# MUNKRES TOPOLOGY: CHAPTER 32

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## 1. Problem 32.6

*Show that $X$ is completely normal if and only if for every pair of sets $A$, $B$ with $A \cap \overline{B} = \overline{A} \cap B = \emptyset$, there exist disjoint open sets containing them.*

Suppose first that $X$ is completely normal. Note that $Y = X - (\overline{A} \cap \overline{B})$ is an open subspace of $X$. It is easy to see that both $A$ and $B$ are in $Y$: if this weren't the case then we could pick a point in both $A$ and $\overline{B}$, or $B$ and $\overline{A}$. Thus, we can pick open sets $U$ and $V$ in $Y$ containing $\overline{A} \cap Y$ and $\overline{B} \cap Y$, which themselves contain $A$ and $B$.

Since $Y$ is open, these sets will also be open in $X$.

Suppose the converse holds. Let $C$ be a subspace of $X$, and let $Q$ and $P$ be closed