

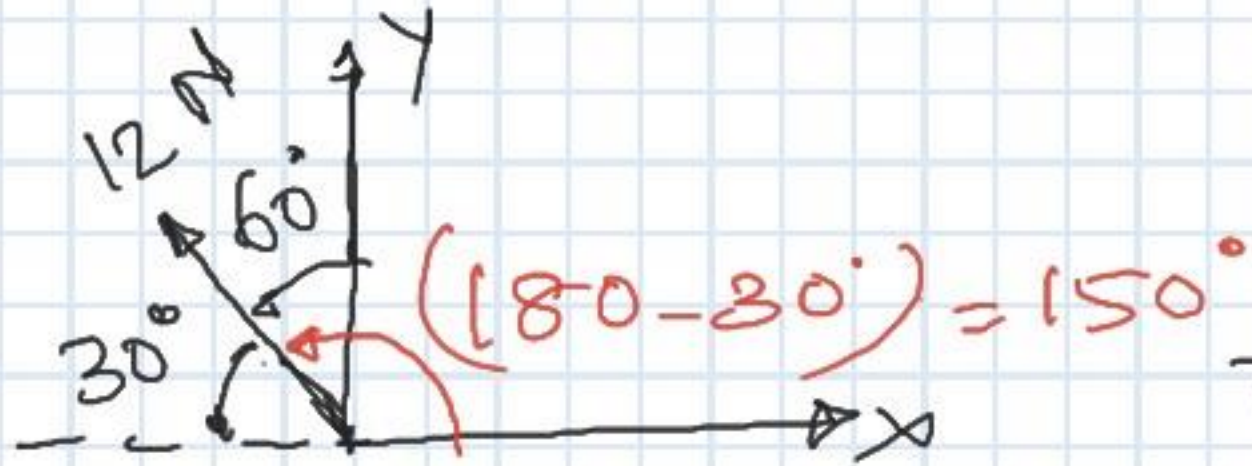
Represent the force in vector form

$$\vec{F} = F_x \vec{i} + F_y \vec{j}$$

$$F_x = 12 \cos 30^\circ$$

$$F_y = 12 \cos 60^\circ$$

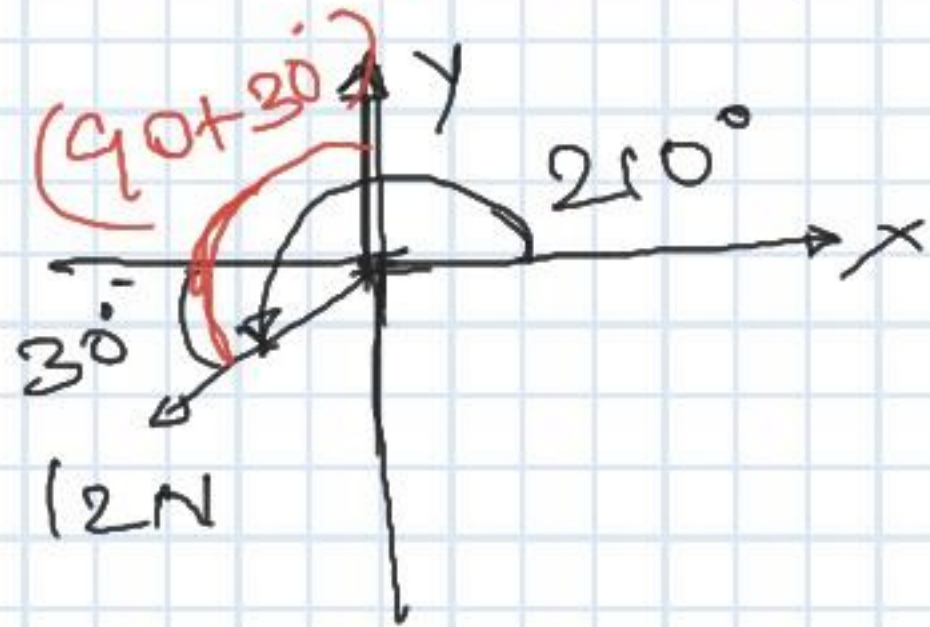
$$\vec{F} = 12 \cos 30^\circ \vec{i} + 12 \cos 60^\circ \vec{j}$$



$$\theta_x = 150^\circ$$

$$\theta_y = 60^\circ$$

$$\vec{F} = 12 \cos 150^\circ \vec{i} + 12 \cos 60^\circ \vec{j}$$



$$\theta_x = 210^\circ$$

$$\theta_y = 90 + 30 = 120^\circ$$

$$\vec{F} = 12 \cos 210^\circ \vec{i} + 12 \cos 120^\circ \vec{j}$$

2. A force \vec{F} has components as $F_x = 36 \text{ N}$ $F_y = -60 \text{ N}$
Get the mag & dir made by \vec{F} with the axis.

sol Magnitude of the force vector

$$F = \sqrt{\sum F_x^2 + \sum F_y^2} = \sqrt{(36)^2 + (60)^2} = 69.97 \text{ N}$$

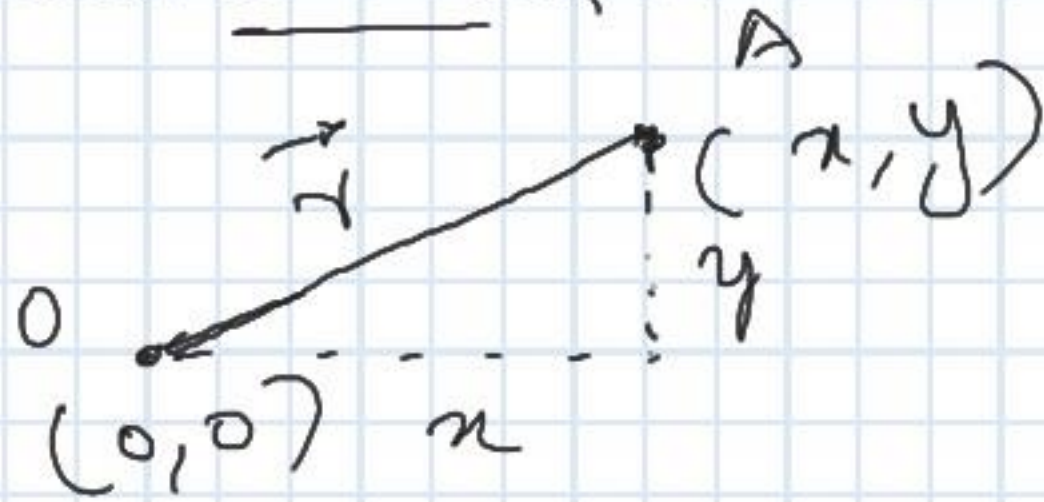
$$F_x = F \cos \theta_x$$

$$F_y = F \cos \theta_y$$

$$\theta_x = \cos^{-1} \frac{F_x}{F} = \cos^{-1} \frac{36}{69.97} = 59.03^\circ$$

$$\theta_y = \cos^{-1} \frac{F_y}{F} = \cos^{-1} \frac{-60}{69.97} = 149.03^\circ$$

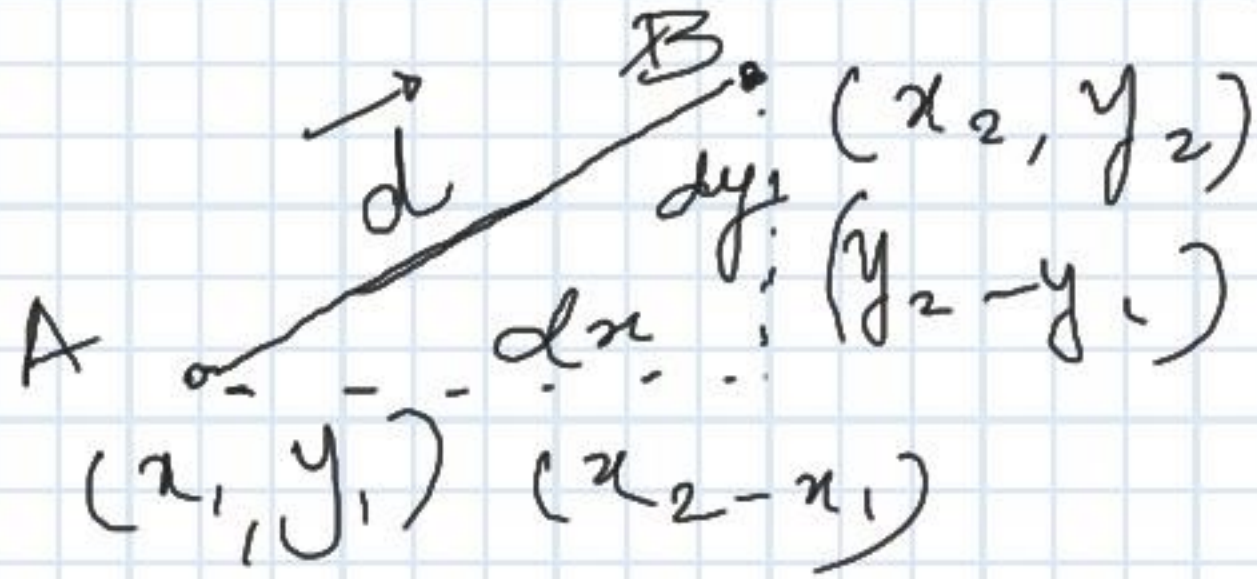
Position vector



The position vector of point A w.r.t origin O is \vec{OA} and it is measured by $\vec{r} = x\vec{i} + y\vec{j}$

Magnitude of position vector

$$r = \sqrt{x^2 + y^2}$$



$$\vec{d} = dx\vec{i} + dy\vec{j}$$

$$= (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j}$$

Magnitude

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Unit vector

$$\lambda_{AB} = \frac{\vec{d}}{d} = \frac{dx\vec{i} + dy\vec{j}}{d}$$

$$= \frac{dx}{d}\vec{i} + \frac{dy}{d}\vec{j}$$

$$\vec{F} = (F)(\lambda_{AB})$$

$$\vec{F} = \frac{Fdx}{d}\vec{i} + \frac{Fdy}{d}\vec{j}$$

$$F_x = \frac{Fdx}{d}$$

$$F_y = \frac{Fdy}{d}$$

But $F_x = F \cos \theta_x$

$$F_y = F \cos \theta_y$$

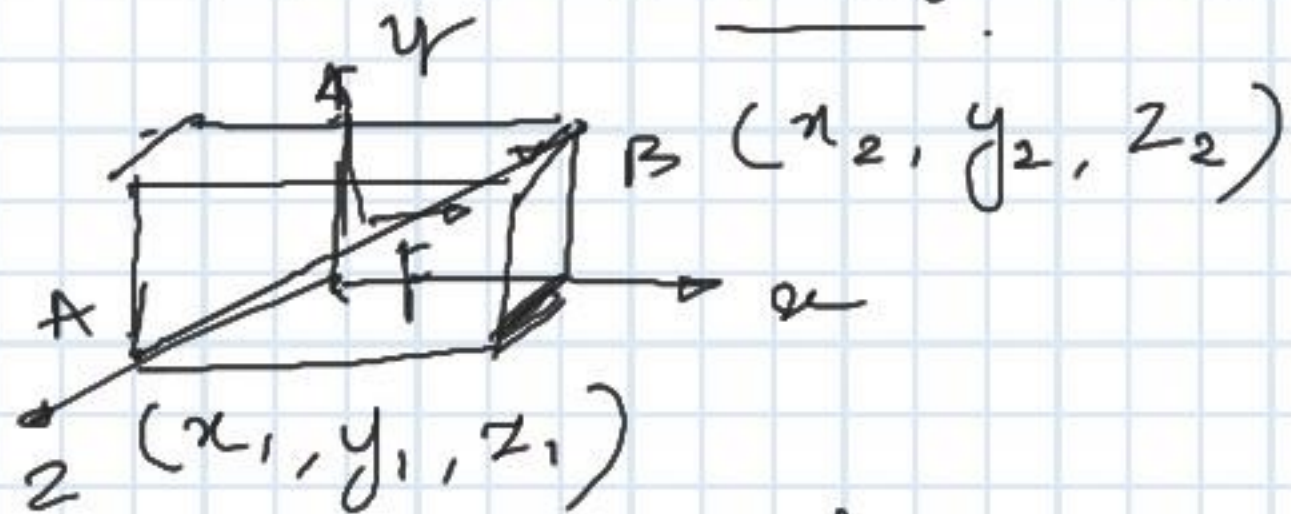
$$\therefore F \cos \theta_x = F \frac{dx}{d}$$

$$F \cos \theta_y = F \frac{dy}{d}$$

$$\Rightarrow \theta_x = \cos^{-1} \frac{dx}{d}$$

$$\theta_y = \cos^{-1} \frac{dy}{d}$$

3D - Unit vector.



Unit vector along AB.

$$\vec{AB} = (AB) (\lambda_{AB})$$

$$\lambda_{AB} = \frac{\vec{AB}}{AB} = \frac{dx \vec{i} + dy \vec{j} + dz \vec{k}}{d}$$

$$\lambda_{AB} = \frac{dx}{d} \vec{i} + \frac{dy}{d} \vec{j} + \frac{dz}{d} \vec{k}$$

$$\vec{F} = (F) (\lambda_{AB})$$

$$= F \frac{dx}{d} \vec{i} + F \frac{dy}{d} \vec{j} + F \frac{dz}{d} \vec{k}$$

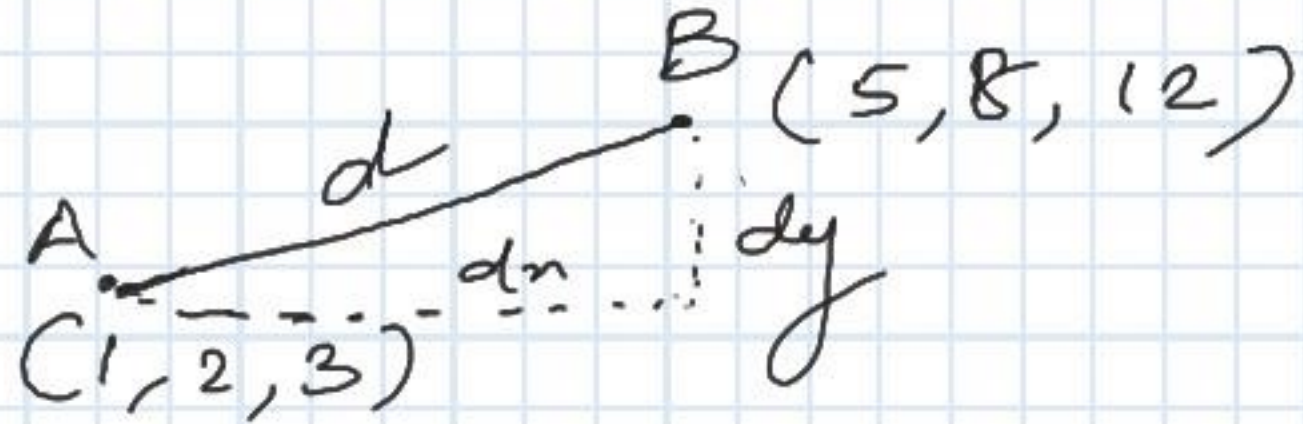
$$\vec{F} = F \cos \theta_x \vec{i} + F \cos \theta_y \vec{j} + F \cos \theta_z \vec{k}$$

Cont

1. $F = 100\text{ N}$ and passing through a line AB with coordinates $A(1, 2, 3)$ & $B(5, 8, 12)$ Determine
- (i) The components of the force along x, y & z axes
 - (ii) The angle of inclination made by the force with x, y & z axes
 - (iii) Represent F as a vector.

Sol

$$F = 100\text{ N}$$



$$\begin{aligned} (i) \quad dx &= (x_2 - x_1) = 5 - 1 = 4 \\ dy &= (y_2 - y_1) = 8 - 2 = 6 \\ dz &= (z_2 - z_1) = 12 - 3 = 9 \end{aligned}$$

$$d = \sqrt{dx^2 + dy^2 + dz^2} = \sqrt{4^2 + 6^2 + 9^2} = 11.53$$

$$F_x = F \frac{dx}{d} = 100 \left(\frac{4}{11.53} \right) = 34.69 \text{ N}$$

$$F_y = F \frac{dy}{d} = 100 \left(\frac{6}{11.53} \right) = 52.03 \text{ N}$$

$$F_z = F \frac{dz}{d} = 100 \left(\frac{9}{11.53} \right) = 78.05 \text{ N}$$

(ii) Inclination of the force with respect to x, y & z axes

$$\cos \theta_x = \frac{dx}{d}$$

$$\theta_x = \cos^{-1} \frac{dx}{d}$$

$$\cos \theta_y = \frac{dy}{d}$$

$$\theta_y = \cos^{-1} \frac{dy}{d}$$

$$\cos \theta_z = \frac{dz}{d}$$

$$\theta_z = \cos^{-1} \frac{dz}{d}$$

$$\theta_x = \cos^{-1}\left(\frac{4}{11.53}\right) = 69.7^\circ$$

$$\theta_y = \cos^{-1}\left(\frac{6}{11.53}\right) = 58.64^\circ$$

$$\theta_z = \cos^{-1}\left(\frac{9}{11.53}\right) = 38.68^\circ$$

$$(ii) \vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$$

$$\vec{F} = 34.69 \vec{i} + 52.03 \vec{j} + 78.05 \vec{k}$$

Resultant of concurrent forces in 3D

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = R_x \vec{i} + R_y \vec{j} + R_z \vec{k}$$

$$= (F_{1x} + F_{2x} + F_{3x}) \vec{i} + (F_{1y} + F_{2y} + F_{3y}) \vec{j} + (F_{1z} + F_{2z} + F_{3z}) \vec{k}$$

Magnitude of the resultant

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

Direction of the resultant

$$\phi_x = \cos^{-1} \frac{R_x}{R}$$

$$\phi_y = \cos^{-1} \frac{R_y}{R}$$

$$\phi_z = \cos^{-1} \frac{R_z}{R}$$



$$\alpha = \tan^{-1} \frac{\Sigma F_y}{\Sigma F_x}$$

Under equilibrium condition

$$\vec{R} = 0$$

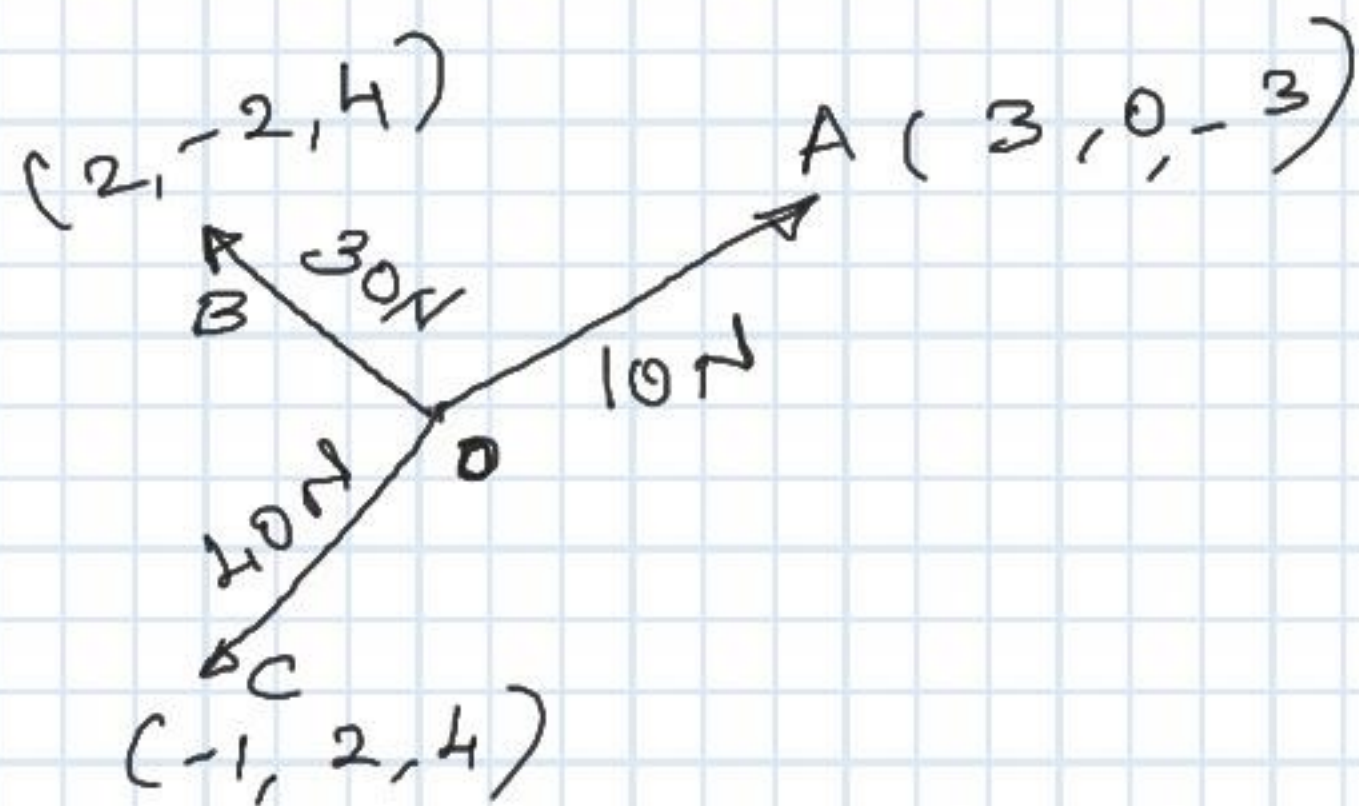
$$\therefore R_x = 0; R_y = 0; R_z = 0$$

$$\sum F_x = 0 \quad - (1)$$

$$\sum F_y = 0 \quad - (2)$$

$$\sum F_z = 0 \quad - (3)$$

2.



The line of action of three forces are concurrent at origin O. Find the magnitude and direction of the resultant force.

Sol

$$R = ?$$

$$\phi_x = ?$$

$$\phi_y = ?$$

$$\phi_z = ?$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$R_x = \sum F_x$$

$$R_y = \sum F_y$$

$$R_z = \sum F_z$$

Given data $F_1 = 10\text{ N}$ $F_2 = 30\text{ N}$ $F_3 = 40\text{ N}$

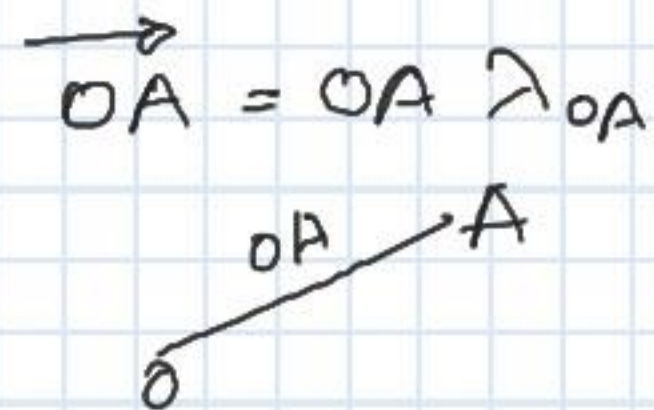
$$\vec{F}_1 = (F_1)(\lambda_{OA}) = 10 \left(\frac{\vec{OA}}{OA} \right)$$

$$\vec{F}_2 = (F_2)(\lambda_{OB}) = 30 \left(\frac{\vec{OB}}{OB} \right)$$

$$\vec{F}_3 = (F_3)(\lambda_{OC}) = 40 \left(\frac{\vec{OC}}{OC} \right)$$

$$O(0, 0, 0) \quad B(2, -2, 4)$$

$$A(3, 0, -3) \quad C(-1, 2, 4)$$



$$OA = \sqrt{(3-0)^2 + (0-0)^2 + (-3-0)^2}$$

$$= 4.24$$

$$\vec{OA} = (3-0)\vec{i} + (0-0)\vec{j} + (-3-0)\vec{k}$$

$$= 3\vec{i} - 3\vec{k}$$

$$\lambda_{OA} = \frac{3\vec{i} - 3\vec{k}}{4.24} = \frac{3}{4.24}\vec{i} - \frac{3}{4.24}\vec{k}$$

$$\vec{F}_1 = 10 \left(\frac{3}{4.24}\vec{i} - \frac{3}{4.24}\vec{k} \right) = 7.07\vec{i} - 7.07\vec{k}$$

$$\vec{F}_1 = 7.07\vec{i} - 7.07\vec{k} \quad - \textcircled{1}$$

$$OB = \sqrt{(2-0)^2 + (-2-0)^2 + (4-0)^2} = 4.899$$

$$\vec{OB} = (2-0)\vec{i} + (-2-0)\vec{j} + (4-0)\vec{k}$$

$$\lambda_{OB} = \frac{\vec{OB}}{OB} = \frac{2\vec{i} - 2\vec{j} + 4\vec{k}}{4.899}$$

$$\begin{aligned}\vec{F}_2 &= (F_2)(\lambda_{OB}) \\ &= 30 \left(\frac{2\vec{i} - 2\vec{j} + 4\vec{k}}{4.899} \right) = 12.25\vec{i} - 12.25\vec{j} + 24.5\vec{k}\end{aligned}$$

$$\vec{F}_2 = 12.25\vec{i} - 12.25\vec{j} + 24.5\vec{k} \quad \text{--- (2)}$$

$$OC = \sqrt{(-1-0)^2 + (2-0)^2 + (4-0)^2} = 4.582$$

$$\vec{OC} = (-1-0)\vec{i} + (2-0)\vec{j} + (4-0)\vec{k}$$

$$\lambda_{OC} = \frac{\vec{OC}}{OC} = \frac{-\vec{i} + 2\vec{j} + 4\vec{k}}{4.582}$$

$$\vec{F}_3 = 40 \left(\frac{-\vec{i} + 2\vec{j} + 4\vec{k}}{4.582} \right) = -8.73\vec{i} + 17.46\vec{j} + 34.92\vec{k} \quad \text{--- (3)}$$

$$R = \sqrt{\sum F_x^2 + \sum F_y^2 + \sum F_z^2}$$

$$\begin{aligned}\vec{R} &= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = F_{x1}\vec{i} + F_{y1}\vec{j} + F_{z1}\vec{k} \\ &= \sum F_x \vec{i} + \sum F_y \vec{j} + \sum F_z \vec{k}\end{aligned}$$

$$\vec{R} = 10.59\vec{i} + 5.212\vec{j} + 52.343\vec{k}$$

$$R = \sqrt{(10.59)^2 + (5.212)^2 + (52.343)^2}$$

Magnitude

$$R = 53.66 \text{ N}$$

Direction

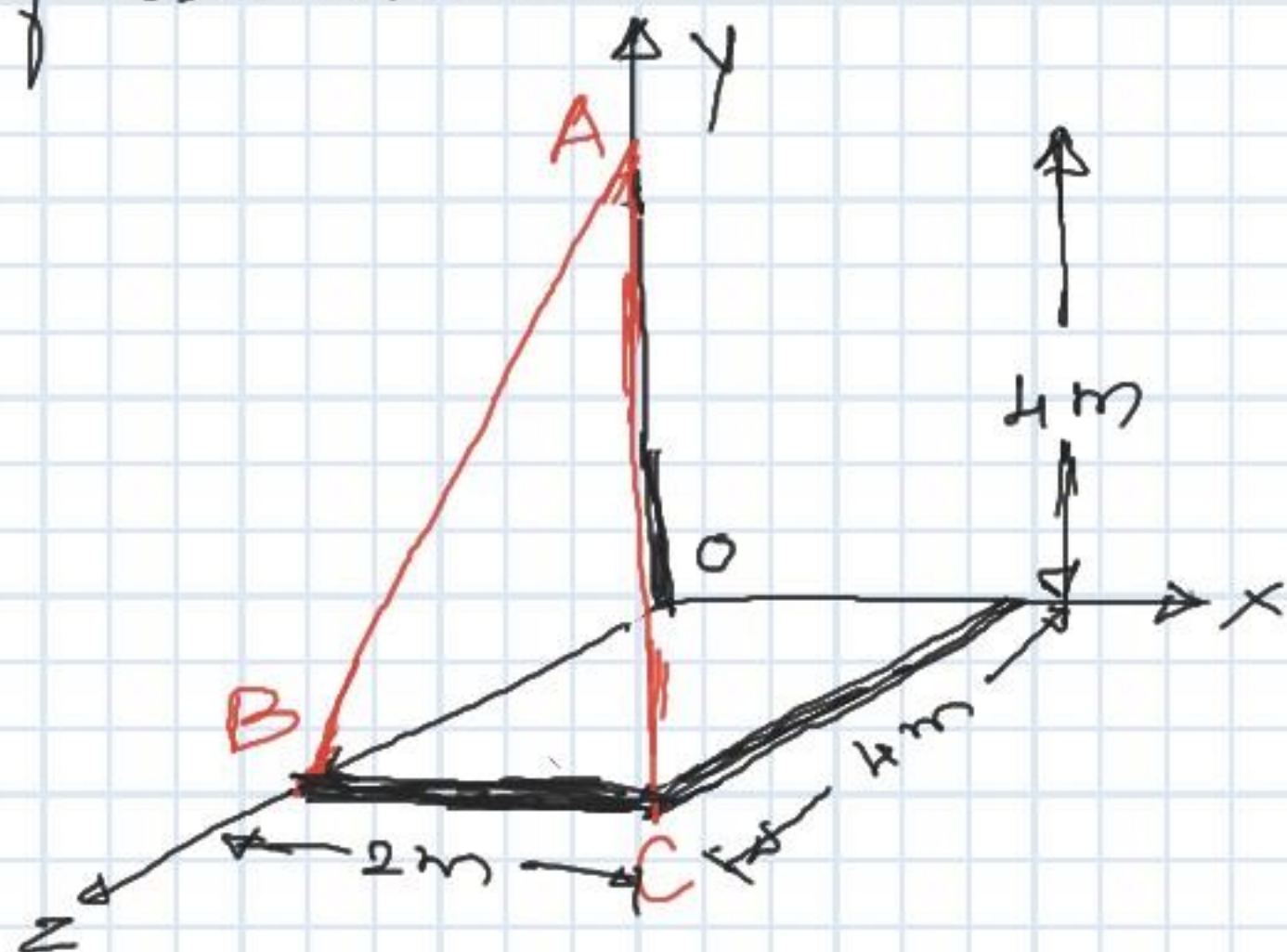
$$\phi_x = \cos^{-1} \frac{R_x}{R} = \cos^{-1} \frac{10.59}{53.66} = 78.61^\circ$$

$$\phi_y = \cos^{-1} \frac{R_y}{R} = \cos^{-1} \frac{5.212}{53.66} = 84.42^\circ$$

$$\begin{aligned}\phi_z &= \cos^{-1} \frac{R_z}{R} \\ &= \cos^{-1} \frac{52.343}{53.66} = 12.7^\circ\end{aligned}$$

~~1. The tension in cables AB and AC are 100N and 120N resp. Evaluate the magnitude of resultant force and with the direction of inclination.~~

1. The tension in cables AB and AC are 100N and 120N resp. Evaluate the magnitude of resultant force and with the direction of inclination.



Sol

$A(0, 4, 0)$
 $B(0, 0, 4)$
 $C(2, 0, 4)$

$R = ?$
 $\phi_x = ? \quad \phi_y = ? \quad \phi_z = ?$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$R_x = \sum F_x$$

$$R_y = \sum F_y$$

$$R_z = \sum F_z$$



$$\begin{aligned} A & (0, 4, 0) \\ B & (0, 0, 4) \\ C & (2, 0, 4) \end{aligned}$$

$$\vec{T}_{AB} = T_{AB} (\lambda_{AB})$$

$$\vec{T}_{AC} = T_{AC} (\lambda_{AC})$$

$$\lambda_{AB} = \frac{\vec{AB}}{|AB|} = \frac{(0-0)\vec{i} + (0-4)\vec{j} + (4-0)\vec{k}}{\sqrt{(4)^2 + (4)^2}} = \frac{-4\vec{j} + 4\vec{k}}{5.656}$$

$$\hat{\lambda}_{AC} = \frac{(2-0)\vec{i} + (0-4)\vec{j} + (4-0)\vec{k}}{\sqrt{2^2 + 4^2 + 4^2}} = \frac{2\vec{i} - 4\vec{j} + 4\vec{k}}{6}$$

$$\vec{T}_{AB} = 100 \left(\frac{-4\vec{j} + 4\vec{k}}{5.656} \right) = -70.72\vec{j} + 70.72\vec{k}$$

$$\vec{T}_{AC} = 120 \left(\frac{2\vec{i} - 4\vec{j} + 4\vec{k}}{6} \right) = 40\vec{i} - 80\vec{j} + 80\vec{k}$$

Magnitude of resultant force

$$\begin{aligned} \vec{R} &= \vec{T}_{AB} + \vec{T}_{AC} \\ &= 40\vec{i} - 150.72\vec{j} + 150.72\vec{k} \end{aligned}$$

$$R = \sqrt{40^2 + 150.72^2 + 150.72^2} = 216.87 \text{ N}$$