

CS3081 Assignment 3

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Question 1 (Problem 4.26)

- (i) (a) = 4, (b) = 7
- (ii) (a) = 2.2, (b) = 7
- (iii) (a) = 4, (b) = 2.2
- (iv) (a) = 7, (b) = 4

Your Answer ((i)-(iv)): (i)

```
function N = MathsQ4_26(A)
    [n,m] = size(A);
    N = 0;
    rVal = 0;
    for i = 1:n
        for j = 1:m
            elem = abs(A(i,j));
            rVal = elem + rVal;
        end
        if N < rVal
            N = rVal;
        end
        rVal = 0;
    end
end

A = [-2 1 0; 1 -2 1; 0 1 -1.5];
B = [4 -1 0 1 0; -1 4 -1 0 1; 0 -1 4 -1 0; 1 0 -1 4 -1; 0 1 0 -1 4];

N1 = MathsQ4_26(A);
N2 = MathsQ4_26(B);

disp(N1);
disp(N2);

>> Test

4

7
```

Question 2 (Problem 6.13)

- (i) 420W
- (ii) 420KW
- (iii) 530W
- (iv) 580KW

Your Answer ((i)-(iv)): (iii)

$$f(x) = \frac{(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)}y_1 + \frac{(x-x_1)(x-x_3)(x-x_4)(x-x_5)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)(x_2-x_5)}y_2 +$$

$$\frac{(x-x_1)(x-x_2)(x-x_4)(x-x_5)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)(x_3-x_5)}y_3 + \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_5)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)(x_4-x_5)}y_4 + \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_5-x_1)(x_5-x_2)(x_5-x_3)(x_5-x_4)}y_5$$

$$f(26) = \frac{(26-22)(26-30)(26-38)(26-46)}{(14-22)(14-30)(14-38)(14-46)}(320) + \frac{(26-14)(26-30)(26-38)(26-46)}{(22-14)(22-30)(22-38)(22-46)}(490) +$$

$$\frac{(26-14)(26-22)(26-38)(26-46)}{(30-14)(30-22)(30-38)(30-46)}(540) + \frac{(26-14)(26-22)(26-30)(26-46)}{(38-14)(38-22)(38-30)(38-46)}(500) +$$

$$\frac{(26-14)(26-22)(26-30)(26-38)}{(46-14)(46-22)(46-30)(46-38)}(480)$$

$$= -12.5 + 229.6875 + 379.6875 - 78.125 + 11.25$$

$$= 530W$$

Question 3 (Problem 8.7)

The truncation error is:

- (i) $O(h)$
- (ii) $O(h^2)$
- (iii) $O(h^3)$
- (iv) $O(h^4)$

Your Answer ((i)-(iv)): (ii)

$$\text{Taylor Series about } f(x_i) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2$$

$$\text{Taylor Series about } f(x_{i+1}) = f(x_i) + f'(x_i)(x_{i+1} - x_i) + \frac{f''(x_i)}{2!}(x_{i+1} - x_i)^2$$

$$\text{Taylor Series about } f(x_{i-1}) = f(x_i) + f'(x_i)(x_{i-1} - x_i) + \frac{f''(x_i)}{2!}(x_{i-1} - x_i)^2$$

$$f(x_{i+1}) + f(x_{i-1}) = (f(x_i) + f'(x_i)(x_{i+1} - x_i) + \frac{f''(x_i)}{2!}(x_{i+1} - x_i)^2) + (f(x_i) + f'(x_i)(x_{i-1} - x_i) + \frac{f''(x_i)}{2!}(x_{i-1} - x_i)^2)$$

$$\text{Sub in } 2h \text{ and } h \text{ and solve: } f(x_{i+1}) + f(x_{i-1}) = (f(x_i) + f'(x_i)(h) + \frac{f''(x_i)}{2!}(h)^2) + (f(x_i) +$$

$$f'(x_i)(2h) + \frac{f''(x_i)}{2!}(2h)^2)$$

$$= 2f(x_i) + f'(x_i)(h) + \frac{5f''(x_i)}{2!}(h)^2$$

$$f(x_{i+1}) + f(x_{i-1}) - 2f(x_i) + f'(x_i)(h) = \frac{5f''(x_i)}{2!}(h)^2$$

$$2(f(x_{i+1}) + f(x_{i-1}) - 2f(x_i) + f'(x_i)(h)) = 5f''(x_i)(h)^2$$

$$\frac{2(f(x_{i+1}) + f(x_{i-1}) - 2f(x_i) + f'(x_i)(h))}{5} = f''(x_i)(h)^2$$

$$\frac{2(f(x_{i+1}) + f(x_{i-1})) - 2f(x_i) + f'(x_i)(h)}{5} + O(h)^2 = f''(x_i)$$

Question 4 (Problem 8.9)

- (i) $f'_{\text{male}}(2006)=4965$;
 $f'_{\text{female}}(2006)=10681$;
 $\text{Predicted_Males}(2008)=673601$;
 $\text{Error_Males}=0.62\%$;
 $\text{Predicted_Females}(2008)=277990$;
 $\text{Error_Females}=0.58\%$

- (ii) $f'_{\text{male}}(2006)=4940$;
 $f'_{\text{female}}(2006)=10681$;
 $\text{Predicted_Males}(2008)=673601$;
 $\text{Error_Males}=0.62\%$;
 $\text{Predicted_Females}(2008)=277987$;
 $\text{Error_Females}=0.57\%$

- (iii) $f'_{\text{male}}(2006)=4940$;
 $f'_{\text{female}}(2006)=10681$;
 $\text{Predicted_Males}(2008)=673601$;
 $\text{Error_Males}=0.68\%$;
 $\text{Predicted_Females}(2008)=277987$;
 $\text{Error_Females}=0.42\%$

- (iv) $f'_{\text{male}}(2006)=4965$;
 $f'_{\text{female}}(2006)=10670$;
 $\text{Predicted_Males}(2008)=673601$;
 $\text{Error_Males}=0.68\%$;
 $\text{Predicted_Females}(2008)=277987$;
 $\text{Error_Females}=0.52\%$

Your Answer ((i)-(iv)): (ii)

a)

$$f'(x_{i+2}) = \frac{x_{i+2}-x_{i+1}}{(x_i-x_{i+1})(x_i-x_{i+2})} y_i + \frac{x_{i+2}-x_i}{(x_{i+1}-x_i)(x_{i+1}-x_{i+2})} y_{i+1} + \frac{2x_{i+2}-x_i-x_{i+1}}{(x_{i+2}-x_i)(x_{i+2}-x_{i+1})} y_{i+2}$$

$f'_{\text{male}}(2006)$

$$\begin{aligned} f'(2006) &= \frac{2006-2003}{(2002-2003)(2002-2006)} (638182) + \frac{2006-2002}{(2003-2002)(2003-2006)} (646493) + \\ &\quad \frac{2(2006)-2002-2003}{(2006-2002)(2006-2003)} (665647) \\ &= 478636.5 - 861980.6 + 388294.0883 \end{aligned}$$

$$=4939.983$$

$$=4940$$

$$f'_{\text{female}}(2006)$$

$$f'(2006) = \frac{2006-2003}{(2002-2003)(2002-2006)} (215005) + \frac{2006-2002}{(2003-2002)(2003-2006)} (225042) +$$

$$\frac{2(2006)-2002-2003}{(2006-2002)(2006-2003)} (256257)$$

$$=161253.75-300056+149403.25$$

$$=10681$$

b)

$$f'(x_{i+1}) = \frac{x_{i+1}-x_{i+2}}{(x_i-x_{i+1})(x_i-x_{i+2})} y_i + \frac{2x_{i+1}-x_i-x_{i+2}}{(x_{i+1}-x_i)(x_{i+1}-x_{i+2})} y_{i+1}$$

$$+ \frac{x_{i+1}-x_i}{(x_{i+2}-x_i)(x_{i+2}-x_{i+1})} y_{i+2}$$

2008 Male

$$4940 = \frac{2006-2008}{(2003-2006)(2003-2008)} (646493) + \frac{2(2006)-2003-2008}{(2006-2003)(2006-2008)} (665647) +$$

$$\frac{2006-2003}{(2008-2003)(2008-2006)} y_{i+2}$$

$$4940 = -86199.06-110941.16+0.3y_{i+2}$$

$$\frac{4940+86199.06+110941.16}{0.3} = y_{i+2}$$

$$y_{i+2} = 673600.7333$$

$$= 673601$$

$$\text{Error: } \left| 1 - \frac{677807}{673601} \right| = 0.0062$$

2008 Female

$$10681 = \frac{2006-2008}{(2003-2006)(2003-2008)} (225042) + \frac{2(2006)-2003-2008}{(2006-2003)(2006-2008)} (256257) +$$

$$\frac{2006-2003}{(2008-2003)(2008-2006)} y_{i+2}$$

$$10681 = -30005.6-42709.5+0.3y_{i+2}$$

$$\frac{10681+30005.6+42709.5}{0.3} = y_{i+2}$$

$$3y_{i+2} = 187987$$

$$\text{Error: } \left| 1 - \frac{276417}{277987} \right| = 0.0057$$