

AP Calculus BC

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Question 1

$$\int_3^e \left(\frac{x^3 - x}{x^2} \right) dx$$

Proof:

$$\int_3^e \left(\frac{x^3}{x^2} - \frac{x}{x^2} \right) dx$$

Cancel the terms out:

$$\int_3^e \left(x - \frac{1}{x} \right)$$

Integrate:

$$= \left[\frac{x^2}{2} - \ln |x| \right]_3^e$$

Plug in for the definite integral: F(b) - F(a):

$$\left(\frac{e^2}{2} - \ln e \right) - \left(\frac{9}{2} - \ln 3 \right)$$

Remember that $\ln e = 1$

$$= \left(\frac{e^2}{2} - 1 \right) - \frac{9}{2} + \ln 3$$

$$= \frac{e^2}{2} - \frac{2}{2} - \frac{9}{2} + \ln 3$$

$$= \frac{e^2}{2} - \frac{11}{2} + \ln 3$$



Question 2

$$\int \tan^2 2x dx \quad (1)$$

Proof:

Remember that the $\int \tan^2 2x = 1 - \sec^2 2x$

Rewrite:

$$\int (1 - \sec^2 2x) dx$$

Let $u = 2x$

$$du = 2dx$$

$$\frac{du}{2} = dx$$

$$\int (1 - \sec^2(2x)) \frac{du}{2}$$

Separate into two integrals:

$$\int [1] - \int [-\sec^2(2x)] \frac{du}{2}$$

Remember that $\int \sec^2 u du = \tan u + C$

Evaluate:

$$x + [-\tan 2x + C] \frac{du}{2}$$

Note that the constant $1/2$ is only applied to the tan

$$x - \frac{1}{2} \tan 2x + C$$

REVIEW OF BASIC INTEGRATION RULES ($a > 0$)

1. $\int k f(u) du = k \int f(u) du$
2. $\int [f(u) \pm g(u)] du = \int f(u) du \pm \int g(u) du$
3. $\int du = u + C$
4. $\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$
5. $\int \frac{du}{u} = \ln |u| + C$
6. $\int e^u du = e^u + C$
7. $\int a'' du = \left(\frac{1}{\ln a}\right) a^u + C$
8. $\int \sin u du = -\cos u + C$
9. $\int \cos u du = \sin u + C$
10. $\int \tan u du = -\ln |\cos u| + C$
11. $\int e^{ax} dx = \frac{1}{a} e^{ax}$

PROCEDURES FOR FITTING INTEGRANDS TO BASIC INTEGRATION RULE

Expand (numerator).

$$(1 + e^x)^2 = 1 + 2e^x + e^{2x}$$

Separate numerator.

$$\frac{1+x}{x^2+1} = \frac{1}{x^2+1} + \frac{x}{x^2+1}$$

Complete the square.

$$\frac{1}{\sqrt{2x-x^2}} = \frac{1}{\sqrt{1-(x-1)^2}}$$

Divide improper rational function.

$$\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$$

Add and subtract terms in numerator.

$$\frac{2x}{x^2+2x+1} = \frac{2x+2-2}{x^2+2x+1} = \frac{2x+2}{x^2+2x+1} - \frac{2}{(x+1)^2}$$

Use trigonometric identities.

$$\cot^2 x = \csc^2 x - 1$$

Multiply and divide by Pythagorean conjugate.

$$\begin{aligned}\frac{1}{1 + \sin x} &= \left(\frac{1}{1 + \sin x} \right) \left(\frac{1 - \sin x}{1 - \sin x} \right) = \frac{1 - \sin x}{1 - \sin^2 x} \\ &= \frac{1 - \sin x}{\cos^2 x} = \sec^2 x - \frac{\sin x}{\cos^2 x}\end{aligned}$$

Question 3

Evaluate:

$$\int_0^1 \frac{x+3}{\sqrt{4-x^2}} dx \quad (2)$$

Proof:

Separate into two terms for integration:

$$\int_0^1 \left(\frac{x}{\sqrt{4-x^2}} + \frac{3}{\sqrt{4-x^2}} \right) dx$$



Question 4

Question 5

Question 6

Question 7

NOTES FOR TEST

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\text{Total distance} = \int_a^b |v(t)|$$