MATLAB ASSIGNMENT-8

SYSTEM OF FIRST ORDER LDEs BY MATRIX METHOD

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1.Solve the following.

(a) $x_1' = 3x_1 - 2x_2; x_2' = 2x_1 - 2x_2; x_1(0) = 1, x_2(0) = -1.$

Matlab code:

```
clc
clear all
syms t C1 C2
A=input('Enter the matrix A:= ');
[P,D]=eig(A);
L1=D(1);L2=D(4);
y1=C1*exp(L1*t);
y2=C2*exp(L2*t);
Y=[y1;y2];
X=P*Y;
Cond=input('Enter the initial conditions [t0, x10,x20]:= ');
t0=Cond(1);
x10=Cond(2);
x20=Cond(3);
eq1=subs(X(1)-x10,t0);
eq2=subs(X(2)-x20,t0);
[C1, C2] = solve(eq1,eq2);
X=subs(X)
figure
ezplot(X(1));
figure
ezplot(X(2));
```

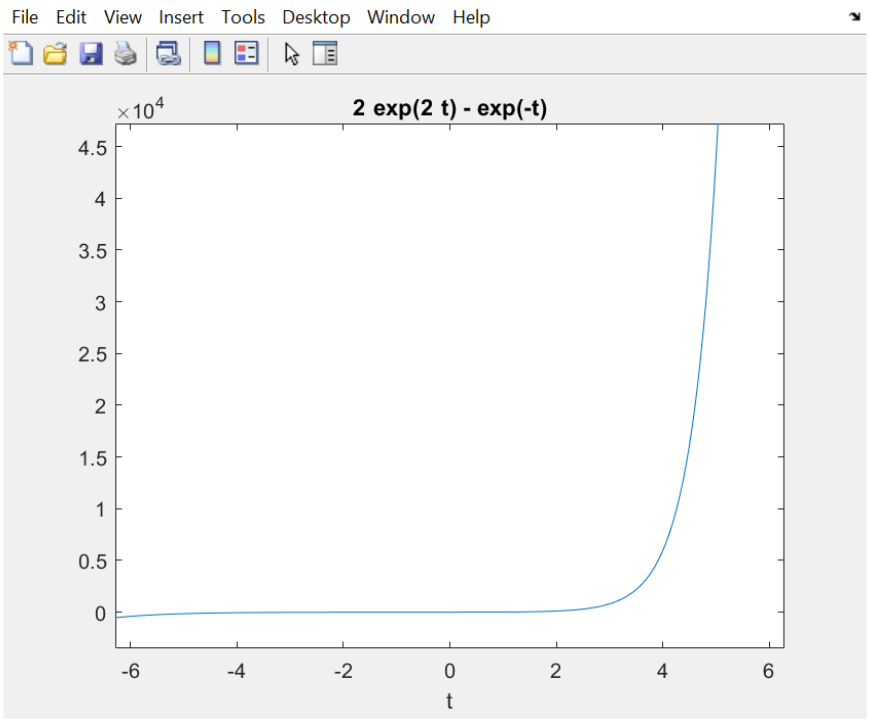
Input:

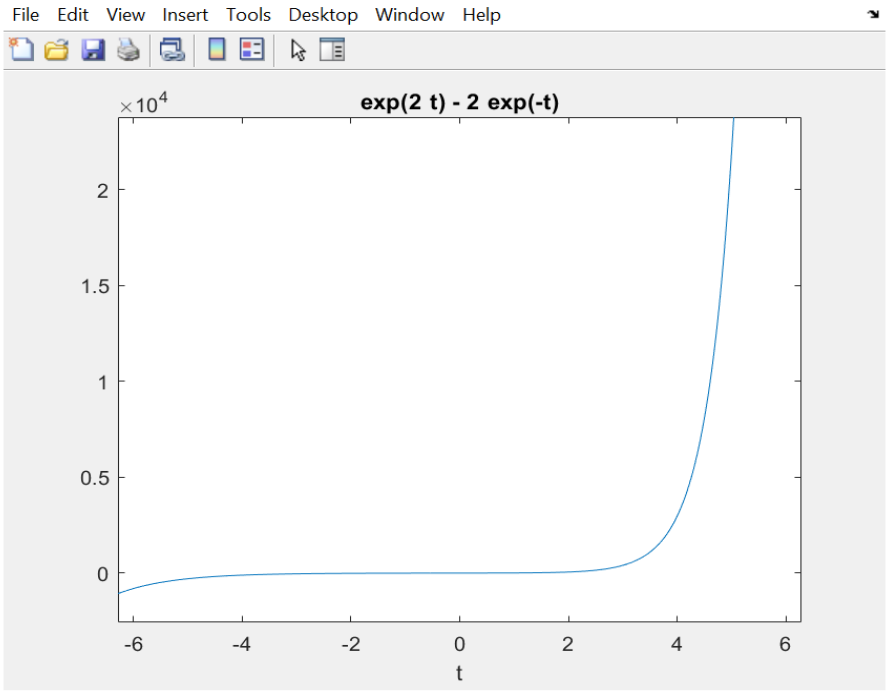
```
Enter the matrix A:= [3 -2 ;2 -2]
Enter the initial conditions [t0, x10,x20]:= [0 1 -1]
```

Output:

```
X = 2*exp(2*t) - exp(-t)
    exp(2*t) - 2*exp(-t)
```

Graph:





(b) $x_1' = -x_2 + x_3; x_2' = 4x_1 - x_2 - 4x_3; x_3' = -3x_1 - x_2 + 4x_3.$

Matlab code:

```
clc
clear all
syms t C1 C2 C3
A=input('Enter the matrix A:= ');
[P,D]=eig(A);
L1=D(1);L2=D(5);L3=D(9);
y1=C1*exp(L1*t);
y2=C2*exp(L2*t);
y3=C3*exp(L3*t);
Y=[y1;y2;y3];
X=P*Y;
X=subs(X)
```

Input:

```
Enter the matrix A:= [0 -1 1;4 -1 -4;-3 -1 4]
```

Output:

```
X =

(6^(1/2)*C3*exp(-t))/6 - (6^(1/2)*C1*exp(3*t))/6
+(2^(1/2)*C2*exp(t))/2
(6^(1/2)*C1*exp(3*t))/6-
(7165183131591449*C2*exp(t))/10141204801825835211973625643008 +
(2^(1/2)*3^(1/2)*C3*exp(-t))/3
(6^(1/2)*C3*exp(-t))/6 + (2^(1/2)*C2*exp(t))/2 -
(2^(1/2)*3^(1/2)*C1*exp(3*t))/3
```

Matlab code for 2nd order eqn

```
clc
clear all
A=input('Enter the matrix A: ');
[P D]=eig(A);
Sol1 = dsolve(['D2y = ',num2str(D(1)),'*y']);
Sol2 = dsolve(['D2y = ',num2str(D(4)),'*y']);
X = P*[Sol1;Sol2];
disp('x1=');
disp(X(1))
disp('x2=');
disp(X(2))
```

4. Solve the following:

(a) $x_1'' = -5x_1 + 2x_2; x_2'' = 2x_1 - 2x_2.$

(b) $x_1'' + 2x_1 - x_2 = 0; x_2'' - x_1 + 2x_2 = 0.$

(a)

Input:

```
Enter the matrix A: [-5 2;2 -2]
```

Output:

```
x1=
   -(5^(1/2)*(C3*cos(t) - C4*sin(t)))/5 -
(2*5^(1/2)*(C1*cos(6^(1/2)*t) - C2*sin(6^(1/2)*t)))/5

x2=
(5^(1/2)*(C1*cos(6^(1/2)*t) - C2*sin(6^(1/2)*t)))/5-
(2*5^(1/2)*(C3*cos(t) - C4*sin(t)))/5
```

(b)

Input:

```
Enter the matrix A: [2 -1;-1 2]
```

Output:

```
x1=
- (2^(1/2)*(C2*exp(t) + C1*exp(-t)))/2 -
(2^(1/2)*(C3*exp(3^(1/2)*t) + C4*exp(-3^(1/2)*t)))/2

x2=
(2^(1/2)*(C3*exp(3^(1/2)*t) + C4*exp(-3^(1/2)*t)))/2 -
(2^(1/2)*(C2*exp(t) + C1*exp(-t)))/2
```

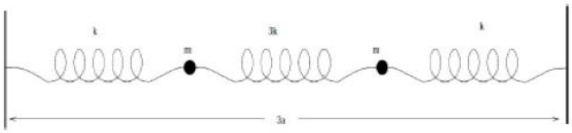
5.

Two particles of equal mass $m = 1$ move in one dimension at the junction of three springs. The springs each have unstretched length $a = 1$ and have spring stiffness constants, k, $3k$ and k (with $k = 1$) respectively see Figure. Applying Newton's second law and Hooke's, this mass-spring system gives rise to the differential equation system

$$x_1'' = -4x_1 + 3x_2$$

$$x_2'' = 3x_1 - 4x_2$$

Find the displacements $x_1(t)$ and $x_2(t)$.



Input:

```
Enter the matrix A: [-4 3 ;3 -4]
```

Output:

```
x1=
(2^(1/2)*(C3*cos(t) - C4*sin(t)))/2 +
(2^(1/2)*(C1*cos(7^(1/2)*t) - C2*sin(7^(1/2)*t)))/2

x2=
(2^(1/2)*(C3*cos(t) - C4*sin(t)))/2 -
(2^(1/2)*(C1*cos(7^(1/2)*t) - C2*sin(7^(1/2)*t)))/2
```

6.

Reduce the third order equation $y''' + 2y'' - y' - 2y = 0$ to the system of first order linear equations and solve by matrix method.

Matlab code:

```
clc
clear all
syms t C1 C2 C3
A=input('Enter the matrix A:= ');
[P,D]=eig(A);
L1=D(1);L2=D(5);L3=D(9);
y1=C1*exp(L1*t);
y2=C2*exp(L2*t);
y3=C3*exp(L3*t);
Y=[y1;y2;y3];
X=P*Y;
X=subs(X)
```

Input:

```
Enter the matrix A:= [0 1 0; 0 0 1 ; 2 1 -2]
```

Output:

```
X =

     (3^(1/2)*C2*exp(-t))/3 + (21^(1/2)*C1*exp(-2*t))/21 -
(3^(1/2)*C3*exp(t))/3
 - (3^(1/2)*C2*exp(-t))/3 - (2*21^(1/2)*C1*exp(-2*t))/21 -
(3^(1/2)*C3*exp(t))/3
   (3^(1/2)*C2*exp(-t))/3 + (4*21^(1/2)*C1*exp(-2*t))/21 -
(3^(1/2)*C3*exp(t))/3
```

7. Consider tanks T_1 and T_2 which contain initially 100 gallons of water each. In T_1 water is pure whereas 150 pounds of salt is dissolved in T_2. By circulating the liquid at the rate of 2 gallons per minute and stirring, the amount of salt $y_1(t)$ in T_1 and $y_2(t)$ in T_2 change with time t, find the amount of salt in the two tanks after a time t.

Sol: -

Input and Output: -

```
Enter the matrix A: =
[-0.02 0.02;0.02 -0.02]

X =

 (2^ (1/2) *C2)/2 + (2^(1/2)*C1*exp(-t/25))/2
 (2^(1/2)*C2)/2 - (2^(1/2)*C1*exp(-t/25))/2
```

--------THANK YOU---------