E and z

for a photon emitted at λ_e wavelength and observed as 'λ_o'.

$$z = \frac{\lambda_o}{\lambda_e} - 1 \Rightarrow \frac{\lambda_o}{\lambda_e} = 1+z \quad \text{and} \quad \frac{\lambda_o}{\lambda_e} = \frac{E_e}{E_o}$$

($\because$ energy of photons is inversely proportional to λ).

$$\Rightarrow \frac{E_e}{E_o} = 1+z$$

$$\Rightarrow E_o = \frac{E_e}{1+z} \Rightarrow \text{Energy}_{\substack{\text{observation} \\ \text{current time}}} = \frac{\text{Energy at previous time}}{(1+z)}.$$

Q.2.

Hubble's constant ~~so~~ is given by

$$H(t) = \frac{1}{a}\frac{da}{dt} \quad \& \quad \text{also } H(t) = \frac{1}{t}.$$

$$\frac{1}{t} = \frac{1}{a}\frac{da}{dt}$$ (for free expansion)

using $a = \frac{1}{1+z}$, $\Rightarrow$ $\frac{1}{t} = \frac{-(1+z)}{(1+z)^2}\frac{dz}{dt}$

$$\frac{da}{dt} = \frac{-dz/dt}{(1+z)^2}$$

$$\Rightarrow \int_{t_0}^{t} \frac{dt}{t} = -\int_0^z \frac{dz}{(1+z)} \quad \left(\frac{z = 0}{t = t_0}\right)$$

$$\Rightarrow \ln\left(\frac{t}{t_0}\right) = \ln\left(\frac{1}{1+z}\right)$$

$$\Rightarrow \frac{t}{t_0} = \frac{1}{1+z}$$

At different times in expansion diagram.

current time $a(t_0) = 1$ & $z = 0$ $a(t) = \frac{1}{1+z}$

using $T \propto \frac{1}{a(t)}$ for any time

$$\frac{T(t)}{T(t_0)} = \frac{a(t_0)}{a(t)} = \frac{1}{a(t)}$$

$T(t_0) = 2.3 \times 10^{-13}$ GeV $\quad \frac{T(t)}{T(t_0)} = 1+z$

$$\Rightarrow \quad z = \frac{T(t)}{T(t_0)} - 1$$

where $t_0 = 13.8 \times 10^9$ years

~~$T(t_0) = 2.3 \times 10^{-13}$ K~~

(Natural units)

$\therefore$ at $t = 10^9$ years, $T = 10^{-12}$ GeV

$$z = \frac{10^{-12}}{2.3 \times 10^{-13}} - 1 = 3.35$$

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at $t = 0.3 \times 10^6$ years, $T = 3 \times 10^{-10}$ GeV

$$z = \frac{T(t)}{T(t_0)} - 1 = \frac{3 \times 10^{-10}}{2.3 \times 10^{-13}} - 1 = 1303$$

WEEK 2 Q4.

at $t = 100\,s$, $T = 10^{-4}$ GeV

$$z = \frac{T(t)}{T(t_0)} - 1 = \frac{10^{-4}}{2.3 \times 10^{-13}} - 1 = \text{order of } (10^9).$$

at $t = 10^{-10}\,s$, $T_5 = 100$ GeV

$$z = \frac{100}{2.3 \times 10^{-13}} - 1 \approx (10^{15}).$$