Introduction to K-theory and C*-algebras

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I. Basics of C* algebra theory

We will assume familiarity with the basic notions of C^*-algebras (the definition, and sub-C^*-algebras). We will however recap one basic definition.

Definition I.1 (Algebra)**.** A ring with identity A is said to be a R-algebra, for ring R, if A is a unital R-module when A is treated as an Abelian group, and the multiplication in A is compatible and commutes with the action of R on A. That is, for $a, b \in A$ and $r \in R$, $r \cdot (ab) = (r \cdot a)b = a(r \cdot b)$.

The first interesting result that we present is due to Gelfand and Naimark. Recall that a Hilbert space H is an inner product space which is complete with respect to the topology induced by the metric induced by the inner product.

Theorem I.1 (Gelfand-Naimark)**.** For each C^*-algebra, there exists a Hilbert space H and an isometric $*$-homomorphism from A into $B(H)$ (the space of bounded linear operators on H, which itself has a canonical involution (the adjoint) and a canonical norm ($||v|| = \sqrt{\langle v, v \rangle}$). If A is topologically separable, then H can be chosen to be separable as well.

II. A collection of exercises: RLL chapter 1