Q. 4. (a)

$\Gamma = n_e \cdot \sigma_T \cdot c$ for ~~photon~~ $e^{\oplus} + e^{\ominus} \rightarrow \gamma + \gamma$

$\sigma_T = 6.65 \times 10^{-25}\ cm^2$

$n_e = 2.2 \times 10^{-7}\ cm^{-3}$

$\therefore \Gamma = (2.2 \times 10^{-7}) \times (6.65 \times 10^{-25}) \times (3 \times 10^{10})$

$\Gamma \cong 43.89 \times 10^{-22}\ 1/s$

& $H_0 = \dfrac{70}{sec}\left(\dfrac{Km}{MPc}\right) = \dfrac{70}{3.086 \times 10^{19}} \dfrac{1}{Sec.} = 22.68 \times 10^{-19}\ 1/sec$

Mpc to Km

$\therefore \dfrac{\Gamma}{H_0} \simeq 1.9 \times 10^{-3}$ or $\Gamma \simeq 1.9 \times 10^{-3} H_0$

1 (b)

$$\Gamma = n_e \sigma_T \cdot c$$

$$\Gamma = n_{e_{z=0}} \times (1+z)^3 \times \sigma_T \times c$$

now,

$$\Gamma = (6.65\times10^{-25}) \times (2.2\times10^{-7}) \times 3\times10^{10}\ sec^{-1} \times (1+z)^3$$

$$\Gamma = 43.89 \times 10^{-22}\ sec^{-1} \times (1+z)^3$$

$$\frac{\Gamma}{H_0} = 1.9\times10^{-3} \quad \text{(part a)}$$

, $\sigma_T = 6.65\times10^{-25}\ cm^2$

$n_{e_{(z=0)}} = \cancel{6.65}\ 2.2\times10^{-7}\ cm^{-3}$

$c = 3\times10^{10}\ cm/s$

$n_e = n_{e_{z=0}} \times (1+z)^3$

& $H = H_0\sqrt{\Omega_{m,0}(1+z)^3}$

$$\Rightarrow \frac{\Gamma}{H} = \left(\frac{\Gamma}{H_0}\right) \times \frac{(1+z)^3}{\sqrt{\Omega_{m,0}(1+z)^3}}$$

$$\frac{\Gamma}{H} = 1.9\times10^{-3} \times \sqrt{\frac{(1+z)^3}{\Omega_{m,0}}}$$

$$z = 1100 \qquad \simeq 1.9\times10^{-3} \times \frac{(1101)^{3/2}}{\sqrt{\Omega_{m,0}}}$$

taking $\Omega_{m,0} = 1$, we get

$$\frac{\Gamma}{H} \simeq 69.4.$$