Applied Statistics – Exercise 3

Goal

To get confident with conditional probability and discrete random variables. To make visualizations of discrete distributions in R.

Problems

T=Theoretical Exercise, R=R Exercise

1. (T)

We are at a train station, waiting for a train. Suppose that the probability of snow is 0.1. If it is snowing, then the probability that the train is delayed is 0.6, otherwise, it is 0.2. Given that the train is delayed, what is the probability that it is snowing? Define appropriate events, and compute the conditional probability.

2. (T)

Consider the following experiment. An urn contains 7 black and 13 red balls. You draw 10 balls from the urn with replacement. The outcome X of the experiment the number of red balls you got.

a) Write down the probability mass function of X.

b) Evaluate the probability $P(X = 7)$.

3. (T)

A pair of fair dice is thrown until the sum of the two equals 2. Let the random variable X be the number of throws needed for this, where throwing the pair of dice once is counted as a single throw. Find the probability mass function of X.

4. (R)

Consider you have two fair coins that you toss simultaneously (fair coins have a 0.5 probability of heads). You repeat the trial 15 times. Let X be the random variable indicating the number of cases, where both coins come with heads up. In the following exercises you can use the `dbinom` and `pbinom` functions.

a) Plot the probability mass function of X. What is the probability $P(X = 5)$?

b) What is the probability $P(X \leq 5)$? What is the visual interpretation of this probabilty in the graphic you plotted in (a)?

5. (R)

You are a collector of soccer players cards. There is just one card missing from your collection. Every day you buy one, and with the probability 1/100 it is the one you are missing. Each purchase is independent from the others. Model the number of days it takes to find the missing card by the random variable $X \sim Geo(1/100)$.

a) Plot the distribution function of X.

b) How many cards do you have to buy so that the chance of finding the missing card is at least 0.5? How about at least 0.95? (play with different ranges for k).

c) Assume you have tried for 20 days, but you have not won yet. For how many days do you need to try further so that you have at least a 0.5 chance of winning?