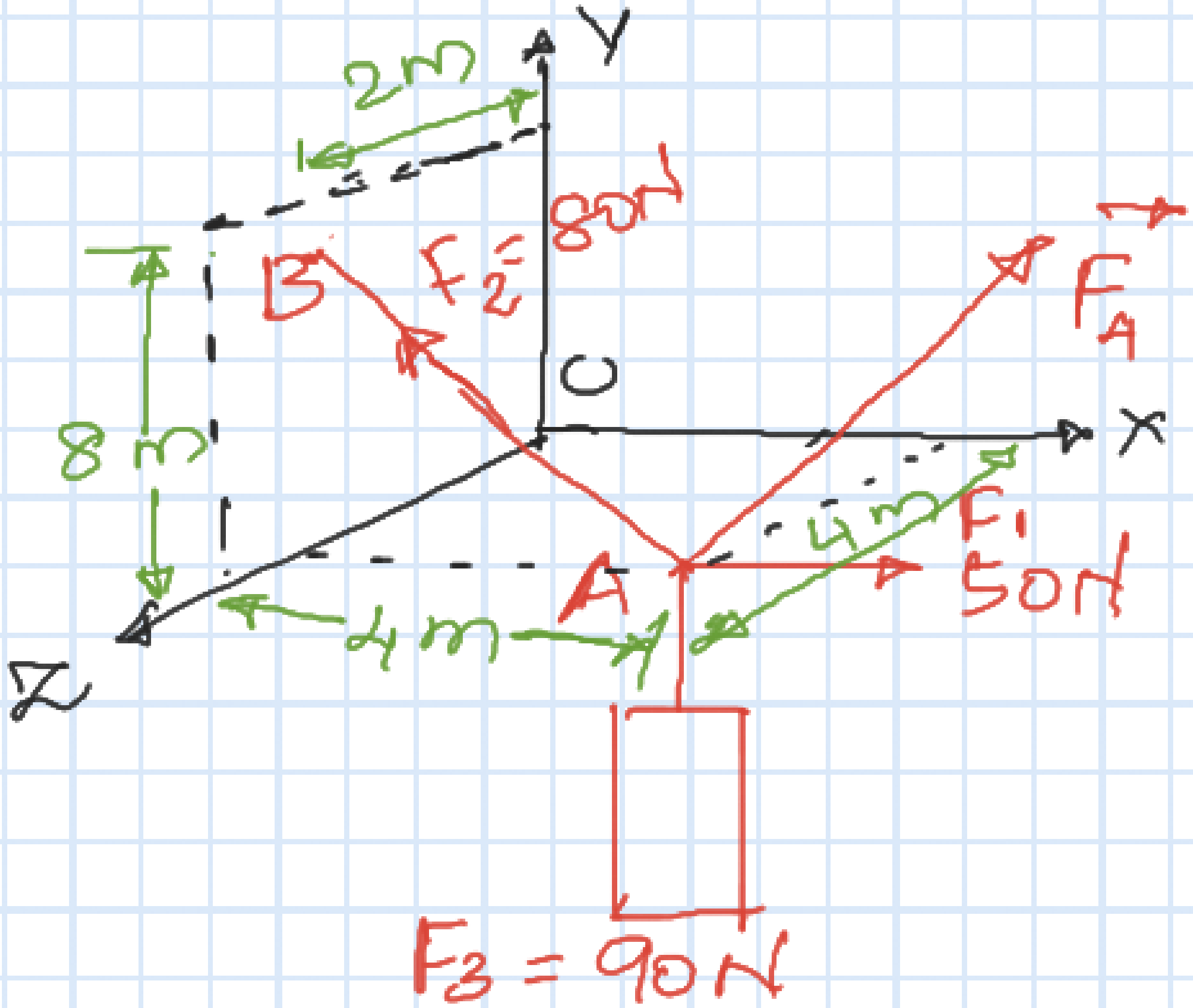
Inclination of the resultant force with the axes.

$$\phi_x = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\left(\frac{40}{216.87}\right) = 79.37^\circ$$

$$\phi_y = \cos^{-1}\frac{R_y}{R} = \cos^{-1}\left(\frac{-150.72}{216.87}\right) = 134.02^\circ$$

$$\phi_z = \cos^{-1}\frac{R_z}{R} = \cos^{-1}\left(\frac{150.72}{216.87}\right) = 46.24^\circ$$

2. Three concurrent forces in space are acting at point A as shown. An unknown force $\vec{F}$ attached to the system makes the particle A to be in equilibrium. Evaluate the magnitude and direction of the unknown force $\vec{F}$.



Sol. Particle is in equilibrium

$R = 0$

$\Sigma F_x = 0$ — (1)

$\Sigma F_y = 0$ — (2)

$\Sigma F_z = 0$ — (3)

Unknowns are

$F_{x_A}, F_{y_A}, F_{z_A}$

A(4, 0, 4)

B(0, 8, 2)

$$\vec{F_1} = F_1\vec{i} = 50\vec{i} \quad — \text{(1)}$$

$$\vec{F_3} = F_3(-\vec{j}) = -90\vec{j} \quad — \text{(2)}$$

$$\vec{F_2} = F_2(\lambda_{AB}) = F_2\left(\frac{\vec{AB}}{AB}\right) = 80\left[\frac{(0-4)\vec{i} + (8-0)\vec{j} + (2-4)\vec{k}}{\sqrt{4^2+8^2+2^2}}\right]$$

$$= -34.92\vec{i} + 69.83\vec{j} - 17.46\vec{k} \quad — \text{(3)}$$

$$\vec{F_4} = F_{x4}\vec{i} + F_{y4}\vec{j} + F_{z4}\vec{k} \quad — \text{(4)}$$

$$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} = 0$$

$$50\vec{i} - 90\vec{j} - 34.92\vec{i} + 69.83\vec{j} - 17.46\vec{k} + F_{x4}\vec{i} + F_{y4}\vec{j} + F_{z4}\vec{k} = 0$$

$$(F_{x4} + 15.085)\vec{i} + (F_{y4} - 20.17)\vec{j} + (F_{z4} - 17.46)\vec{k} = 0$$

$$F_{x4} + 15.085 = 0 \Rightarrow F_{x4} = -15.085$$

$$F_{y4} - 20.17 = 0 \Rightarrow F_{y4} = 20.17$$

$$F_{z4} - 17.46 = 0 \Rightarrow F_{z4} = 17.46$$

Magnitude of $\vec{F_4}$

$$F_4 = \sqrt{15.085^2 + 20.17^2 + 17.46^2}$$

$$= 30.65\ N$$

Inclination of F_4 with the axes

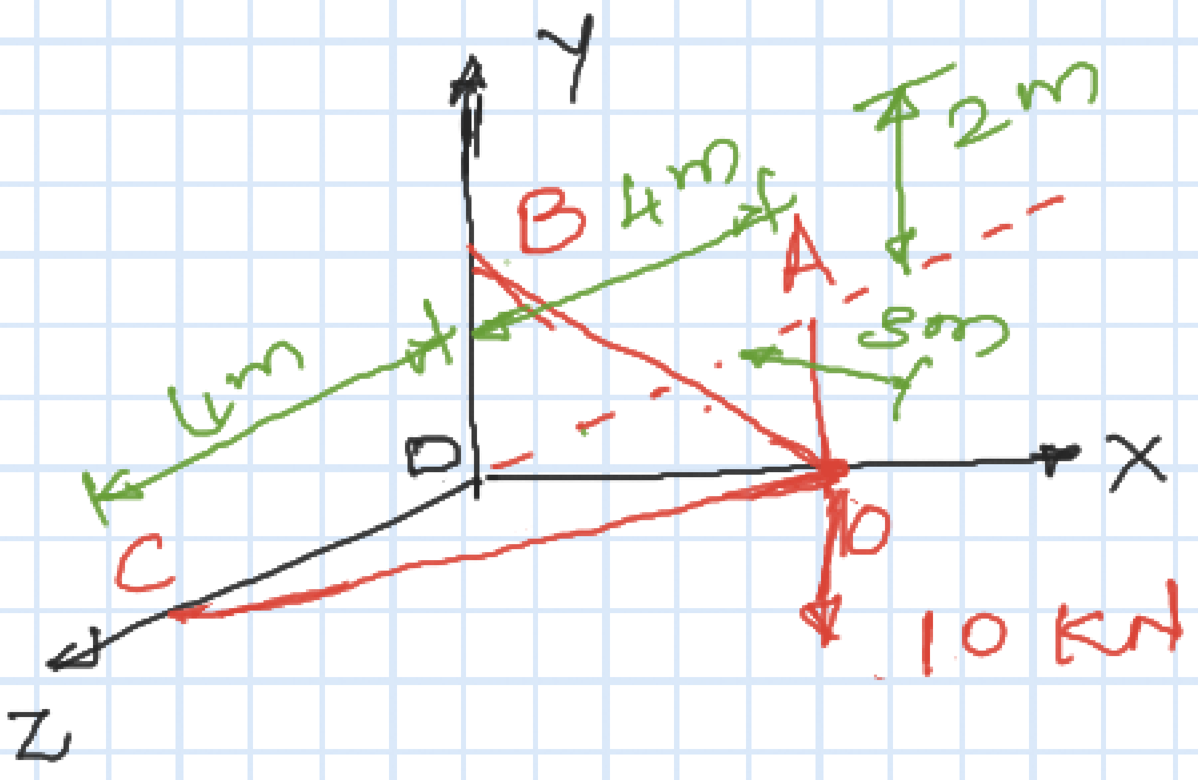
$$\alpha_x = \cos^{-1}\frac{F_x}{F} = 119.48^\circ$$

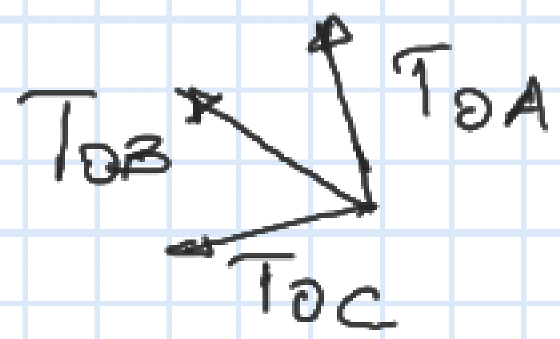
$$\alpha_y = \cos^{-1}\frac{F_y}{F} = 48.84^\circ$$

$$\alpha_z = \cos^{-1}\frac{F_z}{F} = 55.27^\circ$$

3. Problem for practice.

Members OA, OB, OC form a three member truss. A weight of 20 kN is hanging from joint 'O'. Find the force induced in all the three member.





Sol

$$\vec{T_{OA}} = T_{OA}(\lambda_{OA})$$

$$\vec{T_{OB}} = T_{OB}(\lambda_{OB})$$

$$\vec{T_{OC}} = T_{OC}(\lambda_{OC})$$

A(0, 0, −4) D(0, 0, 0)

B(0, 2, 0) O(3, 0, 0)

C(0, 0, 4)

$$\lambda_{OA} = \frac{\overrightarrow{OA}}{OA} = \frac{(0-3)\vec{i} + (0-0)\vec{j} + (-4-0)\vec{k}}{\sqrt{3^2+4^2}}$$

$$= \frac{-3\vec{i} - 4\vec{k}}{5} = -0.6\vec{i} - 0.8\vec{k}$$

$$\lambda_{OB} = \frac{\overrightarrow{OB}}{OB} = \frac{(0-3)\vec{i} + (2-0)\vec{j} + (0-0)\vec{k}}{\sqrt{3^2+2^2}}$$

$$= -0.832\vec{i} + 0.555\vec{j}$$

$$\lambda_{OC} = \frac{\overrightarrow{OC}}{OC} = \frac{(0-3)\vec{i} + (0-0)\vec{j} + (4-0)\vec{k}}{\sqrt{3^2+4^2}}$$

$$= -0.6\vec{i} + 0.8\vec{k}$$

$$\vec{T}_{OA} = T_{OA}(-0.6\vec{i} - 0.8\vec{k}) = -0.6T_{OA}\vec{i} - 0.8T_{OA}\vec{k} \quad -(1)$$

$$\vec{T}_{OB} = T_{OB}(-0.832\vec{i} + 0.555\vec{j}) = -0.832T_{OB}\vec{i} + 0.555T_{OB}\vec{j} \quad -(2)$$

$$\vec{T}_{oc} = T_{oc}(-0.6\vec{i} + 0.8\vec{k}) = -0.6\,T_{oc}\,\vec{i} + 0.8\,T_{oc}\,\vec{k} \quad \text{— (3)}$$

Since the truss is in equilibrium $R = 0$

$\therefore\ R_x = \Sigma F_x = 0$

$R_y = \Sigma F_y = 0$

$R_z = \Sigma F_z = 0$

$$\vec{R} = \vec{T}_{OA} + \vec{T}_{OB} + \vec{T}_{OC} = (-0.6T_{OA} - 0.832\,T_{OB} - 0.6\,T_{OC})\vec{i} + (-10 + 0.555\,T_{OB})\vec{j} + (-0.8T_{OA} + 0.8\,T_{OC})\vec{k} = 0$$

$-0.6\,T_{OA} - 0.832\,T_{OB} - 0.6\,T_{OC} = 0$ — (4)

$+0.555\,T_{OB} - 10 = 0$ — (5)

$-0.8\,T_{OA} + 0.8\,T_{OC} = 0$ — (6)

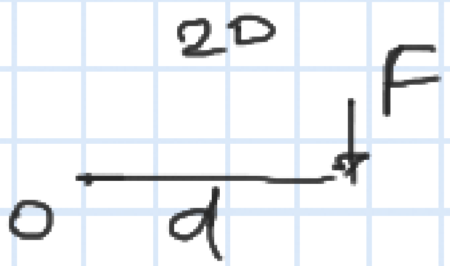
Solve (4), (5) & (6)

$T_{OA} = 12.5$ kN (C)

$T_{OB} = 18.02$ kN (T)

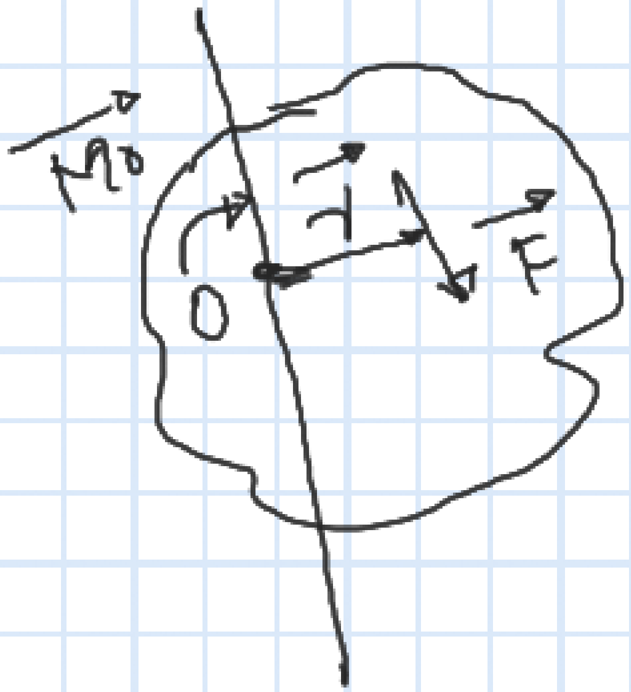
$T_{OC} = 12.5$ kN (C)

Equilibrium of rigid bodies in 3D



$M_O = (F)(d)$

Moment of a force about a point



Position vector

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$\vec{M_O} = \vec{r} \times \vec{F}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= (F_z y - F_y z)\vec{i} - (F_z x - F_x z)\vec{j} + (F_y x - F_z y)\vec{k}$$

$$\vec{M_0} = M_x \vec{i} + M_y \vec{j} + M_z \vec{k}$$

Magnitude

$$M_0 = \sqrt{M_x^2 + M_y^2 + M_z^2}$$

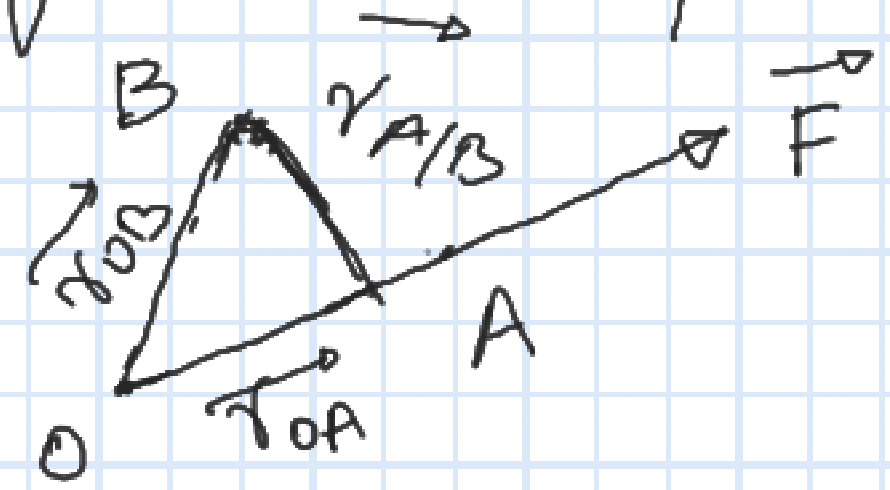
Direction

$$\alpha_x = \cos^{-1} \frac{M_x}{M}$$

$$\alpha_y = \cos^{-1} \frac{M_y}{M}$$

$$\alpha_z = \cos^{-1} \frac{M_z}{M}$$

1. A force $\vec{F} = 9\vec{i} + 3\vec{j} - 6\vec{k}$ passes through a point 'A' whose position vector is $4\vec{i} - 2\vec{j} + 9\vec{k}$. Calculate the moment of the force about point 'B' whose position vector is $6\vec{i} - 3\vec{j} - 7\vec{k}$



$\vec{M}_B = ?$

Sol

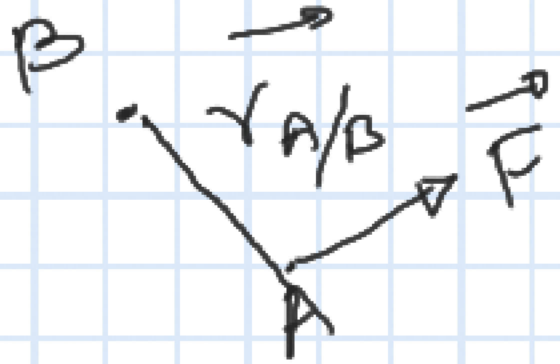
$$\vec{r}_{A/B} = (\vec{r}_{OA} - \vec{r}_{OB}) = (4\vec{i} - 2\vec{j} + 9\vec{k}) - (6\vec{i} - 3\vec{j} - 7\vec{k})$$

$$= (-2\vec{i} + \vec{j} + 16\vec{k})$$

$$\vec{M}_B = \vec{r}_{A/B} \times \vec{F}$$

$$= \begin{vmatrix} i & j & k \\ -2 & 1 & 16 \\ 9 & 3 & -6 \end{vmatrix} = -54\vec{i} + 132\vec{j} - 15\vec{k}$$

2. A force vector $\vec{F} = 6\vec{i} + 2\vec{j} - 3\vec{k}$ acts a point A of coordinate (1, 2, 3). Find the moment of the force about point B (−2, 3, 4).



$$\vec{r}_{A/B} = (x_A - x_B)\vec{i} + (y_A - y_B)\vec{j} + (z_A - z_B)\vec{k}$$

$$= (1+2)\vec{i} + (2-3)\vec{j} + (3-4)\vec{k}$$

$$= 3\vec{i} - 1\vec{j} - 1\vec{k}$$

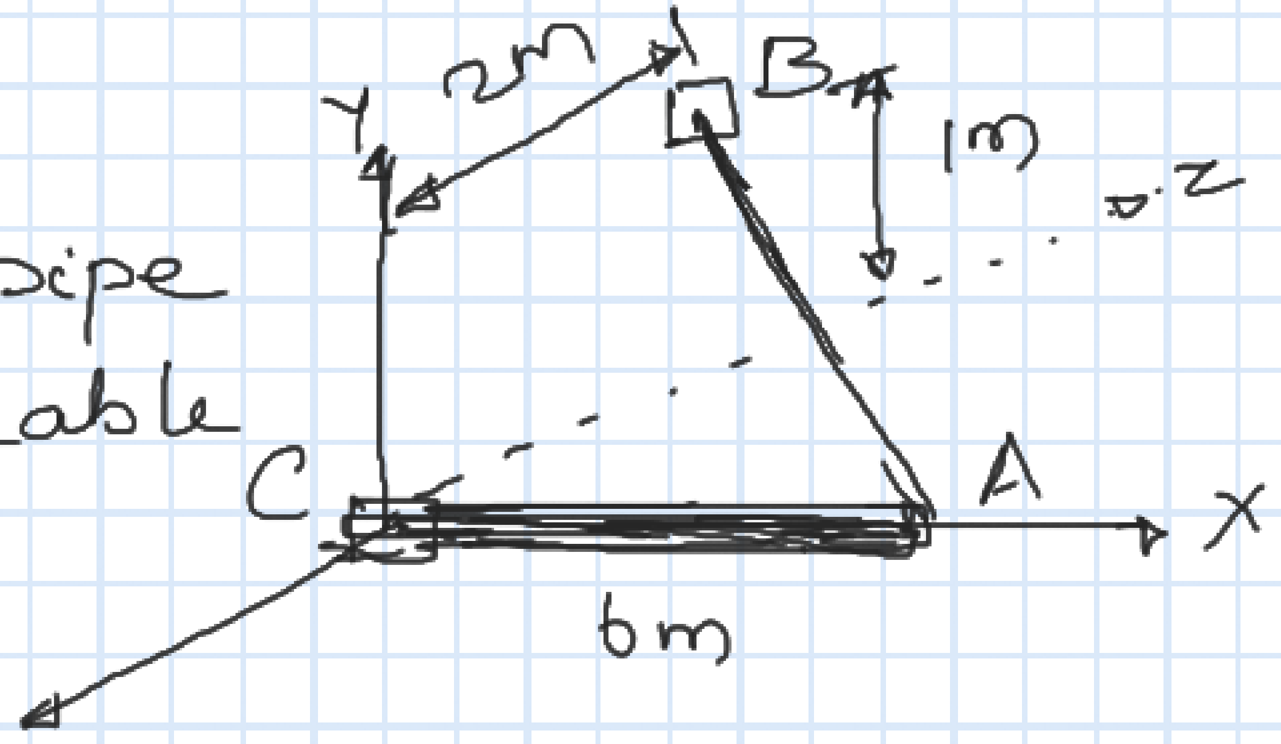
$$\vec{M}_B = \vec{r}_{A/B} \times \vec{F}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & -1 \\ 6 & 2 & -3 \end{vmatrix} = 5\vec{i} + 3\vec{j} + 12\vec{k}$$

3.

Ac_pipe

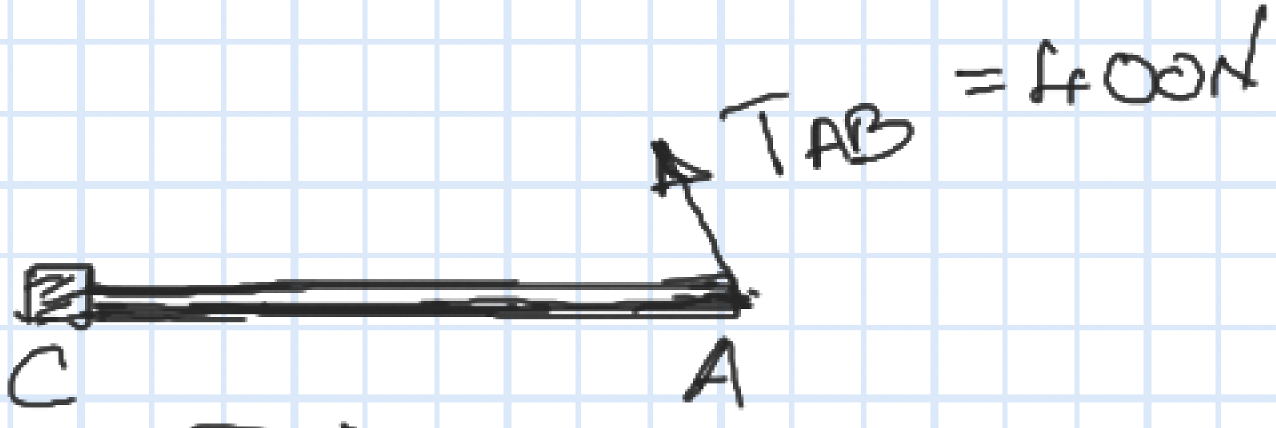
AB_cable



Evaluate the moment of force at A about C

$T_{AB} = 400\text{ N}$

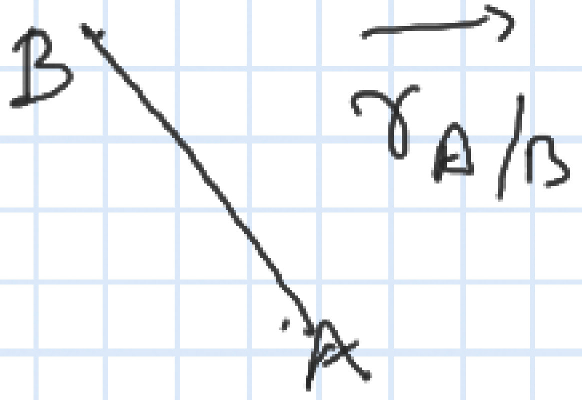
Solution



A (6, 0, 0)

B (0, 1, -2)

C (0, 0, 0)



$$\vec{M}_C = \vec{r}_{AC} \times \vec{T}_{AB}$$

$$\vec{r}_{AC} = (6-0)\vec{i} = 6\vec{i}$$

$$\vec{T}_{AB} = T_{AB}(\lambda_{AB})$$

$$\lambda_{AB} = \frac{\vec{AB}}{AB} = (0-6)\vec{i} + (1-0)\vec{j} + (-2-0)\vec{k} \Big/ \sqrt{6^2+1^2+2^2}$$

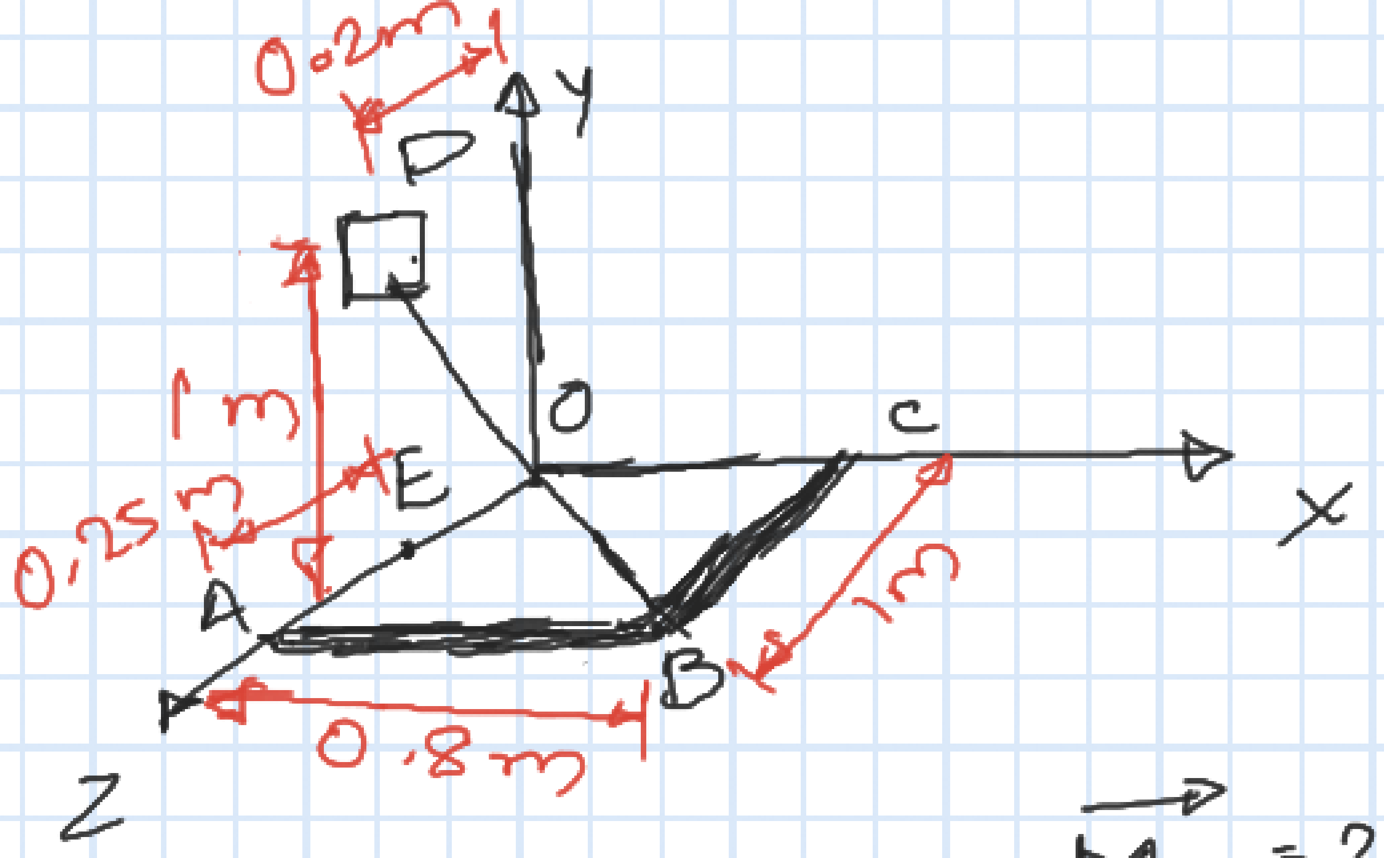
$$\lambda_{AB} = \frac{-6\vec{i} + \vec{j} - 2\vec{k}}{6.4}$$

$$\vec{M_C} = \vec{r_{A/C}} \times \vec{T_{AB}}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & 0 & 0 \\ -375 & 62.5 & -125 \end{vmatrix} = 750\vec{j} + 375\vec{k}$$

$$\vec{M_C} = 750\vec{j} + 375\vec{k}$$

4. A rectangular plate 1 m x 0.8 m is supported by two pins and by a wire BD. If the tension in the wire is 140 N, calculate the moment about 'A'. Also calculate the moment about point 'E'



Sol

O (0, 0, 0)

A (0, 0, 1)

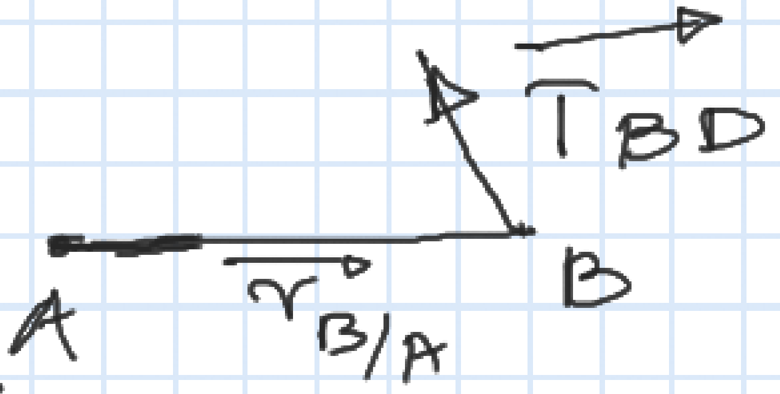
B (0.8, 0, 1)

C (0.8, 0, 0)

D (0, 1, 0.2)

E (0, 0, 0.75)

$\vec{M}_A = ?$



$$\vec{M}_A = \vec{r}_{B/A} \times \vec{T}_{BD}$$

$$\vec{T}_{BD} = T_{BD}(\lambda_{BD})$$

$$\lambda_{BD} = \frac{(0-0.8)\vec{i} + (1-0)\vec{j} + (0.2-1)\vec{k}}{\sqrt{0.8^2 + 1^2 + 0.8^2}}$$

$$= \frac{-0.8\vec{i} + \vec{j} - 0.8\vec{k}}{1.51}$$

$$\vec{T}_{BD} = 140\left(\frac{-0.8\vec{i} + \vec{j} - 0.8\vec{k}}{1.51}\right)$$

$$= -74.17\vec{i} + 92.71\vec{j} - 74.17\vec{k}$$

$$\vec{M}_A = \vec{r}_{B/A} \times \vec{T}_{BD}$$

$$\vec{r}_{B/A} = (0.8-0)\vec{i} + (0-0)\vec{j} + (1-1)\vec{k}$$

$$= 0.8\vec{i}$$

$$\vec{M}_A = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.8 & 0 & 0 \\ -74.17 & 92.71 & -74.17 \end{vmatrix}$$

$$\vec{M}_A = 59.34\vec{j} + 74.17\vec{k}$$

$$\vec{M}_E = \vec{r}_{B/E} \times \vec{T}_{BD}$$

$$\vec{r}_{B/E} = (0.8-0)\vec{i} + (0-0)\vec{j} + (1-0.75)\vec{k}$$

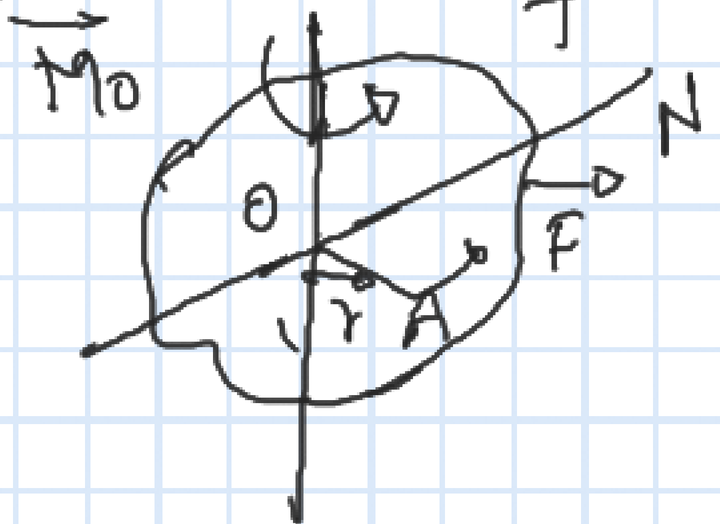
$$= +0.8\vec{i} + 0.25\vec{k}$$

$$\vec{M}_E = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ +0.8 & 0 & 0.25 \\ -74.17 & 92.71 & -74.17 \end{vmatrix}$$

$$= -23.18\vec{i} + 40.8\vec{j} + 74.17\vec{k}$$

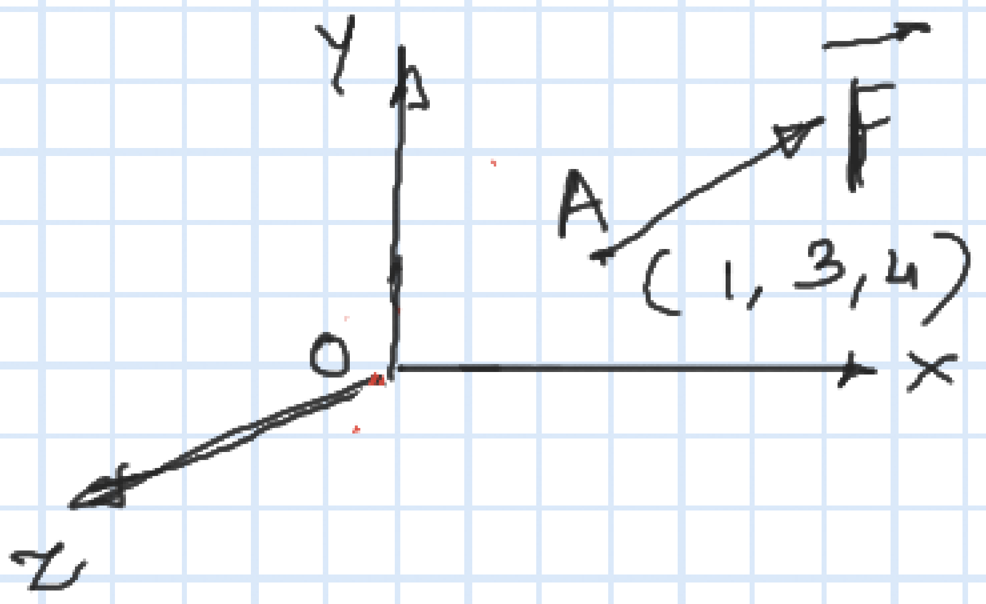
Moment of a force about an axis

(component of moment about any particular direction)



$$\vec{M}_{OA} = \lambda_{OA} \cdot \vec{M}_O$$

1. A force $\vec{F} = 3\vec{i} - 5\vec{j} + 7\vec{k}$ acts at point 'A' (1,3,4). Evaluate the moment produced by $\vec{F}$ about origin 'O' (0,0,0) and about the three coordinates axes.



~~sol~~

$$\vec{M}_O = \vec{r}_{O/A} \times \vec{F}$$

$$\vec{r}_{O/A} = (1-0)\vec{i} + (3-0)\vec{j} + (4-0)\vec{k}$$

$$\vec{M}_O = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & 4 \\ 3 & -5 & 7 \end{vmatrix}$$

$$\vec{M}_O = 41\vec{i} + 5\vec{j} - 14\vec{k}$$

The component of the moment about x-axis

$$M_{ox} = \vec{i} \cdot \vec{M_o} = \vec{i} \cdot (41\vec{i} + 5\vec{j} - 14\vec{k})$$

$$M_{ox} = 41$$

Similarly along y axis

$$M_{oy} = \vec{j} \cdot \vec{M_o} = \vec{j} \cdot (41\vec{i} + 5\vec{j} - 14\vec{k})$$

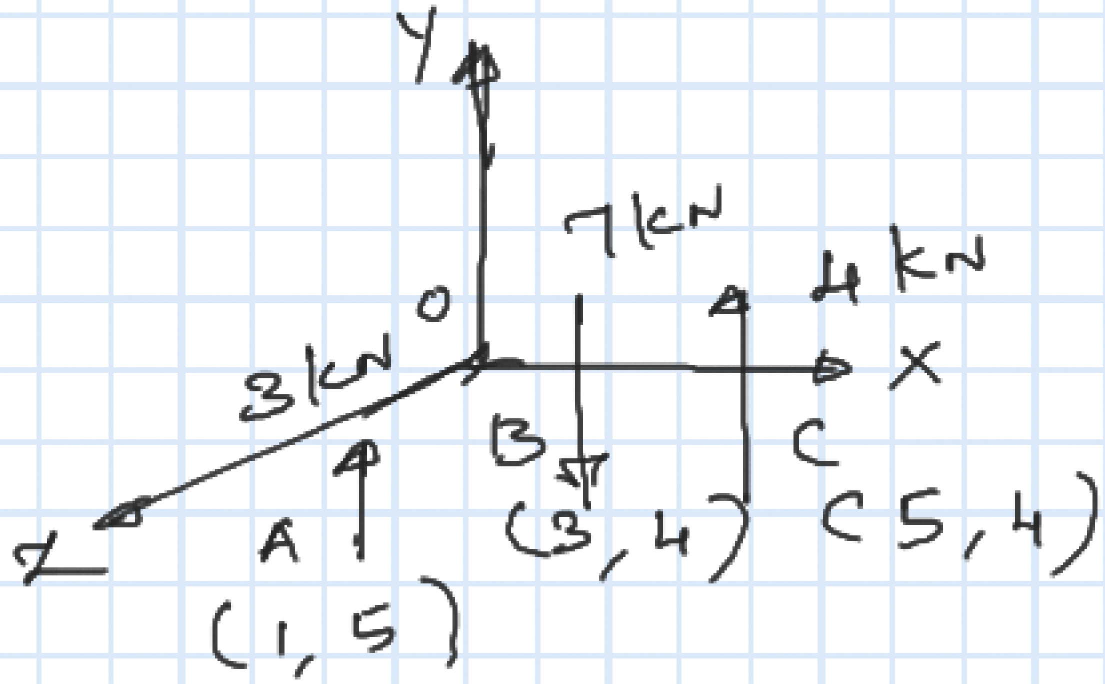
$$M_{oy} = 5$$

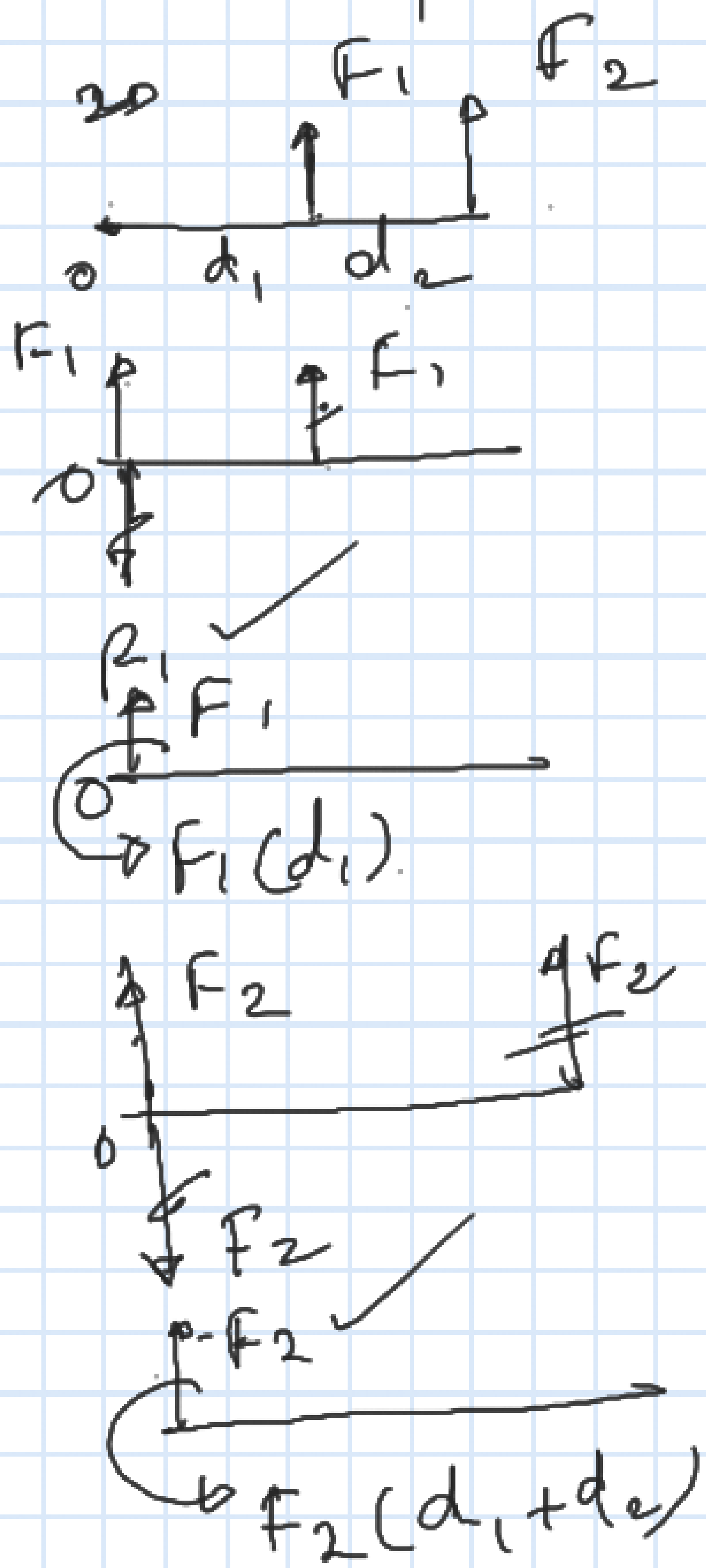
along z-axis

$$M_{oz} = \vec{k} \cdot \vec{M_o} = \vec{k} \cdot (41\vec{i} + 5\vec{j} - 14\vec{k})$$

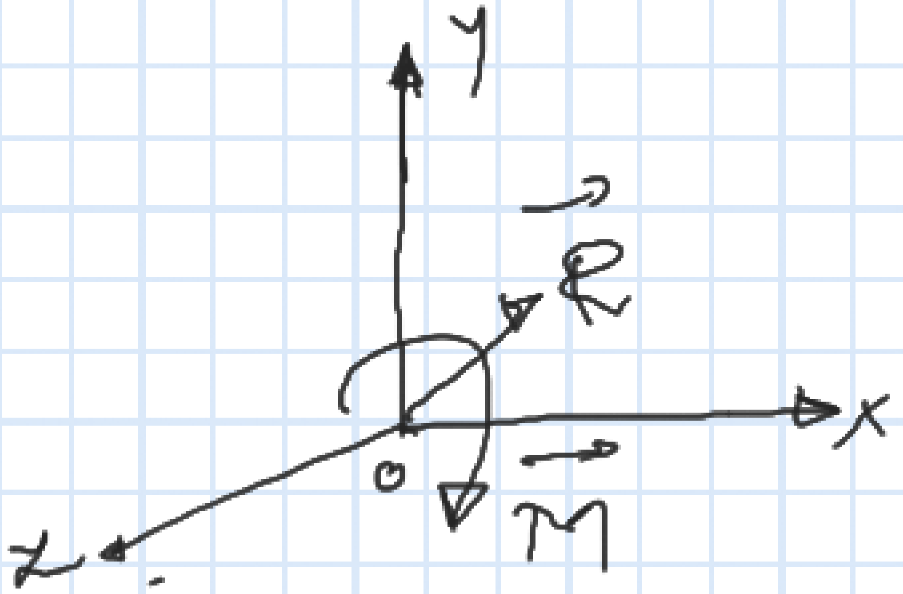
$$= -14$$

2. Determine the resultant of the given force with respect to origin





Sol



$$\vec{R} = \vec{F_A} + \vec{F_B} + \vec{F_C}$$

$$= 3\vec{j} - 7\vec{j} + 4\vec{j} = 0$$

$$\vec{M} = (\vec{r}_{OA} \times \vec{F_A}) + (\vec{r}_{O/B} \times \vec{F_B}) + (\vec{r}_{O/C} \times \vec{F_C})$$

$$\vec{r}_{O/A} = (\vec{i} + 5\vec{k})$$

$$\vec{r}_{OB} = (3\vec{i} + 4\vec{k})$$

$$r_{O/C} = (5\vec{i} + 4\vec{k})$$

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 5 \\ 0 & 3 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 0 & 4 \\ 0 & -7 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 0 & 4 \\ 0 & 4 & 0 \end{vmatrix}$$

$$= -3\vec{i} + 2\vec{k}$$

$$M = \sqrt{3^2 + 2^2} = 3.6 \text{ kNm}$$