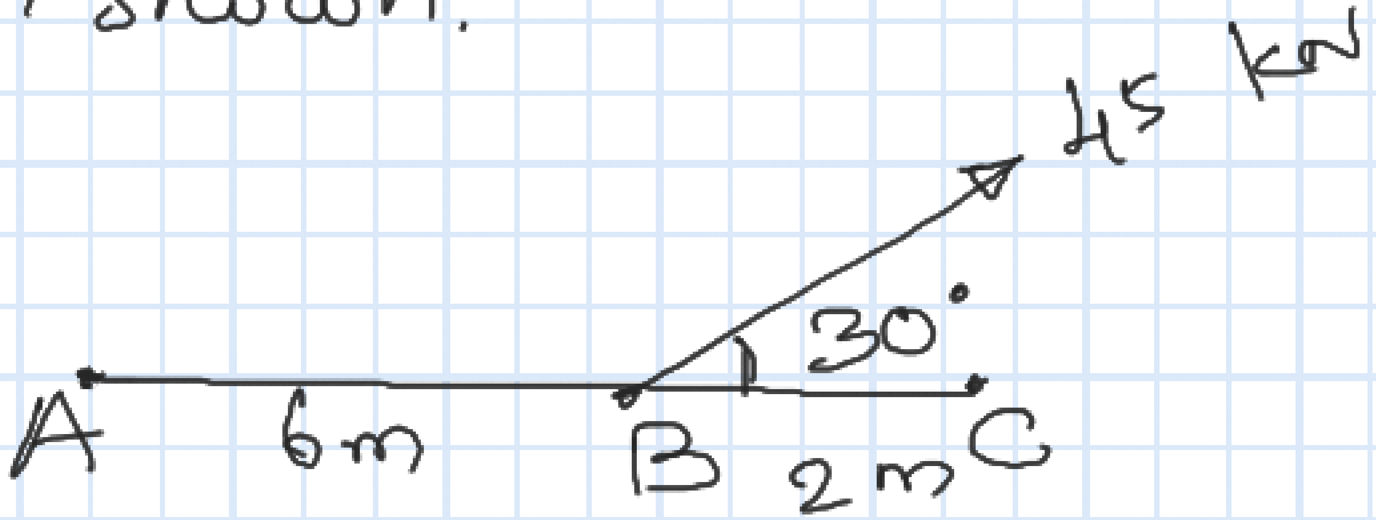
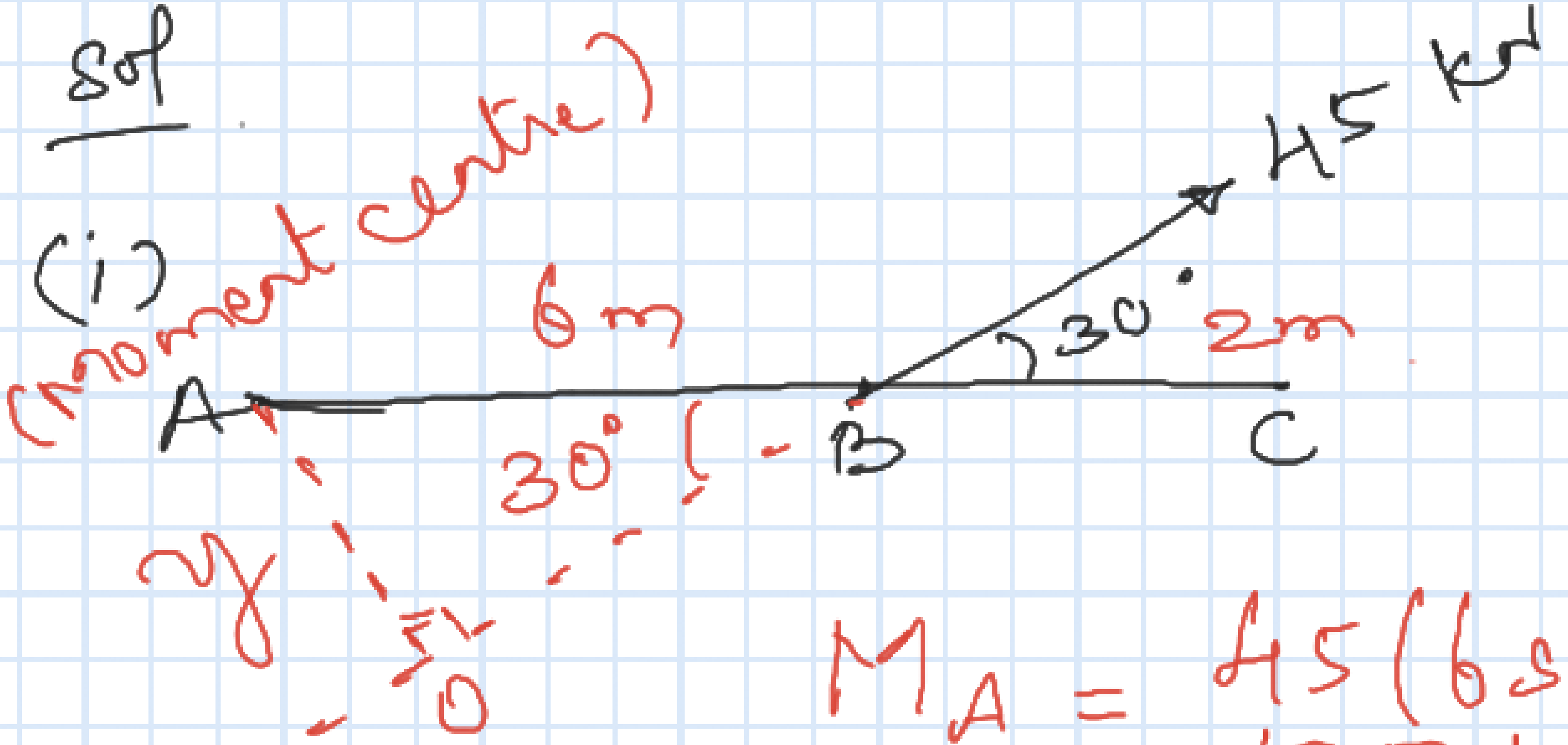
Problems

1. Find the moment about point 'A' of the force 45 kN as shown.

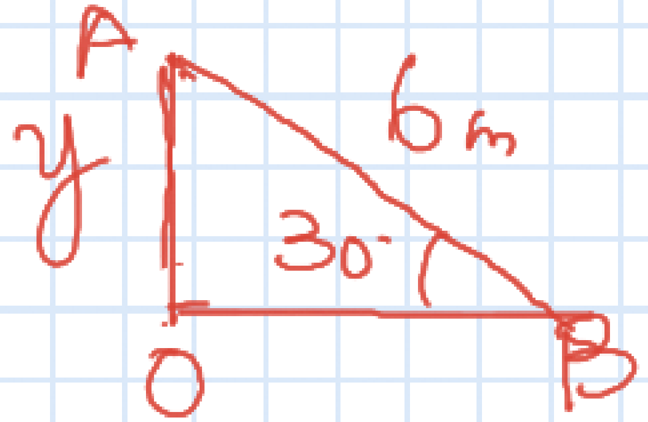


Sol

(i)



In Δ^{le} AOB

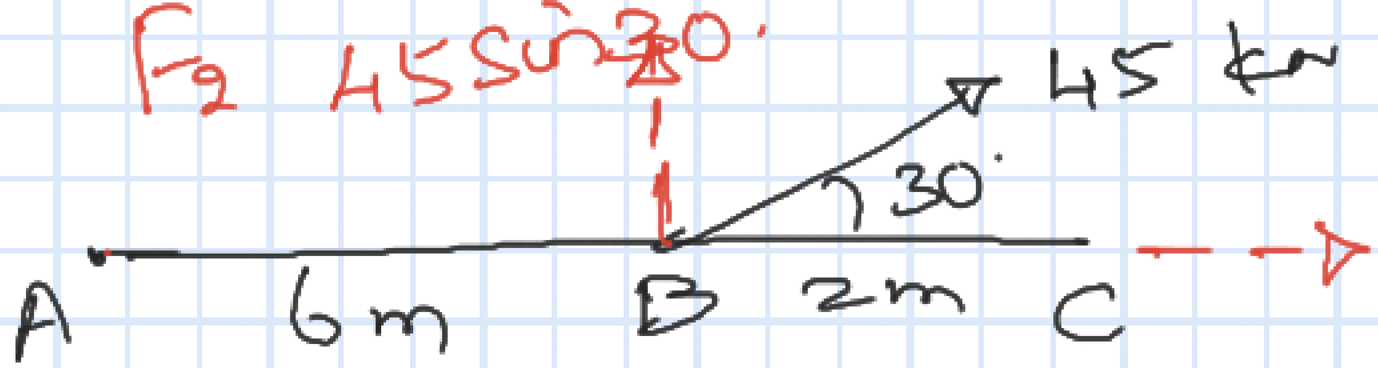


$$\sin 30° = \frac{y}{6}$$

$$y = 6 \sin 30°$$

$$M_A = 45(6 \sin 30°)$$
$$= 135 \text{ kNm} \quad \text{—} \; (1)$$

(ii)



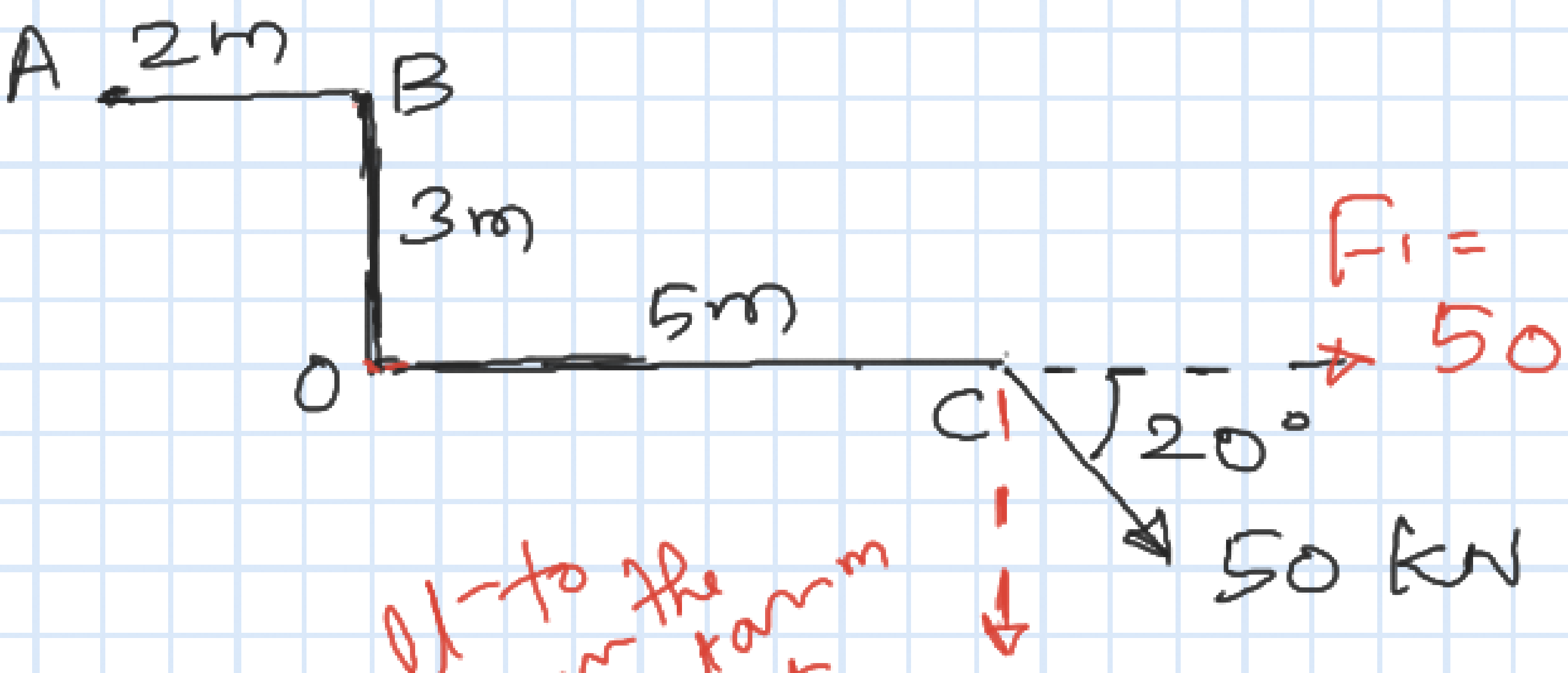
F_1
$45\cos 30°$

Let $R = 45$ kN
$F_x = R\cos\theta$
$F_y = R\sin\theta$

$$M_{A(R)} = M_{A(F_1)} + M_{A(F_2)}$$

$$M_{A(F_1)} = F_1(0) + F_2(6)$$
$$= 45\sin 30°(6)$$
$$= 135 \text{ kNm} \quad - (2)$$

2. Find the moment of the force about point 'A' and point 'O'



$M_A = ?$

$M_O = ?$

$F_1 = 50\cos 20°$

$F_2 = 50\sin 20°$

difficult to measure the momentarm

$\cancel{M_A}_{(50)} = M_{A(F_1)} + M_{A(F_2)}$

$\cancel{M_O}(50) = M_O(F_1) + M_O(F_2)$

$M_A(F_1) = F_1(3) = 50\cos 20°(3) =$

$M_A(F_2) = F_2(7) = 50\sin 20(7) =$

$M_O(F_1) = F_1(0)$

$M_O(F_2) = F_2(5)$

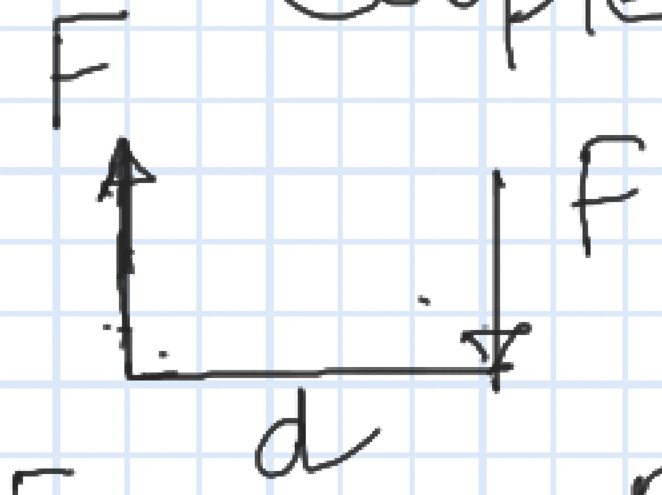
$= 50\sin 20(5)$

$= 85.51$ kNm

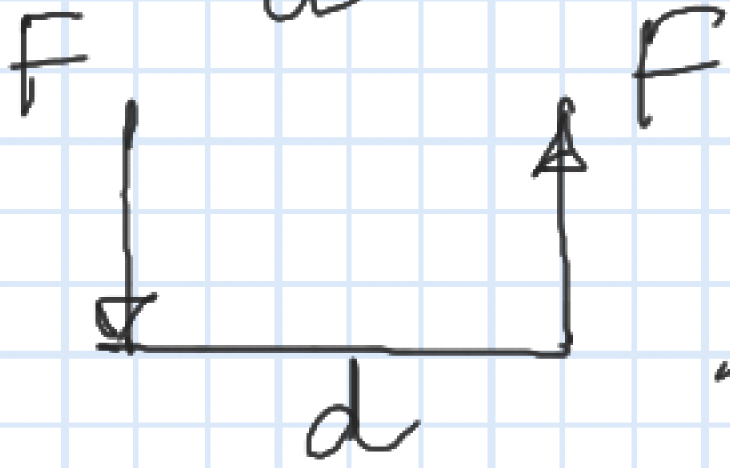
$M_A = -21.25$ kNm

Moment of a couple

Couple - Two forces of equal magnitude, parallel line of action and of opposite sense



moment - Sum of the force is zero



- Forces won't translate but it will rotate the body

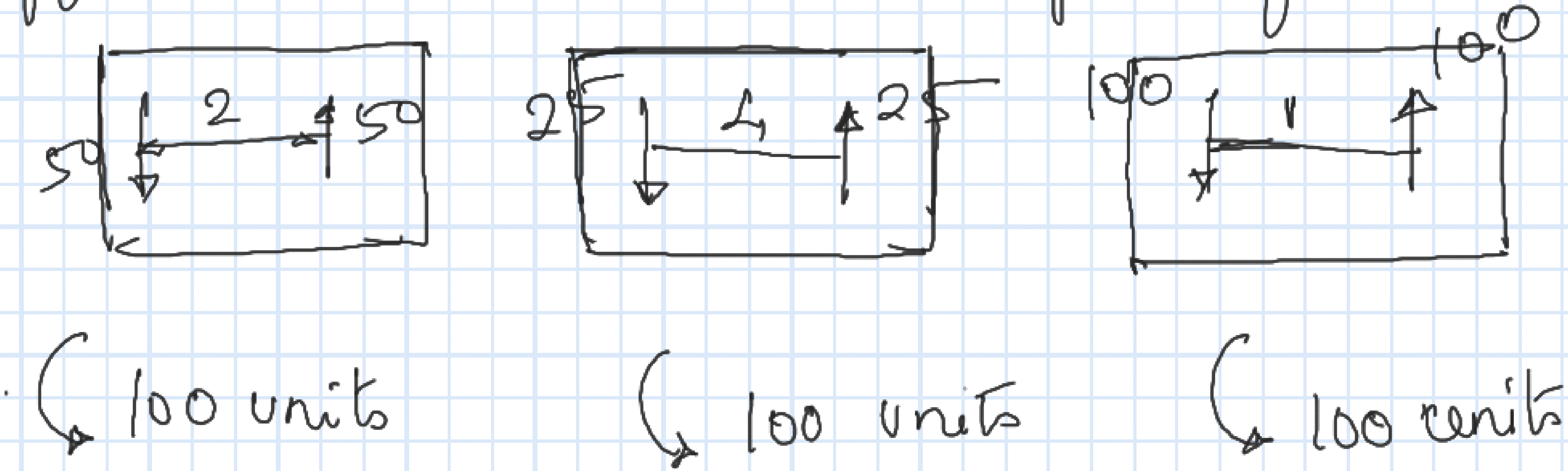
moment

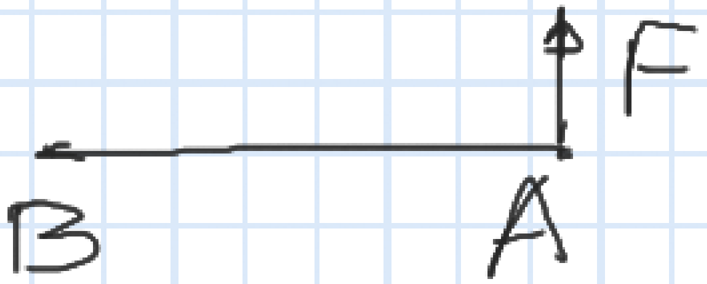
Properties of the couple

1. It can be rotated through any angle

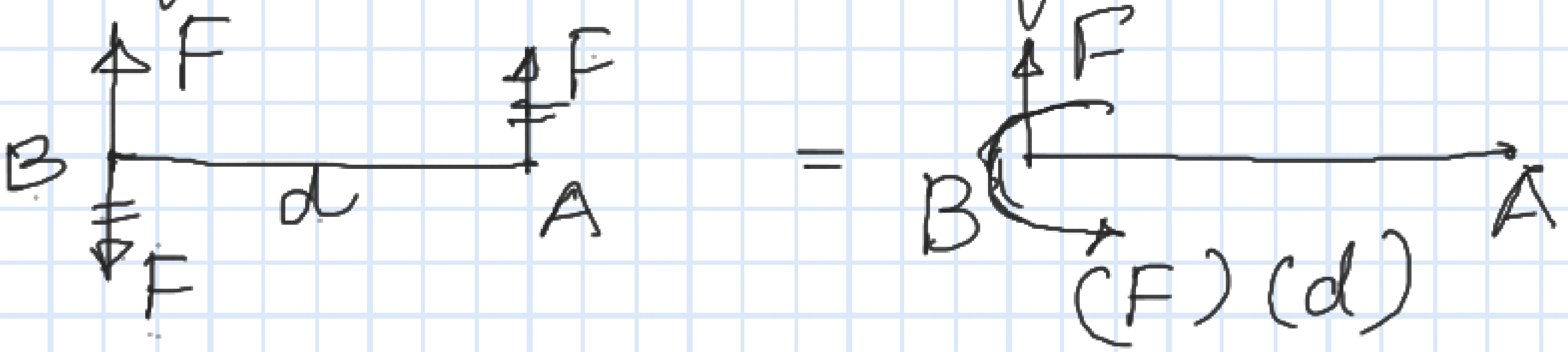
2. The couple can be shifted to any position as a free vector

3. Replaced by another pair of force whose rotational effect is same as the original forces





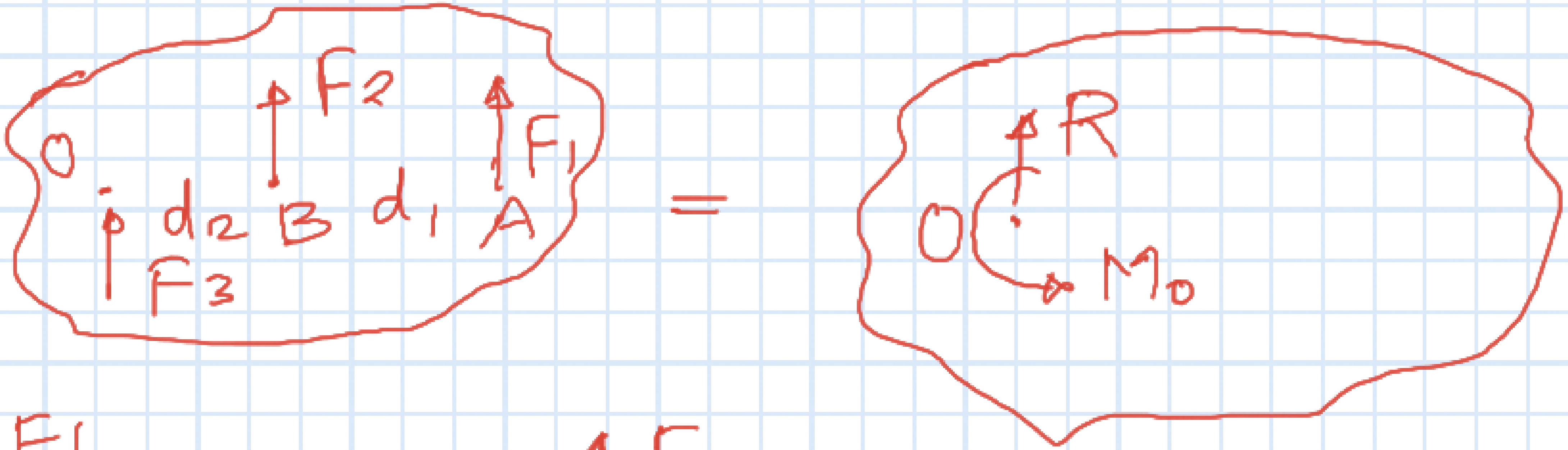
If F is to be shifted to B

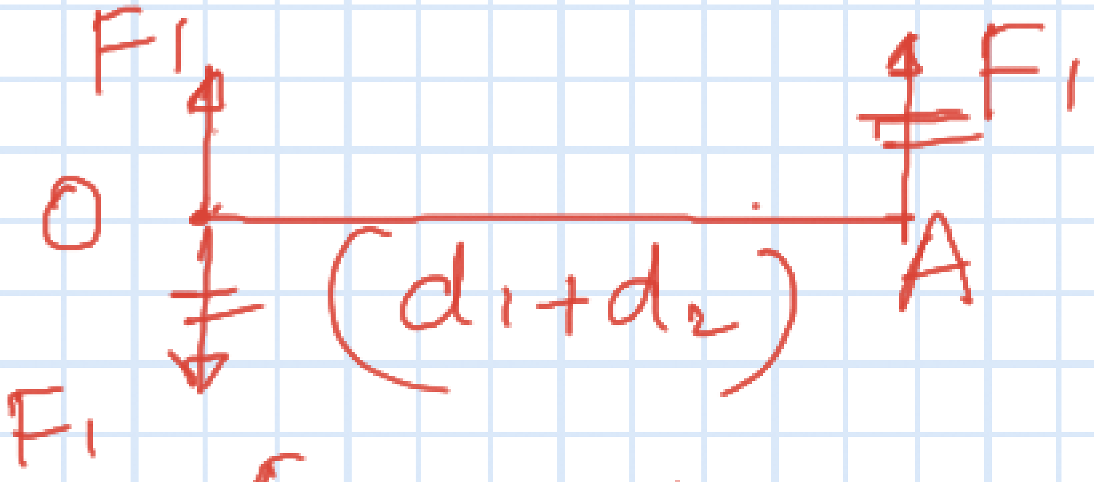


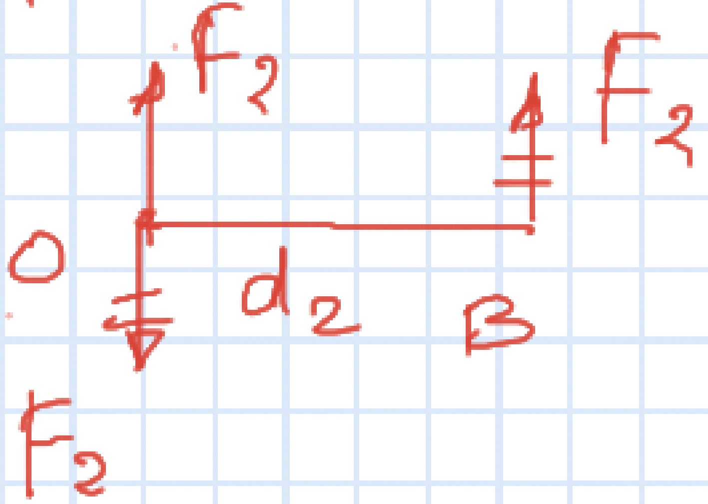
It should be added with a moment produced by a couple

Force system

(i) Coplanar parallel force system





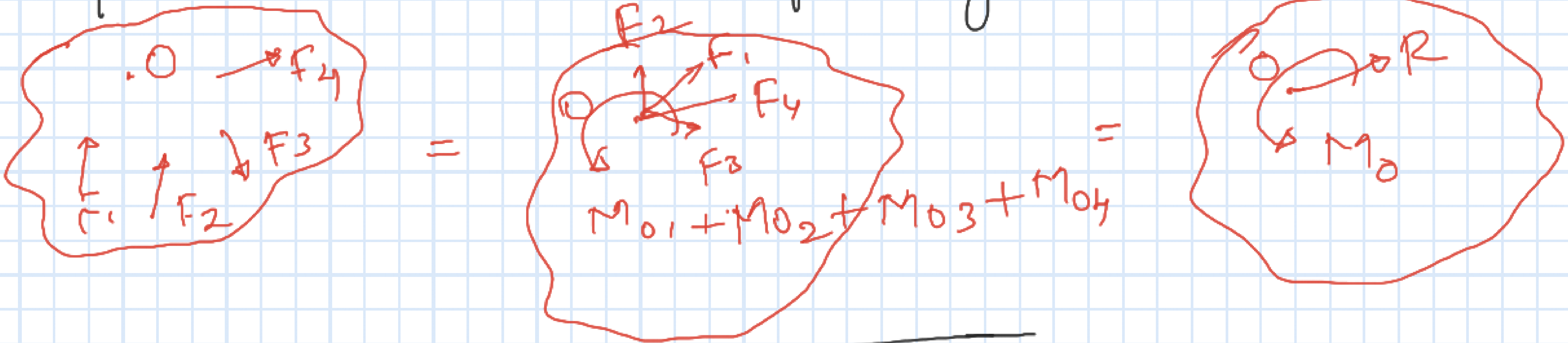


$$R = F_1 + F_2 + F_3$$

$$M_o = \cancel{(F_3)(0)}^{0} + F_2(d_2) + F_1(d_1 + d_2)$$

(ii) Coplanar Concurrent force system

(iii) Coplanar non-concurrent force system



Resultant force $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

$$\alpha = \tan^{-1} \frac{\Sigma F_y}{\Sigma F_x}$$

Problems

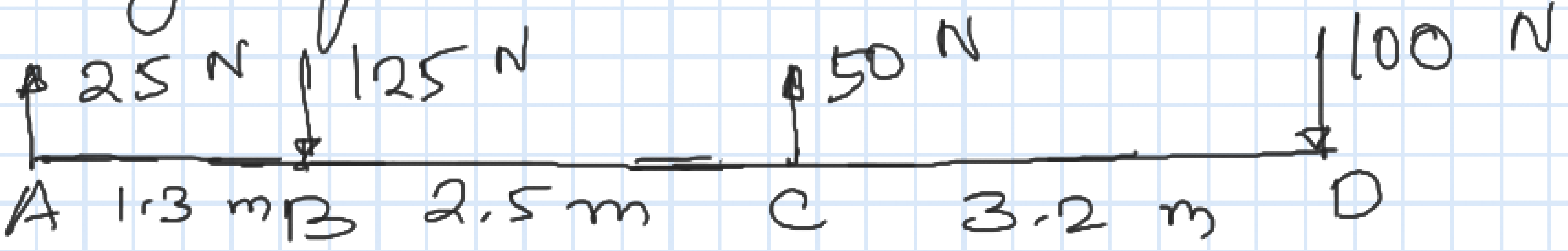
1. ABCD is a rigid bar, subjected to a system of forces. Reduce the system of forces to

(i) Single force and couple at A

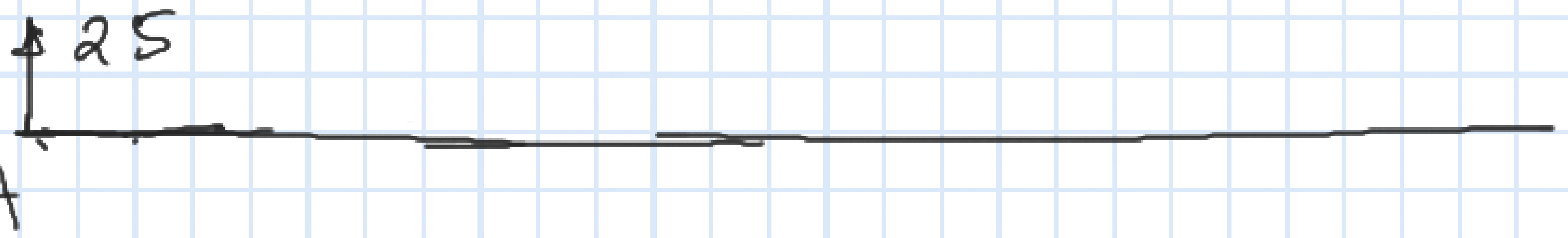
(ii) Single force and couple at D

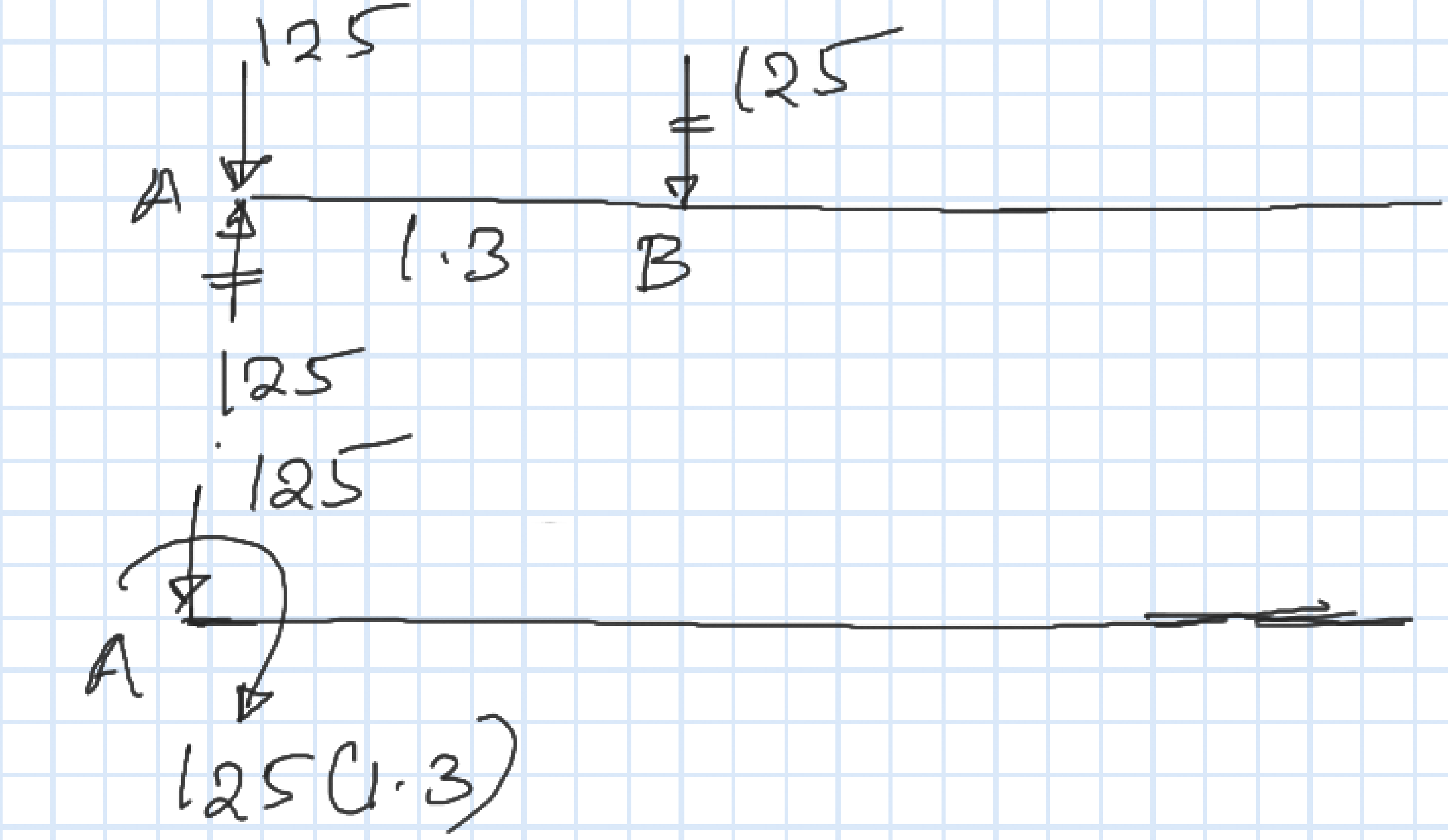
(iii) Single force and couple at C

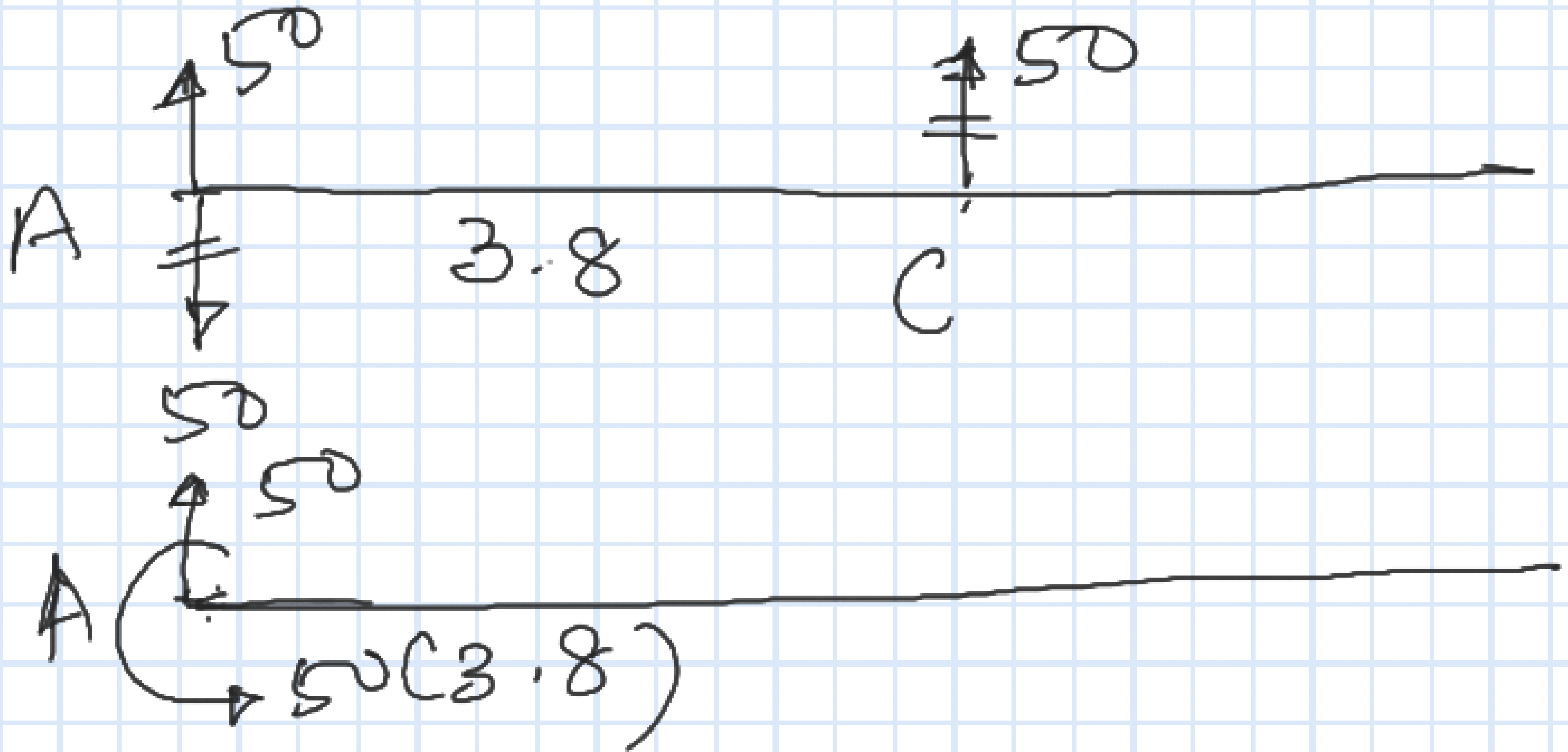
(iv) Single force

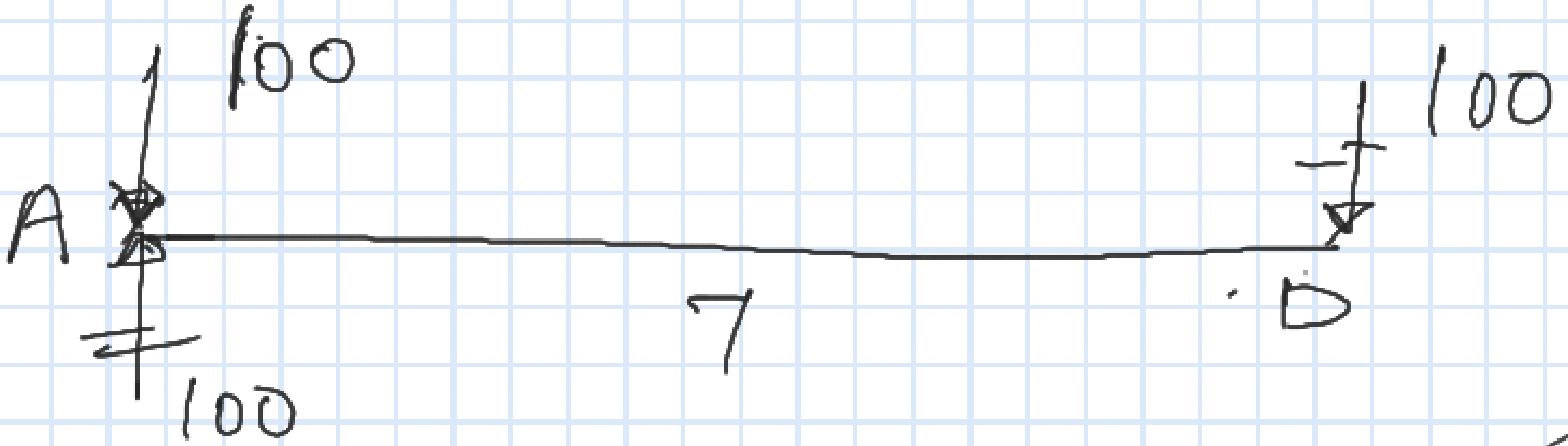


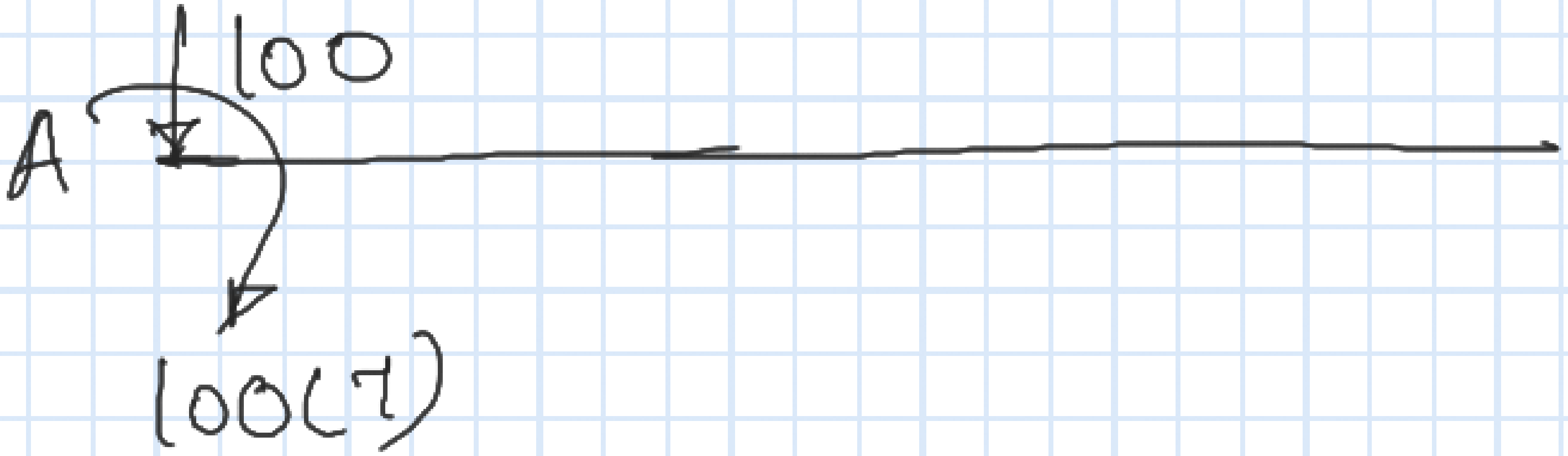
Sol - (i)

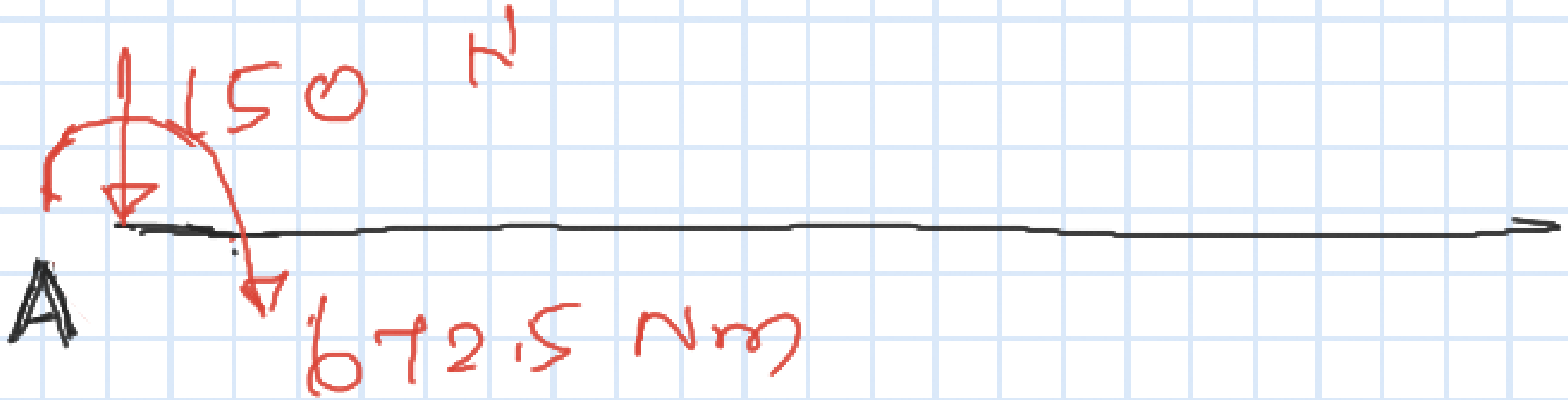



125
125
A
1.3
B
125
125
A
125(1.3)


50
50
A
3.8
C
50
50
A
50(3.8)



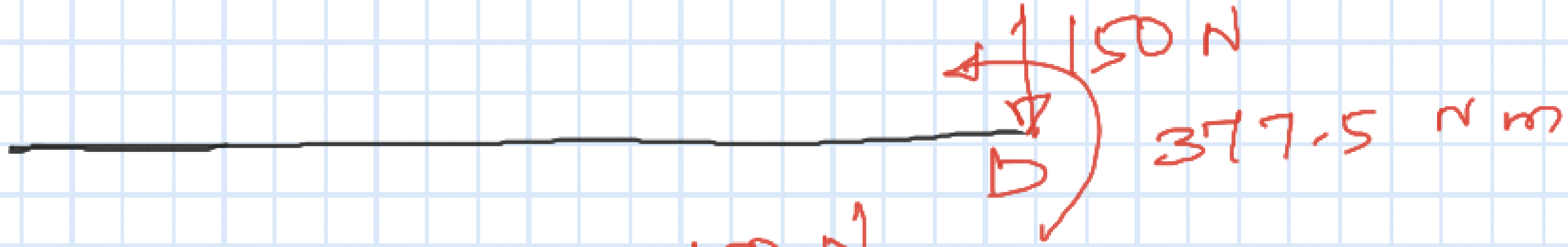




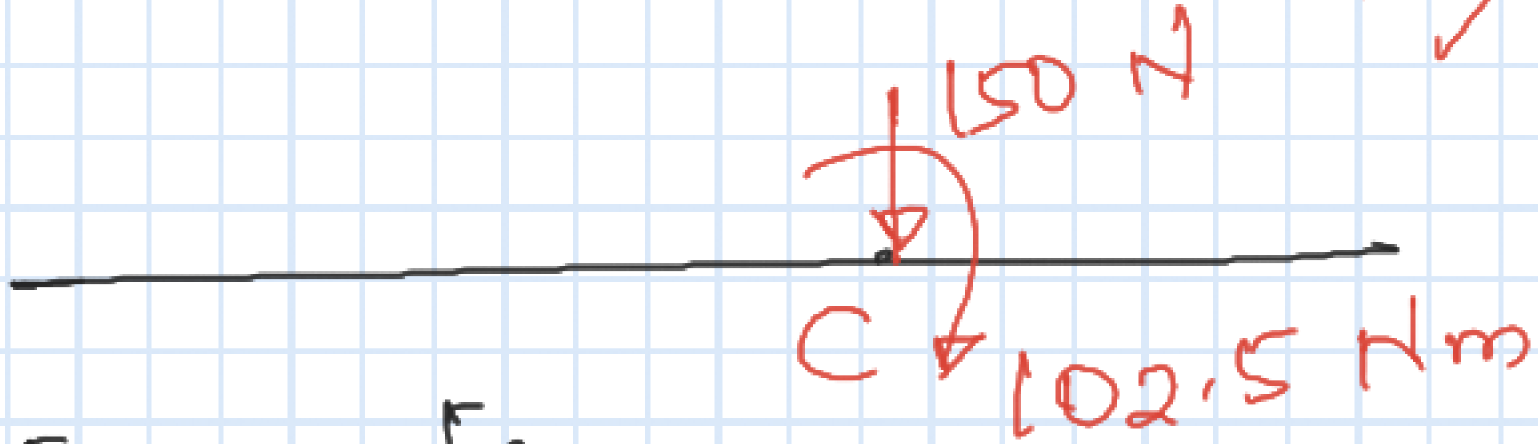
$$F_A = 25 - 125 + 50 - 100$$
$$= -150 \text{ N}$$

$$M_A = 125(1.3) - 50(3.8)$$
$$+ 100(7)$$
$$= -672.5 \text{ Nm}$$

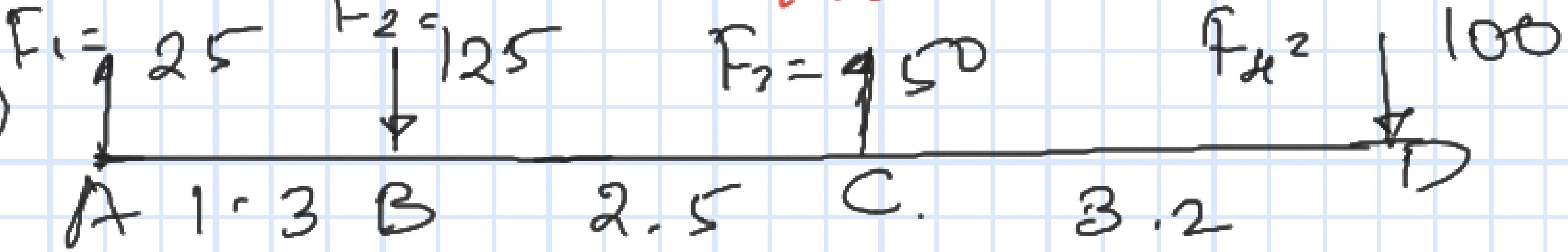
(ii)



(iii)

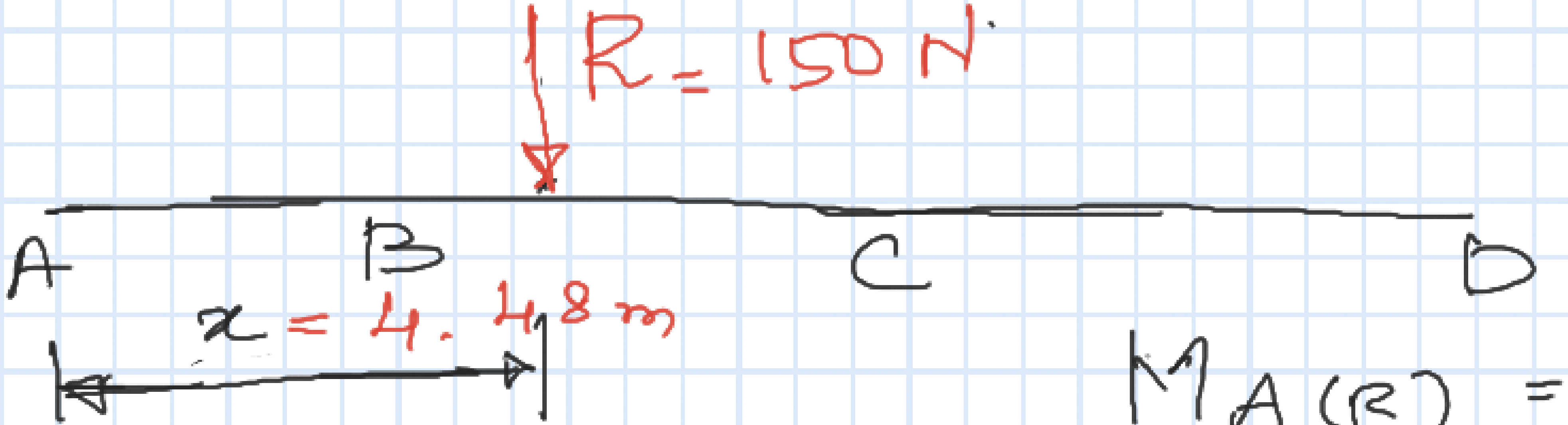


(iv)



Single force = R

$$R = 25 - 125 + 50 - 100$$
$$= -150\ N$$

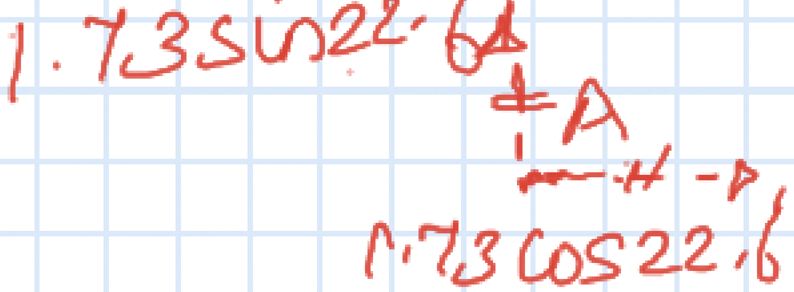


Using Varignons theorem

$$M_{A(R)} = M_{A(F_1)} + M_{A(F_2)} + M_{A(F_3)} + M_{A(F_4)}$$

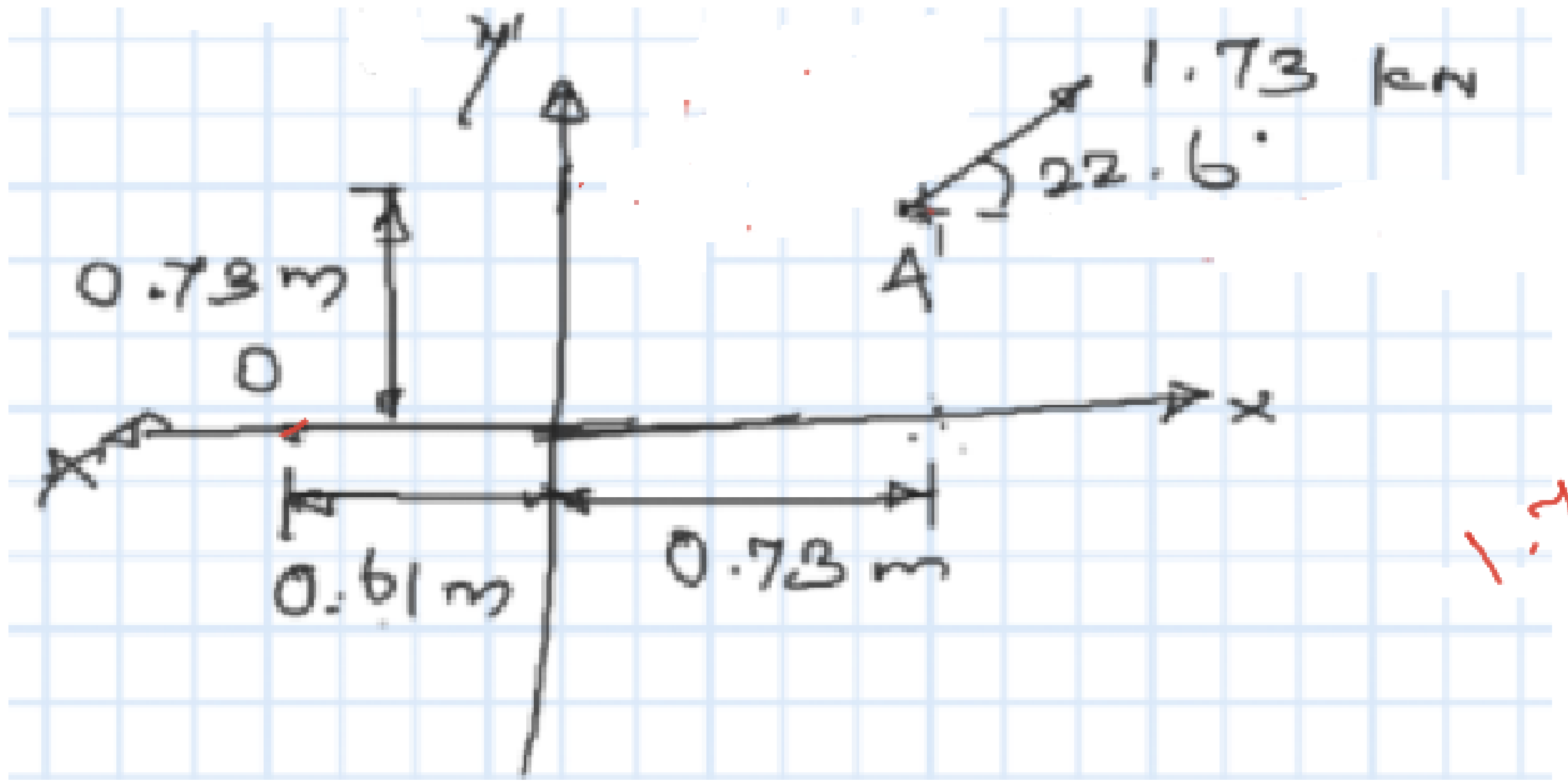
$$150(x) = 0 + 125(1.3) - 50(3.8) + 100(7)$$

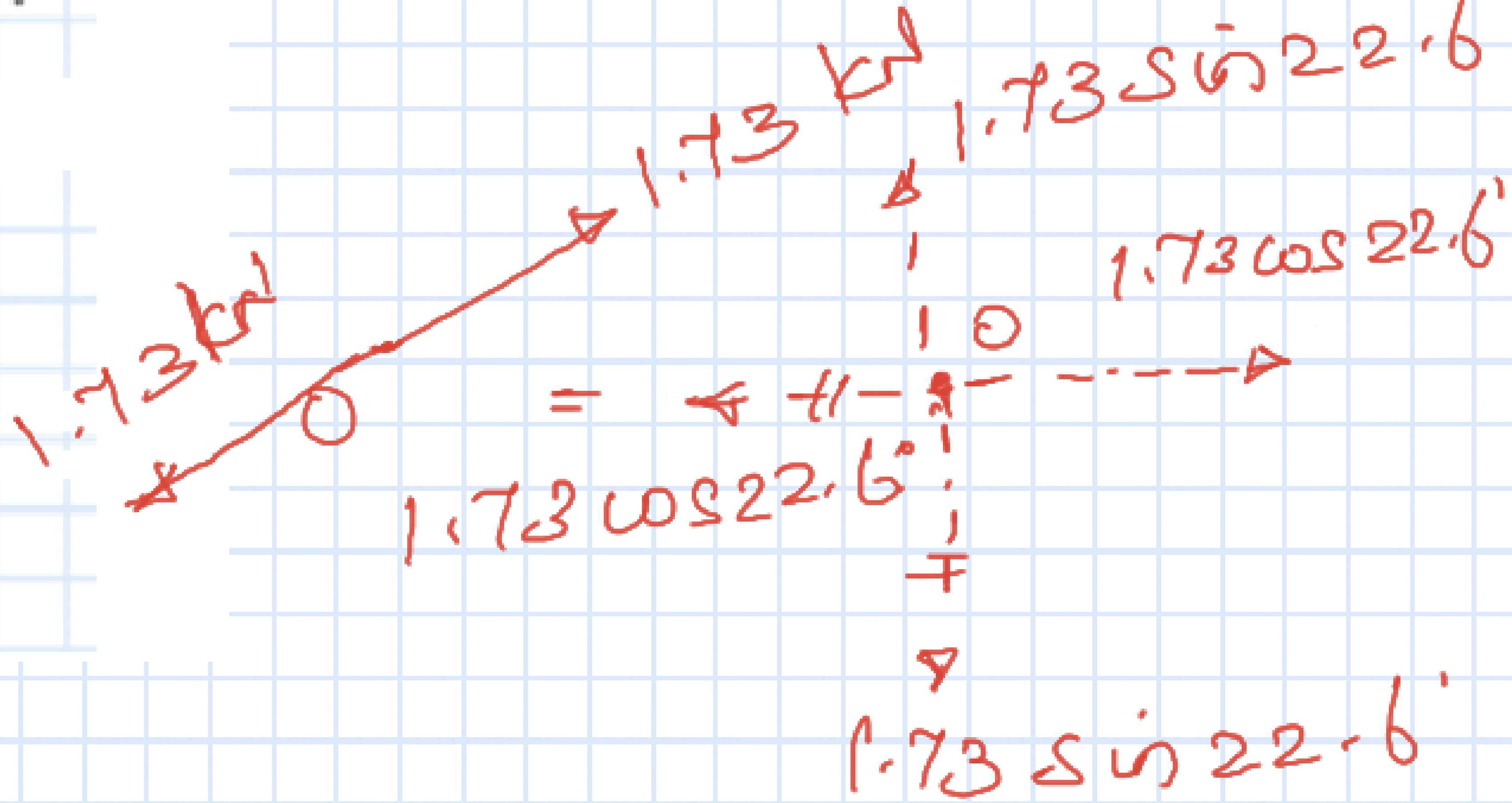
$$x = 4.48\ m$$



1.Shift the force as a force couple system at O

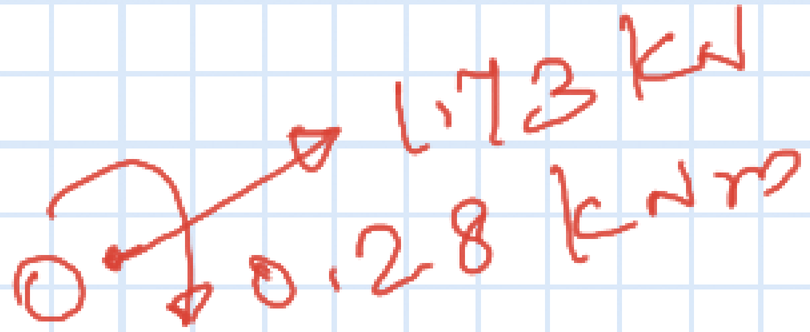
1.Shift the force as a force couple system at O



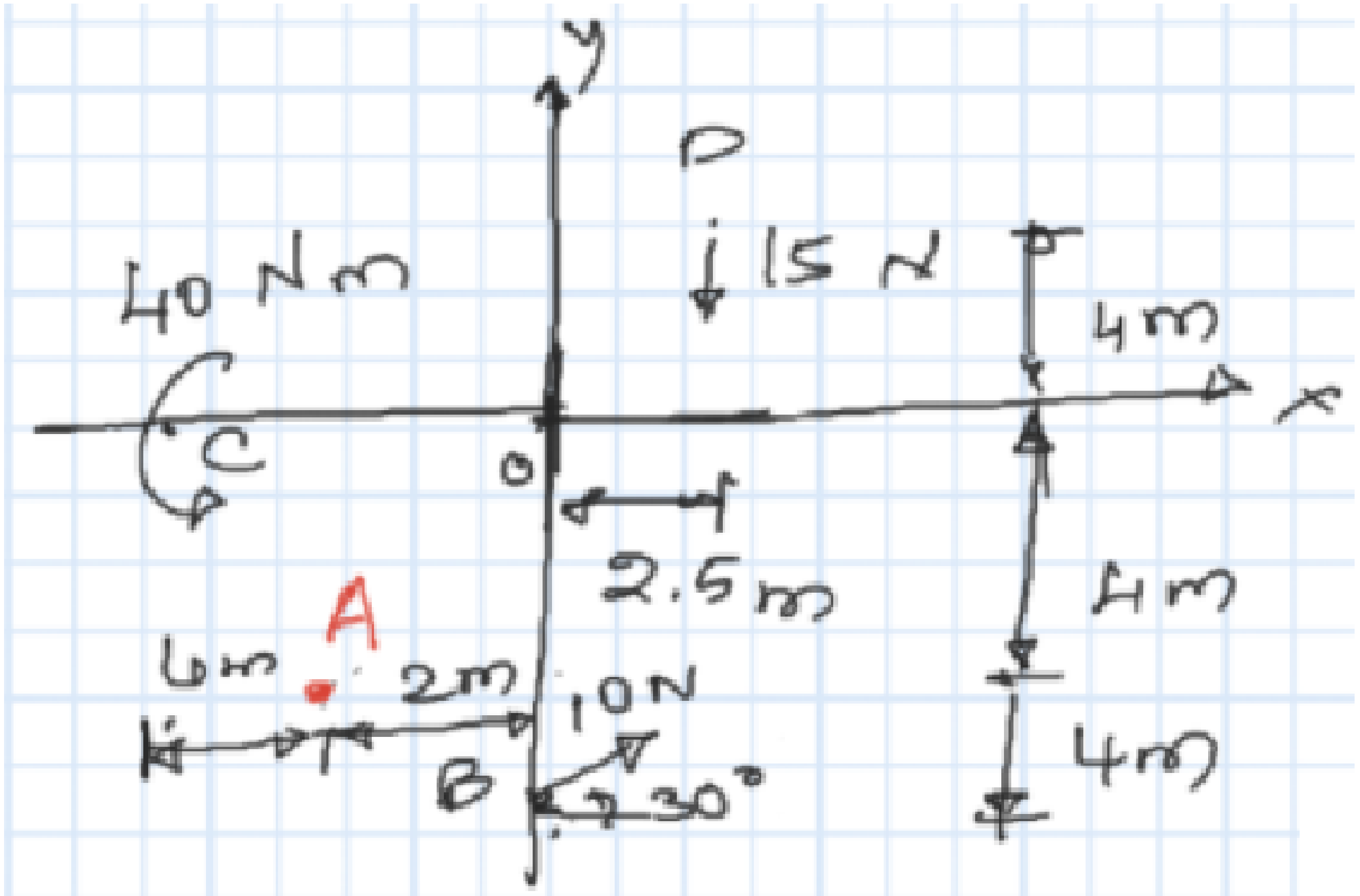


$$M_O = 1.73\cos 22.6\,(0.73) - 1.73\sin 22.6^\circ\,(0.61+0.73)$$

$$= 0.28\,\text{kNm}$$



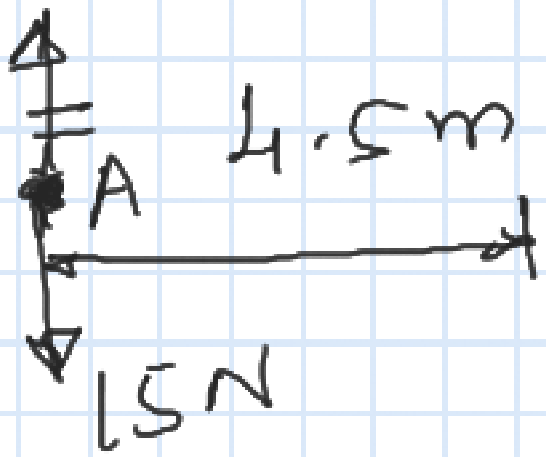
2. Have an equivalent force-couple system at A.

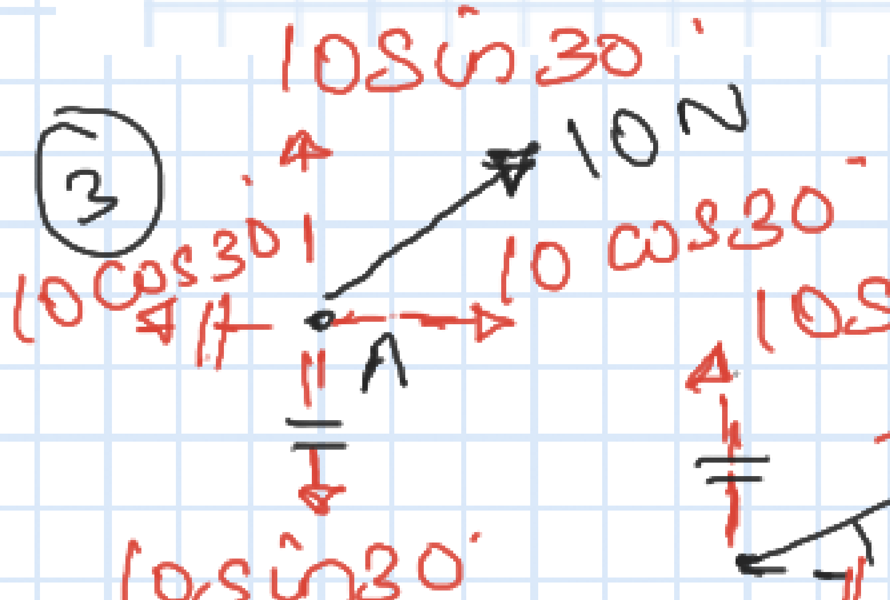


sol ① 40 Nm → It is shifted to A as a free vector

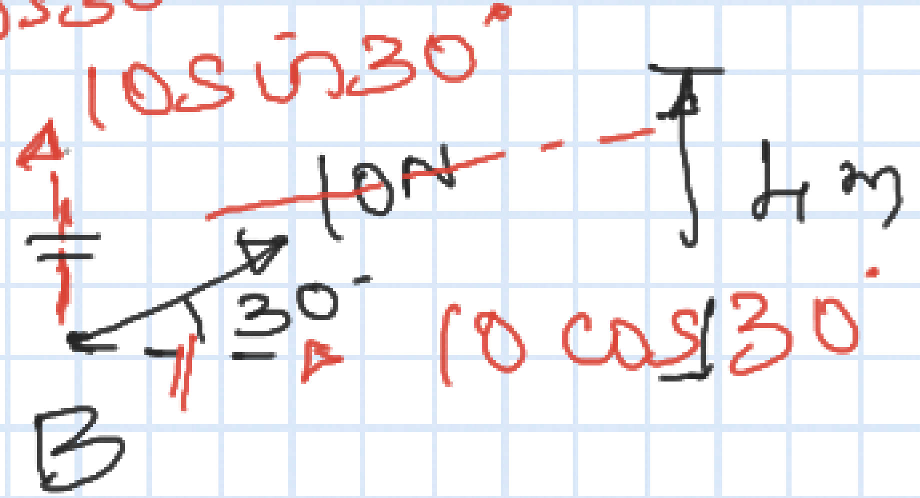
$M_{A(1)} = -40$ Nm

②





③



$M_{A(2)} = 15(4.5)$

$M_{A(3)} = -10\cos 30°(4) - 10\sin 30°(2)$

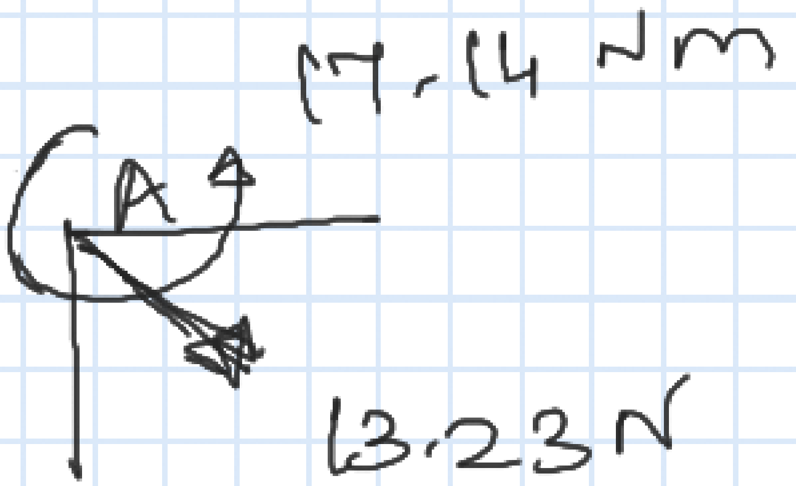
=

$$F_A = \sqrt{(-15 + 10\sin 30^\circ)^2 + (10\cos 30)^2}$$

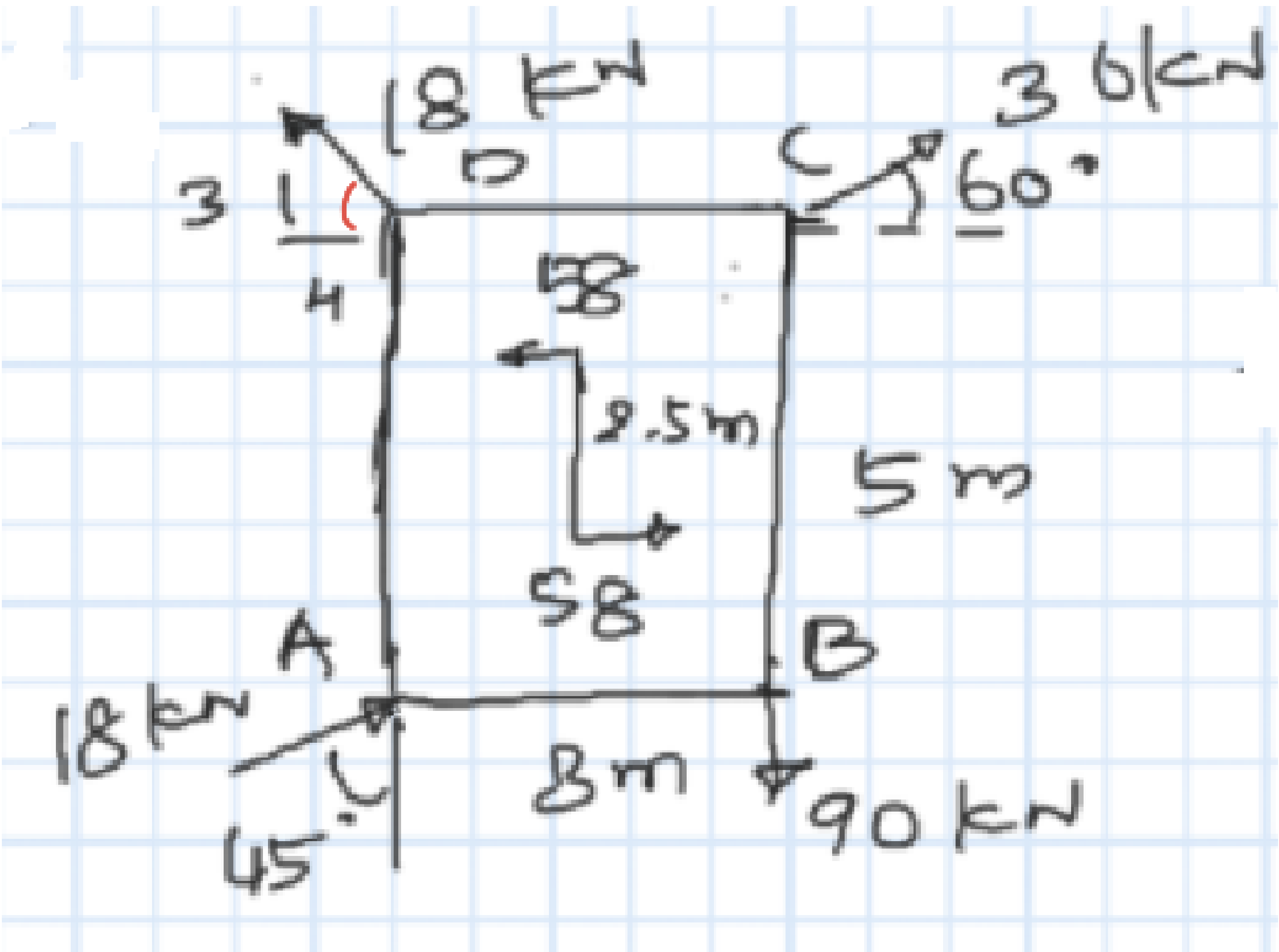
$$= 13.23 \text{ N}$$

$$M_A = M_{A(1)} + M_{A(2)} + M_{A(3)}$$

$$= -17.14 \text{ Nm}$$



3. Find the magnitude and direction of the resultant and locate its position with respect to the sides AB and AD.



1. ΣM_A ✓
2. R
3. ΣF_x, ΣF_y ✓ ✓

$\Sigma M_A = \Sigma F_y (x)$ ✓

$\Sigma M_A = \Sigma F_x (y)$ ✓

18 sin45°

18 kN

45°

18 cos45°

90 kN

36 sin60°

36 kN

60°

36 cos60°

18 sin36.87°

18 kN

θ

18 cos 36.87°

$\tan\theta = 3/4 \quad \theta = 36.87°$

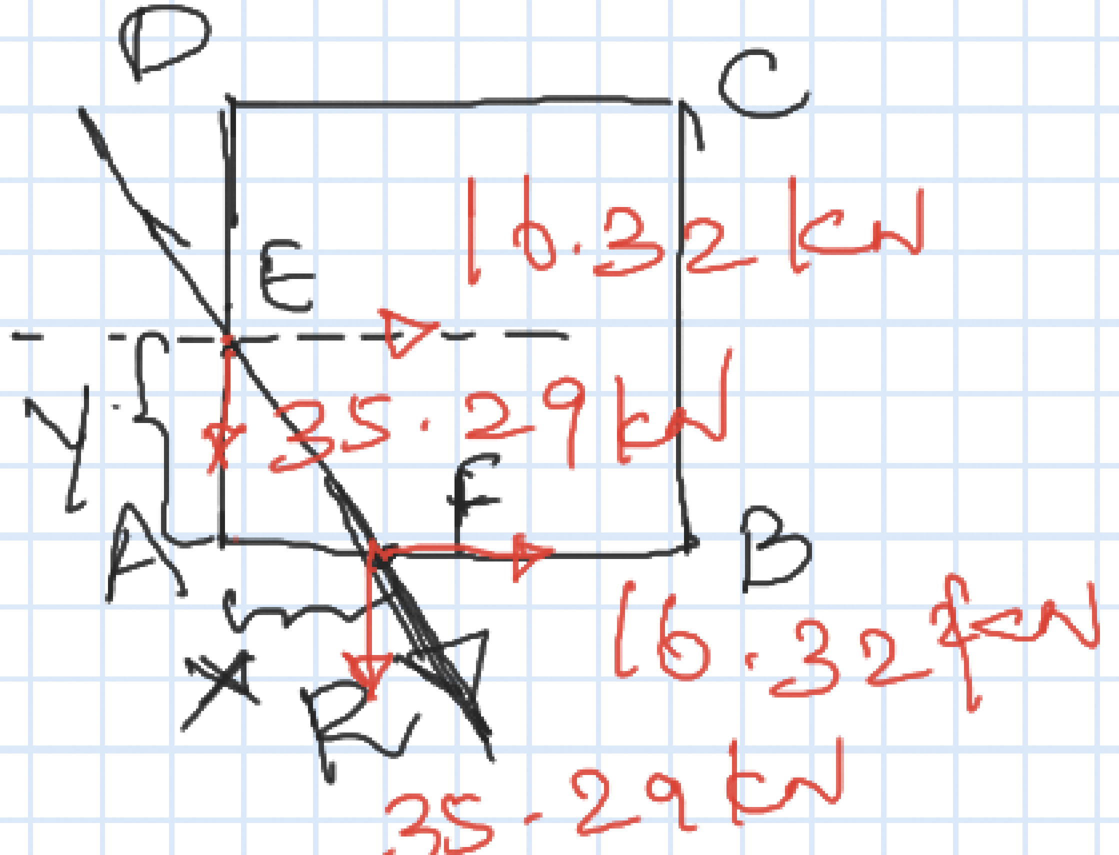
$\Sigma F_x = 18\sin45° + 36\cos60° - 18\cos36.87° = 16.32 \text{ kN}$

$\Sigma F_y = 18\cos45° + 36\sin60° + 18\sin36.87° = -35.29 \text{ kN}$

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = 38.88 \text{ kN}$

$\tan\alpha = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{-35.29}{16.32}$

$\alpha = 65.17°$



$\Sigma M_A = ?$

$$\Sigma M_A = 90(3) + 18(5) - 31.777(3) - 14.4(5) - 58(2.5)$$

$$= 47.66 \text{ kNm}$$

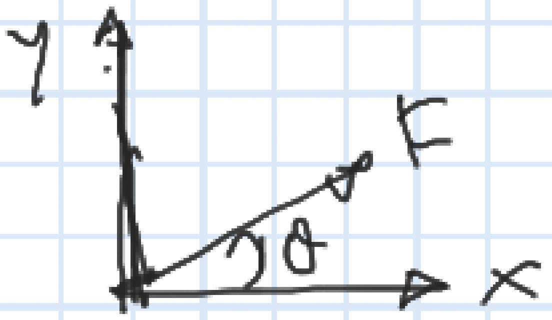
To find intercepts in AB and AD

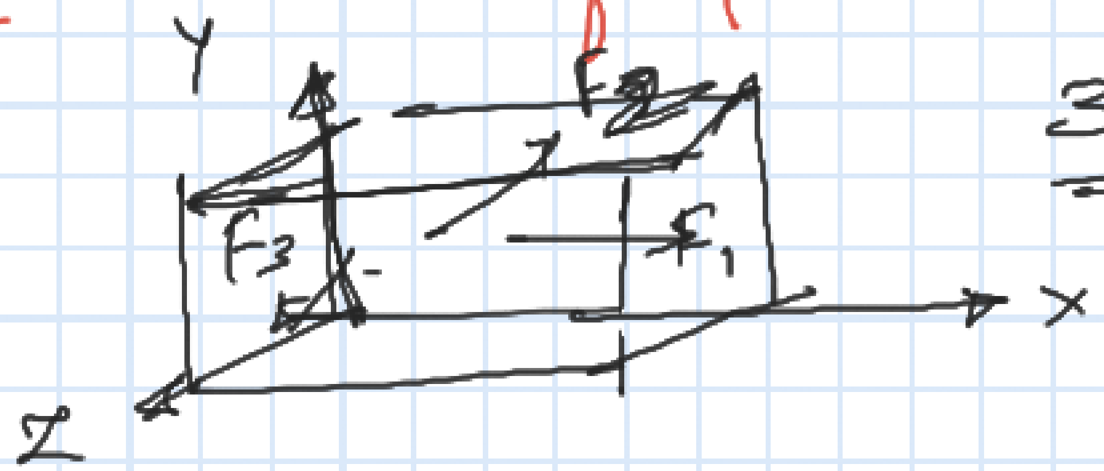
$$X = \frac{\Sigma M_A}{\Sigma F_y} = 1.35 \text{ m}$$

$$Y = \frac{\Sigma M_A}{\Sigma F_x} = 2.9 \text{ m}$$

Resultant and Equilibrium of particles (3D)

2D



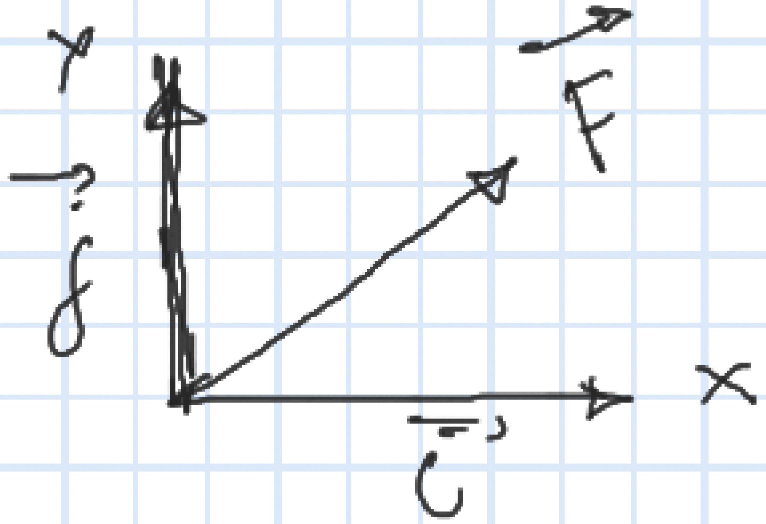


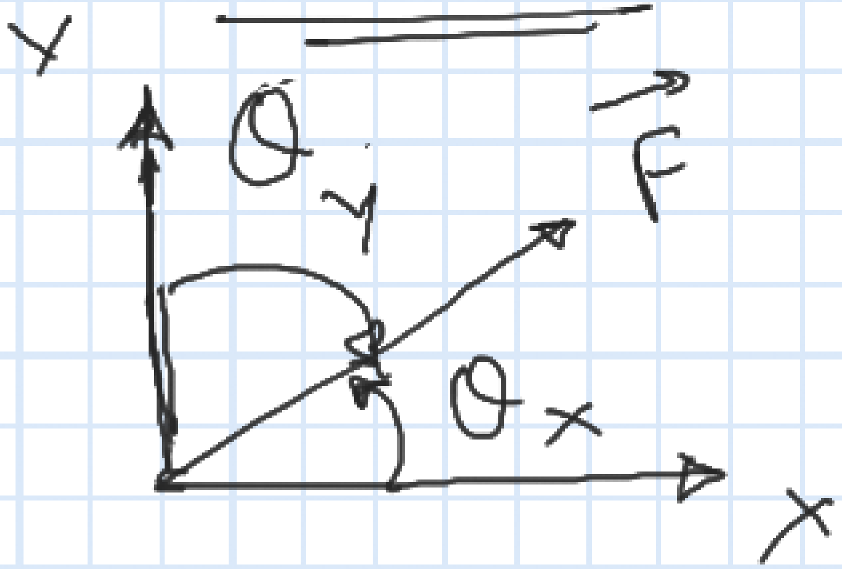
3D

$F_x = F\cos\theta$

$F_y = F\sin\theta$

Vector



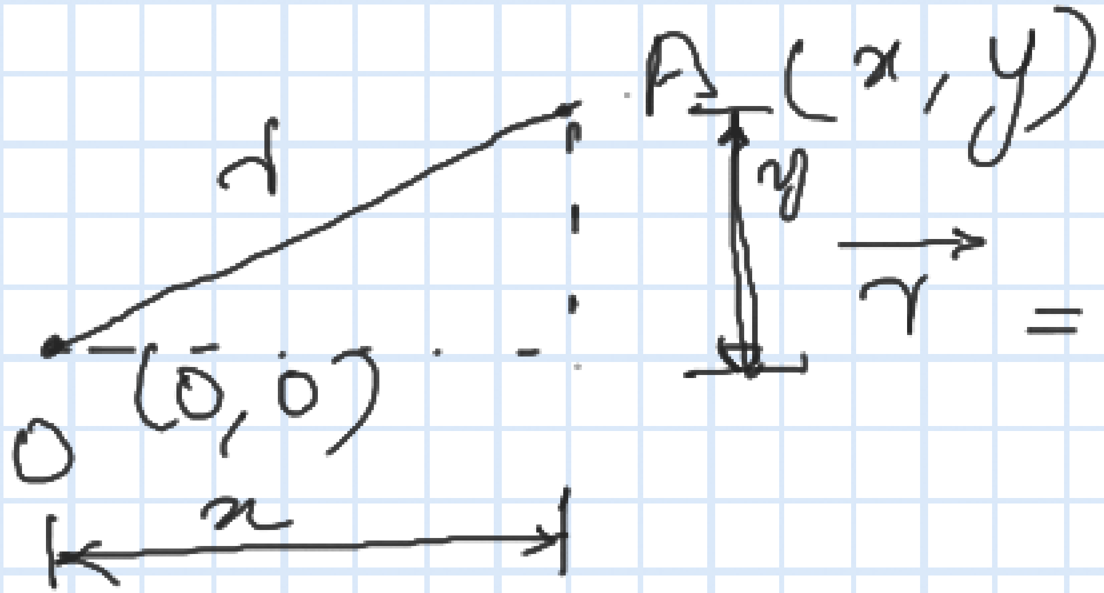


$$\vec{F} = \underbrace{F_x}_{mag}\,\underbrace{\vec{i}}_{dir} + \underbrace{F_y}_{mag}\,\underbrace{\vec{j}}_{dir}$$

$$\vec{F} = F\cos\theta_x\,\vec{i} + F\cos\theta_y\,\vec{j}$$

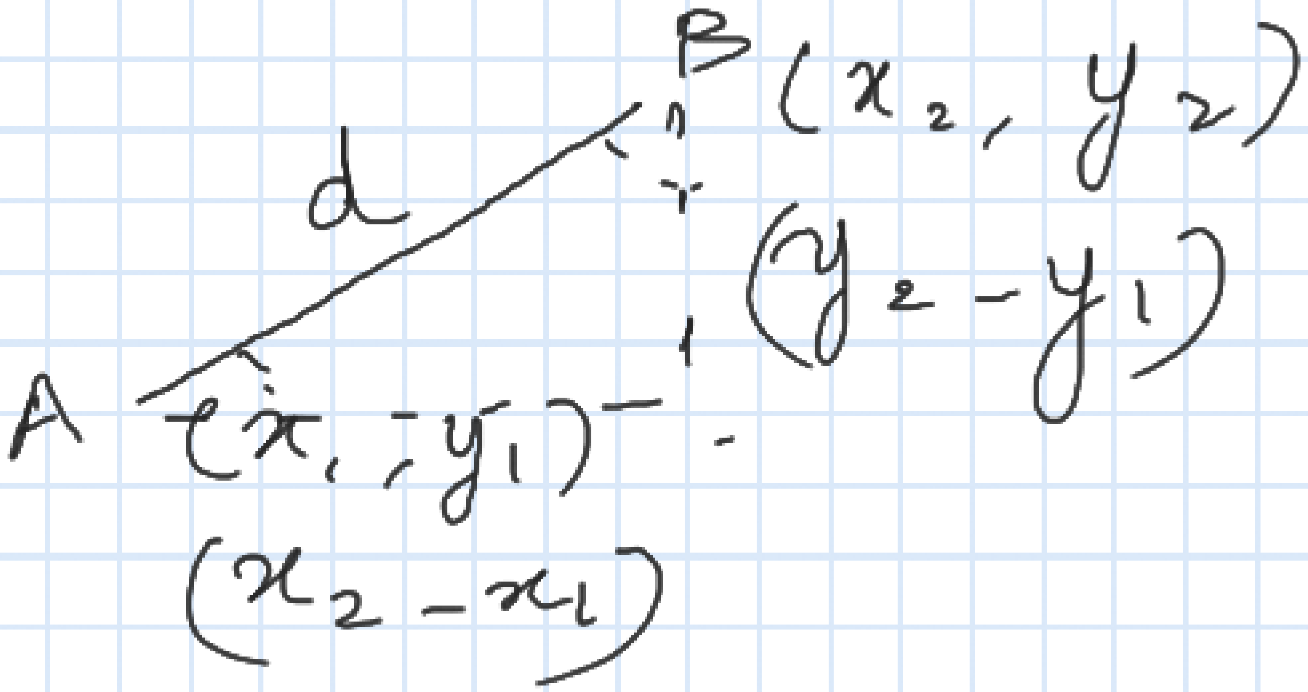
$\underline{\underline{3D}}$ $\vec{F} = F\cos\theta_x \vec{i} + F\cos\theta_y \vec{j} + F\cos\theta_z \vec{k}$

Position vector



$$\vec{r} = x\vec{i} + y\vec{j}$$

$$|\vec{r}| = \sqrt{x^2 + y^2} = r$$

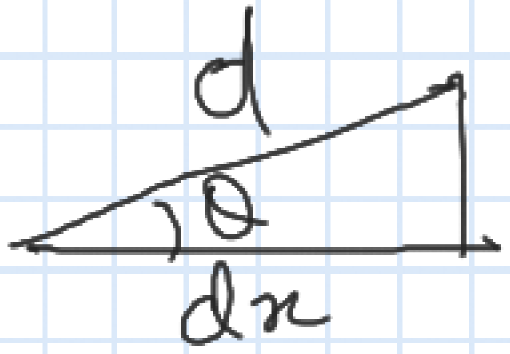


$$\vec{d} = d_x\vec{i} + d_y\vec{j}$$

$$d = \sqrt{d_x^2 + d_y^2}$$

Unit vector

$$\lambda_{AB} = \frac{\vec{d}}{d} = \frac{dx\,\vec{i} + dy\,\vec{j}}{\sqrt{dx^2 + dy^2}} = \frac{dx}{d}\vec{i} + \frac{dy}{d}\vec{j}$$



To have a force as a vector

$$\vec{F} = F(\lambda_{AB})$$

$$= F\left(\frac{dx}{d}\right)\vec{i} + F\left(\frac{dy}{d}\right)\vec{j} \quad - (1)$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j}$$

But $F_x = F\cos\theta_x$ $\qquad F_y = F\cos\theta_y$

Sub in (1) $\qquad F\cos\theta_x = F\frac{dx}{d}$; $F\cos\theta_y = F\frac{dy}{d}$