A primer on cyclic cohomology

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(Dated: Saturday 28th October, 2023)

I. Introduction

This essay was written for the Fall 2023 session of MAT437 at the University of Toronto.

The goal of this essay is to introduce, explain, and place in broader context the basic theory of cyclic cohomology. The main reference for this essay is the book by Moriyoshi and Natsume (Ref. []), as well as review articles [CITE]. This book provides a brief introduction to non-commutative geometry: we attempt to expand upon some of their main points in this work.

II. Motivation and background

A. Trace maps

The K_0-group of a C^*-algebra A is a broadly powerful invariant which encodes the underlying structure of A, in such a way that lends itself well to being analyzed via a large toolbox of different techniques. Among such techniques are extracting numerical invariants from $K_0(A)$ via *trace maps*.

Definition II.1 (Trace map on a C^*-algebra)**.**

Using the functoriality of K_0, it is possible to then obtain a trace map $K_o(\tau) : K_0(A) \to \mathbb{C}$.

Claim II.1 (Induced trace map)**.**

Proof. □

Trace maps of this form are vital to the analysis of K_0-groups. However, as it turns out, one will often encounter C^*-algebras which *admit no non-trivial trace*. For example, one of the most important types of C^*-algebras are the so-called *Cuntz algebras*, which are defined as follows.

Definition II.2 (Cuntz algebra)**.**

This is a problem, as in such cases, it is very much unclear how one should go about extracting numerical invariants from $K_0(A)$. Cyclic cohomology, discovered by Alain Connes, can be thought of as a remedy to this situation: it provides a different technique for mapping from K_0-groups to numbers.

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