APSTA Week 04 Exercises

1. Let X be a continuous random variable. And let $F(x)$ be the distribution function of X. Let $F(a) = 1/5$ and $F(b) = 3/5$.

(a) What is $P(a \leq X \leq b)$?
Solution:

$$\begin{aligned} F(a < X \leq b) &= F(b) - F(a) \\ &= 3/5 - 1/5 \\ &= 2/5 \end{aligned}$$

(b) What is $P(X > b)$?
Solution:

$$\begin{aligned} P(X > b) &= 1 - P(X \leq b) \\ &= 1 - F(b) \\ &= 1 - 3/5 \\ &= 2/5 \end{aligned}$$

(c) What is $P(X < a \vee X > b)$?
Solution:
The $\vee$ means "or".

$$\begin{aligned} P(X < a \vee X > b) &= 1 - P(a \leq X \leq b) \\ &= 1 - 2/5 \\ &= 3/5 \end{aligned}$$

(d) What is $P(X = a)$?

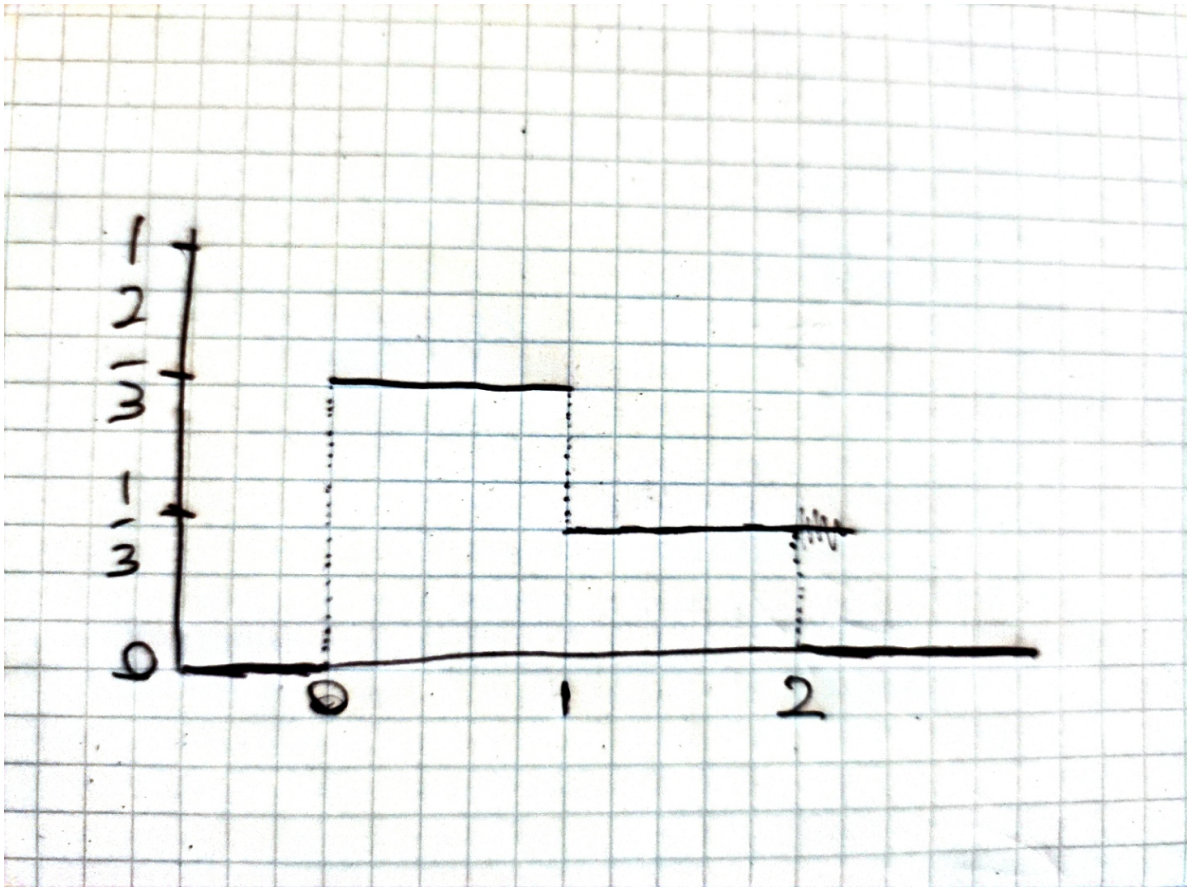
$$P(X = a) = 0$$

The probability approaches zero, as the integral of a to a is 0.

2. Consider the continuous random variable X with the following probability density function

$$f(x) = \begin{cases} 2/3 & \text{if } 0 \leq x < 1 \\ 1/3 & \text{if } 1 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Draw, with pen and paper, the probability density function of X, i.e. $f(x)$.
Solution:


1
2
1/3
1/3
0
0
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2