(b) now, the limits change to

$\underbrace{\mu >> T >> m}$, $\rightarrow$ due to this inequality, $E \approx p$.

& $f_A = \dfrac{g_A}{(2\pi)^3} \dfrac{1}{\exp\left(\frac{p-\mu}{T}\right)+1}$ (for fermions)

we will use the inequality,

$$\int_0^\infty \frac{x^{s-1}}{\frac{e^x}{z}-1}\,dx = Li_s(z)\,\Gamma(s)$$ $\rightarrow$ gamma function

now, $$n_A(T) = \oint \frac{g_A}{(2\pi)^3} \int_0^\infty \frac{4\pi p^2\,dp}{\exp\left(\frac{p-\mu}{T}\right)+1}$$

$$= \frac{g_A \cdot 4\pi}{(2\pi)^3} \int_0^\infty \frac{p^2\,dp}{\frac{-e^{p/T}}{\left(-e^{+\mu/T}\right)}+1}$$

and putting $p/T = x$ & $-e^{\mu/T} = z$

$$n(T) = \frac{gT^3}{2\pi^2} \int_0^\infty \frac{x^2\,dx}{-\frac{e^x}{z}+1}$$

$$n(T) = -\frac{gT^3}{2\pi^2} \int_0^\infty \frac{x^2\,dx}{\frac{e^x}{z}-1}$$

$$n(T) = \frac{-gT^3}{2\pi^2} \int_0^\infty \frac{x^{3-1}}{\frac{e^x}{z} - 1} = \frac{-gT^3}{2\pi^2} Li_3(e^{-\mu/T})\, \Gamma(3)$$

$$n(T) = -Li_3(e^{-\mu/T})\, \Gamma(3)\, \frac{gT^3}{2\pi^2}$$

under the limit $\mu >> T$ or $\frac{\mu}{T} >> 1$

$$n(T) = \frac{gT^3}{2\pi^2} \Gamma(3) \lim_{\mu/T \to \infty} -Li_3(e^{-\mu/T})$$

using $\lim_{Re(\mu) \to \infty} Li_s(\pm e^{\mu}) = \frac{-\mu^s}{\Gamma(s+1)}$

$$\Rightarrow \lim_{\mu/T \to \infty} Li_3(\pm e^3) = \frac{-\mu^3}{\Gamma(4)}$$

$$\Rightarrow n(T) = \frac{gT^3}{2\pi^2} \cancel{\frac{\Gamma(3)}{T^3 \Gamma(4)}} \left(\frac{1}{3}\right) \cdot \mu^3 = \frac{g}{2\pi^2} \frac{\mu^3}{3}$$

$$\rho(T) = \frac{g \cdot 4\pi}{(2\pi)^3} \int_0^\infty \frac{p \cdot p^2 dp}{\exp\left(\frac{p-\mu}{T}\right) + 1}$$

$$\rho(T) = \frac{g}{2\pi^2} \int_0^\infty \frac{p^3 dp}{\exp\left(\frac{p-\mu}{T}\right) + 1}$$

following similar procedure as above,

$$p(T) = \frac{g}{2\pi^2} \int_0^{\infty} \frac{p^3\,dp}{\exp\left(\frac{p-\mu}{T}\right)+1} \qquad x = p/T$$

$$\& \quad z = -e^{\mu/T}$$

$$\Rightarrow p(T) = \frac{gT^4}{2\pi^2} \int_0^{\infty} \frac{x^3\,dx}{-\frac{e^x}{z}+1}$$

$$\Rightarrow p(T) = -\frac{gT^4}{2\pi^2} \int_0^{\infty} \frac{x^3\,dx}{\frac{e^x}{z}-1}$$

$$\Rightarrow p(T) = -\frac{gT^4}{2\pi^2} \left(-Li_4(e^{-\mu/T}) \cdot \Gamma(4)\right)$$

$$\Rightarrow p(T) = \frac{gT^4}{2\pi^2} \cdot \Gamma(4) \cdot Li_4(e^{-\mu/T})$$

taking similar limits $\mu/T \gg 1$, we get

$$\lim_{\mu/T \to \infty} Li_4(\pm e^{-\mu/T}) = -\frac{\mu^4/T^4}{\Gamma(5)}$$

$$\Rightarrow p(T) = \frac{gT^4}{2\pi^2} \cdot \frac{1}{T^4} \mu^4 \cdot \frac{\Gamma(4)}{\Gamma(5)} \frac{1}{4} = \frac{g}{8\pi^2} \mu^4 .$$

now, the pressure limits are calculated as

$$P_{ress}(T) = \frac{g}{2\pi^2} \int_0^{\infty} \frac{p^2 \cdot p^2\,dp}{3p\left(\exp\left(\frac{p-\mu}{T}\right)+1\right)} = \frac{1}{3}\left[\frac{g}{2\pi^2} \int_0^{\infty} \frac{p^3\,dp}{e^{\left(\frac{p-\mu}{T}\right)}+1}\right]$$

→ Press. $(T) = \dfrac{\rho(T)}{3}$.