Given

$$z = (t_0 - t)H_0 + (t_0 - t)^2\left(1 + \frac{q_0}{2}\right)H_0^2 + \cdots$$

$$H_0(t_0 - t) = A_1 z + A_2 z^2 + \cdots$$

$(t_0 - t)$ in terms of $z$,

$$z = (A_1 z + A_2 z^2 \cdots) + (A_1 z + A_2 z^2 + \cdots)^2\left(1 + \frac{q_0}{2}\right)$$

$$z = A_1 z + \left(A_2 + A_1\left(1 + \frac{q_0}{2}\right)\right)z^2 + \cdots$$

$$\Rightarrow A_1 = 1$$

$$\& \quad A_2 + A_1\left(1 + \frac{q_0}{2}\right) = 0$$

$$\Rightarrow \quad A_2 = -\left(1 + \frac{q_0}{2}\right)$$

$$t_0 - t = H_0^{-1} z - \left(1 + \frac{q_0}{2}\right) H_0^{-1} z^2 + \cdots$$