WEEK 5 ASSIGNMENT

Ashmita Panda 1811042
Pulkit Ojha 1811118

Answer 2.9) [James Rich]

Consider a sphere of ionised hydrogen. There are N_p electrons and N_p protons. The sphere has uniform density and is in hydrostatic equilibrium.

Mean pressure $\equiv P$
Mean volume $\equiv V$

So equilibrium equation:

$$\frac{dP}{dr} = -\frac{GM(r)\,\rho(r)}{r^2}$$

Now,

$$PV = \int_0^V P(r)\, dV = \int_0^R P(r)\, 4\pi r^2\, dr$$

Using integration by parts:

$$PV = \int_0^R P(r) \frac{d}{dr}\left\{\frac{4}{3}\pi r^3\right\} dr$$

$$= \left[\left(\frac{4}{3}\pi r^3\right) P(r)\right]_0^R - \int_0^R \frac{4}{3}\pi r^3 \cdot \frac{dP}{dr}\, dr$$

$P(R) = 0$, so we get:

$$PV = 0 + \int_0^R \frac{4}{3}\pi r^3 \cdot \frac{GM(r)}{r^2}\, \rho(r)\, dr$$

$$\Rightarrow PV = \frac{1}{3}\int_0^R \frac{GM(r)}{r}\rho(r)\, 4\pi r^2 dr \;;$$

$$\therefore \quad 3PV = -E_g$$

From ideal gas law:

$$2N_p kT = PV = -\frac{E_g}{3}$$

$$\text{Mean KE} = \frac{3}{2}kT$$

$$\Rightarrow 2N_p\left(\frac{3}{2}k\right)T = -\frac{E_g}{2}$$

Uniform density $\Rightarrow \rho(r) = 1$

$$\Rightarrow \int_0^R \frac{GM(r)}{r} 4\pi r^2 dr = \int_0^R GM(r)\, 4\pi r\, dr$$