Q.1 HW _ WEEK II

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~~for light, from special relativity~~

$$\cancel{ds^2 = c^2dt^2 - a^2(t)[dr^2 + r^2 d\Omega^2]}$$

for expansion of universe,
photon geodesic would be

$ds = 0$ in $ds^2 = c^2dt^2 - a^2(t)\left[dr^2 + r^2 d\Omega^2\right]$

& $d\Omega = 0$ for (assuming photon follows a path of constant θ & ϕ).

$$\Rightarrow |c\,dt| = a(t)\,dr$$

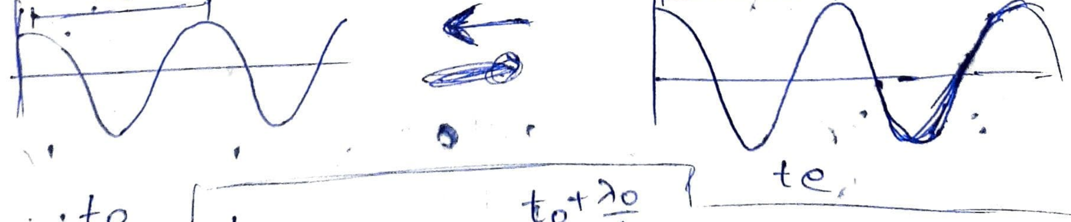
$$\Rightarrow dr = \frac{1}{c}\frac{dt}{a(t)} \qquad \because c = 1$$

$$\Rightarrow dr = \frac{dt}{a(t)} \quad \text{for photons.}$$

$\therefore$ for a photon emitted at $(t_o, r_o, \theta_o, \phi_o)$ & observed at $(t_e, r_e, \theta_o, \phi_o)$

$$\Delta r = r_e - r_o = \int_{t_e}^{t_o} \frac{dt}{a(t)}$$

applying this on two wave crests / troughs.



$$\Delta r = r_o - r_e = \int_{t_e}^{t_o} \frac{dt}{a(t)} = \int_{t_e + \frac{\lambda_e}{c}}^{t_o + \frac{\lambda_o}{c}} \frac{dt}{a(t)}$$

now, $\int_{t_e}^{t_o} \frac{dt}{a(t)} = \int_{t_e+\frac{\lambda_e}{c}}^{t_o+\frac{\lambda_o}{c}} \frac{dt}{a(t)}$

$$\Rightarrow \int_{t_e}^{t_o} \frac{dt}{a(t)} = \int_{t_e}^{t_o} \frac{dt}{a(t)} - \int_{t_e}^{t_e+\frac{\lambda_e}{c}} \frac{dt}{a(t)} + \int_{t_o}^{t_o+\frac{\lambda_o}{c}} \frac{dt}{a(t)}$$

$$\Rightarrow \int_{t_e}^{t_e+\lambda_e/c} \frac{dt}{a(t)} = \int_{t_o}^{t_o+\lambda_o/c} \frac{dt}{a(t)}$$

$\because$ the time $\frac{\lambda_e}{c}$ or $\frac{\lambda_o}{c}$ (between two successive wavecrests) is extremely small, $a(t)$ can be taken to be constant.

$$\Rightarrow \frac{1}{a(t_e)} \int_{t_e}^{t_e+\lambda_e/c} dt = \frac{1}{a(t_o)} \int_{t_o}^{t_o+\lambda_o/c} \cancel{a(t)}\, dt$$

$$\Rightarrow \frac{1}{a(t_e)} \frac{\lambda_e}{c} = \frac{1}{a(t_o)} \frac{\lambda_o}{c} \Rightarrow \frac{\lambda_e}{\lambda_o} = \frac{a(t_e)}{a(t_o)}$$

now, using $z = \frac{\lambda_o}{\lambda_e} - 1$ and $a(t_o) = 1$ (observing at current time)

$$\Rightarrow \frac{\lambda_o}{\lambda_e} = 1 + z = \frac{a(t_o)}{a(t_e)} \Rightarrow a(t_e) = \frac{a(t_o)}{1+z}$$

$$\text{or, } a(t_e) = \frac{1}{1+z}.$$