1 Special types of diffential equations of the first order

1.1 Differential equations with separable variable

$$Q(x,y)\frac{dy}{dx} + P(x,y) = 0 \tag{1}$$

$$= \tag{2}$$

$$P(x,y)dx + Q(x,y)dy \tag{3}$$

1.2 First order differential equation with homogeneous coefficients

1.2.1 Definition of a homogeneous function

Let $z = f(x,y)$ define z as a function of x and y in a region R. The function $f(x,y)$ is said to be **homogeneous of order** n if it can be written as

$$f(x,y) = x^n g(u) \tag{4}$$

where $u = \frac{y}{x}$ or alternatively it can be written as

$$\tag{5}$$

$$f(x,y) = y^n h(u) \tag{6}$$

where $u = \frac{x}{y}$

1.2.2 Differential equations with homogeneous coefficients

The differential equation $P(x,y)dx + Q(x,y)dy = 0$ is homogeneous if P,Q are homogeneous functions of the same order.

Solution

$$P(x,y) = x^n g_1(u), \qquad Q(x,y) = x^n g_2(u)$$

and then

$$\frac{dx}{x} + \frac{g_2(u)}{g_1(u) + g_2(u)}du = 0 \tag{7}$$

a separable differential equation

1.2.3 Differential equations in which the coefficients of dx and dy are linear, nonhomogeneous and when equated to zero represent non parallel lines