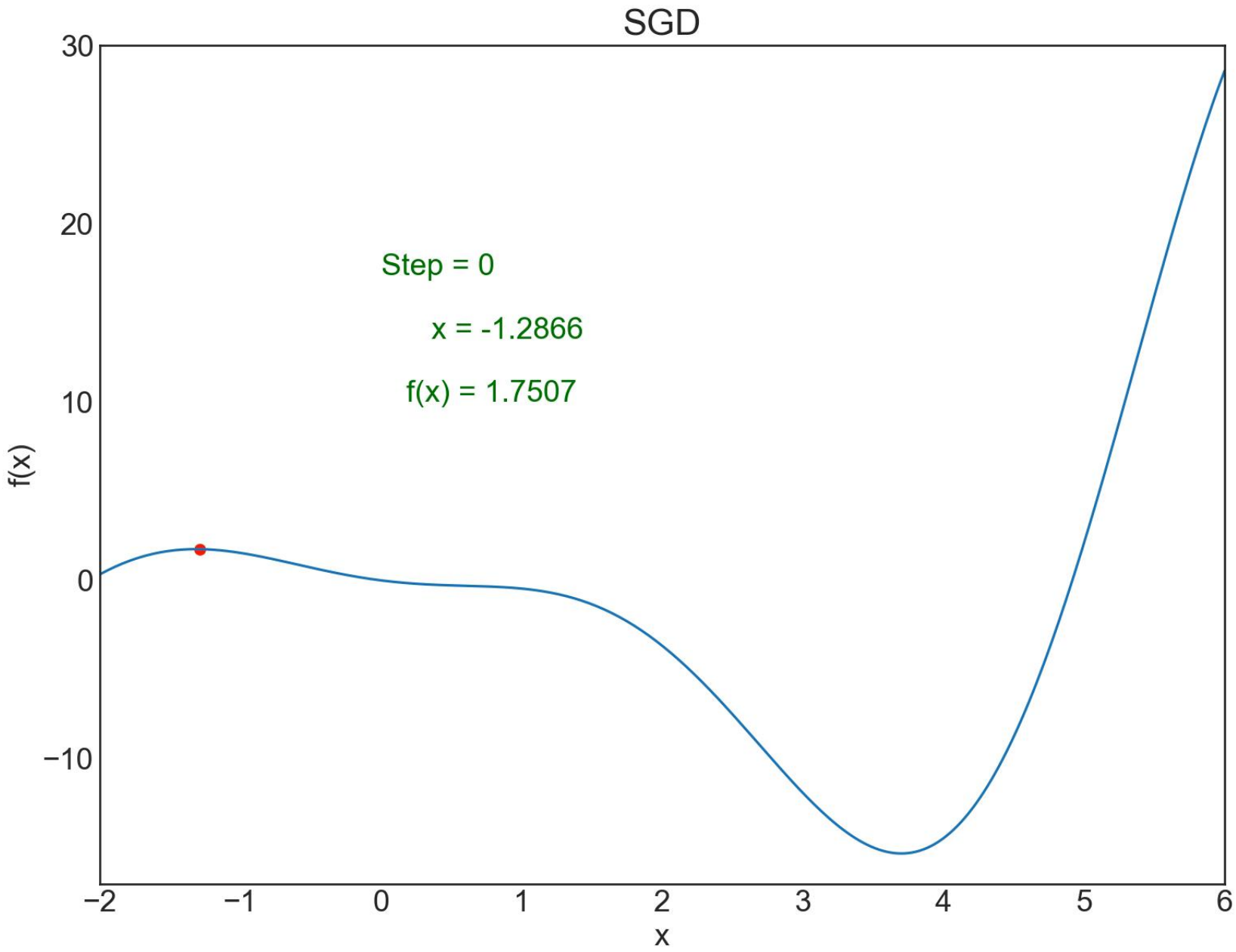
Adaptive Learning Rate

Learning rate decay

$$x_0 = -1.5$$

$$\eta = 0.2$$

$$x_t = x_{t-1} - \eta f'(x)$$



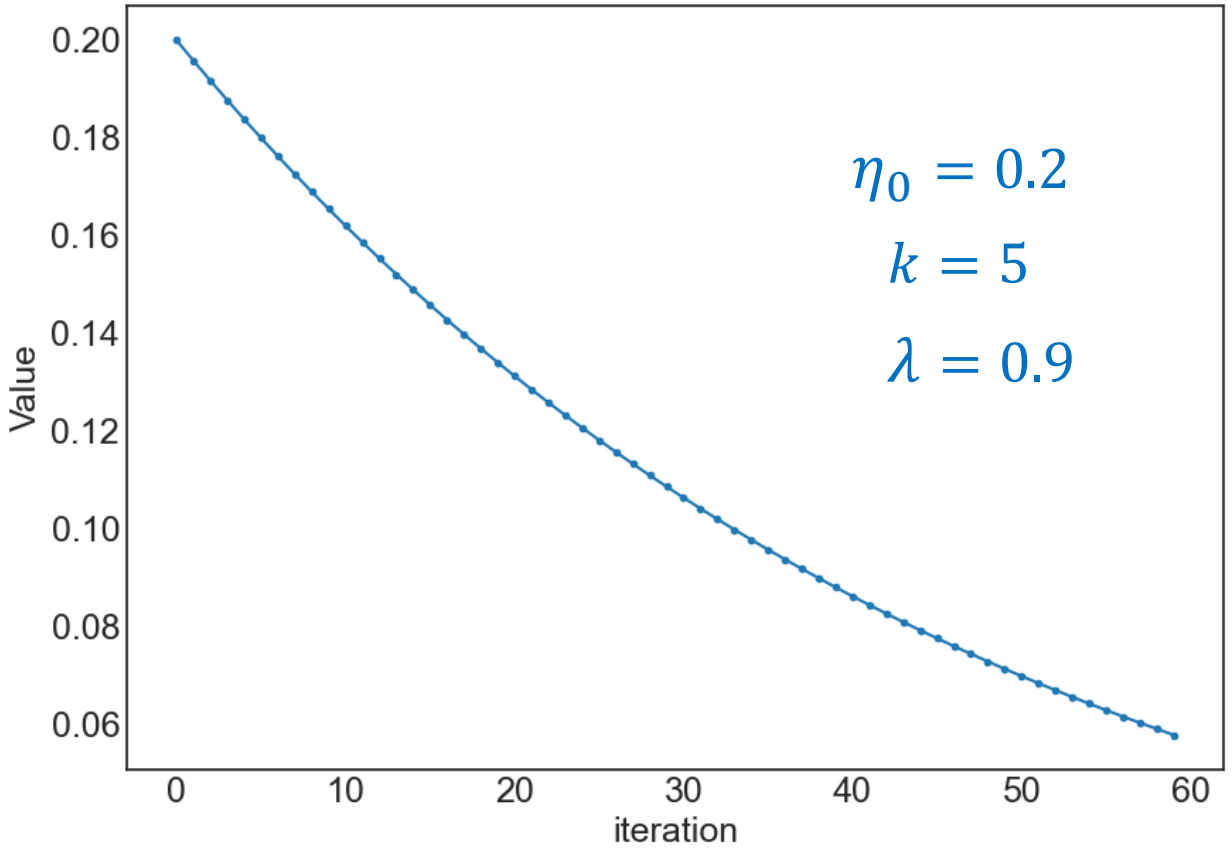
Adaptive Learning Rate

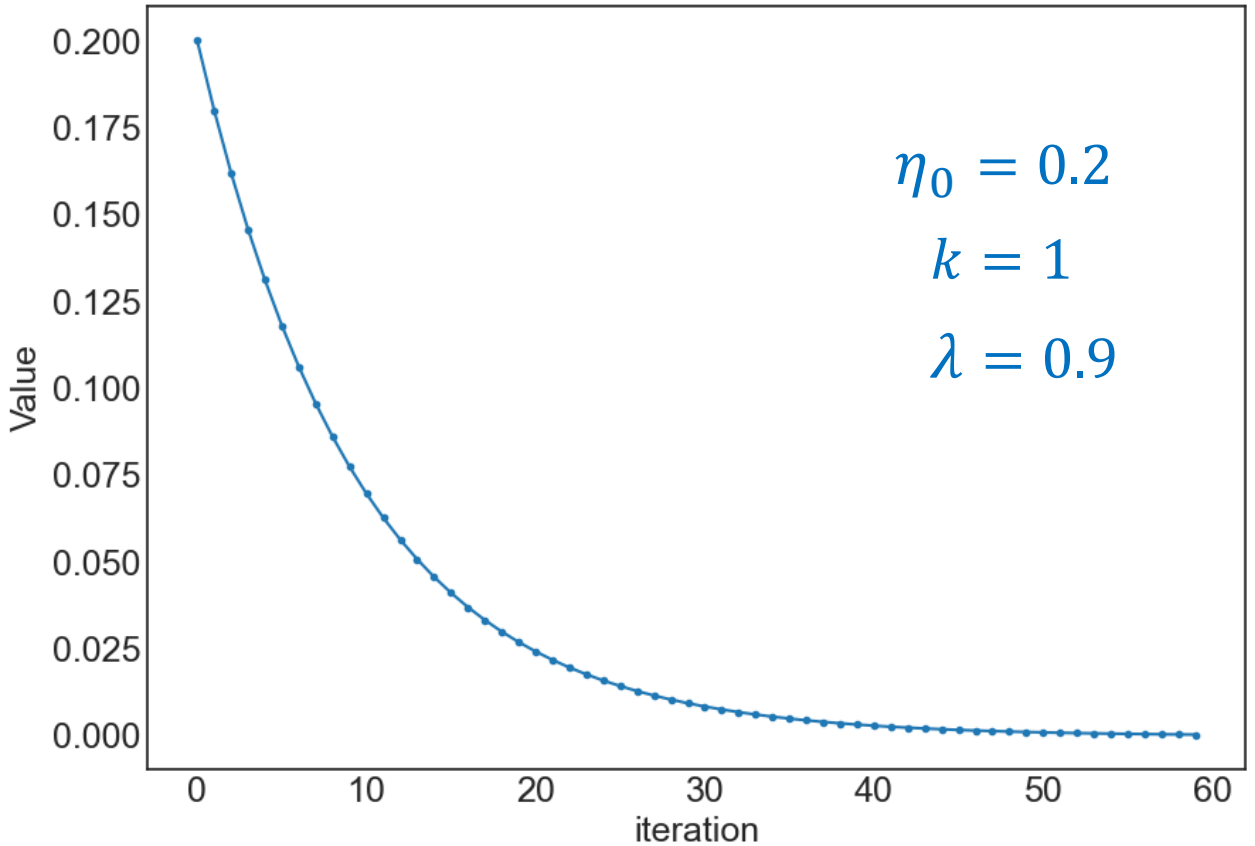
Learning rate decay

$$\eta = \eta_0 \times \lambda^{\frac{s}{k}}$$

torch.optim.lr_scheduler

initial_learning_rate * decay_rate ^ (step / decay_steps)





Adaptive Learning Rate

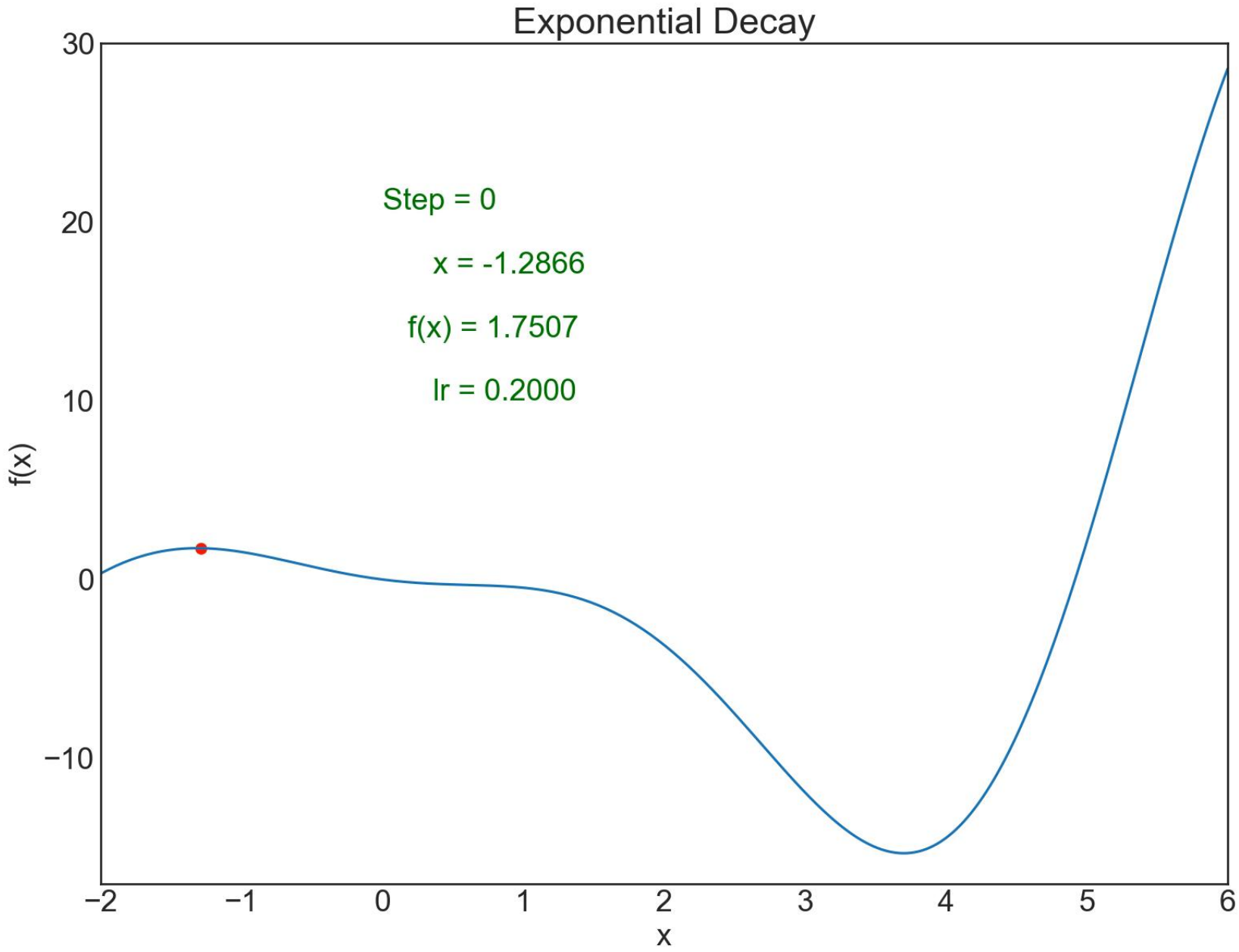
Learning rate decay

$$x_0 = -1.5$$

$$\eta = 0.2$$

$$x_t = x_{t-1} - \eta f'(x)$$

$$\eta = \eta_0 \times \lambda^{\frac{s}{k}}$$



Adaptive Learning Rate

❖ Using derivative values

Adagrad (one variable functions)

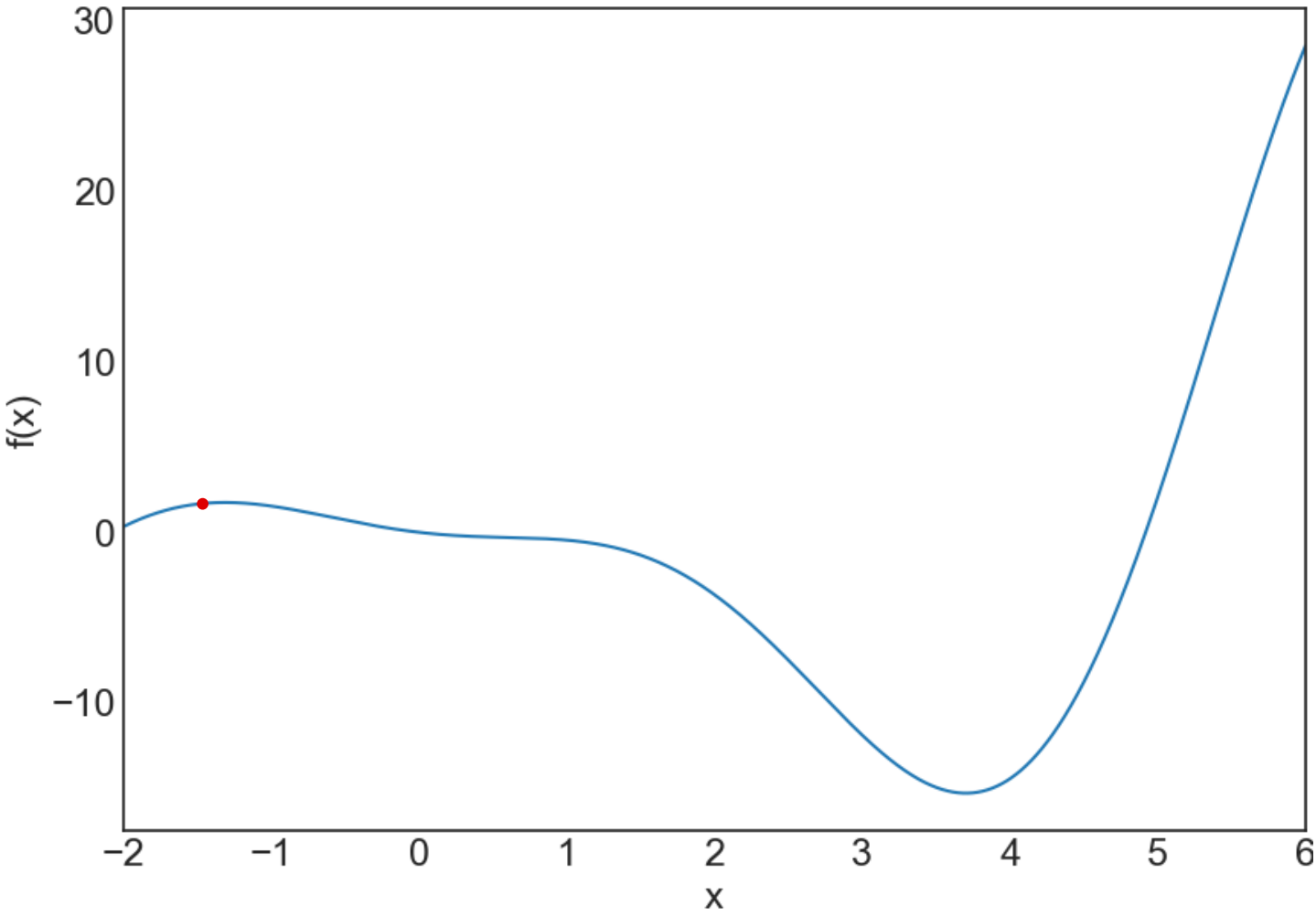
$s_0 = 0.1$ (or $s_0 = 0.0$)

$\epsilon = 10^{-7}$

$$g_t = f'(x_{t-1})$$

$$s_t = s_{t-1} + g_t^2$$

$$x_t = x_{t-1} - \frac{\eta}{\sqrt{s_t + \epsilon}} g_t$$



Adaptive Learning Rate

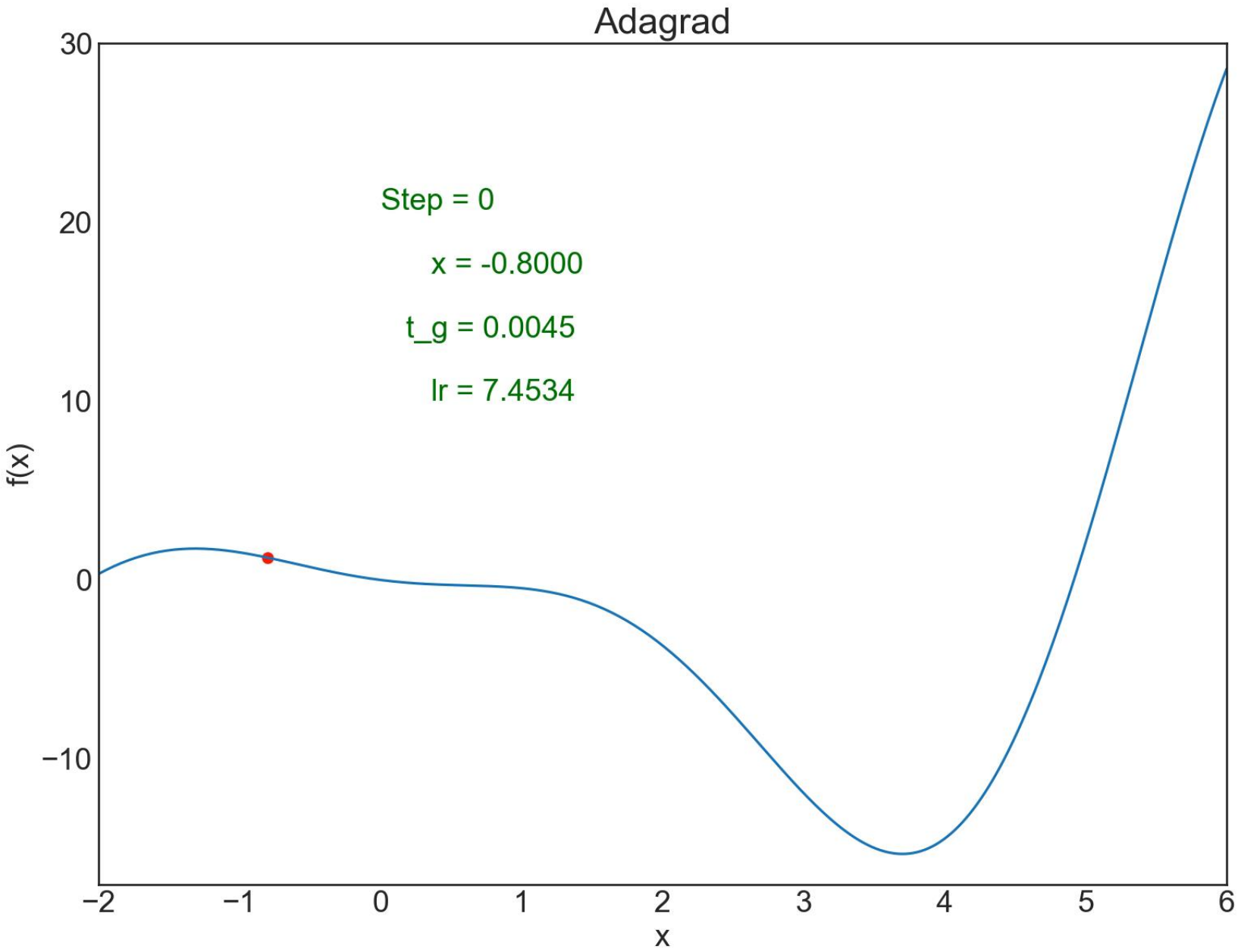
Adagrad

(one variable functions)

$\eta = 0.5$

$s_0 = 0.0$

$\epsilon = 10^{-7}$



Adaptive Learning Rate

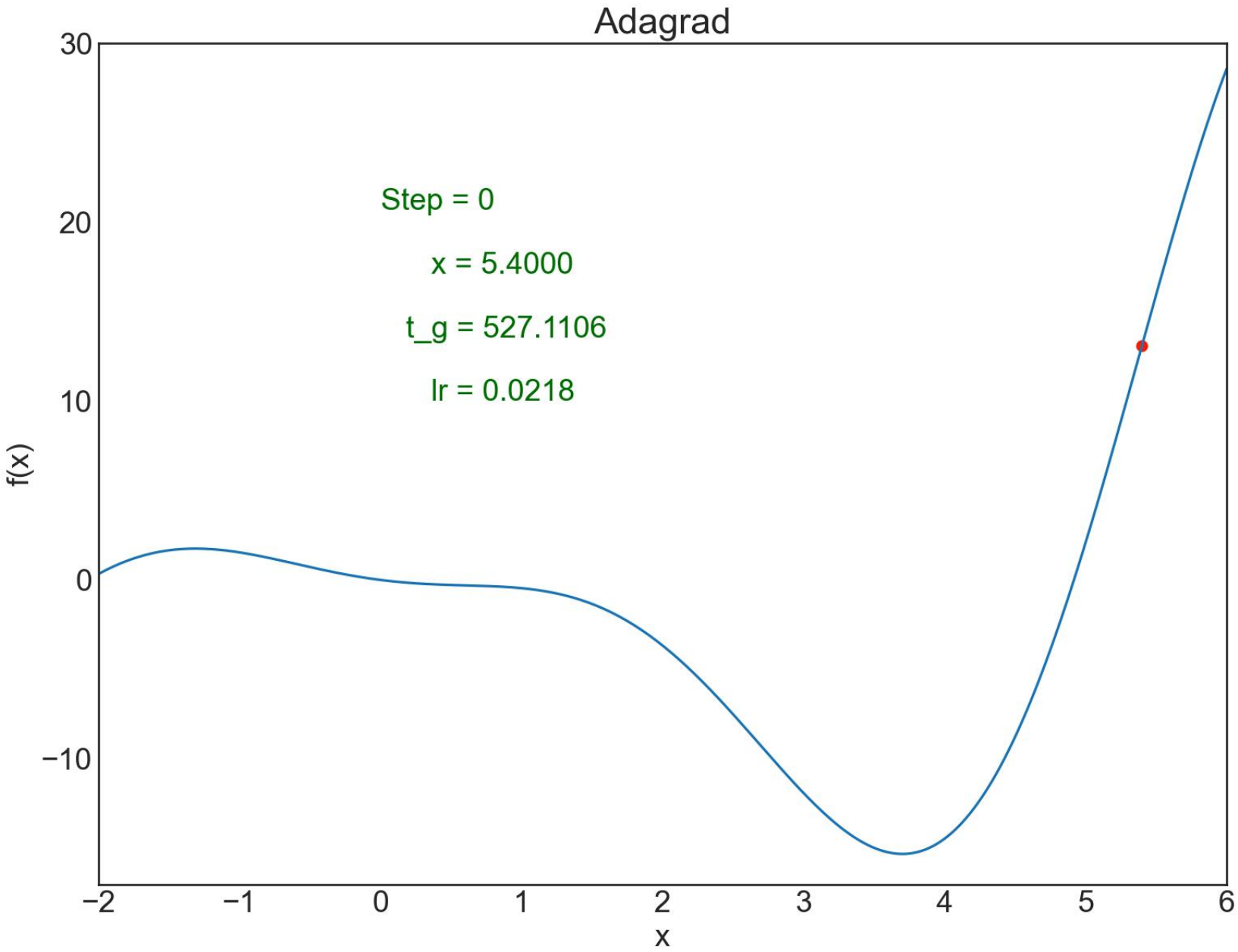
Adagrad

(one variable functions)

$\eta = 0.5$

$s_0 = 0.0$

$\epsilon = 10^{-7}$



Adaptive Learning Rate

Using derivative values

Adagrad (one variable functions)

$s_0 = 0.1$

$\epsilon = 10^{-7}$

$$g_t = f'(x_{t-1})$$

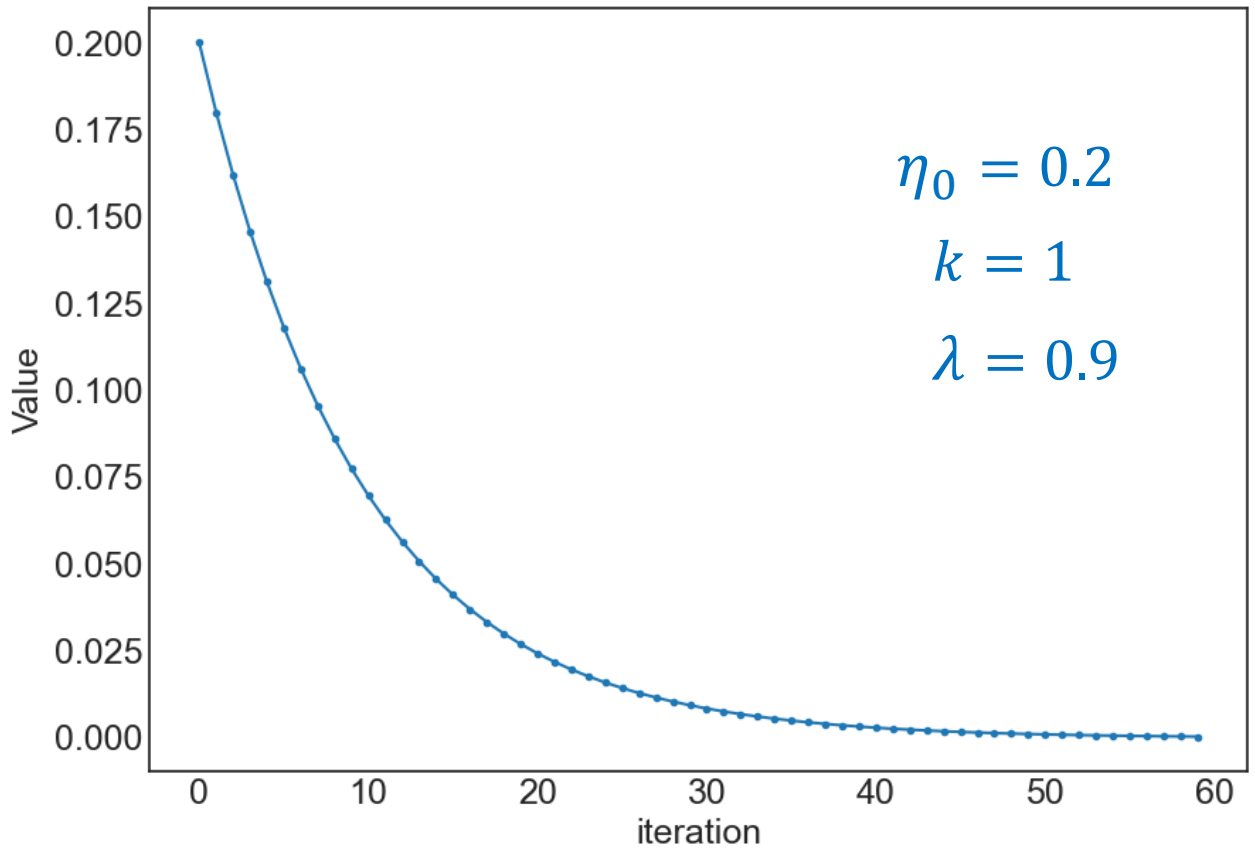
$$s_t = s_{t-1} + g_t^2$$

$$x_t = x_{t-1} - \frac{\eta}{\sqrt{s_t + \epsilon}} g_t$$

Differences?

Learning rate decay

$$\eta = \eta_0 \times \lambda^{\frac{s}{k}}$$



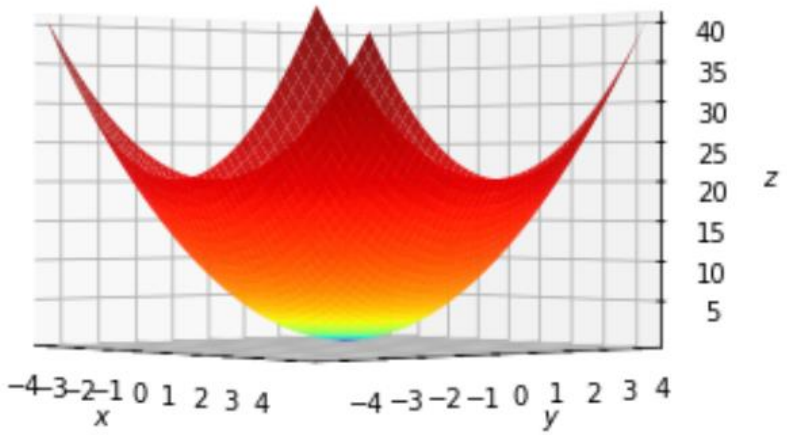
Adaptive Learning Rate

❖ Optimization: 2D function

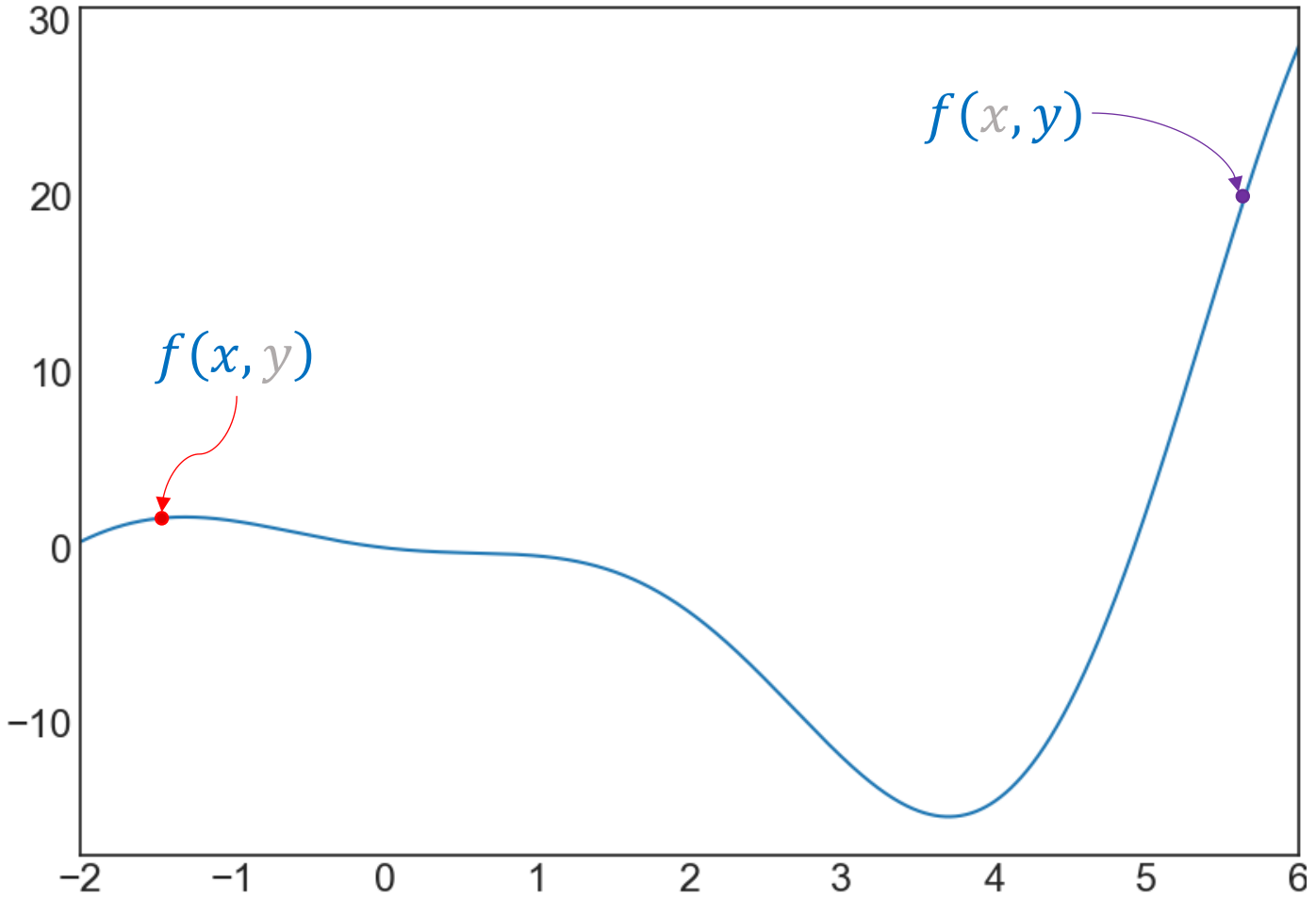
$$f(x, y) = x^2 + y^2$$

$$-100 \leq x, y \leq 100$$

$$x, y \in \mathbb{N}$$



imagine



Adaptive Learning Rate

Adagrad (2D function)

$$g_{t,x} = \frac{\partial f(x,y)}{\partial x}$$

$$s_{t,x} = s_{t-1,x} + g_{t,x}^2$$

$$x_t = x_{t-1} - \frac{\eta}{\sqrt{s_{t,x} + \epsilon}} g_{t,x}$$

$$g_{t,y} = \frac{\partial f(x,y)}{\partial x}$$

$$s_{t,y} = s_{t-1,y} + g_{t,y}^2$$

$$y_t = y_{t-1} - \frac{\eta}{\sqrt{s_{t,y} + \epsilon}} g_{t,y}$$

Adagrad

$$g_t = \nabla_\theta L$$

$$s_t = s_{t-1} + g_t^2$$

$$\theta_t = \theta_{t-1} - \frac{\eta}{\sqrt{s_t + \epsilon}} g_t$$

Adaptive Learning Rate

Adagrad: Limitation

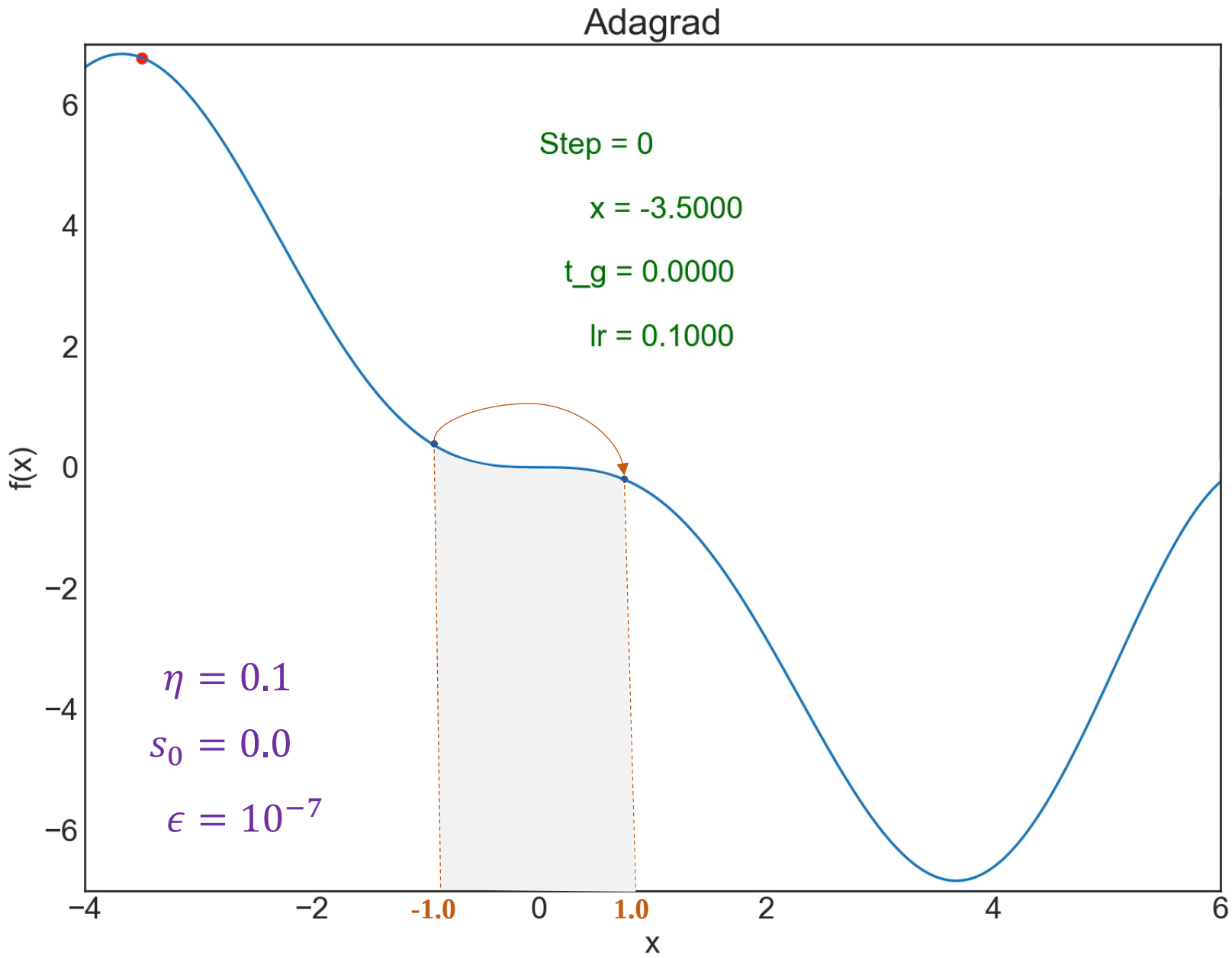
$$g_t = \nabla_\theta L$$

$$s_t = s_{t-1} + g_t^2$$

$$\theta_t = \theta_{t-1} - \frac{\eta}{\sqrt{s_t + \epsilon}} g_t$$

current summation

Expectation



How to Use historical Data

❖ Moving average

$k = 2$

3	8	6	5	1	7	9	0	8	4
	5.5	7.0	5.5	3.0	4.0	8.0	4.5	4.0	6.0

$$SMA_t = \frac{s_{t-1} + s_{t-2} + \cdots + s_{t-k}}{k}$$

3	8	6	5	1	7	9	0	8	4
3.0	5.5	5.8	5.4	3.2	5.1	7.0	3.5	5.8	4.9

$$EMA_t = \rho EMA_{t-1} + (1 - \rho)s_t$$

How to Use historical Data

❖ Exponentially weighted averages

3	8	6	5	1	7	9	0	8	4
3.0	5.5	5.8	5.4	3.2	5.1	7.0	3.5	5.8	4.9

$$V_t = \rho V_{t-1} + (1-\rho)s_t$$

$$V_1 = \rho V_0 + (1-\rho)s_1$$

$$V_2 = \rho V_1 + (1-\rho)s_2$$

$$V_3 = \rho V_2 + (1-\rho)s_3$$

$$V_3 = \rho[\rho V_1 + (1-\rho)s_2] + (1-\rho)s_3$$

$$V_3 = \rho[\rho[\rho V_0 + (1-\rho)s_1] + (1-\rho)s_2] + (1-\rho)s_3$$

Given $V_0 = 0$, we have

$$V_3 = \rho[\rho(1-\rho)s_1 + (1-\rho)s_2] + (1-\rho)s_3$$

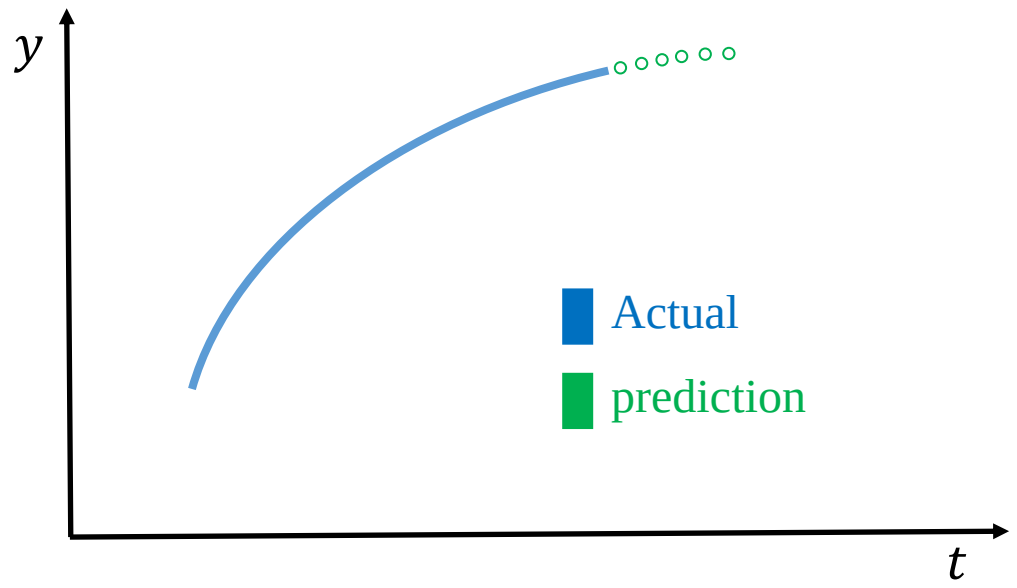
$$V_3 = \rho^2(1-\rho)s_1 + \rho(1-\rho)s_2 + (1-\rho)s_3$$

How to Use historical Data

❖ Exponentially weighted averages

$$V_t = \rho V_{t-1} + (1-\rho)s_t$$

data	3	8	6	5	1	7	9	0	8	4
EWA	3.0	5.5	5.8	5.4	3.2	5.1	7.0	3.5	5.8	4.9



Example

$$V_3 = \rho^2 (1-\rho)s_1 + \rho(1-\rho)s_2 + (1-\rho)s_3$$

With $\rho = 0.5$

$$V_3 = 0.125s_1 + 0.25s_2 + 0.5s_3$$

With $\rho = 0.9$

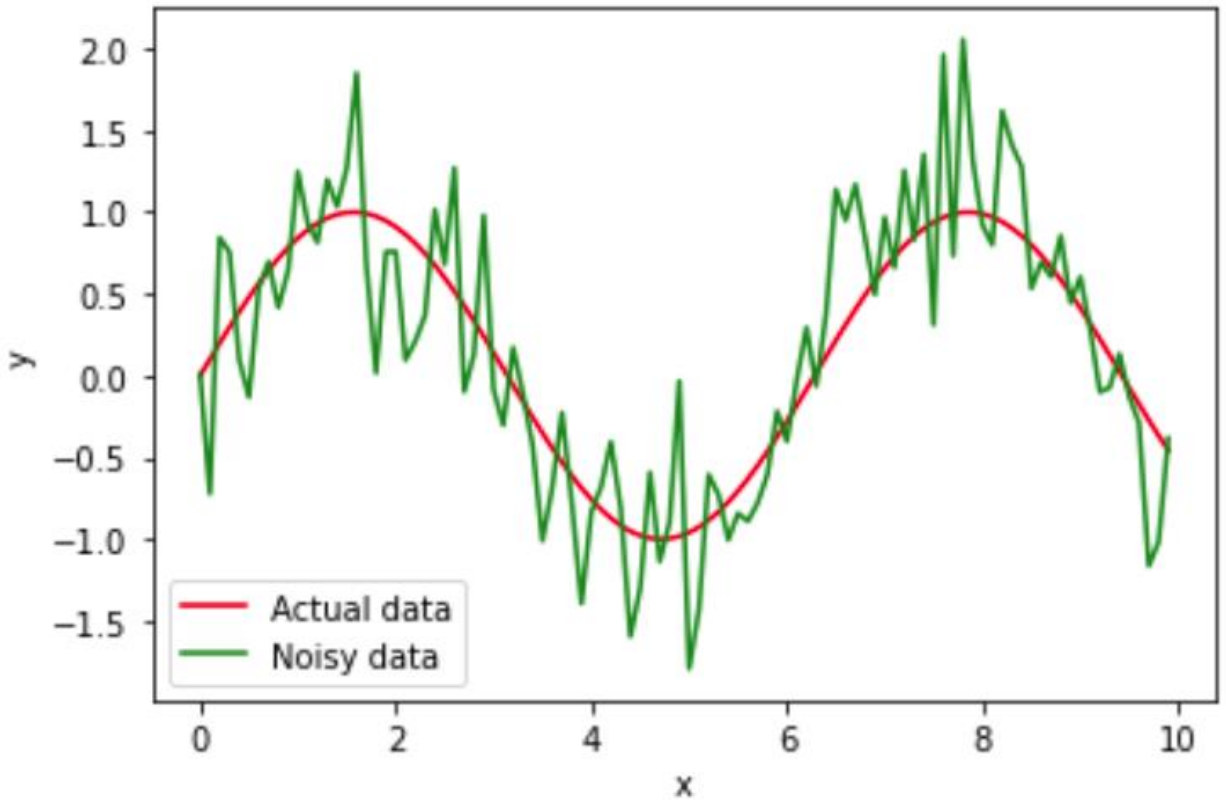
$$V_3 = 0.081s_1 + 0.09s_2 + 0.1s_3$$

With $\rho = 0.98$

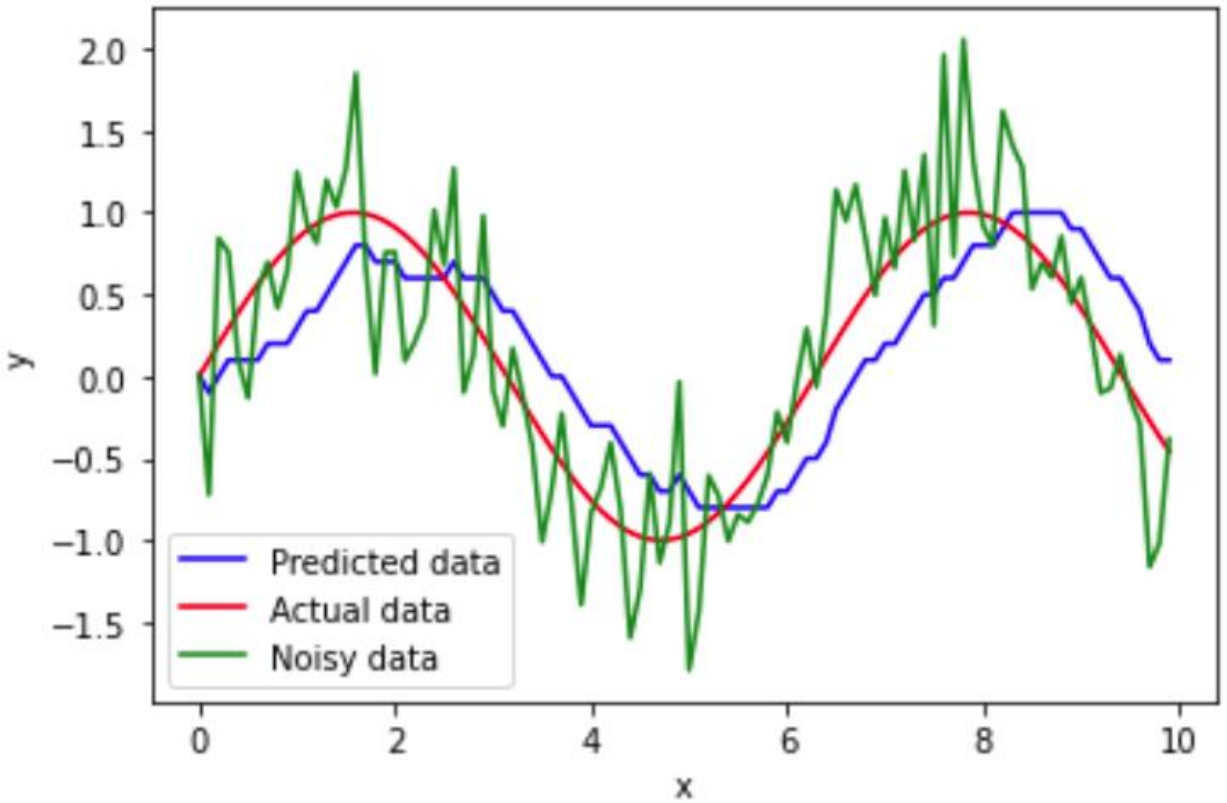
$$V_3 = 0.0392s_1 + 0.0196s_2 + 0.02s_3$$

Optimization Algorithms

❖ Exponentially weighted averages



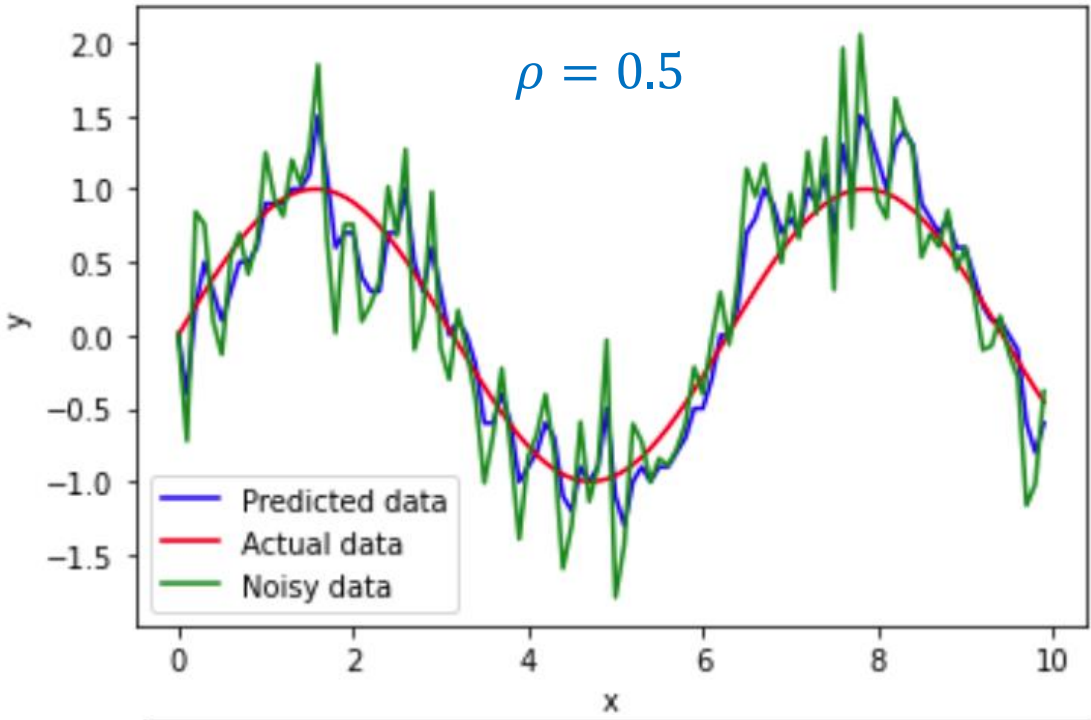
$\rho = 0.9$

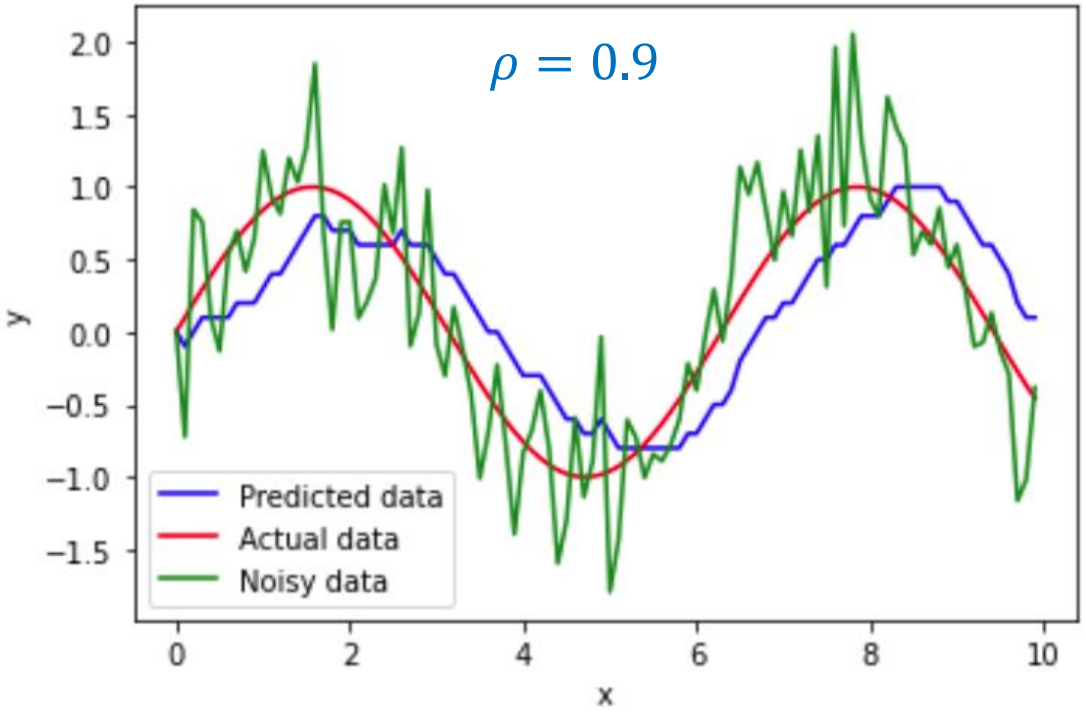


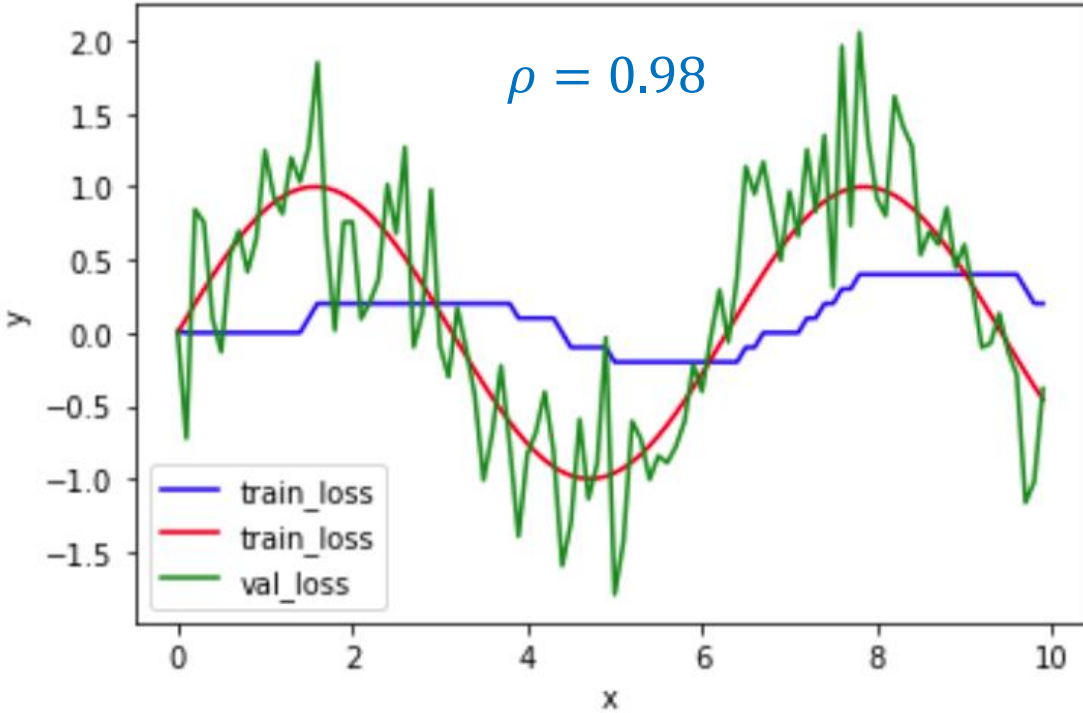
Optimization Algorithms

❖ Exponentially weighted averages

$$V_t = \rho V_{t-1} + (1-\rho)s_t$$







Back to Adaptive Learning Rate

❖ How to apply to adagrad

Adagrad

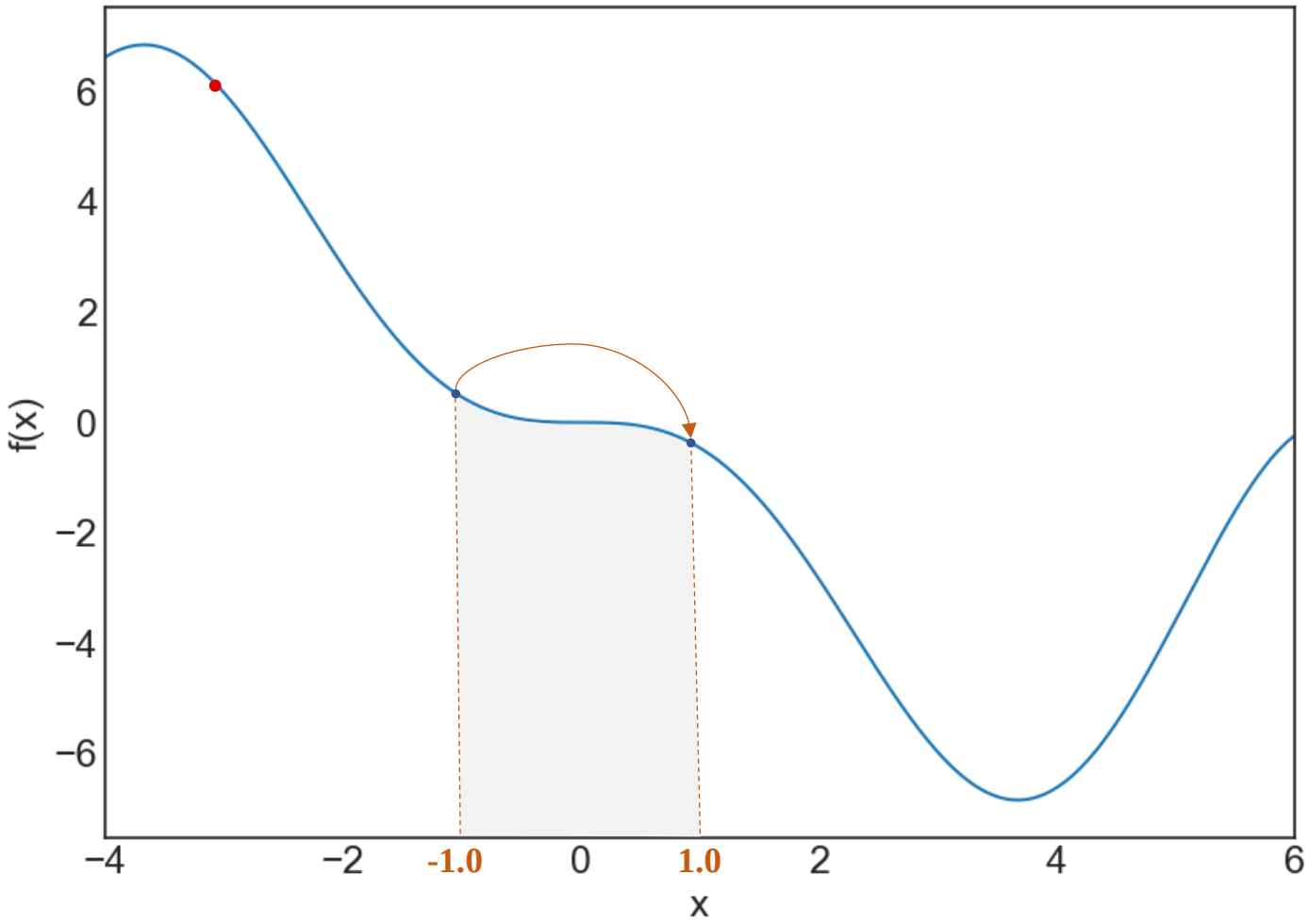
$$g_t = f'(x_{t-1})$$

$$s_t = s_{t-1} + g_t^2$$

$$x_t = x_{t-1} - \frac{\eta}{\sqrt{s_t + \epsilon}} g_t$$

Exponentially weighted averages

$$V_t = \rho V_{t-1} + (1-\rho) s_t$$



Back to Adaptive Learning Rate

❖ How to apply to adagrad

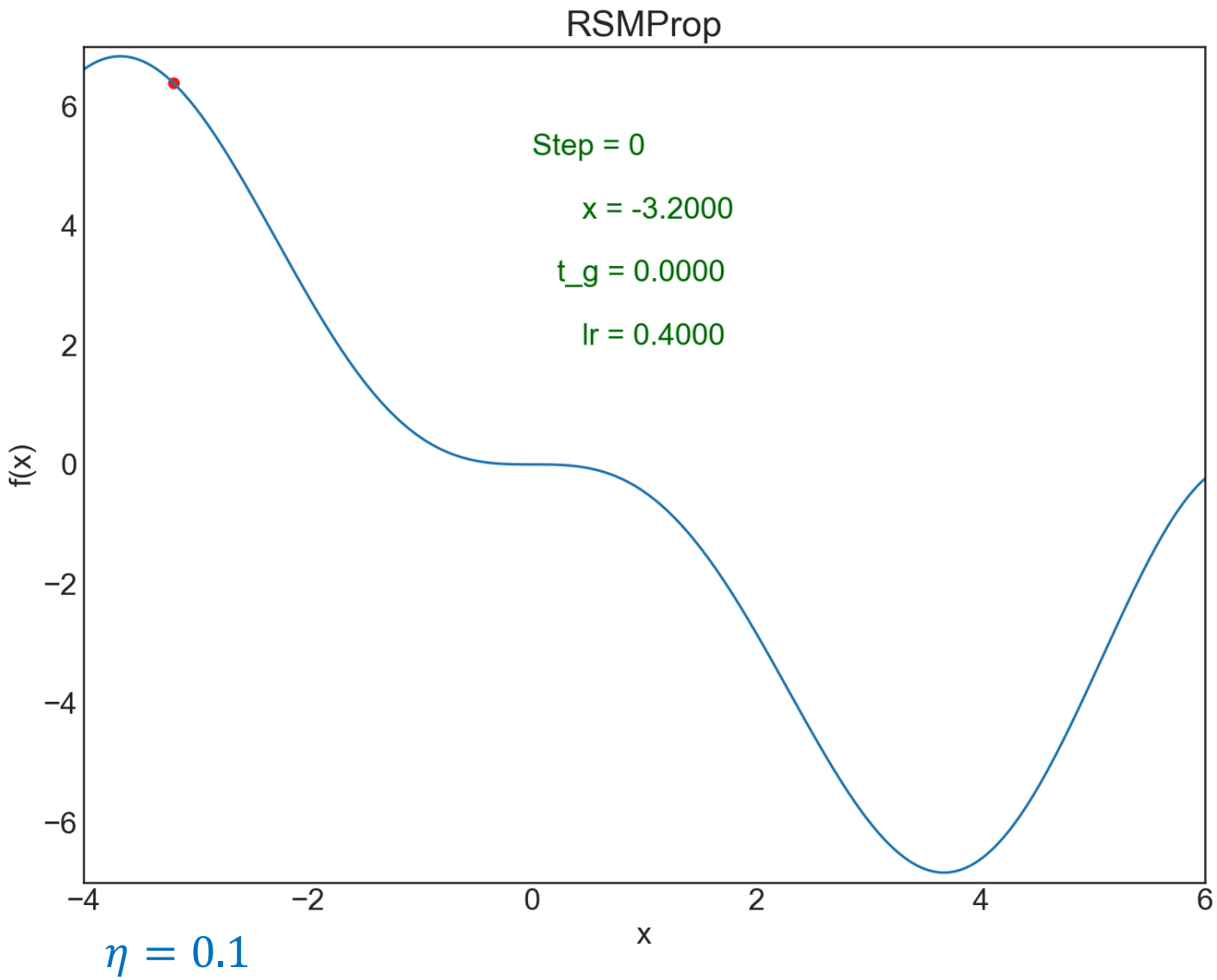
~~Adagrad~~ RMSProp

$$g_t = f'(x_{t-1})$$

$$s_t = \rho s_{t-1} + (1-\rho) g_t^2$$

$$x_t = x_{t-1} - \frac{\eta}{\sqrt{s_t + \epsilon}} g_t$$

Expectation



$\eta = 0.1$

Back to Adaptive Learning Rate

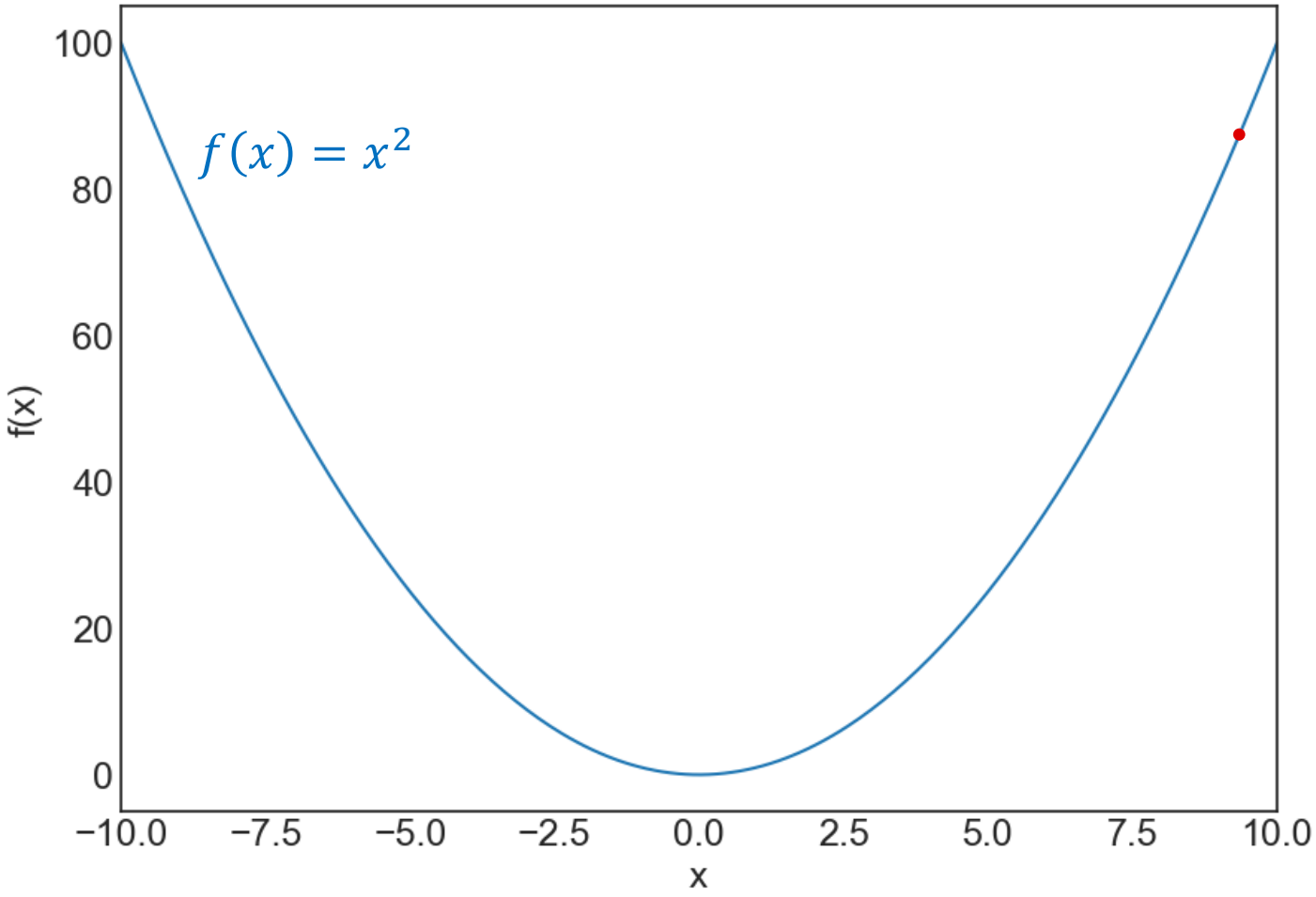
❖ For square function

RMSProp

$$g_t = f'(x_{t-1})$$

$$s_t = \rho s_{t-1} + (1-\rho) g_t^2$$

$$x_t = x_{t-1} - \frac{\eta}{\sqrt{s_t + \epsilon}} g_{t-1}$$



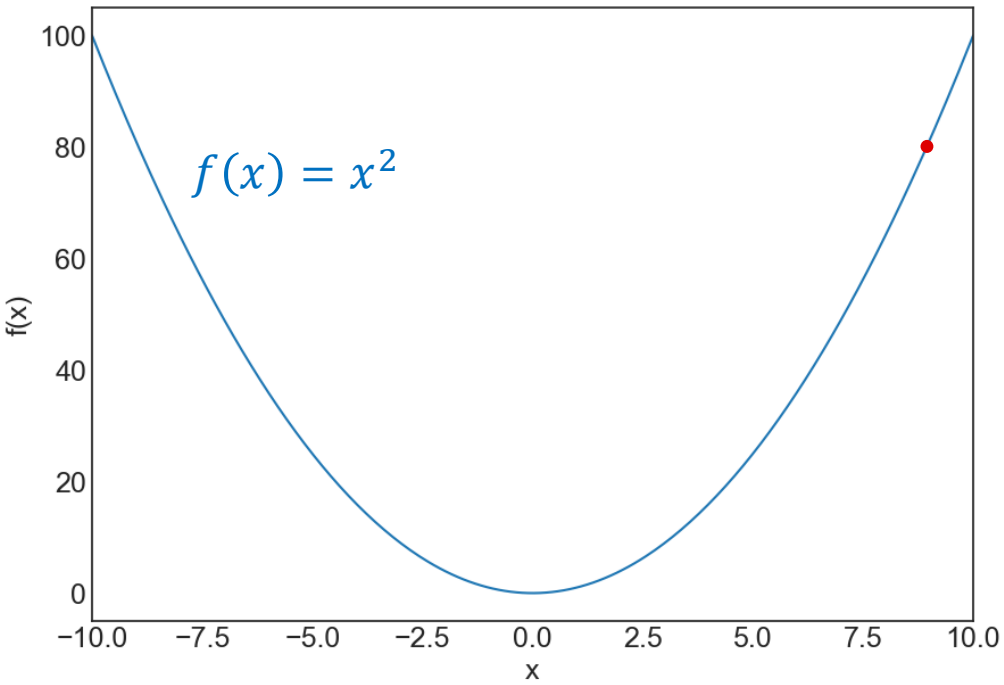
For square function

RMSProp

$$g_t = f'(x_{t-1})$$

$$s_t = \rho s_{t-1} + (1-\rho) g_t^2$$

$$x_t = x_{t-1} - \frac{\eta}{\sqrt{s_t + \epsilon}} g_t$$



$x_0 = 9.0$ $\eta = 0.1$ $s_0 = 0.0$

$\rho = 0.9$ $\epsilon = 10^{-7}$

$f'(x_0) = 18.0$ $s_1 = 32.4$

$$x_1 = 9.0 - \frac{0.1}{\sqrt{s_1 + \epsilon}} f'(x_0) = 8.6837$$

$f'(x_1) = 17.36$ $s_2 = 59.3$

$x_2 = 8.458$

$f'(x_2) = 16.916$ $s_3 = 82.0$

$x_2 = 8.27$

Back to Adaptive Learning Rate

❖ For square function

RMSProp

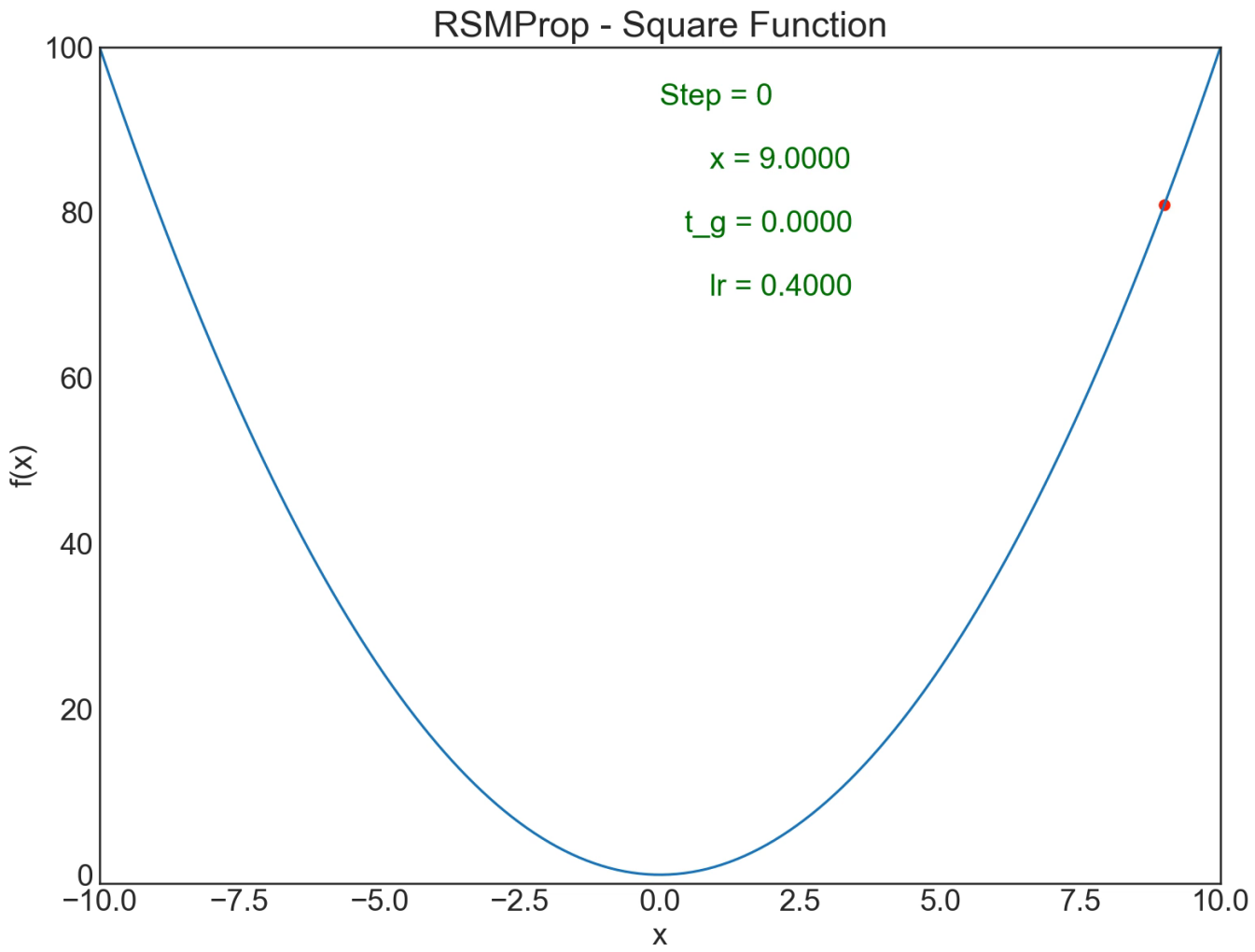
$$g_t = f'(x_{t-1})$$

$$s_t = \rho s_{t-1} + (1-\rho) g_t^2$$

$$x_t = x_{t-1} - \frac{\eta}{\sqrt{s_t + \epsilon}} g_t$$

$x_0 = 9.0$ $\eta = 0.4$ $s_0 = 0.0$

$\rho = 0.9$ $\epsilon = 10^{-7}$



Back to Adaptive Learning Rate

❖ For another function

RMSProp

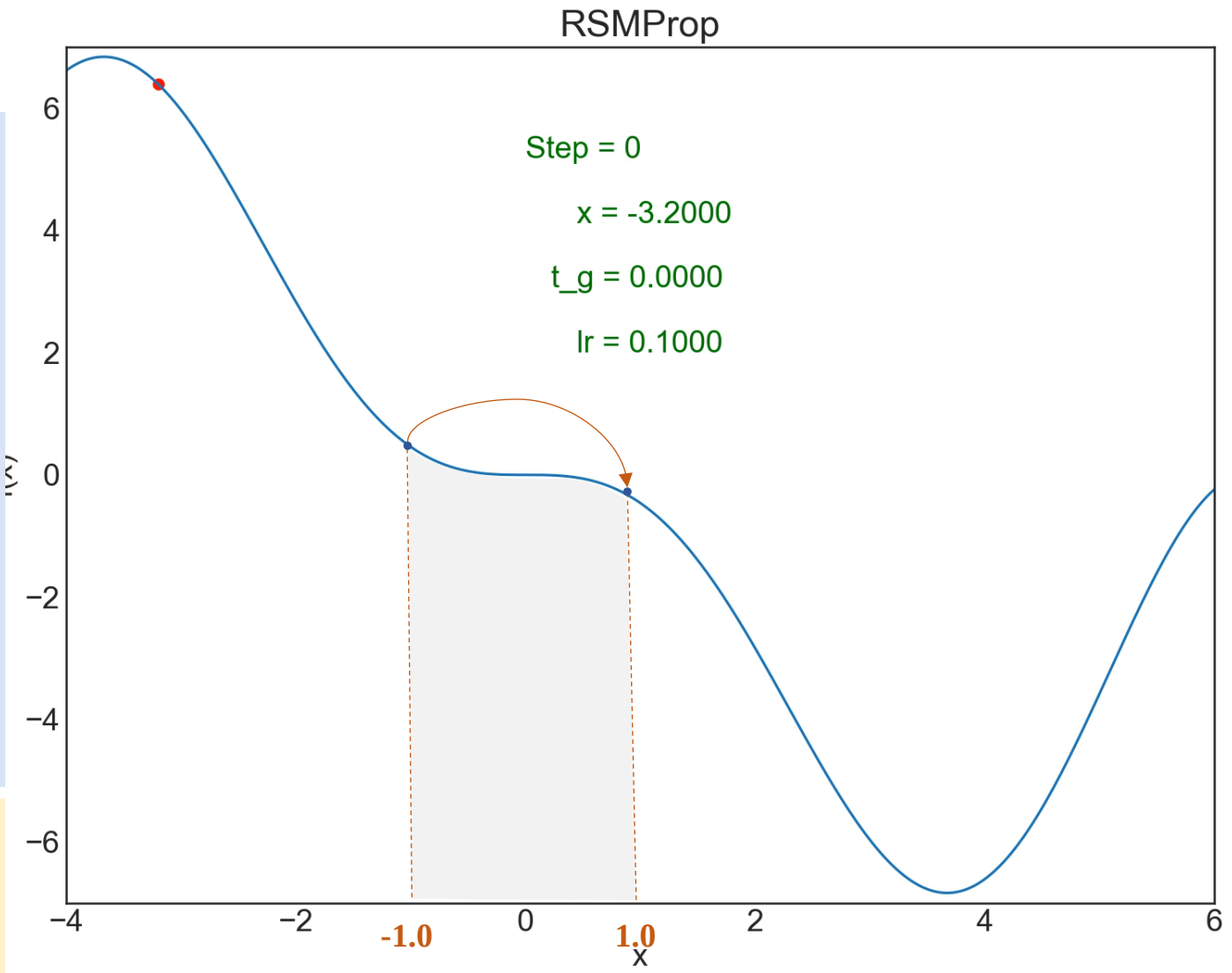
$$g_t = f'(x_{t-1})$$

$$s_t = \rho s_{t-1} + (1-\rho) g_t^2$$

$$x_t = x_{t-1} - \frac{\eta}{\sqrt{s_t + \epsilon}} g_t$$

$x_0 = -3.5$ $\eta = 0.1$ $s_0 = 0.0$

$\rho = 0.9$ $\epsilon = 10^{-7}$



Adaptive Learning Rate

❖ Generalization

RMSProp (One variable Function)

$$g_t = f'(x_{t-1})$$

$$s_t = \rho s_{t-1} + (1-\rho) g_t^2$$

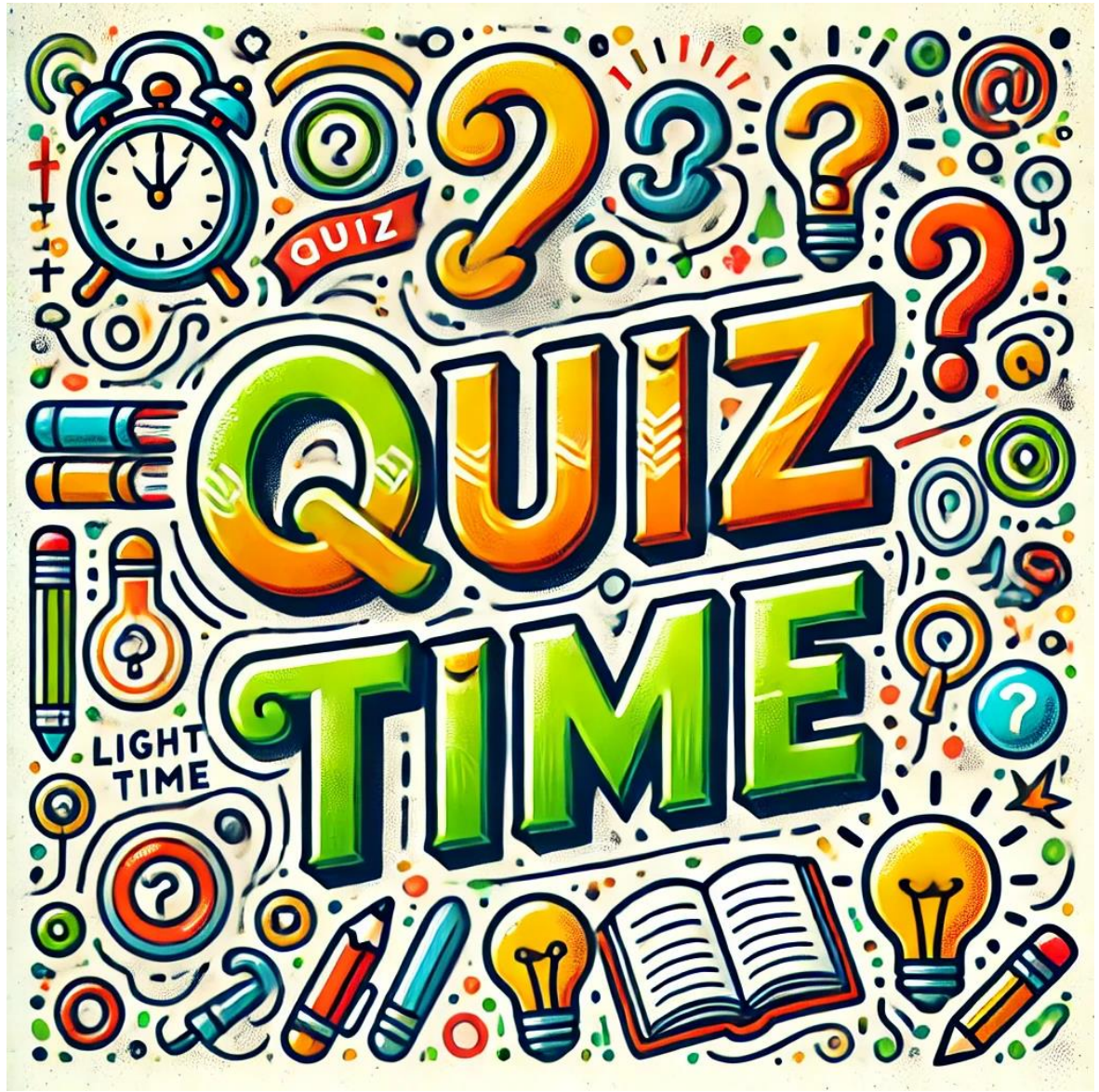
$$x_t = x_{t-1} - \frac{\eta}{\sqrt{s_t + \epsilon}} g_t$$

RMSProp (Multi-variable Function)

$$g_t = \nabla_\theta L$$

$$s_t = \rho s_{t-1} + (1-\rho) g_t^2$$

$$\theta_t = \theta_{t-1} - \frac{\eta}{\sqrt{s_t + \epsilon}} g_{t,i}$$


QUIZ
QUIZ
TIME
LIGHT
TIME

Question 1

❖ Hàm nào trong các hàm loss của các mô hình sau là non-convex?

a) Linear regression

b) Logistic regression (sigmoid+BCE)

c) Softmax regression (softmax+CE)

d) MLP with ReLU activation

Question 2

❖ Đối với một trọng số (tham số của model), các thông tin thu được qua quá trình huấn luyện có thể được xem là dữ liệu gì?

a) Dữ liệu bảng thông thường (như adversiting data)

b) Dữ liệu ảnh

c) Dữ liệu time-series

d) Kiểu dữ liệu khác

Question 3

❖ Với một mô hình học sâu nào đó, optimizer nào cho kết quả tốt nhất?

a) SGD

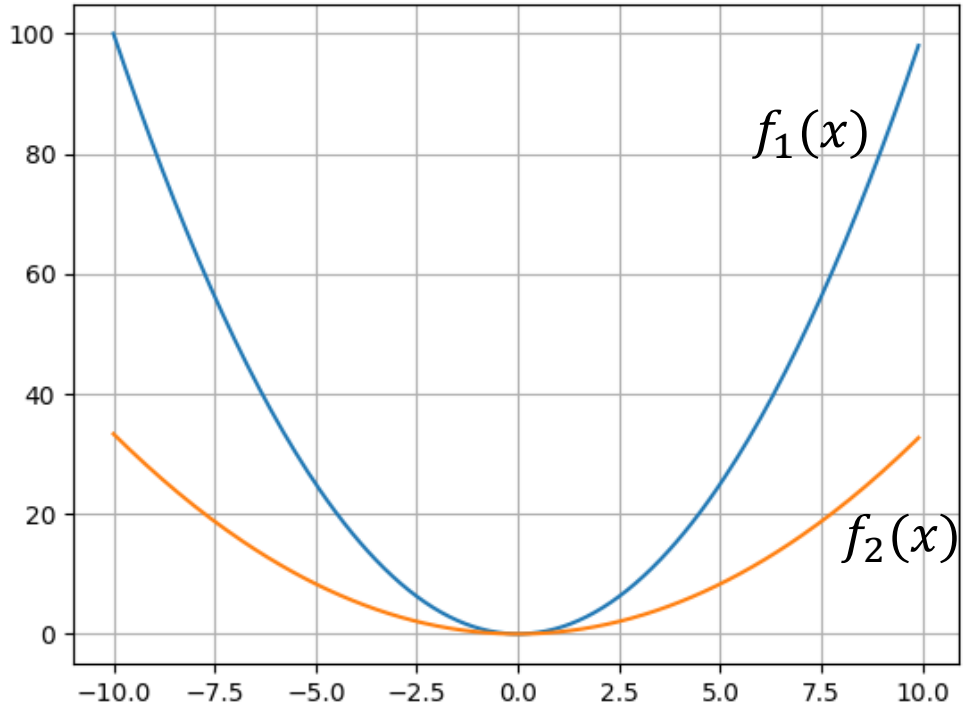
b) Adagrad

c) Adam

d) Không xác định

Question 4

❖ Giả sử learning rate lr_1 là phù hợp cho hàm $f_1(x)$. Chọn learning rate lr_2 thế nào để phù hợp cho hàm $f_2(x)$?



a) Tăng giá trị lr_1 lên rất nhiều

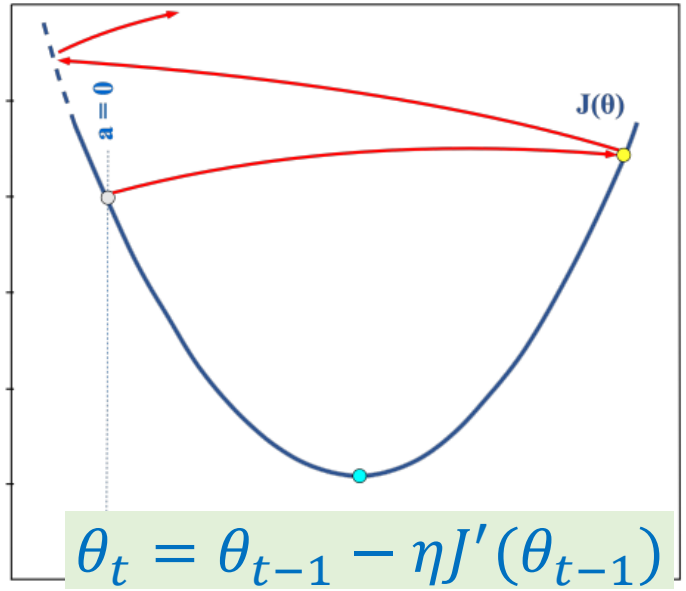
b) Tăng giá trị lr_1 lên một ít

c) Giảm giá trị lr_1 xuống rất nhiều

d) Giảm giá trị lr_1 xuống một ít

Question 5

❖ Chọn 1 nguyên nhân **tiêu biểu nhất** dẫn đến hiện tượng như hình minh họa?



a) Hàm loss là hàm dạng bình phương

b) Learning rate quá lớn

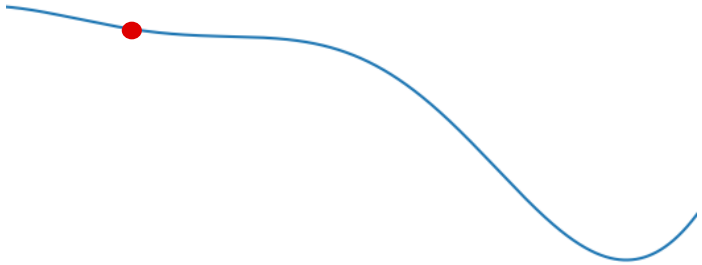
c) Giá trị x (input) quá lớn

d) Giá trị biến theta lớn

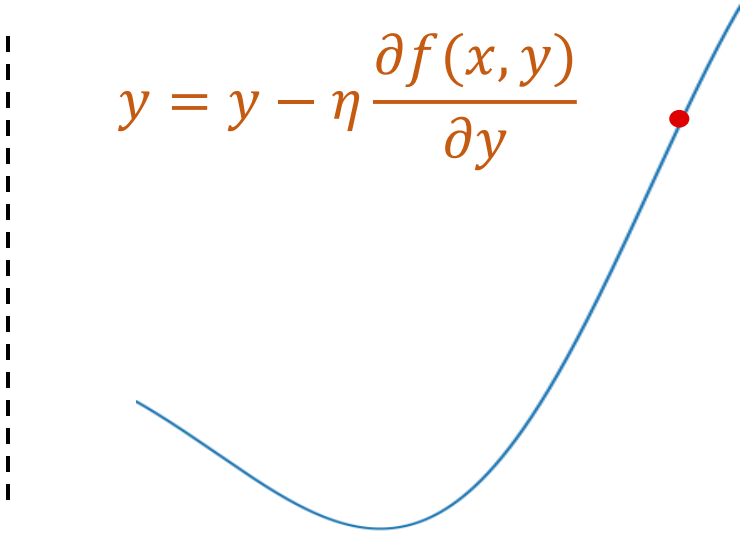
Question 6

❖ Cho hàm f(x,y). Hai hình dưới lần lượt là đồ thị cho x và cho y. Kỹ thuật nào hợp lý nhất để cải thiện tình huống này?

$$x = x - \eta \frac{\partial f(x,y)}{\partial x}$$



$$y = y - \eta \frac{\partial f(x,y)}{\partial y}$$



a) Dùng momentum

b) Cho learning rate lớn

c) Cho learning rate nhỏ

d) Mỗi biến có một lr riêng biệt

Outline

SECTION 1

SGD Insight

SECTION 2

Adaptive Learning Rate

SECTION 3

Momentum and Towards Adam

Simpler version of Adam

$$g_t = \nabla_\theta L$$

$$m_t = \beta_1 m_{t-1} + (1 - \beta_1) g_t$$

$$v_t = \beta_2 v_{t-1} + (1 - \beta_2) g_t^2$$

$$\theta_t = \theta_{t-1} - \frac{\eta}{\sqrt{v_t + \epsilon}} m_t$$

Adaptive Learning Rate

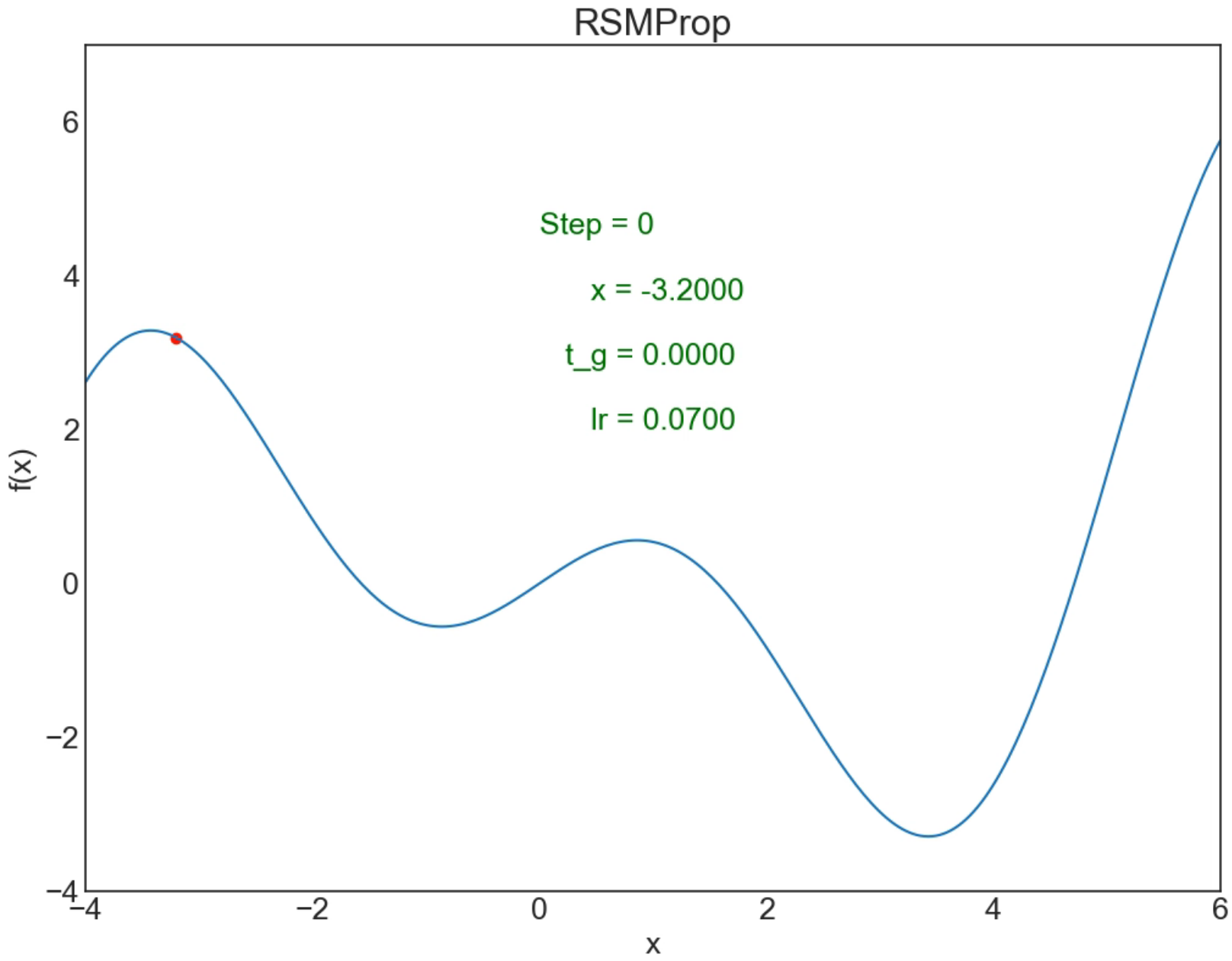
A common limitation so far

$$\theta_t = \theta_{t-1} - \eta \nabla_\theta L$$

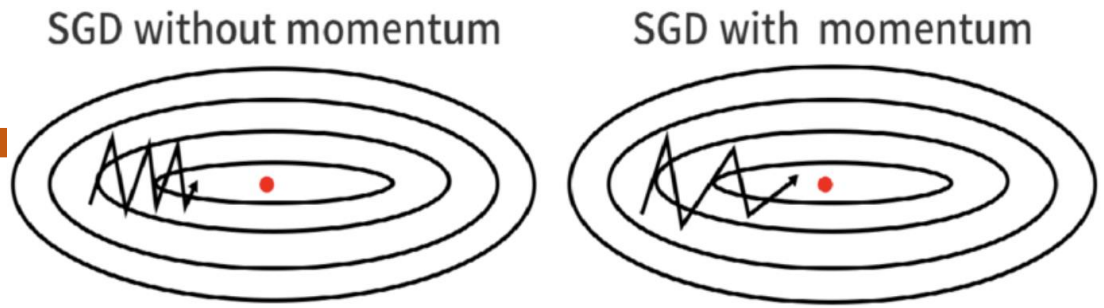
$$g_t = \nabla_\theta L$$

$$s_t = \rho s_{t-1} + (1-\rho) g_t^2$$

$$\theta_t = \theta_{t-1} - \frac{\eta}{\sqrt{s_t + \epsilon}} g_t$$



Using Momentum



Genevieve B. Orr

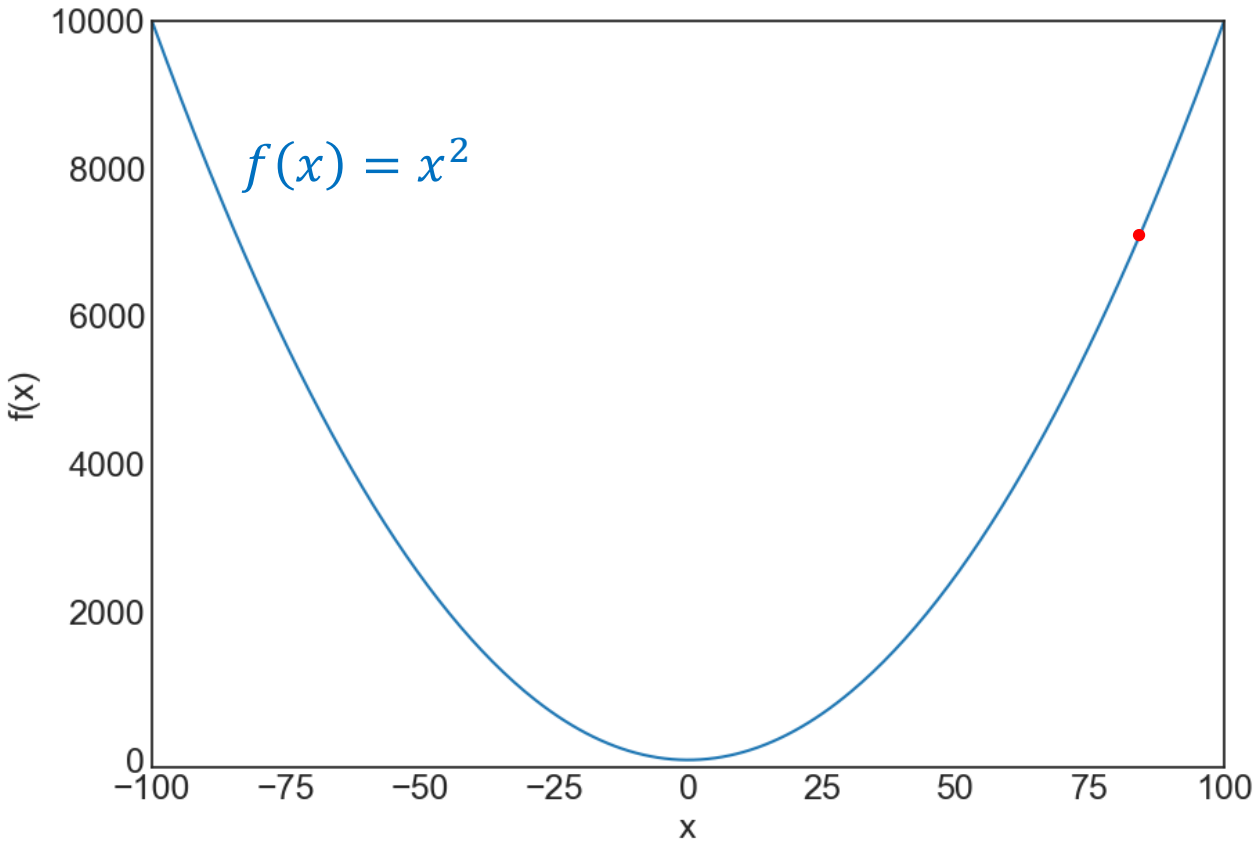
SGD

$$\theta_t = \theta_{t-1} - \eta \nabla_\theta L$$

SGD + Momentum

$$v_t = \rho v_{t-1} - (1-\rho)\alpha \nabla_\theta L$$

$$\theta_t = \theta_{t-1} + v_t$$



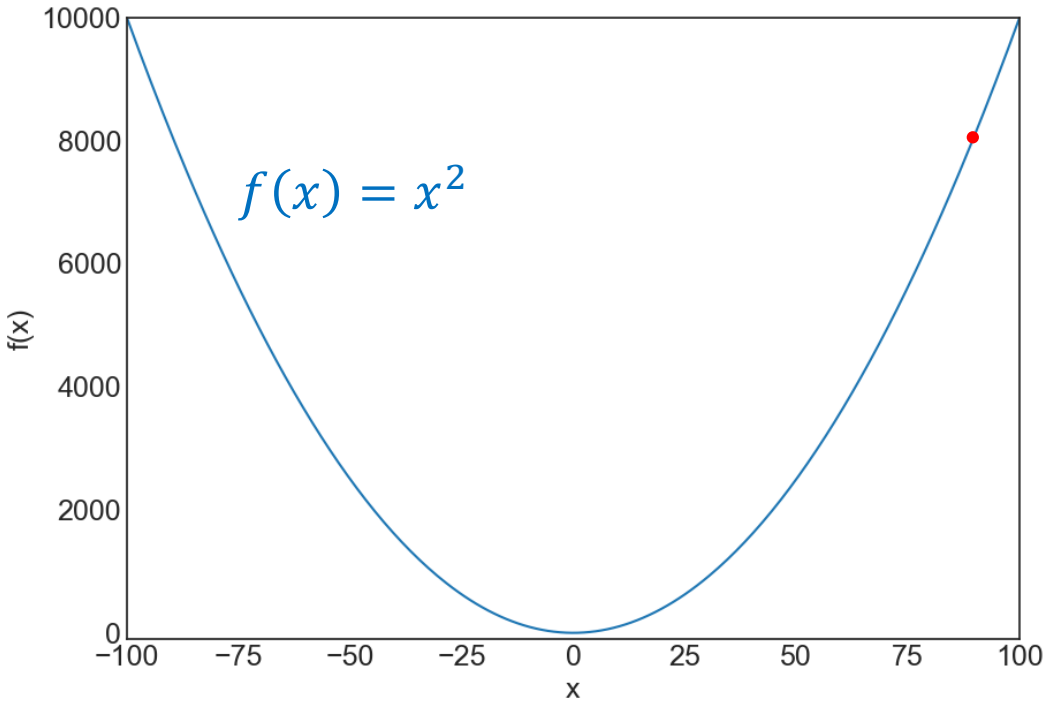
$$v_t = m v_{t-1} - \eta \nabla_\theta L$$

$$\theta_t = \theta_{t-1} + v_t$$

SGD + Momentum

$$v_t = mv_{t-1} - \eta \nabla_\theta L$$

$$\theta_t = \theta_{t-1} + v_t$$



$x_0 = 90.0$ $m = 0.9$

$\eta = 0.1$ $v_0 = 0.0$

$f'(x_0) = 180.0$ $v_1 = -18.0$

$x_1 = x_0 + v_1 = 72.0$

$f'(x_1) = 144.0$ $v_2 = -30.59$

$x_2 = x_1 + v_2 = 41.4$

$f'(x_2) = 82.8$ $v_3 = -35.82$

$x_3 = x_2 + v_3 = 5.58$

$f'(x_3) = 11.16$ $v_4 = -33.354$

$x_4 = x_3 + v_4 = -27.77$

$f'(x_4) = -55.54$ $v_5 = -24.46$

$x_5 = x_4 + v_5 = -52.23$

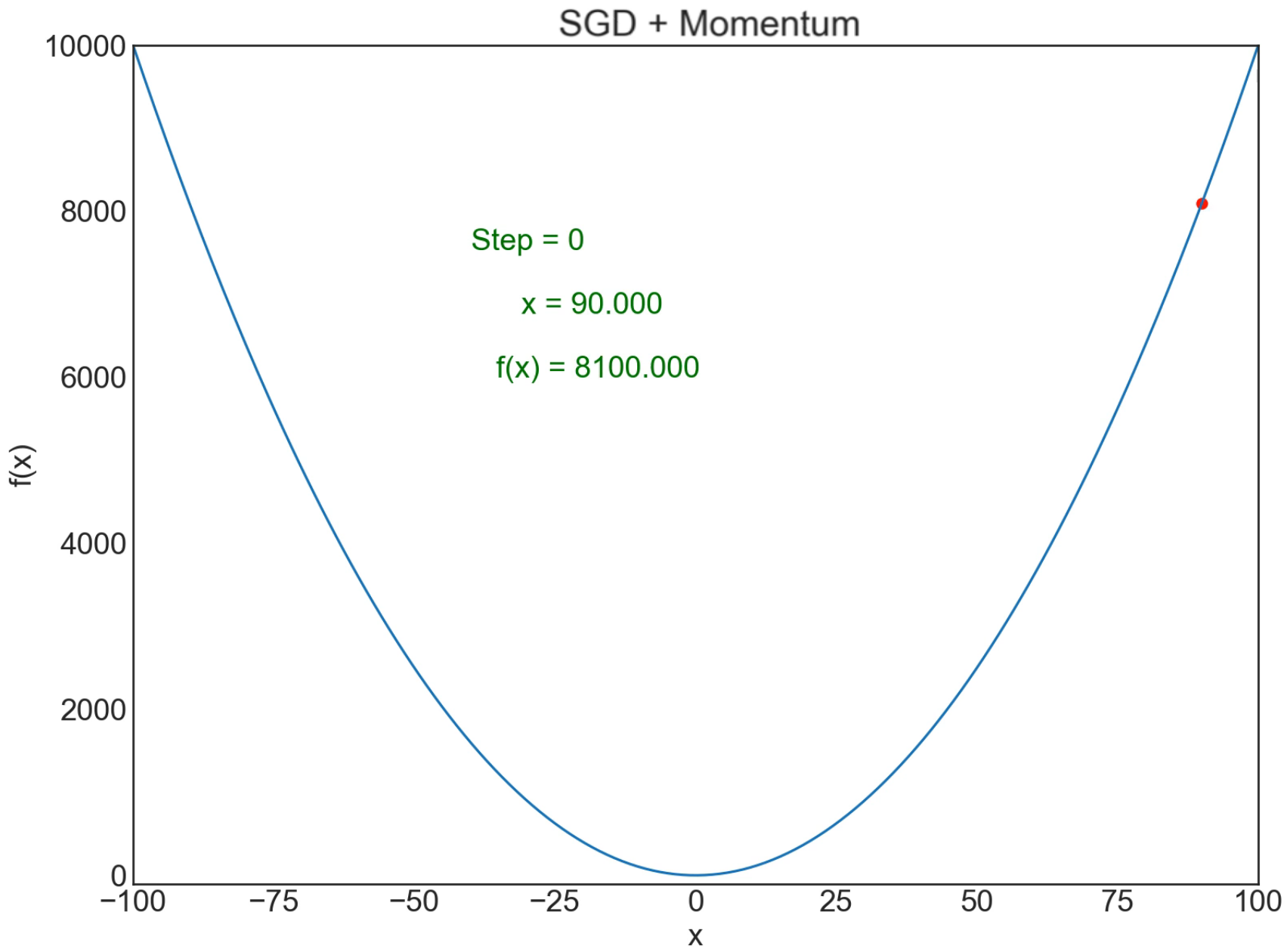
SGD + Momentum

$$v_t = mv_{t-1} - \eta \nabla_\theta L$$

$$\theta_t = \theta_{t-1} + v_t$$

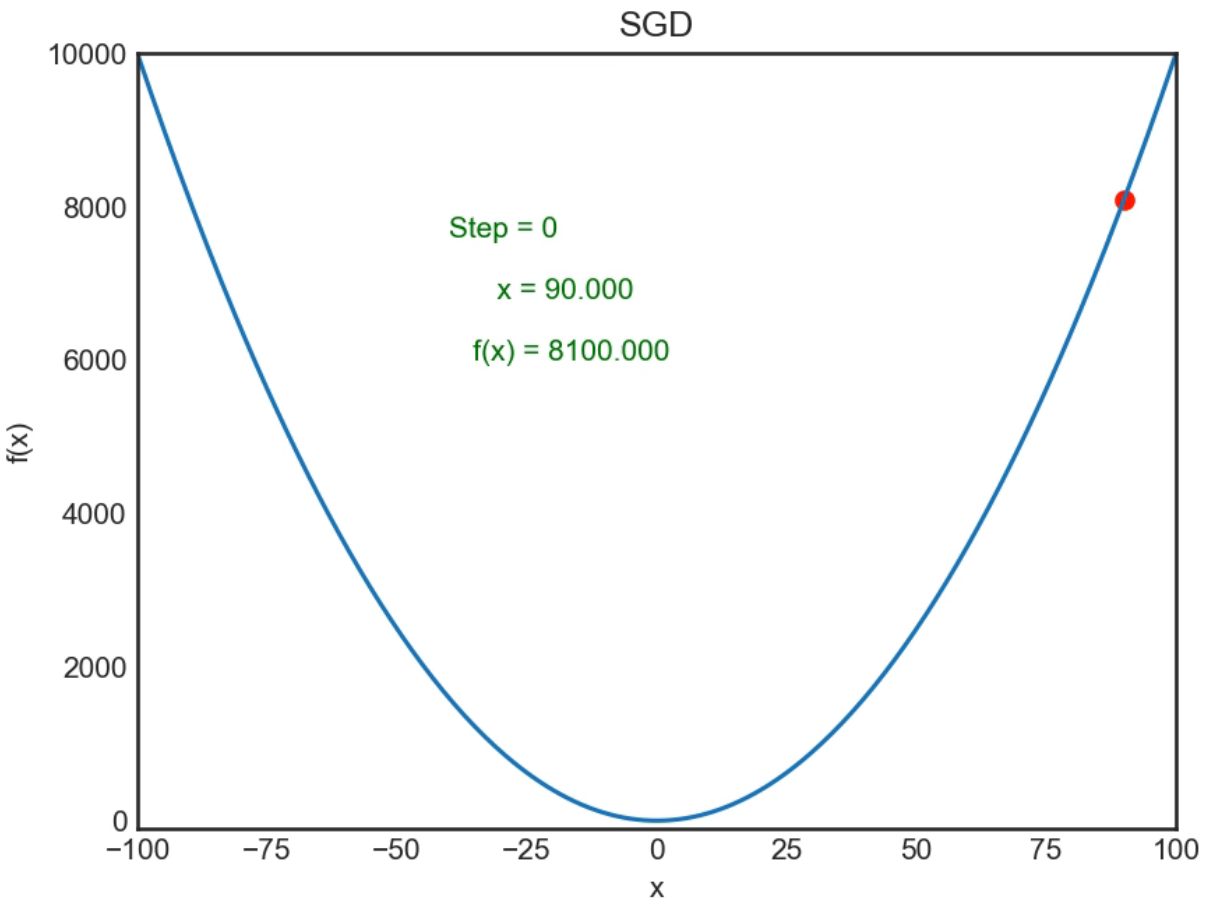
$x_0 = 90.0$ $\quad m = 0.9$

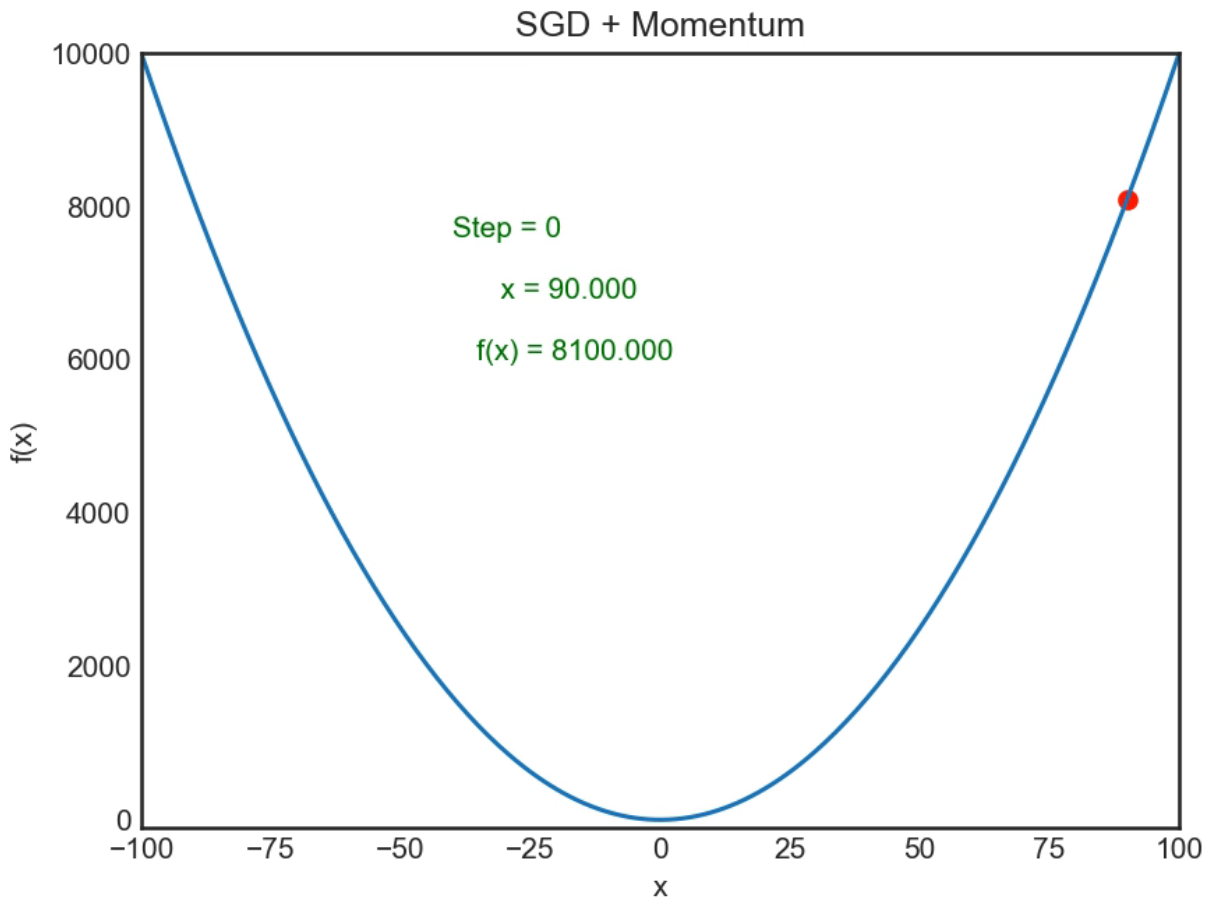
$\eta = 0.01$ $\quad v_0 = 0.0$



SGD + Momentum

❖ Mimic a ball rolling





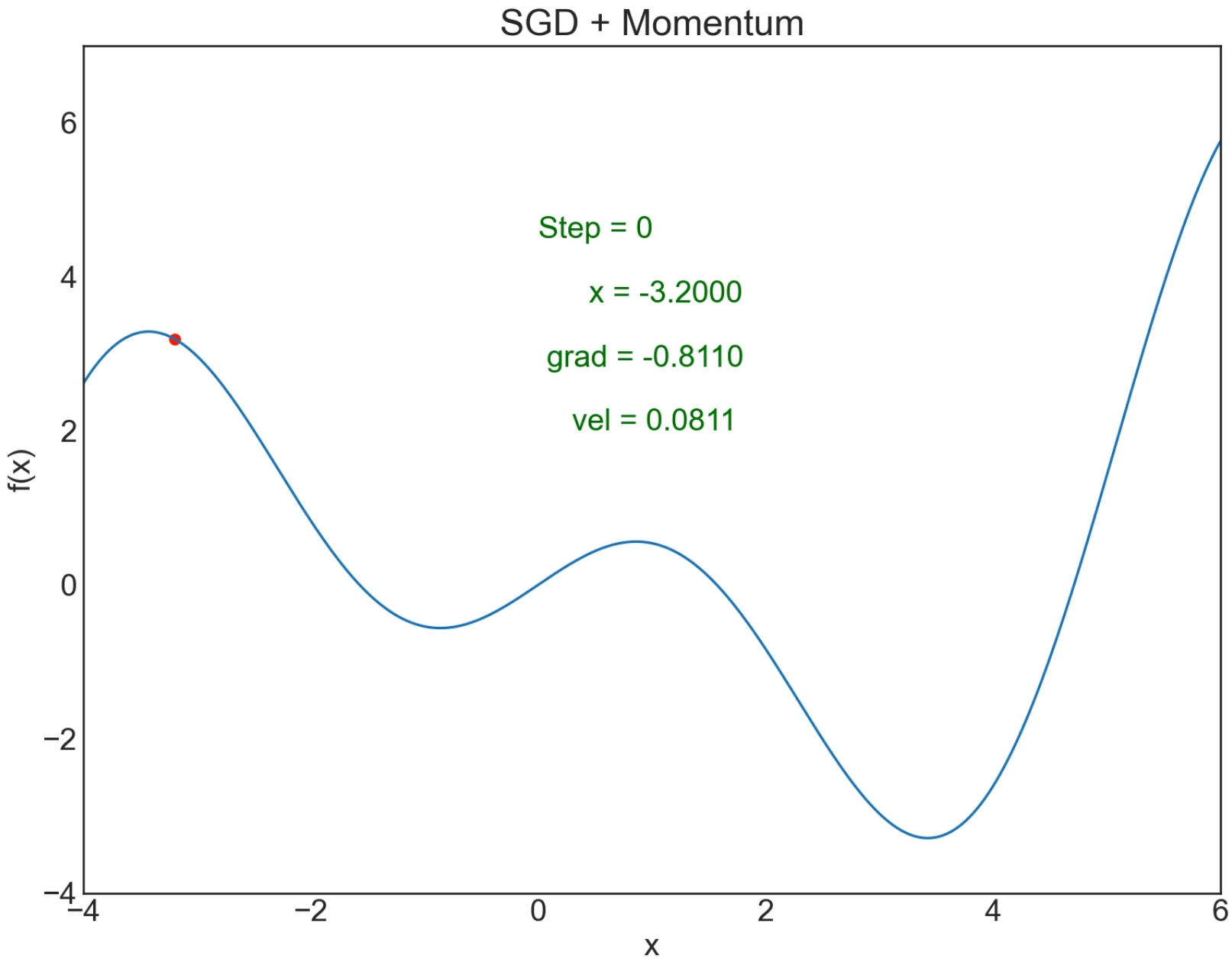
SGD + Momentum

$$v_t = mv_{t-1} - \eta \nabla_\theta L$$

$$\theta_t = \theta_{t-1} + v_t$$

$x_0 = -3.2$ $\quad m = 0.9$

$\eta = 0.1$ $\quad v_0 = 0.0$



What about RSMProp+Momentum

SGD

$$\theta_t = \theta_{t-1} - \eta \nabla_\theta L$$

SGD + Momentum

$$v_t = m v_{t-1} - \eta \nabla_\theta L$$

$$\theta_t = \theta_{t-1} + v_t$$

$$v_t = \rho v_{t-1} - (1-\rho)\alpha \nabla_\theta L$$

$$\theta_t = \theta_{t-1} + v_t$$

RMSProp

$$g_t = \nabla_\theta L$$

$$s_t = \rho s_{t-1} + (1-\rho) g_t^2$$

$$\theta_t = \theta_{t-1} - \frac{\eta}{\sqrt{s_t + \epsilon}} g_t$$

?

What about RSMProp+Momentum

SGD + Momentum

$$m_t = \rho m_{t-1} - \eta \nabla_\theta L$$

$$\theta_t = \theta_{t-1} + m_t$$

$$m_t = \rho m_{t-1} - (1-\rho)\alpha \nabla_\theta L$$

$$\theta_t = \theta_{t-1} + m_t$$

$$m_t = \beta_1 m_{t-1} + (1-\beta_1)\nabla_\theta L$$

$$\theta_t = \theta_{t-1} - \eta m_t$$

idea equivalence

RMSProp

$$g_t = \nabla_\theta L$$

$$v_t = \beta_2 v_{t-1} + (1-\beta_2) g_t^2$$

$$\theta_t = \theta_{t-1} - \frac{\eta}{\sqrt{v_t + \epsilon}} g_t$$

Simpler version of Adam

$$g_t = \nabla_\theta L$$

$$m_t = \beta_1 m_{t-1} + (1-\beta_1) g_t$$

$$v_t = \beta_2 v_{t-1} + (1-\beta_2) g_t^2$$

$$\theta_t = \theta_{t-1} - \frac{\eta}{\sqrt{v_t + \epsilon}} m_t$$

What about RSMProp+Momentum

SGD + Momentum

$$m_t = \rho m_{t-1} - \eta \nabla_\theta L$$

$$\theta_t = \theta_{t-1} + m_t$$

$$m_t = \rho m_{t-1} - (1-\rho)\alpha \nabla_\theta L$$

$$\theta_t = \theta_{t-1} + m_t$$

$$m_t = \beta_1 m_{t-1} + (1-\beta_1)\nabla_\theta L$$

$$\theta_t = \theta_{t-1} - \eta m_t$$

idea equivalence

RMSProp

$$g_t = \nabla_\theta L$$

$$v_t = \beta_2 v_{t-1} + (1-\beta_2) g_t^2$$

$$\theta_t = \theta_{t-1} - \frac{\eta}{\sqrt{v_t + \epsilon}} g_t$$

Adam

$$g_t = \nabla_\theta L$$

$$m_t = \beta_1 m_{t-1} + (1-\beta_1) g_t$$

$$v_t = \beta_2 v_{t-1} + (1-\beta_2) g_t^2$$

$$\theta_t = \theta_{t-1} - \frac{\eta}{\sqrt{\frac{v_t}{1-\beta_2^t} + \epsilon}} \times \frac{m_t}{1-\beta_1^t}$$

Further Reading

Bias Correction

http://manjeetdahiya.com/posts/exponential-weighted-average/

https://stats.stackexchange.com/questions/232741/why-is-it-important-to-include-a-bias-correction-term-for-the-adam-optimizer-for

Adam

https://towardsdatascience.com/complete-guide-to-adam-optimization-1e5f29532c3d

https://optimization.cbe.cornell.edu/index.php?title=Adam