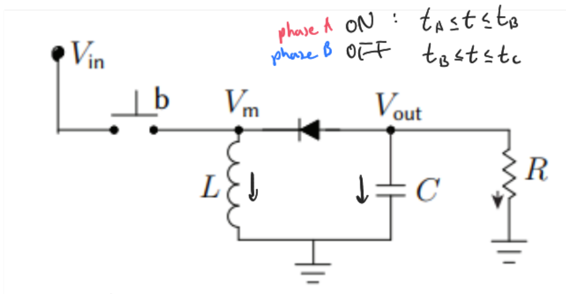

phase A ON : $t_A \le t \le t_B$
phase B OFF $t_B \le t \le t_C$

<50% duty : buck
>50% duty : boost

4 possible scenarios:

Brian Kong A16996735

non zero ICs: $f' = sF(s) - f(0)$

(1) ON Phase w/ no diode flow

$$\left.\begin{aligned} V_m &= V_i/s \\ V_m - 0 &= L[sI_L - I_L^A] \\ I_c &= C[sV_o - V_o^A] \\ V_o &= I_R R \\ 0 &= I_c + I_R \end{aligned}\right\} \quad \begin{aligned} V_o(s) &= \frac{CRV_o^A}{CRs+1}\left(\frac{1/CR}{1/CR}\right) = \frac{V_o^A}{s + 1/CR} \\ I_L(s) &= (V_i + I_L^A Ls)/(Ls^2) \\ V_o(t) &= V_o^A e^{-(t-t_A)/CR} \\ I_L(t) &= I_L^A + V_i(t-t_A)/L \end{aligned}$$

$$\left.\begin{aligned} V_o^B &= V_o(t_B) = V_o^A e^{-(t_B - t_A)/CR} \\ I_L^B &= I_L(t_B) = I_L^A + V_i(t_B - t_A)/L \end{aligned}\right\} \text{ IC's for next phase}$$

(2): ON phase w/ diode flow

$$\left.\begin{aligned} V_o - V_m &= V_d/s \\ V_m - 0 &= L[sI_L - I_L^A] \\ I_c &= C[sV_o - V_o^A] \\ -I_c - I_R &= I_L \\ V_o &= I_R R \end{aligned}\right\} \quad \begin{aligned} V_o(s) &= \frac{CLRV_o^A s^2 - LRI_L^A s + RV_d}{s(CLRs^2 + Ls + R)} \\ &= \frac{V_o^A s^2 - I_L^B C^{-1} s + C^{-1}L^{-1}V_d}{s(s^2 + C^{-1}R^{-1}s + C^{-1}L^{-1})} \\ V_o(s) &= \frac{b_2 s^2 + b_1 s + b_0}{s[s^2 + a_1 s + a_0]} \end{aligned}$$

$$I_L(s) = \frac{CLRI_L^A s^2 + (CR(V_o^A - V_d) + LI_L^A)s - V_d}{s(CLRs^2 + Ls + R)} = \frac{c_2 s^2 + c_1 s + c_0}{s[s^2 + a_1 s + a_0]} = \frac{\text{'' + '' + ''}}{s[(s+\sigma)^2 + \omega_d^2]}$$

$$= \frac{C_2}{s} + C_1 \frac{s+\sigma}{(s+\sigma)^2 + \omega_d^2} + C_0 \frac{\omega_d}{(s+\sigma)^2 + \omega_d^2}$$

$b_2 = V_o^A$, $a_1 = C^{-1}R^{-1}$, $\sigma = a_1/2 = \frac{1}{2CR}$, $c_2 = I_L^A$

$b_1 = -I_L^A/C$, $a_0 = C^{-1}L^{-1}$, $\omega_d = (a_0 - a_1^2/4)^{1/2}$, $c_1 = I_L^A/CR + (V_o^A - V_d)/L$

$b_0 = V_d/(CL)$, $= \left(\frac{1}{CR} - \frac{1}{4C^2R^2}\right)^{1/2}$, $c_0 = -V_d/(CLR)$

following steps for $\{B_2, B_1, B_0\}$:

$C_2 = -V_d/R$

$C_1 = I_L^A + V_d/R$

$C_0 = \frac{c_1 - c_2\sigma}{\omega_d} - C_2\frac{\sigma}{\omega_d}$

$$V_o(s) = \frac{B_2}{s} + B_1 \frac{s+\sigma}{(s+\sigma)^2 + \omega_d^2} + B_0 \frac{\omega_d}{(s+\sigma)^2 + \omega_d^2} \rightarrow V_o(t) = B_2 + B_1 e^{-\sigma(t-t_A)} \cos[\omega_d(t-t_A)] + B_0 e^{-\sigma(t-t_A)} \sin[\omega_d(t-t_A)]$$

$$B_2 = \lim_{s\to 0} s\left(\frac{b_2 s^2 + b_1 s + b_0}{s[(s+\sigma)^2 + \omega_d^2]}\right) = \frac{b_0}{\sigma^2 + \omega_d^2} = V_d$$

$$B_1 = b_2 - B_2 = V_o^A - V_d$$

$$B_0 = \frac{b_1 - b_2\sigma}{\omega_d} - B_2\frac{\sigma}{\omega_d} = -(I_L^A/C + V_o^A\sigma)/\omega_d - V_d\frac{\sigma}{\omega_d}$$

$$I_L(t) = C_2 + C_1 e^{-\sigma(t-t_A)} \cos[\omega_d(t-t_A)] + C_0 e^{-\sigma(t-t_A)} \sin[\omega_d(t-t_A)]$$

Find new IC's: $V_o^B = V_o(t_B)$, $I_L^B = I_L(t_B)$

(3) OFF phase w/o diode flow

$$\left.\begin{aligned} V_m - 0 &= L(sI_L - I_L^B) \\ I_c + I_R &= 0 \quad \text{KCL @ } V_o \text{ node} \\ V_o &= I_R R \\ -I_L &= I_c + I_R \\ I_c &= C(sV_o - V_o^B) \end{aligned}\right\} \quad \begin{aligned} V_o(s) &= \frac{CRV_o^B}{CRs+1} = \frac{V_o^B}{s + 1/CR} \xrightarrow{\mathcal{L}^{-1}} V_o(t) = V_o^B e^{-(t-t_B)/CR} \\ I_L(s) &= 0 \rightarrow I_L(t) = 0 \\ V_o^C &= V_o(t_C) = V_o^B e^{-(t_C - t_B)/CR} \end{aligned}$$

for $t \in [t_B, t_C]$

(4) OFF phase w/ diode flow

$$\left.\begin{aligned} V_o - V_m &= V_d/s \\ V_m - 0 &= L(sI_L - I_L^B) \\ I_c &= C(sV_o - V_o^B) \\ I_L &= -I_R - I_c \\ V_o &= I_R R \end{aligned}\right\} \quad \begin{aligned} V_o(s) &= \frac{CLRV_o^B s^2 - LRI_L^B s + RV_d}{s(CLRs^2 + Ls + R)} = \frac{V_o^B s^2 - I_L^B C^{-1} s + V_d C^{-1}L^{-1}}{s(s^2 + C^{-1}R^{-1}s + C^{-1}L^{-1})} = \frac{b_2 s^2 + b_1 s + b_0}{s(s^2 + a_1 s + a_0)} \\ & s[(s+\sigma)^2 + \omega_d^2] \\ I_L(s) &= \frac{CLRI_L^A s^2 + (CR(V_o^A - V_d) + LI_L^A)s - V_d}{s(CLRs^2 + Ls + R)} = \frac{c_2 s^2 + c_1 s + c_0}{s[s^2 + a_1 s + a_0]} \end{aligned}$$

We notice that the sys of eqn and thus our functions are equivalent to those in (2),

$a_1 = C^{-1}R^{-1}$, $b_2 = V_o^B$, $c_2 = I_L$

$a_0 = C^{-1}L^{-1}$, $b_1 = -I_L^B/C$, $c_1 = I_L/CR + (V_o - V_d)/L$

$\sigma = a_1/2 = \frac{1}{2CR}$, $b_0 = V_d/(CL)$, $c_0 = -V_d/(CLR)$

$\omega_d = (a_0 - a_1^2/4)^{1/2}$

$= \left(\frac{1}{CR} - \frac{1}{4C^2R^2}\right)^{1/2}$

hence we get similar results w/ the PFE and $\mathcal{L}^{-1}$:

$B_2 = V_d$

$B_1 = V_o^B - V_d$

$$B_0 = \frac{b_1 - b_2\sigma}{\omega_d} - B_2\frac{\sigma}{\omega_d} = \frac{I_L^B/C + V_o^B/(2CR)}{\left(\frac{1}{CL} - \frac{1}{4C^2R^2}\right)^{1/2}}$$

$C_2 = -V_d/R$

$C_1 = I_L^B + V_d/R$

$C_0 = \frac{c_1 - c_2\sigma}{\omega_d} - C_2\frac{\sigma}{\omega_d}$

for $t \in [t_B, t_C]$

$$V_o(t) = B_2 + B_1 e^{-\sigma(t-t_B)} \cos[\omega_d(t-t_B)] + B_2 e^{-\sigma(t-t_B)} \sin[\omega_d(t-t_B)],$$

$$I_L(t) = C_2 + C_1 e^{-\sigma(t-t_B)} \cos[\omega_d(t-t_B)] + C_2 e^{-\sigma(t-t_B)} \sin[\omega_d(t-t_B)]$$

scenarios (2) and (4) are when $V_o > V_m + V_d$; when the V_o is able to overcome the cutin voltage to have current thru the diode

for (1), current flows through V_{in} to gnd only through the L, and C and R are in series, driven by the discharge of the capacitor. I_L increases over time (so we can't just leave the V_{in} on or else it breaks), the V_o^B and I_L^B carry on to either OFF phase (3 or 4).

for (2), current flows through the L from both the diode and V_{in} into gnd, and from C and R into the diode. this would occur w/ sufficient charge built up in the C such that $V_o^A > V_m + V_d$.

the V_o^B and I_L^B carry on to either OFF phase (3 or 4).

for (3), the setup is almost identical to (1), w/ R and C in series, and the L being isolated. however, the difference is that V_{in} is no longer connected, resulting in the I_L starting to decrease (but not instant since magnetic field)

for (4), the setup is almost identical to (2), w/ current flowing from R and C into the diode, and the inductor maintaining a current that flows into ground.

I_L remains (+) and lags the voltage drop and its field induces a ΔV in response to the $\frac{dI}{dt}$ being (−), in turn $\Delta(0 - V_m)$ is (−) $\therefore$ V_m being (−), and $V_{out} = 0$ puts the diode in fwd bias and lets current flow through assumption for (4).

As a result, the capacitor accumulates (+) charge on the gnd terminal, and its voltage drop $\Delta(0 - V_{out})$ increases in magnitude

for periodic analysis where we reach a stabilized V_o, $V_o^A = V_o^C$ and $I_L^A = I_L^C$ We can solve by using

$V_o(t_A) = V_o(t_C)$
$I_L(t_A) = I_L(t_C)$
$pA = t_B - t_A = f^{-1} D$
$pB = t_C - t_B = (1-D)/f$

w/ the functions and inputs:
$V_o^B(pA, V_o^A)$, $V_o^C(pB, V_o^B)$
$I_L^B(pA, I_L^A)$, $I_L^C(pB, I_L^B)$

evaluate the following
$V_o^C(pB, V_o^B(pA, V_o^A)) = V_o^A$
$I_L^C(pB, I_L^B(pB, I_L^A)) = I_L^A$

$V_o^A = B_2 + e^{-\sigma pB}[B_1 \cos\omega pB + B_0 \sin\omega pB]$, w/ B_1, B_0, C_1, C_0 needing $V_o^B(pA, V_o^A)$ & $I_L^B(pA, I_L^A)$ simplified
$I_L^A = C_2 + e^{-\sigma pB}[C_1 \cos\omega pB + C_0 \sin\omega pB]$ and then isolating V_o^A & I_L^A for a sys of 2 eqn and 2 unknown.

the on and off phases are a decaying exponential or sinusoid, and a higher duty cycle lets more charge accumulate on the C and gives more time for a stronger magnetic field to form from L, resulting in behavior like a boost converter.