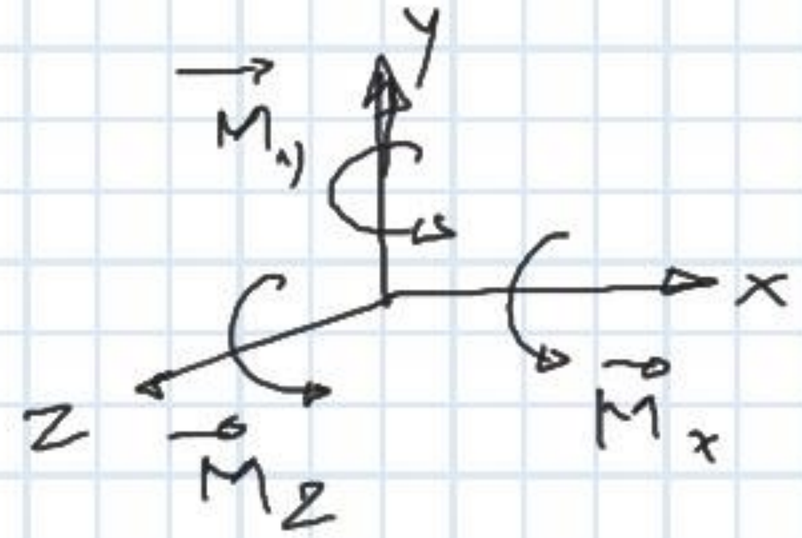
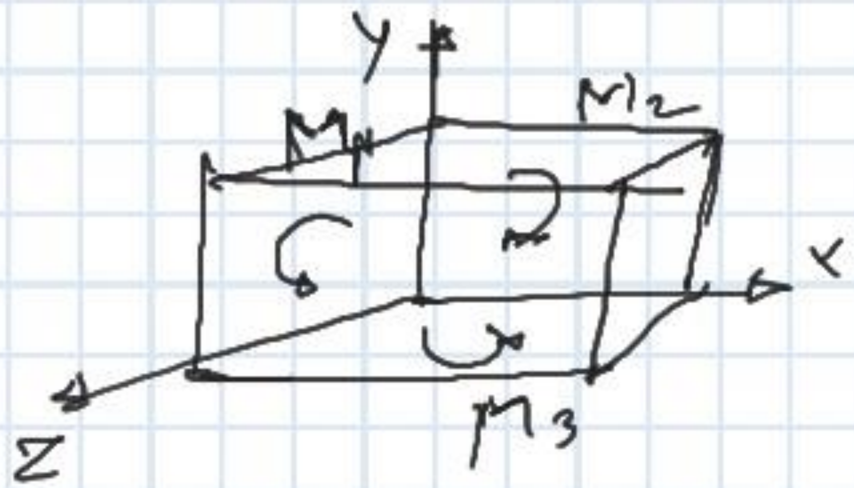
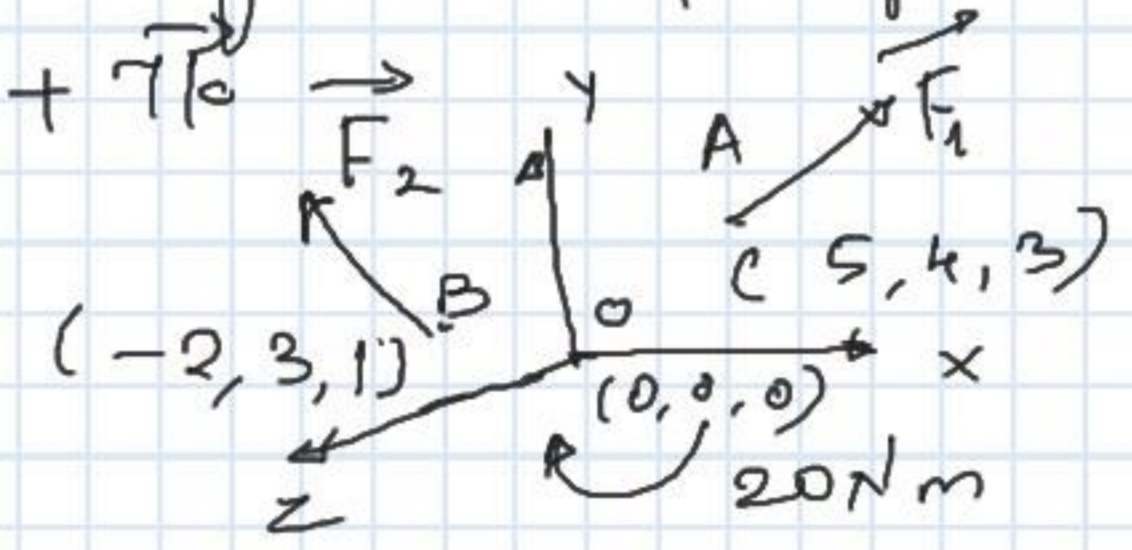
Couple in vector form in 3D





Couple	Plane	Vector form
M_1	ZY	$M_1 \vec{i}$
M_2	XY	$-M_2 \vec{k}$
M_3	ZX	$M_3 \vec{j}$

(1). For the force system, reduce it to an equivalent force-couple system at origin:- $\vec{F_1} = 12\vec{i} - 6\vec{j} + 8\vec{k}$ $\vec{F_2} = -4\vec{i} + 9\vec{j} + 7\vec{k}$



[redacted]

Equivalent force couple system at the origin

sol.

$$\vec{R} = \vec{F_1} + \vec{F_2} = (12\vec{i} - 6\vec{j} + 8\vec{k}) + (-4\vec{i} + 9\vec{j} + 7\vec{k})$$

$$= 8\vec{i} + 3\vec{j} + 15\vec{k}$$

$$\vec{M_o} = (\vec{r_{OA}} \times \vec{F_1}) + (\vec{r_{OB}} \times \vec{F_2}) - 20\vec{j}$$

$$\vec{r_{OA}} = 5\vec{i} + 4\vec{j} + 3\vec{k}$$

$$\vec{r_{OB}} = -2\vec{i} + 3\vec{j} + \vec{k}$$

$$\vec{M_o} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 4 & 3 \\ 12 & -6 & 8 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 3 & 1 \\ -4 & 9 & 7 \end{vmatrix} - 20\vec{j}$$

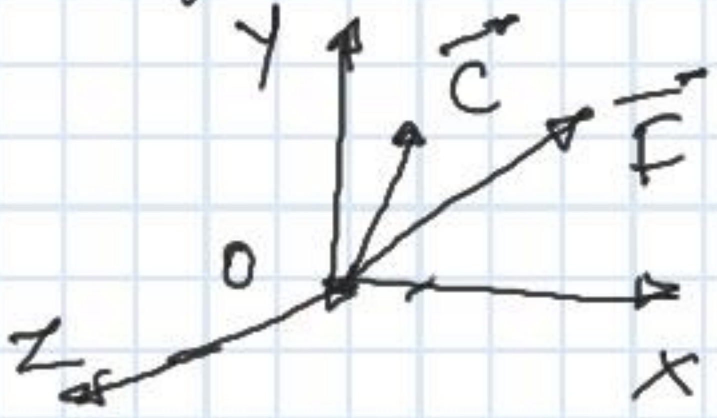
$$= 62\vec{i} - 14\vec{j} - 84\vec{k}$$

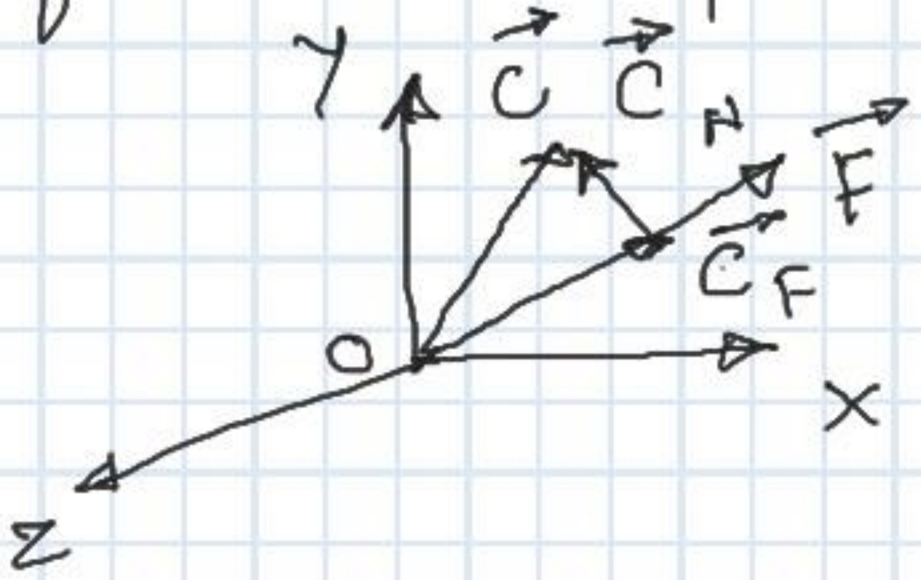
$M_{ox} = 62 \qquad M_{oy} = -14 \qquad M_{oz} = -84$

Wrench - It consists of a force and collinear couple

Procedure

If we have force and couple acting at origin



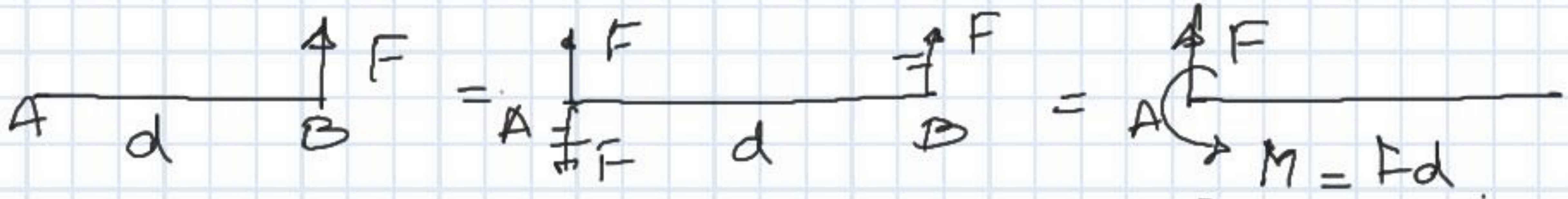


$$\vec{C} = \vec{C}_N + \vec{C}_F$$

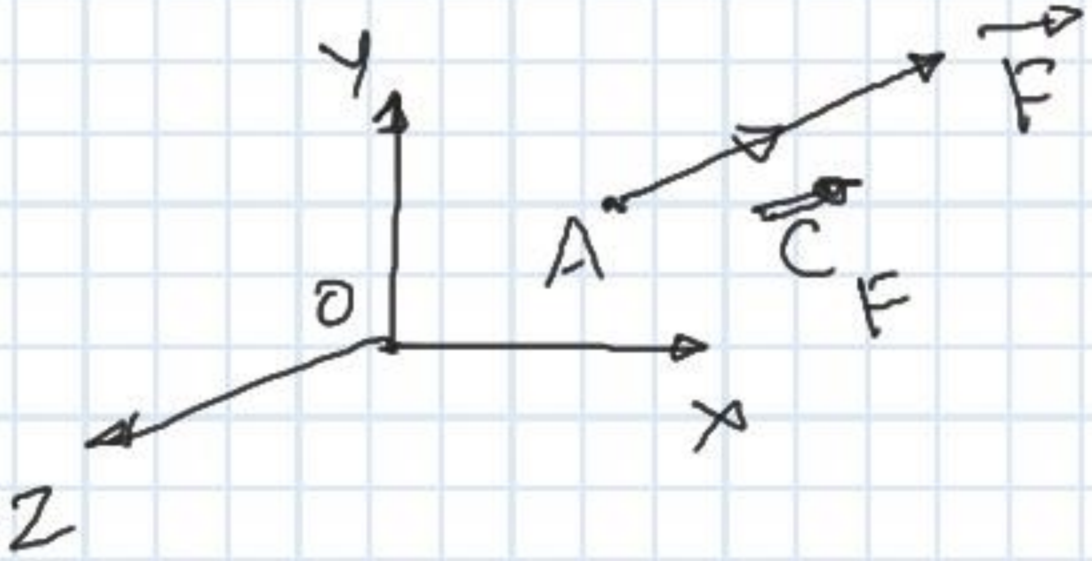
i) $\vec{C}_F$ = Component of $\vec{C}$ on $\vec{F}$
$= (\vec{C} \cdot \vec{\lambda}_F)(\vec{\lambda}_F)$

ii) $\vec{C}_N$ is the additional couple and it can be cancelled if we shift $\vec{F}$ to a new point A.

2D



$\vec{C}_N = \vec{OA} \times \vec{F}$

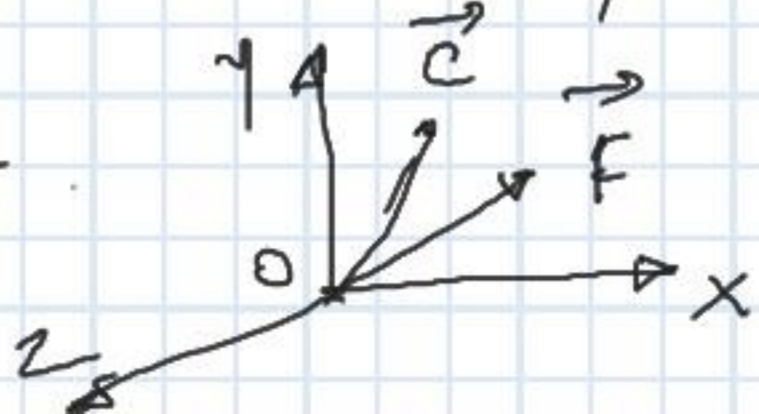


(wrench passing though A)

Problem

1. A force vector $\vec{F} = 20\vec{i} - 30\vec{j} + 10\vec{k}$ and a couple $\vec{C} = 56\vec{i} + 42\vec{j} - 28\vec{k}$ are acting at the origin, find the coordinate of point A lying in x-z plane through which the wrench is passed.

Sol



i) $\vec{C_F} = (\vec{C} \cdot \vec{\lambda_F})(\vec{\lambda_F})$

ii) $\vec{C_N} = \vec{C} - \vec{C_F}$

iii) $\vec{C_N} = \vec{OA} \times \vec{F}$

i) $\vec{C_F}$ - Collinear couple

$$\vec{C_F} = \left[(56\vec{i} + 42\vec{j} - 28\vec{k}) \cdot \left(\frac{20\vec{i} - 30\vec{j} + 10\vec{k}}{\sqrt{20^2 + 30^2 + 10^2}}\right)\right]\left(\frac{20\vec{i} - 30\vec{j} + 10\vec{k}}{\sqrt{20^2 + 30^2 + 40^2}}\right)$$

$$= -0.3\,(20\vec{i} - 30\vec{j} + 10\vec{k})$$

$$\vec{C_F} = -6\vec{i} + 9\vec{j} - 3\vec{k}$$

ii) $\vec{C_N} = (56\vec{i} + 42\vec{j} - 28\vec{k}) - (-6\vec{i} + 9\vec{j} - 3\vec{k})$

$$= 62\vec{i} + 23\vec{j} - 25\vec{k}$$

iii) $\vec{C_N} = \vec{OA} \times \vec{F}$

$$62\vec{i} + 23\vec{j} - 25\vec{k} = \begin{vmatrix} i & j & k \\ x & 0 & z \\ 20 & -30 & 10 \end{vmatrix}$$

Solve for $z = 2.067$ units

$x = 0.833$ units