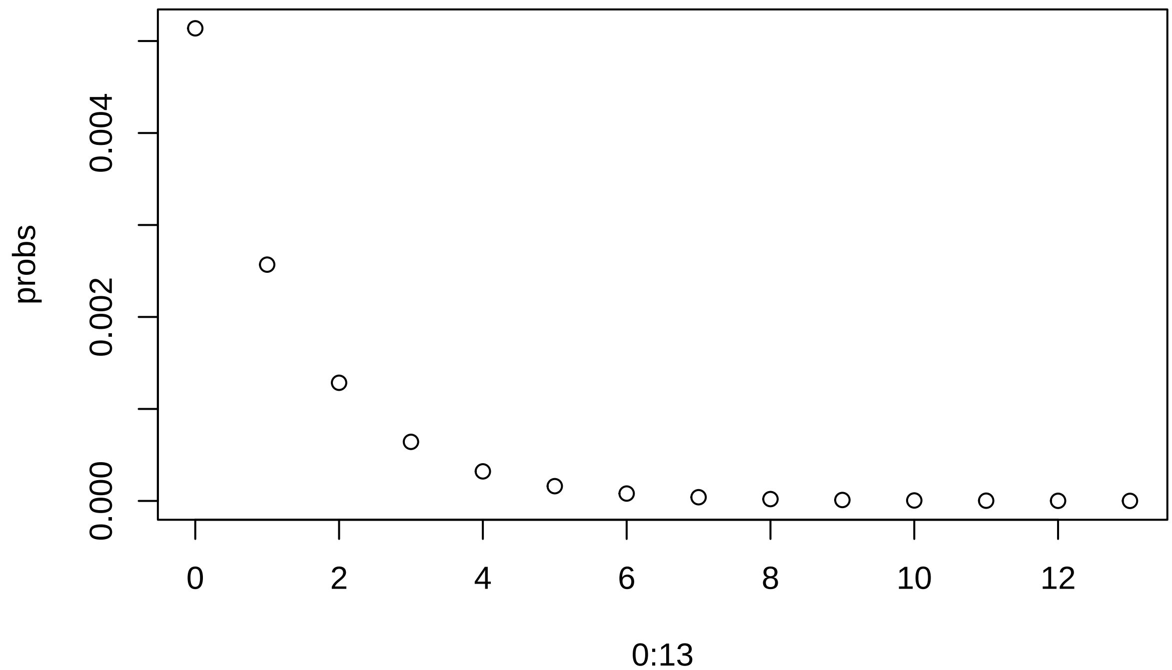
Various exam questions

2 Football Betting (8pt) Imagine a football betting setting where there are 13 football games. Each game can have three possible outcomes: home team wins (1), the teams play even (E) or the visitor team wins (2). Model the outcome of each game as a random process where each of the three outcomes are equally probable and independent from other games. Let the random variable X characterise the number of correct guesses for the 13 outcomes in one betting. a) (Ipt) Write the analytic forms for the probability mass function of X.
Solution:
bla bla

$$p(a) = \left(\frac{2}{3}\right)^{13-a} \cdot \left(\frac{1}{3}\right)^{a}$$

b) (Ipt) Illustrate the probability mass function by plotting it in a figure.
Solution:

```
prob <- function(a) {
  return(
    (
      (2/3)^(13-a)
    ) * (
      (1/3)^a
    )
  )
}

probs <- c()
for (i in 0:13) {
  prub <- prob(i)
  probs <- c(probs, prub)
}

plot(y=probs, x=0:13)
```



c) (Ipt) What is the probability that one get all the 13 outcomes right?
Solution:

```
prob(13)
```

```
## [1] 6.272255e-07
```

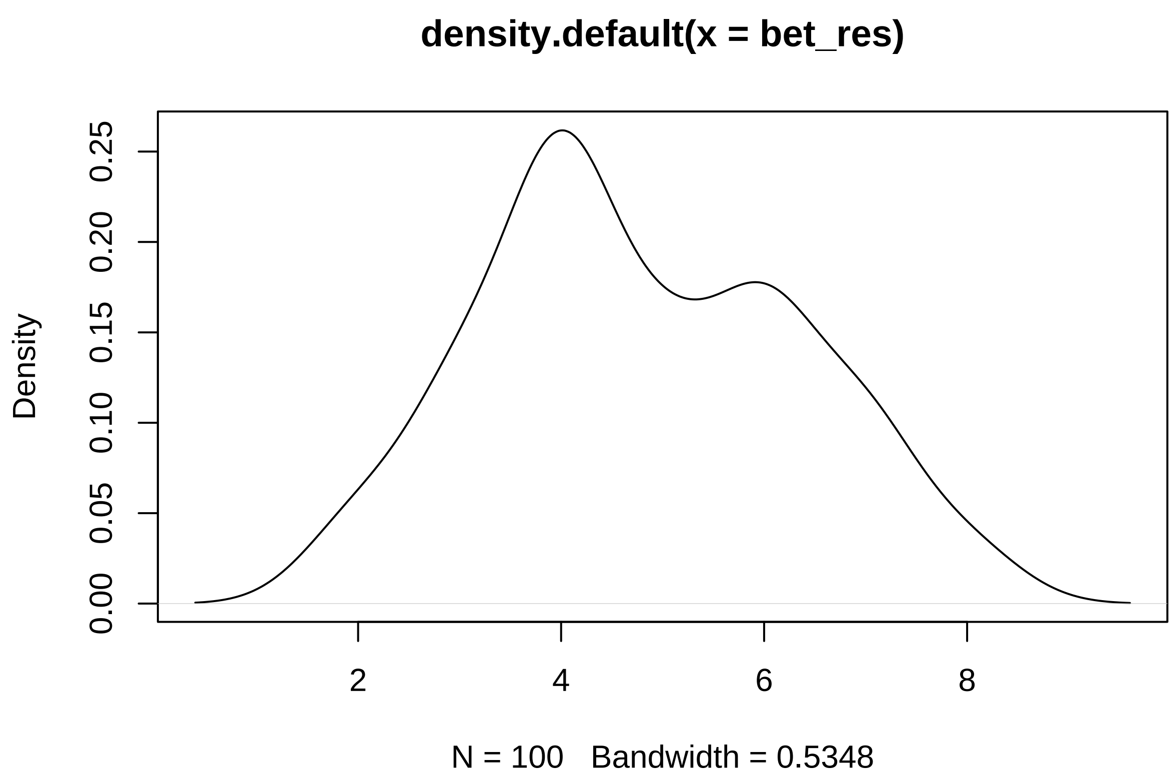
d) (3pt) Simulate the betting by "playing" the betting 100 times. Present the results you got.
Solution:

```
game_result <- function() {
  res <- runif(1, min=-1, max=1)
  return(round(res))
}

setting_result <- function() {
  game_results <- c()
  for (i in 1:13) {
    game_results <- c(game_results, game_result())
  }
  return(game_results)
}

bet_res <- c()
```

```r
for (i in 1:100) {
  set <- setting_result()
  bet <- setting_result()
  bet_res <- c(bet_res, sum(set == bet))
}

summary(bet_res)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    2.00    4.00    4.50    4.79    6.00    8.00
```

```r
plot(density(bet_res))
```



e) (2pt) Assume, one betting costs 0.5 EUR. How would you characterise your chances of getting profit by betting in these games by guessing the results in random? Assume that wins are distributed according to the table below (price is in EUR).

Hits	Price
13	113101
12	7761
11	373
10	63

Solution:

```
total_cost <- 100*0.5
total_profit <- 0
for (i in 1:100) {
  res <- bet_res[i]
  if (res == 10) {
    total_profit = total_profit + 63
  } else if (res == 11) {
    total_profit = total_profit + 373
  } else if (res == 12) {
    total_profit = total_profit + 7761
  } else if (res == 13) {
    total_profit = total_profit + 113101
  }
}
net_gain <- total_profit - total_cost
sprintf("Total profit: %s", total_profit)
```

```
## [1] "Total profit: 0"
```

```
sprintf("Net gain: %s", net_gain)
```

```
## [1] "Net gain: -50"
```

Expected value calculation. First make new random variable, Y. It a discrete random variable with same distribution as X that gives the money gained from a betting round. It takes the numbers 0 to 13, indicating number of correct guesses, and gives out the prizes defined in the table above.

$$
\begin{aligned}
E[Y] &= \sum_i a_i p(a) \\
&= \left(\frac{2}{3}\right)^{13} \cdot \left(\frac{1}{3}\right)^{0} \cdot 0 + \left(\frac{2}{3}\right)^{12} \cdot \left(\frac{1}{3}\right)^{1} \cdot 0 + \dots \\
&+ \left(\frac{2}{3}\right)^{1} \cdot \left(\frac{1}{3}\right)^{12} \cdot 7761 + \left(\frac{2}{3}\right)^{0} \cdot \left(\frac{1}{3}\right)^{13} \cdot 113101 \\
&= \left(\frac{2}{3}\right)^{3} \cdot \left(\frac{1}{3}\right)^{10} \cdot 63 + \left(\frac{2}{3}\right)^{2} \cdot \left(\frac{1}{3}\right)^{11} \cdot 373 \\
&+ \left(\frac{2}{3}\right)^{1} \cdot \left(\frac{1}{3}\right)^{12} \cdot 7761 + \left(\frac{2}{3}\right)^{0} \cdot \left(\frac{1}{3}\right)^{13} \cdot 113101 \\
&= \frac{130619}{1594323} \approx 0.0819
\end{aligned}
$$

Expected value for playing is 0.0819 euro. Seeing as it also costs 0.5 euro to play, it means that our actual expected value is $0.0819 - 0.5 = -0.4181$, that is, we loose money. Don't gamble kids.