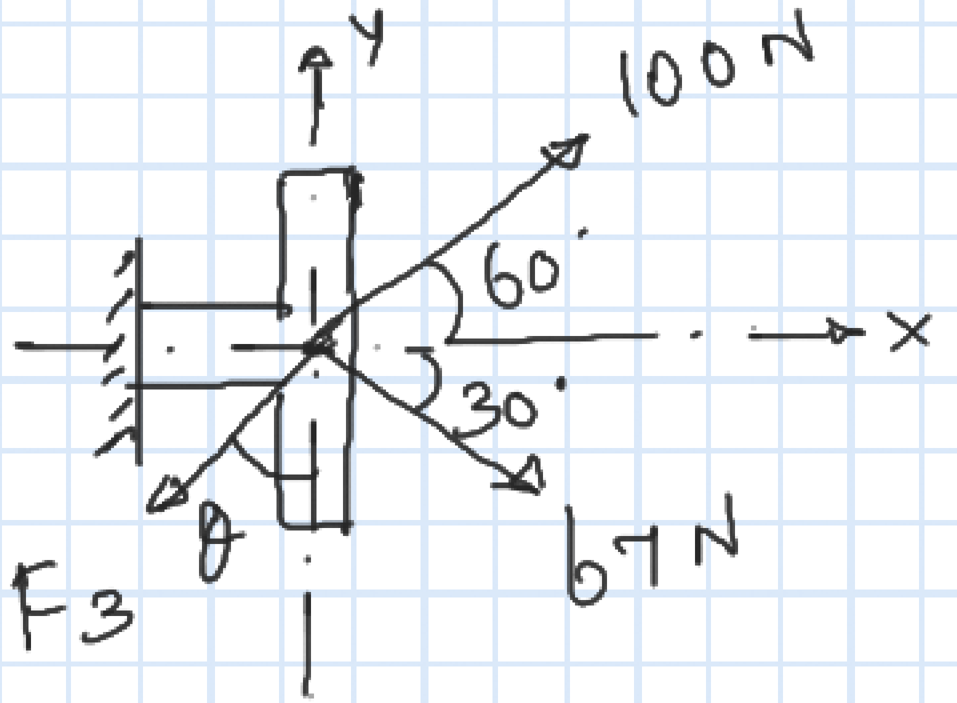
4. If the resultant of three concurrent forces acting on the bolt as shown is zero, determine the Θ and F3.



$R = 0$, $F_3 = ?$, $\theta = ?$

Sol. Since $R = 0$

$\Sigma F_x = 0$; $\Sigma F_y = 0$

Force (N)	F_x	F_y
1. 100	$100 \cos 60°$	$100 \sin 60°$
2. 67	$67 \cos 30°$	$-67 \sin 30°$
3. F_3	$-F_3 \sin\theta$	$-F_3 \cos\theta$

$\Sigma F_x =$

$\Sigma F_y =$

$\Sigma F_x = 0$

$\Sigma F_y = 0$

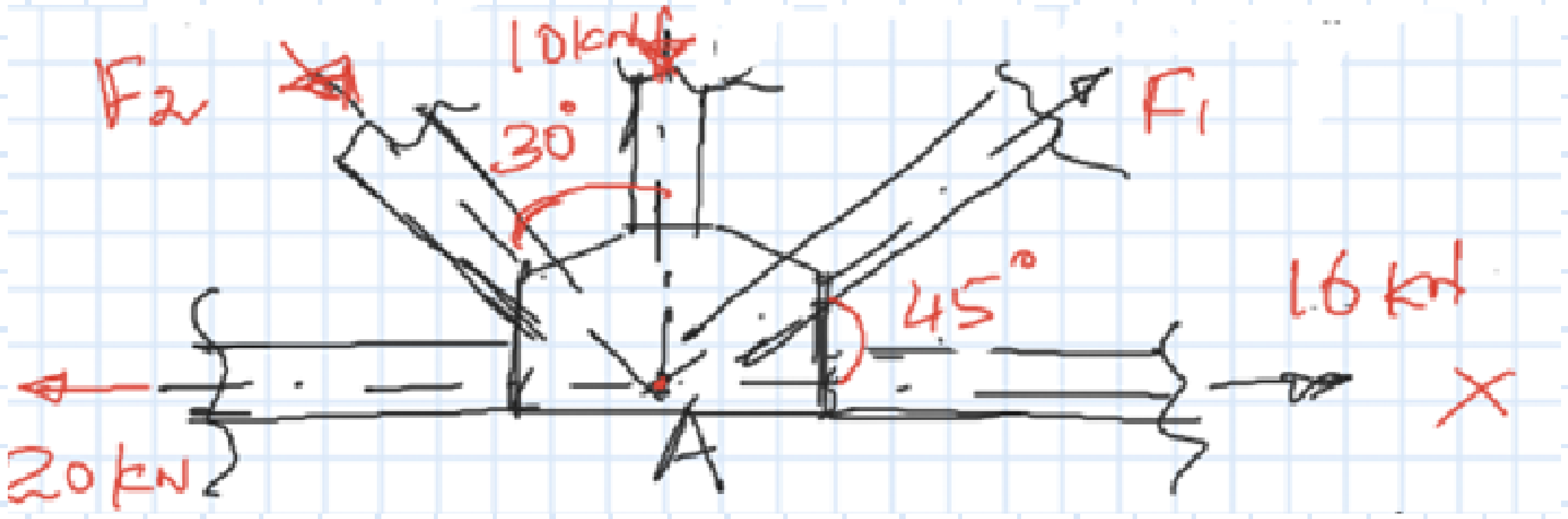
$100\cos 60 + 67\cos 30^\circ - F_3 \sin\theta = 0 \quad - (1)$

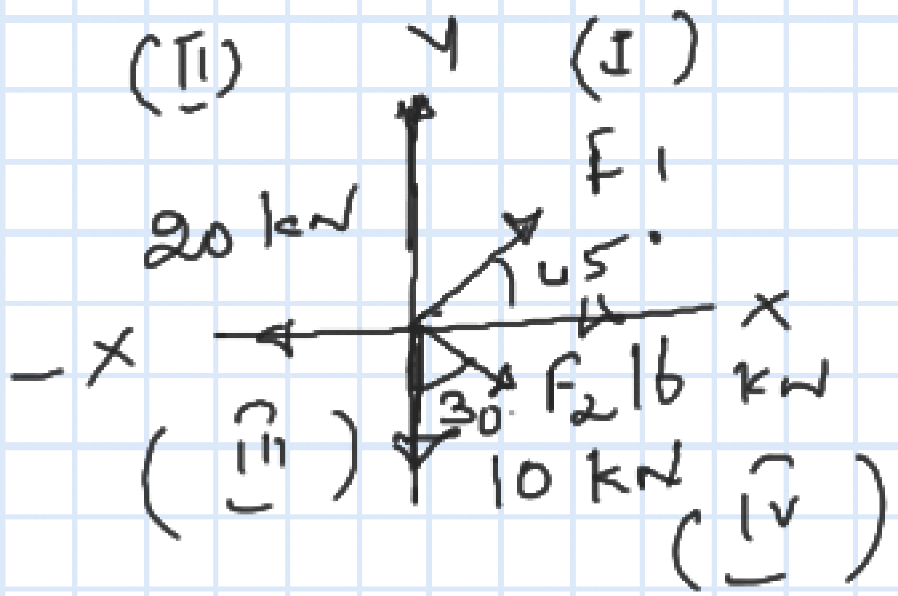
$100\sin 60^\circ - 67\sin 30^\circ - F_3\cos\theta = 0 \quad - (2)$

Solve (1) & (2)

$$\boxed{\begin{aligned} F_3 &= 120.36\ N \\ \theta &= 63.82^\circ \end{aligned}}$$

5. Determine the magnitude of forces F1 and F2 in the bridge truss joint as shown, when point A is in equilibrium.





sol

Forces (kN)	F_x	F_y
1. 16	16	0
2. F_1	$F_1 \cos 45$	$F_1 \sin 45$
3. +10	0	−10
4. $+F_2$	$F_2 \sin 30$	$-F_2 \cos 30°$
5. 20	−20	0
	$\Sigma F_x = ?$	$\Sigma F_y = ?$

Since particle A is in equilibrium $R = 0$

$\therefore \Sigma F_x = 0$ — (1) $\qquad \Sigma F_y = 0$ — (2)

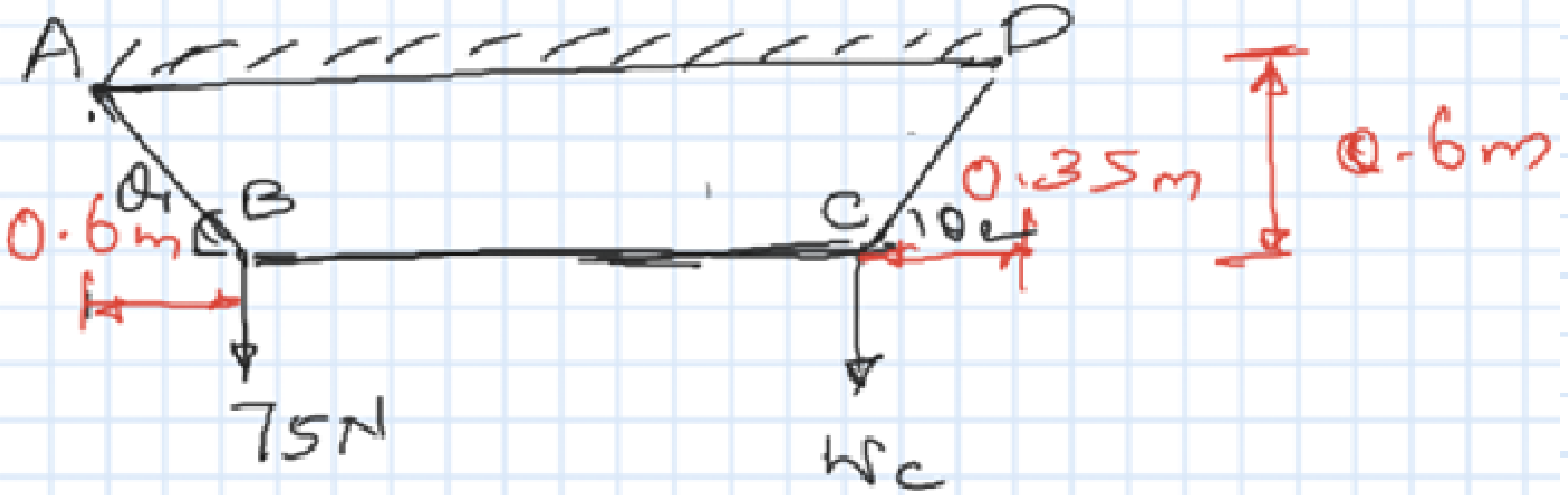
$$F_1 \cos 45 + F_2 \sin 30 = 4 \quad \text{— (1)}$$

$$F_1 \sin 45 - F_2 \cos 30 = 10 \quad \text{— (2)}$$

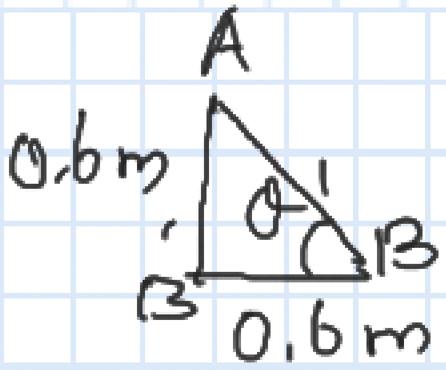
Solve (1) & (2)

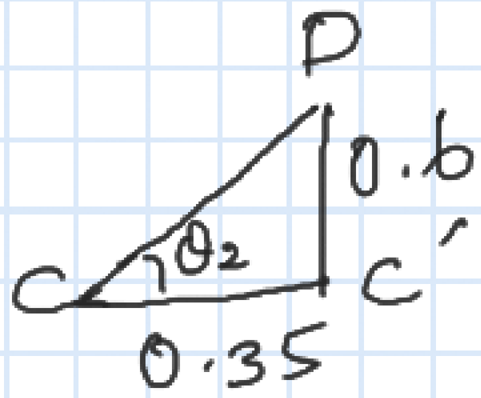
$\boxed{F_1 = 8.76 \text{ kN}, F_2 = -4.4 \text{ kN}}$

6. ABCD is a string hung from a horizontal ceiling at A and D. Weight of 75 N is hung from point B. Determine the magnitude of weight that should be hung from point C such that portion BC of the string is horizontal. Also calculate the tension in various portions AB, BC and CD of the string.



Sol. Angle of inclination of the string





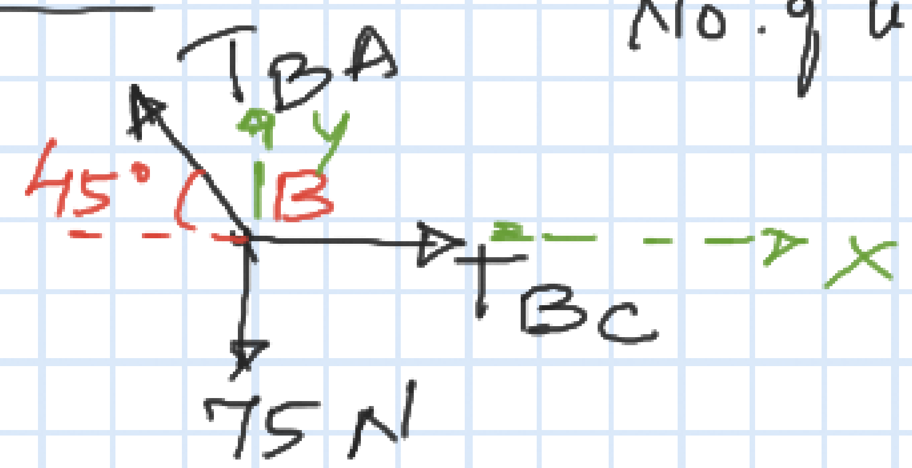
$\tan\theta_1 = 1$; $\theta_1 = 45°$

$\tan\theta_2 = \dfrac{0.6}{0.35} =$; $\theta_2 = 60°$

FBD For point B and point C

Point B

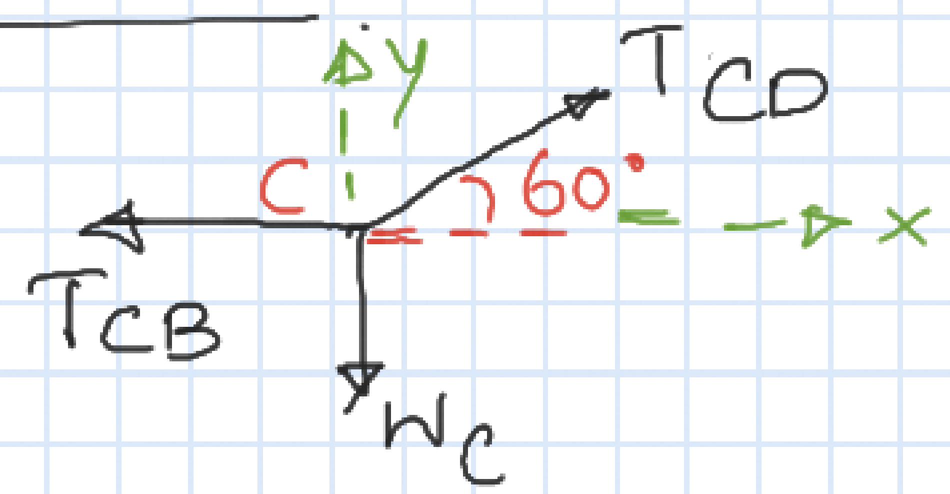
No. of unknown = 2



$\Sigma F_x = 0 \; ; \; \Sigma F_y = 0$

Point C

No. of unknown = 3



$T_{CB} = T_{BC} = 75\,N$

Point B

$T_{BC} - T_{BA}\cos 45° = 0 \quad - (1)$

$T_{BA}\sin 45° - 75 = 0 \quad - (2)$

From (2)

Sub in (1)

$$\boxed{\begin{aligned} T_{BA} &= 106\ N \\ T_{BC} &= 75\,N \end{aligned}}$$

Point C

$T_{CD}\cos 60° - \underset{=75N}{T_{CB}} = 0 \quad - (3)$

$T_{CD}\sin 60° - W_C = 0 \quad - (4)$

from (3)

$T_{CD} = 75/\cos 60° = 148.8\ N$

sub in equ (4)

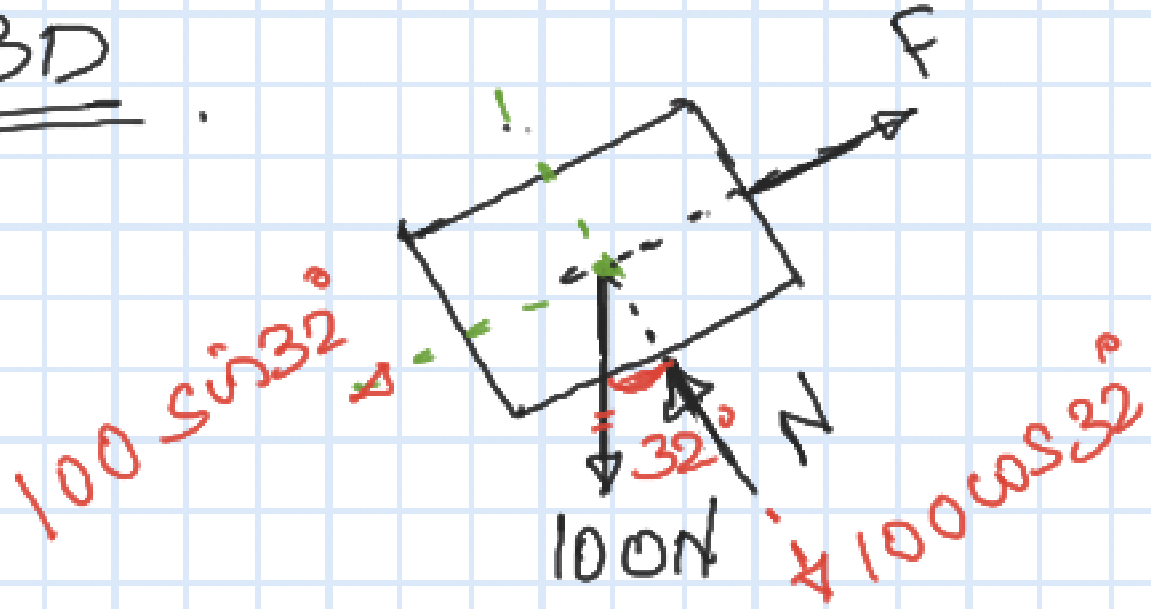
$W_C = T_{CD}\sin 60° = 128.5\,N$

$$\boxed{\begin{aligned} T_{CD} &= 148.8\ N \\ W_C &= 128.5\,N \end{aligned}}$$

7. The block of weight 100 N rest on a smooth inclined plane. The plane makes an angle of 32° with horizontal and force F is applied parallel to the plane. Find the value of F and N.

Sol

FBD



$\Sigma F_{\parallel}$ to the plane $= 0$ — ①

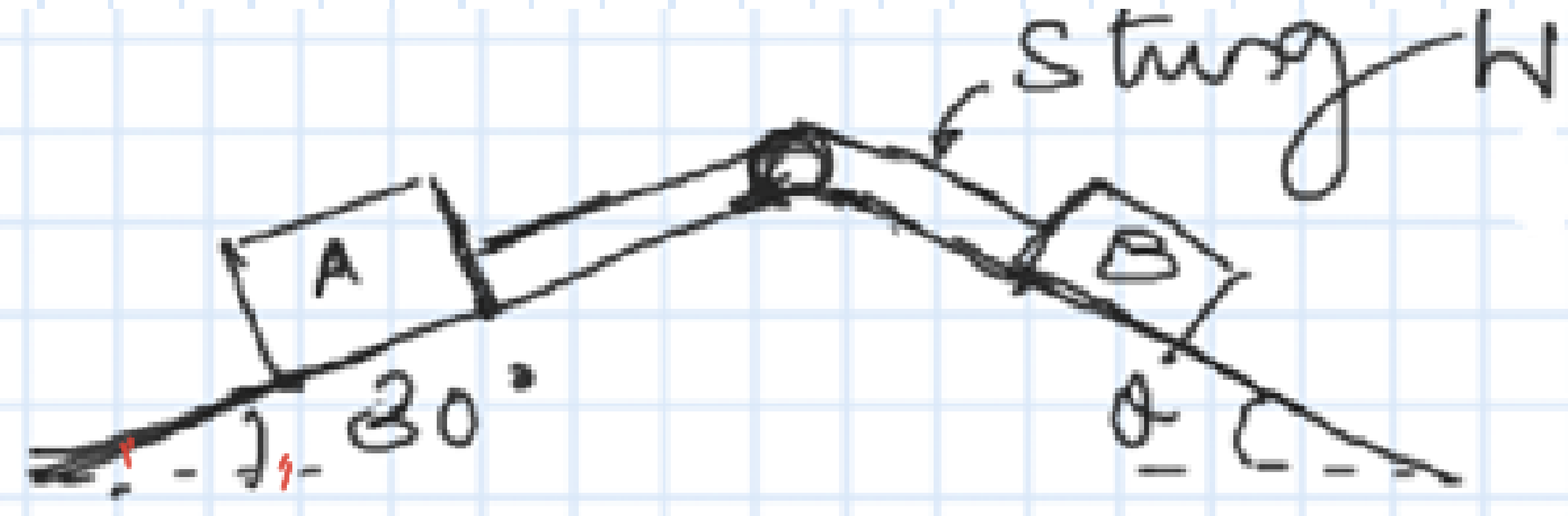
$\Sigma F_{\perp}$ to the plane $= 0$ — ②

$F - 100\sin 32° = 0$ — ①

$N - 100\cos 32° = 0$ — ②

From ① & ② $\boxed{F = 53\ N\ ;\quad N = 84.8\ N}$

8. Two blocks are connected by a string which passes over a smooth pulley as shown. To maintain equilibrium of the blocks, evaluate tension in the string and inclination of the plane.

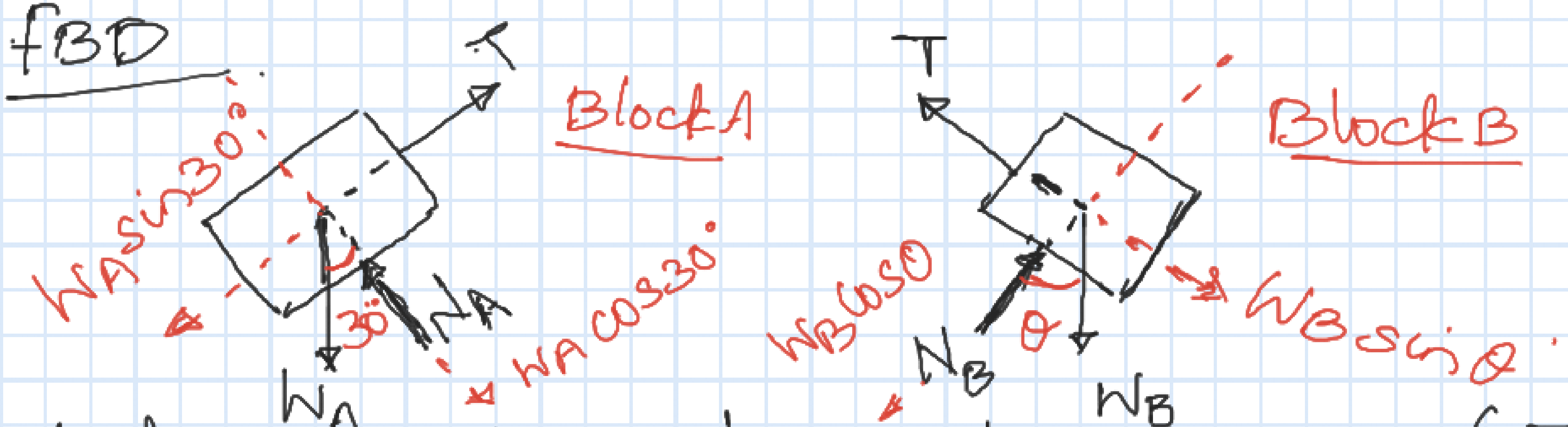


$W_A = 40$ kN

$W_B = 30$ kN

Sol

FBD



Start from Block A. No. of unknown = 2 (T, N_A)

$\Sigma F_{\parallel}$ to plane $= 0$; $\Sigma F_{\perp}$ to plane $= 0$

$$T - W_A \sin 30^\circ = 0 \quad - (1)$$

$$N_A - W_A \cos 30^\circ = 0 \quad - (2)$$

$$\boxed{\begin{aligned} T &= 40 \sin 30^\circ = 20 \text{ kN} \\ N_A &= 40 \cos 30^\circ = 34.6 \text{ kN} \end{aligned}}$$

20 kN

Block B No. of unknown = 2 (T, θ, N_B)

$$T - W_B \sin\theta = 0 \quad - (3)$$

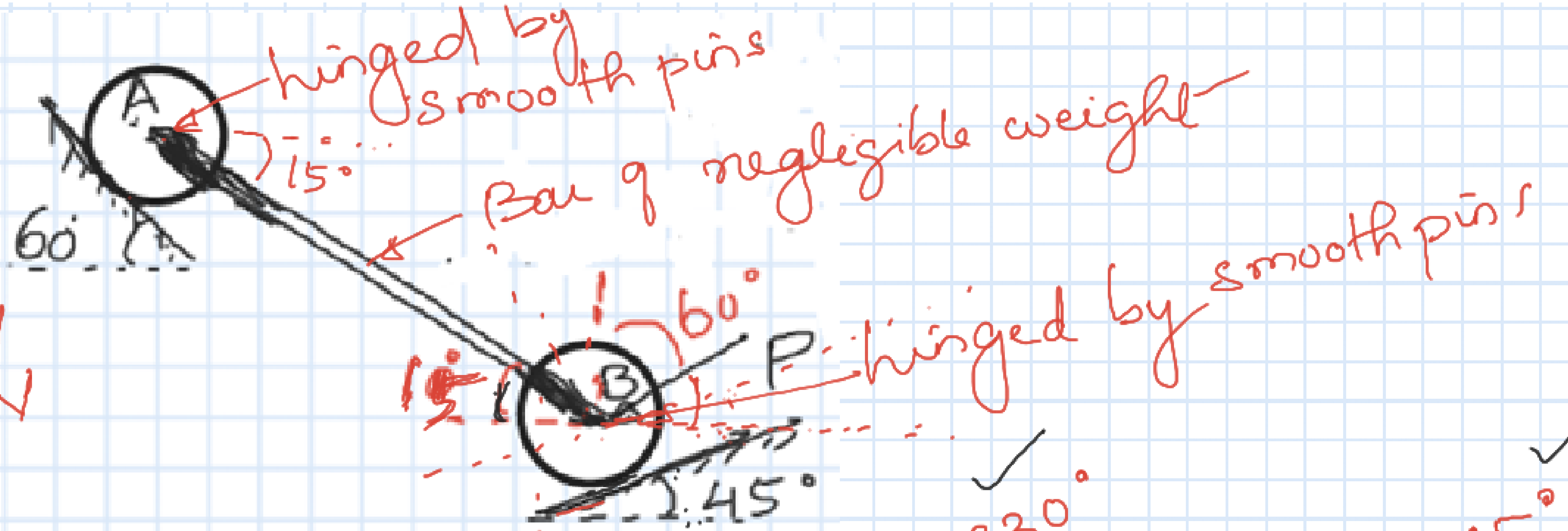
$$N_B - W_B \cos\theta = 0 \quad - (4)$$

$$\sin\theta = T / W_B = \frac{20}{30}$$

$$\boxed{\theta = 41.8^\circ}$$

$$N_B = W_B \cos\theta = 30 \cos(41.8^\circ) = 22.36 \text{ kN}$$

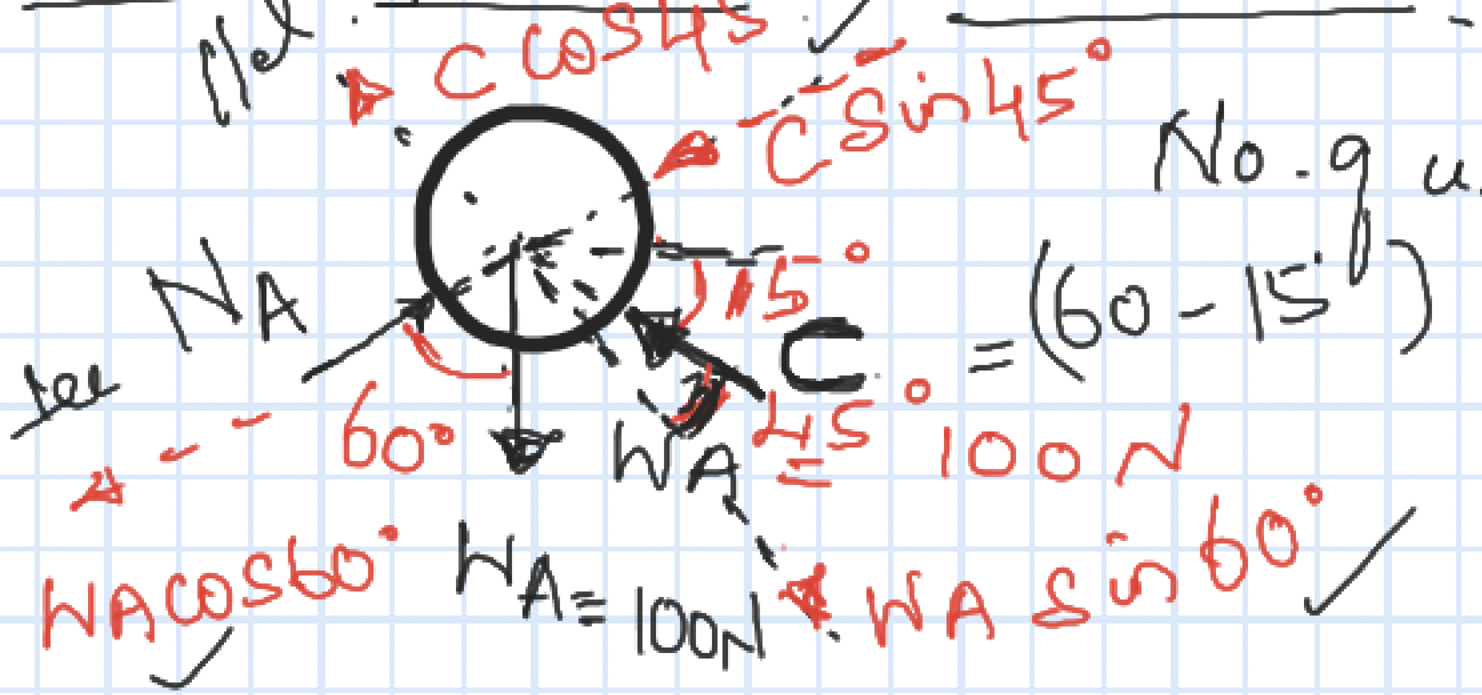
9. To keep the system in equilibrium, evaluate the value of P.



$W_A = 100\ N$

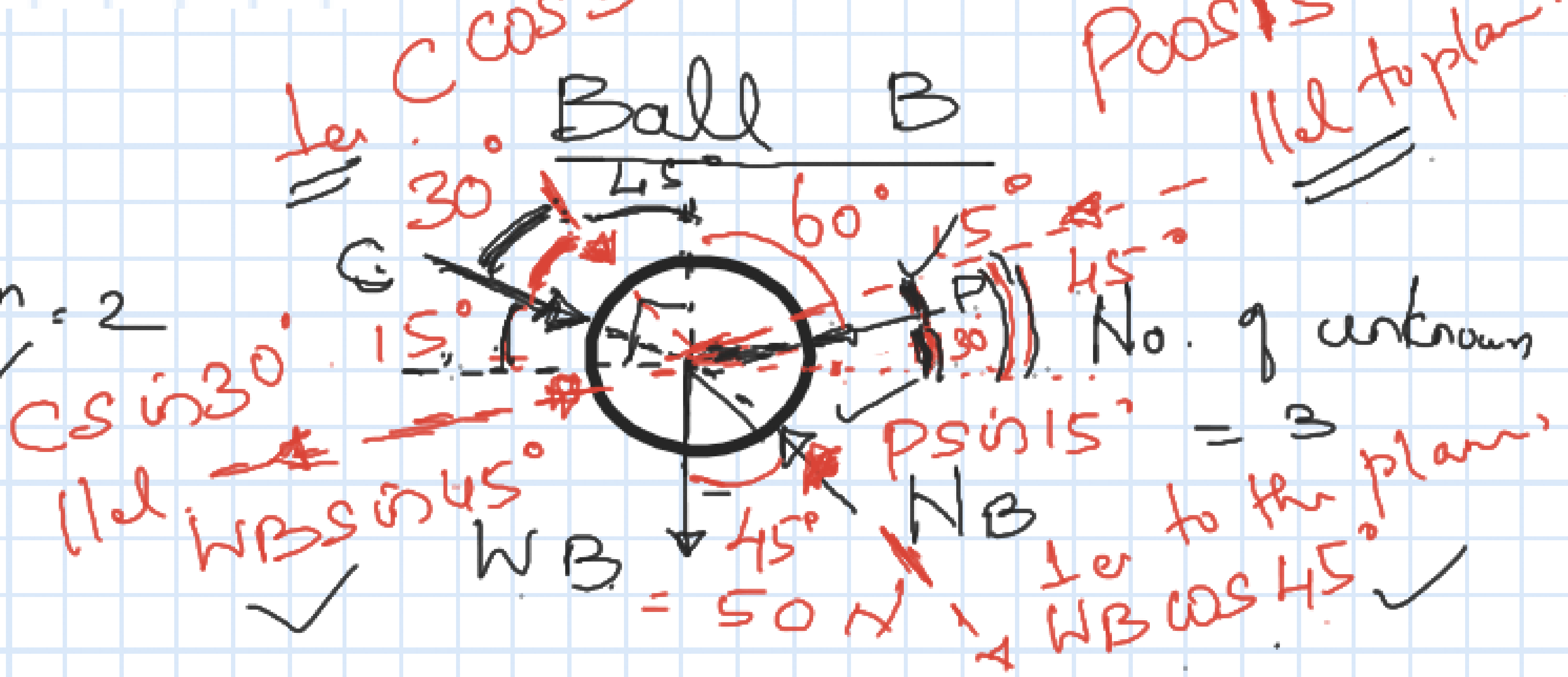
$W_B = 50\ N$

Sol FBD: Ball A



No. of unknown = 2

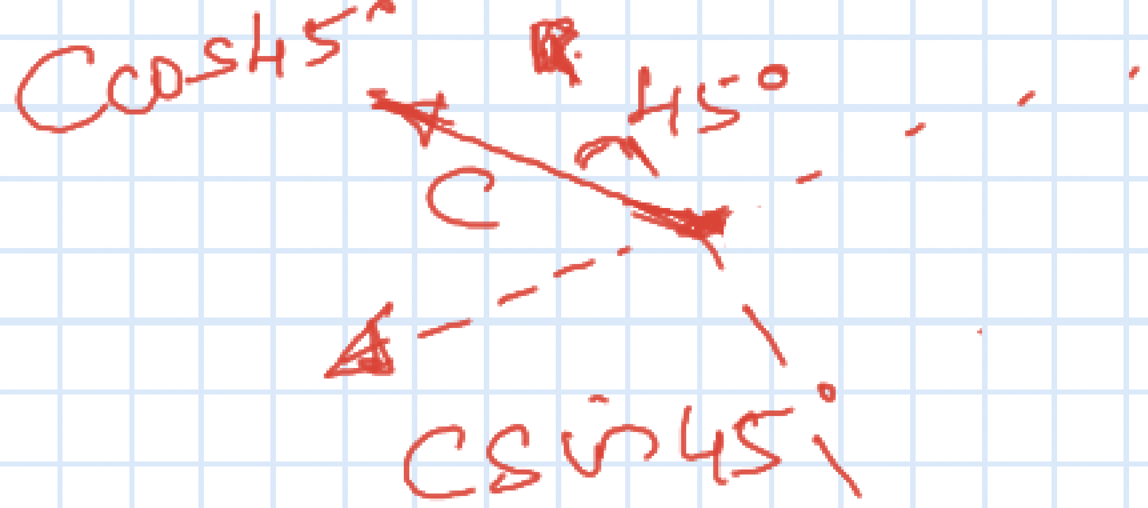
Ball B



No. of unknown = 3

Ball A

$\Sigma F_{\parallel}$ to plane $= 0$ — ①
$\Sigma F_{\perp}$ to plane $= 0$ — ②



$C\cos 45° - 100\sin 60° = 0$ — ①
$N_A - C\sin 45° - 100\cos 60° = 0$ — ②

$C = 100\sin 60° / \cos 45° = 122.5\ N$

$$\boxed{\begin{aligned} N_A &= 136.6\ N \\ C &= 122.5\ N \end{aligned}}$$

Ball B

$\Sigma F_{\parallel}$ to plane $= 0$ — ③
$\Sigma F_{\perp}$ to plane $= 0$ — ④

$$-W_B \sin 45^\circ + C\sin 30 - P\cos 15^\circ = 0 \quad -(3)$$

$$N_B - W_B \cos 45 + P\sin 15^\circ - C\cos 30^\circ = 0 \quad -(4)$$

$$-50\sin 45 + 122.5\sin 30^\circ = P\cos 15 \quad -(3)$$

$$N_B - 50\cos 45 - 122.5\cos 30^\circ = -P\sin 15^\circ \quad -(4)$$

from (3)

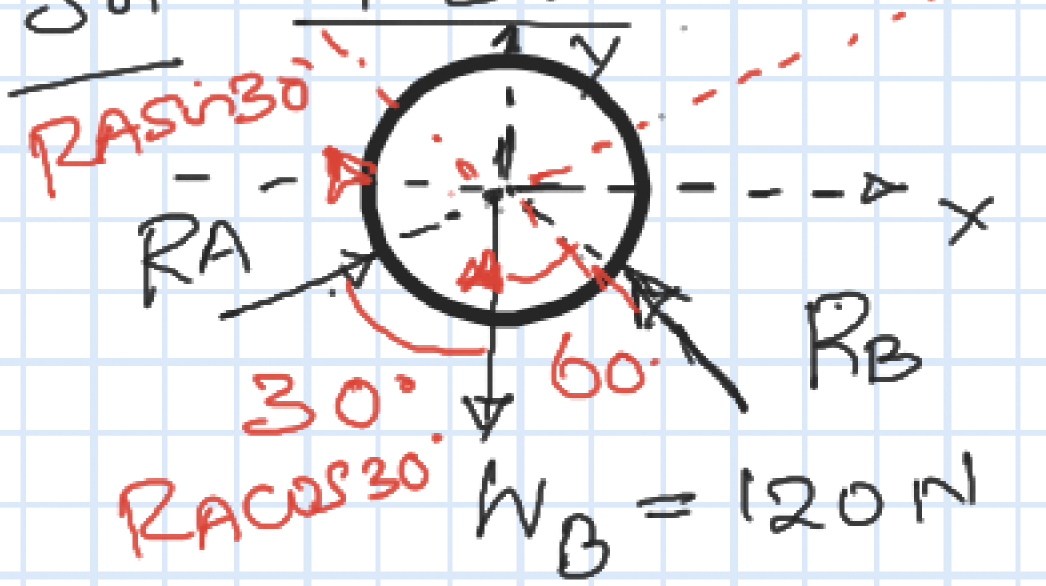
$$\boxed{\begin{aligned} P &= 26.8 \text{ N} \\ N_B &= 134.48 \text{ N} \end{aligned}}$$

10. A ball is resting in a groove. Under equilibrium evaluate the reaction forces.

$W_B = 120$ N

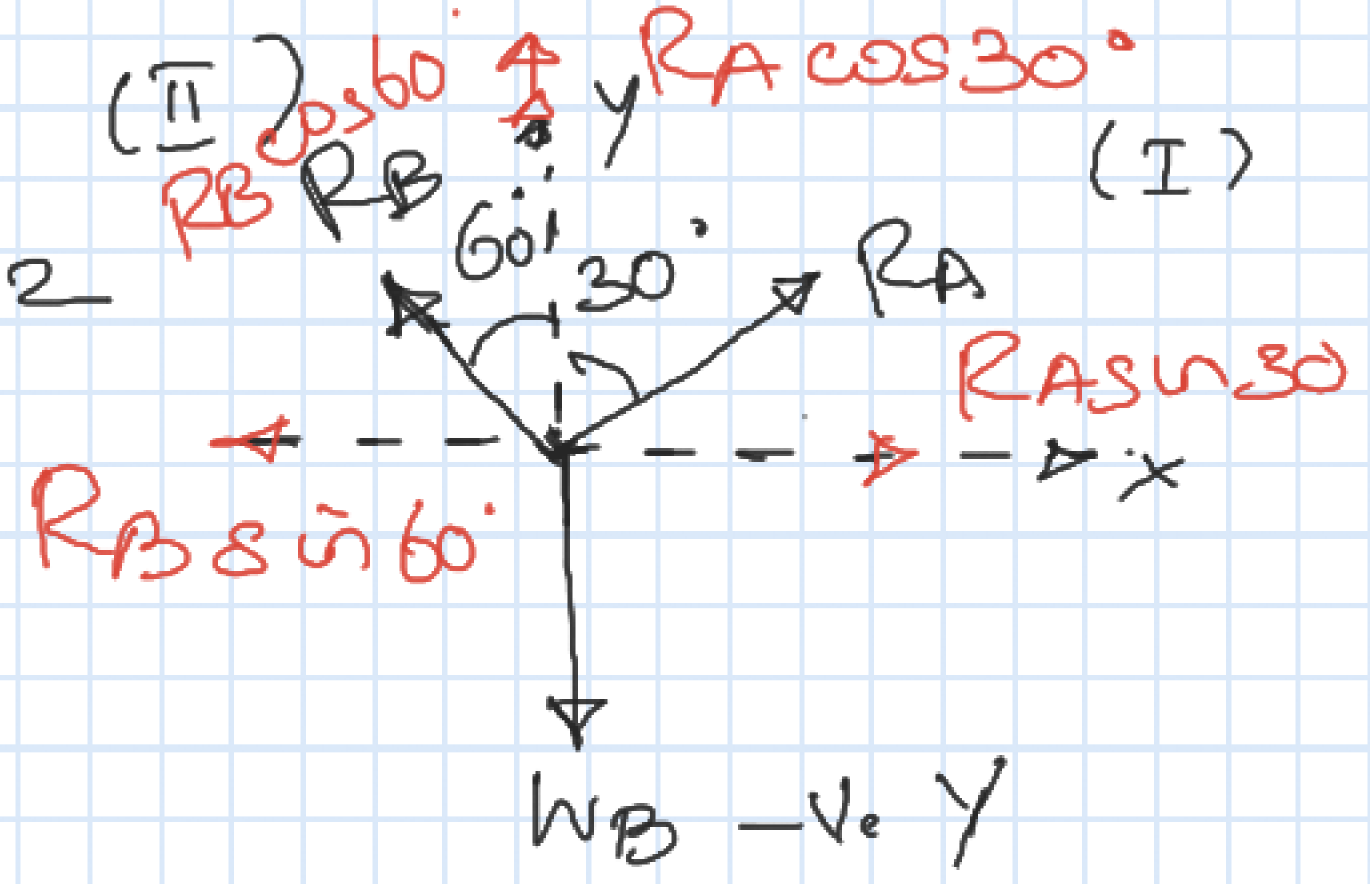
Sol

FBD



No. of unknown = 2

R_A, R_B



(use principle of transmissibility)

$\Sigma F_x = 0$ — (1)

$\Sigma F_y = 0$ — (2)

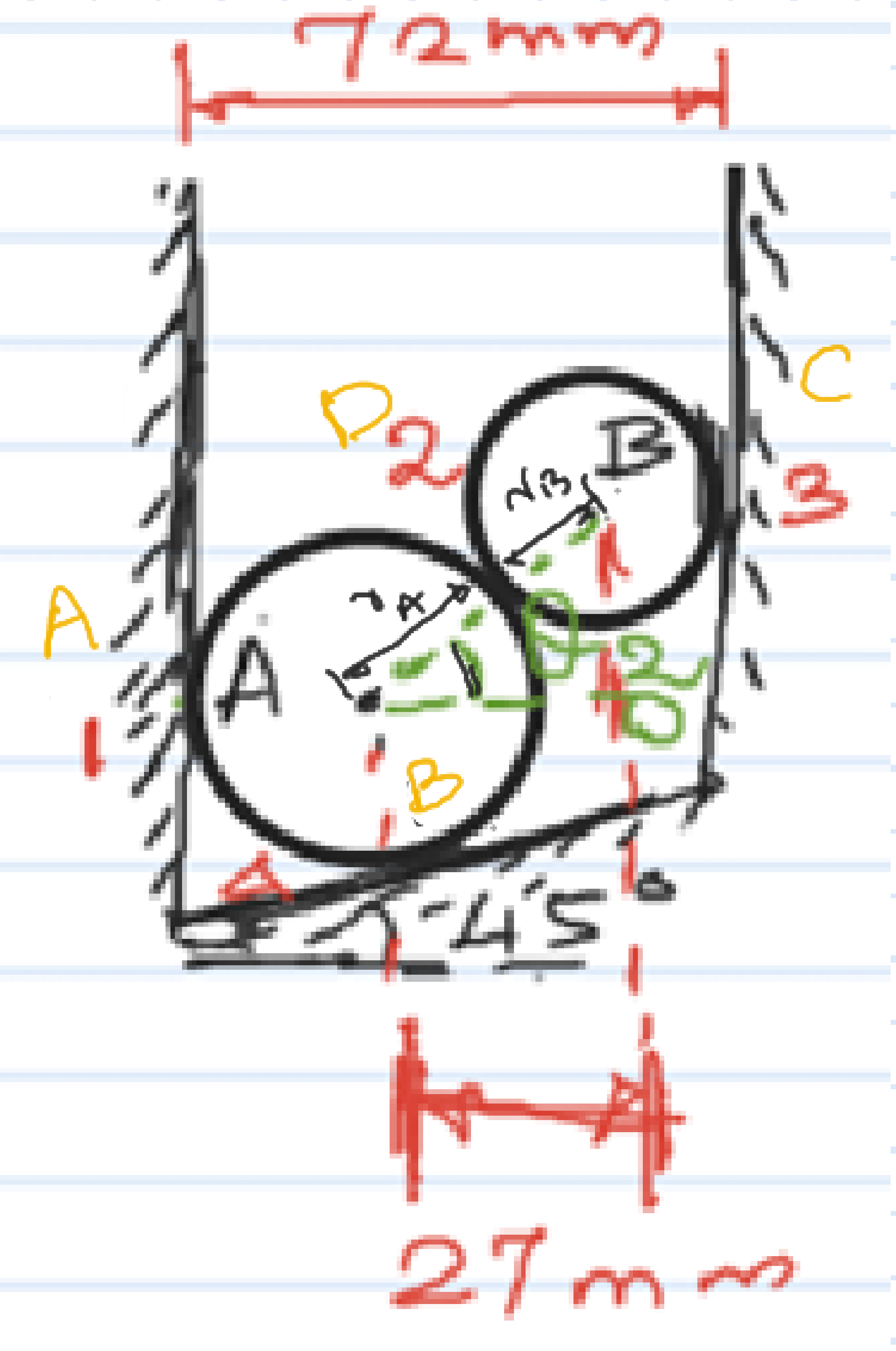
$R_A \sin 30° - R_B \sin 60° = 0$ — (1)

$R_A \cos 30° + R_B \cos 60° - W_B = 0$ — (2)

Solve (1) & (2)

$$\boxed{\begin{aligned} R_A &= 103.93\ N \\ R_B &= 60\ N \end{aligned}}$$

11. Two cylinders of diameter 60mm and 30 mm weighing 160 N and 40 N respectively are placed as shown. Assuming all contact surfaces as smooth, find the reactions at A.B and C.



$W_A = 160\ N$

$W_B = 40\ N$

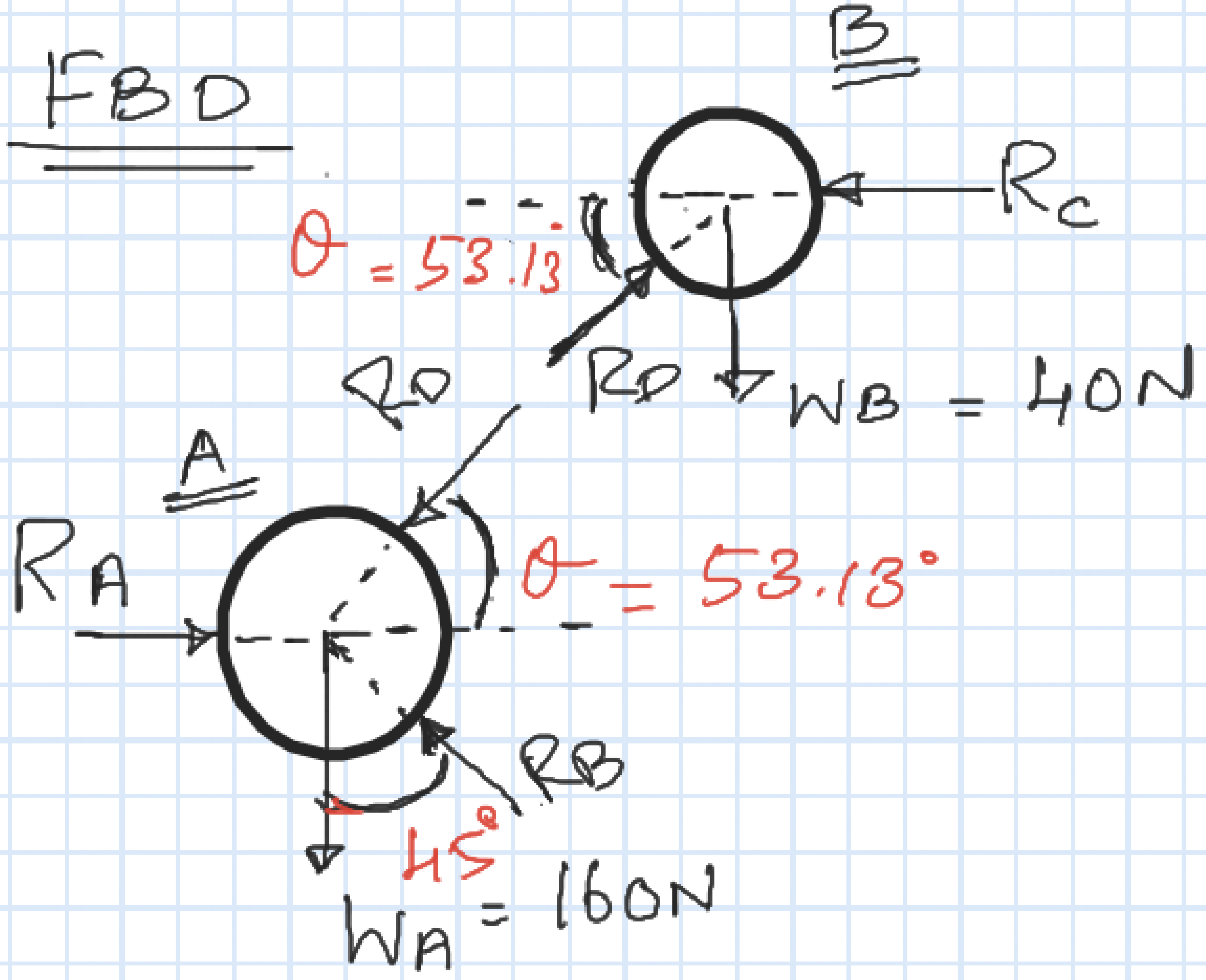
$\phi_A = 60mm$ $\quad r_A = 30mm$

$\phi_B = 30mm$ $\quad r_B = 15mm$

$R_A = ?$ $\quad R_B = ?$ $\quad R_C = ?$ $\quad R_D = ?$

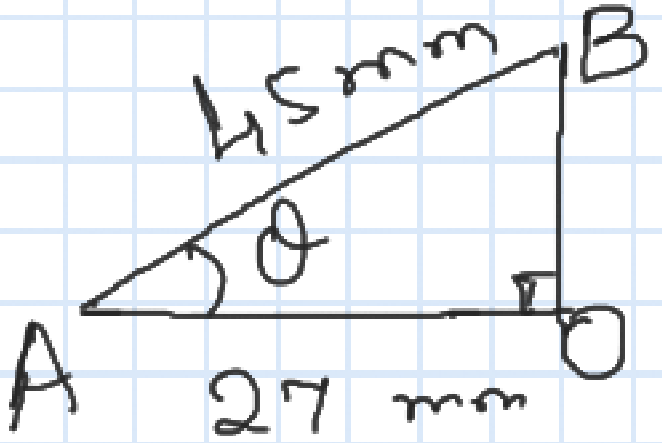
Sol

FBD



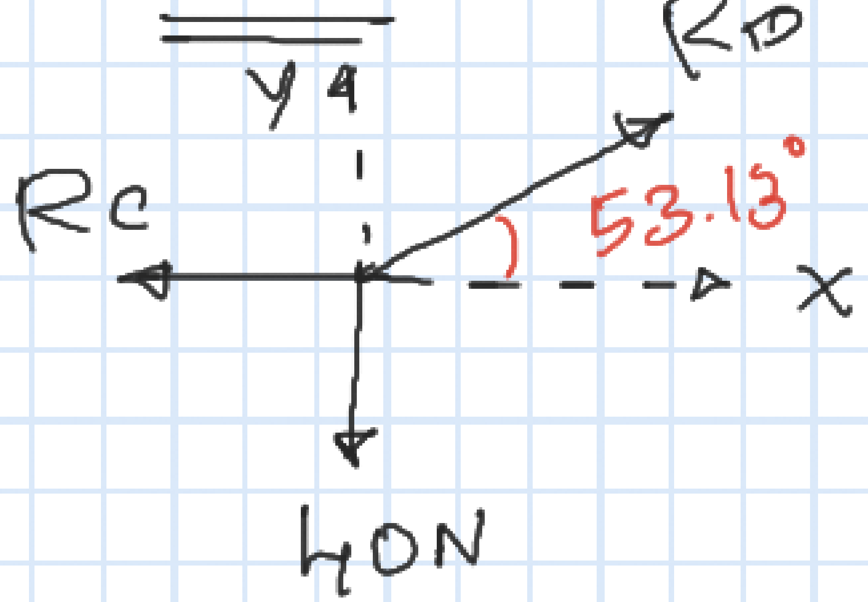
To calculate θ

$\triangle^{le}$ AOB



$$\cos\theta = \frac{27}{45} \qquad \theta = 53.13°$$

Ball B



$\Sigma F_x = 0$ — (1)

$\Sigma F_y = 0$ — (2)

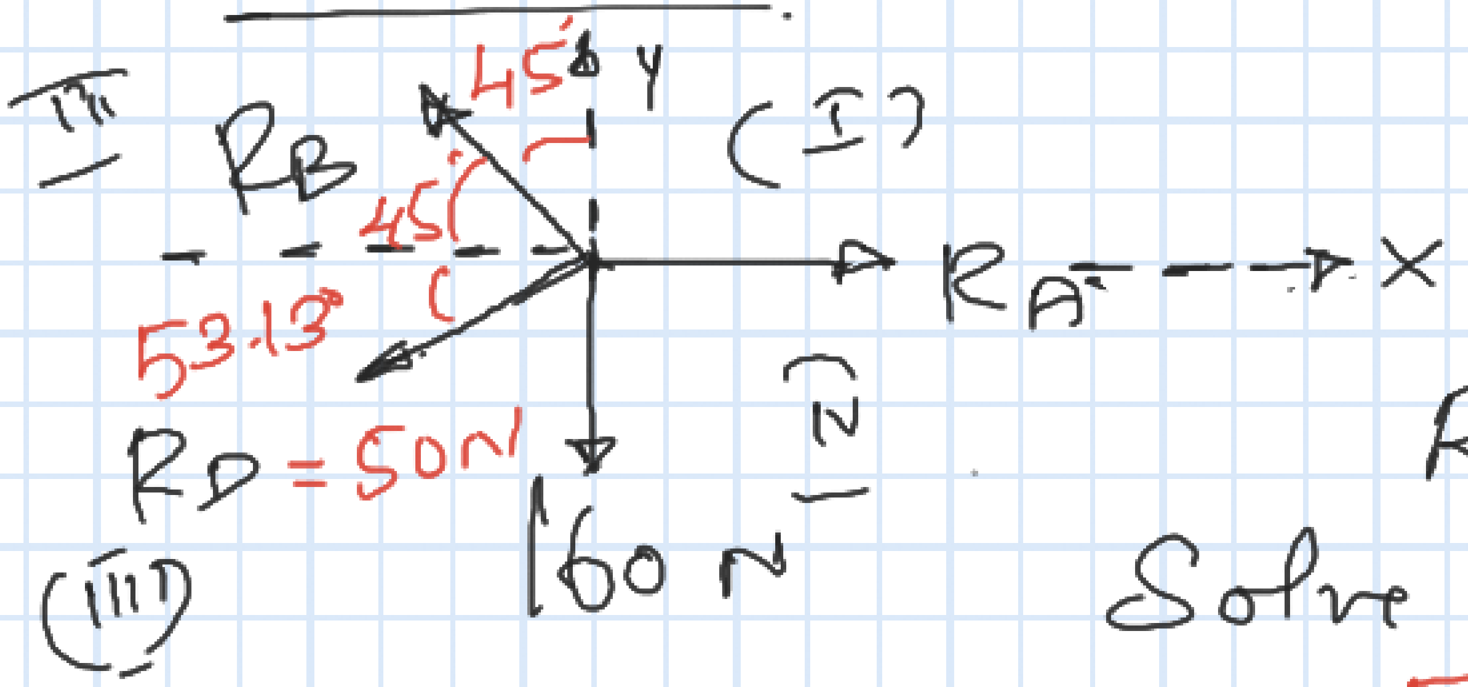
$R_D \cos 53.13° - R_C = 0$ — (1)

$R_D \sin 53.13° - 40 = 0$ — (2)

Solve (1) & (2)

$\boxed{R_C = 30 \text{ N} \quad R_D = 50 \text{ N}}$

Ball A



$\Sigma F_x = 0$ — (3)

$\Sigma F_y = 0$ — (4)

$R_A - R_B \cos 45° - 50 \cos 53.13° = 0$ — (3)

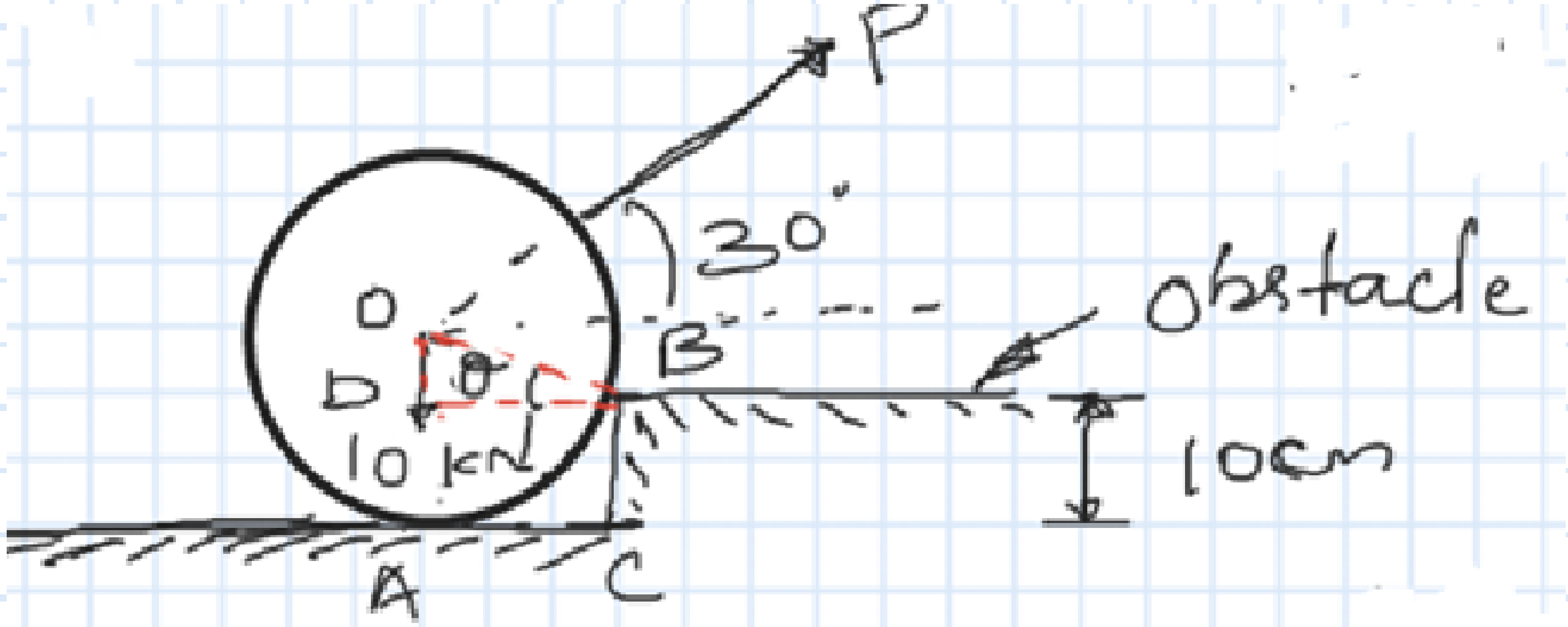
$R_B \cos 45° - 50 \sin 53.13° - 160 = 0$ — (4)

Solve (3) & (4)

$$\boxed{\begin{aligned} R_A &= 230\ N \\ R_B &= 282.8\ N \end{aligned}}$$

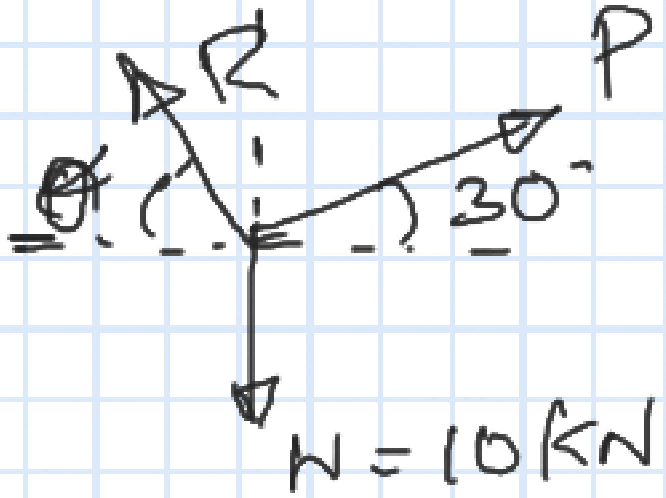
Practice problem

12. Force P is applied to move the roller over the obstacle. To maintain the equilibrium what should be the value of P.

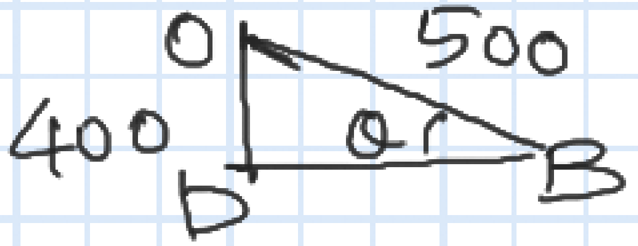


$\phi = 1m$

Sol



△le ODB



OA = Radius of the ball
= 500mm

OD = OA - DA = 500 - 100

OD = 400mm

$$\sin\theta = \frac{400}{500}$$

$$\theta = 53.13^\circ$$

$\Sigma F_x = 0$

$$P\cos 30^\circ - R\cos 53.13^\circ = 0 \quad -(1)$$

$$R\sin 53.13^\circ - 10 + P\sin 30^\circ = 0 \quad -(2)$$

Solve equ (1) & (2)

P = 6.04 kN

Equilibrium of Rigid bodies (2D)

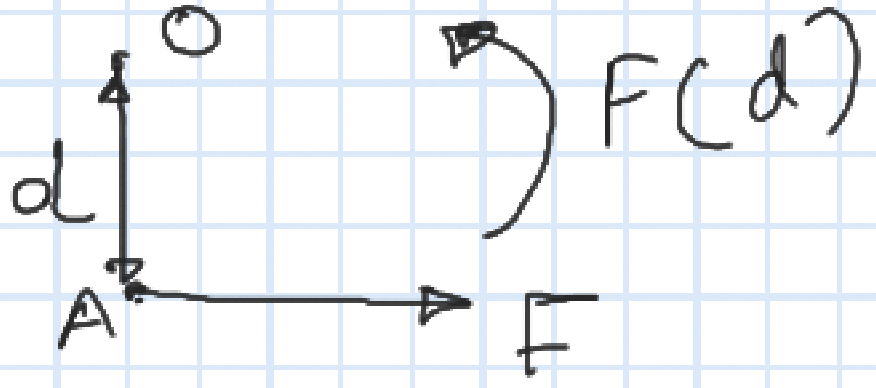
Rigid body - Assumption is that body does not deform

- The deformations are small and do not affect the equilibrium condition
- Forces tend to translate and rotate

Moment of a force

Moment of a force about a point - It is a measure of the tendency of a force to rotate a body about that point. It is a Vector.

$M = (F)(d)$ unit is Nm

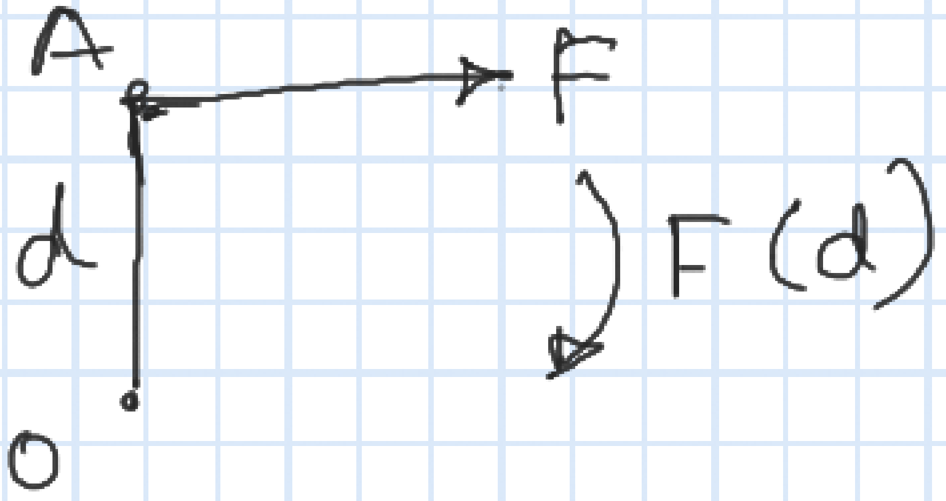


F - Force in N

O - Moment centre

d - Moment arm in m

$M_O = (F)(d)$ -ve



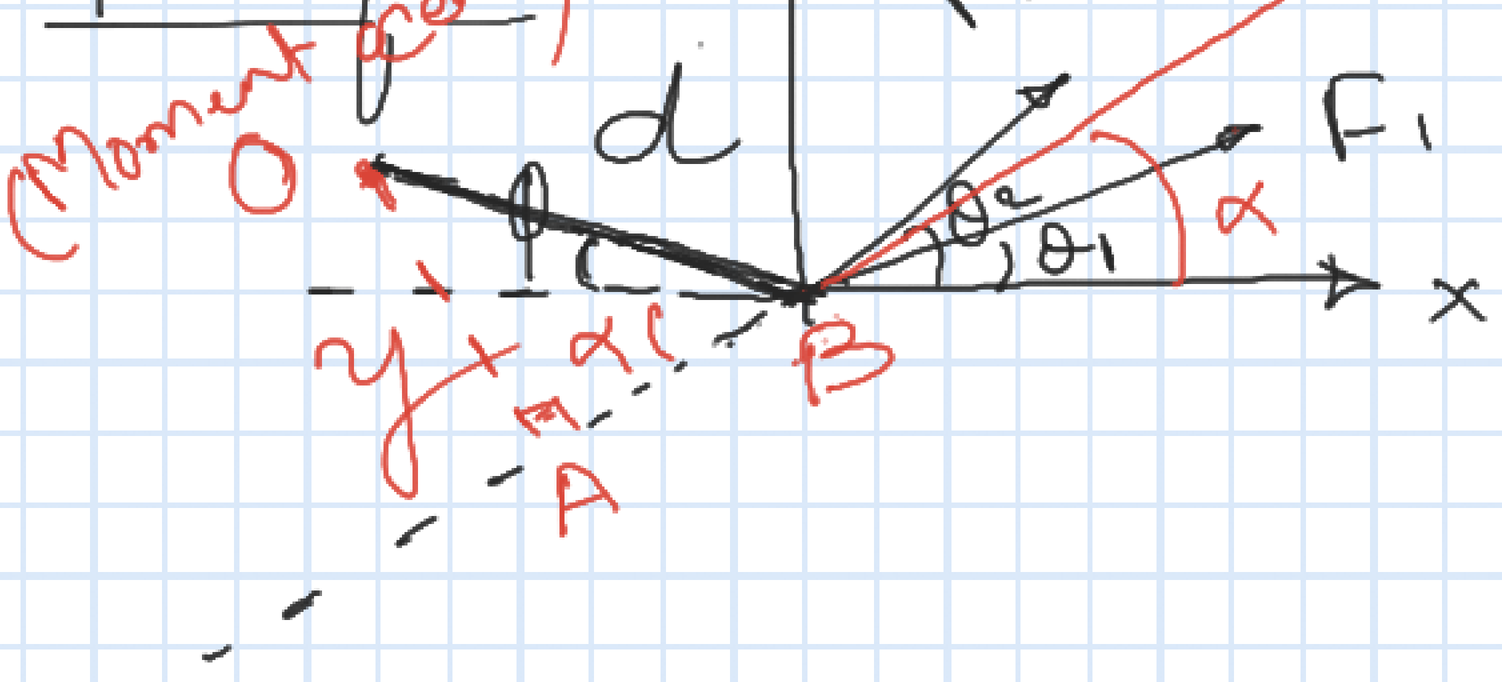
$M_O = (F)(d)$ +ve.

Varignon's Theorem: (By French Mathematician)

used to determine the moment of resultant of several concurrent forces

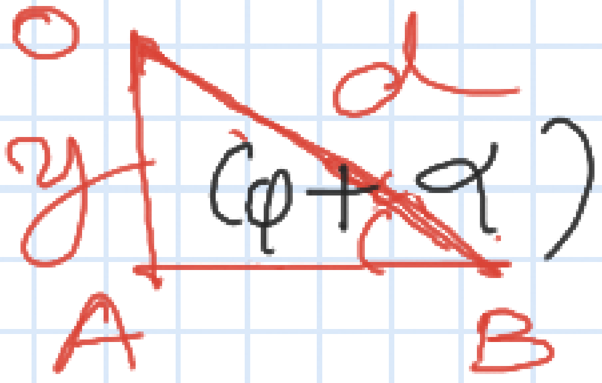
It states ' The moment about a given point O of the resultant of several concurrent forces is equal to sum of moments of the various forces about the same point O."

Proof



$M_O = R(y)$

In Δ^s OAB



$$\sin(\varphi+\alpha) = \frac{y}{d}$$

$$y = d(\sin(\varphi+\alpha))$$

Moment produced by R about 'O' $= d[\sin\varphi \cos\alpha + \cos\varphi \sin\alpha]$

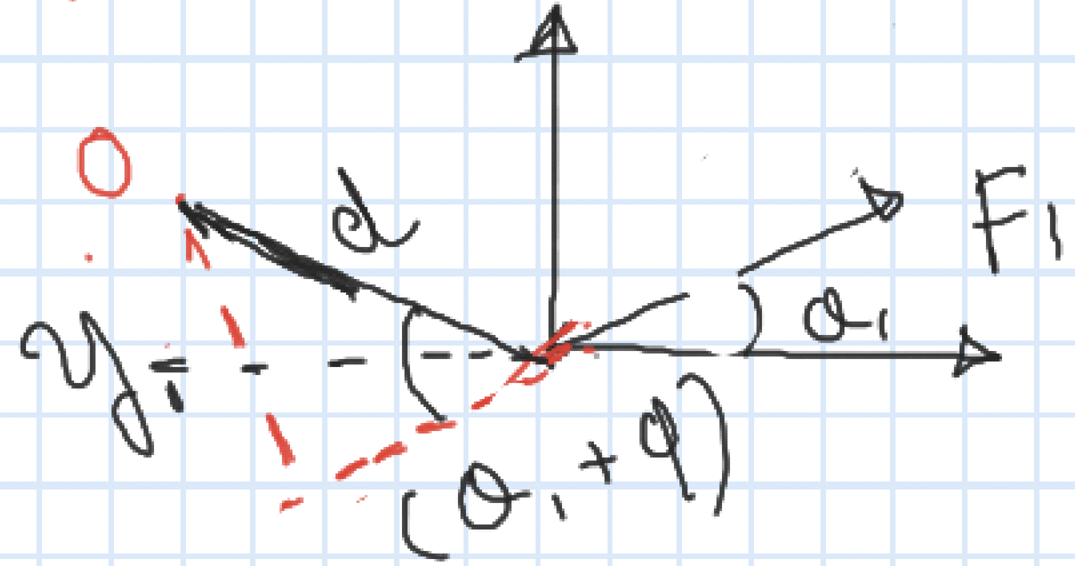
$$M_O = R[d(\sin\varphi \cos\alpha + \cos\varphi \sin\alpha)]$$

$$= d\sin\varphi (R\cos\alpha) + d\cos\varphi (R\sin\alpha) \quad \text{— (1)}$$

Moment produced by F_1 about 'O'

$$= F_1(y_1)$$

$$= F_1 d \sin(\theta_1+\varphi)$$



$$M_{O_1} = F_1 d[\sin\phi \cos\theta_1 + \cos\phi \sin\phi_1]$$

$||^{ly}$. Moment produced by F_2 about 'O'

$$M_{O_2} = F_2 \, d[\sin(\theta_2 + \phi)]$$

$$= F_2 d[\sin\phi \cos\theta_2 + \cos\phi \, \sin\theta_2]$$

$$M_{O_1} + M_{O_2} = F_1 d[\sin\phi \cos\theta_1 + \cos\phi \sin\theta_1] + F_2 d[\sin\phi \cos\theta_2 + \cos\phi \sin\theta_2]$$

$$= d\sin\phi[F_1 \cos\theta_1 + F_2 \cos\theta_2] + d\cos\phi[F_1 \sin\theta_1 + F_2 \sin\theta_2]$$

$$M_{O_1} + M_{O_2} = d\sin\phi[R\cos\alpha] + d\cos\phi[R\sin\alpha] \quad - (2)$$

Proved (1) = (2)