PLOT

~~$T \gg m$~~

$T \gg m$ $\quad \rho_\nu \propto T_\nu^4$

$$\rho_\nu = \frac{21}{8}\left(\frac{\pi^2}{15} T_\nu^4\right) \rightarrow \rho_\gamma = \frac{\pi^2}{15} T_\gamma^4$$

$$\Rightarrow \frac{\rho_\nu}{\rho_\gamma} = \frac{21}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 \qquad \frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)$$

$$\frac{\rho_\nu}{\rho_\gamma} = \frac{21}{8}\left(\frac{4}{11}\right)^{4/3}$$

when temperature is higher the y-axis value remains constant.

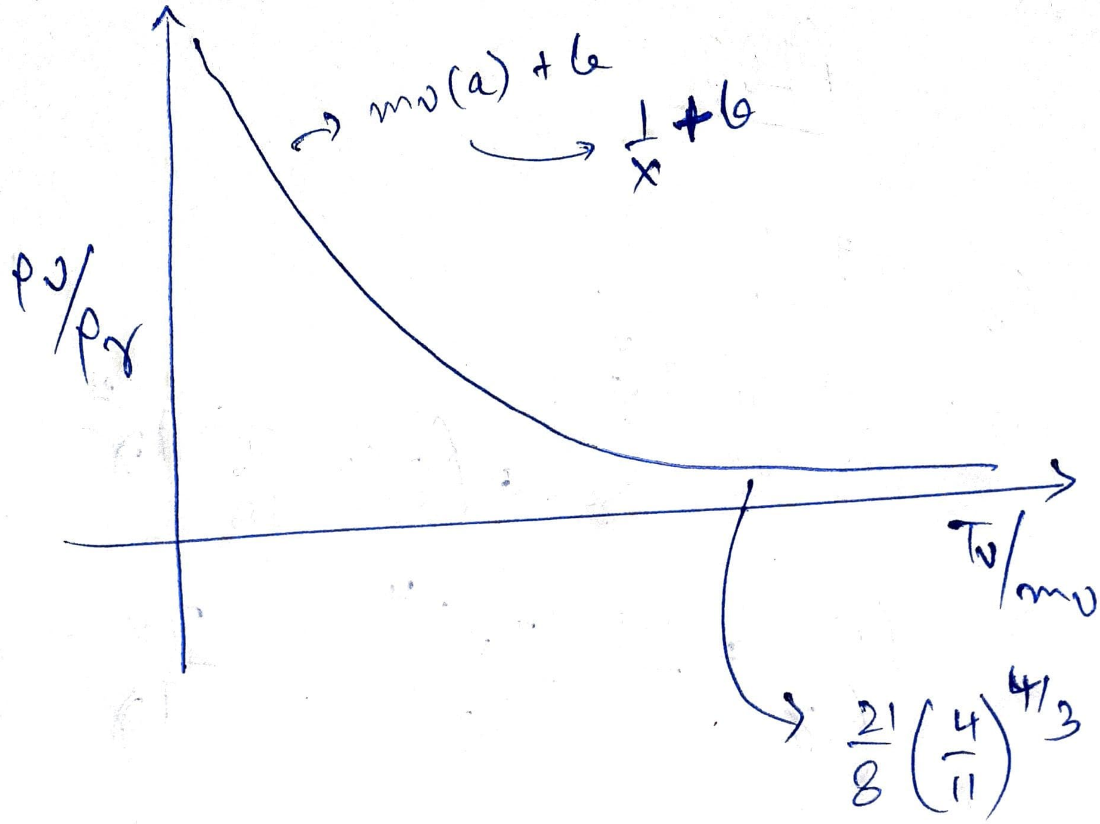
$T \ll m$ $\quad \rho_\gamma \propto T_\gamma^4 . \quad \rho_\nu = \left(m_\nu + \frac{3}{2} T_\nu\right) n_\nu$

now, $\frac{T_\nu}{m_\nu} \propto \frac{1}{a}$ $\qquad \Rightarrow \frac{\rho_\nu}{\rho_\gamma} = \frac{\left(m_\nu + \frac{3 T_\nu}{2}\right) n_\nu}{\frac{\pi^2}{15} T_\gamma^4}$

& $\frac{n_\nu}{n_\gamma} = \frac{9}{11}$ $\qquad = \frac{1}{\left(\frac{\pi^2}{15}\right)}\left(\frac{m_\nu}{T_\gamma^4} + \frac{3}{2}\left(\frac{4}{11}\right)^{1/3} \frac{1}{T_\gamma^3}\right) n_\gamma$

$$\frac{\rho_\nu}{\rho_\gamma} \propto \frac{15}{\pi^2}\left(m_\nu a^4 + \left(\frac{3}{2}\left(\frac{4}{11}\right)^{1/3}\right) a^3\right)\left(a^{-3}\right)$$

$\propto$ $(m_\nu a + (b)) \rightarrow$ linear w.r.t a



$$\left(\frac{m_\nu}{T_\nu}\right) + 6$$

$$\frac{1}{x} + 6$$