HW 1 :

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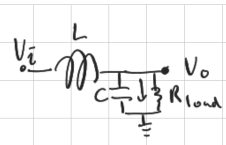
1)




$I_L = I_C + I_R$,
$L\frac{dI_L}{dt} = V_i - V_o$
$I_C = C\frac{d(V_o - 0)}{dt}$
$V_o - 0 = I_R R_{load}$

$\rightarrow$

$$\begin{aligned} I_L - I_C - I_R &= 0 \\ LsI_L + V_o &= V_i \\ CsV_o - I_C &= 0 \\ I_R R_{load} - V_o &= 0 \end{aligned}$$

$\rightarrow$

$$\underbrace{\begin{pmatrix} 1 & -1 & -1 & 0 \\ Ls & 0 & 0 & 1 \\ 0 & -1 & 0 & Cs \\ 0 & 0 & R_{load} & -1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} I_L \\ I_C \\ I_R \\ V_o \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} 0 \\ V_i \\ 0 \\ 0 \end{pmatrix}}_{b}$$

$x = A\backslash b$ $\xrightarrow{}$ solving for $V_o$ as a funct. of $V_i$, $x(4)$

$$V_o = V_i \frac{R_{load}}{CLR_{load}s^2 + Ls + R_{load}}$$

2)

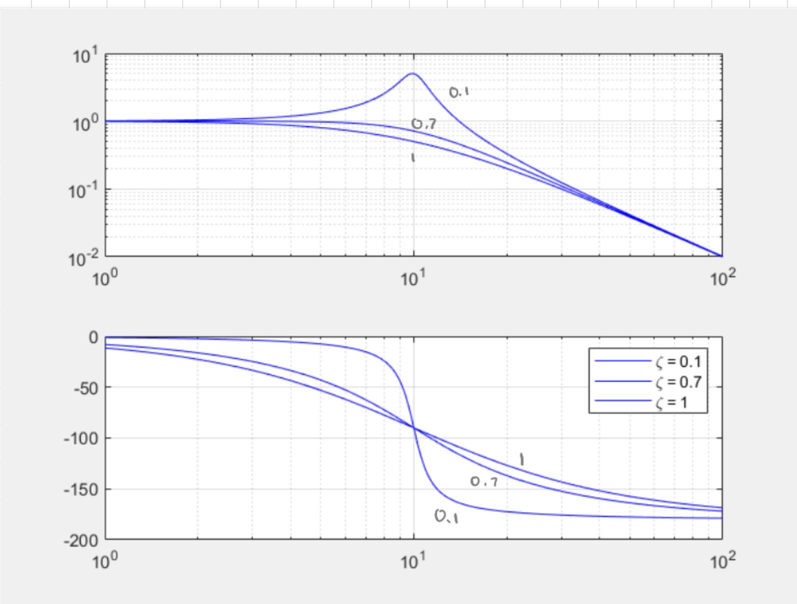
$$\frac{V_o(s)}{V_i(s)} = \frac{R_{load}/(R_{load}CL)}{s^2 + \frac{L}{R_{load}CL}s + \frac{R_{load}}{R_{load}CL}} = \frac{1/CL}{s^2 + \frac{1}{RC}s + \frac{1}{CL}} = \frac{\omega_4^2}{s^2 + 2\zeta\omega_4 s + \omega_4^2}$$

$\omega_4^2 = 1/CL \rightarrow \omega_4 = 1/\sqrt{CL}$

$2\zeta\omega_4 s = \frac{1}{RC}s$

$\zeta = \frac{1}{2RC}\frac{(CL)^{1/2}}{1} = \frac{1}{2}L^{1/2}R_{load}^{-1}C^{-1/2}$

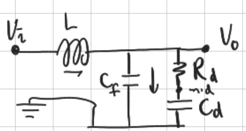
$= \frac{1}{2R_{load}}\sqrt{L/C}$




the $\zeta$ value acts like a damping coeff., with smaller values increasing the degree of "resonance" of the system near certain frequencies.

(degree in a figurative sense, not mathematically)

3)




$I_L = I_{Cf} + I_{Rd}$
$I_{Cd} = I_{Rd}$
$L\frac{dI_L}{dt} = V_i - V_o$
$C_f\frac{d(V_o - 0)}{dt} = I_{Cf}$
$I_{Rd}R_d = V_o - V_{mid}$
$C_d\frac{d(V_{mid} - 0)}{dt} = I_{Cd}$

$\xrightarrow{\mathcal{L}}$

$$\begin{aligned} I_L - I_{Cf} - I_{Rd} &= 0 \\ I_{Rd} - I_{Cd} &= 0 \\ LsI_L + V_o &= V_i \\ -I_{Cf} + C_f s V_o &= 0 \\ R_d I_{Rd} - V_o + V_{mid} &= 0 \\ -I_{Cd} + C_d s V_{mid} &= 0 \end{aligned}$$

$\rightarrow$

$$\begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ Ls & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & C_f s & 0 \\ 0 & 0 & 0 & R_d & -1 & 1 \\ 0 & 0 & -1 & 0 & 0 & C_d s \end{bmatrix} \begin{bmatrix} I_L \\ I_{Cf} \\ I_{Cd} \\ I_{Rd} \\ V_o \\ V_{mid} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_i \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$V_o = \frac{C_d R_d s + 1}{C_d C_f L R_d s^3 + (C_d + C_f)Ls^2 + C_d R_d s + 1} V_i$$

$$\frac{V_o(s)}{V_i(s)} = \frac{C_d R_d s + 1}{C_d C_f L R_d s^3 + (C_d + C_f)Ls^2 + C_d R_d s + 1} = \frac{s + (C_d R_d)^{-1}}{C_f L s^3 + \left(\frac{C_d + C_f}{C_d R_d}\right)Ls^2 + s + (C_d R_d)^{-1}} = \frac{s + \omega}{C_f L s^3 + L\omega(C_d + C_f)s^2 + s + \omega}$$

$\omega = \frac{1}{C_d R_d}$

```
F_PDF =
  RR_tf with properties:
num:[1/(Cf*L), 1/(4*Cf^2*L*(L/Cf)^(1/2))]
den:[1, 5/(4*Cf*(L/Cf)^(1/2)), 1/(Cf*L), 1/(4*Cf^2*L*(L/Cf)^(1/2))]
Continuous-time transfer function
  m=1, n=3, n_r=n-m=2, strictly proper, K=1/(Cf*L)
  z:-1/(4*Cf*(L/Cf)^(1/2))
  p:[((12*87^(1/2)*Cf^3*(L/Cf)^(3/2)*(1/(Cf^3*L^3))^(1/2) + 19)/(Cf^3*(L/Cf
```

we can see that the modified circuit now has 1 zero and 3 poles, compared to the previous which had no zeros and 2 poles. this still gives us a strictly proper transfer funct.

```
>> F_undamped
F_undamped =
  RR_tf with properties:
num:    100
den:      1    20    100
Continuous-time transfer function
  m=0, n=2, n_r=n-m=2, strictly proper, K=   100
  z:  p:   -10   -10
```

$\leftarrow$ Q2

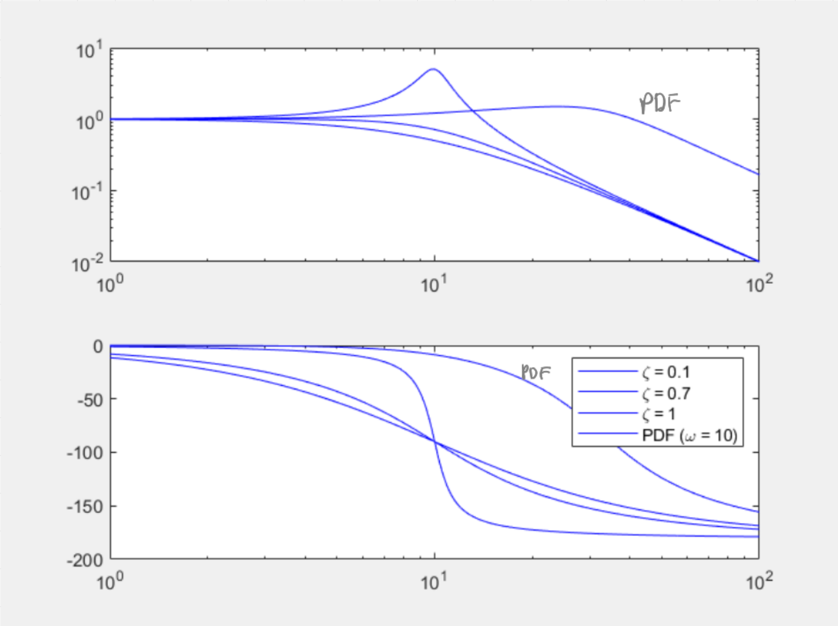
$R_d^2 C_f = L$

4) $R_d = \sqrt{L/C_f}$, $C_d = 4C_f$ $\rightarrow$ $\omega = \frac{1}{C_d R_d} = \frac{1}{4C_f\sqrt{L/C_f}} = \frac{1}{4C_f^{+1/2}L^{1/2}} = \frac{1}{4\sqrt{C_f L}}$

$$\frac{V_o(s)}{V_i(s)} = \frac{s+\omega}{C_f L s^3 + L\omega(C_d+C_f)s^2 + s + \omega} = \frac{s+\omega}{C_f L s^3 + 5C_f L\omega s^2 + s + \omega}$$

test: $\omega = 10 = 1/4\sqrt{C_f L}$ $\rightarrow$ $C_f L = \frac{1}{40^2}$ $\rightarrow$ $\frac{V_o(s)}{V_i(s)} = \frac{s+10}{\frac{1}{1600}s^3 + \frac{5}{160}s^2 + s + 10}$

```
F_PDF_2 =
  RR_tf with properties:
num:          1600        16000
den:             1           50         1600        16000
Continuous-time transfer function
  m=1, n=3, n_r=n-m=2, strictly proper, K=         1600
  z:    -10
  p: -14.8389 + 0.0000i -17.5806 -27.7340i -17.5806 +27.7340i
```




we can observe that for the same frequency $\omega = 10$, the response is more subdued, which can be attributed to larger poles (toward LHP), and as a consequence, the breakpoint has shifted further right, and while there's a complex conjugate pair, its effects are not as noticeable because of how close it is to the other breakpoint.
the "$5C_f L\omega s^2$" term in the denominator behaves similarly to $\zeta$, as reducing its value makes the complex conj. pair have a greater presence in the plot.