week 4 q 4.6.

in the radiation epoch

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0\left(\frac{\Omega_R}{a^4} + \frac{1-\Omega_T}{a^2}\right)$$

$$\therefore \Omega_R(a) = \frac{\rho_R(a)}{\rho_c(a)} = \frac{\rho_R(a)\,8\pi G_1}{3\left(\frac{\dot{a}}{a}\right)^2}$$

using $\rho_R(a) = \rho_{R_0}/a^4$

$$\Rightarrow \Omega_R(a) = \frac{\rho_{R_0}/a^4\; 8\pi G_1}{3\left(\frac{\dot{a}^2}{a^2}\right)}$$

$$\Rightarrow \Omega_R(a) = \frac{\rho_{R_0} 8\pi G_1}{3a^4 H_0^2\left(\frac{\Omega_R}{a^4} + \frac{(1-\Omega_T)}{a^2}\right)}$$

$$\Omega_R(a) = \frac{\rho_{R_0}}{\left(\frac{3H_0^2}{8\pi G_1}\right)\left(\Omega_R + (1-\Omega_T)a^2\right)}$$

$$\Omega_R(a) = \frac{\Omega_{R_0}}{\Omega_{R_0} + (1-\Omega_T)a^2}$$

$$\Omega_R(a) = \frac{1}{1 + \left(\frac{1-\Omega_T}{\Omega_{R_0}}\right)a^2}$$

$$\simeq \left(1 + \left(\frac{1-\Omega_T}{\Omega_{R_0}}\right)a^2\right)^{-1}$$

$$\simeq 1 - \left(\frac{1-\Omega_{T_0}}{\Omega_{R_0}}\right)a^2$$

$$(1-\Omega_T)_{a=a_0} = 10^{-2}$$

$$a \approx 10^{-9}, \quad \Omega_{R_0} = 8.4 \times 10^{-5} \approx 10^{-4}$$

$$\simeq 1 - \frac{10^{-2} \times 10^{-18}}{10^{-4}}$$

$$\simeq 1 - 10^{-16}$$