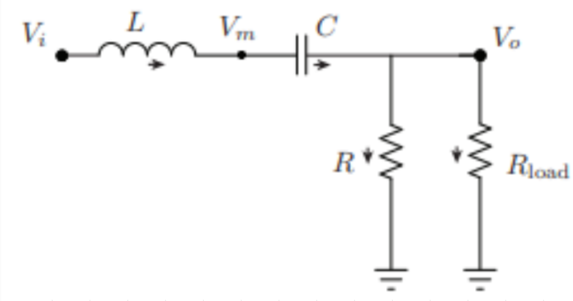
MAE 40 HW3 — I don't hate myself enough to do this all in LaTeX

Brian Kong
A16996735

1. Consider the "anti-notch" filter shown in Figure 1, assuming that:
(i) the voltage $V_i(t)$ at the input is specified precisely, regardless of the current drawn by the rest of the circuit, and
(ii) the current at the output $V_o(t)$ drives a resistive load such that $V_o = I_{\text{load}} R_{\text{load}}$, where the resistance of the load is taken as $R_{\text{load}} = R/c_1$, where R is the value of the resistor in the filter itself.

1a. Write down the component equations across each of the 4 components in this circuit, as well as the appropriate KCL, and take their Laplace transform, assuming zero initial conditions on all variables.



(1) $V_i - V_m = L\frac{dI_L}{dt} \xrightarrow{\mathcal{L}} V_i(s) - V_m(s) = Ls\, I_L(s)$

(2) $I_C = C\frac{d(V_m - V_o)}{dt} \xrightarrow{\mathcal{L}} I_C(s) = Cs(V_m - V_o)$

(3) $V_o = I_R R$

(4) $V_o = I_{load} R_{load} = I_{load} R c_1^{-1}$

(5) $I_L - I_C = 0$

(6) $I_C = I_R + I_{load}$

1b. Combine the equations governing this system by hand, and rearrange to determine $F(s) = V_o(s)/V_i(s)$ in terms of $\{R, C, L, c_1\}$. This is the transfer function of this filter circuit (including its resistive load).
Writing the denominator of this transfer function as $(s^2 + 2\zeta\omega_0 s + \omega_0^2)$, determine ω_0 and ζ in terms of $\{R, C, L, c_1\}$.
Writing the denominator of this transfer function as $(s^2 + (1/Q)\omega_0 s + \omega_0^2)$, determine Q in terms of $\{R, C, L, c_1\}$.
Finally, rewrite the entire transfer function (including the numerator) as a function of ω_0 and Q only.

$(1,2) \to 5:$ $(V_i - V_m)\frac{1}{Ls} = Cs(V_m - V_o)$

$V_i - V_m = LCs^2(V_m - V_o)$

(7) $V_i + LCs^2 V_o = (LCs^2+1)V_m$

$(2,3,4) \to 6:$ $Cs(V_m - V_o) = V_o R^{-1} + V_o R^{-1} c_1$

$CRs(V_m - V_o) = V_o(1 + c_1)$

$CRs\, V_m = V_o(CRs + 1 + c_1)$

(8) $V_m = \frac{1}{CRs} V_o(CRs + 1 + c_1)$

$(8) \to 7:$ $V_i + LCs^2 V_o = \frac{LCs^2+1}{CRs}(CRs + 1 + c_1)V_o$

$CRs V_i + LC^2Rs^2 V_o = (LCs^2+1)(CRs+1+c_1)V_o$

$CRs V_i = [LC^2Rs^2 + CRs + (LCs^2+1)(1+c_1) - LC^2Rs^2]\, V_o$

$$\frac{V_o(s)}{V_i(s)} = \frac{CRs}{LC(1+c_1)s^2 + CRs + 1 + c_1}\left(\frac{[LC(c_1+1)]^{-1}}{[LC(c_1+1)]^{-1}}\right) = \frac{\frac{R}{L(c_1+1)}s}{s^2 + \frac{R}{L(c_1+1)}s + \frac{1}{LC}}$$

$$F(s) = \frac{V_o(s)}{V_i(s)} = \frac{\alpha s}{s^2 + \alpha s + 1/LC}, \quad \alpha = \frac{R}{L(c_1+1)}$$

(to think I messed this up in the test is embarrassing)

$s^2 + \alpha s + 1 = s^2 + 2\zeta\omega_0 s + \omega_0^2$

$s^1: \alpha = 2\zeta\omega_0$

$s^0: \frac{1}{LC} = \omega_0^2$ $\quad \omega_0 = \sqrt{\frac{1}{LC}}$

$\zeta = \frac{\alpha}{2\omega_0} = \frac{\sqrt{LC}}{2}\left(\frac{R}{L(c_1+1)}\right) = \frac{R}{2(c_1+1)}\sqrt{\frac{C}{L}}$

$s^2 + \alpha s + 1 = s^2 + Q^{-1}\omega_0 s + \omega_0^2$

$s^1: \alpha = Q^{-1}\omega_0$

$s^0: \frac{1}{LC} = \omega_0^2$

$\omega_0 = \sqrt{\frac{1}{LC}}$

$Q = \omega_0/\alpha = (LC)^{-1/2} L(c_1+1)R^{-1} = \frac{c_1+1}{R}\left(\frac{L}{C}\right)^{1/2}$

$$F(s) = \frac{V_o(s)}{V_i(s)} = \frac{Q^{-1}\omega_0 s}{s^2 + Q^{-1}\omega_0 s + \omega_0^2}$$

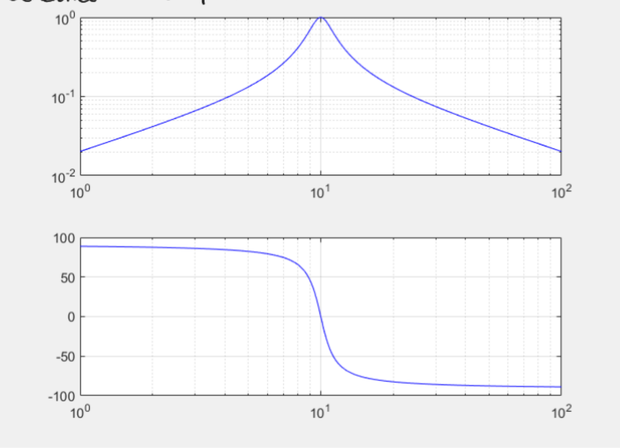
1c. Given the last result in Q.1b, write the commands in Matlab that can be used to generate the corresponding Bode plot, which for your convenience is also given for you in Figure 1.

```
%% Q1
clear, clc, close all
syms s L V_m C V_o R I_L I_C I_R I_load c_1 V_i omega_0

%x= {V_o    V_m     I_L    I_C     I_R     I_load}

A = [0      1       L*s     0       0       0;      % L*s*I_L + V_m = Vi
     -1     1       0    -1/(C*s)   0       0;      % I_C = C*s(V_m - V_o)
     1      0       0       0      -R       0;      % V_o = I_R*R
     -1     0       0       0       0       R/c_1   % V_o = I_load*R/c_1
     0      0       1      -1       0       0;      % I_L - I_C = 0
     0      0       0      -1       1       1;      % I_C = I_R + I_load
    ];
b = [V_i; 0; 0; 0; 0; 0];

x = A\b;
V_o_resp = simplify(x(1))
%{
Here we get that V_o/V_i = [C*R*V*s] / [C*L*(c_1+1)*s^2 + C*R*s + (1+c_1)]
We can simplify this with the following:
%}
Q = L/R*(c_1+1);
num1= [omega_0/Q 0]; den1 = [1 omega_0/Q omega_0^2];
% F_1 = RR_tf(num1, den1);
% substitute values for Q=5 and omega_0=10
F_1 = RR_tf(subs(num1,[Q omega_0],[5 10]), ...
            subs(den1,[Q omega_0],[5 10]));
RR_bode(F_1)
```

values of Q and ω_0 given by the following:

Figure 1: (upper-left) The "anti-notch" filter circuit considered in Q.1; (right) its associated Bode plot, and (lower-left) a close up of the magnitude part of this Bode plot, for (solid) $\{R, C, L\}$ such that $\omega_0 = 10$ rad/s and $Q = 5$ with $R_{\text{load}} = \infty$ [thus, $I_{\text{load}} = 0$], and (dot-dashed) the same $\{R, C, L\}$ but with $R_{\text{load}} = R/c_1$ finite [thus, $I_{\text{load}} \neq 0$].

Generated bode plot



1e. In the future, I hope you will choose to consistently solve systems of algebraic equations on a computer (as in Q.1d) rather than solving them solely by hand (as in Q.1b). Why?

1f. In the future, I also hope you will choose to use GitHub for anything important that you do on a computer (e.g., the writing of code, documents, ...). Why? (Hint: remember Nickel's computer, may God have mercy on its soul...)

1e) It is much more preferable to solve systems of algebraic eqn's on a computer as it can eliminate computational errors and allow quicker and complex calculations for a similar amount of effort. this lets us focus less on lin. algebra and more on the actual problem at hand.

1f) Github lets us reap benefits from version control and having a cloud-based repository:
1. transfer and sync files between devices
2. data recovery and controlling changes made locally and different forks.

if you want something more suited to developing images and containers in order to encapsulate everything needed to run a software in development, then stuff like Docker exist, but I irrationally hate it.

1g. Assuming $c_1 = 0$ (i.e., $R_{\text{load}} = \infty$, so negligible output current), and that $\{R, C, L\}$ are such that $\omega_0 = 10$ rad/s and $Q = 5$, the Bode plot of $F(s)$ is given as shown as the solid curve in the Bode plot in Figure 1. Explain the behavior of this Bode plot, and discuss how it is similar to, and different from, the notch filter considered in Quiz 5 and problem 1 of HW2. Again, this filter was designed with $Q = 5$. How might you define the BW (bandwidth) of this filter, and how does what you see in the closeup of the Bode plot provided reconcile with this value of Q?

the notch filter in Quiz 5 and Q1 of HW2 are 2nd order like this system, giving a peak or trough at a given frequency, and a phase shift bound within $\angle G(j\omega) \in (-\frac{\pi}{2}, \frac{\pi}{2})$.

however, this system affects the response for the entire range of frequencies, fading the gain for frequencies the further they are from ω_0, having an anti-notch effect where we want a response only near ω_0, rather than wanting to lessen the response near ω_0 and leaving the rest untouched.

to define Q, we decide a set bandwidth w/ a ω_H and ω_L as bounds, then compute $Q = \frac{\omega_0}{\omega_H - \omega_L}$.
ω_L and ω_H are determined by the upper and lower frequencies for which the magnitude of the bode plot is equal to $\frac{\sqrt{2}}{2}$
for the solid curve, we observe that $\omega_L \simeq 9.05$, $\omega_H \simeq 11.05$, $\omega_0 = 10$ $\rightarrow$ $Q = \frac{10}{11.05 - 9.05} = 5$
to obtain BW from Q and ω_0, we simply solve for $\omega_H - \omega_L = \omega_0 / Q = 10/5 = 2$, and then center ω_L and ω_H around ω_0.

1h. Taking the same $\{R, C, L\}$ values as in Q.1g, Figure 1 also shows, as (dot-dashed), the Bode plot of this filter when $c_1 \neq 0$ (i.e., R_{load} is now finite, so there is now a non-negligible output current). Based on your discussion in Q.1g of the BW of the notch, as well as your derivation in Q.1b of how Q depends on $\{R, C, L, c_1\}$, what value of c_1 was used when plotting the (dot-dashed) curve in Figure 1?

$$Q_2 = \frac{\omega_0}{BW} = \frac{10}{10.5 - 9.5} = 10 = 2Q,$$

$$\frac{1}{R}\left(\frac{L}{C}\right)^{1/2}(c_{1,2}+1) = 2(c_{1,1}+1)\frac{1}{R}\left(\frac{L}{C}\right)^{1/2}, \quad C, L, R \text{ are const.}$$

$$1 + c_{1,2} = 2(\cancel{c_{1,1}}+1), \quad c_{1,1} \text{ is } 0 \text{ in the solid curve}$$

$$c_{1,2} = 1$$

1i. What might this filter be useful for?

this filter would be useful for isolating data from a narrow range and can have applications in the biomedical industry (i.e. EKG's) and signal processing.

1j. Taking $R = 10\,$kOhm, what values of L and C from the E24 family of inductors and capacitors (see Figure 2) give $\omega_0 = 10$ rad/s and $Q = 5$ when $c_1 = 0$? Indicate clearly the units used.

1.0	**1.1**	**1.2**	**1.3**	**1.5**	**1.6**	**1.8**	**2.0**	**2.2**	**2.4**	**2.7**	**3.0**
3.3	**3.6**	**3.9**	**4.3**	**4.7**	**5.1**	**5.6**	**6.2**	**6.8**	**7.5**	**8.2**	**9.1**

Figure 2: The 24 values per decade in the **E24** families of resistors, capacitors, and inductors.

$$Q = \frac{c_1+1}{R}\left(\frac{L}{C}\right)^{1/2}$$

$$5 = \frac{1}{10^4}\left(\frac{L}{C}\right)^{1/2}$$

$$5 \times 10^4 = (L/C)^{1/2}$$

$$2.5 \times 10^9 C = L$$

the decade of L is 9 times that of C, and the ratio of C:L must be 5:2
we can brute force this by finding pairs that match E24 values after multiplying each E24 val by 2.5 and ignoring decade offsets ($x \times 10$, $\times 100$, etc)
the working value pairs (C, L) are:
(1.2, 3.0E9),
(3.0, 7.5E9),
w/ C in F and L in H.

2. Consider next the full wave bridge rectifier circuit shown in Figure 3, where the cut-in voltage is $V_d = 1$ volt (when under positive bias) and the breakdown voltage is $V_{br} = 100$ volts (when under negative bias) for the diodes used, assuming negligible leakage current. In Q.2d and Q.2e, we will take the input voltage as $V_i(t) = 10\sin(\omega t)$ volts [that is, with $V_i(t)$ varying between -10 volts and $+10$ volts].

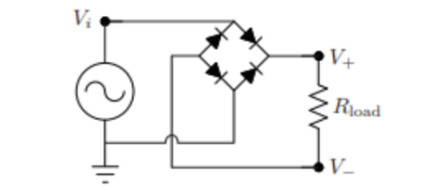
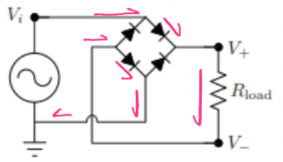
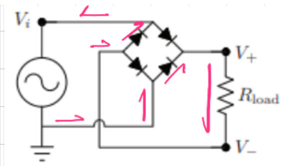


Figure 3: A full wave bridge rectifier circuit.

2a. Draw the circuit in Figure 3 on your exam sheet, and indicate carefully (using several arrows) by what path the current flows when $V_i = +10$ volts.



2b. Draw the circuit in Figure 3 on your exam sheet again, and now indicate carefully (using several arrows) by what path the current flows when $V_i = -10$ volts.



2c. There is a small range of inputs $-V_m \le V_i \le V_m$ over which something else happens. How big is V_m, what happens over this range, and why?

in this small range there's not enough voltage to overcome the cut-in voltage of the two diodes in series, there is no current flowing in this range of inputs as a result. → $V_m = 2V_d$

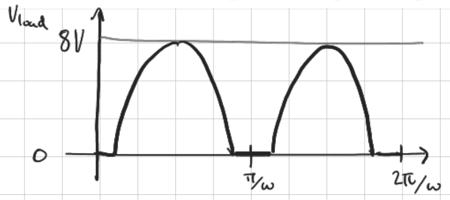
2d. Based on the above analysis, carefully plot $V_{load}(t) = V_+(t) - V_-(t)$ and $I_{load}(t)$ and $P_{load}(t)$ [that is, the voltage across and the current through and the power expended by the load resistor R_{load}] when $V_i(t) = 10\sin(\omega t)$.

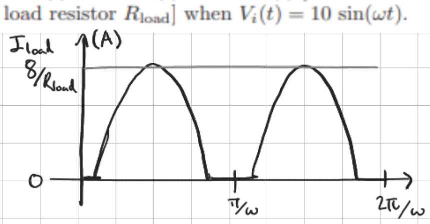
$V_{load} = V_i - 2V_d$ for $|V_i| > V_m$

$V_{load,max} = 10 - 2 = 8$

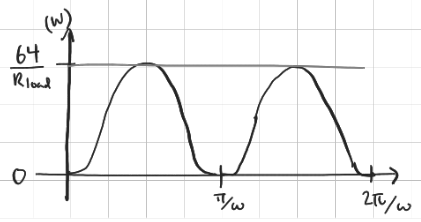
$I_{load} = V_{load}/R_{load}$ for $|V_i| > V_m$,

$= 0$ for $|V_i| \le V_m$



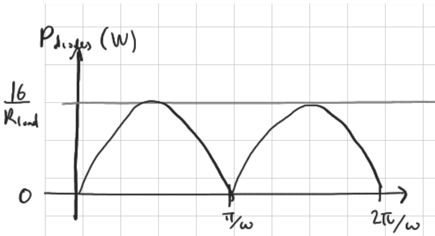


$P_{load} = I_{load}V_{load} = I_{load}^2 R_{load}$

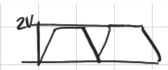


the points near 0 are smoother as we multiply fractions

2e. Finally, carefully plot $P_{diodes}(t)$ [that is, the total power expended over the 4 diodes as a function of time], as well as $\epsilon = P_{diodes}(t)/[P_{load}(t) + P_{diodes}(t)]$ [that is, the proportion of the power in this circuit that is lost in the diodes, which of course is defined only when current is actually flowing]. Does ϵ depend on the size of R_{load}? Discuss.

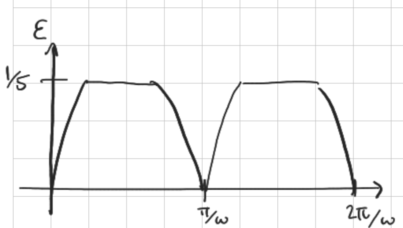


$V_{diodes} = 2V$ for when $|V_i(t)| \ge V_m$



$P_{diodes} = IV = V^2/R$ $\quad I_{diodes} = I_{load}$

$= \frac{V_{load}}{R_{load}} \cdot V_{diodes}$



$$\epsilon = \frac{P_{diodes}}{P_{load} + P_{diodes}} = \frac{I_{load}V_{diodes}}{I_{load}V_{load} + I_{load}V_{diodes}} = \frac{V_{diode}}{V_{load} + V_{diodes}}$$

$$\epsilon_{max} = \frac{2}{8+2} = 1/5$$

ϵ doesn't depend on R_{load}, because the I_{load} (and hence R_{load}) are able to be factored out of the proportion, making it purely reliant on the voltage drops.

2f. What might this circuit be useful for? Any concerns?

anything that needs a DC while using an AC source. chargers, fans, almost any device we plug in the outlet. however, this doesn't give us a consistent I, V, or P plot, and also has power loss from the diodes.