1.2 James Rich.

(a)

$$\left(\frac{\dot a}{a}\right)^2 = H_0^2\left(\Omega_M \hat a^{-3} + \Omega_R \hat a^{-4} + \Omega_\Lambda + (1-\Omega_T)\right)$$

where $\hat a = \dfrac{a}{a_0}$.

(a) Vacuum epoch.

approximation of only dominant term in the expression.

$$\Rightarrow \left(\frac{\dot a}{a}\right)^2 = H_0^2 \Omega_\Lambda \quad \text{or} \quad \frac{da}{a\,dt} = H_0\sqrt{\Omega_\Lambda}$$

$$\text{or} \int_{a_{vac.}}^{a} \frac{da}{a} = \int_{t_{vac.}}^{t} H_0\sqrt{\Omega_\Lambda}\, dt$$

where

$$a_{vac.} = a_0\left(\frac{\Omega_M}{\Omega_\Lambda}\right)^{1/3}$$

$$\text{or} \quad \ln\left(\frac{a}{a_{vac.}}\right) = H_0\sqrt{\Omega_\Lambda}\,(t - t_{vac.})$$

$$\Rightarrow t - t_{vac.} = \frac{1}{H_0\sqrt{\Omega_\Lambda}} \ln\left(\frac{a}{\left(\frac{\Omega_M}{\Omega_\Lambda}\right)^{1/3}}\right)$$

for time spent $t = t_0$, $a = a_0 = 1$

$$\Rightarrow t_0 - t_{vac.} = \frac{1}{H_0\sqrt{\Omega_\Lambda}} \ln\left(\left(\frac{\Omega_\Lambda}{\Omega_M}\right)^{1/3}\right)$$

or,

$$t_0 - t_{vacc.} = \frac{1}{3H_0\sqrt{\Omega_\Lambda}} \ln\left(\frac{\Omega_\Lambda}{\Omega_M}\right)$$

putting $\Omega_\Lambda = 0.73$, $\Omega_M = 0.27$

& $\frac{1}{H_0} = t_H$, we get

$$t_0 - t_{vacc.} = \frac{1}{3\sqrt{0.73}} \ln\left(\frac{0.73}{0.27}\right) t_H$$

$$= 0.39\, t_H$$

(b) in the matter dominated epoch

approximation of only dominant term in the expression.

$$\frac{\dot{a}^2}{a^2} = H_0^2 \frac{\Omega_M}{a^3}.$$

or, $$\dot{a}^2 = H_0^2 \frac{\Omega_M}{a}$$

or, $$\frac{da}{dt} = \frac{H_0\sqrt{\Omega_M}}{\sqrt{a}}$$

or. $$\int_{a_{eq.}}^{a_{vac.}} \sqrt{a}\, da = H_0\sqrt{\Omega_M}\int_{t_{eq.}}^{t_{vac.}} dt$$

or. $$\frac{2}{3}\left(a_{vac.}^{3/2} - a_{eq.}^{3/2}\right) = H_0\sqrt{\Omega_M}\left(t_{vac.} - t_{eq.}\right)$$

or, $t_{vac.} - t_{eq.} = \frac{1}{H_0\sqrt{\Omega_M}} \frac{3}{2}\left(a_{vacc.}^{3/2} - a_{eq.}^{3/2}\right)$

using $a_{eq.} = \frac{\Omega_R}{\Omega_M} \simeq 10^{-4}$

$a_{eq.}^{3/2} \simeq 10^{-6}$ (ignoring)

$t_{vac.} - t_{eq.} = \frac{2 a_{vacc.}}{3 H_0 \sqrt{\Omega_M}}$

$= \frac{2}{3}\left(\frac{\Omega_M}{\Omega_\Lambda}\right)^{1/2} \times \frac{t_H}{\sqrt{\Omega_M}}$

$= \frac{2}{3} \frac{t_H}{\sqrt{\Omega_\Lambda}} = 0.78\, t_H$

$\because \Omega_\Lambda = 0.73$

(c) in the radiation dominated epoch, with the same approx.

$$\frac{\dot{a}^2}{a^2} = H_0 \frac{\Omega_R}{a^4} \quad \text{or,} \quad \dot{a}^2 = H_0^2 \frac{\Omega_R}{a^2}$$

$$\Rightarrow \frac{da}{dt} = \frac{H_0 \sqrt{\Omega_R}}{a}$$

$$\Rightarrow \int_{a_{inf.}}^{a_{eq.}} a \, da = H_0 \sqrt{\Omega_R} \int_{t_{inf.}}^{t_{eq.}} dt$$

$$\Rightarrow \frac{a_{eq.}^2 - a_{inf.}^2}{2} = H_0 \sqrt{\Omega_R} \left(t_{eq.} - t_{inf.}\right)$$

$$\text{or,} \quad t_{eq.} - t_{inf.} = \frac{1}{2 H_0 \sqrt{\Omega_R}} \left(a_{eq.}^2 - a_{inf.}^2\right)$$

$$a_{inf.} = 10^{-28} a_0$$

$$a_{inf.}^2 = 10^{-56} a_0^2 \quad (\text{ignoring})$$

$$\Rightarrow t_{eq.} - t_{inf} \approx \frac{a_{eq}^2}{2 H_0 \sqrt{\Omega R}}$$

$$a_{eq.} = \frac{\Omega R}{\Omega M}$$

$$\Rightarrow t_{eq.} - t_{inf.} \approx \frac{1}{2} \frac{\Omega R^{3/2}}{\Omega M} t_H$$

$$= \frac{1}{2} (a_{eq.}) \sqrt{\Omega R}\, t_H$$

$$= \frac{1}{2} \times 10^{-6} \sqrt{\Omega R}\, t_H$$

using $\Omega R = 1.68 \times 5 \times 10^{-5}$

$$t_{eq.} - t_{inf.} \approx 10^{-9} t_H$$

?

NOT SURE

(d) Nuclei formation happened in radiation epoch

hence, copying the above expression, we get

$$t_{nuc.} - t_{inf.} = \frac{1}{2H_0\sqrt{\Omega_R}}\left(a^2_{nuc.} - a^2_{inf.}\right)$$

using $a_{nuc.} = 3 \times 10^{-9}\, a_0$

& $a_{inf.} = 10^{-56}\, a_0$

$$t_{nuc.} - t_{inf.} = \frac{(9 \times 10^{-18} - 10^{-56})}{2H_0\sqrt{\Omega_R}}$$

using $\Omega_R = 8.4 \times 10^{-5}$

$$t_{nuc.} - t_{inf.} = \frac{4.5 \times 10^{-18}}{9 \times 10^{-3}} t_H$$

$$= 0.5 \times 10^{-15} t_H$$

$$= 0.5 \times 10^{-15} \times 7.31 \times 10^{15}$$

$$= 3.22 \text{ minutes}$$