Exercise: 5.1

1) $x^2 - y^2 = a^2$

=> Differentiate with respect to 'x'.

$$\Rightarrow 2x - 2yy_1 = 0 \qquad \left[y_1 = \frac{dy}{dx}\right]$$

$$\Rightarrow \cancel{2}yy_1 = \cancel{2}x$$

$$\Rightarrow yy_1 = x$$

$$\Rightarrow y_1 = \frac{x}{y} \quad \text{—(1)}$$

=> Differentiate with respect to 'x'.

$$\Rightarrow y_2 = \frac{y - xy_1}{y^2} \qquad \left[y_2 = \frac{d^2y}{dx^2}\right]$$

$$\Rightarrow y_2 = \frac{y - \frac{x^2}{y}}{y^2}$$

$$\Rightarrow y_2 = \frac{y^2 - x^2}{y^3} \quad \text{—(2)}$$

we know that,

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{\left(1+\frac{x^2}{y^2}\right)^{3/2}}{\frac{y^2-x^2}{y^3}}$$

$$\Rightarrow = \left(1+\frac{x^2}{y^2}\right)^{3/2} \times \frac{y^3}{y^2-x^2}$$

$$\Rightarrow \left(\frac{y^2+x^2}{y^2}\right)^{3/2} \times \frac{y^3}{y^2-x^2}$$

$$\Rightarrow \frac{(y^2+x^2)^{3/2}}{\cancel{y^3}} \times \frac{\cancel{y^3}}{y^2-x^2}$$

$$\Rightarrow \frac{(y^2+x^2)^{3/2}}{y^2-x^2}$$

we know that,

$x^2-y^2=a^2$

$$\Rightarrow \rho = \frac{(y^2+x^2)^{3/2}}{-a^2}$$ (as ρ is always positive)

$$= \frac{(x^2+y^2)^{3/2}}{a^2}$$

2) $y = \log(\sin x)$.

Differentiate with respect to 'x'.

$$\Rightarrow y_1 = \frac{1}{\sin x}\cdot \cos x \qquad \left[y_1 = \frac{dy}{dx}\right]$$

$$\Rightarrow y_1 = \cot x \; - ①$$

Differentiate with respect to 'x'.

$$\Rightarrow y_2 = -\operatorname{cosec}^2 x \; - ② \qquad \left[y_2 = \frac{d^2y}{dx^2}\right]$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$$\Rightarrow \frac{(1+\cot^2 x)^{3/2}}{-\operatorname{cosec}^2 x}$$

$$\left[\operatorname{cosec}^2 x - \cot^2 x = 1\right]$$

$$\Rightarrow \frac{(\operatorname{cosec}^2 x)^{3/2}}{-\operatorname{cosec}^2 x}$$

$$\Rightarrow \frac{\operatorname{cosec}^3 x}{-\operatorname{cosec}^2 x}$$

$$\Rightarrow -\operatorname{cosec} x$$

As ρ is always positive

$$\Rightarrow \rho = \operatorname{cosec} x$$

3) $x = a\cos^3\theta$, $y = b\sin^3\theta$.

$$\Rightarrow x = a\cos^3\theta$$

Differentiate with respect to 'θ'.

$$\Rightarrow \frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta \quad -(1)$$

$$\Rightarrow y = b\sin^3\theta$$

Differentiate with respect to 'θ'.

$$\frac{dy}{d\theta} = 3b\sin^2\theta\cos\theta \quad -(2)$$

from (1) and (2)

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3b\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\frac{b}{a}\operatorname{Tan}\theta \quad -(3)$$

Differentiate with respect to 'x'.

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(-\frac{b}{a}\operatorname{Tan}\theta\right)$$

$$\Rightarrow -\frac{b}{a}\frac{d}{d\theta}(\operatorname{Tan}\theta) \times \frac{d\theta}{dx}$$

$$\Rightarrow -\frac{b}{a}\sec^2\theta \times \frac{d\theta}{dx}$$

$$\Rightarrow -\frac{b}{a}\sec^2\theta \times -\frac{1}{-3a\cos^2\theta\sin\theta}$$

$$\Rightarrow \frac{b}{3a^2} \times \frac{1}{\cos^4\theta\sin\theta}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{b}{3a^2\cos^4\theta\sin\theta} \quad -(4)$$

$$\rho = \frac{\left[1+y_1^2\right]^{3/2}}{y_2} = \left[1+\left(\frac{b^2}{a^2}\operatorname{Tan}^2\theta\right)\right]^{3/2} \times \frac{3a^2\cos^4\theta\sin\theta}{b}$$

$\Rightarrow$

$$\Rightarrow \left[1+\left(\frac{b^2}{a^2}\frac{\sin^2\theta}{\cos^2\theta}\right)^{\cancel{3/2}}\right]^{3/2} \times \frac{3a^2\cos^4\theta\sin\theta}{b}$$

$$\Rightarrow \left[\frac{a^2\cos^2\theta+b^2\sin^2\theta}{a^2\cos^2\theta}\right]^{3/2} \times \frac{3a^2\cos^4\theta\sin\theta}{b}$$

$$\Rightarrow \frac{\left[a^2\cos^2\theta+b^2\sin^2\theta\right]^{3/2}}{a^{\cancel{3}}\cancel{\cos}^3\theta} \times \frac{3\cancel{a}^2\cos^{\cancel{4}}\theta\sin\theta}{b}$$

$$\Rightarrow \frac{3}{ab}\left[a^2\cos^2\theta+b^2\sin^2\theta\right]^{3/2}\sin\theta\cos\theta.$$

$$\therefore \rho = \frac{3}{ab}\left[a^2\cos^2\theta+b^2\sin^2\theta\right]^{3/2}\sin\theta\cos\theta$$

4) $x = ct,\ y = c/t.$

$\Rightarrow x = ct$

Differentiate with respect to 't'.

$$\Rightarrow \frac{dx}{dt} = c \quad -(1)$$

$\Rightarrow y = c/t$

Differentiate with respect to 't'.

$$\Rightarrow \frac{dy}{dt} = -\frac{c}{t^2} \quad -(2)$$

$\Rightarrow$ From ① & ②

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{-\cancel{c}}{t^2} \times \frac{1}{\cancel{c}} = \frac{-1}{t^2} \quad ③$$

$\Rightarrow$ Differentiate with respect to x.

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{-1}{t^2}\right) = \frac{d}{dt}\left(\frac{-1}{t^2}\right)\frac{dt}{dx}$$

$$\Rightarrow \frac{2}{t^3} \times \frac{1}{c}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{2}{ct^3} \quad — ④$$

$$\rho = \frac{\left[1+y_1^2\right]^{3/2}}{y_2} = \frac{\left[1+\frac{1}{t^4}\right]^{3/2}}{2/ct^3}$$

$$\Rightarrow \left[1+\frac{1}{t^4}\right]^{3/2} \times \frac{ct^3}{2}$$

$$= \left[\frac{t^4+1}{t^4}\right]^{3/2} \times \frac{ct^3}{2} \qquad \therefore \rho = \frac{c(t^4+1)^{3/2}}{2t^3}$$

$$= \frac{(t^4+1)^{3/2}}{t^{\cancel{6}3}} \times \frac{c\cancel{t^3}}{2}$$

$$= \frac{c(t^4+1)^{3/2}}{2t^3}$$

5) TO Prove:

ρ at $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is three times the length of perpendicular from the origin to the tangent.

sol) we know:

$x = a\cos^3\theta$; $y = a\sin^3\theta$

(Parametric coordinates of the given curve).

$\Rightarrow x = a\cos^3\theta$

Differentiate with respect to 'θ'.

$$\frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta \quad - (1)$$

$y = a\sin^3\theta$

Differentiate with respect to 'θ'

$$\frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta \quad - (2)$$

from (1) & (2)

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\tan\theta$$

m = slope

$$\frac{dy}{dx} = -\tan\theta = m$$

we Know that,

Equation of tangent:

$y - y_1 = m(x - x_1)$

$\Rightarrow y - a\sin^3\theta = -\tan\theta\,[x - a\cos^3\theta]$

$\Rightarrow y + x\tan\theta - a\sin^3\theta - a\tan\theta\cos^3\theta = 0$

$\Rightarrow y + x\tan\theta - a\sin^3\theta - a\sin\theta\cos^2\theta = 0$

$\Rightarrow y + x\tan\theta - a\sin\theta\,[\sin^2\theta + \cos^2\theta] = 0$

$\Rightarrow y + x\tan\theta - a\sin\theta = 0$ $[\sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow x\tan\theta + y - a\sin\theta = 0$ — 3

we Know that,

formula for length of perpendicular distance at a point:

$P(x_1, y_1)$, $Ax + By + C = 0$ is.

$d = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$ — 4,

from eq 3 and 4

$A = \tan\theta$, $B = 1$

$C = -a\sin\theta$

$p = \dfrac{|0 + 0 - a\sin\theta|}{\sqrt{1^2 + \tan^2\theta}}$ $[\sqrt{1 + \tan^2\theta} = \sec\theta]$

$= \dfrac{a\sin\theta}{\sec\theta} = a\sin\theta\cos\theta$

$p = a\sin\theta\cos\theta$ — 5

$$\Rightarrow \frac{dy}{dx} = -\tan\theta$$

Differentiate with respect to 'x'.

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}(-\tan\theta)$$

$$\Rightarrow \frac{d}{d\theta}(-\tan\theta) \times \frac{d\theta}{dx}$$

$$\Rightarrow = -\sec^2\theta \times \frac{1}{-3a\cos^2\theta\sin\theta}$$

$$\frac{d^2y}{dx^2} = \frac{1}{3a\cos^4\theta\sin\theta}$$

$$\rho = \frac{[1+y_1]^{3/2}}{y_2} = \frac{[1+\tan^2\theta]^{3/2}}{\frac{1}{3a\cos^4\theta\sin\theta}}$$

$$= [\sec^2\theta]^{3/2} \times 3a\sin\theta\cos^4\theta$$

$$= \sec^3\theta \times 3a\sin\theta\cos^4\theta$$

$[1+\tan^2\theta = \sec^2\theta]$

$$\rho = 3a\sin\theta\cos\theta \quad - (6)$$

from (5) & (6)

$$P = a\sin\theta\cos\theta$$

$$\rho = 3a\sin\theta\cos\theta$$

$$\therefore \rho = 3P$$

(P- length of perpendicular from origin to tangent)

Hence proved.

6) $x = at^2$, $y = 2at$

(Parametric coordinates of parabola $y^2 = 4ax$).

$x = at^2$

Differentiate with Respect to 't'.

$$\frac{dx}{dt} = 2at \quad -①$$

$y = 2at$

Differentiate with respect to 't'

$$\frac{dy}{dt} = 2a \quad -②$$

from ① & ②

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$$

Differentiate with respect to 'x'.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{t}\right) = \frac{d}{dt}\left(\frac{1}{t}\right)\frac{dt}{dx}$$

$$\Rightarrow -\frac{1}{t^2} \times \frac{1}{2at} = \frac{-1}{2at^3}$$

$$= \frac{d^2y}{dx^2} = \frac{-1}{2at^3}$$

$$e = \frac{[1+y_1^2]^{3/2}}{y_2} = \frac{\left[1+\frac{1}{t^2}\right]^{3/2}}{\frac{-1}{2at^3}} = \left[\frac{t^2+1}{t^2}\right]^{3/2} \times -2at^3$$

$$= \left[t^2+1\right]^{3/2} \times \frac{-2a\cancel{t^3}}{\cancel{t^3}}$$

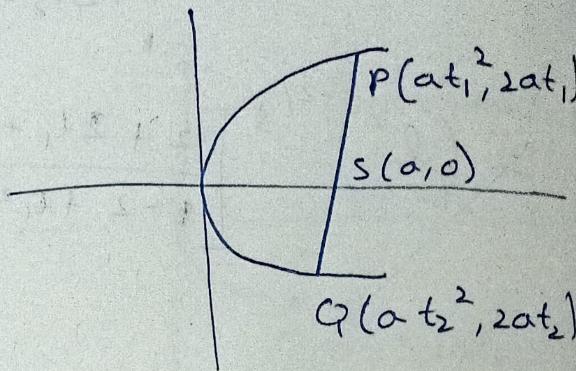
$$= (t^2+1)^{3/2} \times -2a$$

$\rho = 2a(t^2+1)^{3/2}$ (as ρ is always positive).

Now,

$$\rho_1 = 2a(t_1^2+1)^{3/2}$$

$$\rho_2 = 2a(\cancel{t_1}^2+1)^{3/2}$$



$$\Rightarrow (\rho_1)^{-2/3} = (2a)^{-2/3}(t_1^2+1)^{-1}$$

$$\Rightarrow (\rho_2)^{-2/3} = (2a)^{-2/3}(t_2^2+1)^{-1}$$

we know,

latus rectum $= 4a$

$$4a = 2\ell$$

$$a = \frac{\ell}{2}$$

$$t_1 \cdot t_2 = -1.$$

$$(\rho_1)^{-2/3} + (\rho_2)^{-2/3} = (\ell)^{-2/3}\left[\frac{1}{t_1^2+1} + \frac{1}{t_2^2+1}\right]$$

$$= (\ell)^{-2/3}\left[\frac{t_1^2+1+t_2^2+1}{t_1^2t_2^2+t_1^2+1+t_2^2}\right]$$

$$= (\ell)^{-2/3}\left[\frac{2+\cancel{t_1^2}+t_2^2}{2+\cancel{t_1^2}+t_2^2}\right]$$

$$\Rightarrow (\rho_1)^{-2/3} + (\rho_2)^{-2/3} = (\ell)^{-2/3}$$

Hence Proved

7) ρ at $y=0$, $y^2 = x^3+8$

Differentiate with respect to 'x'

$\Rightarrow 2y\,y_1 = 3x^2 \cancel{+8}$ $\quad \left[y_1 = \frac{dy}{dx}\right]$

$\Rightarrow y_1 = \frac{3x^2 \cancel{+8}}{2y}$

$\Rightarrow y_{1(y=0)} = \frac{3x^2 \cancel{+8}}{0} = \infty$

Hence, Differentiate $y^2 = x^3+8$ with 'y'

$\Rightarrow 2y = 3x^2 x_1 \cancel{+}$ $\quad \left[x_1 = \frac{dx}{dy}\right]$

$\Rightarrow x_1 = \frac{2y}{3x^2}$

$\Rightarrow x_{1(y=0)} = 0$

Differentiate with respect to 'y'

$\Rightarrow \frac{d^2x}{dy^2} = \frac{2(3x^2) - 2y(6x x_1)}{9x^4}$

$\Rightarrow \left(\frac{d^2x}{dy^2}\right)_{y=0} = \frac{6x^2}{9x^{4\,2}} = \frac{2}{3x^2}$

$\rho = \frac{(1+x_1^2)^{3/2}}{x_2} = (1+0)^{3/2} \times \frac{3x^2}{2}$

$\rho = \frac{3x^2}{2}$

at $y=0$,

$x^3=-8$

$x=-2$

$$\rho = \frac{3x^2}{2}$$

$$\rho = \frac{3(-2)^2}{2}$$

$= 6$

8) $y^2 = \left(\frac{a^3 - x^3}{x}\right)$, $(a, 0)$.

Differentiate with respect to 'x'.

$\Rightarrow 2yy_1 = -\frac{a^3}{x^2} - 2x$ $\left[y_1 = \frac{dy}{dx}\right]$

$\Rightarrow y_1 = \frac{-a^3 - 2x^3}{2yx^2}$

at point $(a, 0)$

$(y_1)_{(a,0)} = \infty$

$\Rightarrow$ Differentiate with respect to y:

$$xy^2 = a^3 - x^3$$

$$\Rightarrow x_1 y^2 + 2xy = -3x^2 x_1 \qquad \left[x_1 = \frac{dx}{dy}\right]$$

$$\Rightarrow x_1 y^2 + 3x^2 x_1 = -2xy$$

$$\Rightarrow x_1 [y^2 + 3x^2] = -2xy$$

$$\Rightarrow x_1 = \frac{-2xy}{y^2+3x^2}$$

$$\Rightarrow x_{1(a,0)} = 0$$

$$\frac{d^2x}{dy^2} = -2\left[\frac{[x + yx_1][3x^2+y^2] - (-2xy)(6xx_1 + 2y)}{(3x^2+y^2)^2}\right]$$

$$\frac{d^2x}{dy^2}_{(a,0)} = -2\left[\frac{a[3a^2] - 0}{9a^4}\right] = \frac{-2}{3a}$$

$$\rho = \frac{\left[1 + \left(\frac{dx}{dy}\right)^2\right]^{3/2}}{\frac{d^2x}{dy^2}}$$

as $\frac{dx}{dy} = 0$

$$\rho = [1+0]^{3/2} \times \frac{3a}{-2} = \frac{3a}{2}$$

as ρ is always positive.

$x^3 = -8$

$x = -2$

$e = \frac{3x^2}{2}$

$e = \frac{3(-2)^2}{2}$

$= 6$

a) $x = e^t \cos t$

$y = e^t \sin t$.

$x = e^t \cos t$

Differentiate with respect to 't'.

$\frac{dx}{dt} = e^t \cos t \cancel{+} - e^t \sin t$

$y = e^t \sin t$

Differentiate with respect to 't'.

$\frac{dy}{dt} = e^t \sin t + e^t \cos t$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{e^t \sin t + e^t \cos t}{e^t \cos t - e^t \sin t} = \frac{\sin t + \cos t}{\cos t - \sin t}$

Differentiate with respect to 'x'.

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left[\frac{\sin t + \cos t}{\cos t - \sin t}\right]\frac{dt}{dx}$$

$$= \left[\frac{\cos t - \sin t(\cos t - \sin t) - (\sin t + \cos t)(-\sin t - \cos t)}{(\cos t - \sin t)^2}\right] \times \frac{1}{\cancel{x} e^t(\cos t - \sin t)}$$

$$\Rightarrow \left[\frac{(\cos t - \sin t)^2 + (\sin t + \cos t)^2}{(\cos t - \sin t)^2}\right] \times \frac{1}{\cancel{x} e^t(\cos t - \sin t)}$$

$$\Rightarrow \left[\frac{\cos^2 t + \sin^2 t - 2\cos t \sin t + \sin^2 t + \cos^2 t + 2\cos t \sin t}{e^t(\cos t - \sin t)^3}\right]$$

$$\Rightarrow \left[\frac{2}{e^t(\cos t - \sin t)^3}\right] = \frac{d^2y}{dx^2}$$

$$\rho = \frac{\left(1 + \left(\frac{\sin t + \cos t}{\cos t - \sin t}\right)^2\right)^{3/2}}{\frac{2}{e^t(\cos t - \sin t)^3}}$$

$$\rho = \left(1 + \left(\frac{\sin t + \cos t}{\cos t - \sin t}\right)^2\right)^{3/2} \times \frac{e^t(\cos t - \sin}{2}$$

$$\rho = \left(\frac{(\cos t - \sin t)^2 + (\sin t + \cos t)^2}{(\cos t - \sin t)^2}\right)^{3/2} \times \frac{e^t(\cos t -}{2}$$

$$\rho = \left(\frac{2}{(\cos t - \sin t)^2}\right)^{3/2} \times \frac{e^t(\cos t - \sin t)^3}{2}$$

$$\rho = \frac{\cancel{2}\sqrt{2}}{(\cos t - \sin t)^3} \times \frac{e^t(\cos t - \sin t)^3}{\cancel{2}}$$

$$\rho = \sqrt{2}\, e^t$$

10. P.T $\rho = 3a\sin\theta$

for $x = 3a\cos\theta - a\cos3\theta$, $y = 3a\sin\theta - a\sin3\theta$

$$\frac{dx}{d\theta} = a(-3\sin\theta + 3\sin3\theta)$$

$$= 3a(\sin3\theta - \sin\theta)$$

$$\frac{dy}{d\theta} = 3a\cos\theta - 3a\cos3\theta$$

$$= 3a(\cos\theta - \cos3\theta)$$

Hence,

$$y_1 = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cancel{3a}(\cos\theta - \cos3\theta)}{\cancel{3a}(\sin3\theta - \sin\theta)}$$

$$y_1 = \frac{\cos\theta - \cos3\theta}{\sin3\theta - \sin\theta} = \frac{-2\sin\left(\frac{\theta+3\theta}{2}\right)\sin\left(\frac{3\theta-\theta}{2}\right)}{2\cos\left(\frac{3\theta+\theta}{2}\right)\sin\left(\frac{3\theta-\theta}{2}\right)}$$

$$\cancel{y_1} = \frac{\cancel{dy/d\theta}}{\cancel{dx/d\theta}} \qquad y_1 = -\frac{\sin(4\theta/2)\sin(2\theta/2)}{\cos(4\theta/2)\sin(2\theta/2)}$$

$$y_1 = -\frac{\sin2\theta\sin\theta}{(\cos2\theta)(\sin\theta)} = -\frac{\sin2\theta}{\cos2\theta} \times \frac{\cancel{\sin\theta}}{\cancel{\sin\theta}}$$

Hence, $\boxed{y_1 = -\tan2\theta}$

$$y_2 = \frac{\frac{dy_1}{d\theta}}{\frac{dx}{d\theta}} = \frac{-\sec^2 2\theta}{3a(\sin3\theta - \sin\theta)}$$

$$y_2 = \frac{-\sec^2 2\theta}{3a\cos\left(\frac{3\theta+\theta}{2}\right)\sin\left(\frac{3\theta-\theta}{2}\right)}$$

$$y_2 = \frac{-\sec^2 2\theta}{3a\cos2\theta\sin\theta}$$

$$\boxed{y_2 = \frac{-\sec^3 2\theta}{3a\sin\theta \cancel{\cos^3 2\theta}}}$$

W.K.T

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$$\rho = \frac{(1+\tan^2 2\theta)^{3/2}}{\frac{-\sec^3 2\theta}{3a\sin\theta \cancel{\cos^3 2\theta}}}$$

$$\rho = \frac{(\sec^2 2\theta)^{3/2}}{\frac{-\sec^3 2\theta}{3a\sin\theta}} = -\frac{\cancel{\sec^3 2\theta}}{\frac{\cancel{\sec^3 2\theta}}{3a\sin\theta}}$$

$$\rho = \frac{1}{\frac{1}{3a\sin\theta}}$$

Hence, $\boxed{\rho = 3a\sin\theta}$,

11. $r = a\cos\theta$

$$\frac{dr}{d\theta} = -a\sin\theta$$

$$\boxed{\rho = \frac{(r^2+(r')^2)^{3/2}}{r^2 - rr'' + 2(r')^2}}$$

$$\rho = \frac{(a^2\cos^2\theta + a^2\sin^2\theta)^{3/2}}{a^2\cos^2\theta + (a\cos\theta)(a\cos\theta) + 2(-a\sin\theta)^2}$$

$$\frac{d^2r}{d\theta^2} = -a\cos\theta$$

$$\rho = \frac{\left[a^2(\cos^2\theta + \sin^2\theta)\right]^{3/2}}{a^2\cos^2\theta + a^2\cos^2\theta + 2a^2\sin^2\theta}$$

$$\rho = \frac{(a^2)^{3/2}}{2a^2(\cos^2\theta + \sin^2\theta)} = \frac{a^{\cancel{3}}}{2\cancel{a^2}} = a/2$$

12. $a^2 = r^2\cos 2\theta$

To prove, $\rho = \dfrac{r^3}{a^2}$

$$r^2 = \frac{a^2}{\cos 2\theta}$$

$$r = a(\sec^{1/2} 2\theta)$$

$$r_1 = a\left(\frac{1}{2}\right)\frac{((2)\sec 2\theta \tan 2\theta)}{\sec^{1/2} 2\theta}$$

$$r_1 = \frac{a\sec 2\theta\tan 2\theta}{\sec^{1/2} 2\theta} = a\sec^{1/2} 2\theta \tan 2\theta,$$

Hence, $\boxed{r_1 = r\tan 2\theta}$,

$$r_2 = a\left[\tan 2\theta \left(\frac{\cancel{2}}{2}\right)\left(\frac{\sec 2\theta\tan 2\theta}{\sec^{1/2} 2\theta}\right) + (\sec^{1/2} 2\theta)(\sec^2 2\theta)\right]$$

$$r_2 = \frac{a}{\cancel{2}}\left[\frac{\tan^2 2\theta \sec^{1/2} 2\theta}{\cancel{2}} + (\sec^{1/2} 2\theta)(\sec 2\theta)\right.$$

$$r_2 = (a\sec^{1/2} 2\theta)\left[\frac{\tan^2 2\theta}{\cancel{2}} + \sec 2\theta\right]$$

$$\boxed{r_2 = r\left[\frac{\tan^2 2\theta}{\cancel{2}} + \sec 2\theta\right]},$$

$$\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 - rr_2 + 2r_1^2},$$

$$\rho = \frac{(a^2\sec 2\theta + a^2\sec 2\theta \tan^2 2\theta)^{3/2}}{a\sec^{1/2} 2\theta\left(\frac{\tan^2 2\theta}{\cancel{2}} + \sec 2\theta\right)}$$

$$\rho = \frac{(r^2(1+\tan^2 2\theta))^{3/2}}{r\left(\frac{\sin^2 2\theta}{\cancel{2}\cos^2 2\theta} + \frac{1}{\cos 2\theta}\right)}$$

$$\rho = \frac{r^3(\sec^2 2\theta)^{3/2}}{r\left(\frac{\sin^2 2\theta + \cos^2 2\theta}{\cos^2 2\theta}\right)}$$

$$\rho = \frac{r^{\cancel{3}}\,(\sec^2 2\theta)^{3/2}}{\cancel{r}\,(\sec^2 2\theta)}$$

$$\rho = \frac{r\,\sec^{\cancel{3}} 2\theta}{\cancel{\sec^2 2\theta}}$$

$$\rho = r\sec 2\theta$$

$$\rho = \frac{r}{\cos 2\theta}$$

W.K.T, ~~sec~~ $\boxed{\cos 2\theta = \frac{a^2}{r^2}}$

Therefore,

$$\boxed{\rho = \frac{r^3}{a^2}}$$ //

Hence proved,

13. Find ρ, for $r = a\theta$ and $r\theta = a$.

i) $r = a\theta$,

$\frac{dr}{d\theta} = r_1 = a$,

$\frac{d^2 r}{d\theta^2} = r_2 = 0$

W.K.T,

$$\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 - r r_2 + 2r_1^2}$$

$$\rho = \frac{(a^2\theta^2 + a^2)^{3/2}}{a^2\theta^2 - a\theta(0) + 2a^2}$$

$$\rho = \frac{a^3(\theta^2+1)^{3/2}}{a^2\theta^2 + 2a^2}$$

$$\rho = \frac{a^{\cancel{3}}(\theta^2+1)^{3/2}}{\cancel{a^2}(\theta^2+2)}$$, Hence, $\boxed{\rho = \frac{a(\theta^2+1)^{3/2}}{\theta^2+2}}$ //

at pole $(\theta = 0)$

$$\rho = \frac{a(0+1)^{3/2}}{0+2} = \frac{a(1)^{3/2}}{2}$$

Hence, $\boxed{\rho = a/2}$ //

ii) $r\theta = a$

$r = a/\theta$

$\frac{dr}{d\theta} = a(-1)\left(\frac{1}{\theta^2}\right) \Rightarrow \boxed{\frac{dr}{d\theta} = r_1 = -\frac{a}{\theta^2}}$

$\frac{d^2r}{d\theta^2} = r_2 = -a(-2)\left(\frac{1}{\theta^3}\right)$

$\boxed{r_2 = \frac{2a}{\theta^3}}$,

$$\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 - r r_2 + 2 r_1^2}$$

$$\rho = \frac{\left(\frac{a^2}{\theta^2} + \frac{a^2}{\theta^4}\right)^{3/2}}{\frac{a^2}{\theta^2} - \frac{a}{\theta}\left(\frac{2a}{\theta^3}\right) + \frac{2a^2}{\theta^4}}$$

$$\rho = \frac{\left(\frac{a^2\theta^2 + a^2}{\theta^4}\right)^{3/2}}{\left(\frac{a^2\theta^2 - 2a^2 + 2a^2}{\theta^4}\right)}$$

$$\rho = \frac{(a^2\theta^2 + a^2)^{3/2}}{a^2\theta^2 - 2a^2 + 2a^2} = \frac{(a^2(\theta^2+1))^{3/2}}{a^2\theta^2 - \cancel{2a^2} + \cancel{2a^2}}$$

$$\rho = \frac{a^3(\theta^2+1)^{3/2}}{a^2\theta^2}$$

$$\boxed{\rho = \frac{a(\theta^2+1)^{3/2}}{\theta^2}}$$

14. P.T $y^2(3-x)(x+1) = 8(x-1)$ at $(1,0)$

has curvature as 1.

$$y^2(3x+3-x^2-x) = 8x-8$$

$$y^2(2x+3-x^2) = 8x-8$$

$$2yy_1(2x+3-x^2) + y^2(2-2x) = 8 \quad (\text{diff. w.r.t } x)$$

$$2yy_1(2x+3-x^2) = 8 - y^2(2-2x)$$

$$y_1 = \frac{8-2y^2(1-x)}{2y(2x+3-x^2)} \quad \text{at } (1,0)$$

$$y_1 = \frac{8-2(0)}{0}$$

Hence, diff w.r.t y.

$$\cancel{2y(2x+3-x^2) + y^2(2x x_1 - 2x x_1) = 8x_1}$$

$$\cancel{x_1(y^2(2-2x)) = 8x_1 - 2y(}$$

$$3y^2+8 = 8x - 2xy^2 + x^2y^2$$

$$6y = 8x_1 - 2(x_1y^2 + 2yx) + x^2(2y) + y^2(2x x_1)$$

$$6y = 8x_1 - 2x_1y^2 + 2x x_1 y^2 - 4xy + 2x^2y$$

$$6y + 4xy - 2x^2y = x_1(8-2y^2+2xy^2)$$

$$\therefore x_1 = \frac{y(6+4x-2x^2)}{8-2y^2+2xy^2} \quad (\text{at } (1,0)).$$

$$\boxed{x_1 = 0}$$

$$x_2 = \frac{(8-2y+2xy^2+)(6+4[(x)(1)+yx_1] - 2(xx_1y+x^2) - (6+4x-2x^2)(y)(-4y+2(x(2y)+y^2+1)}{(8-2y^2+2xy^2)^2}$$

at (1,0).

$$x_2 = \frac{(8-0)(6+4[1+0]-(2)-0}{(8-0)^2}$$

$$x_2 = \frac{(8)(8)}{8^2} = 1$$

$$\therefore \rho = \frac{(1+x_1^2)^{3/2}}{x_2}$$

$$\rho = \frac{(1+0^2)^{3/2}}{1}$$

$$\rho = 1$$

Hence curvature $\Rightarrow k = 1/\rho = 1$

Hence proved

15. $y^2 = 4ax$

at $\rho = 4\sqrt{2}$, S.T at points $(1,2)$, $(1,-2)$.

Here, w.k.T,

$x = at^2$, $y = 2at$

from $(1,2)$, $(1,-2)$

$1 = at^2$, $2 = 2at$, $-2 = 2at$

Hence, $t^2 = 1/a$, $t = 1/a$, $t = -1/a$

Here, $a = 1/t = -1/t = 1/t^2$

only at $\boxed{a=1}$

$\therefore$ In eqn $y^2 = 4ax$ $(a=1)$

$\cancel{2yy_1} = \cancel{4}(1)$ $\qquad y = \sqrt{4x}$

$\therefore \cancel{y_1} = \cancel{\frac{4}{2y}}^2$ $\qquad y_1 = \sqrt{4}\left(\frac{1}{2}\right)x^{1/2 - 1}$

$\cancel{y_1 = 2/y}$ $\qquad y_1 = \frac{2}{2\sqrt{x}}$

Hence, $\boxed{y_1 = 1/\sqrt{x}}$

$y_2 = \left(-\frac{1}{2}\right)\left(\frac{1}{x^{3/2}}\right)$

$\therefore \boxed{y_2 = -\frac{1}{2x^{3/2}}}$

w.k.T,

$$\rho = \frac{(1 + \cancel{a} y_1^2)^{3/2}}{y_2}$$

$$\rho = \frac{(1 + 1/x)^{3/2}}{\frac{-1}{2x^{3/2}}} = \frac{\left(\frac{x+1}{x}\right)^{3/2} \times 2x^{3/2}}{-1}$$

$$\rho = \frac{2(x+1)^{3/2}}{\cancel{x^{3/2}}} \times \cancel{x^{3/2}}$$

at $\rho = 4\sqrt{2}$

$\cancel{4}^{2}\sqrt{2} = \cancel{2}(a+1)^{3/2}$

$2\sqrt{2} = (a+1)^{3/2}$

$(2)^{3/2} = (a+1)^{3/2}$

$\therefore \boxed{a=1}$

at $a=1$,

$y^2 = 4ax \quad (a=1)$

$y^2 = 4(1)$

$\boxed{y = \pm 2}$

$\therefore$ The points are, $(1,2)$ and $(1,-2)$.

Hence proved.

K-SECTION

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