Q.4.

$$n_\gamma = \frac{2\zeta(3)(K_BT)^3}{\pi^2\hbar^3c^3} \rightarrow \text{photon number density}$$

units should be $1/m^3$ $\left(\frac{N}{V}\right)$ and dimensions should be $\frac{1}{\ell^3}$ $\hookrightarrow$ length

now, <u>dimensions</u>

numerator $\rightarrow [(K_BT)^3] = [E^3]$ $\hookrightarrow$ Energy

& ~~using~~ denominator $\rightarrow [\hbar^3 c^3]$

from the energy $E = \hbar f$, we can see

$E \rightarrow$ energy, $f \rightarrow$ frequency

$$[\hbar] = \left[\frac{E}{f}\right] = [Et]$$

$$\therefore [\hbar^3 \cdot c^3] = \cancel{E^3 \cdot \frac{\ell^3 \cdot t^3}{t^3}} \; E^3 \cdot t^3 \cdot \frac{\ell^3}{t^3} = E^3\ell^3.$$

$$\Rightarrow [n_\gamma] = \left[\frac{2\zeta(3)(K_BT)^3}{\pi^2\hbar^3c^3}\right] = \frac{E^3}{E^3\cdot\ell^3\cdot 1} = \frac{1}{\ell^3}$$

$$[n_\gamma] = \frac{1}{\ell^3} \Rightarrow \text{units will be } 1/m^3$$

$$\rho_\gamma = \frac{\pi^2}{15} \frac{(k_B T)^4}{\hbar^3 c^3 \cdot c^2} \quad kg/m^3$$

dimensions of ρ_γ should be $\left[\frac{(k_B T)^4}{\hbar^3 c^5}\right] = \frac{M}{l^3}$

$$\frac{\text{mass}}{\text{length}^3} \quad \text{or} \quad \frac{\text{Energy}}{\text{speed}^2 \ \text{length}^3}$$

$\because \ E = mc^2$

$[m] = [E/c^2]$

now, numerator $\rightarrow$ $[k_B T^4] = E^4$

& denominator $\rightarrow$ $\hbar^3 c^3 \cdot c^2 = [\hbar^3 c^3] \cdot (\text{speed})^2$

$\because [\hbar^3 c^3] = E^3 \cdot l^3$ (from n_γ)

$\Rightarrow [\hbar^3 c^3 \cdot c^2] = E^3 \cdot l^3 \cdot (\text{speed})^2$

$$\Rightarrow [\rho_\gamma] = \frac{\text{Energy}^4}{\cancel{\text{Energy}^3} \cdot l^3 (\text{speed})^2} = \left[\frac{\text{Energy}}{(\text{speed})^2}\right] \cdot \frac{1}{l^3}$$

$$[\rho_\gamma] = \frac{\text{mass}}{l^3}$$

& units should be kg/m^3

~~Week2~~ $n_\gamma = \dfrac{g\zeta(3)(K_B T_\gamma)^3}{\pi^2 \hbar^3 c^3}$ $\zeta(3) = 1.202$

$$= \frac{2 \times 1.202 \times (1.38 \times 10^{-23} \times 2.725)^3}{(3.14159)^2 \times (1.055 \times 10^{-34})^3 \times (3 \times 10^8)^3}$$

$$= 4.21 \times 10^8 / m^3$$

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in SI units of mass,

$$\rho_\gamma = \frac{\pi^2}{15} \times \frac{(K_B T_\gamma)^4}{\hbar^3 c^5} = \frac{\pi^2 \times (1.38 \times 10^{-23} \times 2.725)^4}{15 \times (1.055 \times 10^{-34})^3 \times (3 \times 10^8)^5}$$

$$\rho_\gamma = 4.611 \times 10^{-31} \ kg/m^3$$

critical density of the universe is

$$\rho_c = 9.4 \times 10^{-27} \ kg/m^3.$$

$$\Rightarrow \Omega_\gamma = \frac{4.611 \times 10^{-31}}{9.4 \times 10^{-27}} = 0.51 \times 10^{-4}$$

$\hookrightarrow$ contribution of gamma photons.