Differential geometry

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II. Basic differential geometry

A. Introduction to manifolds

We begin with a rigorous discussion of manifolds, heavily inspired by the introduction provided by Lee in his introductory text.

B. Differential forms

C. Tangent space, cotanget space, de Rahm complex

D. Integration over manifolds

I want to preface this section by stating that it my goal to provide decent intuition as to why we wish to formalize integrals over lines, areas, and volumes via the language of differential forms. Classical texts like Spivak are brief in this regard: they define the integral over a form in the particular way, but never elaborate on why we actually require the underlying power of a differential form in the first place. In some sense, this is because the main utility of a differential form, when it comes to integrating, is actually it algebraic properties, rather than the fact that it actually "eats vectors" as an element of the cotangent space.

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