

A primer on cyclic cohomology

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I. Introduction

This essay was written for the Fall 2023 session of MAT437 at the University of Toronto.

The goal of this essay is to introduce, explain, and place in broader context the basic theory of cyclic cohomology. The main reference for this essay is the book by Moriyoshi and Natsume (Ref. []). This book provides a brief introduction to non-commutative geometry: we attempt to expand upon some of their main points in this work.

II. Motivation and background

A. Trace maps

The K_0 -group of a C^* -algebra A is a broadly powerful invariant which encodes the underlying structure of A , in such a way that lends itself well to being analyzed via a large toolbox of different techniques. Amongst such techniques are extracting numerical invariants from $K_0(A)$ via *trace maps*.

Definition II.1 (Trace map on a C^* -algebra).

Using the functoriality of K_0 , it is possible to then obtain a trace map $K_0(\tau) : K_0(A) \rightarrow \mathbb{C}$.

Claim II.1 (Induced trace map).

Proof.

□

Trace maps of this form are vital to the analysis of K_0 -groups. However, as it turns out, one will often encounter C^* -algebras which *admit no non-trivial trace*.

B. A brief aside: the Cuntz algebra

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