Searching in the Dark

Chasing Magnum Opus

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1 Introduction

The detection of gravitational waves (GW) on September 14, 2015 (**2016PhRvL.116f1102A**), opened a new into the Universe. Joint observations of electromagnetic (EM) counterparts to the GW signals will result in a deeper understanding of the physics involved. We anticipate EM counterparts to binary sources involving matter i.e. neutron star-neutron star (NSNS) and neutron star-black hole (NSBH) binaries. In an effort to measure these joint signals, GW detectors are working in conjunction with traditional observatories. Some of these will yield weak, nearly isotropic electromagnetic counterparts and others will not.

This report is organized as follows. In §2, we describe the estimation of the chirp mass distribution. In §3 we descibe an EM followup classifier based on the data.

2 Chirp Mass Distribution

2.1 Density Estimation

We use a histogram to estimate the merger rate as a function of $\mathcal{M}_c$ (See Figure 5). Since we are interested in the intrinsic rate, not just that of detected events, we weigh each point by the inverse of the spacetime volume in which we are sensitive to it, $w = 1/VT$. Binaries with a higher chirp mass are easier to detect, so we do not want to count them as heavily. For a given chirp mass, we are sensitive out to a distance of

$$D(\mathcal{M}_c) = 200\,\mathrm{Mpc}\big(\mathcal{M}_c/1.2\,\mathrm{M}_\odot\big)^{5/6} \tag{1}$$

which corresponds to a volume of

$$V(\mathcal{M}_c) = \frac{4}{3}\pi D^3(\mathcal{M}_c). \tag{2}$$

Multiplying this by the time spent observing, $T = 0.6\,\mathrm{yr}$, gives us the spacetime volume $(VT)(\mathcal{M}_c)$.

To obtain uncertainties in our histogram, we take the square root of the sum-of-squares of the weights within that bin, i.e.

$$\sigma_k = \sqrt{\sum_i w_i^2}. \tag{3}$$

We also overplot a pure power law. To do this, we employ Bayesian linear regression, fitting a straight line to $\log r$ versus $\log \mathcal{M}_c$, and transforming back to linear space.

2.2 The Likelihood of Fitting Parameters

The likelihood estimator of each fitting parameter is:

$$\ln P(\{d\}|\lambda) = \frac{-1}{2}\sum_k \left\{ \frac{[r(x_k) - r_{\mathrm{model}}(x_k)]^2}{\sigma_r^2} \right\} \tag{4}$$

where p_{smooth} is the smoothing prior, defined to be:

$$p_{\mathrm{smooth}} = \exp\left\{ -\int \left[\frac{\mathrm{d}^n(r)}{\mathrm{d}(x)^n} \right]^2 \mathrm{d}x \right\} \tag{5}$$

In theory, p_{smooth} can be any n^{th} derivative. To make our code robust, we define a function that takes n as an argument. The function then calls `numpy.polynomial.polynomial.polyder()` to find the n^{th} derivative. Next, we square the n^{th} derivative and integrate it between the minimum and maximum of x. Here we choose $n = 3$.

2.3 Model fitting

We employed a Markov Chain Monte Carlo (MCMC) script to fit the coefficients of a polynomial model for $r(\mathcal{M}_c)$. The MCMC uses the smoothing prior as defined in §2.2. We performed a least square fit first in order to obtain the initial guess for the coefficients of the model.

The model we used is a polynomial of degree 4. Because of the smoothing function, using higher order does not change the shape of the fit in a significant way. The MCMC best fit can be seen in Figure 2. The MCMC walkers and triangle plot are in Figures 3 and 4.

Best fit model is:

$$r(\mathcal{M}_c) = \sum_{k=0}^{9} \alpha_k (\log \mathcal{M}_c)^k \tag{6}$$

Where the coefficients are listed in table 1.

Table 1: Coefficients from the MCMC fit.

k	α_k
0	-1.903×10^{-6}
1	1.842×10^{-5}
2	9.999×10^{-1}
3	1.748×10^{-4}
4	-2.441×10^{-4}
5	2.122×10^{-4}
6	-1.130×10^{-4}
7	3.470×10^{-5}
8	-5.416×10^{-6}
9	3.470×10^{-7}

3 Classifying GW Events that have Electromagnetic Countparts

3.1 Overview

The GW observatory, the Laser Interferometer Gravitational Wave Observatory (LIGO), can provide very rapid mass estimates of candidate GW events. Since most of these detections are mostly binary black holes and EM followup is extremely expensive, only a few events can be followed up. We have therefore trained a classifier to determine if an event will have a EM followup. We trained this classifier on the first half of the data; we simply took the mid-way point betweewn the maximum chirp mass for the EM counterpart group and the nonEM counterpart group. This can be seen in Figure 7.

3.2 Method

The classifier was constructed simply by taking the minimum chirp mass event of the nonEM counterpart group plus the maximum chirp mass event of the EM counterpart group and finding the mid-point between those two events. This classifier was trained from the first half of the dataset. This is visiualized by the filled points in Figure 7.

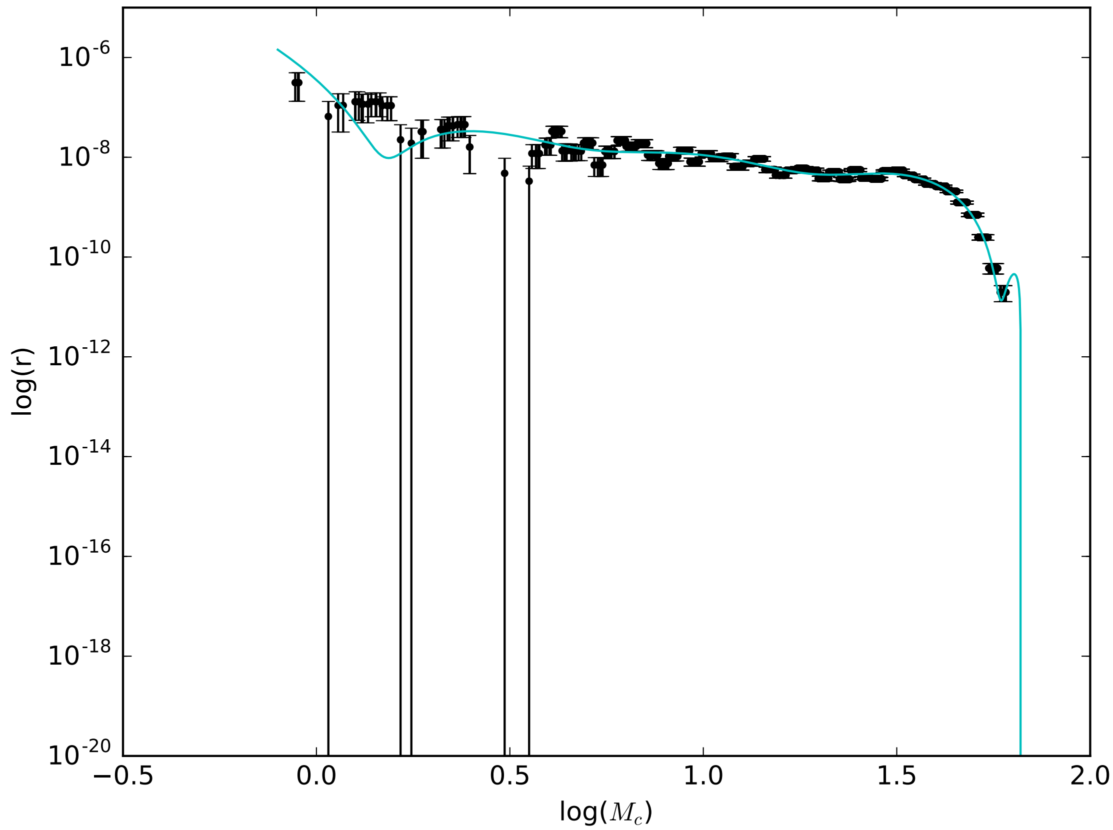


Figure 1: The cyan line is an example of a fit from MCMC.

3.3 Results

The classifier correctly classified the two groups without any contamination. More importantly, this was also the case when classifying the full data set. As you can see in Figure 7, the classifier correctly classified the two groups without any contamination. In the Table 2, you can see numerically that the classifier worked completely. Listed are the chirp mass for the maximum EM counterpart event and the minimum of the other group along with the chirp mass of the line that divides the group. The classifier shows a clear distinction between the two groups.

Figure 5 shows the rate vs the chirp mass with the dividing line from the classifier overplotted. This correlates to the two hump structure in the graph that represents the two

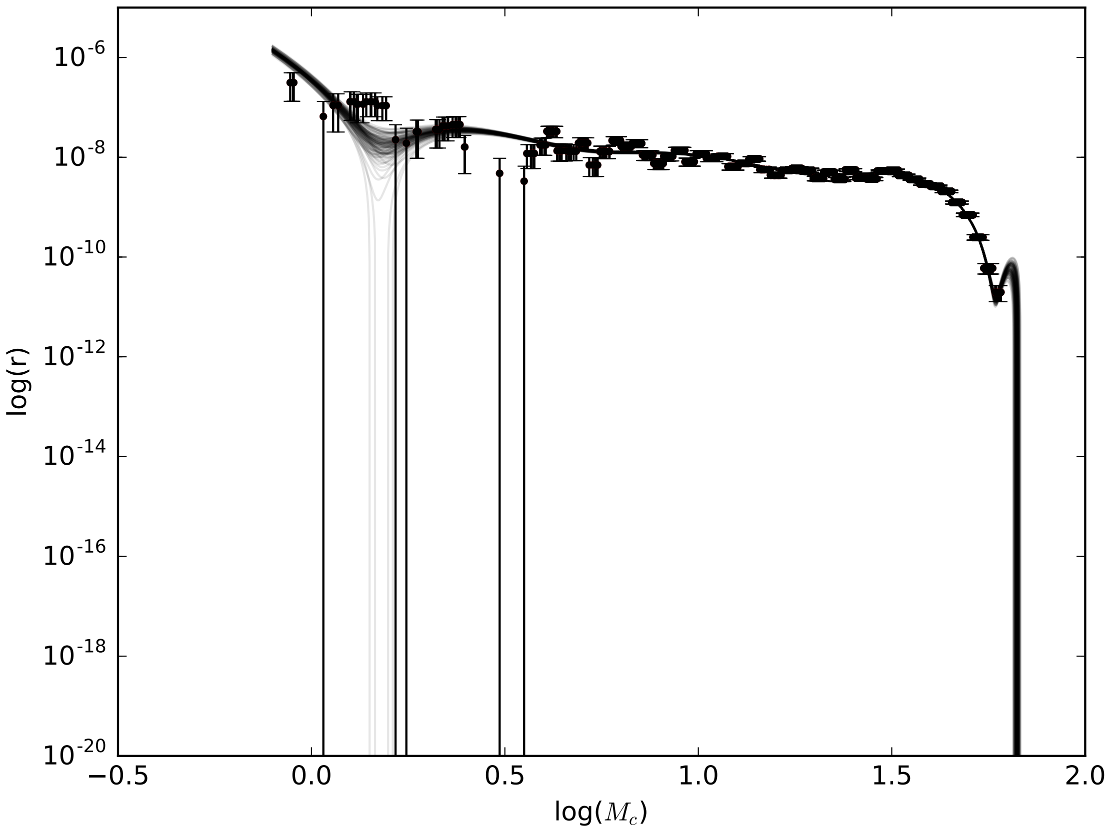


Figure 2: 100 randomly chosen fits from the MCMC.

groups (EM counterparts and nonEM counterparts). Figure 6 shows a similar correlation; the dividing clearly separates the two groups in m_1-m_2 parameter space.

We also created a classifier based on the 2D distributions i.e. m_1 vs m_2. We use the scikit-learn's SVM.LinearSVC classifier, and we again used half the dataset to train the classifier. The classifier completely classified the whole training set completely without any contamination. The classifier was then used on the whole training set. As shown in Figure 8, the classifier correctly identifies the two groups completely without any contamination.

Table 2: Chirp masses of the maximum event from the EM counterpart group, the minimum event from nonEM counterpart group, and the vertical dividing line, respectively.

3.545980	3.065123	3.378711

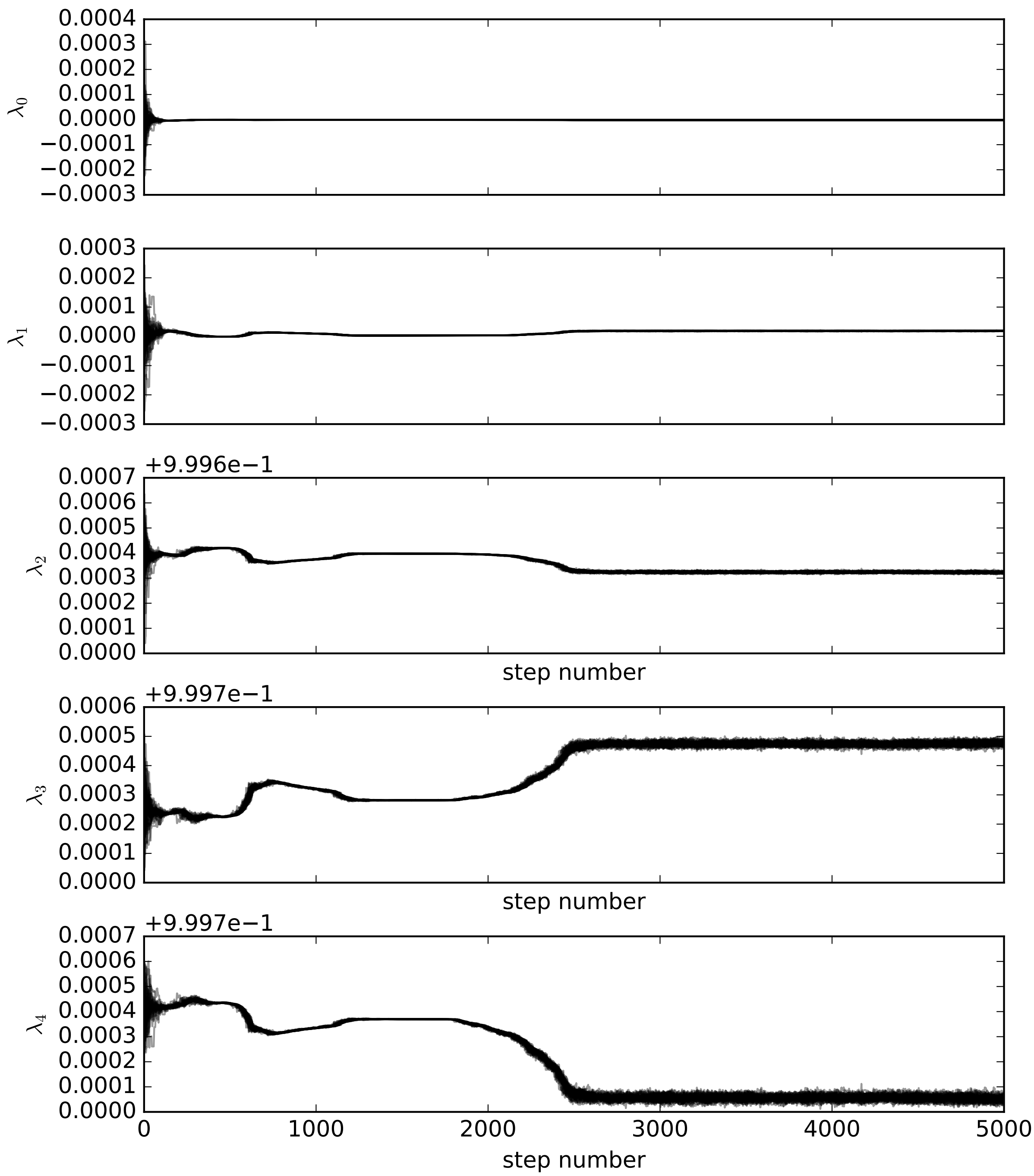


Figure 3: MCMC workers converging.

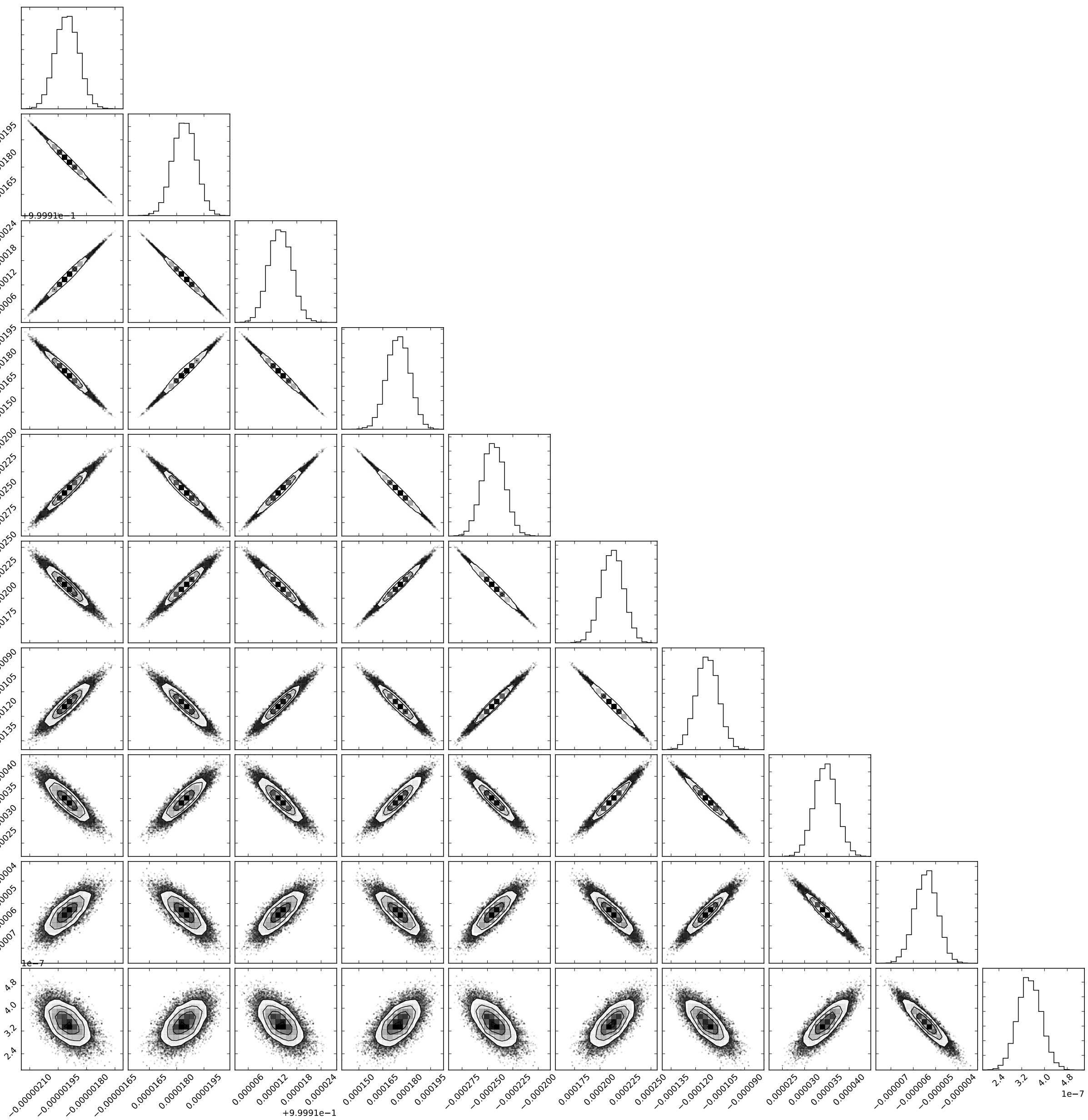
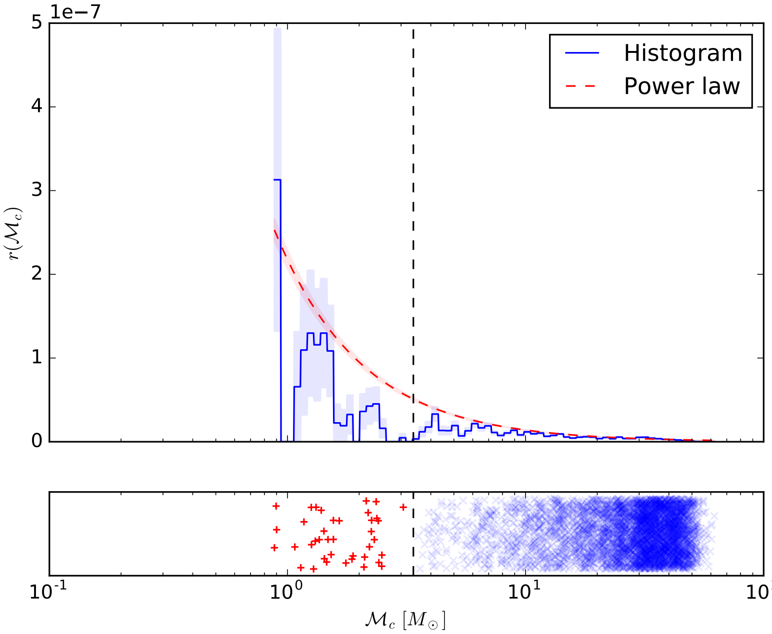
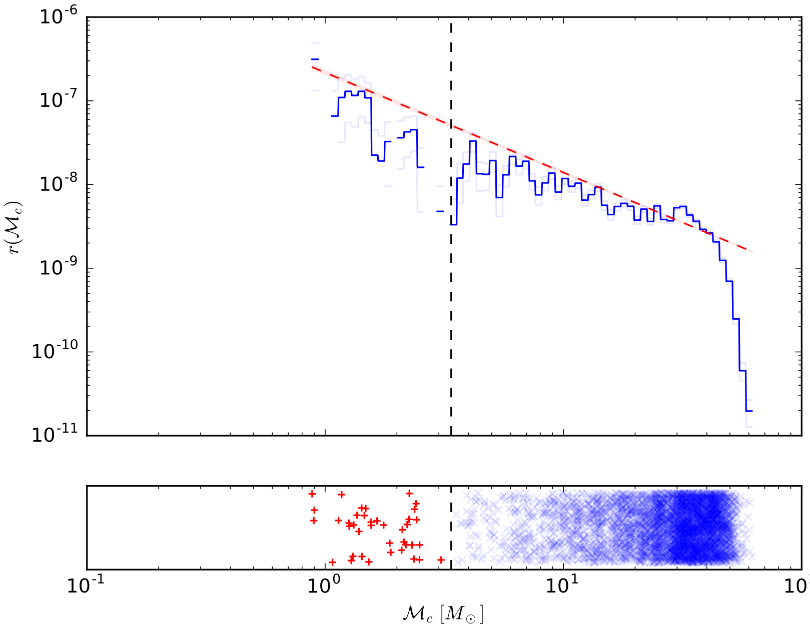


Figure 4: Corner plot of MCMC fit coefficients.



(a)



(b)

Figure 5: Estimated rate of compact binary mergers, based on 5000 synthetic observations. Rate is shown in (a) linear and (b) log scale. Blue line is weighted histogram fit. Red curve is power law fit. Shaded regions are 1-σ error bars. Vertical dashed line is boundary between events with counterparts and without.

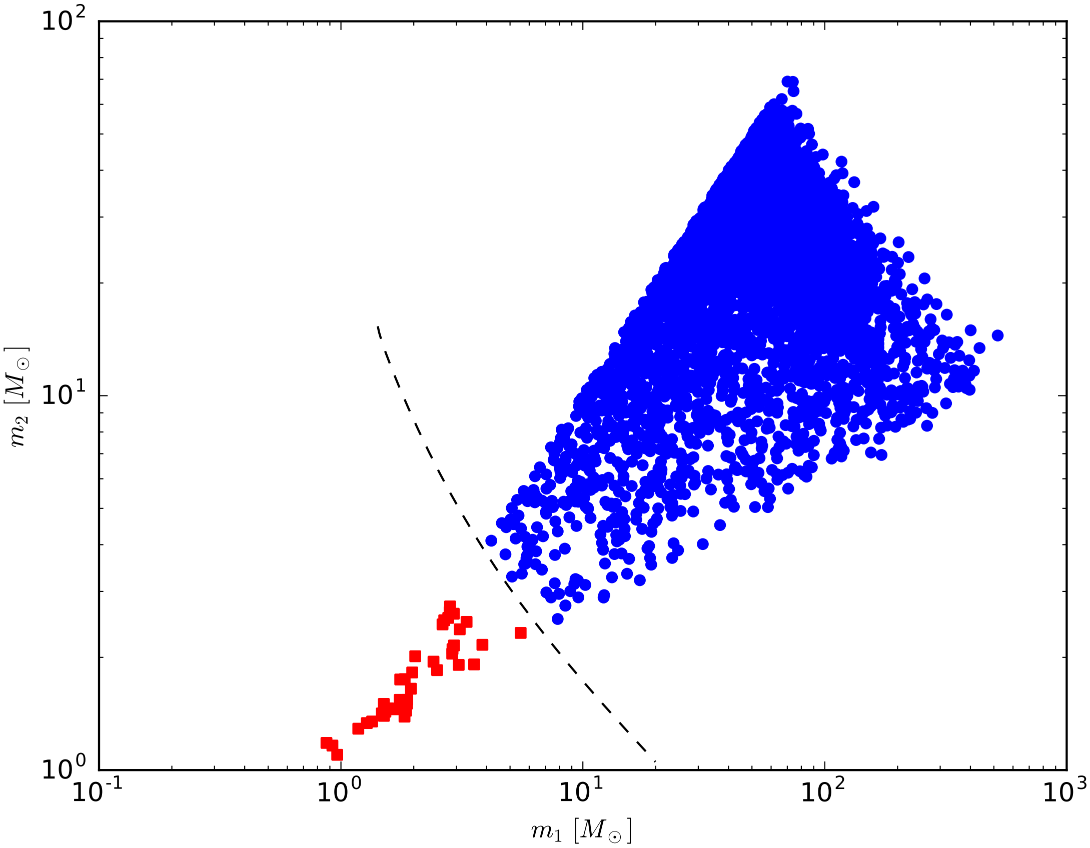


Figure 6: Boundary between masses in two dimensions. Line is $m_2(m_1, \mathcal{M}_c)$, where $\mathcal{M}_c$ is fixed at the value of the decision boundary from the 1D plot.

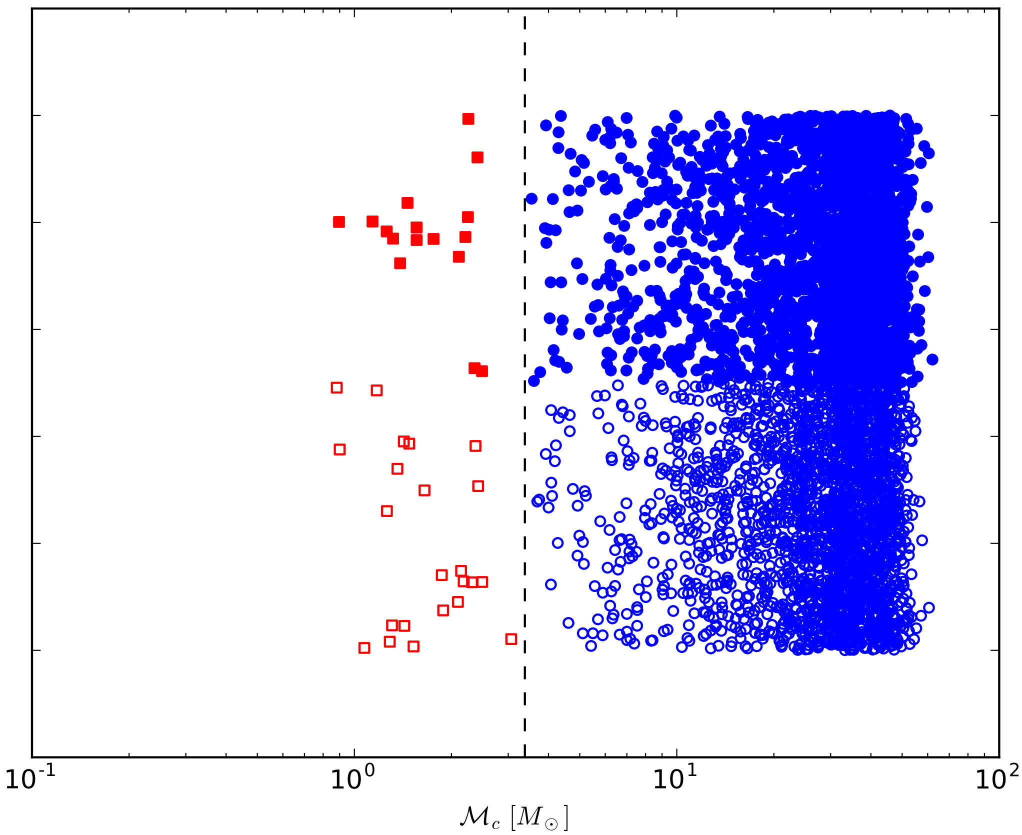


Figure 7: This shows a selected set of the data. The red points represent the events with EM counterparts, and the blue points represent the events without EM counterparts. The filled points represent represent the train dataset, and the open points represent the rest of the data. The vertical line indicates the division between the two groups indicated by the classifier.

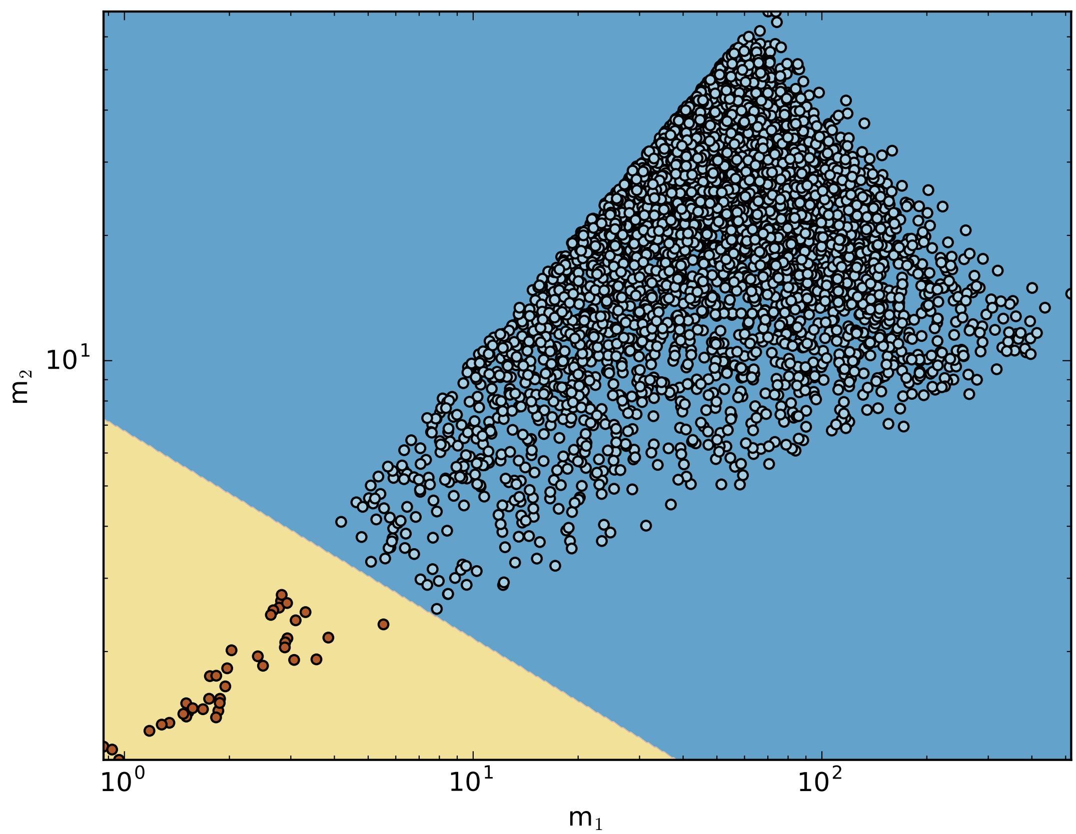


Figure 8: This shows the classifier on the whole dataset. The red points represent the events with EM counterparts, and the blue points represent the events without EM counterparts. The dividing line can be seen by the changing of the colored regions.