A primer on generalized complex geometry

Jack Ceroni*
(Dated: Saturday 28th October, 2023)

The purpose of these notes is to introduce, with minimal assumed knowledge, the field of generalized complex geometry. These notes were written in the Fall of 2023.

Proposition .1. Let $\langle \cdot, \cdot \rangle$ denote a bilinear form on n-dimensional vector space V. Then, there exists a basis $v_1, \dots, v_n$ for V such that

$$\langle x_1 v_1 + \cdots + x_n v_n, x_1 v_1 + \cdots x_n v_n \rangle = x_1^2 + \cdots x_p^2 - x_{p+1}^2 - \cdots x_{p+q}^2 \tag{1}$$

where $p + q = n$. Moreover, the pair (p, q) is independent of choice of basis, and is called the *signature* of the bilinear form.

Proof. $\square$

Definition .1. Given vector space V endowed with bilinear form $\langle \cdot, \cdot \rangle$, a subspace W is said to be isotropic if the form vanishes for pairs $w_1, w_2 \in W$.

Given n-dimensional vector space V, we take a double $\mathcal{D}V$ to be a $2n$-dimensional vector space endowed with $\langle \cdot, \cdot \rangle$ and projection $\pi : \mathcal{D}V \to V$ such that $\mathrm{Ker}(\pi)$ is isotropic with respect to $\langle \cdot, \cdot \rangle$. Note that $\mathrm{Ker}(\pi)$ being isotropic implies that $\mathrm{Ker}(\pi) \subset \mathrm{Ker}(\pi)^{\perp}$.

* jackceroni@gmail.com