Q.2.

Hubble's constant H_0 is given by

$$H(t) = \frac{1}{a}\frac{da}{dt} \quad \& \quad \text{also} \quad H(t) = \frac{1}{t}.$$

(a)

$$\frac{1}{t} = \frac{1}{a}\frac{da}{dt} \qquad \text{(for free expansion)}$$

using $a(t) = \frac{1}{1+z}$, $\Rightarrow$ $\frac{1}{t} = \frac{-(1+z)}{(1+z)^2}\frac{dz}{dt}$

$$\frac{da}{dt} = \frac{-dz/dt}{(1+z)^2}$$

$$\Rightarrow \int_{t_0}^{t}\frac{dt}{t} = -\int_{0}^{z}\frac{dz}{(1+z)} \qquad \left(\begin{array}{c} z = 0 \\ t = t_0 \end{array}\right)$$

$$\Rightarrow \ln\left(\frac{t}{t_0}\right) = \ln\left(\frac{1}{1+z}\right)$$

$$\Rightarrow \frac{t}{t_0} = \frac{1}{1+z}$$