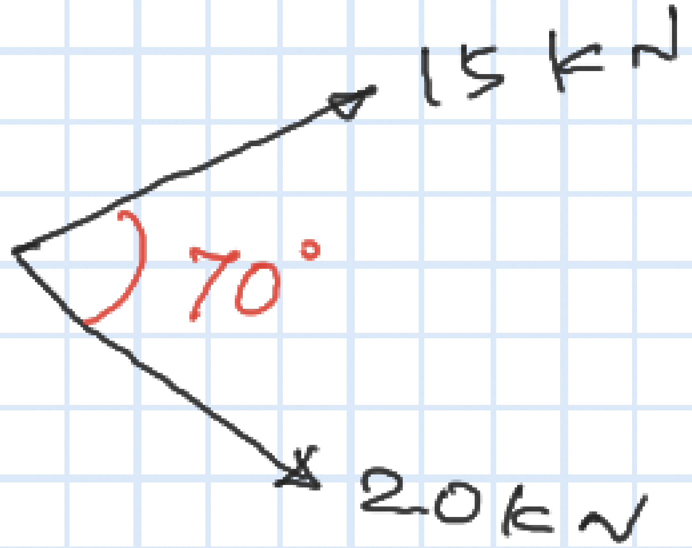
(1) Resultant of Forces (2D - Particles)

1.Two forces 20 kN and 15 kN act on a particle as shown. Find the magnitude and direction of the resultant force using (i) Parallelogram law (ii) Triangle law.



Sol. (i) Parallelogram law

Magnitude of R

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$$

$F_1 = 15$ kN

$F_2 = 20$ kN

$\theta = 70°$

$$\boxed{R = 28.813 \text{ kN}}$$

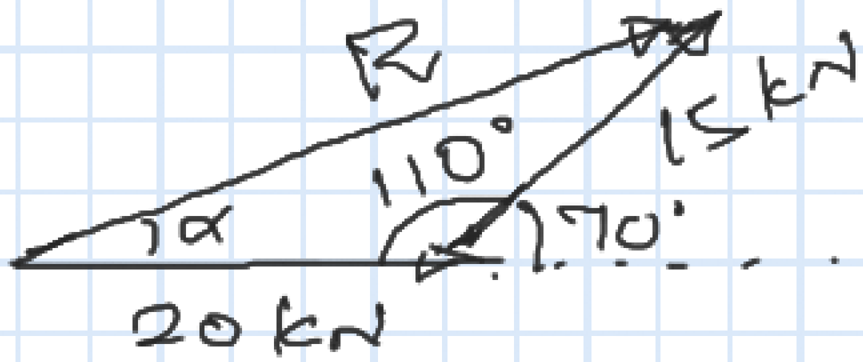
Direction of R

$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

$$\alpha = \tan^{-1}(\quad)$$

$$\boxed{\alpha = 29.3°}$$

(ii) Triangle law of forces



Mag

$$R^2 = 20^2 + 15^2 - 2(20)(15)\cos 110^\circ$$

$$\boxed{R = 28.813 \text{ kN}}$$

Direction

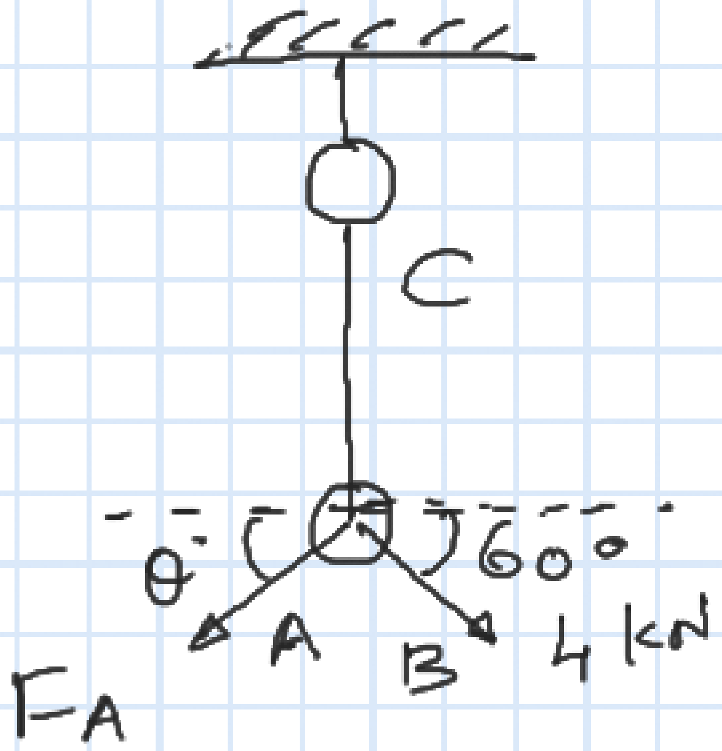
$$\frac{R}{\sin 110^\circ} = \frac{15}{\sin\alpha}$$

$$\frac{28.813}{\sin 110^\circ} = \frac{15}{\sin\alpha}$$

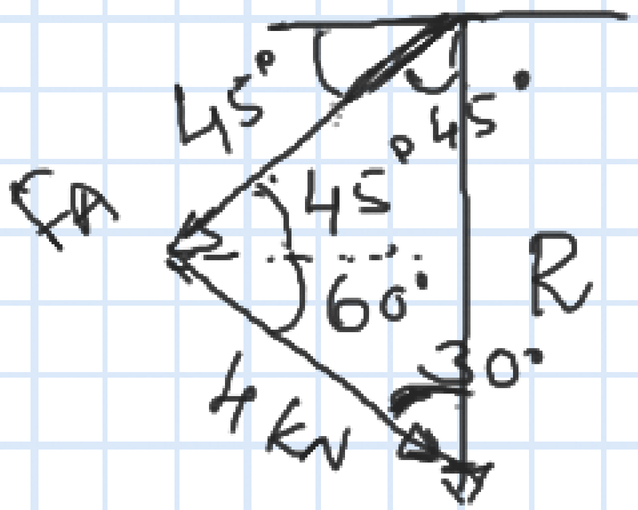
$$\sin\alpha = \frac{15(\sin 110^\circ)}{28.813}$$

$$\boxed{\alpha = 29.3^\circ}$$

2.A cable B is subjected to a force of 4 kN and is directed as shown. Determine the force in cable A when Θ = 45°, if the resultant of the two forces is directed vertically downwards along cable C. Also calculate the magnitude of resultant force.



Sol



Force in cable A

$$\frac{F_A}{\sin 30^\circ} = \frac{4}{\sin 45^\circ}$$

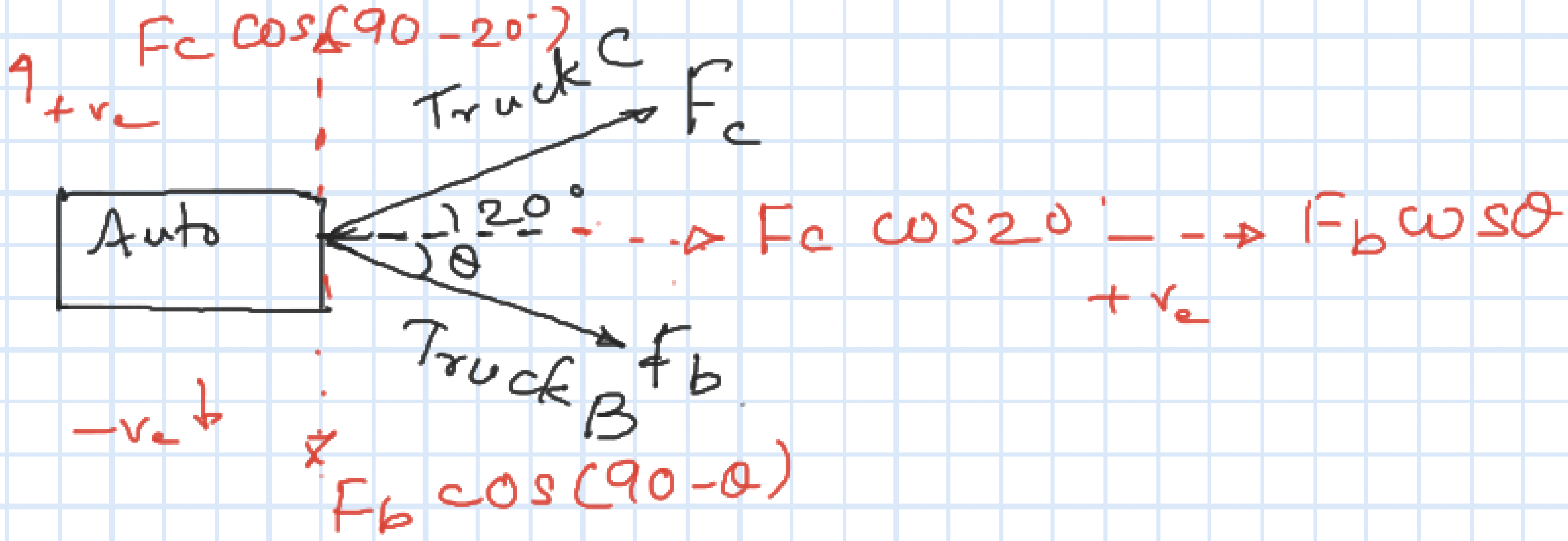
$$\boxed{F_A = 2.82 \text{ kN}}$$

Resultant force

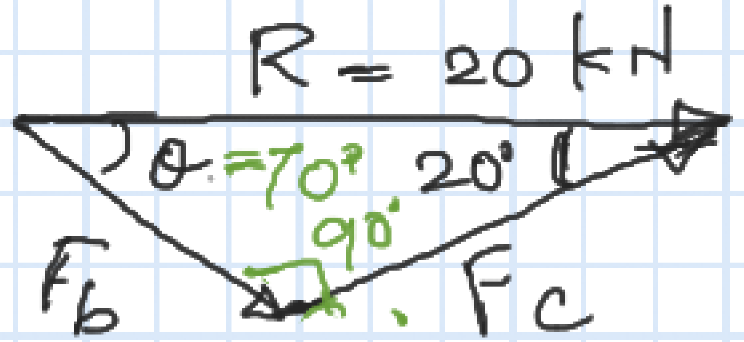
$$R^2 = F_A^2 + F_B^2 - 2F_AF_B\cos(105^\circ)$$

$$\boxed{R = 5.47 \text{ kN}}$$

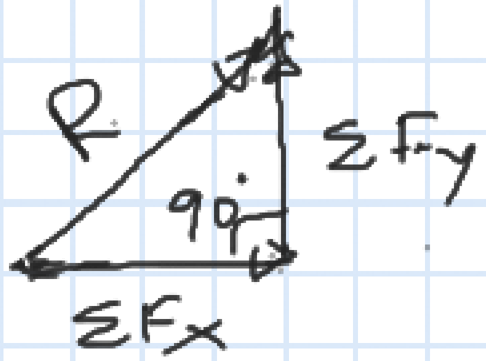
3. An automobile is pulled by means of two trucks as shown. If the resultant of two forces acting on the automobile is 20 kN being directed along positive direction of X-axis, determine the angle Θ of the cable attached to the truck at B such that the force Fb in this cable is minimum. What is the magnitude of force in each cable when this occurs?



Sol



Mathematically $F_{b\,min} = \frac{Nu}{(De)_{max}}$



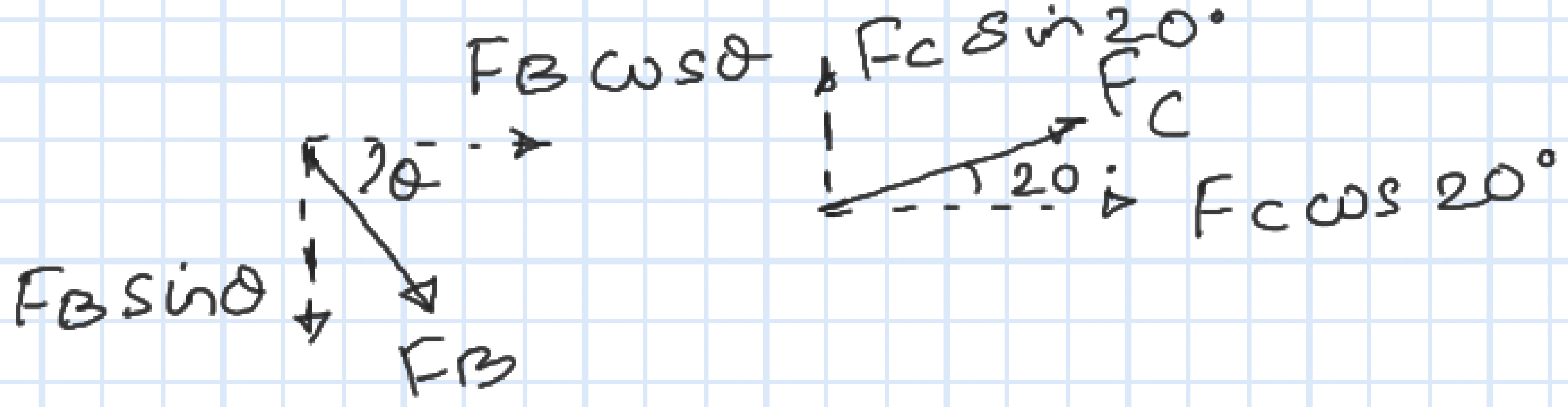
Since R is along x direction

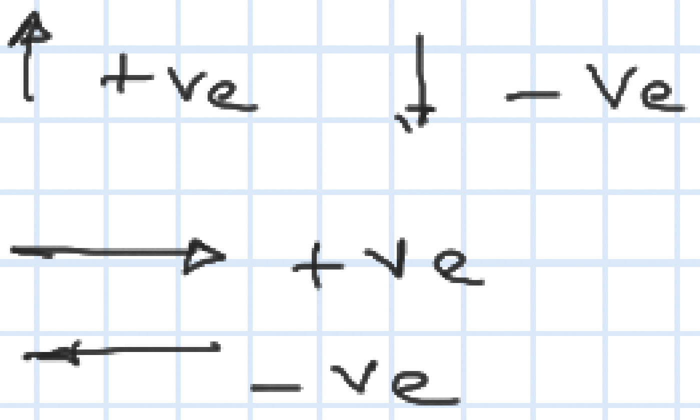
$\Sigma F_y = 0$

$\Sigma F_x = R$

$R^2 = \Sigma F_x^2 + \cancel{\Sigma F_y^2}^{\,0}$

$R = \Sigma F_x$





$\Sigma F_y = 0$

$F_c \sin 20° - F_b \sin\theta = 0$ — (1)

$\Sigma F_x = R$

$F_b \cos\theta + F_c \cos 20° = R$ — (2)

From (1) $\quad F_c = \dfrac{F_b \sin\theta}{\sin 20°}$

sub in (2)

$$F_b \cos\theta + \frac{F_b \sin\theta}{\sin 20°}(\cos 20°) = 20$$

$$F_b(\cos\theta + \sin\theta \cot(20°)) = 20$$

$$F_b = \frac{(20)}{(\cos\theta + \sin\theta \cot(20°))}$$

Maximise the denominator to minimise F_b.

$$\frac{d}{d\theta}(\cos\theta + \sin\theta \cot 20) = 0$$

$$\cos\theta \cot 20^\circ = \sin\theta$$

$$\cot 20^\circ = \tan\theta$$

$$\boxed{\theta = 70^\circ}$$

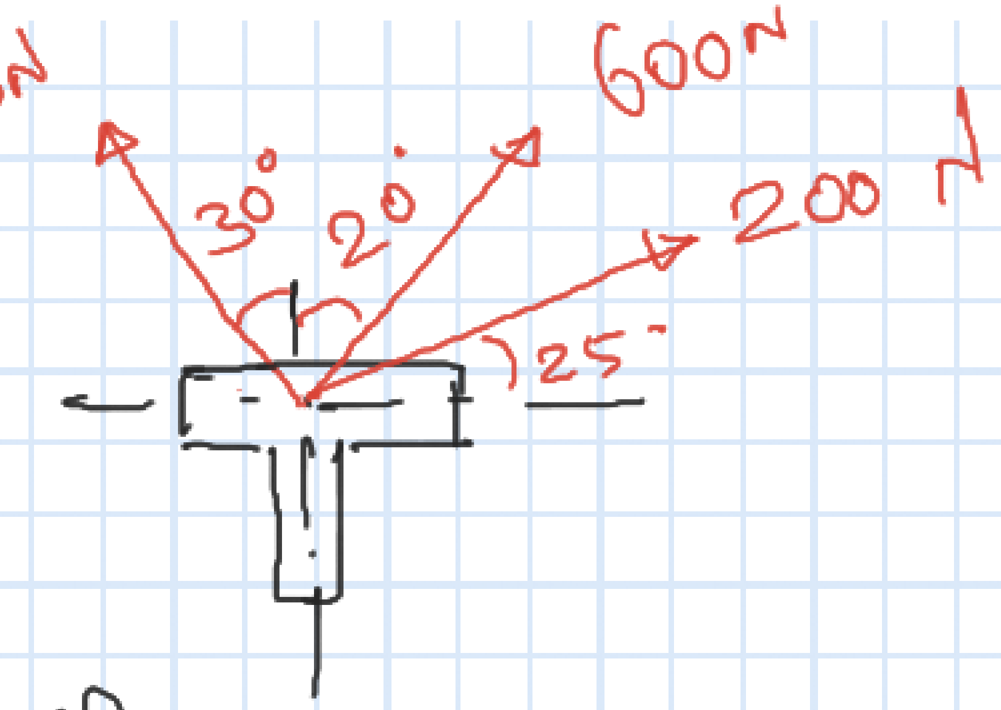
The apply triangle law of forces to solve F_b and F_c.

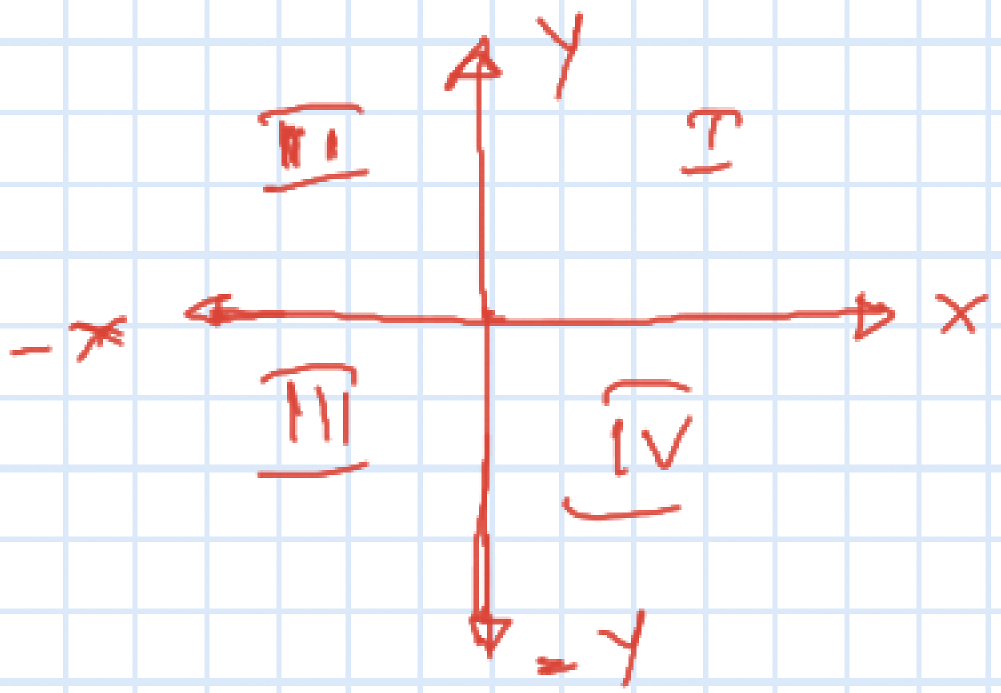
$$\frac{R}{\sin 90} = \frac{F_B}{\sin 20^\circ} = \frac{F_C}{\sin 70^\circ}$$

$$F_B = \frac{20}{\sin 90^\circ}(\sin 20^\circ) = 6.84 \text{ kN}$$

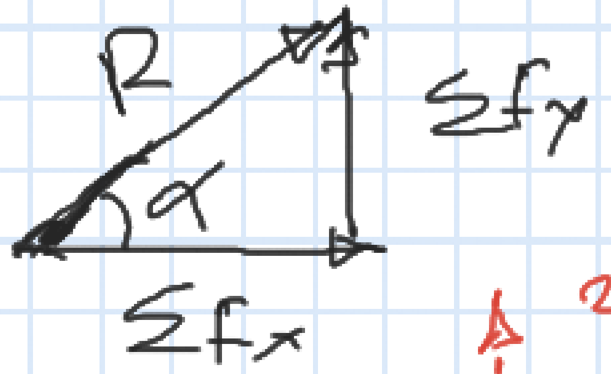
$$F_C = \frac{20}{\sin 90^\circ}(\sin 70^\circ) = 18.77 \text{ kN}$$

4. Determine the magnitude and direction of the resultant of the forces acting on the bolt as shown.

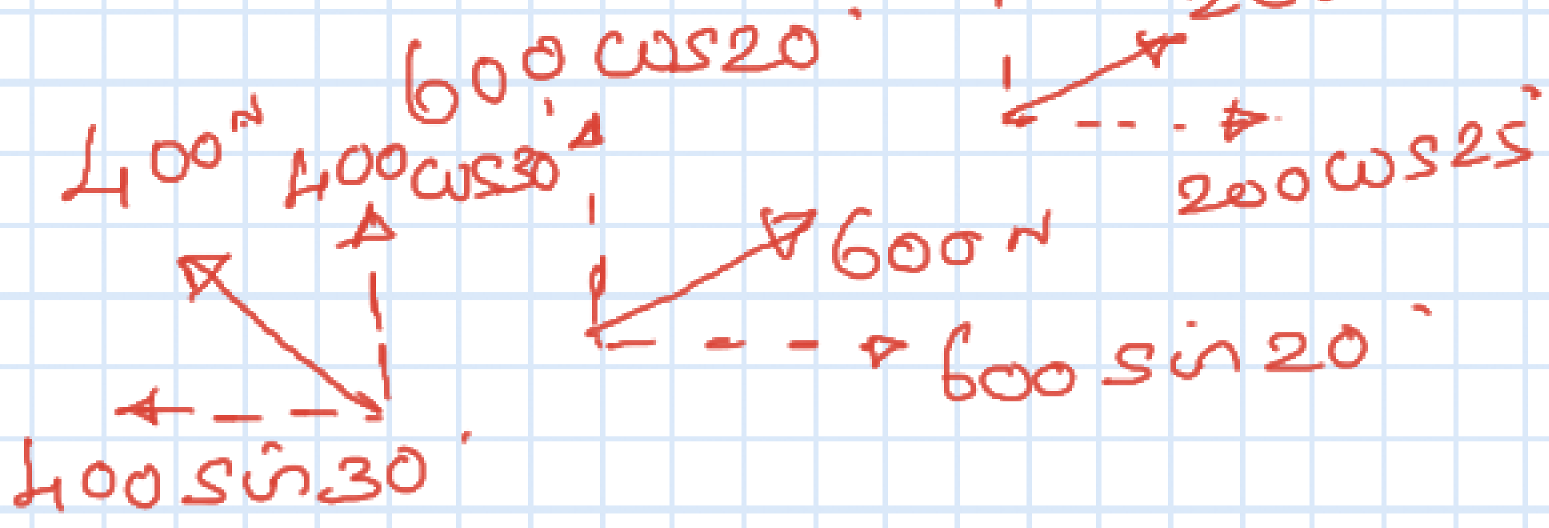


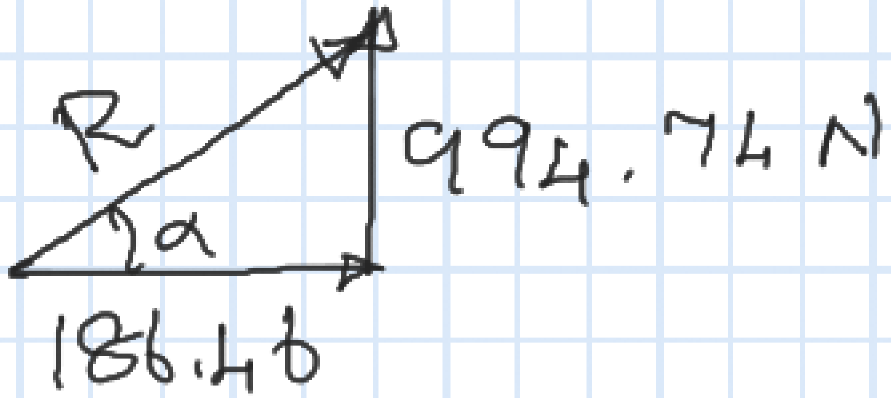


Sol Since the no. of forces ≥ 2
Resolve the forces in x & Y directions to have their Components



	Forces	Fx	Fy
1.	200 N	200 cos 25°	200 sin 25
2.	600 N	600 sin 20°	600 cos 20°
3.	400 N	− 400 sin 30°	400 cos 30°
		$\Sigma f_x = 186.46N$	$\Sigma f_y = 994.74N$





Magnitude of the resultant

$$R^2 = 186.46^2 + 994.74^2$$

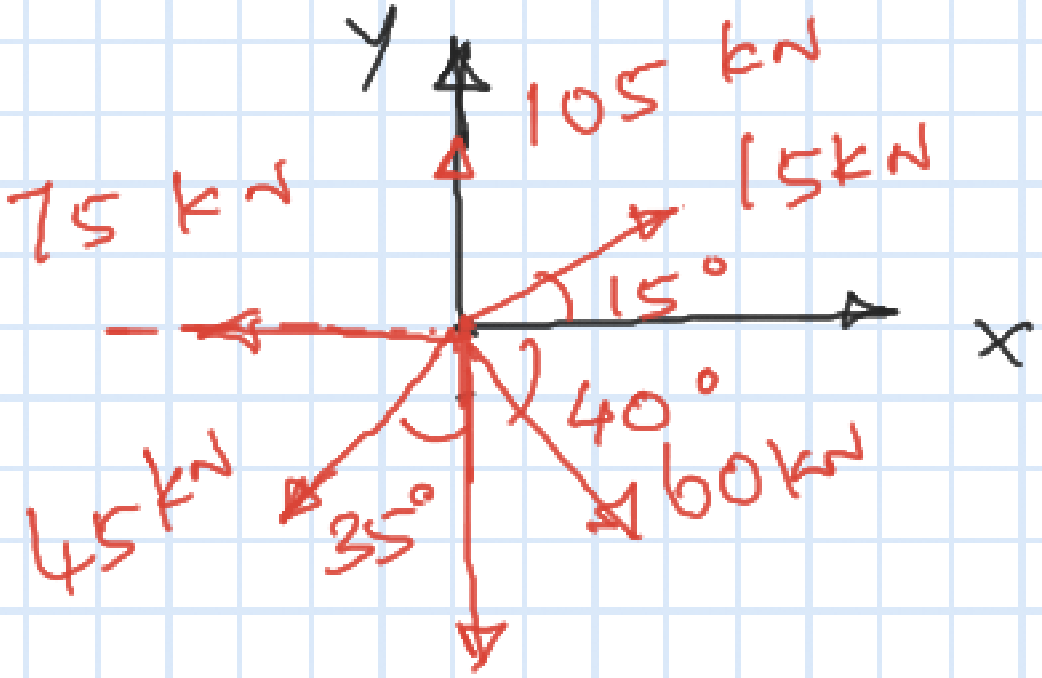
$$R = 1012.07\ N$$

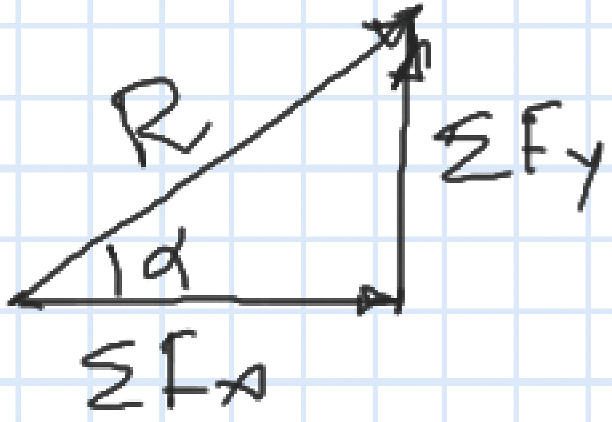
Direction of the resultant

$$\tan\alpha = \frac{994.74}{186.46}$$

$$\alpha = 79.38°$$

5. Five forces are acting on a particle as shown. Get the resultant force in magnitude and direction.

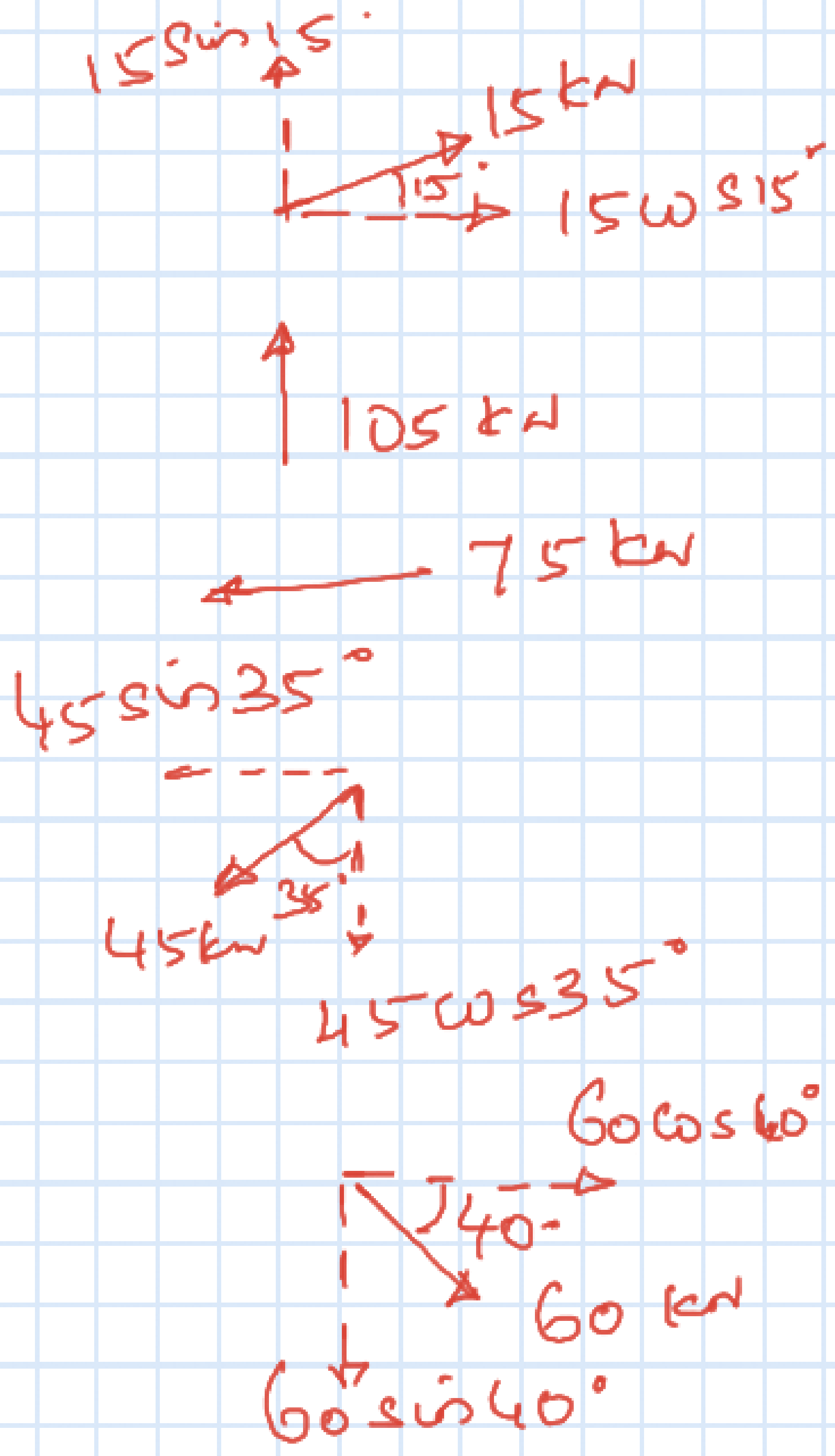


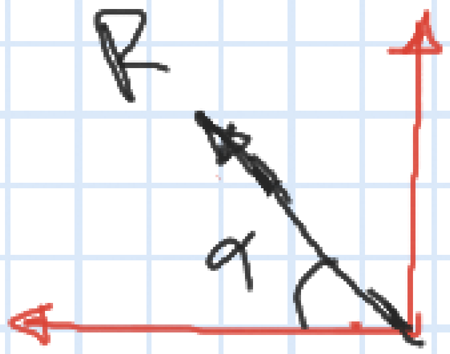


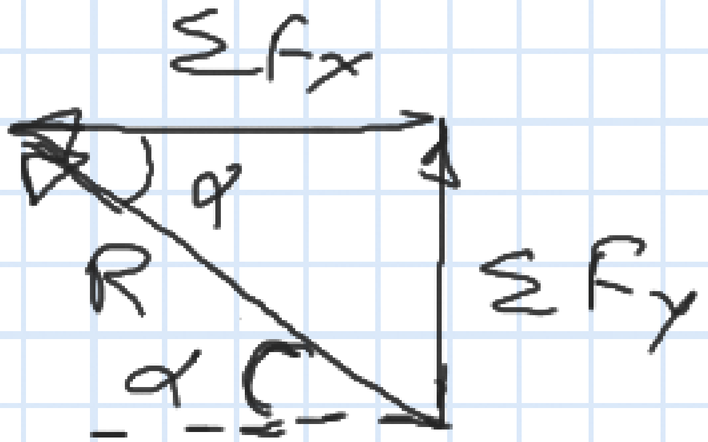
Forces (kN)	F_x	F_y
15	$15\cos 15^\circ$	$15\sin 15^\circ$
105	0	105
75	-75	0
45	$-45\sin 35^\circ$	$-45\cos 35^\circ$
60	$60\cos 40^\circ$	$-60\sin 40^\circ$

$\Sigma F_x = -40.37$ kN

$\Sigma F_y = 33.46$ kN







Mag

$$R^2 = \Sigma F_x^2 + \Sigma F_y^2$$

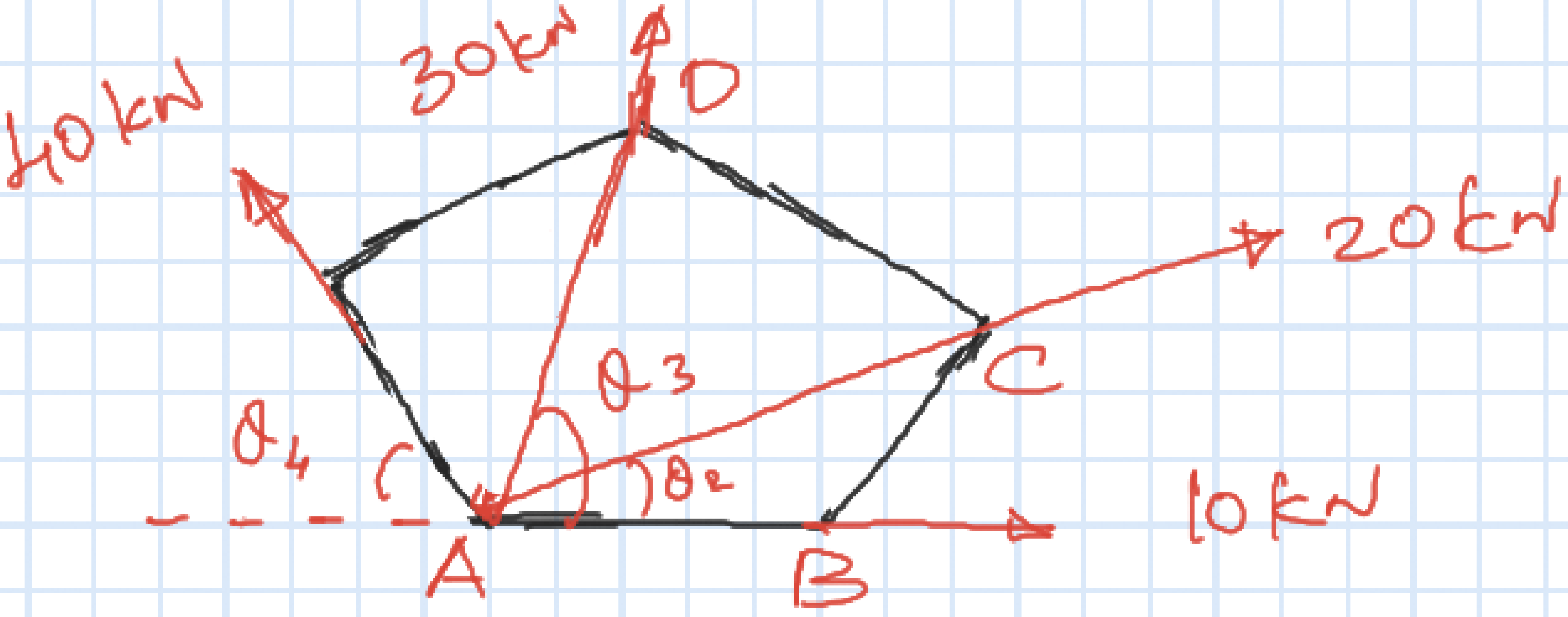
$$R = 52.45 \text{ kN} ✓$$

Dir

$$\tan\alpha = \frac{\Sigma F_y}{\Sigma F_x}$$

$$\alpha = 39.69° ✓$$

6. Four forces are acting in a pentagon as shown. Find the magnitude and direction of the resultant force.



sol Sum of the interior angles $= (2n-4)\,90^\circ$

$= (2\times5-4)\,90^\circ$

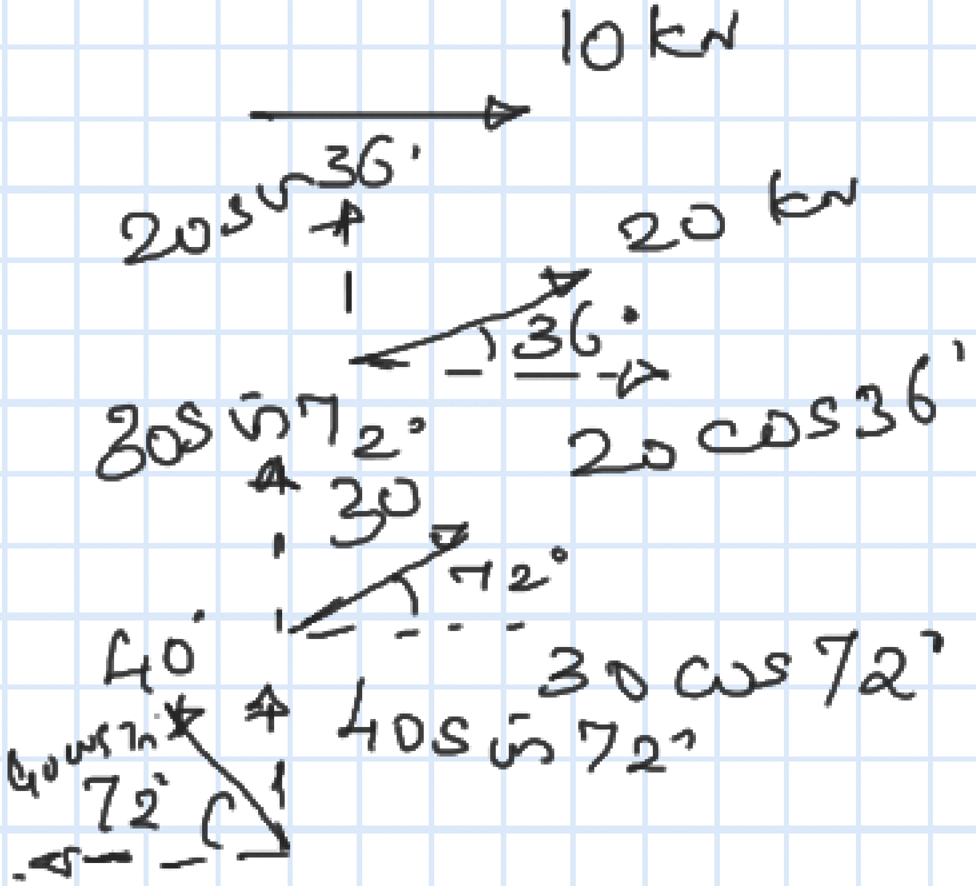
$= 540^\circ$

Each included angle $= \frac{540}{5} = 108^\circ$

$\theta_2 = \frac{108}{3} = 36^\circ$ $\qquad \theta_1 = 0^\circ$

$\theta_3 = 2(\theta_2) = 72^\circ$

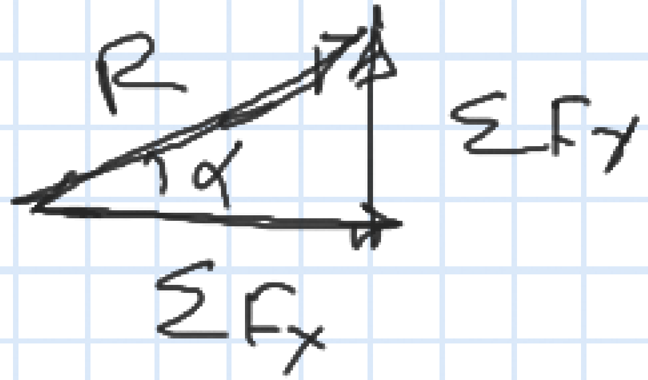
$\theta_4 = 180^\circ - 108^\circ = 72^\circ$



	Forces (kN)	F_x	F_y
1.	10	10	0
2.	20	$20\cos 36^\circ$	$20\sin 36^\circ$
3.	30	$30\cos 72^\circ$	$30\sin 72^\circ$
4.	40	$-40\cos 72^\circ$	$40\sin 72^\circ$

$\Sigma F_x =$ $\qquad \Sigma F_y =$

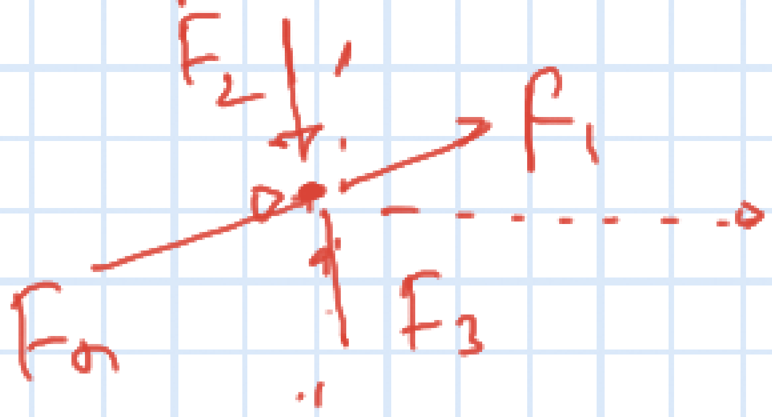
$\Sigma F_x = 23.08$ kN $\Sigma F_y : 78.3$ kN



Mag $R = 81.6$ kN

Dir $\alpha = 73.6^\circ$

Equilibrium of particle - 2D



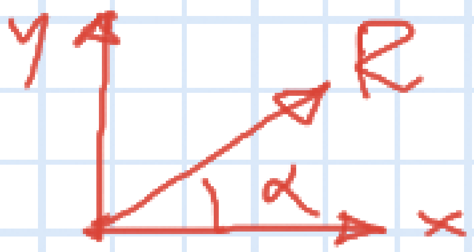
$\Sigma F_x \neq 0$, $\Sigma F_x = 0$ — (1)

$\Sigma F_y \neq 0$, $\Sigma F_y = 0$ — (2)

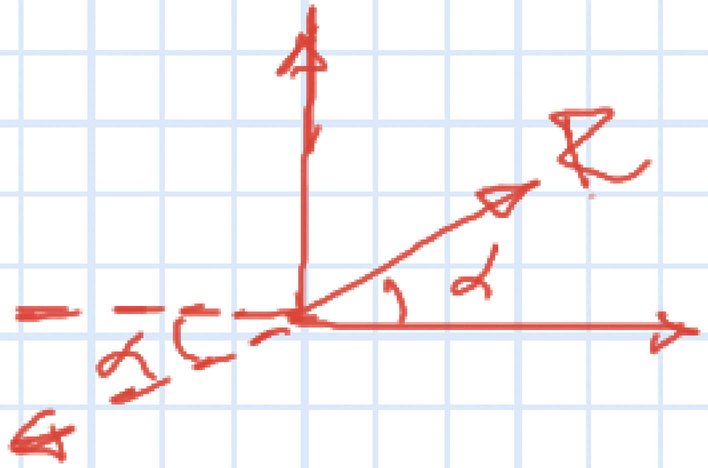
Two equations of equilibrium are available to solve for Two unknown

A particle is said to be in equilibrium, when the resultant of the force system acting on it is zero. Such a particle will be in state of rest or of uniform motion.

Equilibrant



Resultant



E = Equilibrant

It is a single force which acts along with other force to keep the particle in equilibrium

$R = 0$

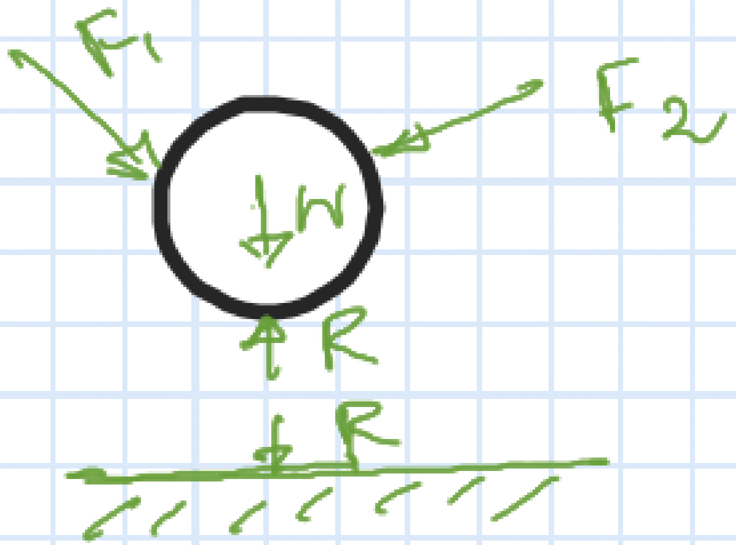
$\Sigma F_x = 0 \quad \Sigma F_y = 0.$

Free Body diagram

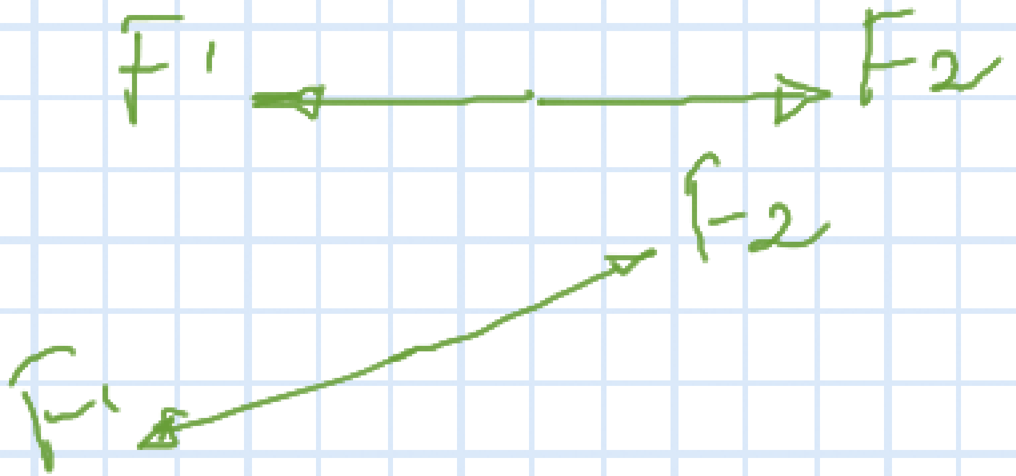
Free body - Body isolated from all other attached members

FBD — It is an isolated body showing external forces on the body and reactions exerted on it by removed elements

FBD



Equilibrium of Two forces



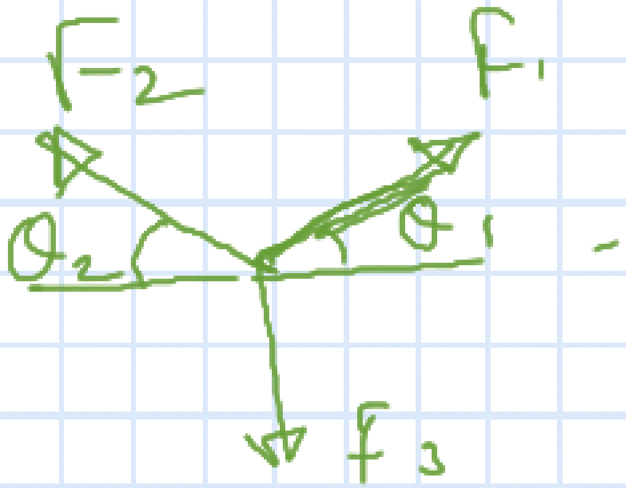
$R = 0$

$-F_1 + F_2 = 0$

$\boxed{F_1 = F_2}$

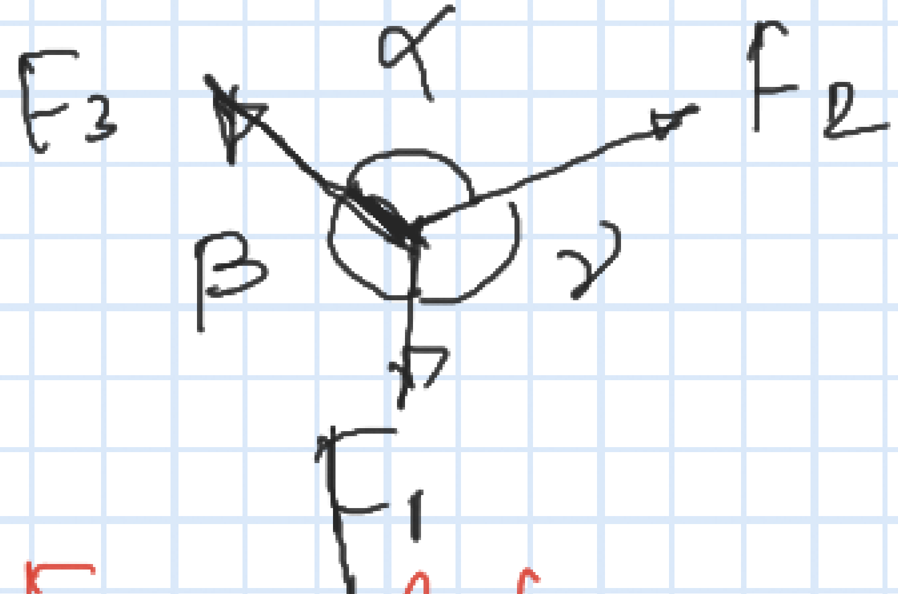
Collinear and in opposite direction

Equilibrium of Three forces



Lami's theorem

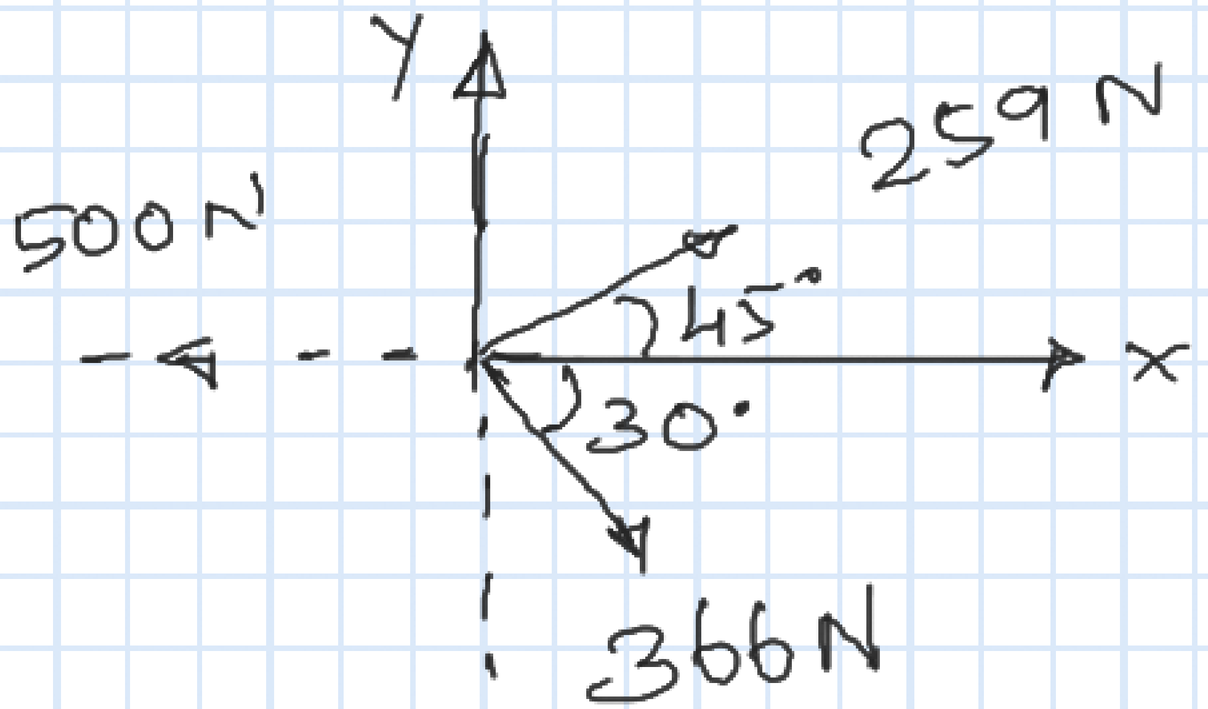
If three forces acting on a particle are in equilibrium, then each force is proportional to the sine of the angle included between the other two forces.



$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

Equilibrium of particle (2D) - Problems

1. Examine the equilibrium condition for the particle A shown in Figure.



Sol When a particle is in equilibrium $R = 0$

Since $R = 0$

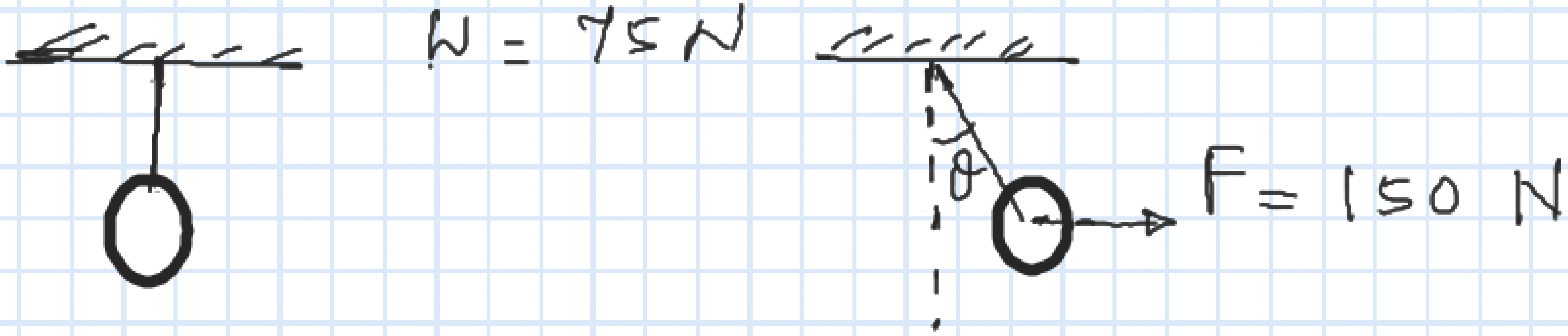
$\Sigma F_x = 0$ — (1)

$\Sigma F_y = 0$ — (2)

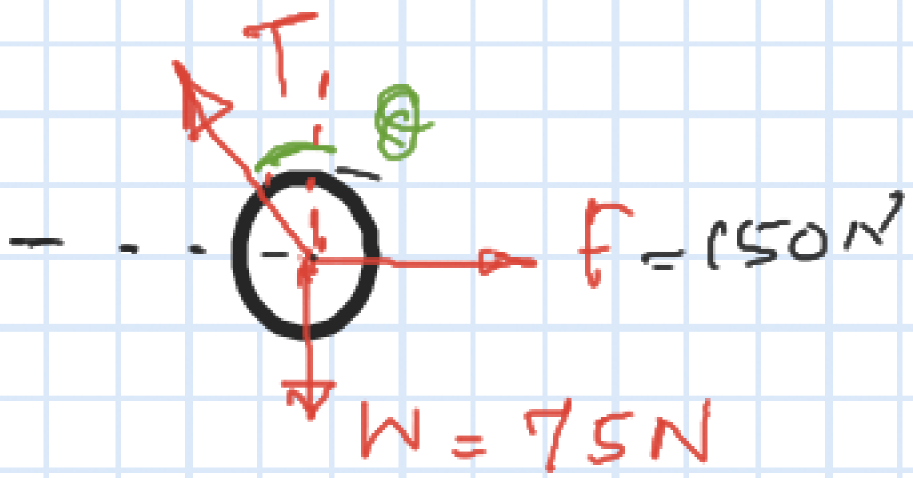
Force (N)	F_x	F_y
1. 259	$259 \cos 45°$	$259 \sin 45°$
2. 500	-500	0
3. 366	$366 \cos 30°$	$-366 \sin 30°$
	$\Sigma F_x = 0$	$\Sigma F_y = 0$

Therefore the particle is in equilibrium.

2. A spherical ball of weight 75 N is attached to a string and is suspended from the ceiling as shown. Find the tension in the string, if the horizontal force is applied to the ball. Determine the angle the string makes with the vertical and also the tension in the string if F = 150 ~~k~~N.



Sol.: FBD for the ball



Equilibrium equation

$\Sigma F_x = 0$; $\Sigma F_y = 0$

Force (N)	F_x	F_y
150	150	0
T	$-T\sin\theta$	$T\cos\theta$
75	0	-75

$$-T\sin\theta + 150 = 0 \quad - (1)$$

$$T\cos\theta - 75 = 0 \quad - (2)$$

solve (1) & (2)

$$T\sin\theta = 150$$

$$T\cos\theta = 75$$

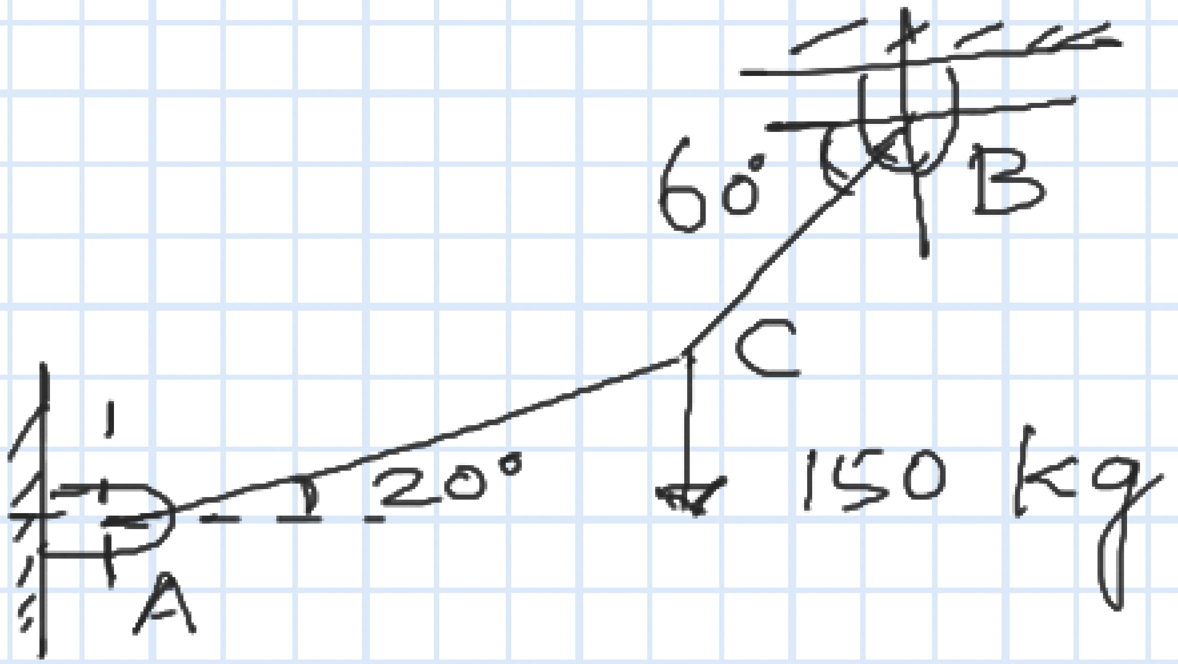
$$T = 168 \text{ N}$$

$$\theta = 63.5^\circ$$

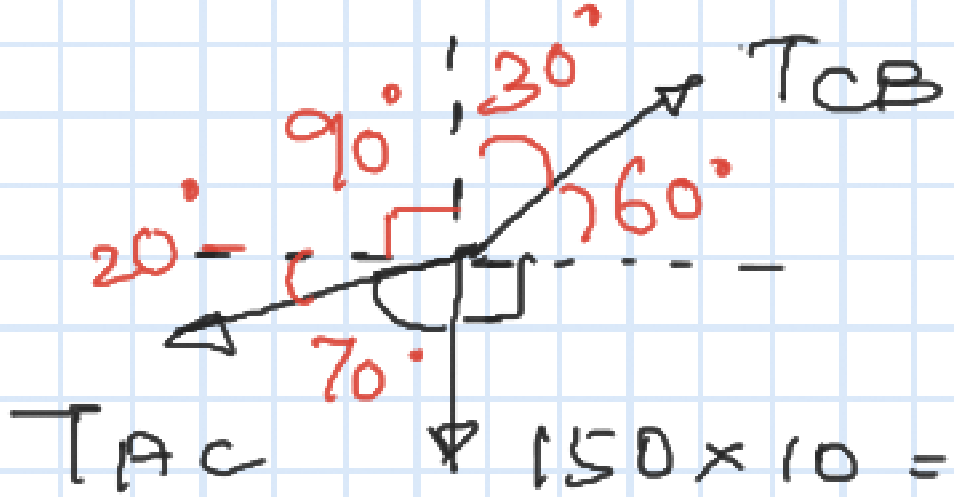
Lamis theorem

$$\frac{75}{\sin(90+\theta)} = \frac{T}{\sin 90^\circ} = \frac{150}{\sin(180-\theta)}$$

3. Two cables are tied together at C and loaded as shown. Determine the tension in cables AC and BC.



Sol. FBD of the string



$150 \times 10 = 1500\ N$

Resolution of forces

$\Sigma F_x = 0$ — (1)

$\Sigma F_y = 0$ — (2)

Lami's theorem

$$\frac{1500}{\sin 140^\circ} = \frac{T_{AC}}{\sin 150^\circ} = \frac{T_{CB}}{\sin 70^\circ}$$

$$T_{AC} = \frac{1500}{\sin 140^\circ}(\sin 150^\circ) = 1166.8\,N$$

$$T_{CB} = \frac{1500}{\sin 140^\circ}(\sin 70^\circ) = 2192.8\,N$$

$T_{CB} \cos 60^\circ - T_{AC} \cos 20^\circ = 0 \quad - (1)$

$T_{CB} \sin 60^\circ - T_{AC} \sin 20^\circ - 1500 = 0 \quad - (2)$

solve (1) & (2)

From (1)

$$T_{CB} = T_{AC} \frac{\cos 20^\circ}{\cos 60^\circ}$$

Sub in (2)

$$T_{AC} \frac{\cos 20^\circ}{\cos 60^\circ} (\sin 60^\circ) - T_{AC} \sin 20^\circ = 1500$$

$$\boxed{\begin{aligned} T_{AC} &= 1166.8 \text{ N} \\ T_{BC} &= 2192.8 \text{ N} \end{aligned}}$$