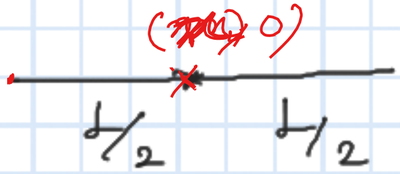
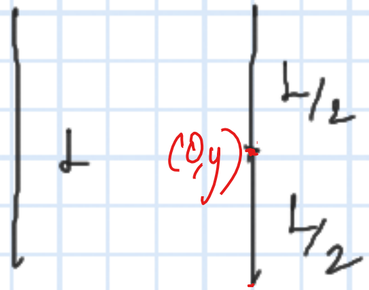
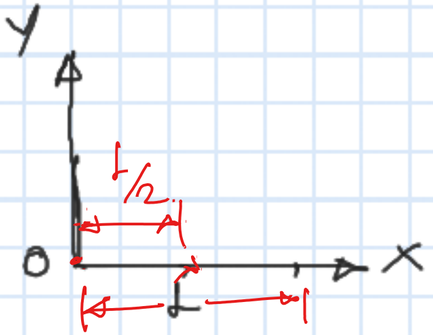
Unit II – Properties of lines, areas and volumes

1D – Centroid – $(\bar{x}, \bar{y})$

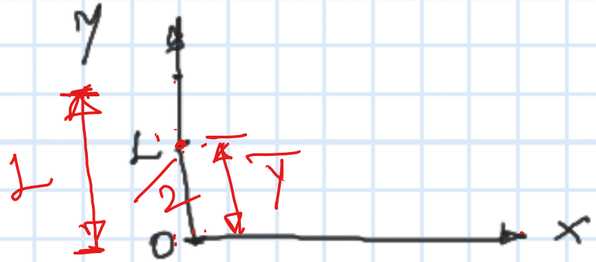
Straight line



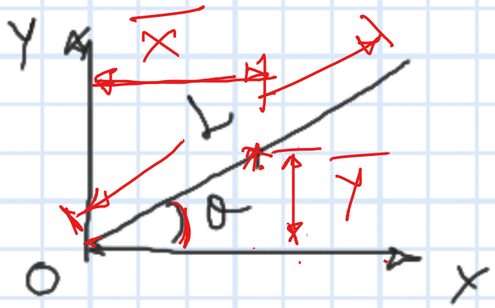


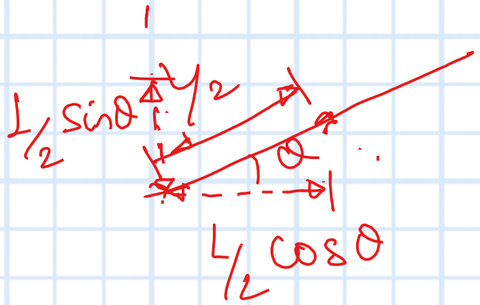


$\bar{x} = L/2$ from the origin O



$\bar{y} = L/2$ from the origin O

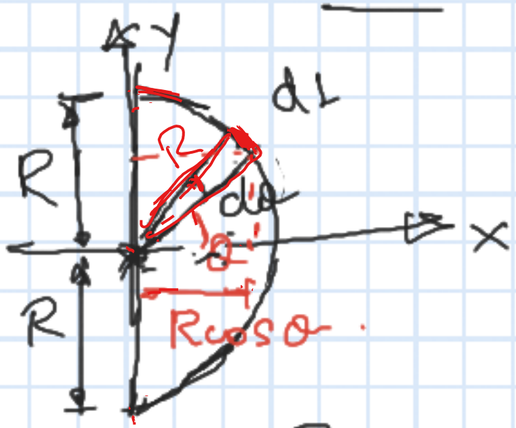




$$\bar{x} = \frac{L}{2}\cos\theta$$

$$\bar{y} = \frac{L}{2}\sin\theta$$

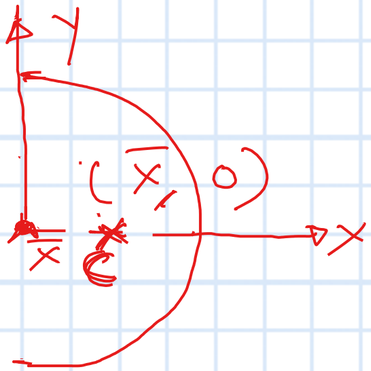
Semicircular arc



$$dL = R\,d\theta$$

$$L = \int_{-\pi/2}^{\pi/2} R\,d\theta = \pi R$$

=



$\bar{y} = 0$

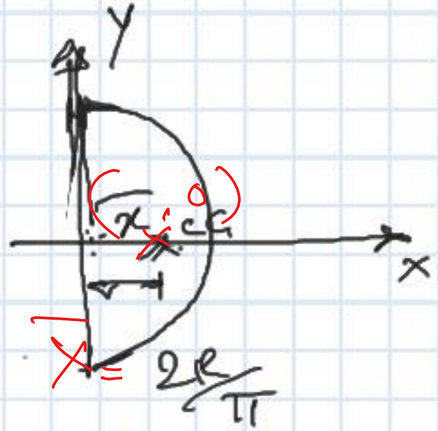
First moment of dL about y axis = $dL(x)$

~~$\pi R(\bar{x})$~~ $= R\,d\theta\,(R\cos\theta)$

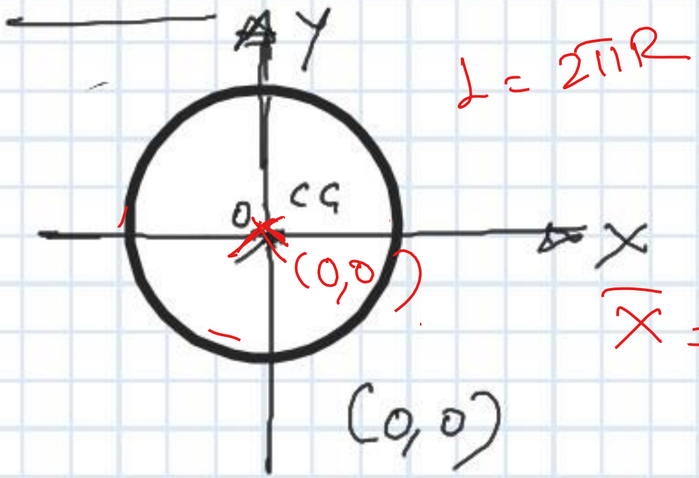
First moment of the semicircular arc about y axis

$$\int_{-\pi/2}^{\pi/2} R^2 \cos\theta \, d\theta = 2R^2 = \pi R(\bar{x})$$

$$\bar{x} = \frac{2R^2}{\pi R} = \frac{2R}{\pi}$$

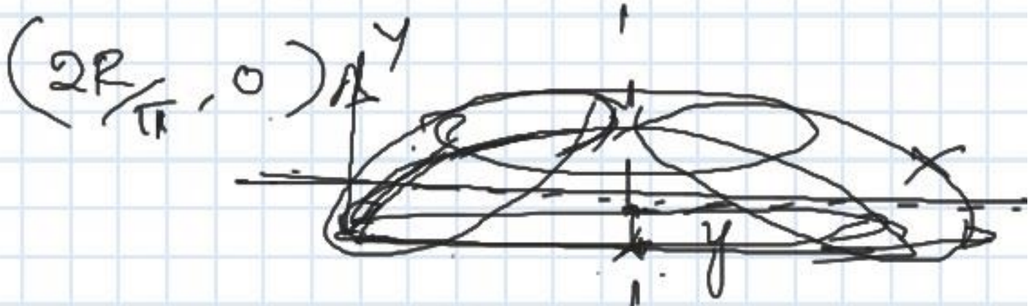


Circular arc



$l = 2\pi R$

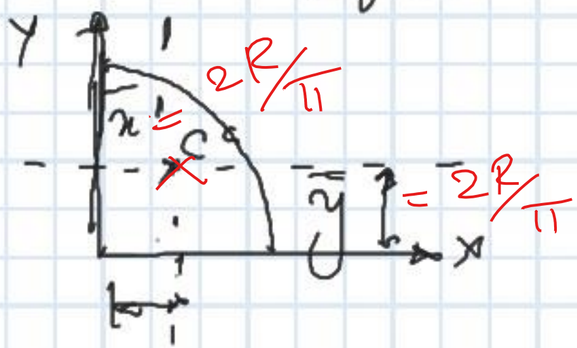
$\bar{x} = 0, \bar{y} = 0$



$$\bar{y} = 2R/\pi$$

$(R, 2R/\pi)$

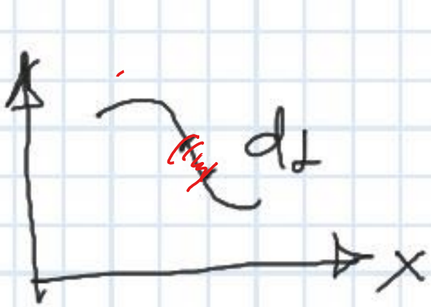
Quadrant of a circle



$(\bar{x}, \bar{y})$ ✓ ✓

$\left(\frac{2R}{\pi}, \frac{2R}{\pi}\right)$

In general Line



$L = \int dL$

$\bar{x} = \frac{\int dL\, \bar{x}}{L}$

$\bar{y} = \frac{\int dL\, \bar{y}}{L}$

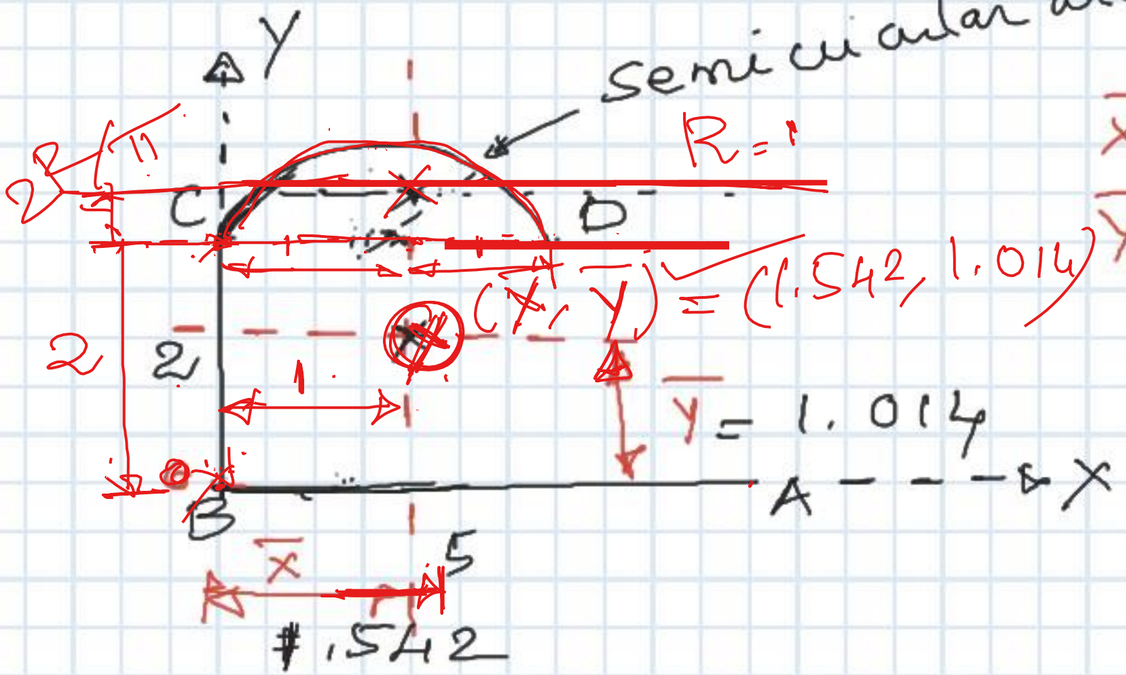
$L\bar{x} = \int dL\, \bar{x}$

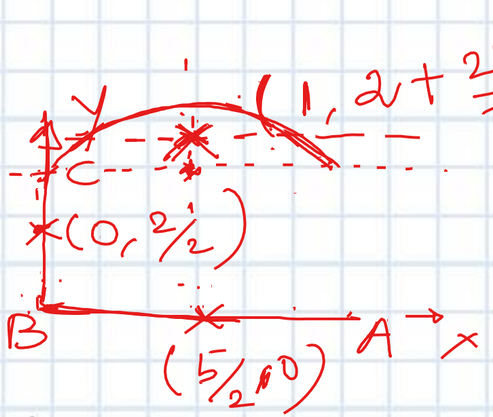
$L\bar{y} = \int dL\, \bar{y}$

Location of Centroid for 1D composite area/ bents.

1. Locate the centroid for the given bent ABCD

semicircular arc.





$$\bar{x} = \frac{\int d L \bar{x}}{L} \qquad \bar{y} = \frac{\int d L \bar{y}}{L}$$

$$\bar{x} = \frac{\Sigma L_i \bar{x}_i}{\Sigma L_i} \qquad \bar{y} = \frac{\Sigma L_i \bar{y}_i}{\Sigma L_i}$$

S.No.	Segment	L_i	$\bar{x}_i$	$\bar{y}_i$	$L_i \bar{x}_i$	$L_i \bar{y}_i$
1.	AB	5	5/2	0	(25/2)	(0)
2.	BC	2	0	2/2	(0)	(1/2)
3.	CD	$\frac{2\pi(1)}{2}$	1	$\left(\frac{2(1)}{\pi} + 2\right)$	$\frac{\pi(1)}{}$	( )

$\Sigma L_i = 10.14$

$\Sigma L_i \bar{x}_i = 15.64$ $\Sigma L_i \bar{y}_i = 10.28$

$$\Sigma L_i(\bar{x}) = \Sigma L_i \bar{x}_i \qquad \Sigma L_i(\bar{y}) = \Sigma L_i \bar{y}_i$$

$$\bar{x} = \frac{\Sigma L_i \bar{x}_i}{\Sigma L_i} = \frac{15.64}{10.14} \qquad \bar{y} = \frac{\Sigma L_i \bar{y}_i}{\Sigma L_i} = \frac{10.28}{10.14}$$

$= 1.542$ units $\qquad = 1.014$ units

ABCD

$(\bar{x}, \bar{y})$