18EES101J- BASIC ELECTRICAL & ELECTRONICS ENGINEERING

LEC-03

Overview

- ✓ Circuit Analysis
  - ✓ Cramer’s rule
  - ✓ Mesh current method
  - ✓ Node voltage method
- ✓ Problems
- ✓ Practice problems

Circuit Analysis

Cramer's rule

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = y_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = y_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = y_3$$

- $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$
- $AX = Y$
- $Det(A) = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$
- $Det(A_1) = \begin{bmatrix} y_1 & a_{12} & a_{13} \\ y_2 & a_{22} & a_{23} \\ y_3 & a_{32} & a_{33} \end{bmatrix}$
- $Det(A_2) = \begin{bmatrix} a_{11} & y_1 & a_{13} \\ a_{21} & y_2 & a_{23} \\ a_{31} & y_3 & a_{33} \end{bmatrix}$
- $Det(A_3) = \begin{bmatrix} a_{11} & a_{12} & y_1 \\ a_{21} & a_{22} & y_2 \\ a_{31} & a_{32} & y_3 \end{bmatrix}$
- $x_1 = \frac{Det(A_1)}{Det(A)}$
- $x_2 = \frac{Det(A_2)}{Det(A)}$
- $x_3 = \frac{Det(A_3)}{Det(A)}$

Circuit Analysis

- $A.X = Y$

Using Ohm's Law

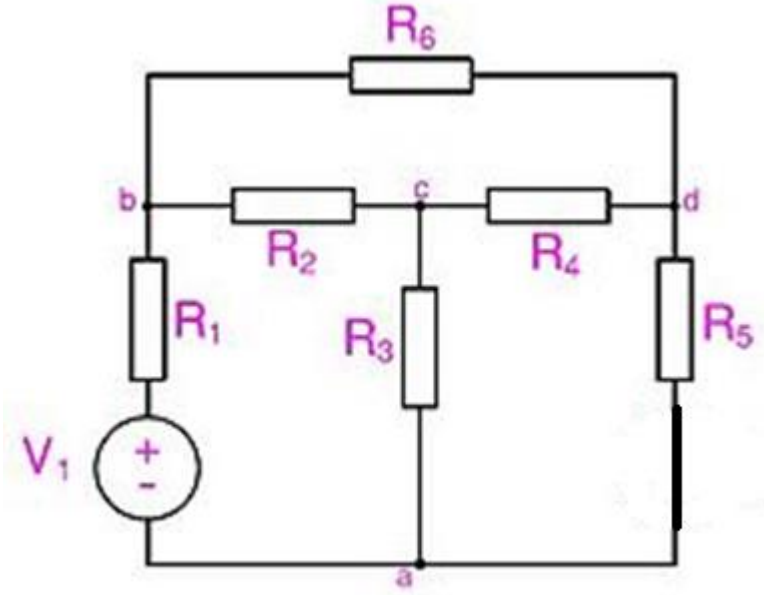
- $R.I = V$ Mesh current analysis

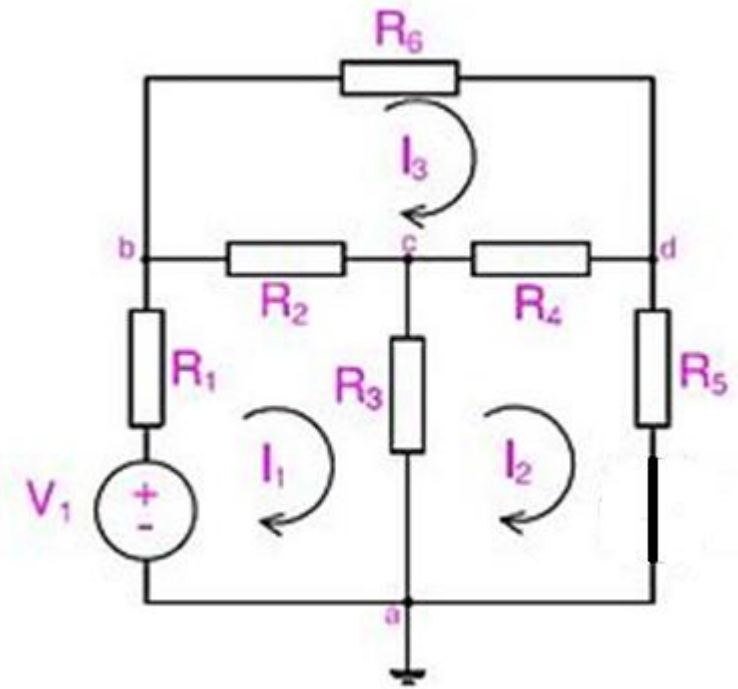
- $I = V\left(\frac{1}{R}\right)$

- $I = V.G$

- $G.V = I$ Node voltage analysis

Mesh Analysis





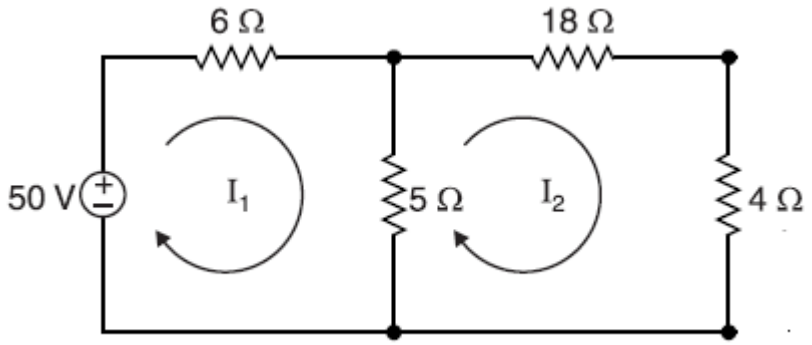
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} (R_1+R_2+R_3) & (-R_3) & (-R_2) \\ (-R_3) & (R_3+R_4+R_5) & (-R_4) \\ (-R_2) & (-R_4) & (R_2+R_4+R_6) \end{bmatrix}.$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Mesh Analysis

- Find the mesh currents I_1 and I_2



- Loop ABDA

$$-50 + 6I_1 + 5(I_1 - I_2) = 0 \qquad (1)$$

$$11I_1 - 5I_2 = 50 \qquad (2)$$

- Loop BCDB

$$+18I_2 + 4I_2 + 5(I_2 - I_1) = 0 \qquad (3)$$

$$+27I_2 - 5I_1 = 0 \qquad (4)$$

$$I_1 = \frac{27}{5} I_2$$

Sub I_1 in (2)

$$I_2 = 0.92\ A$$

$$I_1 = 4.96\ A$$

Mesh Analysis

Using Cramer's rule

$$\begin{bmatrix} 11 & -5 \\ -5 & 27 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 11 & -5 \\ -5 & 27 \end{bmatrix}$$

$$\text{Det}(A) = 272$$

$$A_1 = \begin{bmatrix} 50 & -5 \\ 0 & 27 \end{bmatrix}$$

$$\text{Det}(A_1) = 1350$$

$$A_2 = \begin{bmatrix} 11 & 50 \\ -5 & 0 \end{bmatrix}$$

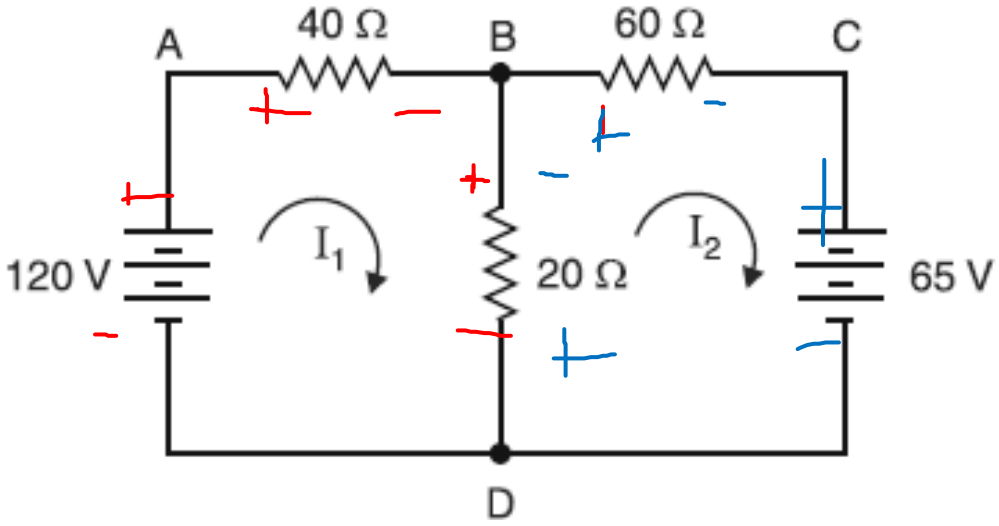
$$\text{Det}(A_2) = 250$$

$$I_1 = \frac{\text{Det}(A_1)}{\text{Det}(A)} = \frac{1350}{272} = 4.96\ A$$

$$I_2 = \frac{\text{Det}(A_2)}{\text{Det}(A)} = \frac{250}{272} = 0.92\ A$$

Mesh Analysis

- Find the Mesh currents I_1 and I_2



$$-120 + 40I_1 + 20(I_1 - I_2) = 0 \qquad (1)$$

$$+60I_2 + 20(I_2 - I_1) + 65 = 0 \qquad (2)$$

$$60I_1 - 20I_2 = 120 \qquad (3)$$

$$-20I_1 + 80I_2 = -65 \qquad (4)$$

$(4) \times 3$

$$-60I_1 + 240I_2 = -195 \qquad (5)$$

$(3) + (5)$

$$I_2 = -0.34\ A$$

Sub I_2 in (3) $\qquad I_1 = 1.87\ A$

Nodal Analysis

Ground

- The voltage at one point in a circuit is always measured relative to another point in the circuit.
- For example, if we say that voltage at a point in a circuit is + 10V, we mean that the point is 10V more positive than some reference point in the circuit.
- This reference point in a circuit is usually called the *ground point*.
- Thus ground is used as reference point (zero reference point) for specifying voltages.

Symbols for ground

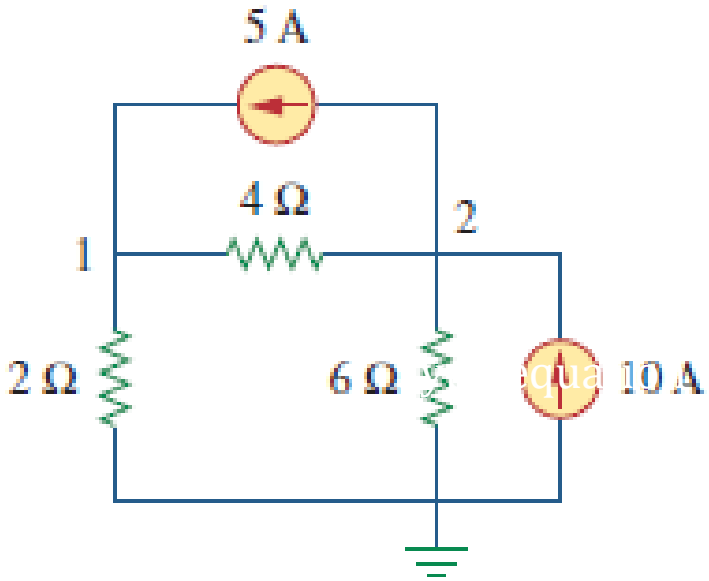
Nodal Analysis

- Current always flows from higher potential to lower potential

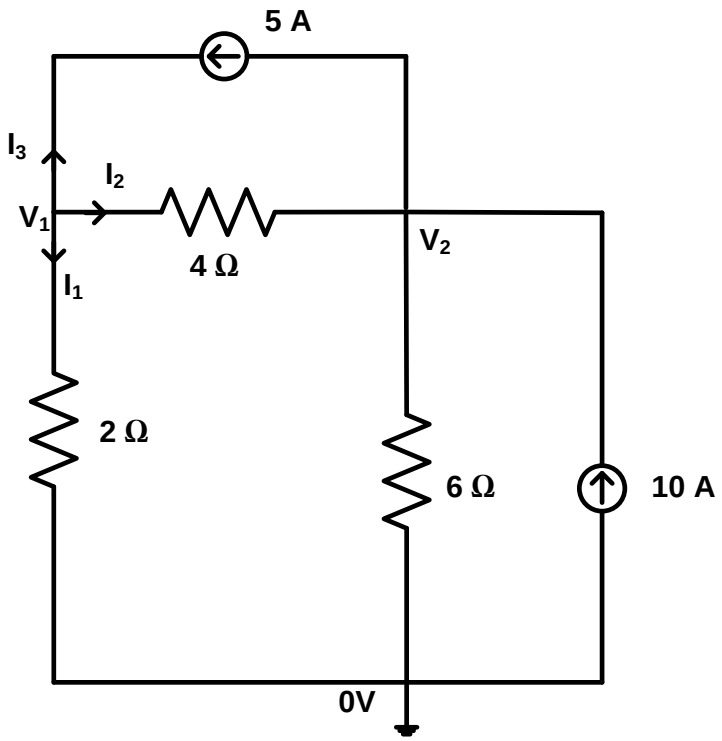
$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}$$

Nodal Analysis

- Find the node voltages at nodes 1 and 2



Nodal Analysis



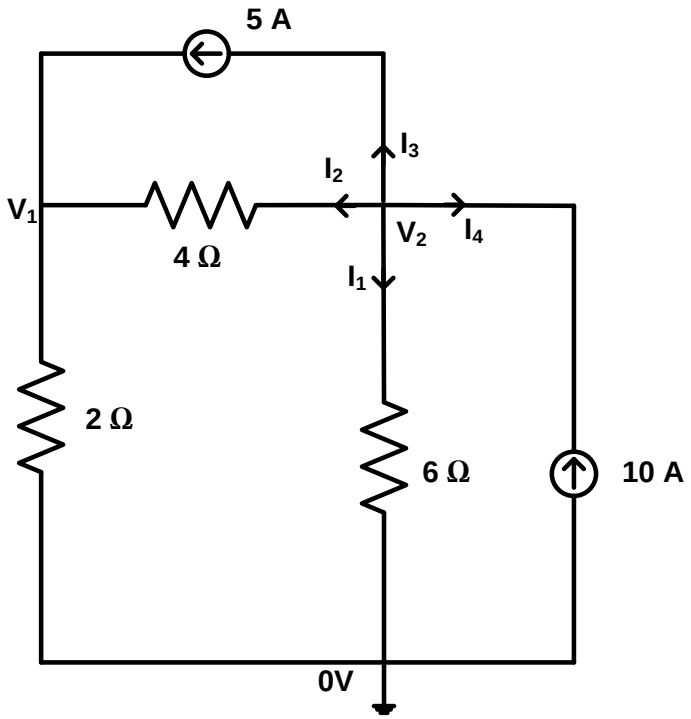
At node 1

$$\frac{V_1}{2} + \frac{V_1 - V_2}{4} - 5 = 0 \qquad (1)$$

$$3V_1 - V_2 = 20 \qquad (2)$$

V_1=13.33 V

V_2 = 20V



At node 2

$$\frac{V_2}{6} + \frac{V_2 - V_1}{4} + 5 - 10 = 0 \qquad (2)$$

$$-3V_1 + 5V_2 = 60 \qquad (4)$$

Nodal Analysis

- Using Cramer's rule

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

$$v_1 = \frac{\Delta_1}{\Delta}$$

$$v_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - 3 = 12$$

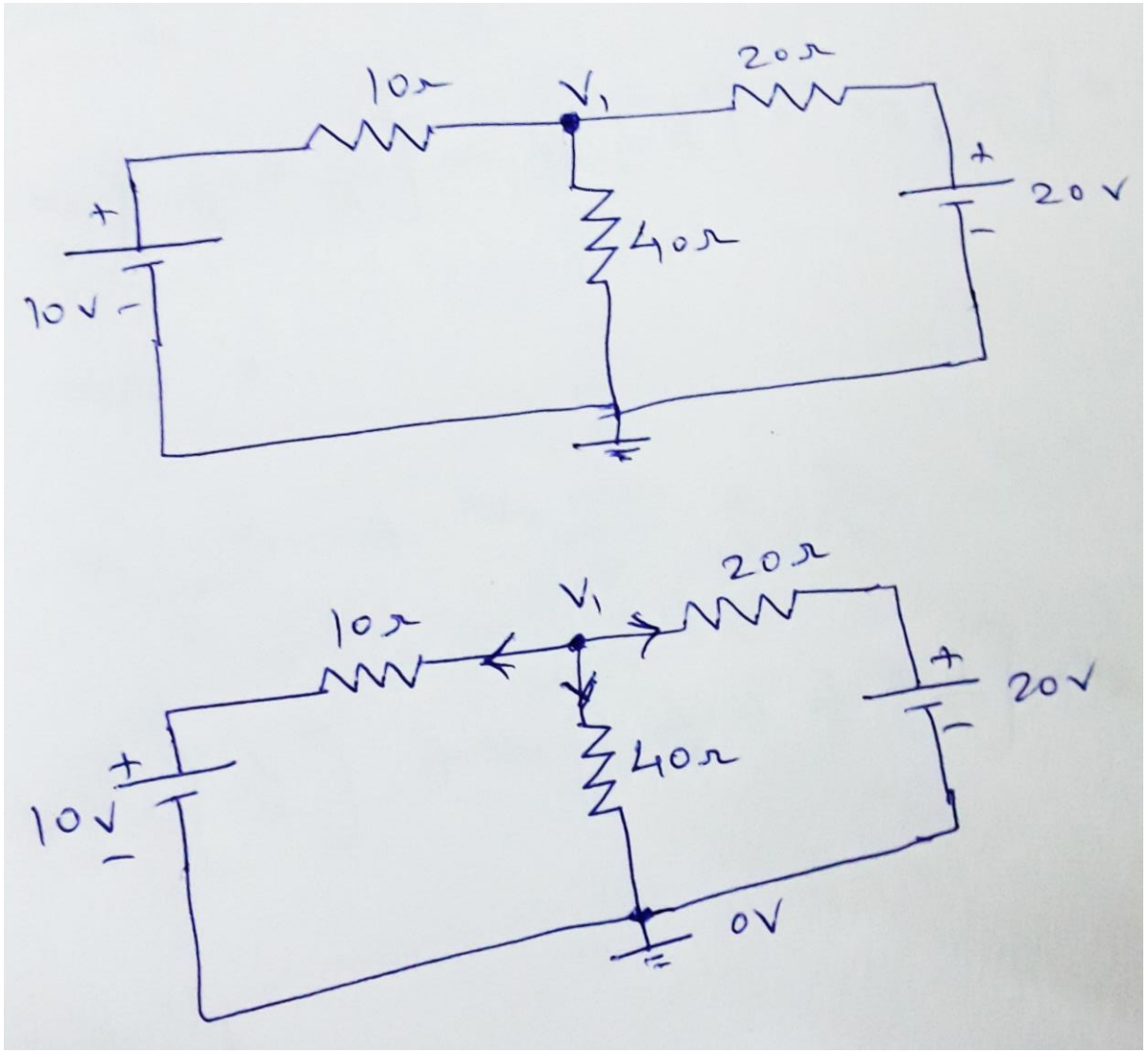
$$\Delta_1 = \begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix} \qquad \Delta_2 = \begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}$$

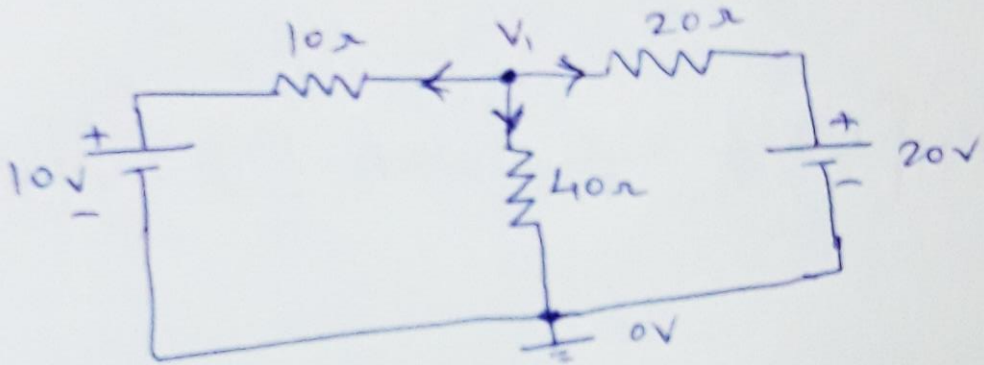
V_1=13.33 V

V_2 = 20V

Nodal Analysis

Find the node voltage V_1





$$\frac{V_1 - 10}{10} + \frac{V_1 - 20}{20} + \frac{V_1 - 0}{40} = 0$$

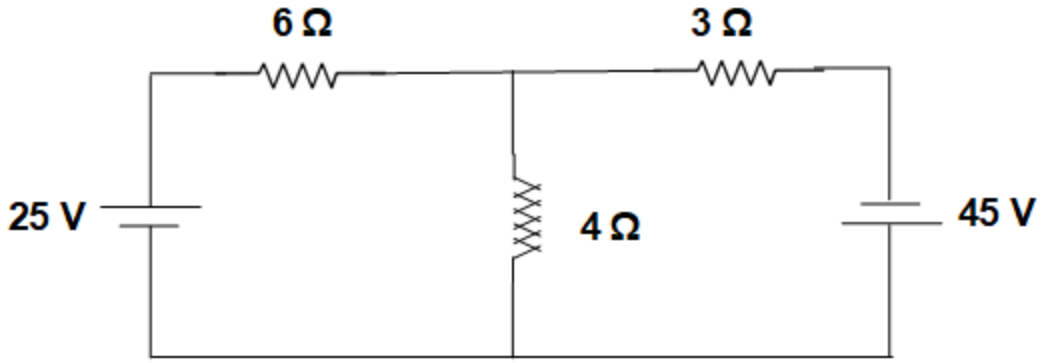
$$V_1 \left[\frac{1}{10} + \frac{1}{20} + \frac{1}{40} \right] - 2 = 0$$

$$V_1 \left[\frac{1}{10} + \frac{1}{20} + \frac{1}{40} \right] = 2$$

$$\boxed{V_1 = 11.43\ V}$$

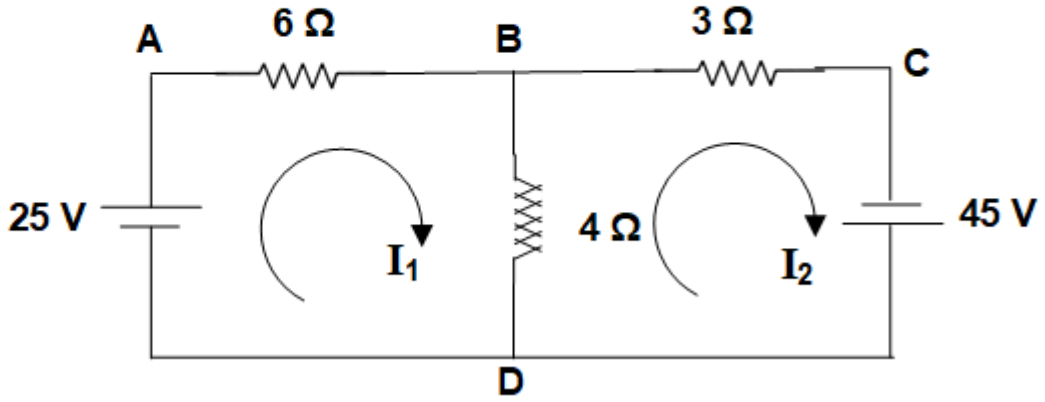
Example 11

Using Kirchhoff's laws, find the current in various resistors in the circuit shown.

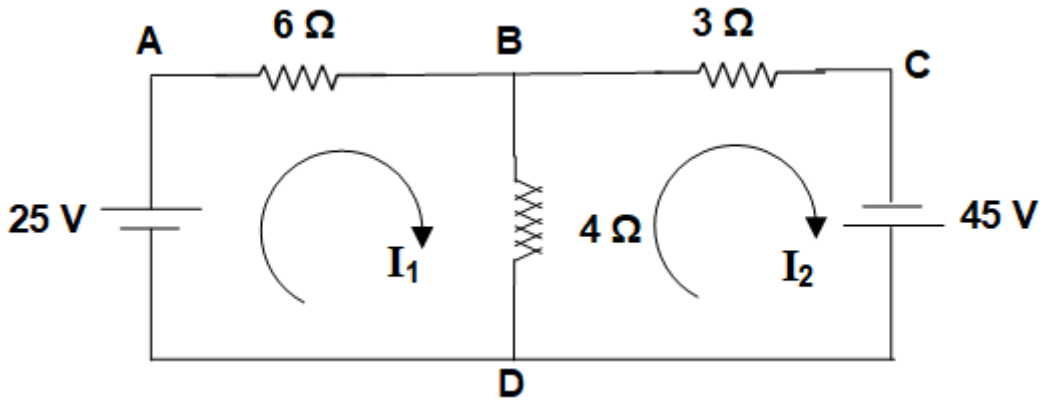


Solution

Let the loop current be I_1 and I_2



Problems



For the loop ABDA

$6 I_1 + 4 (I_1 - I_2) - 25 = 0$ ⇒ $10 I_1 - 4 I_2 = 25$

For the loop BCDB

$3 I_2 - 45 + 4 (I_2 - I_1) = 0$ ⇒ $-4 I_1 + 7 I_2 = 45$

$I_1 = 6.574$ A; $I_2 = 10.1852$ A

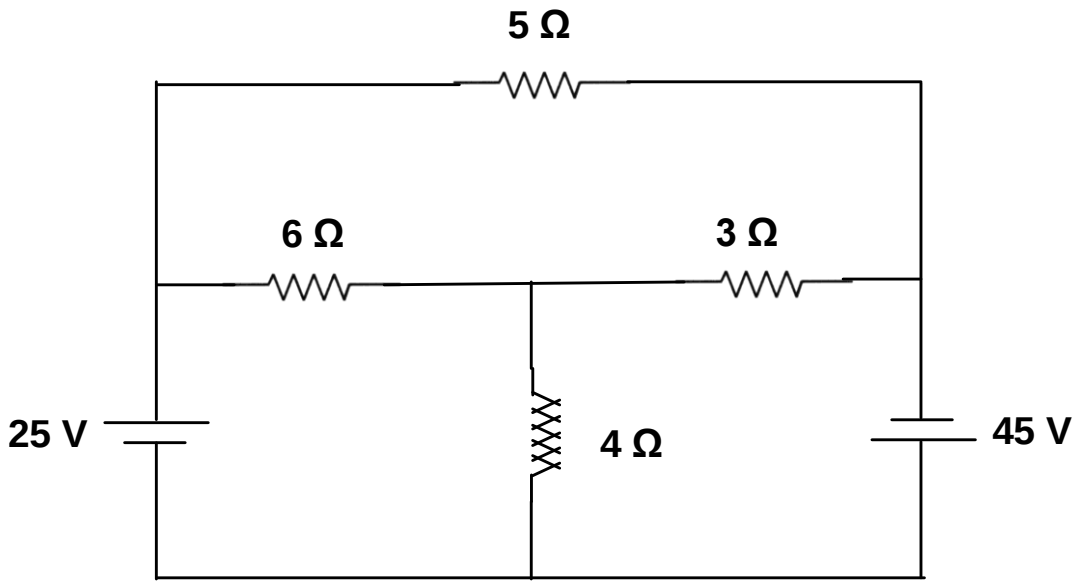
Current in 4Ω resistor = $I_1 - I_2$ = 6.574 – 10.1852 = - 3.6112 A

Thus the current in 4Ω resistor is from A to C

Current in 6 Ω resistor = 6.574 A; Current in 3 Ω resistor = 10.185

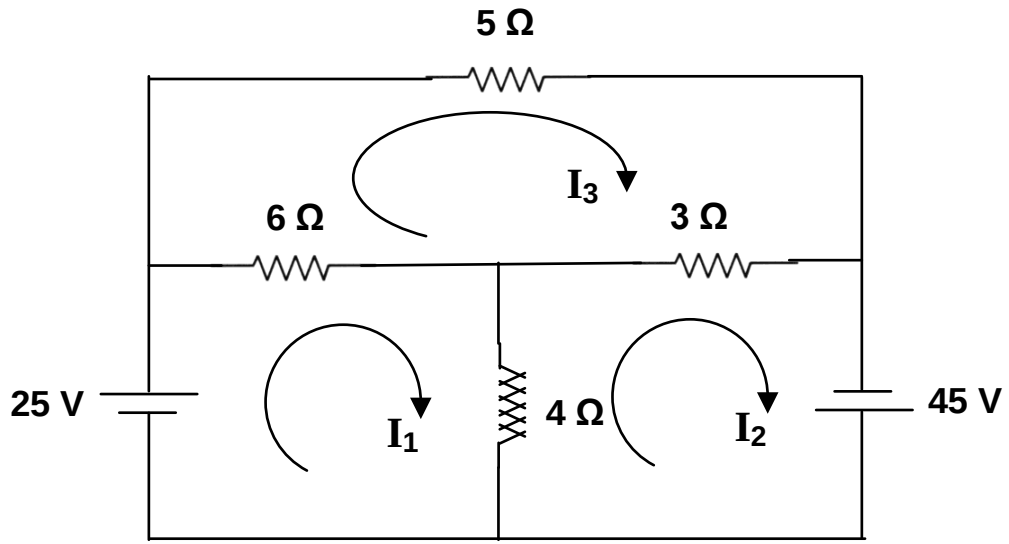
Example 12

Find the current in 5 Ω resistor in the circuit shown.



Solution

Let the loop current be I_1, I_2 and I_3.



Three loops equations are:

$$6 (I_1 - I_3) + 4 (I_1 - I_2) - 25 = 0$$

$$4 (I_2 - I_1) + 3 (I_2 - I_3) - 45 = 0$$

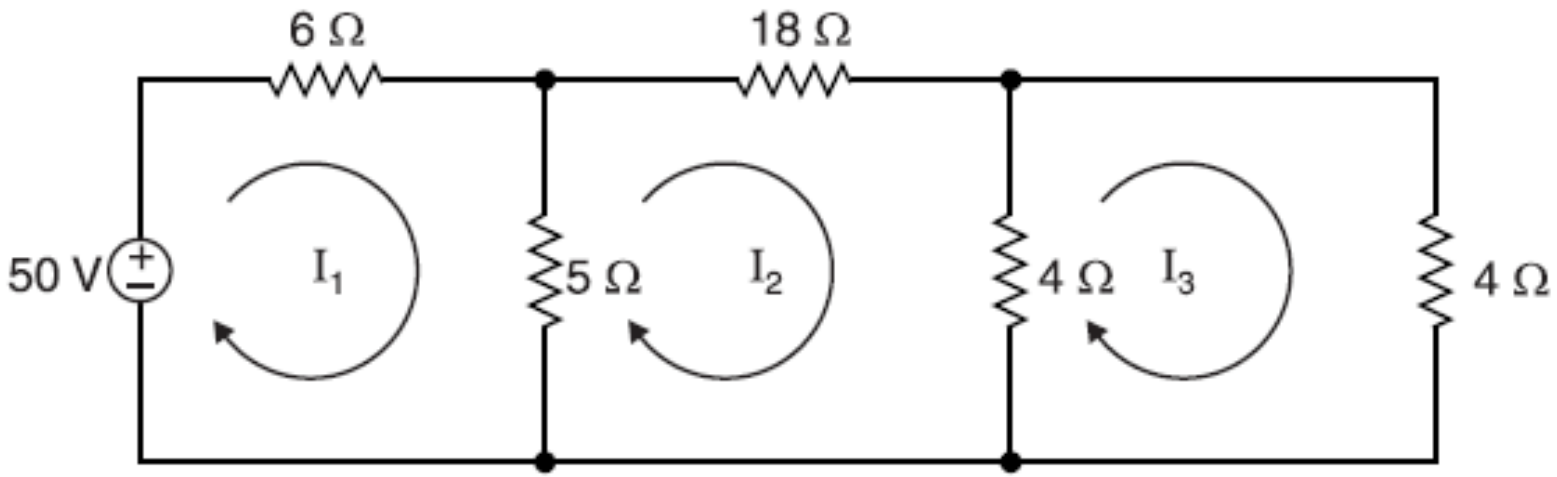
$$5 I_3 + 3 (I_3 - I_2) + 6 (I_3 - I_1) = 0$$

On solving

Current in 5 Ω resistor, I_3 = 14 A

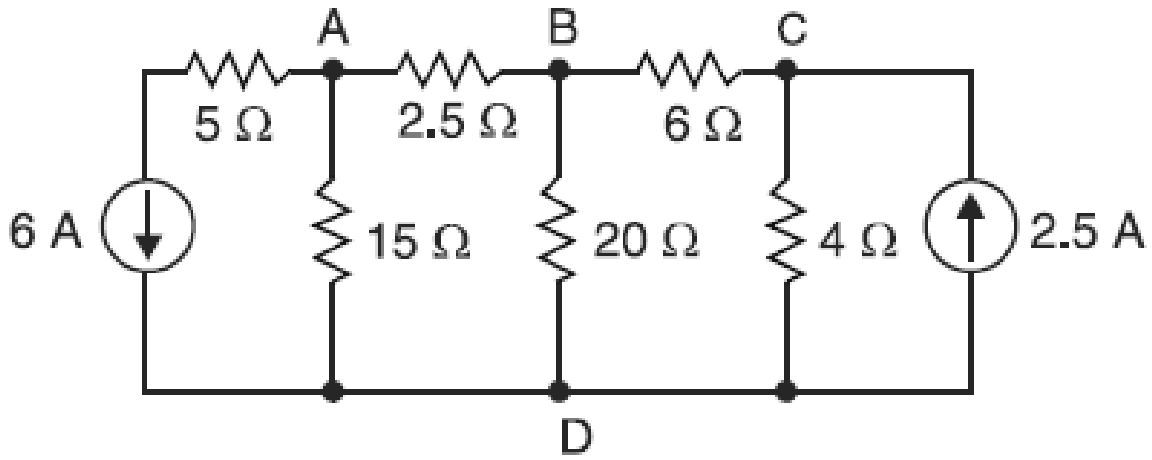
Practice Problems

- Find Mesh currents I_1, I_2, I_3



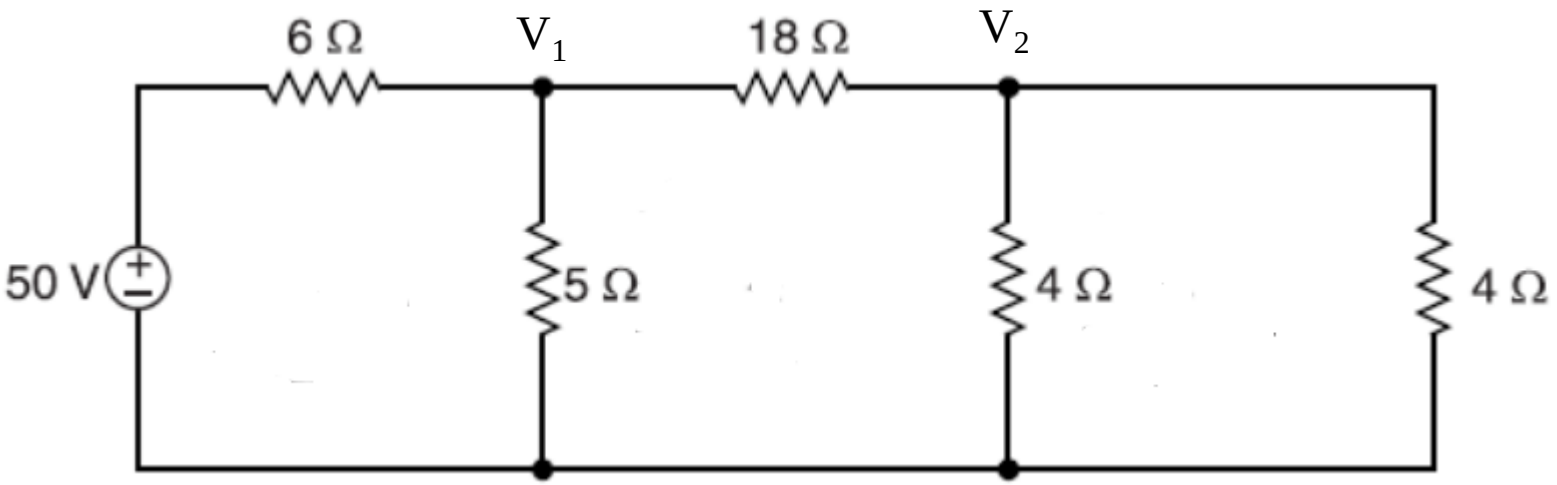
Practice Problems

- Find the node voltages at Nodes A, B, C



Practice Problems

- Find node voltages V_1, V_2



Practice Problems

- Find the node voltages V_1 and V_2