Ans) For non relativistic case, we can derive the temperature evolution of the Universe by assuming the ~~adiabatic~~ expansion to be adiabatic process.

$$P = K\rho^{\gamma}$$

γ : adiabatic index ; K : constant

P : pressure ; ρ : density

Pressure of an ideal gas can be written as :

$$P = \frac{\rho k_B T}{\bar{m}}$$

$$\Rightarrow \quad \frac{\rho k_B T}{\bar{m}} = K\rho^{\gamma}$$

$$\Rightarrow \quad T \propto \rho^{\gamma - 1}$$

We also know that for non relativistic masses :

$$\rho_M \propto a^{-3}$$

$$\Rightarrow \quad T \propto a^{-3(\gamma-1)} \quad \Rightarrow T \propto a^{-3\gamma+3}$$

$$\therefore \quad T \propto a^{-3\gamma+3}$$

Now we can assume the Universe to be primarily made of Hydrogen and Helium.

$\gamma_H = \frac{7}{5}$ (diatomic gas)

$\gamma_{He} = \frac{5}{3}$ (monoatomic gas)

The matter content of the universe can be approximated to be 75% Hydrogen and 25% Helium.

So, $\gamma_{avg} = \dfrac{\gamma_H \times 3 + \gamma_{He} \times 1}{4}$

$= \dfrac{\frac{7}{5} \times 3 + \frac{5}{3} \times 1}{4}$

$= \dfrac{\frac{21}{5} + \frac{5}{3}}{4} = \dfrac{63 + 25}{15 \times 4}$

$= \dfrac{88}{15 \times 4} = \dfrac{22}{15}$

$\therefore \gamma_{avg} = \dfrac{22}{15}$

as $T \propto a^{-3\gamma + 3}$

$-3\gamma + 3 = -3 \times \dfrac{22}{15} + 3 = -\dfrac{7}{5}$

$\therefore \boxed{T \propto a^{-7/5}}$ Ans

for non relativistic matter.