Advanced information, computation, communication I
EPFL - Fall semester 2020-2021

Week 9

November 17, 2020

Exercise 1. Prove that if a and b are nonzero integers, a divides b, and $a + b$ is odd, then a is odd.

Exercise 2. Let m be a positive integer. Show that $a \equiv b \pmod{m}$ if a **mod** $m = b$ **mod** m.

Exercise 3. Suppose that a and b are integers, $a \equiv 11 \pmod{19}$, and $b \equiv 3 \pmod{19}$. Find the integer c with $0 \leq c \leq 18$ such that:

1. $c \equiv 13a \pmod{19}$.
2. $c \equiv 7a + 3b \pmod{19}$.
3. $c \equiv a^3 + 4b^3 \pmod{19}$.

Exercise 4. Show that the hexadecimal expansion of a positive integer can be obtained from its binary expansion by grouping together blocks of four binary digits, adding initial zeros if necessary, and translating each block of four binary digits into a single hexadecimal digit.

Exercise 5. Find the sum and product of each of these pairs of numbers. Express your answers as a hexadecimal expansion.

1. $(1AE)_{16}, (BBC)_{16}$
2. $(ABCDE)_{16}, (1111)_{16}$

Exercise 6. Suppose that n and b are positive integers with $b \geq 2$ and the base b expansion of n is $n = (a_m a_{m-1} ... a_1 a_0)_b$. Find the base b expansion of:

1. b^n
2. $\lfloor n/b \rfloor$

Exercise 7. Find the decimal expansion of the number with the $n-$digit base seven expansion $(111...111)_7$ (with n 1's). [Hint: Use the formula for the sum of the terms of a geometric progression.]

Exercise 8. Express in pseudocode the trial division algorithm for determining whether an integer is prime.

Exercise 9. Express in pseudocode an algorithm for finding the prime factorization of an integer.

Exercise 10. Show that a positive integer is divisible by 3 if and only if the sum of its decimal digits is divisible by 3.

Exercise 11. Determine how we can use the decimal expansion of an integer n to determine whether n is divisible by:

1. 4
2. 25

3. 20

Exercise 12. Describe an algorithm for finding the difference of two binary expansions.

Exercise 13. Show that $a^m + 1$ is composite if a and m are integers greater than 1 and m is odd. [Hint: Show that $x + 1$ is a factor of the polynomial $x^m + 1$ if m is odd.]

Exercise 14. Find gcd(92928, 123552) and lcm(92928, 123552), and verify that gcd(92928, 123552)·lcm(92928, 123552) = 92928·123552.