MAE40: Linear Circuits, midterm (2023)

2 hours, no electronics, closed book, 1 page of handwritten notes allowed

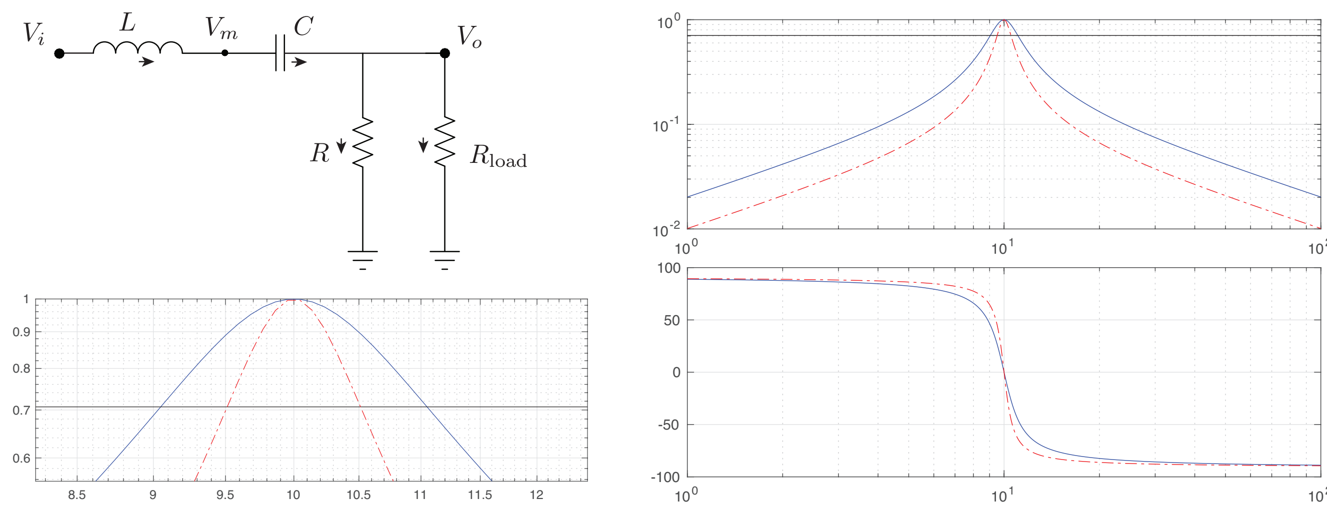

Figure 1: (upper-left) The "anti-notch" filter circuit considered in Q.1; (right) its associated Bode plot, and (lower-left) a close up of the magnitude part of this Bode plot, for (solid) $\{R, C, L\}$ such that $\omega_0 = 10$ rad/s and $Q = 5$ with $R_{\text{load}} = \infty$ [thus, $I_{\text{load}} = 0$], and (dot-dashed) the same $\{R, C, L\}$ but with $R_{\text{load}} = R/c_1$ finite [thus, $I_{\text{load}} \neq 0$].

1. Consider the "anti-notch" filter shown in Figure 1, assuming that:
(i) the voltage $V_i(t)$ at the input is specified precisely, regardless of the current drawn by the rest of the circuit, and
(ii) the current at the output $V_o(t)$ drives a resistive load such that $V_o = I_{\text{load}} R_{\text{load}}$, where the resistance of the load is taken as $R_{\text{load}} = R/c_1$, where R is the value of the resistor in the filter itself.

1a. Write down the component equations across each of the 4 components in this circuit, as well as the appropriate KCL, and take their Laplace transform, assuming zero initial conditions on all variables.

1b. Combine the equations governing this system by hand, and rearrange to determine $F(s) = V_o(s)/V_i(s)$ in terms of $\{R, C, L, c_1\}$. This is the transfer function of this filter circuit (including its resistive load).
Writing the denominator of this transfer function as $(s^2 + 2\,\zeta\,\omega_0\,s + \omega_0^2)$, determine ω_0 and ζ in terms of $\{R, C, L, c_1\}$.
Writing the denominator of this transfer function as $(s^2 + (1/Q)\,\omega_0\,s + \omega_0^2)$, determine Q in terms of $\{R, C, L, c_1\}$.
Finally, rewrite the entire transfer function (including the numerator) as a function of ω_0 and Q only.

1c. Given the last result in Q.1b, write the commands in Matlab that can be used to generate the corresponding Bode plot, which for your convenience is also given for you in Figure 1.

1d. Write the equations governing this circuit as $A\mathbf{x} = \mathbf{b}$ (that is, define A and $\mathbf{b}$), where:
- the unknown vector $\mathbf{x}$ lists $\{V_o, V_m\}$ first, followed by the currents in the various components, and
- the Laplace variable s, the parameters $\{R, C, L, c_1\}$, and the transform of the input, $V_i(s)$, are taken as symbolics.
Note: to avoid algebra errors, please write the $\mathbf{x}$ vector horizontally above your definition of A, and write each eqn to the right of the corresponding line of A that incorporates that eqn, with the correct terms on the LHS and RHS, to make certain you don't make any transcription errors (just as we did in the several example codes that we studied). Solving $\mathbf{x} = A\backslash\mathbf{b}$ in Matlab after the exam (which you will do in HW3 - see bottom of next page), and focusing on the first component, should (it does!) give the same answer in Q.1d as found by hand in Q.1b.

1e. In the future, I hope you will choose to consistently solve systems of algebraic equations on a computer (as in Q.1d) rather than solving them solely by hand (as in Q.1b). Why?

1f. In the future, I also hope you will choose to use GitHub for anything important that you do on a computer (e.g., the writing of code, documents, ...). Why? (Hint: remember Nickel's computer, may God have mercy on its soul...)

1g. Assuming $c_1 = 0$ (i.e., $R_{\text{load}} = \infty$, so negligible output current), and that $\{R, C, L\}$ are such that $\omega_0 = 10$ rad/s and $Q = 5$, the Bode plot of $F(s)$ is given as shown as the solid curve in the Bode plot in Figure 1. Explain the behavior of this Bode plot, and discuss how it is similar to, and different from, the notch filter considered in Quiz 5 and problem 1 of HW2. Again, this filter was designed with $Q = 5$. How might you define the BW (bandwidth) of this filter, and how does what you see in the closeup of the Bode plot provided reconcile with this value of Q?

1h. Taking the same $\{R, C, L\}$ values as in Q.1g, Figure 1 also shows, as (dot-dashed), the Bode plot of this filter when $c_1 \neq 0$ (i.e., R_{load} is now finite, so there is now a non-negligible output current). Based on your discussion in Q.1g of the BW of the notch, as well as your derivation in Q.1b of how Q depends on $\{R, C, L, c_1\}$, what value of c_1 was used when plotting the (dot-dashed) curve in Figure 1?

1i. What might this filter be useful for?

1.0	**1.1**	**1.2**	**1.3**	**1.5**	**1.6**	**1.8**	**2.0**	**2.2**	**2.4**	**2.7**	**3.0**
3.3	**3.6**	**3.9**	**4.3**	**4.7**	**5.1**	**5.6**	**6.2**	**6.8**	**7.5**	**8.2**	**9.1**

Figure 2: The 24 values per decade in the **E24** families of resistors, capacitors, and inductors.

1j. Taking $R = 10\,\text{kOhm}$, what values of L and C from the E24 family of inductors and capacitors (see Figure 2) give $\omega_0 = 10$ rad/s and $Q = 5$ when $c_1 = 0$? Indicate clearly the units used.

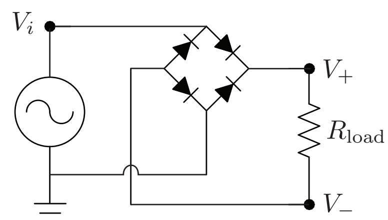

Figure 3: A full wave bridge rectifier circuit.

2. Consider next the full wave bridge rectifier circuit shown in Figure 3, where the cut-in voltage is $V_d = 1$ volt (when under positive bias) and the breakdown voltage is $V_{br} = 100$ volts (when under negative bias) for the diodes used, assuming negligible leakage current. In Q.2d and Q.2e, we will take the input voltage as $V_i(t) = 10\,\sin(\omega t)$ volts [that is, with $V_i(t)$ varying between -10 volts and $+10$ volts].

2a. Draw the circuit in Figure 3 on your exam sheet, and indicate carefully (using several arrows) by what path the current flows when $V_i = +10$ volts.

2b. Draw the circuit in Figure 3 on your exam sheet again, and now indicate carefully (using several arrows) by what path the current flows when $V_i = -10$ volts.

2c. There is a small range of inputs $-V_m \leq V_i \leq V_m$ over which something else happens. How big is V_m, what happens over this range, and why?

2d. Based on the above analysis, carefully plot $V_{\text{load}}(t) = V_+(t) - V_-(t)$ and $I_{\text{load}}(t)$ and $P_{\text{load}}(t)$ [that is, the voltage across and the current through and the power expended by the load resistor R_{load}] when $V_i(t) = 10\,\sin(\omega t)$.

2e. Finally, carefully plot $P_{\text{diodes}}(t)$ [that is, the total power expended over the 4 diodes as a function of time], as well as $\epsilon = P_{\text{diodes}}(t)/[P_{\text{load}}(t) + P_{\text{diodes}}(t)]$ [that is, the proportion of the power in this circuit that is lost in the diodes, which of course is defined only when current is actually flowing]. Does ϵ depend on the size of R_{load}? Discuss.

2f. What might this circuit be useful for? Any concerns?

Hint: for resistors, capacitors, and inductors, respectively, $\Delta V = I\,R$, $I = C\,d(\Delta V)/dt$, $\Delta V = L\,d(I)/dt$, where I denotes the current through each component and ΔV denotes the voltage drop across it in the same direction. Recall also that the power $P = I\,\Delta V$ for any component (resistor, capacitor, inductor, diode, transistor, ...).

Also, as reviewed on discord: to analyze circuits with diodes, assume at any moment that any given diode is in one of two states: (a) either the circuit outside of the diode is such that it can bring the magnitude of the voltage across the diode just high enough (V_d if under forward bias, V_{br} if under reverse bias) such that current flows through the diode, or (b) it isn't, and the current through the diode is zero. If case (a), the current through the diode is established by the rest of the circuit. If case (b), the voltage across the diode is established by the rest of the circuit.

NOTE: After the exam, please keep both this exam sheet and your page of handwritten notes (aka "cheat sheet", so you can use it on the final). **HW3** in MAE40 this year (**due Mon Aug 28 2023 at 11:59pm**) is for you to write up a neat solution set to this exam yourself, using Matlab to solve $A\mathbf{x} = \mathbf{b}$ in Q.1d, to make all plots, and anywhere else that you might find it to be useful. Though HW3 is open book and it is ok to look at things on the interweb, there is strictly **NO COLLABORATION** with other humans, or with ChatGPT, on this HW (only).