Hyperbolic geometry: preparing for MAT1305

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(Dated: Monday 7th August, 2023)

I. Introduction

These notes will be in much more of a "stream-of-concious" style than many of my others.

A. Brief reivew on differential forms

The material for this section is mostly drawn from Spivak's Calculus on Manifolds.

To begin, we review some simple notions in differential geometry.

Definition I.1 (Differential Form)**.** A *differential k-form* on a manifold M is a map $\omega : M \to \bigwedge^k(TM)$, where $\bigwedge^k(TM)$ denotes the disjoint union of all collections of alternating k-tensors at each tangent space of the manifold, such that $\omega(x) \in \bigwedge^k(T_x M)$. Let $\Omega^k(M)$ denote the collection of all k-forms on manifold M.

Recall that we are able to write a particular k-form in terms of a sum over wedge products of the standard coordinate 1-forms,

$$\omega(p) = \sum_I f_I(p) dx_{i_1} \wedge \cdots \wedge dx_{i_k} \tag{1}$$

where the sum is over all ascending k-tuples drawn from $\{1, \dots, n\}$, and we recall that coordinate one-forms are defined locally via pullback of the coordinate one-forms in $\mathbb{R}^n$ via the chat at a particular point.

Definition I.2 (Differential)**.** The *differential* is a map $d : \Omega^k(M) \to \Omega^{k+1}(M)$ such that $d\omega$ takes the differential of each coefficients function when ω is expanded, as in Eq. (1),

$$d\omega = \sum_I \sum_{k=1}^{n} \frac{\partial f_I}{\partial x_k} dx_k \wedge dx_{i_1} \wedge \cdots \wedge dx_{i_k} \tag{2}$$

B. Chapter 1

Lemma I.1. Any complex analytic map with non-singular derivative is conformal.

Proof. TODO □

Exercise I.1 (Series 1.12)**.** We summarize in a list:

- This is clear in the case of $\hat{\mathbb{C}}$: take the rotation action about point a, and note that it fixes a.
- In the case of two-variable tuples of distinct points, the transitive property is obviouswe can take the action which performs the exchange $z \mapsto 1/z$, followed by rotation of π about a point $\frac{1}{2}\left(a + \frac{1}{a}\right)$. It is easy to check that this action will fix $(a, 1/a)$, and is not the identity.

[Series 1.13] $\text{Aut}(\hat{\mathbb{C}})$ acts transitively and freely