Q.4.

$$n_\gamma = \frac{2\zeta(3)\,(K_B T)^3}{\pi^2 \hbar^3 c^3}$$ → photon number density

units should be $1/m^3$ $\left(\frac{N}{V}\right)$ and dimensions should be $\frac{1}{\ell^3}$.
↳ length

now, dimensions

numerator → $[(K_B T)^3] = [E^3]$
↳ Energy

& ~~using~~ denominator → $[\hbar^3 c^3]$

from the energy $E = \hbar f$, we can see
energy, frequency

$$[\hbar] = \left[\frac{E}{f}\right] = [Et]$$

$$\therefore [\hbar^3 \cdot c^3] = \cancel{E^3 \frac{\ell^3 \cdot t^3}{t^3}} \; E^3 \cdot t^3 \cdot \frac{\ell^3}{t^3} = E^3 \ell^3.$$

$$\Rightarrow [n_\gamma] = \left[\frac{2\zeta(3)\,(K_B T)^3}{\pi^2 \hbar^3 c^3}\right] = \frac{E^3}{E^3 \cdot \ell^3} = \frac{1}{\ell^3}$$

$$[n_\gamma] = \frac{1}{\ell^3} \Rightarrow$$ units will be $1/m^3$

$$\rho_\gamma = \frac{\pi^2}{15} \frac{(k_B T)^4}{\hbar^3 c^3 \cdot c^2} \quad kg/m^3$$

dimensions of $\rho_\gamma$ should be $\left[\frac{(k_B T)^4}{\hbar^3 c^5}\right] = \frac{M}{l^3}$

$$\frac{\text{mass}}{\text{length}^3} \quad \text{or} \quad \frac{\text{Energy}}{\text{speed}^2 \ \text{length}^3}$$

$\because E = mc^2$

$[m] = [E/c^2]$

now, numerator $\rightarrow [k_B T^4] = E^4$

& denominator $\rightarrow \hbar^3 c^3 \cdot c^2 = [\hbar^3 c^3] \cdot (\text{speed})^2$

$\because [\hbar^3 c^3] = E^3 \cdot l^3$ (from $n_\gamma$)

$\Rightarrow [\hbar^3 c^3 \cdot c^2] = E^3 \cdot l^3 \cdot (\text{speed})^2$

$$\Rightarrow [\rho_\gamma] = \frac{\text{Energy}^4}{\cancel{\text{Energy}^3} \cdot l^3 (\text{speed})^2} = \left[\frac{\text{Energy}}{(\text{speed})^2}\right] \cdot \frac{1}{l^3}$$

$$[\rho_\gamma] = \frac{\text{mass}}{l^3}$$

& units should be $kg/m^3$

now, in SI units of mass.

$$\rho_\gamma = \frac{\pi^2}{15}\frac{(K_B T_\gamma)^4}{\hbar^3 c^3 \cdot c^2} = \frac{\pi^2 \times (1.38\times10^{-23} \times 2.725)^4}{15\times(1.055\times10^{-24})^3 \times (3\times10^8)^5}\ kg/m^3$$

$$\rho_\gamma = 4.611 \times 10^{-31}\ kg/m^3.$$

critical density of universe is $\rho_c = 9.47\times10^{-27}\ kg/m^3$.

$$\Rightarrow \Omega_\gamma = \frac{4.611\times10^{-31}}{9.47\times10^{-27}} = 0.49\times10^{-4}$$

$\therefore$ the contribution of CMB photons is close to $0.5\times10^{-4}$.