

Week 6

October 20, 2020

Exercise 1.

<pre>function f1() { x=0 i=1 while (i ≤ n) { x=x+1 i=x+x } a=x }</pre>	<pre>function f2() { y=0 j=1 while (j ≤ n) { y=y+1 j=y*y } b=y }</pre>
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After execution of the two program fragments **f1** and **f2**, it is the case that

- ☐ $a \approx \frac{n}{2}, b \approx \sqrt{n}.$
☐ $a \approx n, b \approx \log_2(n).$
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☐ $a \approx n, b \approx \sqrt{n}.$

Exercise 2. The following three algorithms sort the input sequence a_0, a_1, \dots, a_n of real numbers in ascending order.

Algorithm 1 Bubble Sort	Algorithm 2 Selection Sort	Algorithm 3 Insertion Sort
<pre>repeat swapped ← FALSE for i = 1 to n - 1 do if $a_{i-1} > a_i$ then swap a_{i-1} and a_i swapped ← TRUE end if end for until swapped = FALSE</pre>	<pre>for i = 0 to n - 1 do min ← i + 1 for j = i + 1 to n do if $a_{\min} > a_j$ then min ← j end if end for if $a_i > a_{\min}$ then swap a_i and a_{\min} end if end for</pre>	<pre>for j = 1 to n do i ← 0 while $a_j > a_i$ do i ← i + 1 end while m ← a_j for k = 0 to j - i - 1 do $a_{j-k} \leftarrow a_{j-k-1}$ end for $a_i \leftarrow m$ end for</pre>

- Use Bubble Sort, Selection Sort and Insertion Sort to sort the following sequence:

$9, 12, -43, 20, -2, 3, 7, 28, 19.$
- How many comparisons are done in each of the algorithms?
- How many swaps are done in each of the algorithms?
- What is the approximate overall cost of the two algorithms for an input sequence of length $n + 1$?

Exercise 3. Recall the definition of Quicksort given in the lecture notes.

1. Using Quicksort with the first element as divider, sort the following sequence:

-7, 19, 11, -5, -4, 0, 18, -8, 16, 3, -3, 5, 1, 20, 6.

2. How many comparisons are done in total?
3. How many swaps are done in total?

Exercise 4. Recall the definition of Mergesort given in the lecture notes.

1. Using Mergesort, sort the following sequence:

-5, 11, -7, 15, -1, 7, 10, 9, 8, -4, 18, 17, -8, -10, 19, 2.

2. How many comparisons are done in total?
3. How many swaps are done in total?

Exercise 5.

(*français*) La raison d'avoir une mémoire cache est

(*English*) The reason to have a cache is

- ☐ $\left\{ \begin{array}{l} \text{de rendre possible de servir rapidement des futures demandes de données de mémoire.} \\ \text{so that it may be possible to serve future requests for data from memory in a fast manner.} \end{array} \right.$
- ☐ $\left\{ \begin{array}{l} \text{de laisser les programmeurs décider eux-mêmes comment organiser au mieux l'accès à la mémoire.} \\ \text{to let programmers decide for themselves how to best organize memory access.} \end{array} \right.$
- ☐ $\left\{ \begin{array}{l} \text{d'exploiter la localité temporelle avec des lignes de cache courtes et simultanément} \\ \text{la localité spatiale avec de longues lignes de cache.} \\ \text{to exploit temporal locality with short cache lines and simultaneously spatial locality with long cache lines.} \end{array} \right.$
- ☐ $\left\{ \begin{array}{l} \text{pour aider les programmeurs à transposer des matrices.} \\ \text{to help programmers transpose matrices.} \end{array} \right.$

Exercise 6. A palindrome is a string that reads the same forward and backward. Describe an algorithm for determining whether a string of n characters is a palindrome.

Exercise 7. Describe an algorithm that determines whether a function from a finite set to another finite set is one-to-one.

Exercise 8. Adapt the bubble sort algorithm so that it stops when no interchanges are required. Express this more efficient version of the algorithm in pseudocode.

Exercise 9. Two strings are anagrams if each can be formed from the other string by rearranging its characters. Devise an algorithm to determine whether two strings are anagrams