Quiz Questions: Sets and Functions

1) Let S = ∅. Which of the following is not a subset of **P**(**P**(S)) (The power set of the power set of S)?
   a. ∅
   b. {∅}
   c. {{∅}}
   d. {∅, {{∅}} }

2) If you need to prove that S is a proper subset of T, it is sufficient to show which of the following?
   a. | T – S | > 0
   b. | S | < |T|
   c. There is an element of T that is not an element of S
   d. None of these

3) According to De Morgan's laws $\overline{A \cup (B \cap C)}$ =
   a. $\overline{A} \cap (B \cap C)$
   b. $\overline{A} \cup (\overline{B} \cap \overline{C})$
   c. $\overline{A} \cap (\overline{B} \cup \overline{C})$
   d. $\overline{A} \cup (B \cap C)$

4) Which of these rules defines a function f from the set of all letter strings of length 6 to the set {1, 2, 3, 4, 5, 6}?
   a. The number of vowels in the string. For example, f(TAZNAV) = 2.
   b. The reverse of the string. For example, f(BAQKDU) = UDKQAB.
   c. The number of distinct letters in the string. For example, f(TNVRRN) = 4.
   d. The position in which the first Z occurs. For example, f(PPABZY) = 5.

5) Suppose f: A → B is a function. Which one of these statements is true?
   a. If a1 and a2 are distinct elements of A, then f (a1) ≠ f (a2).
   b. If b ∈ B, then there is at least one element a ∈ A such that f(a) = b.
   c. If b ∈ B, then there is exactly one a ∈ A such that f(a) = b.
   d. For each element a ∈ A, there is exactly one element b ∈ B such that f (a) = b.

6) Let f: A → B where B = {0, 1, 4, 9} and f is defined by the rule f(x) = x^2. For which set A is "f" a correctly defined function from A to B and one-to-one
   a. {1, 2, 3}
   b. {-3, -1, 0, 2, 3}
   c. {0, 1, 4, 9}
   d. {-1, 0, 1, 2}

7) Let S be the set of all bit strings of length at least 2. Which of the following functions f: S → S is NOT one-to-one?
   a. f(s) = the string s with a 1 bit appended at the end. (For example, f(1101) = 11011.)
   b. f(s) = the string obtained by moving all 0's (if any) in s to the end of the string. (For example, f(101101) = 111100.)
   c. f(s) = the reversal of s. (For example, f(110) = 011.)
   d. f(s) = the string obtained from interchanging 0's and 1's. (For example, f(11000) = 00111.)

8) Suppose f: **R** → **R** has the following property for all real numbers x and y: if $x < y$ then $f(x) < f(y)$. Which is true?
   a. f must be 1-1 but is not necessarily onto R.
   b. f is onto R, but is not necessarily 1-1.
   c. f must be both 1-1 and onto R.
   d. f is not necessarily 1-1 and not necessarily onto R.