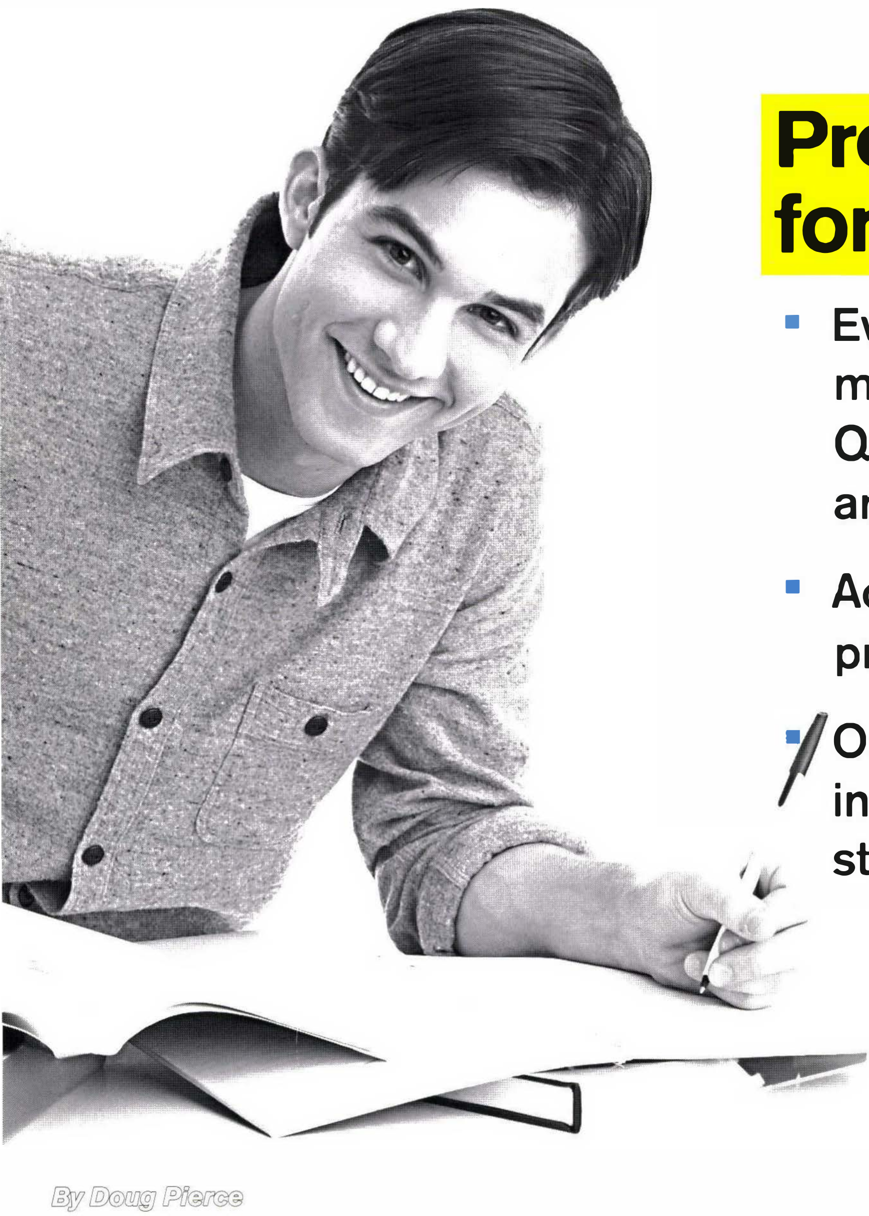
The Princeton Review

Cracking the GRE® 2013



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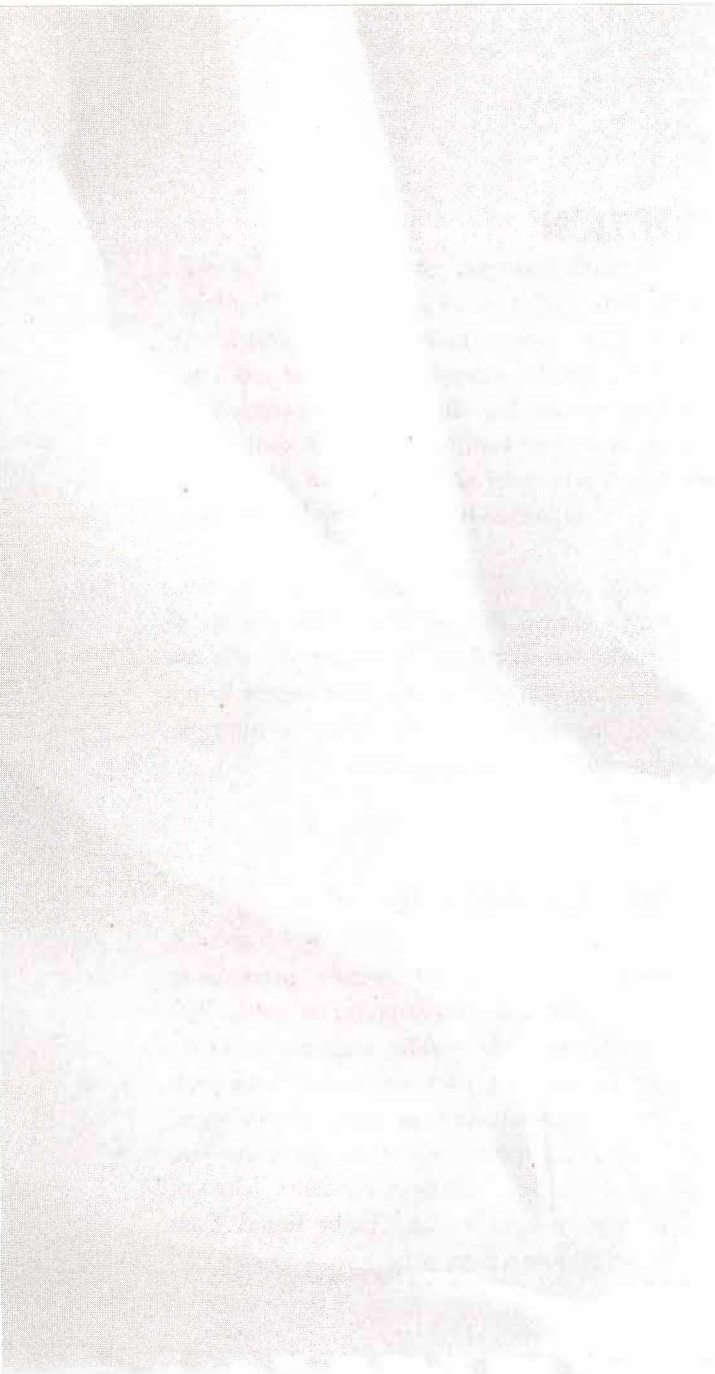
By Doug Pierce

Part III
How to Crack the Math Section



Chapter 8
The Geography of the Math Section

This chapter contains an overview of the content and structure you'll see on the Math sections of the GRE. It provides valuable information on pacing strategies and the various question formats you'll encounter on the GRE. It also goes over how to use basic test-taking techniques such as Process of Elimination and Ballparking as they relate to math questions. After finishing this chapter, you'll have a good idea of what the Math section of the GRE looks like and some basic approaches to help you navigate it.

WHAT'S IN THE MATH SECTION

The GRE Math section primarily tests math concepts you learned in seventh through tenth grades, including arithmetic, algebra, and geometry. ETS alleges that the Math sections on the new version of the exam better test the reasoning skills that you'll use in graduate school, but what the Math section primarily tests is your comfort level with some basic math topics and your ability to take a test with strange-looking questions under timed conditions.

The Math section of the exam consists of two 35-minute sections, each of which will consist of 20 questions. The first 7 or 8 questions of each section will be *quantitative comparisons* (quant comp, for short). The remainder will consist of multiple-choice or numeric-entry questions.

> Junior High School?
>
> The Math section of the GRE mostly tests how much you remember from the math courses you took in seventh, eighth, ninth, and tenth grades. But here's some good news: GRE math is easier than SAT math. Why? Because many people study little or no math in college. If the GRE tested college-level math, everyone but math majors would bomb the test.
>
> If you're willing to do a little work, this is good news for you. By brushing up on the modest amount of math you need to know for the test, you can significantly increase your GRE Math score. All you have to do is shake off the dust.

Predictable Triggers

The beauty of a standardized test is that it is, well, standardized. Standardized means predictable. We know exactly what ETS is going to test and how they're going to test it. The math side of the test consists of a series of utterly predictable triggers, to which we have designed a series of highly scripted responses. ETS wants you to see each problem as a new challenge to solve. What you will find, however, is that there are only about 20 math concepts that are being tested. All of the questions you will see are just different ways of asking about these different concepts. Most of these concepts you already know. Once you recognize what's being tested, even the trickiest questions become familiar and easy to solve.

It's Really a Reading Test

In constructing the Math section, ETS is limited to the math that nearly everyone has studied: arithmetic, basic algebra, basic geometry, and elementary statistics. There's no calculus (or even precalculus), no trigonometry, and no major-league algebra or geometry. Because of these limitations, ETS has to resort to traps in order to create hard problems. Even the most commonly missed GRE math problems are typically based on relatively simple principles. What makes the problems difficult is that these simple principles are disguised.

Many test takers have no problem doing the actual calculations involved in the math questions on the GRE; in fact, you'll even be allowed to use a calculator (more on that soon). However, on this test your ability to carefully read the problems and figure out how to set them up is more important than your ability to make calculations.

As you work through this section, don't worry about how quickly you're doing the problems. Instead, take the time to really understand what the questions are asking; pay close attention to the wording of the problems. Most math errors are the result of careless mistakes caused by not reading the problem carefully enough!

Read and Copy Carefully

You can do all the calculations right and still get a question wrong. How? What if you solve for x but the question asked for the value of $x + 4$? Ugh. Always reread the question before you choose an answer. Take your time and don't be careless. The problem will stay on the screen as long as you want it to, so reread the question and double-check your work before answering it.

Or how about this? The radius of the circle was 5, but when you copied the picture onto your scratch paper, you accidentally made it 6. Ugh! If you make a mistake copying down information from the screen, you'll get the question wrong no matter how perfect your calculations are. You have to be extra careful when copying down information.

THE CALCULATOR

As we mentioned before, on this new GRE you'll be given an on-screen calculator. The calculator program on the GRE is a rudimentary one that gives you the five basic operations: addition, subtraction, multiplication, division, and square root, plus a decimal function and a positive/negative feature. It also follows the order of operations, or PEMDAS (more on this topic in Chapter 9). The calculator also has the ability to transfer the answer you've calculated directly into the answer box for certain questions. The on-screen calculator can be a huge advantage—if it's used correctly!

As you might have realized by this point, ETS is not exactly looking out for your best interests. Giving you a calculator might seem like an altruistic act, but rest assured that ETS knows that there are certain ways in which calculator use can be exploited. Keep in mind the following:

1. **Calculators Can't Think.** Calculators are good for one thing and one thing only: calculation. You still have to figure out how to set up the problem correctly. If you're not sure what to calculate, then a calculator isn't helpful. For example, if you do a percent calculation on your calculator and then hit "Transfer Display," you will have to remember to move the decimal point accordingly, depending on whether the question asks for a percent or a decimal.
2. **The Calculator as a Liability.** ETS will give you questions that you can solve with a calculator, but the calculator can actually be a liability. You will be tempted to use it. For example, students who are uncomfortable adding, subtracting, multiplying, or dividing fractions may be tempted to convert all fractions to decimals using the calculator. Don't do it. You are better off mastering fractions than avoiding

them. Working with exponents and square roots is another way in which the calculator will be tempting but may yield really big and awkward numbers or long decimals. You are much better off learning the rules of manipulating exponents and square roots (there are only five rules). Most of these problems will be faster and cleaner to solve with rules than with a calculator. The questions may also use numbers that are too big for the calculator. Time spent trying to get an answer out of a calculator for problems involving really big numbers will be time wasted. Find another way around.

3. **A Calculator Won't Make You Faster.** Having a calculator should make you more accurate, but not necessarily faster. You still need to take time to read each problem carefully and set it up. Don't expect to blast through problems just because you have a calculator.
4. **The Calculator Is No Excuse for Not Using Scratch Paper.** Scratch paper is where good technique happens. Working problems by hand on scratch paper will help to avoid careless errors or skipped steps. Just because you can do multiple functions in a row on your calculator does not mean that you should be solving problems on your calculator. Use the calculator to do simple calculations that would otherwise take you time to solve. Make sure you are still writing steps out on your scratch paper, labeling results, and using set-ups. Accuracy is more important than speed!

You will score higher if you spend your time working carefully. Double-check your work before you hit confirm.

Of course, ·you should not fear the calculator; by all means, use it and be grateful for it. Having a calculator should help you eliminate all those careless math mistakes.

GEOGRAPHY OF A MATH SECTION

Math sections contain 20 questions each. Test takers are allowed 35 minutes per section. The first 7 or 8 questions of each math section are quantitative comparisons, while the remainder are a mixed bag of problem solving, all that apply, numeric entry, and charts and graphs. Each section covers a mixture of algebra, arithmetic, quantitative reasoning, geometry, and real-world math.

QUESTION FORMATS

Much like the Verbal section, the Math section on the GRE contains a variety of different question formats. Let's go through each question format and discuss how to crack it.

Standard Multiple Choice

These questions are the basic five-answer multiple-choice questions. These are great candidates for POE (Process of Elimination) strategies we will discuss later in this chapter.

Multiple Choice, Multiple Answer

These questions appear similar to the standard multiple-choice questions; however, on these you will have the opportunity to pick more than one answer. There may be anywhere from three to eight answer choices. Here's an example of what these will look like:

If $\frac{1}{12} < x < \frac{1}{6}$, then x could equal which of the following?

Indicate all such values.

- ☐ $\frac{2}{9}$
- ☐ $\frac{1}{5}$
- ☐ $\frac{1}{10}$
- ☐ $\frac{2}{15}$
- ☐ $\frac{2}{25}$

Your approach on these questions won't be radically different from the approach you use on standard multiple-choice questions. But obviously, you'll have to consider all of the answers—make sure you read each question carefully and remember that more than one answer can be correct. For example, for this question, you'd click on choices (C) and (D). You must select *every* correct choice to get credit for the problem.

Quantitative Comparison Questions

Quantitative comparison questions, hereafter affectionately known as "quant comp" questions, ask you to compare some Quantity A to some Quantity B. These questions have four answer choices instead of five, and all quant comp answer choices are the same. Here they are:

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Your job is to compare the two quantities and choose one of these answers.

Quant comp problems test the same basic arithmetic, algebra, and geometry concepts as do the other GRE math problems. So, to solve these problems, you'll apply the same techniques you use on the other GRE math questions. But quant comps also have a few special rules you need to remember.

There Is No "E"

Because there are only four choices on quant comp questions, after you use POE to eliminate all of the answer choices you can, your odds of guessing correctly are even better. Think about it this way: Eliminating even one answer on a quant comp question will give you a one-in-three chance of guessing correctly.

If a Quant Comp Question Contains Only Numbers, the Answer Can't Be (D)

Any quant comp problem that contains only numbers and no variables must have a single solution. Therefore, on these problems, you can eliminate choice (D) immediately because the larger quantity can be determined. For example, if you're asked to compare $\frac{3}{2}$ and $\frac{3}{4}$, you can determine which fraction is larger, so the answer cannot be (D).

Compare, Don't Calculate

Do only as much work as you need to.

You don't always have to calculate the exact value of each quantity before you compare them. After all, your mission is simply to compare the two quantities. It's often helpful to treat the two quantities as if they were two sides of an equation. Anything you can do to both sides of an equation, you can also do to both quantities. You can add the same number to both sides, you can multiply both sides by the same positive number, and you can simplify a single side by multiplying it by one.

If you can simplify the terms of a quant comp, you should always do so.

Here's a quick example:

Quantity A	**Quantity B**
$\frac{1}{16}+\frac{1}{7}+\frac{1}{4}$	$\frac{1}{4}+\frac{1}{16}+\frac{1}{6}$

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Here's How to Crack It

Don't do any calculating! Remember: Do only as much work as you need to in order to answer the question! The first thing you should do is eliminate (D). After all, there are only numbers here. After that, get rid of numbers that are common to both columns (think of this as simplifying). Both columns contain a $\frac{1}{16}$ and a $\frac{1}{4}$, so because we're talking about addition, they can't make a difference to the outcome. With them gone, you're merely comparing the $\frac{1}{7}$ in column A to the $\frac{1}{6}$ in column B. Now we can eliminate (C) as well—after all, there is no way that $\frac{1}{7}$ is equal to $\frac{1}{6}$. So, we're down to two choices, (A) and (B). If you don't remember how to compare fractions, don't worry—it's covered in Chapter 10 (Real World Math). The answer to this question is (B).

Okay, let's talk about another wacky question type you'll see in the Math section.

Numeric Entry

Some questions on the GRE won't even have answer choices, and you'll have to generate your own answer. Here's an example:

> Renaldo earns a monthly commission of 10.5% of his total sales for the month, plus a flat salary of $2,500. If Renaldo earns $3,025 in a certain month, what were his total sales? Disregard the $ sign when entering your answer.
>
> Click on the answer box and type in a number. Backspace to erase.

On this type of question, POE is not going to help you! That means if you're not sure how to do one of these questions, you should immediately move on. Leave it blank and come back to it in your second pass through the test.

To answer this question, you'd enter 5,000 into the box. Alternately, you could transfer your work directly from the on-screen calculator to the text box.

MAXIMIZE YOUR SCORE

As you're probably aware by now, doing well on the Math section will involve more than just knowing some math. It will also require the use of some good strategies. Let's go through some good strategies now; make sure you read this section carefully; it will be important for you to keep these techniques in mind as you work through the content chapters that follow this one!

The Two Roles of Techniques

The techniques are there to ensure that the questions that you should get right, you do get right. A couple of careless errors on easy questions will kill your score. The techniques are not just tools; they are proven standard approaches that save time and effort and guarantee points. Use these techniques on every question. Turn them into a habitual approach that you use every time.

Take the Easy Test First

The new GRE offers the opportunity to mark a question and return to it. Since all questions count equally toward your score, why not do the easy ones first? Getting questions right is far more important than getting to every question, so start with the low hanging fruit. There is no law that says you have to take the test in the order in which it is given. If you see a question you don't like, keep moving. Play to your strengths and get all of the questions that you're good at in the bank, before you start spending time on the hard ones. It makes no sense to spend valuable minutes wrestling with hard questions while there are still easy ones on the table. It makes even less sense if you end up having to rush some easy ones (making mistakes in the process), as a result. Free yourself from numerical hegemony! Take the easy test first!

Bend, Don't Push

Eighty percent of the errors on the math side of the test are really reading errors. It is a four-hour test and at some point during these four hours your brain is going to get tired. When this happens you will read, see, or understand questions incorrectly. Once you see a problem wrong, it is nearly impossible to see it correctly. When this happens, even simple problems can become extremely frustrating. If you solve a problem and your answer is not one of the choices, this is what has happened. When you would swear that a problem can't be solved, this is what has happened. When you have absolutely no idea how to solve a problem, this is what has happened. If you find yourself with half a page full of calculations and are no closer to the answer, this is what has happened. You are in La La Land. Once you are in La La Land, you can continue to push on that problem all day and you won't get any closer.

There is a good chance that you are already familiar with this frustration. The first step is to learn to recognize it when it is happening. Here are some keys to recognizing when you are off track.

You know you are in La La Land when...

- You have spent more than three minutes on a single problem.
- Your hand is not moving.
- You don't know what to do next.
- Your answer is not one of the choices.
- You're spending lots of time with the calculator and working with some ugly numbers.

Once you recognize that you are in La La Land, get out. Continuing to push on a problem, at this point, is a waste of your time. You could easily spend three or four precious minutes on this problem and be no closer to the answer. Spend those three or four minutes on other questions. That time should be yielding you points, not frustration.

After you have done two or three other questions, return to the one that was giving you trouble. Most likely, the reason it was giving you trouble is that you missed something or misread something the first time around. If the problem is still difficult, walk away again.

This is called Bend, Don't Push. The minute you encounter any resistance on the test, walk away. Bend. There are plenty of other easier points for you to get with that time. Then return to the problem a few questions later. It's okay to take two or three runs at a tough problem. If you run out of time before returning to the question, so be it. Your time is better spent on easier problems anyway, since all problems count the same.

Forcing yourself to walk away can be difficult, especially when you have already invested time in a question. You will have to train yourself to recognize resistance when it occurs, to walk away, and then to remember to come back. Employ this technique anytime you are practicing for the GRE. It will take some time to master. Be patient and give it a chance to work. With this technique, there are no questions that are out of your reach on the GRE.

POE: Ballparking and Trap Answers

Use Process of Elimination whenever you can on questions that are in standard multiple-choice format. Always read the answer choices before you start to solve a math problem because often they will help guide you—you might even be able to eliminate a couple of answer choices before you begin to calculate the answer.

Two effective POE tools are Ballparking and Trap Answers.

You Know More Than You Think

Say you were asked to find 30 percent of 50. Wait—don't do any math yet. Let's say that you glance at the answer choices and you see these:

- ○ 5
- ○ 15
- ○ 30
- ○ 80
- ○ 150

Think about it. Whatever 30 percent of 50 is, it must be less than 50, right? So any answer choice that's greater than 50 can't be right. That means you should eliminate both (D) and (E) before you even do any calculations! Thirty percent is less than half, so we can get rid of anything greater than 25, which means that choice (C) is gone too. What is 10% of 50? Eliminate choice (A). You're done. The only answer left is (B). This process is known as Ballparking. Remember that the answers are part of the question. There are more than four times the number of wrong answers on the GRE as there are right ones. If it were easy to find the right ones, you wouldn't need this book. It is almost always easier to identify and eliminate the wrong answers than it is to calculate the right one. Just make sure that you are using your scratch paper to eliminate answer choices instead of keeping track in your head.

Ballparking helps you eliminate answer choices and increases your odds of zeroing in on the correct answer. The key is to eliminate any answer choice that is "out of the ballpark."

Let's look at another problem:

A 100-foot rope is cut so that the shorter piece is $\frac{2}{3}$ the length of the longer piece. How many feet long is the shorter piece?

- 75
- $66\frac{2}{3}$
- 50
- 40
- $33\frac{1}{3}$

Here's How to Crack It

Now, before we dive into the calculations, let's use a little common sense. The rope is 100 feet long. If we cut the rope in half, each part would be 50 feet. However, we didn't cut the rope in half; we cut it so that there's a longer part and a shorter part. What has to be true of the shorter piece then? It has to be smaller than 50 feet. If it weren't, it wouldn't be shorter than the other piece. So looking at our answers, we can eliminate (A), (B), and (C) without doing any real math. That's Ballparking. By the way, the answer is (D) and you'll learn how to tackle this type of problem when you get to Chapter 9.

Trap Answers

ETS likes to include "trap answers" in the answer choices to their math problems. Trap answers are answer choices that appear correct upon first glance. Often these answers will look so tempting that you'll choose them without actually bothering to complete the necessary calculations. Watch out for this! If a problem seems way too easy, be careful and double-check your work.

What are the trap answers in the problem about the rope?

Look at the next problem:

The price of a jacket was reduced by 10%. During a special sale, the price was discounted another 10%. What was the total percentage discount from the original price of the jacket?

- ○ 15%
- ○ 19%
- ○ 20%
- ○ 21%
- ○ 25%

Here's How to Crack It

The answer might seem like it should be 20 percent. But wait a minute: Does it seem likely that the GRE is going to give you a problem that you can solve just by adding 10 + 10? Probably not. Choice (C) is a trap answer.

To solve this problem, imagine that the original price of the jacket was $100. After a 10 percent discount the new price is $90. But now when we take another 10 percent discount, we're taking it from $90, not $100. 10 percent of 90 is 9, so we take off another $9 from the price and our final price is $81. That represents a 19 percent total discount because we started with a $100 jacket. The correct answer is (B).

HOW TO STUDY

Make sure you learn the content of each of the following chapters before you go on to the next one. Don't try to cram everything in all at once. It's much better to do a small amount of studying each day over a longer period; you will master both the math concepts and the techniques if you focus on the material a little bit at a time.

Practice, Practice, Practice

Practice may not make perfect, but it sure will help. Use everyday math calculations as practice opportunities. Balance your checkbook without a calculator! Make sure your check has been added correctly at a restaurant, and figure out the exact percentage you want to leave for a tip. The more you practice simple adding, subtracting, multiplying, and dividing on a day-to-day basis, the more your arithmetic skills will improve for the GRE.

After you work through this book, be sure to practice doing questions on our online tests and on real GREs. There are always sample questions at **www.gre.org**, and practice will rapidly sharpen your test-taking skills.

Finally, unless you trust our techniques, you may be reluctant to use them fully and automatically on the real GRE. The best way to develop that trust is to practice before you get to the real test.

Need more practice?
The Princeton Review's *Math Workout for the New GRE*, Second Edition, includes hundreds of drill questions.

Summary

- The GRE contains two 35-minute Math sections. Each section has 20 questions.
- The GRE tests math concepts up to about the tenth-grade level of difficulty.
- You will be allowed to use a calculator on the GRE. The calculator is part of the on-screen display.
- The Math section employs a number of different question formats, including multiple choice, numeric entry, and quantitative comparison questions.
- Use the Two-Pass system on the Math section. Find the easier questions and do them first. Use your remaining time to work some of the more difficult questions.
- When you get stuck on a problem, walk away. Do a few other problems to distract your brain, and then return to the question that was giving you problems.
- Ballpark or estimate the answers to math questions and eliminate answers that don't make sense.
- Watch out for trap answers. If an answer seems too easy or obvious, it's probably a trap.
- Always do your work on your scratch paper, not in your head. Even when you are Ballparking, make sure that you are eliminating answer choices on your scratch paper. If your hand isn't moving, you're stuck and you need to walk away, or you're doing work in your head, which leads to errors.

Chapter 9
Numbers and Equations

Numbers and equations form the basis of all the math questions on the GRE. Simply put, the more comfortable you are working with numbers and equations, the easier the math portion of the exam will be. This chapter gives you a review of all the basic mathematical concepts including properties of numbers, factors and multiples, exponents and square roots, and lessons on manipulating and solving equations. This chapter also introduces you to one of The Princeton Review's key mathematical strategies: Plugging In.

IN THE BEGINNING...

...there were numbers. If you wish to do well on the GRE Math section, you'll have to be comfortable working with numbers. The concepts tested on the GRE are not exceptionally difficult, but if you are even the least bit skittish about numbers you'll have a harder time working the problems.

You may be a little rusty when it comes to working with numbers but you'll be surprised at how quickly you'll become comfortable again.

This chapter will familiarize you with all the basics you need to know about numbers and how to work with them. If you're a mathphobe or haven't used math in a while, take it slowly and make sure you're comfortable with this chapter before moving onto the succeeding ones.

GRE MATH VOCABULARY

Quick—what's an integer? Is 0 even or odd? How many even prime numbers are there?

Before we go through our techniques for specific types of math problems, we'll acquaint ourselves with some basic vocabulary and properties of numbers. The GRE loves to test your knowledge of integers, fractions, decimals, and all those other concepts you probably learned years ago. Make sure you're comfortable with the topics in this chapter before moving on. Even if you feel fairly at ease with number concepts, you should still work through this chapter. ETS is very good at coming up with questions that require you to know ideas forwards and backwards.

Learn your vocabulary!

The math terms we will review in this section are very simple, but that doesn't mean they're not important. Every GRE math question uses simple terms, rules, and definitions. You absolutely need to know this math "vocabulary." Don't worry; we will cover only the math terms that you *must* know for the GRE.

Digits

Digit refers to the numbers that make up other numbers. There are 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and every number is made up of one or more digits. For example, the number 10,897 has five digits: 1, 0, 8, 9, and 7. Each of the digits in a number has its own name, which is designated by a place value. In the number 10,897

- 7 is the ones or units digit.
- 9 is the tens digit.
- 8 is the hundreds digit.
- 0 is the thousands digit.
- 1 is the ten-thousands digit.

Numbers

A **number** is simply a digit or a collection of digits. There are, of course, an infinite number of numbers. Basically, any combination of digits you can imagine is a number, which includes 0, negative numbers, fractions and decimals, and even weird numbers such as $\sqrt{2}$.

GRE problems like to try to trip you up on the difference between a number and an integer.

Integers

The **integers** are the counting numbers, such as –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, and so on.

Notice that fractions, such as $\frac{1}{2}$, are not integers.

Remember: Fractions are NOT integers.

Remember that the number zero is an integer! Positive integers get bigger as they move away from 0 (6 is bigger than 5); negative integers get smaller as they move away from zero (–6 is smaller than –5).

PROPERTIES OF NUMBERS AND INTEGERS

Now that you've learned the proper names for various types of numbers, let's look at properties of numbers and integers.

Positive or Negative

Numbers can be positive or negative. Negative numbers are less than zero, while positive numbers are greater than zero. Zero, itself, is neither positive nor negative—all other numbers are one or the other.

Even or Odd

Only integers possess the property of being even or odd. Fractions, decimals, and other non-integers can never be even or odd. Integers that are even are those that are divisible by 2; odd integers are those integers that are not divisible by 2.

- Here are some even integers: –4, –2, 0, 2, 4, 6, 8, 10.
- Here are some odd integers: –3, –1, 1, 3, 5, 7, 9, 11.

Zero has a number of special properties that are tested frequently on the GRE. Technically, zero is a multiple of every number, but this fact is rarely tested on the GRE.

Zero

Zero is a special little number. It is an integer, but it is neither positive nor negative. However, try to remember these facts about zero:

- 0 is even.
- 0 plus any other number is equal to that other number.
- 0 multiplied by any other number is equal to 0.
- You cannot divide by 0.

Keep in Mind

- Fractions are neither even nor odd.
- Any integer is even if its units digit is even; any integer is odd if its units digit is odd.
- The results of adding and multiplying odd and even integers are as follows:
  - even + even = even
  - odd + odd = even
  - even + odd = odd
  - even × even = even
  - odd × odd = odd
  - even × odd = even

Be careful: Don't confuse odd and even with positive and negative!

If you have trouble remembering some of these rules for odd and even, don't worry. As long as you remember that there are rules, you can always figure them out by plugging in numbers. Let's say you forget what happens when an odd number is multiplied by an odd number. Just pick two odd numbers, say 3 and 5, and multiply them. $3 \times 5 = 15$. Now you know: odd × odd = odd.

Consecutive Integers

Consecutive integers are integers listed in order of increasing value without any integers missing in between them. Here are some examples:

- 0, 1, 2, 3, 4, 5
- −6, −5, −4, −3, −2, −1, 0
- −3, −2, −1, 0, 1, 2, 3

By the way, fractions and decimals cannot be consecutive, only integers can be consecutive. However, you can have different types of consecutive integers. For example consecutive even numbers could be 2, 4, 6, 8, 10. Consecutive multiples of four could be 4, 8, 12, 16.

Absolute Value

The **absolute value** of a number is equal to its distance from 0 on the number line, which means that the absolute value of any number is always positive, whether the number itself is positive or negative. The symbol for absolute value is a set of double lines: $|\ \ |$. Thus $|-5| = 5$, and $|5| = 5$.

FACTORS, MULTIPLES, AND DIVISIBILITY

Now let's look at some ways that integers are related to each other.

Factors

A **factor** of a particular number is a number that will divide evenly into the number in question. For example, 1, 2, 3, 4, 6, and 12 are all factors of 12 because each number divides evenly into 12. In order to find all the factors of a particular number, write down the factors systematically in pairs of numbers that, when multiplied together, make 12, starting with 1 and the number itself:

- 1 and 12
- 2 and 6
- 3 and 4

If you always start with 1 and the number itself and work your way up, you'll make sure you get them all.

Multiples

A **multiple** of a number is one that the number itself is a factor of. For example, the multiples of 8 are all the numbers of which 8 is a factor: 8, 16, 24, 32, 40 and so on and so on. Note that there are an infinite number of multiples for any given number. Also, zero is a multiple of every number, although this concept is rarely tested on the GRE.

There are only a few factors of any number; there are many multiples of any number.

Prime Numbers

A **prime number** is an integer that only has two factors: itself and one. Thus, 37 is prime because the only integers that divide evenly into it are 1 and 37, while 10 is not prime because its factors are 1, 2, 5, and 10.

Here is a list of all the prime numbers that are less than 30: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

- 0 is not a prime number.
- 1 is not a prime number.
- 2 is the only even prime number.
- Prime numbers are positive integers. There's no such thing as a negative prime number or a prime fraction.

1 is not prime!

DIVISIBILITY

A number is always divisible by its factors. If you're not sure if one number is divisible by another, a surefire way to find out is to use the calculator. However, there are also certain rules you can use to determine whether one number is a factor of another.

- An integer is divisible by 2 if its units digit is divisible by 2. For example, we know just by glancing at it that 598,447,896 is divisible by 2, because the units digit, 6, is divisible by 2.
- An integer is divisible by 3 if the sum of its digits is divisible by 3. For example, we know that 2,145 is divisible by 3 because 2 + 1 + 4 + 5 = 12, and 12 is divisible by 3.
- An integer is divisible by 4 if its last two digits form a number that's divisible by 4. For example, 712 is divisible by 4 because 12 is divisible by 4.
- An integer is divisible by 5 if its units digit is either 0 or 5.
- An integer is divisible by 6 if it's divisible by both 2 and 3.
- An integer is divisible by 8 if its last three digits form a number that's divisible by 8. For example, 11,640 is divisible by 8 because 640 is divisible by 8.
- An integer is divisible by 9 if the sum of its digits is divisible by 9.
- An integer is divisible by 10 if its units digit is 0.

Remainders

If one number is not divisible by another—meaning that the second number is not a factor of the first number—you'll have a number left over when you divide. This left-over number is called a **remainder**; you probably remember working with remainders in grade school.

If a question asks about a remainder, don't use the calculator. Use long division.

For example, when 4 is divided by 2, there's nothing left over so there's no remainder. In other words, 4 is divisible by 2. You could also say that the remainder is 0.

Five divided by 2 is 2, with 1 left over; 1 is the remainder. Thirteen divided by 8 is 1, with 5 left over as the remainder.

MORE MATH VOCABULARY

In a way, the Math section is almost as much of a vocabulary test as the Verbal section. Below, you'll find some more standard terms that you should commit to memory before you do any practice problems.

Term	Meaning
sum	the result of addition
difference	the result of subtraction
product	the result of multiplication
quotient	the result of division
divisor	the number you divide by
numerator	the top number in a fraction
denominator	the bottom number in a fraction
consecutive	in order from least to greatest
terms	the numbers used in an equation

BASIC OPERATIONS WITH NUMBERS

Now that you've learned about numbers and their properties, you're ready to begin working with them. As we mentioned above, there are four basic operations you can perform on a number: addition, subtraction, multiplication, and division.

Order of Operations

Unfortunately, when you work with numbers you can't just perform the four operations in any way you please. Instead, math has some very specific rules to follow, which are commonly referred to as the order of operations.

It is absolutely necessary that you perform these operations in exactly the right order. In many cases, the correct order will be apparent from the way the problem is written. In cases in which the correct order is not apparent, you need to remember the following mnemonic.

Please Excuse My Dear Aunt Sally, or PEMDAS.

What does PEMDAS stand for?

$$\begin{array}{c} \text{P} \mid \text{E} \mid \text{MD} \mid \text{AS} \\ \qquad\quad \rightarrow \;\; \rightarrow \end{array}$$

P stands for "parentheses." Solve anything in parentheses first.

E stands for "exponents." Solve exponents next. (We'll review exponents soon.)

M stands for "multiplication" and D stands for "division." The arrow indicates that you do all your multiplication and division together in the same step, going from left to right.

A stands for "addition" and S stands for "subtraction." Again, the arrow indicates that you do all your addition and subtraction together in one step, from left to right.

Let's look at an example:

$$12 + 4(2 + 1)^2 \div 6 - 7 =$$

Here's How to Crack It

Start by doing all the math inside the parentheses. $2 + 1 = 3$. Now the problem looks like this:

$$12 + 4(3)^2 \div 6 - 7 =$$

Next we have to apply the exponent. $3^2 = 9$. Now this is what we have:

$$12 + 4(9) \div 6 - 7 =$$

Now we do multiplication and division from left to right. $4 \times 9 = 36$, and $36 \div 6 = 6$, which gives us

$$12 + 6 - 7 =$$

Finally, we do the addition and subtraction from left to right. $12 + 6 = 18$, and $18 - 7 = 11$. Therefore,

$$12 + 4(2 + 1)^2 \div 6 - 7 = 11$$

Multiplication and Division

When multiplying or dividing, keep the following rules in mind:

- positive × positive = positive $\quad 2 \times 2 = 4$
- negative × negative = positive $\quad -2 \times -2 = 4$
- positive × negative = negative $\quad 2 \times -2 = -4$
- positive ÷ positive = positive $\quad 8 \div 2 = 4$
- negative ÷ negative = positive $\quad -8 \div -2 = 4$
- positive ÷ negative = negative $\quad 8 \div -2 = -4$

Before taking the GRE, you should have your times tables memorized from 1 through 15. It will be a tremendous advantage if you can quickly and confidently figure out, for example, what 7 × 12 is (it's 84).

It seems like a small thing, but memorizing your times tables will really help you on test day.

A FEW LAWS

These two basic laws are not necessary for success on the GRE, so if you have trouble with them, don't worry too much. However, ETS likes to use these laws to make certain math problems more difficult to work with. If you're comfortable with these two laws, you'll be able to simplify problems using them, so it's definitely worth it to use them.

Associative Laws

There are two associative laws—one for addition and one for multiplication. For the sake of simplicity, we've lumped them together.

Here's what you need to know:

> When you are adding or multiplying a series of numbers, you can regroup the numbers in any way you'd like.

Write everything down on scratch paper! Don't do anything in your head!

Here are some examples:

$$4 + (5 + 8) = (4 + 5) + 8 = (4 + 8) + 5$$
$$(a + b) + (c + d) = a + (b + c + d)$$
$$4 \times (5 \times 8) = (4 \times 5) \times 8 = (4 \times 8) \times 5$$
$$(ab)(cd) = a(bcd)$$

Distributive Law

This is often tested on the GRE. Here's what it looks like:

$$a(b + c) = ab + ac$$
$$a(b - c) = ab - ac$$

Here's an example:

$$12(66) + 12(24) = ?$$

Here's How to Crack It

This is in the same form as $ab + ac$. Using the distributive law, this must equal $12(66 + 24)$, or $12(90) = 1{,}080$.

EXPONENTS AND SQUARE ROOTS

Exponents and square roots are a popular topic on the GRE. Here's the information you need to know in order to work with them.

What Are Exponents?

Exponents are a sort of mathematical shorthand for repeated multiplication. Instead of writing (2)(2)(2)(2), you can use an exponent and write 2^4. The little 4 is the **exponent** and the 2 is called the **base**. If you're stuck on an exponent problem, it's often helpful to write out the repeated multiplication: When in doubt, expand it out!

There are only five rules for exponents:

1. $a^2 = a \cdot a$
2. $a^2 \cdot a^3 = (a \cdot a)(a \cdot a \cdot a) = a^{2+3} = a^5$
3. $(a^2)^3 = (a \cdot a)\ (a \cdot a)\ (a \cdot a) = a^{2 \cdot 3} = a^6$
4. $\dfrac{a^2}{a^3} = \dfrac{a \cdot a}{a \cdot a \cdot a} = \dfrac{1}{a} = a^{2-3} = a^{-1}$
5. $15^{12} - 15^{11} = 15^{11}(15 - 1) = 15^{11}(14)$

Multiplication with Exponents

It's simple to multiply two or more numbers that are raised to exponents, as long as they have the same base. In this situation, all you have to do is add the exponents. Consider this example:

$$2^2 \times 2^4 =$$
$$2^{2+4} = 2^6$$

You can see that this is true when you expand it out, which is just as good a way to solve the problem:

$$2^2 \times 2^4 =$$
$$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$$

Be careful, though. This rule does not apply to addition. $2^2 + 2^4$ does not equal 2^6. There's no quick and easy method for adding numbers with exponents.

Division with Exponents

Dividing two or more numbers with the same base that are raised to exponents is simple, too. All you have to do is subtract the exponents. Study the following example:

$$2^6 \div 2^2 = 2^{6-2} = 2^4$$

You can see that this is true when you expand it out:

$$2^6 \div 2^2 = (2 \times 2 \times 2 \times 2 \times 2 \times 2) \div (2 \times 2) = 2 \times 2 \times 2 \times 2 = 2^4$$

Once again, don't assume this same shortcut applies to subtraction of numbers with exponents. It doesn't.

Another time you might need to divide with exponents is when you see a negative exponent. In this situation, you just put 1 over it (in other words, take its reciprocal) and get rid of the negative. For example,

$$3^{-2}$$

should be rewritten as

$$\frac{1}{3^2}$$

and this gives us

$$\frac{1}{9}$$

Exponents and Parentheses

When there are exponents inside and outside the parentheses, you simply multiply them:

$$(4^5)^2 =$$
$$4^{5 \times 2} =$$
$$4^{10}$$

This is what the shorthand notation is really telling us to do:

$$(4^5)^2 =$$
$$(4 \times 4 \times 4 \times 4 \times 4)(4 \times 4 \times 4 \times 4 \times 4) = 4^{10}$$

Remember that the exponent applies to *everything* inside the parentheses. For example, $(3x)^2 = (3x)(3x) = 9x^2$, not $3x^2$. The same is true of fractions within parentheses: $\left(\frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)\left(\frac{3}{2}\right) = \frac{9}{4}$.

Factoring with Exponents

ETS loves to give you exponents that are too big to calculate on the calculator they provide. Whenever you see large exponents added or subtracted, look to factor. For example, does $4^4 - 4^3 = 4$? Well, let's see. The expression 4^4 is 256, and 4^3 is 64. Therefore, $256 - 64$ does not equal 4 because the first term, 4^4, is literally four *times* larger than the second term, 4^3. However, inside your 4^4 you have a 4^3 that you can factor out.

So, when you see this: $15^{12} - 15^{11}$

do this: $15^{11}(15 - 1)$, or $15^{11}(14)$

$15^{12} - 15^{11} = 15^{11}(14)$

The Peculiar Behavior of Exponents

- Raising a number greater than 1 to a power greater than 1 results in a bigger number. For example, $2^2 = 4$.
- Raising a fraction that's between 0 and 1 to a power greater than 1 results in a smaller number. For example, $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$.
- A negative number raised to an even power becomes positive. For example, $(-2)^2 = 4$, because $(-2)(-2) = 4$.

- A negative number raised to an odd power remains negative. For example, $(-2)^3 = -8$, because $(-2)(-2)(-2) = -8$.
- A number raised to a negative power is equal to 1 over the number raised to the positive power. For example, $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$.
- A number raised to the 0 power is 1, no matter what the number is. For example, $1{,}000^0 = 1$. Note, however, that 0 to the 0 power is undefined.
- A number raised to the first power is ALWAYS the number itself. For example, $1{,}000^1 = 1{,}000$.

Here's an example of a question you might see on the GRE:

If $a \neq 0$, then $\frac{(a^6)^2}{a \bullet a^2} =$

- ○ a^5
- ○ a^6
- ○ a^7
- ○ a^8
- ○ a^9

Always cross off wrong answer choices on your scratch paper.

Here's How to Crack It

In the numerator, we have $(a^6)^2$, which is a^{12}. In the denominator, we have $a \bullet a^2$, which is a^3. So, $a^{12} \div a^3 = a^9$. That's choice (E).

Let's try another—this time, a quant comp:

Quantity A	Quantity B
27^4	9^6

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

ALWAYS write down A, B, C, D for quant comps.

Here's How to Crack It

Looks scary, huh? But remember what you learned about quant comp problems in the math introduction. Your job is to compare the two quantities, not calculate their values. First of all, eliminate (D)—when just numbers are being compared, the answer can always be determined. Now, as they're written, we can't compare these exponents—they don't have the same base. But we can fix that. Both 27 and 9 are powers of 3: 27 is $3 \times 3 \times 3$, so 27^4 is $(3 \times 3 \times 3)^4$. This equals $(3 \times 3 \times 3)(3 \times 3 \times 3)(3 \times 3 \times 3)(3 \times 3 \times 3)$, also known as 3^{12}. That takes care of Quantity A. In Quantity B, 9 is 3×3, so 9^6 is $(3 \times 3)^6$. This equals $(3 \times 3)(3 \times 3)(3 \times 3)(3 \times 3)(3 \times 3)(3 \times 3)$, also known as 3^{12}. So, we have 3^{12} in Quantity A and 3^{12} in Quantity B. They're equal, and the answer is (C).

What Is a Square Root?

The sign $\sqrt{}$ indicates the **square root** of a number. For example, $\sqrt{2}$ means that some value, squared, equals 2.

If $x^2 = 16$, then $x = \pm 4$. You must be especially careful to remember this on quantitative comparison questions. But when ETS asks you for the value of $\sqrt{16}$, or the square root of any number, it is asking you for the positive root only. Although squaring –5 will result in 25, just as squaring 5 will, when ETS asks for $\sqrt{25}$, the only answer it's looking for is 5.

Playing with Square Roots

You can multiply and divide any square roots, but you can add or subtract roots only when they are the same.

You multiply and divide square roots just like you would any other number.

$$\sqrt{3} \times \sqrt{12} = \sqrt{36} = 6$$

$$\sqrt{\frac{16}{4}} = \frac{\sqrt{16}}{\sqrt{4}} = \frac{4}{2} = 2$$

However, you can't add or subtract square roots unless the roots are the same.

So, $\sqrt{2} + \sqrt{2} = 2\sqrt{2}$. (Just pretend there's an invisible 1 in front of the root sign.) But $\sqrt{2} + \sqrt{3}$ does **not** equal $\sqrt{5}$. In order to add different roots, you need to estimate their values first and then add them. We'll cover how to estimate roots in the pages to come.

Here's an example:

$$z^2 = 144$$

Quantity A	Quantity B
z	$\sqrt{144}$

- Quantity A is greater.
- Quantity B is greater.
- The two quantities are equal.
- The relationship cannot be determined from the information given.

Here's How to Crack It

You want to pick choice (C), don't you? After all, if z^2 is 144, then the square root of 144 must be z, right? Not so fast. If $z^2 = 144$, then z could be either 12 or –12. But when the radical sign ($\sqrt{\ }$) is used, only the positive root is being referred to. Therefore, Quantity A is equal to 12 or –12, but Quantity B is 12. And that gives us (D) as the answer.

Estimating and Simplifying Roots

When you have a perfect square, such as 25 or 36, finding the square root is easy. $\sqrt{25} = 5$ and $\sqrt{36} = 6$. But what about finding $\sqrt{32}$? You could use your calculator, although that may be too time-consuming. Since 32 is between 25 and 36, you can estimate that $\sqrt{32}$ must be between $\sqrt{25}$ and $\sqrt{36}$. So $\sqrt{32}$ is somewhere between 5 and 6. You also know that 32 is closer to 36 than it is to 25, so $\sqrt{32}$ will be closer to 6 than it is to 5, and will probably be about 5.6 or 5.7 (it's actually 5.66). This process of estimating roots for numbers that aren't perfect squares can be extremely helpful in eliminating answer choices through Ballparking.

The other thing you might be able to do with a root is simplify it. As we've seen, 32 isn't a perfect square, but one of its factors is a perfect square. 32 can be split into 16×2, which means that $\sqrt{32}$ is the same thing as $\sqrt{16 \times 2}$. We can get the square root of 16 and move that outside the square root symbol, giving us $4\sqrt{2}$. $4\sqrt{2}$ has exactly the same value as $\sqrt{32}$, it's just written in simpler form. Since, on the GRE, answer choices will nearly always be in simplest terms, it's important to know how to do this.

Try the following problem:

$$\frac{\sqrt{75}}{\sqrt{27}} =$$

- ○ $\frac{5}{3}$
- ○ $\frac{25}{9}$
- ○ 3
- ○ $3\sqrt{3}$
- ○ $3\sqrt{5}$

Here's How to Crack It

First, let's try to simplify each of these roots. $\sqrt{75}$ has a factor that is a perfect square—25, so it can be rewritten as $\sqrt{25 \times 3}$ and simplified to $5\sqrt{3}$. $\sqrt{27}$ has the perfect square 9 as a factor, so it can be written as $\sqrt{9 \times 3}$ and then simplified to $3\sqrt{3}$. This means that $\frac{\sqrt{75}}{\sqrt{27}}$ is equal to $\frac{5\sqrt{3}}{3\sqrt{3}}$; the $\sqrt{3}$ in the top and bottom will cancel, leaving you with $\frac{5}{3}$. The answer is (A).

Learn These Four Values

To make calculations of square roots easier, you should memorize the following values. You should be able to recite them without hesitation.

$$\sqrt{1} = 1$$

$$\sqrt{2} = 1.4$$

$$\sqrt{3} = 1.7$$

$$\sqrt{4} = 2$$

You'll see them again when we discuss geometry, in Chapter 11.

ALGEBRA: OPERATIONS WITH LETTERS

Algebra is simply a way of performing operations without numbers; in algebraic expressions, a **variable** stands in for the missing number or numbers. While the GRE Math section is not by and large an algebra test, you should be comfortable with the basics of working with equations.

Dealing with Variables

Now that you've familiarized yourself with number concepts, it's time to put your knowledge to work. So far, we've been showing you how to manipulate numbers, but many GRE math problems involve variables (such as n, x, or y). It's time to learn how to deal with those.

Manipulating Equations

When working with equations, you can do pretty much anything you want to them as long as you follow the golden rule:

> Whatever you do on one side of the equals sign you must also do on the other side.

Don't assume you'll always need to solve for the variable on the GRE; sometimes you'll simply have to manipulate the equation to get the answer.

Solving for One Variable

You can solve equations that have just one variable. In these cases, you start by isolating the variable on one side of the equation and the numbers on the other side. You can do this by adding, subtracting, multiplying, or dividing both sides of the equation by the same number. Just remember that anything you do to one side of an equation, you must do to the other side. Be sure to write down every step. Let's look at a simple example:

$$3x - 4 = 5$$

Here's How to Crack It

In this case, you can collect all the constants on the right side of the equation by adding 4 to both sides of the equation. (If for some reason you wanted to move the 5 to the left side of the equation, you would have to subtract 5 from both sides. That's just how it works.) In general, you can eliminate negative numbers by adding them to both sides of the equation, just as you can eliminate positives by subtracting them from both sides of the equation.

$$\begin{array}{r} 3x - 4 = 5 \\ \underline{+4 = +4} \\ 3x = 9 \end{array}$$

The above rule also applies to numbers in the equation that are divided or multiplied. So in this case, in order to get rid of the 3 that's multiplied by the variable, x, we would need to divide both sides of the equation by 3 to solve for x.

$$\frac{3x}{3} = \frac{9}{3}$$

$$x = 3$$

Let's try another one:

$$5x - 13 = 12 - 20x$$

Here's How to Crack It

Again, we want to get all the x values on the same side of the equation:

$$\begin{array}{rcl} 5x - 13 &=& 12 - 20x \\ +\ 20x & & +\ 20x \\ \hline 25x - 13 &=& 12 \end{array}$$

Now let's get rid of that negative 13:

$$\begin{array}{rcl} 25x - 13 &=& 12 \\ +\ 13 & & +\ 13 \\ \hline 25x &=& 25 \end{array}$$

It might be pretty obvious that x is 1, but let's just finish it:

$$25x = 25$$

$$\frac{25x}{25} = \frac{25}{25}$$

$$x = 1$$

Let's try another one:

$$5x + \frac{3}{2} = 7x$$

Here's How to Crack It

First multiply both sides by 2 to get rid of the fraction. Remember to multiply all of the members of the equation!

$$10x + 3 = 14x$$

Now collect the x's on the same side:

$$\begin{array}{rcl} 10x + 3 &=& 14x \\ -10x & & -\ 10x \\ \hline 3 &=& 4x \end{array}$$

Always write A, B, C, D, E on your scratch paper to represent the answer choices (or A, B, C, D if it's quant comp).

You must always do the same thing to both sides of an equation.

Now finish it up:

$$3 = 4x$$
$$\frac{3}{4} = \frac{4x}{4}$$
$$\frac{3}{4} = x$$

INEQUALITIES

In an equation, one side is always equal to another. In an inequality, one side of the equation does *not* equal the other. Equations contain equal signs, while inequalities contain one of the following symbols:

The point of the inequality sign always points to the smaller value.

$\neq$	is not equal to
$>$	is greater than
$<$	is less than
$\geq$	is greater than or equal to
$\leq$	is less than or equal to

You can manipulate any inequality in the same way you can an equation, with one important difference. When you multiply or divide both sides of an inequality by a negative number, the direction of the inequality symbol must change. That is, if $x > y$, then $-x < -y$.

To see what we mean, take a look at this simple inequality:

$$12 - 6x > 0$$

Here's How to Crack It

You could manipulate this inequality without ever multiplying or dividing by a negative number by just adding $6x$ to both sides. The sign stays the same. Then divide both sides by positive 6. Again, the sign stays the same.

$$\begin{array}{r} 12 - 6x > 0 \\ \underline{+\ 6x > +\ 6x} \\ 12 > 6x \end{array}$$

$$\frac{12}{6} > \frac{6x}{6}$$

$$2 > x$$

But suppose you subtract 12 from both sides at first:

$$\begin{array}{r} 12 - 6x > 0 \\ \underline{-12 > -12} \\ -6x > -12 \end{array}$$

$$\frac{-6x}{-6} < \frac{-12}{-6}$$

$$x < 2$$

Flip the sign! When you multiply or divide both sides of an inequality by a negative number, the greater than/less than sign points the opposite way.

Notice that the sign flipped because you divided both sides by a negative number. But the answer means the same thing: The first answer says that the number 2 is greater than x, and the second says that x is less than the number 2!

Sometimes, ETS will give you a range for two variables and then combine them in some way. It looks something like this:

If $0 < x < 10$, and $-10 < y < -1$, then what is the range for $x - y$?

Here's How to Crack It.

First, treat the inequality sign like an equal sign. You need all possible combinations of $x - y$, which means that you need the biggest x minus the biggest y, the biggest x minus the smallest y, the smallest x minus the biggest y, and the smallest x minus the smallest y. There is a simple set-up to do this.

On your scratch paper write the following:

	x		y	x-y
B		B		
B		S		
S		B		
S		S		

Now just solve for $x - y$. When you're done, the biggest and smallest numbers are your answers.

	x		y	x-y
B	10	B	-1	11
B	10	S	-10	20
S	0	B	-1	1
S	0	S	-10	10

The range for $x - y$, therefore is $1 < x - y < 20$. Check your answer choices and eliminate.

WORKING WITH TWO VARIABLES

So far we've dealt with simple equations that involve only one variable. But on the GRE you'll sometimes have to deal with equations with two variables. Here's an example:

$$3x + 10y = 64$$

Here's How to Crack It

The important thing to note about this situation is that we cannot solve this equation. Why, you ask? The problem is that since there are two variables, there are many possible solutions to this equation and we have no way of knowing which solutions are correct. For example, the values $x = 8$ and $y = 4$ satisfy the equation. But so do the values $x = 10$ and $y = 3.4$. Which solutions are correct? We just don't know. In order to solve equations with two variables, we need two equations. Having two equations allows us to find definitive values for our variables.

You can't solve an equation with two variables unless you have a second equation.

$$3x + 10y = 64$$
$$6x - 10y = 8$$

When we're given two equations, we can combine them by adding or subtracting them. We do this so that we can cancel out one of the variables, leaving us with a simple equation with one variable. In this case, it's easier to add the two equations together:

$$\begin{array}{l} 3x + 10y = 64 \\ \underline{6x - 10y = 8} \\ 9x \qquad\ \ = 72 \end{array}$$

When we add these two equations we get $9x = 72$. This is a simple equation which we can solve to find $x = 8$. Once we've done that, we plug that value back into one of the equations and solve for the other variable. Substituting $x = 8$ into either equation gives us $y = 4$.

Try this one:

$$4x + 7y = 41$$
$$2x + 3y = 19$$

Here's How to Crack It

You might notice that if we add or subtract the two equations, we won't be left with one variable: Adding the two yields $6x + 10y = 60$. That doesn't help. Subtracting the equations leaves $2x + 4y = 22$. No help there, either. In cases like this one, you'll have to manipulate one of the equations so that subtracting or adding gets rid of one of the variables. In this case, let's multiply the second equation by 2:

$$2(2x + 3y) = 2(19)$$

This gives us the following:

$$4x + 6y = 38$$

Now we can subtract this equation from the first equation, yielding $y = 3$. If we substitute $y = 3$ into either of the equations we find that $x = 5$.

Quadratic Equations

Quadratic equations are special types of equations that involve, as the name suggests, four terms. Here is an example of a quadratic:

$$(x + 4)(x - 7)$$

In order to work with quadratics on the GRE, you must be familiar with two concepts: FOIL and factoring.

FOIL

When you see two sets of parentheses, all you have to do is remember to multiply every term in the first set of parentheses by every term in the second set of parentheses. Use FOIL to remember this method. FOIL stands for *first*, *outer*, *inner*, *last*—the four steps of multiplication. For example, if you see $(x + 4)\ (x + 3)$, you would multiply the first terms $(x \times x)$, the outer terms $(x \times 3)$, the inner terms $(4 \times x)$, and the last terms (4×3), as follows:

$$(x \times x) + (x \times 3) + (4 \times x) + (4 \times 3)$$
$$x^2 + 3x + 4x + 12$$
$$x^2 + 7x + 12$$

This also works in the opposite direction. For example, if you were given $x^2 + 7x + 12 = 0$, you could solve it by breaking it down as follows:

$$(x\ +\ \ )\ (x\ +\ \ ) = 0$$

We know to use plus signs inside the parentheses because both the 7 and the 12 are positive. Now we have to think of two numbers that, when added together, give us 7, and when multiplied together, give us 12. Yep, they're 4 and 3:

$$(x + 4)\ (x + 3) = 0$$

Note that there are two solutions for x. So x can either be -4 or -3.

Quadratic Equations

There are three expressions of quadratic equations that can appear on the GRE. You should know them cold, in both their factored and unfactored forms. Here they are:

1. Factored form: $x^2 - y^2$ (the difference between two squares)
   Unfactored form: $(x + y)(x - y)$
2. Factored form: $(x + y)^2$
   Unfactored form: $x^2 + 2xy + y^2$
3. Factored form: $(x - y)^2$
   Unfactored form: $x^2 - 2xy + y^2$

Let's see how this could be used on the GRE:

If x and y are positive integers, and if $x^2 + 2xy + y^2 = 25$, then $(x + y)^3 =$

- ○ 5
- ○ 15
- ○ 50
- ○ 75
- ○ 125

Here's How to Crack It

Problems like this one are the reason you have to memorize those quadratic equations. The equation in this question is Expression 2 in the box from the previous page: $x^2 + 2xy + y^2 = (x + y)^2$. The question tells us that $x^2 + 2xy + y^2$ is equal to 25, which means that $(x + y)^2$ is also equal to 25. Think of $x + y$ as one unit that, when squared, is equal to 25. Since this question specifies that x and y are positive integers, what positive integer squared equals 25? Right, 5. So $x + y = 5$. The question is asking for $(x + y)^3$. In other words, what's 5 cubed, or $5 \times 5 \times 5$? It's 125. Choice (E).

Here's another one:

Quantity A	Quantity B
$(4 + \sqrt{6})(4 - \sqrt{6})$	10

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Here's How to Crack It

First, eliminate choice (D)—we have only numbers here, so the answer can be determined. Now, Quantity A looks like a job for FOIL! Multiply the first terms, and you get 16. Multiply the outer terms and you get $-4\sqrt{6}$. Multiply the inner terms and you get $4\sqrt{6}$. Multiply the last terms and you get –6. So, we have $16-4\sqrt{6}+4\sqrt{6}-6$. Those two inner terms cancel each other out, and we're left with 16 – 6, or 10. What do you know? That's what we have in Quantity B, too! So, the answer is (C). You might also notice that Quantity A is common quadratic Expression 1 from the box on page 198: $(x+y)(x-y)=x^2-y^2$. Therefore, $(4+\sqrt{6})(4-\sqrt{6}) = 4^2 - \sqrt{6}^2 = 16 - 6 = 10$.

Factoring

The process of factoring "undoes" the FOIL process. Here is a quadratic in its unfactored, or expanded, form:

$$x^2 - 10x + 24$$

From this point, we can factor a quadratic by taking the following steps:

1. Separate the x^2 into $(x \quad)(x \quad)$.
2. Find the factors of the third term that, when added or subtracted, yield the second term.
3. Figure out the signs (+/–) for the terms. The signs have to yield the middle number when added and the last term when multiplied.

If we apply these steps to the expression above, we first set up the problem by splitting x^2 into

$$(x \quad)(x \quad)$$

Next, write down the factors of the third term, 24. The factors are 1 and 24, 2 and 12, 3 and 8, and 4 and 6. Of these pairs of factors, which contains two numbers that we can add or subtract to get the second term, 10? 4 and 6 are the only two that work. That gives us

$$(x \quad 4)(x \quad 6)$$

The final step is to figure out the signs. We need to end up with a negative 10 and a positive 24. If we add –6 and –4, we'll get –10. Similarly, if we multiply –6 and –4, we'll end up with 24. So the answer is

$$(x-4)(x-6)$$

Solving Quadratic Equations

ETS likes to use quadratic equations because they have an interesting quirk; often when you solve a quadratic equation, you get not one answer, but two. This property makes quadratic equations perfect ways for ETS to try to trick you.

Here's an example:

Quadratic equations often have two solutions.

$$x^2 + 2x - 15 = 0$$

Quantity A	Quantity B
2	x

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Here's How to Crack It

In order to solve a quadratic equation, the equation must be set equal to zero. Normally, this will already be the case on the GRE, as it is in this example. But if you encounter a quadratic equation that isn't set equal to zero, you must first manipulate the equation so that it is. Next you must factor the equation; otherwise you cannot solve it. So let's factor the quadratic equation in this example. We need to figure out the factors of 15 that we can add or subtract to give us 2. The only possible factors are 3 and 5. In order to get a negative 15 and a positive 2, we need to use 5 and –3. So that leaves us

$$(x - 3)(x + 5) = 0$$

Next, we're going to solve each of the two expressions within parentheses separately:

$$x - 3 = 0 \text{ and } x + 5 = 0$$

Thus, $x = 3$ and $x = -5$. This means that in this particular problem, the answer must be (D). If $x = 3$, then Quantity B is greater, but if $x = -5$ then Quantity A is greater.

Let's try another one:

If $x^2 + 8x + 16 = 0$, then $x =$

Click on the answer box and type in a number.
Backspace to erase.

Here's How to Crack It

Let's factor the equation. Start with $(x\quad)(x\quad)$. Next, find the factors of 16 that add or subtract to 8. The factors of 16 are 1 and 16, 2 and 8, and 4 and 4. Of these pairs, only 4 and 4 work. Since we have a positive 8 and a positive 16, the signs for both numbers must be positive. Thus, we end up with $(x + 4)(x + 4) = 0$. Now, we need to solve the equation. If $x + 4 = 0$, then $x = -4$. This is the number we'd enter into the text box on the GRE.

Simultaneous Equations

ETS will sometimes give you two equations and ask you to use them to find the value of a given expression. Don't worry; you won't need any math-class algebra; in most cases, all you will have to do to find ETS's answer is to add or subtract the two equations.

Here's an example:

If $5x + 4y = 6$ and $4x + 3y = 5$, then what does $x + y$ equal?

Here's How to Crack It

All you have to do is add together or subtract one from the other. Here's what we get when we add them:

$$\begin{array}{r} 5x + 4y = 6 \\ +\ 4x + 3y = 5 \\ \hline 9x + 7y = 11 \end{array}$$

A dead end. So let's try subtracting them:

$$\begin{array}{r} 5x+4y=6 \\ -\,4x+3y=5 \\ \hline x+\;\;y=1 \end{array}$$

Bingo. The value of the expression $(x + y)$ is exactly what we're looking for. On the GRE, you may see the two equations written horizontally. Just rewrite the two equations, putting one on top of the other, and then simply add or subtract them.

PLUGGING IN

Some of the hardest questions you might encounter on the GRE involve algebra. Algebra questions are generally difficult for two reasons. First, they are often complicated, multistep problems. Second, the answer choices often involve "distractor" choices. These are answer choices that look right, but they are actually wrong. They're designed to tempt you or to influence how you think about a problem.

If you don't like algebra, you're in luck. You don't have to do it. Plugging In will take even the hardest, messiest GRE problem and turn it into a simple arithmetic problem. It will never let you down, and it will never take more than a minute per problem.

Here are the steps:

Step 1: **Recognize the opportunity.** You can Plug In on any problem that has variables in the answer choices. The minute you see variables in the answers, even before you have read the problem, you know you can Plug In.

Why Plug In?

Plugging In is a powerful tool that can greatly enhance your math score, but you may be wondering why you should plug in when algebra works just fine. Here's why:

Plugging In converts algebra problems into arithmetic problems. No matter how good you are at algebra, you're better at arithmetic. Why? Because you use arithmetic every day, every time you go to a store, balance your checkbook, or tip a waiter. Chances are you rarely use algebra in your day-to-day activities.

Plugging In is more accurate than algebra. By Plugging In real numbers, you make the problems concrete rather than abstract. Once you're working with real numbers, it's easier to notice when and where you've messed up a calculation. It's much harder to see where you went wrong (or to even know you've done something wrong) when you're staring at a bunch of x's and y's.

The GRE allows the use of a calculator. A calculator can do arithmetic but it can't do algebra, so Plugging In allows you to take advantage of the calculator function.

ETS expects its students to attack the problems algebraically and many of the tricks and the traps built into the problem are designed to catch students who do the problems with algebra. By Plugging In, you'll avoid these pitfalls.

As you can see, there are a number of excellent reasons for Plugging In. Mastering this technique can have a significant impact on your score.

Step 2: **Engage the Hand.** You cannot solve Plugging In problems in your head. Even if it seems like an easy question of translating a word problem into an algebraic equation, remember that there are trap answer choices. When a question pops up, the minute you see variables, list your answer choices, A–E, on your scratch paper.

Step 3: **Plug In.** If the question asks for "x apples," come up with a number for x. The goal here is to make your life easier, so Plug In something simple and happy, but avoid 1 or 0. If you Plug In a number and the math starts getting creepy (anything involving fractions or negative numbers is creepy), don't be afraid to just change the number you Plug In. Always label each variable on your scratch paper.

Step 4: **ID Target Number.** The Target Number is the value the problem asks you to solve for. Once you you've arrived at a Target Number, write it down on your scratch paper and circle it.

Step 5: **Check All Answer Choices.** Anywhere you see a variable, Plug In the number you have written down for that variable. Do any required math. The correct answer is the one that matches your target number. If more than one answer matches your target number, just Plug In a different number for your variables and test the remaining answer choices.

Can I Just Plug In Anything?

Plug In numbers that will make the math EASY.

You can Plug In any numbers you like, as long as they're consistent with any restrictions stated in the problem, but it's faster if you use easy numbers. What makes a number easy? That depends on the problem. In most cases, smaller numbers are easier to work with than larger numbers. Usually, it's best to start small, with 2, for example. Avoid 0 and 1; both 0 and 1 have special properties, which you'll hear more about later. You want to avoid these numbers because they will often make more than one answer choice appear correct. For example, if we Plug In 0 for a variable, then the answers $2x$, $3x$, and $5x$ would all equal 0. If you avoid these bad number choices, you should also avoid these bad situations. Also, do not Plug In any numbers that show up a lot in the question or answer choices.

Try this one. Read through the whole question before you start to Plug In numbers:

The price of a certain stock increased 8 points, then decreased 13 points, and then increased 9 points. If the stock price before the changes was x points, which of the following was the stock price, in points, after the changes?

- $x - 5$
- $x - 4$
- $x + 4$
- $x + 5$
- $x + 8$

Here's How to Crack It

Let's use an easy number like 10 for the variable (write down "$x = 10$" on your scratch paper!). If the original price was 10, and then it increased 8 points, that's 18. Then it decreased 13 points, so now it's 5 (do everything out on the scratch paper—don't even add or subtract in your head). Then it increased 9 points, so now it's 14. So, it started at 10 and ended at 14. Circle 14 (our target answer) and Plug In 10 for every x in the answer choices. Which one gives you 14?

(A) $10 - 5 = 5$—Nope.
(B) $10 - 4 = 6$—Nope.
(C) $10 + 4 = 14$—Bingo!
(D) $10 + 5 = 15$—Nope.
(E) $10 + 8 = 18$—Nope.

Pretty easy, huh?

Don't skip steps! Use your scratch paper.

Good Numbers Make Life Easier

Small numbers aren't always the best choices for Plugging In, though. In a problem involving percentages, for example, 10 and 100 are good numbers to use. In a problem involving minutes or seconds, 30 or 120 are often good choices. (Avoid 60, however; it tends to cause problems.) You should look for clues in the problem itself to help you choose good numbers.

Always Plug In when you see variables in the answer choices!

On the GRE, Plugging In is often safer, and easier, than doing the algebra.

What's your target number?

Let's work through the following problem, using the steps above:

Mara has six more than twice as many apples as does Robert and half as many apples as does Sheila. If Robert has x apples, then, in terms of x, how many apples do Mara, Robert, and Sheila have?

- $2x + 6$
- $2x + 9$
- $3x + 12$
- $4x + 9$
- $7x + 18$

Here's How to Crack It

Step 1: **Identify the Opportunity.** You're sitting in your cubical at the Prometric testing center and this question pops up. What do you see? The variable, *x*, is in both the question and the answer choices. Good, so what do you do?

Step 2: **Engage the Hand.** On the upper left-hand corner of your scratch paper, list answer choices (A) through (E).

Step 3: **Plug In.** The problem tells us that Robert has *x* apples, so Plug In a number for *x*. Make it something nice and happy. Try 4. On your scratch paper, write $x = 4$.

Step 4: **ID Target Number.** The problem tells us that "Mara has six more than twice as many apples as does Robert." If Robert has 4 apples, then Mara must have 14. On your scratch paper, write $m = 14$. We are also told that Mara has "half as many apples as does Sheila." Ignoring the weird diction, that means that Sheila must have 28 apples. Write down $s = 28$. Now, what does the question ask you to find? It asks for the number of apples that Mara, Robert, and Sheila have. That's no problem; add the three up to come up with 46 apples. This is your target number. Write it down and circle it.

Step 5: **Check All Answer Choices.** You are allowed to perform only one mathematical function in your head at a time. Anything more than that leads to trouble. For the first answer choice, therefore, you can do $2x$ in your head; that's 8, but write down 8 + 6. You don't need to go any farther than that because this clearly will not add up to 46. Cross off choice (A). Choice (B) gives you 8 + 9. Cross that off. Choice (C) is 12 + 12. This is also too small, so cross it off. Choice (D) gives you 16 + 9. That gets you to 25, which is not your target number, so cross it off. Choice (E) is 28 + 18. Do this on your scratch paper or with the calculator. Do NOT do it in your head. It equals 46, which is your target number. Choice (E) is the correct answer.

(A) 2(4) + 6 = 14—This is not 46, so eliminate it.
(B) 2(4) + 9 = 17—No good either.
(C) 3(4) + 12 = 24—Still not 46.
(D) 4(4) + 9 = 25—This isn't 46 either.
(E) 7(4) + 18 = 46—Bingo! This is your answer.

On the GRE, you can Plug In any time the question has variables in the answer choices. You can usually Plug In any number you wish, although you should always pick numbers that will be easy to work with. Some numbers can end up causing more trouble than they're worth.

When a problem has variables in the answer choices, PLUG IN!

When Plugging In, follow these rules:

1. Don't Plug In 0 or 1. These numbers, while easy to work with, have special properties.
2. Don't Plug In numbers that are already in the problem; this might confuse you as you work through it.
3. Don't Plug In the same number for multiple variables. For example, if a problem has x, y, and z in it, pick three different numbers to Plug In for the three variables.

Finally, Plugging In can be a powerful tool, but you must remember to always check all five answer choices when you plug in. In certain cases, two answer choices can yield the same target number. This doesn't necessarily mean you did anything wrong; you just hit some bad luck. Plug In some new numbers, get a new target and recheck the answers that worked the first time.

PLUGGING IN THE ANSWERS (PITA)

Some questions may not have variables in them but will try to tempt you into applying algebra to solve them. We call these Plugging In The Answers, or PITA for short. These are almost always difficult problems. Once you recognize the opportunity, however, they turn into simple arithmetic questions. In fact, the hardest part of these problems is often identifying them as opportunities for PITA. The beauty of these questions is that they take advantage of one of the inherent limitations of a multiple-choice test. ETS has actually given you the answers, and one of them must be correct. In fact, only one can work. The essence of this technique is to systematically Plug In The Answers into the question to see which answer choice works.

Here are the steps:

Step 1: **Recognize the Opportunity.** There are three ways to do this. The first triggers are the phrases "how much…," "how many…," or "what is the value of…." When you see one of these phrases in a question, you can Plug In The Answers. The second tip-off is specific numbers in the answer choices in ascending or descending order. The last tip-off is your own inclination. If you find yourself tempted to write your own algebraic formulas and to invent your own variables to solve the problem, it's a sure bet that you can just Plug In The Answer choices.

Step 2: **Engage the Hand.** The minute you recognize the opportunity, list the numbers in the answer choices in a column in the upper left-hand corner of your scratch paper.

Step 3: **Label the First Column.** What do these numbers represent? The question asks you to find a specific number. The answer choices are this number. At the top of the column, write down what these numbers represent.

Step 4: **Assume (C) to be Correct.** Choice (C) will always be the number in the middle. This the most efficient place to start because it will allow you to eliminate as many as three answer choices if it is wrong.

Step 5: **Create Your Spreadsheet.** Assuming the number in choice (C) is correct, use this number to work through the problem. It is always easier to understand the problem using a specific number. Work through the problem in bite-size pieces, and every time you have to do something with the number, make a new column. You can't have too many columns. Each column is a step in solving the problem.

Step 6: **Rinse and Repeat.** On single-answer multiple-choice questions, only one answer choice can work. If choice (C) is correct, you are done. If it is not correct, you may be able to determine if it is too big or too small. If it is too big, you can eliminate it and every answer choice that is bigger. This very quickly get's you down to a 50/50 shot. It also gives you a little spreadsheet specifically designed to calculate the correct answer. When you need to check the remaining answer choices, let the spreadsheet do the thinking for you. All you need to do is to fill in the cells. As soon as you find an answer choice that works, you're done.

Are you tempted to do algebra? Are there numbers in the answer choices? Plug In The Answers.

The following is an example of a PITA problem:

An office supply store charged $13.10 for the purchase of 85 paper clips. If some of the clips were 16 cents each and the remainder were 14 cents each, how many of the paper clips were 14-cent clips?

- ○ 16
- ○ 25
- ○ 30
- ○ 35
- ○ 65

Here's How to Crack It

Step 1: **Recognize the Opportunity.** The question asks "how many of the paper clips...." That's your first sign. Additionally, you have specific numbers in the answer choices in ascending order.

Step 2: **Engage the Hand.** The minute you recognize this as a PITA question, list your answer choices in a column on your scratch paper.

Step 3: **Label the First Column.** What do those answer choices represent? They are the number of 14-cent clips, so label this column 14¢.

Step 4: **Assume (C) to be Correct.** Start with choice (C) and assume that 30 of the clips were 14 cents each.

Step 5: **Create Your Spreadsheet.** If 30 of the clips were 14 cents each, then the purchaser would have spent \$4.20 on 14-cent clips. Label this column "amount spent." Now you know that there were 85 clips total, so if 30 of the clips were 14 cents each, there must have been 55 clips that were 16 cents each. Write down a 55 and label this column 16¢. The purchaser then spent \$8.80 on 16-cent clips. Write this down and label this column "amount spent." You can now calculate the total spent. 4.20 + 8.80 = 13.00. Write this down and label this column "total."

Step 6: **Rinse and Repeat.** You know that the purchaser spent \$13.10 on paper clips. If answer choice C were correct, then the purchaser would have spent only \$13.00 on paper clips. Since you know this is wrong, choice (C) cannot be correct. Cross it off. You also know that your total is too small. You need a greater portion of your clips to be the more expensive ones to get a higher total, so cross off choices (D) and (E). Now try choice (B). If 25 of the clips cost 14 cents each, the purchaser would have spent \$3.50. There must have been 60 clips that cost 16 cents each (85 – 25 = 60). Then, the purchaser would have spent \$9.60 on them. The total spent on clips, therefore, comes to \$13.10, and you're done.

Make sure to keep your hand moving, to write down all steps, and to use your calculator for simple arithmetic steps like multiplying and adding complex numbers. Here's what your scratch paper should look like after this problem:

14¢	Amt.	16¢	Amt.	Tot.
16				
↘ 25	3.50	60	9.60	\$13.10
→ ~~30~~	4.20	55	8.80	\$13.00
~~35~~				
~~65~~				

On PITA questions, you can stop once you've found the correct answer; you don't have to check all five answer choices. Just make sure you write EVERYTHING down when doing these questions (and, indeed, all math questions).

PLUGGING IN ON QUANTITATIVE COMPARISON QUESTIONS

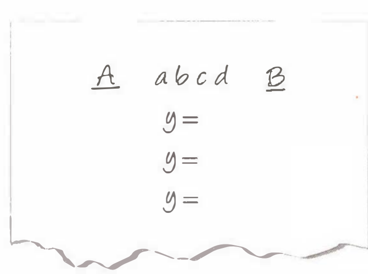
Quantitative Comparison questions with variables can be extremely tricky because the obvious answer is often wrong, whereas the correct answer may be a scenario most people would never think of. On the other hand, there is a simple set-up and approach that you can use that ensures that you get these questions right without taking too much time. As always, whenever you see variables, replace them with real numbers. On quant comp questions, however, it is crucial that you Plug In more than once and specifically that you Plug In all of the weird and obscure numbers that you would never use elsewhere. Always keep the nature of the answer choices in mind. Picking choice (A) means that you believe that the quantity in column A will *always* be bigger—*no matter what you Plug In.* Choice (B) means that the quantity in column B will *always* be bigger—*no matter what you Plug In,* and so forth. To prove that one of these statements is true you have to Plug In every possible number that could change the outcome. Don't worry. We have a simple process to help figure out what to Plug In and how to track your progress as you do.

Quantitative Comparison questions often test your knowledge of the properties of fractions, zero, one, negatives, and other odd numbers.

Here are the steps:

Step 1: **Recognize the Opportunity**. The first six, seven, or eight questions of any math section will be quant comp. When a quant comp question pops up and you see variables, make your set-up.

Step 2: **Engage the Hand**. The minute you see quant comp and variables make your set-up in the upper left hand of your scratch paper. Your set-up looks like this:



Step 3: **Plug In and Eliminate**. Start with something nice and happy. Pay close attention to the rules the question gives you for what you are allowed to Plug In, and start with a simple, happy number. With a number for the variable, calculate the value in Quantity A and write it down. Then calculate the value in Quantity B and write it down. If Quantity A is bigger, eliminate choices (B) and (C). If Quantity B is bigger, eliminate (A) and (C). If they are both the same, eliminate choices (A) and (B). Note that you are already down to a 50/50 shot.

Step 4: **Rinse and Repeat.** There are still two answer choices left, so you're not done yet. The second time you Plug In, you want to try to get a different result. What can you Plug In the second time that messes with the problem? If you're not sure, use this simple check list: ZONE F. This stands for Zero, One, Negative, Extremely Big or Small, and Fractions. You won't always be allowed to Plug In all of these and rarely will you have to. Your goal is to eliminate choices (A), (B), and (C). If you Plug In everything on the checklist and (A), (B), or (C) is still standing, the one that's still standing is your answer.

The easiest way to solve most quant comp questions that involve variables is to Plug In, just as you would on word problems. But because answer choice (D) is always an option, you always have to make sure it isn't the answer. So...

On quant comp, Plug In "normal" numbers, and eliminate two choices. Then Plug In "weird" numbers (zero, one, negatives, fractions, or big numbers) to try to disprove your first answer. If different numbers give you different answers, you've proved that the answer is (D).

Always Plug In at Least Twice in Quant Comp Questions

Plugging In on quant comp questions is just like Plugging In on "must be" problems. The reason for this is (D). On quant comp questions, it's not enough to determine whether one quantity is sometimes greater than, less than, or equal to the other; you have to determine whether it *always* is. If different numbers lead to different answers, then the correct answer is (D). To figure out if one quantity is always bigger, you have to Plug In weird numbers to account for all possible situations.

What makes certain numbers weird? They behave in unexpected ways when added, multiplied, or raised to powers. Here are some examples:

- 0 times any number is 0.
- 0^2 is 0.
- 1^2 is 1.
- $\left(\frac{1}{2}\right)^2$ is less than $\frac{1}{2}$.
- $(-2)(-2)$ is 4.
- A negative number squared is positive.
- Really big numbers (100, 1,000) can make a really big difference in your answer.

Here's how it works:

Quantity A	Quantity B
$2x^3$	$4x^2$

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Here's How to Crack It

Step 1: **Recognize the Opportunity.** First you see quant comp. Second, you see variables. It takes all of three seconds to recognize a quant comp Plug In. You don't even have to understand the problem at this point. Just recognize the opportunity.

Step 2: **Engage the Hand.** The minute you recognize this as a quant comp Plug In, make your set-up on your scratch paper. List "x =" three times down the middle.

Step 3: **Plug In.** Let's start with something nice and happy, like 2. Write down 2 next to your first x. When $x = 2$, the quantity in column A is 16 ($2 \cdot 2^3$), and the quantity in column B is also 16 ($4 \cdot 2^2$). Since you have followed the rules and both quantities are the same, neither (A) nor (B) can be the answer. Cross them off. Note that you haven't worked very hard yet, haven't spent much time at all, and you are already down to a 50/50 shot.

Step 4: **Rinse and Repeat.** Now try something different for x. What if $x = 1$? The quantity in column A will be 2, and the quantity in column B will be 4. In this case, they are not the same, so choice (C) cannot be the correct answer. Cross it off. Only choice (D) is left, so you're done.

Using Set-Ups

Quant comp questions with variables are tricky because they require you to think through every possible number that could be used for each variable. If you forget to account for a few of the less obvious possibilities, you get the question wrong. This is why you can't outthink ETS. They are very good at this, and they have tested and refined all of their questions on thousands of students to figure out how best to fool you. Fortunately, you can out-process ETS. A good process will get you to the right answer every time without taxing your brain and without taking up too much time. This is where set-ups come in. The set-up, or what you do with your hand, will give structure to what you do with your brain when you approach the problem.

To watch a short video of the process in action, register your book at PrincetonReview.com/cracking.

Here is what your scratch paper should look like:

A	~~a~~ ~~b~~ ~~c~~ d	B
16	x = 2	16
2	x = 1	4
	x =	

You might also have noticed that Plugging In $x = 0$ would also yield different results. On quant comp questions, ETS hopes you'll forget to consider what happens when you use numbers such as 0, 1, fractions, and negatives. Therefore, when Plugging In, make sure to use the following ZONE F numbers whenever possible:

Zero
One
Negatives
Extreme Values
Fractions

Make sure you use them aggressively on quant comp problems because they can radically affect the relationship between the two quantities.

Phew. Now we've covered the basics of mathematical operations; hopefully a lot of this material came back to you as we went through it, but if not don't worry! You'll have plenty of opportunities to refresh your memory of this material as you read through the next two chapters and work the problems you see in the drills.

In the next chapter we'll look at some everyday math topics that are tested on the GRE, so practice the techniques in the drill that follows, and move on!

Numbers and Equations Drill

Ready to try out your new skills? Give this drill a shot and then check your answers in Part V.

1 of 10

A prime number, p, is multiplied by itself and is added to the next prime number greater than p. Which of the following could be the resulting sum?

Indicate all possible values.

- ☐ 3
- ☐ 4
- ☐ 7
- ☐ 14
- ☐ 58
- ☐ 60
- ☐ 65
- ☐ 69

2 of 10

$$x^2 + 8x = -7$$

Quantity A	Quantity B
x	0

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

3 of 10

If $3^3 \times 9^{12} = 3^x$, what is the value of x ?

☐

Click on the answer box and type in a number. Backspace to erase.

4 of 10

If $A = 2x - (y - 2c)$ and $B = (2x - y) - 2c$, then $A - B$ is

- ○ $-2y$
- ○ $-4c$
- ○ 0
- ○ $2y$
- ○ $4c$

5 of 10

A merchant sells three different sizes of canned tomatoes. A large can costs as much as 5 medium cans or 7 small cans. If a customer buys an equal number of small and large cans of tomatoes for the exact amount of money that would buy 200 medium cans, how many small cans will she buy?

- ○ 35
- ○ 45
- ○ 72
- ○ 199
- ○ 208

6 of 10

If $6k - 5l > 27$ and $3l - 2k < -13$ and $5k - 5l > j$, what is the value of j ?

☐

Click on the answer box and type in a number. Backspace to erase.

7 of 10

When the integer a is multiplied by 3, the result is 4 less than 6 times the integer b. Therefore, what is $a - 2b$?

- ○ -12
- ○ $-\frac{4}{3}$
- ○ $-\frac{3}{4}$
- ○ $\frac{4}{3}$
- ○ 12

8 of 10

Quantity A	**Quantity B**
The number of books purchased for \$1,750	The number of books purchased for \$2,250

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

9 of 10

If the product of two distinct integers is 91, which of the following could represent the sum of those two numbers?

Indicate all possible values.

- ☐ -92
- ☐ -91
- ☐ 7
- ☐ 13
- ☐ 20

10 of 10

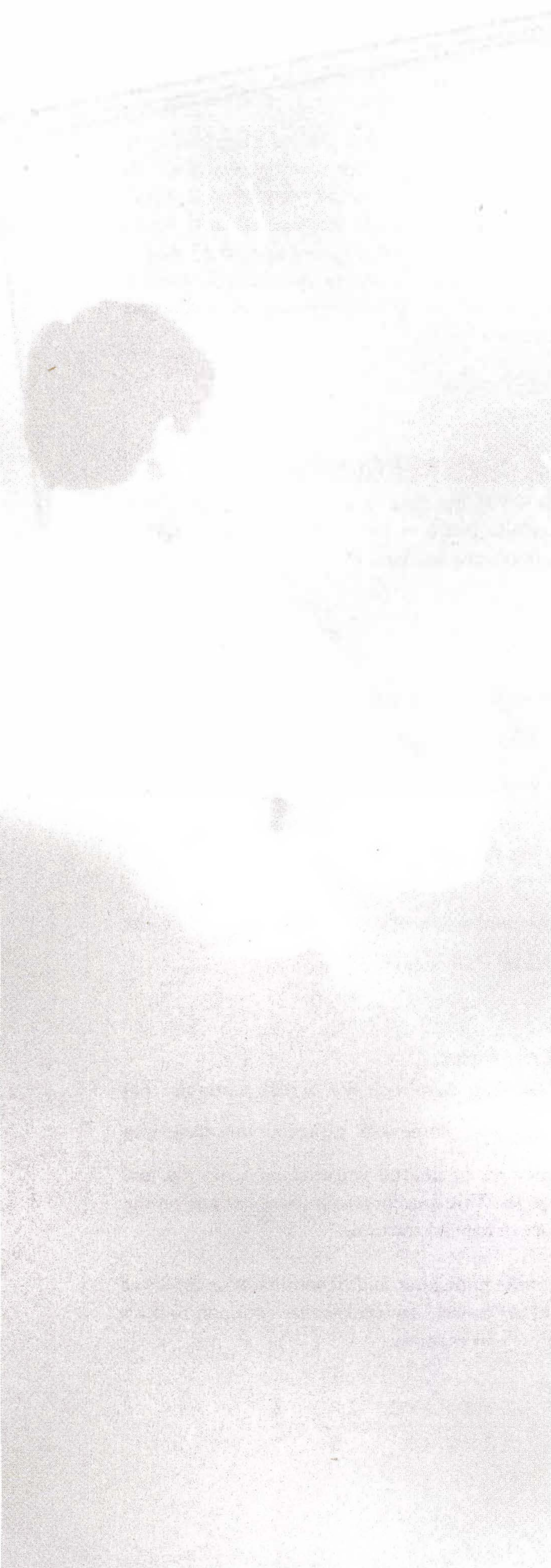
If $x = 3a$ and $y = 9b$, then all of the following are equal to $2(x + y)$ EXCEPT

Indicate all such expressions.

- ☐ $2(3a) + 9b$
- ☐ $12a + 9b$
- ☐ $6a + 18b$
- ☐ $6(a + 3b)$
- ☐ $6a + 21b$

Summary

- Digits are the numbers that make up other numbers. Numbers include whole numbers, fractions, negative numbers, and weird values like the square root of 2. Integers are numbers with no decimal or fractional part.
- Positive numbers are greater than zero and negative numbers less than zero. The number zero is neither positive nor negative.
- Even numbers are divisible by 2; odd numbers aren't. Only integers can be even or odd.
- A factor divides evenly into a number. A multiple is a number that a certain number is a factor of. Every number is a factor and a multiple of itself.
- The order of operations is PEMDAS.
- An exponent is shorthand for repeated multiplication. When in doubt on exponent problems, expand them out.
- The golden rule of equations: Whatever you do to one side of the equation, you must do to the other.
- With inequalities you have to flip the sign when you multiply or divide by a negative number.
- In order to solve an equation with two variables, you need two equations. Stack them up and add or subtract to cancel out one of the variables.
- Use the FOIL process to expand quadratics. To solve a quadratic equation, set it equal to zero and factor.
- Plugging In converts algebra problems to arithmetic problems. Plug In by replacing variables in the question with real numbers or by working backwards from the answer choices provided.
- Use the ZONE F numbers on tricky quant comp questions with variables.



Chapter 10
Real World Math

Real world math is our title for the grab bag of math topics that will be heavily tested on the GRE. This chapter details a number of important math concepts, many of which you've probably used at one point or another in your daily adventures, even if you didn't recognize that you were. After completing this chapter, you'll have brushed up on important topics such as fractions, percents, ratios, proportions, and average. You'll also learn some important Princeton Review methods for organizing your work and efficiently and accurately answering questions on these topics.

The math on the GRE is supposed to reflect the math you use in your day-to-day activities.

EVERYDAY MATH

As we've mentioned, when ETS reconfigured the GRE, one of its goals was to make the Math section reflect more of the kind of math that a typical graduate school student would use. Another of their goals was to test more of what it calls "real-life" scenarios. You can therefore expect the math questions on the GRE to heavily test things such as fractions, percents, proportions, averages, and ratios—mathematical concepts that are theoretically part of your everyday life. Regardless of whether that's true of your daily life or not, you'll have to master these concepts in order to do well on the GRE Math section.

FRACTIONS, DECIMALS, AND PERCENTS

In the previous chapter we spent most of our time working with integers. Now we'll expand our discussion to include concepts like fractions, decimals, and percents—all of which will appear frequently on the GRE.

Fractions

A **fraction** expresses a specific piece of information, namely, the number of parts out of a whole. In the fraction $\frac{2}{3}$, for instance, the top part, or **numerator**, tells us that we have 2 parts, while the bottom part of the fraction, the **denominator**, indicates that the whole, or total, consists of 3 parts. We use fractions whenever we're dealing with a quantity that's less than one.

Fractions are important on the GRE. Make sure you're comfortable with them.

Notice that the fraction bar is simply another way of expressing division. Thus, the fraction $\frac{2}{3}$ is just expressing the idea of "2 divided by 3."

Reducing and Expanding Fractions

Fractions express a relationship between numbers, not actual amounts. For example, saying that you did $\frac{1}{2}$ of your homework expresses the same idea whether you had 10 pages of homework to do and you've done 5, or you had 50 pages to do and you've done 25 pages. This concept is important because on the GRE you'll frequently have to reduce or expand fractions.

To reduce a fraction, simply express the numerator and denominator as the products of their factors. Then cross out, or "cancel," factors that are common to both the numerator and denominator. Here's an example:

$$\frac{16}{20} = \frac{2\times2\times2\times2}{2\times2\times5} = \frac{\cancel{2}\times\cancel{2}\times2\times2}{\cancel{2}\times\cancel{2}\times5} = \frac{2\times2}{5} = \frac{4}{5}$$

You can achieve the same result by dividing the numerator and denominator by the factors that are common to both. In the example you just saw, you might realize that 4 is a factor of both the numerator and the denominator. That is, both the numerator and the denominator can be divided evenly (without remainder) by 4. Doing this yields the much more manageable fraction $\frac{4}{5}$.

When you confront GRE math problems that involve big fractions, always reduce them before doing anything else.

> Remember: You can only reduce across a multiplication sign.

Look at each of the following fractions:

$$\frac{1}{4} \quad \frac{2}{8} \quad \frac{6}{24} \quad \frac{18}{72} \quad \frac{90}{360} \quad \frac{236}{944}$$

What do you notice about each of these fractions? They all express the same information! Each of these fractions expresses the relationship of "1 part out of 4 total parts."

Adding and Subtracting Fractions

Adding and subtracting fractions that have a common denominator is easy—you just add the numerators and put the sum over the common denominator. Here's an example:

$$\frac{1}{10} + \frac{2}{10} + \frac{4}{10} =$$

$$\frac{1+2+4}{10} = \frac{7}{10}$$

> **Why Bother?**
>
> You may be wondering why, if the GRE allows the use of a calculator, you should bother learning how to add or subtract fractions or to reduce them or even know any of the topics covered in the next few pages. While it's true that you can use a calculator for these tasks, for many problems it's actually slower to do the math with the calculator than without. Scoring well on the GRE Math section requires a fairly strong grasp of the basic relationships among numbers, fractions, percents, and so on, so it's in your best interest to really understand these concepts rather than to rely on your calculator to get you through the day. In fact, if you put in the work now, you'll be surprised at how easy some of the problems become, especially when you don't have to refer constantly to the calculator to perform basic operations.

In order to add or subtract fractions that have different denominators, you need to start by finding a common denominator. You may remember your teachers from grade school imploring you to find the "lowest common denominator." Actually, any common denominator will do, so find whichever one you find most comfortable working with.

$$\frac{7}{8} - \frac{5}{12} = \frac{21}{24} - \frac{10}{24} = \frac{11}{24}$$

Here, we expanded the fraction $\frac{7}{8}$ into the equivalent fraction $\frac{21}{24}$ by multiplying both the numerator and denominator by 3. Similarly, we converted $\frac{5}{12}$ to $\frac{10}{24}$ by multiplying both denominator and numerator by 2. This left us with two fractions that had the same denominator, which meant that we could simply subtract their numerators.

When adding and subtracting fractions, you can also use a technique we call the Bowtie. The Bowtie method accomplishes exactly what we just did in one fell swoop. To use the Bowtie, first multiply the denominators of each fraction. This gives you a common denominator. Then multiply the denominator of each fraction by the numerator of the other fraction. Take these numbers and add or subtract them—depending on what the question asks you to do—to get the numerator of the answer. Then reduce if necessary.

The Bowtie method is a convenient shortcut to use when you're adding and subtracting fractions.

$$\frac{2}{3} + \frac{3}{4} =$$

$$\overset{8}{\frac{2}{3}} \times \overset{9}{\frac{3}{4}} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12}$$

and

$$\frac{2}{3} - \frac{3}{4} =$$

$$\overset{8}{\frac{2}{3}} \times \overset{9}{\frac{3}{4}} = \frac{8}{12} - \frac{9}{12} = -\frac{1}{12}$$

Multiplying Fractions

There's nothing tricky about multiplying fractions: All you do is multiply straight across—multiply the first numerator by the second numerator and the first denominator by the second denominator. Here's an example:

$$\frac{4}{5} \times \frac{10}{12} = \frac{40}{60} = \frac{2}{3}$$

At this point, you should probably reduce the fraction. When multiplying fractions, you can make your life easier by reducing before you multiply. Do this once again by dividing out common factors.

Multiplying fractions is a snap: Just multiply straight across, numerator times numerator and denominator times denominator.

$$\frac{4}{5} \times \frac{10}{12} = \frac{4}{5} \times \frac{5}{6} = \frac{20}{30} = \frac{2}{3}$$

Also remember that when you're multiplying fractions, you can even reduce diagonally; as long as you're working with a numerator and a denominator of opposite fractions, they don't have to be in the same fraction. So you end up with

$$\frac{4}{5} \times \frac{5}{6} = \frac{2}{1} \times \frac{1}{3} = \frac{2}{3}$$

Of course, you get the same answer no matter what method you use, so attack fractions in whatever fashion you find easiest.

Dividing Fractions

Dividing fractions is just like multiplying fractions, with one crucial difference: Before you multiply, you have to turn the second fraction upside down (that is, put its denominator over its numerator, or to use fancy math lingo, find its reciprocal). In some cases, you can also reduce before you multiply. Here's an example:

$$\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{1}{3} \times \frac{5}{2} = \frac{5}{6}$$

ETS sometimes gives you problems that involve fractions whose numerators or denominators are themselves fractions. These problems might look intimidating, but if you're careful, you won't have any trouble with them. All you have to do is remember what we said about a fraction being shorthand for division. Always rewrite the expression horizontally. Here's an example:

$$\frac{7}{\frac{1}{4}} = 7 \div \frac{1}{4} = \frac{7}{1} \times \frac{4}{1} = \frac{28}{1} = 28$$

Comparing Fractions

The GRE might also present you with math problems that require that you to compare two fractions and decide which is larger, especially on quant comp questions. There are a couple of ways to accomplish this. One is to find equivalent fractions that have a common denominator. This works with simpler fractions, but on some problems the common denominator might be hard to find or hard to work with.

As an alternative, you can use a variant of the Bowtie technique. In this variant, you don't have to multiply the denominators, just the denominators and the numerators. The fraction with the larger product in its numerator is the bigger fraction. Let's say we had to compare the following fractions:

$$\frac{3}{7} \qquad \frac{7}{12}$$

$$\begin{array}{cc} 36 & 49 \\ \frac{3}{7} & \frac{7}{12} \end{array}$$

You can also use the calculator feature to change the fractions into decimals.

Multiplying the first denominator by the second numerator gives you 49. This means the numerator of the second fraction $\left(\frac{7}{12}\right)$ will be 49. Multiplying the second denominator by the first numerator gives you 36, which means the first fraction will have a numerator of 36. If 49 is bigger than 36, $\frac{7}{12}$ is bigger than $\frac{3}{7}$. Remember that when you use this method, it's the numerators that matter.

Comparing More Than Two Fractions

You may also be asked to compare more than two fractions. On these types of problems, don't waste time trying to find a common denominator for all of them. Simply use the Bowtie to compare two of the fractions at a time.

Here's an example:

Which of the following statements is true?

- ◯ $\frac{3}{8} < \frac{2}{9} < \frac{4}{11}$
- ◯ $\frac{2}{5} < \frac{3}{7} < \frac{4}{13}$
- ◯ $\frac{4}{13} < \frac{2}{5} < \frac{3}{7}$
- ◯ $\frac{3}{7} < \frac{3}{8} < \frac{2}{5}$
- ◯ $\frac{2}{9} < \frac{3}{7} < \frac{3}{8}$

Here's How to Crack It

As you can see, it would be a nightmare to try to find common denominators for all these funky fractions, so instead we'll use the Bowtie method. Simply multiply the denominators and numerators of a pair of fractions and note the results. For example, to check answer choice (A), we first multiply 8 and 2, which gives us a numerator of 16 for the fraction $\frac{2}{9}$. But multiplying 9 and 3 gives us a numerator of 27 for the first fraction. This means that $\frac{3}{8}$ is bigger than $\frac{2}{9}$, and we can eliminate choice (A), because the first part of it is wrong. Here's how the rest of the choices shape up:

- ◯ $\frac{2}{5} < \frac{3}{7} < \frac{4}{13}$ Compare $\frac{3}{7}$ and $\frac{4}{13}$; $\frac{3}{7}$ is larger.
- ◯ $\frac{4}{13} < \frac{2}{5} < \frac{3}{7}$ These fractions are in order.
- ◯ $\frac{3}{7} < \frac{3}{8} < \frac{2}{5}$ $\frac{3}{7}$ is larger than $\frac{3}{8}$.
- ◯ $\frac{2}{9} < \frac{3}{7} < \frac{3}{8}$ $\frac{3}{7}$ is larger than $\frac{3}{8}$.

Make sure you are doing all of this work in an organized fashion on your scratch paper.

Converting Mixed Numbers into Fractions

A **mixed number** is a number that is represented as an integer and a fraction, such as $2\frac{2}{3}$. In most cases on the GRE, you should get rid of mixed fractions by converting them to fractions. How do you do this? By multiplying the denominator of the fraction by the integer, then adding that number to the numerator, and then putting the whole thing over the denominator. In other words, for the fraction above we would get $\frac{3 \times 2 + 2}{3}$ or $\frac{8}{3}$.

The result, $\frac{8}{3}$, is equivalent to $2\frac{2}{3}$. The only difference is that $\frac{8}{3}$ is easier to work with in math problems. Also, answer choices are usually not in the form of mixed numbers.

Decimals

Decimals are just fractions in disguise. Basically, decimals and fractions are two different ways of expressing the same thing. Every decimal can be written as a fraction, and every fraction can be written as a decimal. For example, the decimal .35 can be written as the fraction $\frac{35}{100}$: These two expressions, .35 and $\frac{35}{100}$, have the same value.

To turn a fraction into its decimal equivalent, all you have to do is divide the numerator by the denominator. Here, for example, is how you would find the decimal equivalent of $\frac{3}{4}$:

$$\frac{3}{4} = 3 \div 4 = 4\overset{\displaystyle 0.75}{\overline{)3.00}}$$

Try this problem:

$$7 < x < 8$$

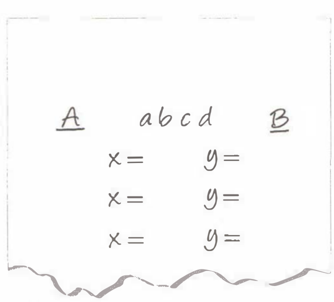
$$y = 9$$

Quantity A	Quantity B
$\frac{x}{y}$	.85

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

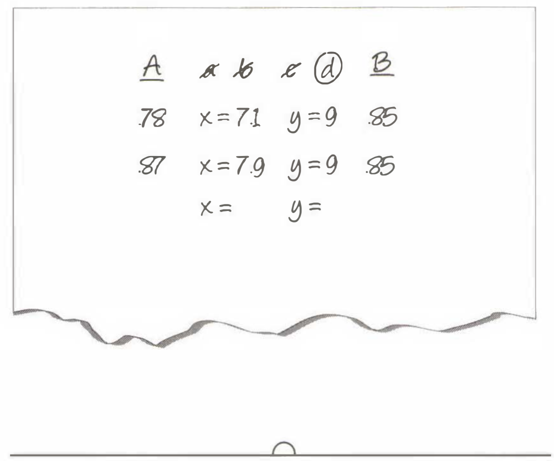
Here's How to Crack It

So, you're sitting at your cubical at the Prometric testing center and this problem pops up. What do you see? Before we even talk about fractions, the first thing you should note is that this is a quant comp with variables. With your hand, on your scratch paper, make your set-up. It should look like this:



Now they've told us that x is going to be seven point something. Try Plugging In the smallest value you can think of for x. Write down $x = 7.1$ and $y = 9$. The value in Quantity A is .78. The value in Quantity B is .85. Quantity B is bigger, so eliminate choices (A) and (C). Now try making x as big as you can make it. Write down $x = 7.9$ and $y = 9$. The value in column A is .87 and the value in Quantity B is .85. Quantity A is bigger so eliminate choice B, and you're done. The answer is (D).

Your scratch paper should look like this:



Comparing Decimals

Which is larger: 0.00099 or 0.001? ETS loves this sort of problem. You'll never go wrong, though, if you follow these easy steps.

- Line up the numbers by their decimal points.
- Fill in the missing zeros.

Here's how to answer the question we just asked. First, line up the two numbers by their decimal points.

0.00099
0.001

Now fill in the missing zeros.

0.00099
0.00100

Can you tell which number is larger? Of course you can. 0.00100 is larger than 0.00099, because 100 is larger than 99.

Digits and Decimals

Remember our discussion about digits, earlier? Well, sometimes the GRE will ask you questions about digits that fall after the decimal point as well. Suppose you have the number 0.584.

- 0 is the units digit.
- 5 is the tenths digit.
- 8 is the hundredths digit.
- 4 is the thousandths digit.

Percentages

The final member of our numbers family is percents. A **percent** is just a special type of fraction, one that always has 100 as the denominator. Percent literally means "per 100" or "out of 100" or "divided by 100." If your best friend finds a dollar and gives you 50¢, your friend has given you 50¢ out of 100, or $\frac{50}{100}$ of a dollar, or 50 percent of the dollar. To convert fractions to percents, just expand the fraction so it has a denominator of 100:

$$\frac{3}{5} = \frac{60}{100} = 60\%$$

For the GRE, you should memorize the following percentage-decimal-fraction equivalents. Use these friendly fractions and percentages to eliminate answer choices that are way out of the ballpark.

$0.01 = \frac{1}{100} = 1\%$ $\quad$ $0.1 = \frac{1}{10} = 10\%$

$0.2 = \frac{1}{5} = 20\%$ $\quad$ $0.25 = \frac{1}{4} = 25\%$

$0.333... = \frac{1}{3} = 33\frac{1}{3}\%$ $\quad$ $0.4 = \frac{2}{5} = 40\%$

$0.5 = \frac{1}{2} = 50\%$ $\quad$ $0.6 = \frac{3}{5} = 60\%$

Percents are another very common topic on the GRE.

$$0.666\ldots = \frac{2}{3} = 66\frac{2}{3}\%$$

$$0.75 = \frac{3}{4} = 75\%$$

$$0.8 = \frac{4}{5} = 80\%$$

$$1.0 = \frac{1}{1} = 100\%$$

$$2.0 = \frac{2}{1} = 200\%$$

Converting Decimals to Percentages

In order to convert decimals to percents, just move the decimal point two places to the right. This turns 0.8 into 80 percent, 0.25 into 25 percent, 0.5 into 50 percent, and 1 into 100 percent.

Translation

These translations apply to any word problem, not just percent problems.

One of the best tricks for handling percentages in word problems is knowing how to translate them into an equation that you can manipulate. Use the following table to help you translate percentage word problems into equations you can work with.

Word	Equivalent Symbol
percent	/100
is	=
of, times, product	×
what (or any unknown value)	any variable (x, k, b)

Here's an example:

56 is what percent of 80 ?

- ○ 66%
- ○ 70%
- ○ 75%
- ○ 80%
- ○ 142%

Here's How to Crack It

To solve this problem, let's translate the question and then solve for the variable. So, "56 is what percent of 80," in math speak, is equal to

$$56 = \frac{x}{100}(80)$$

$$56 = \frac{80x}{100}$$

Don't forget to reduce the fraction: $56 = \frac{4}{5}x$.

Now multiply both sides of the equation by the reciprocal, $\frac{5}{4}$.

Don't forget to reduce again before you calculate:

$$\left(\frac{5}{4}\right)\left(\frac{56}{1}\right) = \left(\frac{5}{4}\right)\left(\frac{4x}{5}\right)$$

$$(5)(14) = x$$

$$70 = x$$

That's answer choice (B). Did you notice choice (E)? Because 56 is less than 80, the answer would have to be less than 100 percent, so 142 percent is way too big, and you could have eliminated it from the get-go by Ballparking.

Let's try a quant comp example.

5 is r percent of 25
s is 25 percent of 60

Quantity A	Quantity B
r	s

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Here's How to Crack It

First translate the first statement.

$$5 = \frac{r}{100}(25)$$

$$5 = \frac{25r}{100}$$

$$5 = \frac{r}{4}$$

$$(4)(5) = \left(\frac{r}{4}\right)(4)$$

$$20 = r$$

That takes care of Quantity A. Now translate the second statement.

$$s = \frac{25}{100}(60)$$

$$s = \frac{1}{4}(60)$$

$$s = 15$$

That takes care of Quantity B. The answer is (A).

Percentage Increase/Decrease

On percent increase problems, the original is always the smaller number. On percent decrease problems, the original is the larger number.

To find the percentage by which something has increased or decreased, use the following formula.

$$\text{Percent Change} = \frac{\text{Difference}}{\text{Original}} \times 100$$

The "difference" is simply what you get when you subtract the smaller number from the larger number. The "original" is whichever number you started with. If the question asks you to find a percent increase, then the original number will be the smaller number. If the question asks you to find a percent decrease, then the original number will be the larger number.

Here's an example.

What number goes on the bottom of the fraction?

Vandelay Industries reported a $6,000 profit over the three-month period from March to May of the current year. If, over the previous three-month period, Vandelay Industries realized a $3,500 profit, by approximately what percent did its profit increase?

- 25%
- 32%
- 42%
- 55%
- 70%

Here's How to Crack It

Let's use the percent change formula we just learned. The first step is to find the difference between the two numbers. The initial profit was $3,500 and the final profit is $6,000. The difference between these two numbers is 6,000 – 3,500 = 2,500. Next, we need to divide this number by the original, or starting, value.

One way to help you figure out what value to use as the original value is to check to see whether you're dealing with a percent increase or a percent decrease question. Remember that on a percent increase question, you should always use the smaller of the two numbers as the denominator and that on percent decrease you need to use the larger of the two numbers as the denominator. Because here we want to find the percent increase, the number we want to use for our denominator is 3,500. So our percent increase fraction looks like this: $\frac{2,500}{3,500}$. We can reduce this to $\frac{25}{35}$ by dividing by 100, and reduce even further by dividing by 5. This leaves us with $\frac{5}{7}$, which is approximately 70% (remember that the fraction bar means divide, so if you divide 5 by 7, you'll get .71). Thus, choice (E) is the answer.

Here's another question.

Model	Original Price	Sale Price
A	$12,000	$9,500
B	$16,000	$13,000
C	$10,000	$7,500
D	$17,500	$13,000
E	$20,000	$15,500
F	$22,000	$16,000

The table above shows the original price and the sale price of six different models of cars. Which models of car would a consumer have to buy to get at least a 25% discount?

Indicate all such models.

☐ A

☐ B

☐ C

☐ D

☐ E

☐ F

Here's How to Crack It

First list A, B, C, D, E, and F in a column in the upper left corner of your scratch paper. You are asked to identify a 25% change or greater between the two numbers. You know the formula for this. It is difference/original × 100. Using your calculator, subtract 9,500 from 12,000. You should get 2,500. This is the difference. Divide it by the original, 12,000, to get 0.2. You don't even need to multiply by 100. You know that this is 20%, which is less than 25%, so cross it off on your scratch paper. Try the next one. 16,000 – 13,000 = 3,000. Divide by 16,000. Too small. Cross it off. Repeat this process for each of the answer choices. Choices (C), (D), and (F) all work.

PLUGGING IN ON FRACTION AND PERCENT PROBLEMS

Now that you've become familiar with fractions and percents, we'll show you a neat trick. When you come to regular multiple-choice questions, or multiple choice, multiple answers, that involve fractions or percents, you can simply Plug In a number and work through the problem using that number. This approach works even when the problem doesn't have variables in it. Why? Because, as you know, fractions and percents express only a relationship between numbers—the actual numbers don't matter. For example, look at the following problem:

Plugging In on fraction and percent problems is a great way to make your life easier.

A recent survey of registered voters in City x found that $\frac{1}{3}$ of the respondents support the mayor's property tax plan. Of those who did not support the mayor's plan, $\frac{1}{8}$ indicated they would not vote to reelect the mayor if the plan was implemented. Of all the respondents, what fraction indicated that it would not vote for the mayor if the plan is enacted?

- ○ $\frac{1}{16}$
- ○ $\frac{1}{12}$
- ○ $\frac{1}{6}$
- ○ $\frac{1}{3}$
- ○ $\frac{2}{3}$

What important information is missing from the problem?

Here's How to Crack It

Even though there are no variables in this problem, we can still Plug In. On fraction and percent problems, ETS will often leave out one key piece of information: the total. Plugging In for that missing value will make your life much easier. What crucial information did ETS leave out of this problem? The total number of respondents. So let's Plug In a value for it. Let's say that there were 24 respondents to the survey. 24 is a good number to use because we'll have to work with $\frac{1}{3}$ and

$\frac{1}{8}$, so we want a number that's divisible by both those fractions. Working through the problem with our number, we see that $\frac{1}{3}$ of the respondents support the plan. $\frac{1}{3}$ of 24 is 8, so that means 16 people do not support the plan. Next, the problem says that $\frac{1}{8}$ of those who do not support the plan will not vote for the mayor. $\frac{1}{8}$ of 16 is 2, so 2 people won't vote for the mayor. Now we just have to answer the question: Of all respondents, how many will not vote for the mayor? Well, there were 24 total respondents and we figured out that 2 aren't voting. So that's $\frac{2}{24}$, or $\frac{1}{12}$. Answer choice (B) is the one we want.

RATIOS AND PROPORTIONS

If you're comfortable working with fractions and percents, you'll be comfortable working with ratios and proportions, because ratios and proportions are simply special types of fractions. Don't let them make you nervous. Let's look at ratios first and then deal with proportions.

What Is a Ratio?

A ratio is just another type of fraction.

Recall that a fraction expresses the relationship of a part to the whole. A **ratio** expresses a different relationship: part to part. Imagine yourself at a party with 8 women and 10 men in attendance. What fraction of the partygoers are female? $\frac{8}{18}$, or 8 women out of a total of 18 people at the party. But what's the ratio of women to men? $\frac{8}{10}$, or as ratios are more commonly expressed, 8 : 10. You can reduce this ratio to 4 : 5, just like you would a fraction.

On the GRE, you may see ratios expressed in several different ways:

$x : y$
the ratio of x to y
x is to y

In each case, the ratio is telling us the relationship between parts of a whole.

Every Fraction Can Be a Ratio, and Vice Versa

Every ratio can be expressed as a fraction. A ratio of 1 : 2 means that there's either a total of three things or a multiple of three, and the fraction $\frac{1}{2}$ means "1 out of 2."

Treat a Ratio Like a Fraction

Anything you can do to a fraction you can also do to a ratio. You can cross-multiply, find common denominators, reduce, and so on.

Find the Total

The key to dealing with ratio questions is to find the whole, or the total. Remember: A ratio tells us only about the parts, not the total. In order to find the total, add the numbers in the ratio. A ratio of 2 : 1 means that there are three total parts. A ratio of 2 : 5 means that we're talking about a total of 7 parts. And a ratio of 2 : 5 : 7 means there are 14 total parts. Once you have a total you can start to do some fun things with ratios.

For example, let's say you have a handful of pennies and nickels. If you have 30 total coins and the pennies and nickels are in a 2 : 1 ratio, how many pennies do you have? The total for our ratio is 3, meaning that out of every 3 coins, there are 2 pennies and 1 nickel. So if there are 30 total coins, there must be 20 pennies and 10 nickels. Notice that $\frac{20}{10}$ is the same as $\frac{2}{1}$, is the same as 2 : 1!

Like a fraction, a ratio expresses a relationship between numbers.

When you are working with ratios, there's an easy way not only to keep track of the numbers in the problem but also to quickly figure out the values in the problem. It's called the Ratio Box. Let's try the same question, but with some different numbers; if you have 24 coins in your pocket and the ratio of pennies to nickels is 2 : 1, how many pennies and nickels are there? The Ratio Box for this question is below, with all of the information we're given already filled in.

The minute you see the word "ratio," draw your box on your scratch paper.

	Pennies	Nickels	Total
ratio	2	1	3
multiply by			
real			24

Remember that ratios are relationships between numbers, not real numbers, so the real total is 24; that is, you have 24 actual coins in your pocket. The ratio total (the number you get when you add the number of parts in the ratio) is 3.

The middle row of the table is for the multiplier. How do you get from 3 to 24? You multiply by 8. Remember when we talked about finding equivalent fractions? All we did was multiply the numerator and denominator by the same value. That's exactly what we're going to do with ratios. This is what the ratio box would look like now:

	Pennies	Nickels	Total
ratio	2	1	3
multiply by	8	8	**8**
real			24

Now let's finish filling in the box by multiplying everything else.

	Pennies	Nickels	Total
ratio	2	1	3
multiply by	8	8	**8**
real	16	8	24

Therefore, of the 24 coins 16 are pennies and 8 are nickles.

Let's try a GRE example.

Flour, eggs, yeast, and salt are mixed by weight in the ratio of 11 : 9 : 3 : 2, respectively. How many pounds of yeast are there in 20 pounds of the mixture?

- ○ $1\frac{3}{5}$
- ○ $1\frac{4}{5}$
- ○ 2
- ○ $2\frac{2}{5}$
- ○ $8\frac{4}{5}$

Here's How to Crack It

The minute you see the word *ratio*, draw your box on your scratch paper and fill in what you know.

	Flour	Eggs	Yeast	Salt	Total
ratio	11	9	3	2	
multiply by					
real					20

First, add all of the numbers in the ratio to get the ratio total.

	Flour	Eggs	Yeast	Salt	Total
ratio	11	9	3	2	25
multiply by					
real					20

Now, what do we multiply 25 by to get 20?

$$25x = 20$$

$$\frac{25x}{25} = \frac{20}{25}$$

$$x = \frac{20}{25}$$

$$x = \frac{4}{5}$$

So $\frac{4}{5}$ is our "multiply by" number. Let's fill it in.

	Flour	Eggs	Yeast	Salt	Total
ratio	11	9	3	2	25
multiply by	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{4}{5}$
real					20

The question asks for the amount of yeast, so we don't have to worry about the other ingredients. Just look at the yeast column. All we have to do is multiply 3 by $\frac{4}{5}$ and we have our answer: $3 \times \frac{4}{5} = \frac{12}{5} = 2\frac{2}{5}$, which is answer choice (D).

What Is a Proportion?

So you know that a fraction is a relationship between part and whole, and that a ratio is a relationship between part and part. A **proportion** is an equivalent relationship between two fractions or ratios. Thus, $\frac{1}{2}$ and $\frac{4}{8}$ are proportionate because they are equivalent fractions. But $\frac{1}{2}$ and $\frac{2}{3}$ are not in proportion because they are not equal ratios.

The GRE often contains problems in which you are given two proportional, or equal, ratios from which one piece of information is missing. These questions take a relationship or ratio, and project it onto a larger or smaller scale. Proportion problems are recognizable because they always give you three values and ask for a fourth value. Here's an example:

The key to proportions is setting them up correctly.

If the cost of a one-hour telephone call is \$7.20, what would be the cost in dollars of a 10-minute telephone call at the same rate?

[ ] dollars

Click on the answer box and type in a number. Backspace to erase.

Here's How to Crack It

It's very important to set up proportion problems correctly. That means using your hand and parking your information on your scratch paper. Be essentially careful to label *everything*. It takes only an extra two or three seconds, but doing this will help you catch lots of errors.

For this question, let's express the ratios as dollars over minutes, because we're being asked to find the cost of a 10-minute call. That means that we have to convert 1 hour to 60 minutes (otherwise it wouldn't be a proportion).

$$\frac{\$}{\text{min}} = \frac{\$7.20}{60} = \frac{x}{10}$$

Now cross-multiply.

$$60x = (7.2)(10)$$
$$60x = 72$$
$$\frac{60x}{60} = \frac{72}{60}$$
$$x = \frac{6}{5}$$

Now we can enter 1.20 into the box.

Relationship Review

You may have noticed a trend in the preceding pages. Each of the major topics covered—fractions, percents, ratios, and proportions—described a particular relationship between numbers. Let's review:

- A fraction expresses the relationship between a part and the whole.
- A percent is a special type of fraction, one that expresses the relationship of part to whole as a fraction with the number 100 in the denominator.
- A ratio expresses the relationship between part and part. Adding the parts of a ratio gives you the whole.
- A proportion expresses the relationship between equal fractions, percents, or ratios.
- Each of these relationships shares all the characteristics of a fraction. You can reduce them, expand them, multiply them, and divide them using the exact same rules you used for working with fractions.

AVERAGES

The average (arithmetic mean) of a set of numbers is the sum, or total value, of all the numbers in the set divided by the number of numbers in the set. The average of the set {1, 2, 3, 4, 5} is equal to the total of the numbers (1 + 2 + 3 + 4 + 5, or 15) divided by the number of numbers in the set (which is 5). Dividing 15 by 5 gives us 3, so 3 is the average of the set.

GRE average problems always give you two of the three numbers needed.

ETS always refers to an average as an "average (arithmetic mean)." This confusing parenthetical remark is meant to keep you from being confused by other kinds of averages, such as medians and modes. You'll be less confused if you simply ignore the parenthetical remark and know that average means total of the elements divided by the number of elements. We'll tell you about medians and modes later.

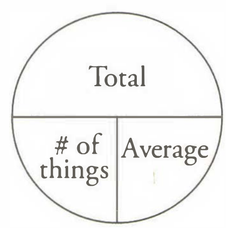
Think Total

Don't try to solve average problems all at once. Do them piece by piece. The key formula to keep in mind when doing problems that involve averages is

$$\text{Average} = \frac{\text{Total}}{\text{\# of things}}$$

The minute you see the word *average*, draw an average pie on your scratch paper.

Drawing an Average Pie will help you organize your information.



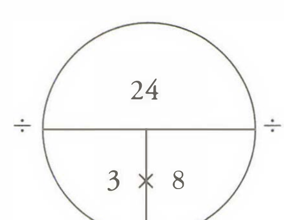
Here's how the Average Pie works. The *total* is the sum of the numbers being averaged. The *number of things* is the number of different elements that you are averaging. And the *average* is, naturally, the average.

Say you wanted to find the average of 4, 7, and 13. You would add those numbers to get the total and divide that total by three.

$$4 + 7 + 13 = 24$$
$$\frac{24}{3} = 8$$

Which two pieces of the pie do you have?

Mathematically, the Average Pie works like this:



The horizontal bar is a division bar. If you divide the *total* by the *number of things*, you get the *average*. If you divide the total by the *average*, you get the *number of things*. If you have the *number of things* and the *average*, you can simply multiply them together to find the *total*. This is one of the most important things you need to be able to do to solve GRE average problems.

Using the Average Pie has several benefits. First, it's an easy way to organize information. Furthermore, the Average Pie makes it clear that if you have two of the three pieces, you can always find the third. This makes it easier to figure out how to approach the problem. If you fill in the number of things, for example, and the question wants to know the average, the Average Pie shows you that the key to unlocking that problem is finding the total.

Try this one.

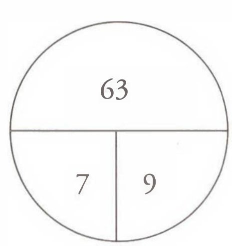
The average of seven numbers is 9. The average of three of these numbers is 5. What is the average of the other four numbers?

- ○ 4
- ○ 5
- ○ 7
- ○ 10
- ○ 12

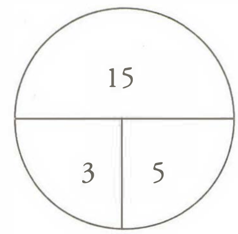
If you see the word *average* twice in a problem, draw two pies. If you see it three times, then draw three pies.

Here's How to Crack It

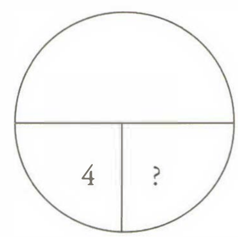
Let's take the first sentence. You have the word *average*, so draw your pie and fill in what you know. We have seven numbers with an average of 9, so plug those values into your Average Pie and multiply to find the total.



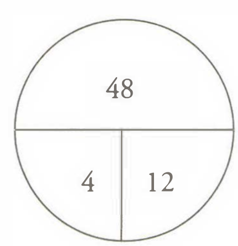
Now we also know that three of the numbers have an average of 5, so draw another Average Pie, plug those values into their places, and multiply to find the total of those three numbers.



The question is asking for the average of the four remaining numbers. Draw one more Average Pie and Plug In 4 for the number of things.



In order to solve for the average, we need to know the total of those four numbers. How do we find this? From our first Average Pie we know that the total of all seven numbers is 63. The second Average Pie tells us that the total of three of those numbers was 15. Thus, the total of the remaining four has to be 63 – 15, which is 48. Plug 48 into the last Average Pie, and divide by 4 to get the average of the four numbers.



The average is 12, which is answer choice (E).

Let's try one more.

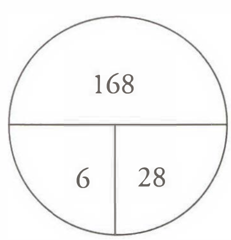
The average (arithmetic mean) of a set of 6 numbers is 28. If a certain number, y, is removed from the set, the average of the remaining numbers in the set is 24.

Quantity A	Quantity B
y	48

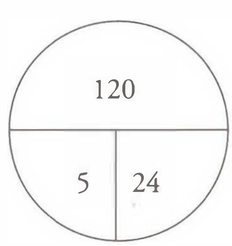
- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Here's How to Crack It

All right, let's attack this one. The problem says that the average of a set of six numbers is 28, so let's immediately draw an average pie and calculate the total.



If a certain number, y, is removed from the set, there are now five numbers left. We already know that the new average is 24, so draw another Average Pie.



The difference between the totals must be equal to y. $168 - 120 = 48$. Thus, the two quantities are equal, and the answer is (C).

Up and Down

Averages are very predictable. You should make sure you automatically know what happens to them in certain situations. For example, suppose you take three tests and earn an average score of 90. Now you take a fourth test. What do you know?

If your average goes up as a result of the fourth score, then you know that your fourth score was higher than 90. If your average stays the same as a result of the fourth score, then you know that your fourth score was exactly 90. If your average goes down as a result of the fourth score, then you know that your fourth score was less than 90.

MEDIAN, MODE, AND RANGE

Don't confuse median and mode!

The **median** is the middle value in a set of numbers; above and below the median lie an equal number of values. For example, in the set {1, 2, 3, 4, 5, 6, 7}, the median is 4, because it's the middle number (and there are an odd number of numbers in the set). If the set contained an even number of integers {1, 2, 3, 4, 5, 6}, the median would be the average of 3 and 4, or 3.5. When looking for the median, sometimes you have to put the numbers in order yourself. What is the median of the set {13, 5, 6, 3, 19, 14, 8}? First, put the numbers in order from least to greatest, {3, 5, 6, 8, 13, 14, 19}. Then take the middle number. The median is 8. Just think *median = middle* and always make sure the numbers are in order.

The minute you see the word *median* in a question, find a bunch of numbers and put them in order.

The **mode** is the number in a set that occurs most frequently. For example, in the set {2, 3, 4, 5, 3, 8, 6, 9, 3, 9, 3} the mode is 3, because 3 shows up the most. Just think *mode = most.*

The **range** is the difference between the biggest and the smallest numbers in your set. So, in the set {2, 6, 13, 3, 15, 4, 9}, the range is 15 (the highest number in the set) – 2 (the lowest number in the set), or 13.

Here's an example:

$$F = \{4, 2, 7, 11, 8, 9\}$$

What do we need to do to the numbers in this list?

Quantity A	Quantity B
The range of Set F	The median of Set F

- Quantity A is greater.
- Quantity B is greater.
- The two quantities are equal.
- The relationship cannot be determined from the information given.

Here's How to Crack It

Let's put the numbers in order first, so it'll be easier to see what we have: {2, 4, 7, 8, 9, 11}. First let's look at Quantity A: The range is the largest number, or 11, minus the smallest number, or 2. That's 9. Now let's look at Quantity B: The minute you see the word *median*, find a bunch of numbers and put them in order. The median is the middle number of the set, but because there are two middle numbers, 7 and 8, we have to find the average. Or do we? Isn't the average of 7 and 8 clearly going to be smaller than the number in Quantity A, which is 9? Yes, in quant comp questions, we compare, not calculate. The answer is (A).

STANDARD DEVIATION

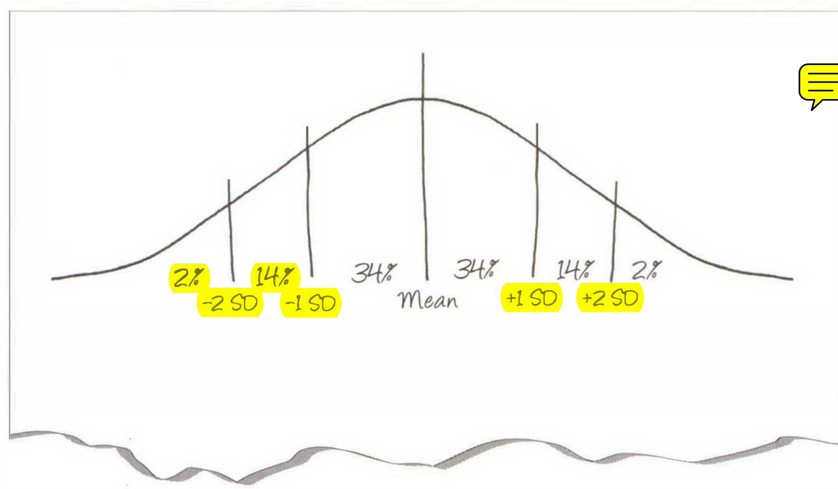
Standard deviation is one of those phrases that math people like to throw around to scare non-math people, but it's really not that scary. The GRE might ask you questions about standard deviation, but you'll never have to actually calculate it; instead, you'll just need a basic understanding of what standard deviation is. In order to understand standard deviation, we must first look at something all standardized testers should be familiar with, the bell curve.

You'll never have to calculate the standard deviation on the GRE.

Your Friend the Bell Curve

The first thing to know about a bell curve is that the number in the middle is the mean, the median, and the mode.

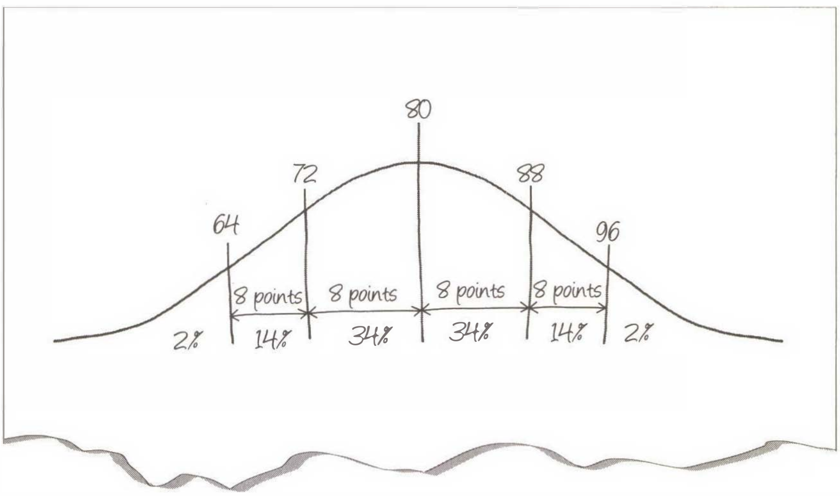
The minute you see the phrase "standard deviation" or "normal distribution," draw your curve and fill in your percentages.



Imagine that 100 students take a test and the results follow a normal distribution. The minute you see the phrase "normal distribution," draw your curve. Let's say that the average score on this test is an 80. That means that the median and the mode must also be 80. Put 80 in the middle of your curve. You know, however, that a few of those students were extremely well prepared and got a really high score, let's say that 2% of them got a 96 or higher. Put a 96 above the right 2% line on your curve.

Standard deviation measures how much a score differs from the norm (the average) in even increments. The curve tells us that a score earned by only 2% of the students is two standard deviations from the norm. If the norm is 80 and 96 is two standard deviations away, then one standard deviation on this test is 8 points. Two standard deviations above the norm is 96, while two standard deviations below the norm is 64. One standard deviation above the norm is 88, and one standard deviation below the norm is 72. Fill these in on your bell curve.

Now you know quite a bit about the distribution of scores on this test. Sixty-eight percent of the students received a score between 72 and 88. Ninety-eight percent scored above a 64. That's all there is to know about standard deviations. The percentages don't change, so memorize those. When you see the phrase, just make your curve and fill in what you know. Here's what the curve would look like for this test:



When it comes to standard deviation, the percentages don't change, so memorize those: 2, 14, and 34.

Here's an example of how ETS might test standard deviation:

Quantity A	Quantity B
The standard deviation of a set of data consisting of 10 integers ranging from –20 to –5	The standard deviation of a set of data consisting of 10 integers ranging from 5 to 20

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Here's How to Crack It

ETS is hoping you'll make a couple of wrong turns on this problem. The first trap they set is that one set of numbers contains negative integers while the other doesn't—but this doesn't mean that one set will have a negative standard deviation. Standard deviation is defined as the distance a point is from the mean, so it can never be negative. The second trap is that ETS hopes you'll waste a lot of time trying to calculate standard deviation based on the information given. But you know better than to try to do that. Remember that ETS won't ask you to calculate standard deviation; it's a complex calculation. Plus, as you know, you need to know the mean in order to figure the standard deviation and there's no way we can find it based on the information here. Thus, we have no way of comparing these two quantities, and our answer is (D).

Now let's try a question that will make use of the bell curve.

The fourth grade at School x is made up of 300 students who have a total weight of 21,600 pounds. If the weight of these fourth graders has a normal distribution and the standard deviation equals 12 pounds, approximately what percentage of the fourth graders weighs more than 84 pounds?

- ○ 12%
- ○ 16%
- ○ 36%
- ○ 48%
- ○ 60%

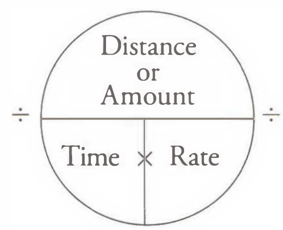
Here's How to Crack It

This one's a little tougher than the earlier standard deviation questions. The first step is to determine the average weight of the students, which is $\frac{21,600}{300} = 72$ pounds. If the standard deviation is 12 pounds, then 84 pounds places us exactly one standard deviation above the mean, or at the 84th percentile (remember the bell curve?). Because 16 percent of all students weigh more than 84 pounds, the answer is (B).

RATE

Rate problems are similar to average problems. A rate problem might ask for an average speed, distance, or the length of a trip, or how long a trip (or a job) takes. To solve rate problems, use the Rate Pie.

A rate problem is really just an average problem.



The Rate Pie works exactly the same way as the Average Pie. If you divide the *distance* or *amount* by the *rate*, you get the *time*. If you divide the *distance* or *amount* by the *time*, you get the *rate*. If you multiply the *rate* by the *time*, you get the *distance* or *amount*.

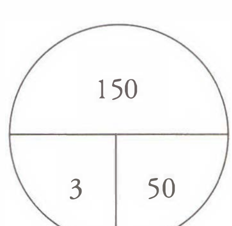
Let's take a look.

It takes Carla three hours to drive to her brother's house at an average speed of 50 miles per hour. If she takes the same route home, but her average speed is 60 miles per hour, how long does it take her to get home?

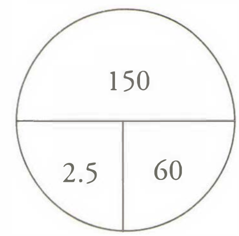
- ○ 2 hours
- ○ 2 hours and 14 minutes
- ○ 2 hours and 30 minutes
- ○ 2 hours and 45 minutes
- ○ 3 hours

Here's How to Crack It

The trip to her brother's house takes three hours, and the rate is 50 miles per hour. Plug those numbers into a Rate Pie and multiply to find the distance.



So the distance is 150 miles. On her trip home, Carla travels at a rate of 60 miles per hour. Draw another Rate Pie and Plug In 150 and 60. Then all you have to do is divide 150 by 60 to find the time.



So it takes Carla two and a half hours to get home. That's answer choice (C).

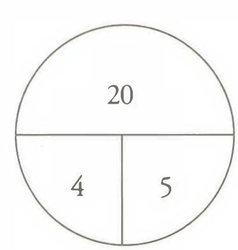
Try another one.

A machine can stamp 20 envelopes in 4 minutes. How many of these machines, working simultaneously, are needed to stamp 60 envelopes per minute?

- ○ 5
- ○ 10
- ○ 12
- ○ 20
- ○ 24

Here's How to Crack It

First we have to find the rate per minute of one machine. Plug 20 and 4 into a Rate Pie and divide to find the rate.



The rate is 5. If one machine can stamp 5 envelopes per minute, how many machines do you need to stamp 60 per minute? $60 \div 5 = 12$, or answer choice (C).

CHARTS

Every GRE Math section has a few questions that are based on a chart or graph (or on a group of charts or graphs). But don't worry; the most important thing that chart questions test is your ability to remember the difference between real-life charts and ETS charts.

Chart questions frequently test percents, percent change, ratios, proportions, and averages.

In real life, charts are often provided in order to display information in a way that's easier to understand. Conversely, ETS constructs charts to hide information you need to know and to make that information harder to understand.

Chart Questions

There are usually two or three questions per chart or per set of charts. Like the Reading Comprehension questions, chart questions appear on split screens. Be sure to click on the scroll bar and scroll down as far as you can; there may be additional charts underneath the top one, and you want to make sure you've seen all of them.

Chart problems just recycle the basic arithmetic concepts we've already covered: fractions, percentages, and so on. This means you can use the techniques we've discussed for each type of question, but there are two additional techniques that are especially important to use when doing chart questions.

On charts, look for the information ETS is trying to hide.

Don't Start with the Questions: Start with the Charts

Take a minute to note the following key bits of information from any chart you see.

- **Information in titles:** Make sure you know what each chart is telling you.
- **Asterisks, footnotes, parentheses, and small print:** Often there will be crucial information hidden away at the bottom of the chart. Don't miss it!
- **Funny units:** Pay special attention when a title says "in thousands" or "in millions." You can usually ignore the units as you do the calculations, but you have to use them to get the right answer.

Don't try to work with huge values. Ballpark instead!

Approximate, Estimate, and Ballpark

Like some of our other techniques, you have to train yourself to estimate when looking at charts and graphs. You should estimate, not calculate exactly, in the following situations:

- Whenever you see the word *approximately* in a question
- Whenever the question has answer choices and when the answer choices are far apart in value
- Whenever you start to answer a question and you justifiably say to yourself, "This is going to take a lot of calculation!"

Review those "friendly" percentages and their fractions from earlier in the chapter. Try estimating this question:

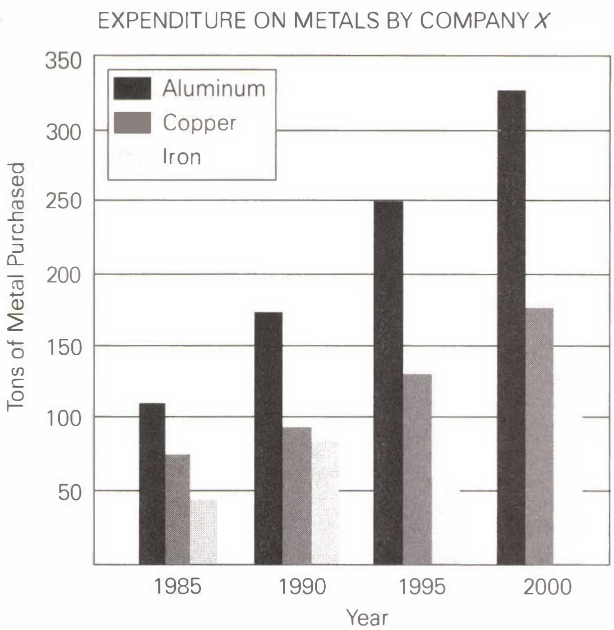
What is approximately 9.6 percent of 21.4?

Here's How to Crack It

Use 10 percent as a friendlier percentage and 20 as a friendlier number. One-tenth of 20 is 2 (it says "approximately"—who are you to argue?). That's all you need to do to answer most chart questions.

Chart Problems

Make sure you've read everything on the chart carefully before you try the first question.



Year	Price of Aluminum Per Ton
1985	$1,900
1990	$2,200
1995	$2,700
2000	$3,400

Note: Graphs drawn to scale.

Approximately how many tons of aluminum and copper combined were purchased in 1995 ?

- ○ 125
- ○ 255
- ○ 325
- ○ 375
- ○ 515

How much did Company X spend on aluminum in 1990 ?

- $675,000
- $385,000
- $333,000
- $165,000
- $139,000

Approximately what was the percent increase in the price of aluminum from 1985 to 1995 ?

- 8%
- 16%
- 23%
- 30%
- 42%

Here's How to Crack the First Question

As you can see from the graph on the previous page, in 1995, the black bar (which indicates aluminum) is at 250, and the dark grey bar (which indicates copper) is at approximately 125. Add those figures and you get the number of tons of aluminum and copper combined that were purchased in 1995: 250 + 125 = 375. That's choice (D). Notice that the question says "approximately." Also notice that the numbers in the answer choices are pretty far apart.

Here's How to Crack the Second Question

We need to use the chart and the graph to answer this question, because we need to find the number of tons of aluminum purchased in 1990 and multiply it by the price per ton of aluminum in 1990 in order to figure out how much was spent on aluminum in 1990. The bar graph tells us that 175 tons of aluminum was purchased in 1990, and the little chart tells us that aluminum was $2,200 per ton in 1990. 175 × $2,200 = $385,000. That's choice (B).

Here's How to Crack the Third Question

Remember that percent increase formula from earlier in this chapter?

$$\text{Percent change} = \frac{\text{Difference}}{\text{Original}} \times 100$$

We'll need to use the little chart for this one. In 1985, the price of aluminum was \$1,900 per ton. In 1995, the price of aluminum was \$2,700 per ton. Now let's use the formula. 2,700 – 1,900 = 800, so that's the difference. This is a percent increase problem, so the original number is the smaller one. Thus, the original is 1,900, and our formula looks like this: Percent change = $\frac{800}{1{,}900} \times 100$. By canceling the 0's in the fraction you get $\frac{8}{19} \times 100$, and multiplying gives you $\frac{800}{19}$. At this point you could divide 800 by 19 to get the exact answer, but because they're looking for an approximation, let's round 19 to 20. What's 800 ÷ 20? That's 40, and answer choice (E) is the only one that's close.

Real World Math Drill

Now it's time to try out what you've learned on some practice questions. Try the following problems and then check your answers in Part V.

1 of 19

If 3 $(r + s) = 7$, then, in terms of r, $s =$

- ○ $\frac{7}{3} - r$
- ○ $\frac{7}{3} + r$
- ○ $7 - 3r$
- ○ $\frac{7}{3} - \frac{r}{3}$
- ○ $\frac{7}{3} + \frac{r}{3}$

2 of 19

If Sadie sells half the paintings in her collection, gives one-third to friends, and keeps the remaining paintings for herself, what fraction of her collection does Sadie keep?

Click on each box and type in a number. Backspace to erase.

3 of 19

During a sale, a store decreases prices on all its scarves by 25 to 50 percent. If all of the scarves in the store originally cost \$20, which of the following could be the sale price of a scarf?

Indicate all such prices.

- ☐ \$8
- ☐ \$10
- ☐ \$12
- ☐ \$14
- ☐ \$16

4 of 19

Quantity A	Quantity B
12 percent of 35	35 percent of 12

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

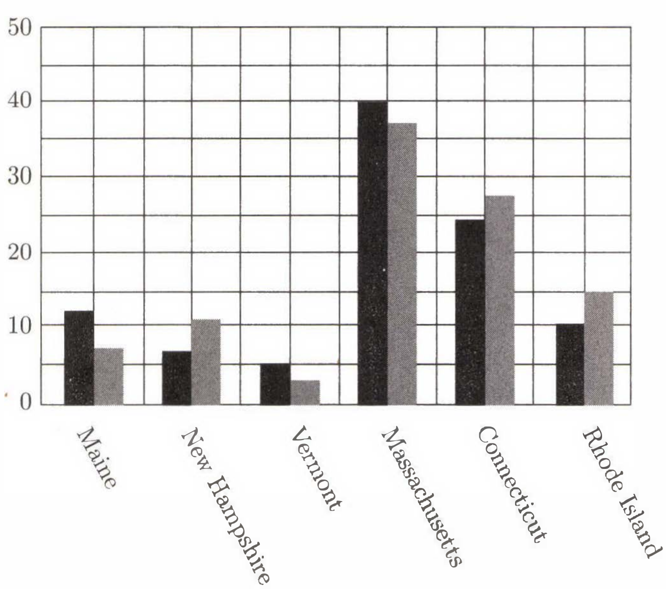
5 of 19

Quantity A	Quantity B
$\dfrac{2.6}{0.259}$	10

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.

Questions 6 through 9 refer to the following graph.

PERCENT OF POPULATION IN NEW ENGLAND BY STATE IN YEAR X AND YEAR Y



Note: Drawn to scale

■ Year X: Total New England population= 15 million
■ Year Y: Total New England population= 25 million

6 of 19

The six New England states are ranked by population in Year X and in Year Y. How many states had a different ranking from Year X to Year Y ?

- ◯ None
- ◯ One
- ◯ Two
- ◯ Three
- ◯ Four

7 of 19

In Year Y, the population of Massachusetts was approximately what percent of the population of Vermont?

- ◯ 50%
- ◯ 120%
- ◯ 300%
- ◯ 400%
- ◯ 1,200%

8 of 19

By approximately how much did the population of Rhode Island increase from Year X to Year Y ?

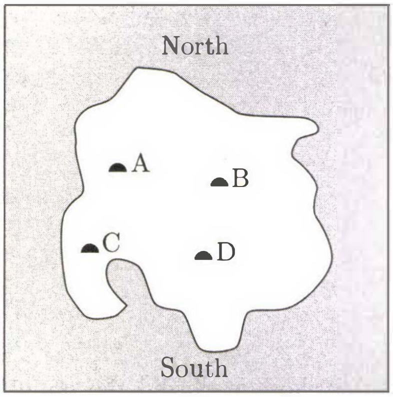
- ◯ 750,000
- ◯ 1,250,000
- ◯ 1,500,000
- ◯ 2,250,000
- ◯ 3,375,000

9 of 19

Approximately what is the difference between the percent change of Connecticut's percent of total New England population from Year X to Year Y and the percent change of Massachusetts percent of total New England population from Year X to Year Y ?

- ○ –12.5
- ○ 5
- ○ 12.5
- ○ 25
- ○ 50

10 of 19



Towns A, B, C, and D are located on the map as shown. Towns A and B have 3,000 people each who support referendum R, and the referendum has an average of 3,500 supporters in towns B and D.

Quantity A	Quantity B
The average number of supporters of referendum R in the two southern-most towns	2,500

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

11 of 19

A company paid \$500,000 in merit raises to employees whose performances were rated A, B, or C. Each employee rated A received twice the amount of the raise that was paid to each employee rated C; each employee rated B received one-and-a-half times the amount of the raise that was paid to each employee rated C. If 50 workers were rated A, 100 were rated B, and 150 were rated C, how much was the raise paid to each employee rated A ?

- ○ \$370
- ○ \$625
- ○ \$740
- ○ \$1,250
- ○ \$2,500

12 of 19

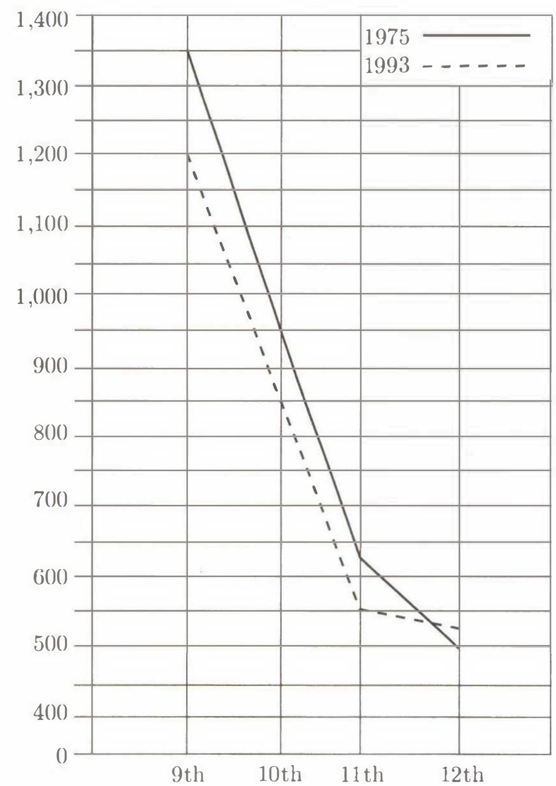
The original price of an item at a store is 40 percent more than the price the retailer paid for it. To encourage sales, the retailer reduces the price of the item by 15 percent from the original selling price. If the retailer sells the item at the reduced cost, his profit is what percent of his cost?

[ ] percent

Click on the answer box and type in a number.
Backspace to erase.

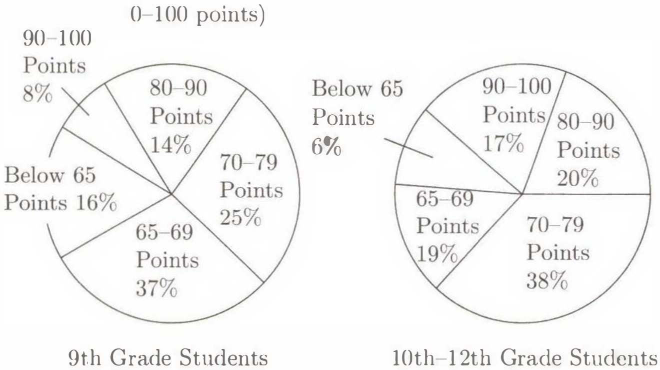
Questions 13 through 15 refer to the following graphs.

NUMBER OF STUDENTS IN GRADES 9 THROUGH 12 FOR SCHOOL DISTRICT *X* IN 1975 AND 1993



DISTRIBUTION OF READING TEST SCORES* FOR SCHOOL DISTRICT *X* STUDENTS IN 1993

(*Reading Test scores can range from 0–100 points)



Note: Drawn to scale.

13 of 19

In 1993, the median reading test score for ninth grade students was in which score range?

- ○ Below 65 points
- ○ 65–69 points
- ○ 70–79 points
- ○ 80–89 points
- ○ 90–100 points

14 of 19

If the number of students in grades 9 through 12 comprised 35 percent of the number of students in School District *X* in 1975, then approximately how many students were in School District *X* in 1975 ?

- ○ 9,700
- ○ 8,700
- ○ 3,400
- ○ 3,000
- ○ 1,200

15 of 19

Assume that all students in School District *X* took the reading test each year. In 1993, approximately how many more ninth grade students had reading test scores in the 70–79 point range than in the 80–89 point range?

- ○ 470
- ○ 300
- ○ 240
- ○ 170
- ○ 130

16 of 19

Quantity A	Quantity B
$2 - \frac{27}{25}$	$\frac{3}{5} + \frac{12}{125}$

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.

17 of 19

Solution X contains only ingredients a and b in a ratio of $2 : 3$. Solution Y contains only ingredients a and b in a ratio of $1 : 2$. If Solution Z is created by mixing solutions X and Y in a ratio of $3 : 11$, then 630 ounces of Solution Z contains how many ounces of a?

- ◯ 68
- ◯ 73
- ◯ 89
- ◯ 219
- ◯ 236

18 of 19

On Sunday, Belmond Public Library has 160 books, none of which have been checked out. On Monday, 40 of the books are checked out. On Tuesday, $\frac{1}{2}$ of the borrowed books are returned. Wednesday, $\frac{1}{2}$ of the books still checked out are returned and then 20 more are checked out. On Thursday, a wealthy patron donates 80 books, and $\frac{1}{6}$ of the books still checked out are returned. On Friday 30 more books are borrowed, and on Saturday 35 are checked out. What is the percent change from the books in the library at the end of the day on Monday to the books in the library at end of the day the following Saturday?

[ ] percent

Click on the answer box and type in a number. Backspace to erase.

19 of 19

Jill has received 8 of her 12 evaluation scores. So far, Jill's average (arithmetic mean) is 3.75 out of a possible 5. If Jill needs an average of 4.0 points to get a promotion, which set of scores will allow Jill to receive her promotion?

Indicate all such sets.

- ☐ 3.0, 3.5, 4.75, 4.75
- ☐ 3.5, 4.75, 4.75, 5.0
- ☐ 3.25, 4.5, 4.75, 5.0
- ☐ 3.75, 4.5, 4.75, 5.0

Summary

- Fractions, decimals, and percents are all ways of expressing parts of integers.
- Translation is a useful tool for converting fraction and percent problems into mathematical equations.
- Percent change is expressed as the difference between two numbers divided by the original number × 100.
- Plug In on questions that ask about percents or fractions of an unknown amount.
- A ratio expresses a part to part relationship. The key to ratio problems is finding the total. Use the ratio box to organize ratio questions.
- A proportion expresses the relationship between equal fractions, percents, or ratios. A proportion problem always provides you with three pieces of information and asks you for a fourth.
- Use the Average Pie to organize and crack average problems.
- The median is the middle number in a set of values. The mode is the value that appears most frequently in a set. The range of a set is the difference between the largest and smallest values in the set.
- You will never have to calculate standard deviation on the GRE.
- Standard deviation problems are really average and percent problems. Make sure you know the percentages associated with the bell curve: 34%–14%–2%.
- Use the Rate Pie for rate questions.
- On chart questions, make sure you take a moment to understand what information the chart is providing. Estimate answers to chart questions whenever possible.

Chapter 11
Geometry

Chances are you probably haven't used the Pythagorean theorem recently or had to find the area of a circle in quite a while. However, you'll be expected to know geometry concepts such as these on the new GRE. This chapter reviews all the important rules and formulas you'll need to crack the geometry problems on the GRE. It also provides examples of how such concepts will be tested on the GRE Math section.

WHY GEOMETRY?

Expect to see a handful of basic geometry problems on each of your Math sections.

Good question. If you're going to graduate school for political science or linguistics or history or practically anything that doesn't involve math, you might be wondering why the heck you have to know the area of a circle or the Pythagorean theorem for this exam. While we may not be able to give you a satisfactory answer to that question, we can help you do well on the geometry questions on the GRE.

WHAT YOU NEED TO KNOW

The good news is that you don't need to know much about actual geometry to do well on the GRE; we've boiled down geometry to the handful of bits and pieces that ETS actually tests.

Before we begin, consider yourself warned: Since you'll be taking your test on a computer screen, you'll have to be sure to transcribe all the figures onto your scrap paper accurately. All it takes is one mistaken angle or line and you're sure to get the problem wrong. So make ample use of your scratch paper and always double-check your figures. Start practicing now, by using scratch paper with this book.

Problem-solving questions will be drawn to scale unless they clearly tell you otherwise. Quant comp questions, on the other hand, may *not* be drawn to scale, so be on your guard!

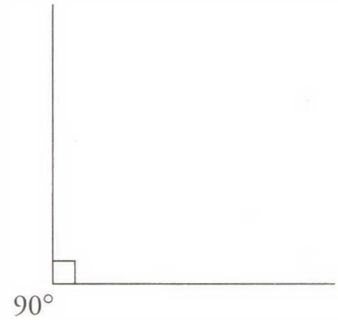
Another important thing to know is that you cannot necessarily trust the diagrams ETS gives you. Sometimes they are very deceptive and are intended to confuse you. Always go by what you read, not what you see.

DEGREES, LINES, AND ANGLES

For the GRE, you will need to know that

1. A line is a 180-degree angle. In other words, a line is a perfectly flat angle.
2. When two lines intersect, four angles are formed; the sum of these angles is 360 degrees.
3. When two lines are perpendicular to each other, their intersection forms four 90-degree angles. Here is the symbol ETS uses to indicate perpendicular lines: $\perp$.

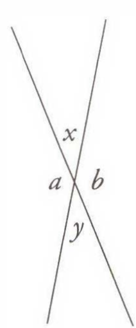
4. Ninety-degree angles are also called *right angles.* A right angle on the GRE is identified by a little box at the intersection of the angle's arms:



5. The three angles inside a triangle add up to 180 degrees.
6. The four angles inside any four-sided figure add up to 360 degrees.
7. A circle contains 360 degrees.
8. Any line that extends from the center of a circle to the edge of the circle is called a *radius* (plural is *radii*).

Vertical Angles

Vertical angles are the angles that are across from each other when two lines intersect. Vertical angles are always equal. In the drawing below, angle x is equal to angle y (they are vertical angles) and angle a is equal to angle b (they are also vertical angles).

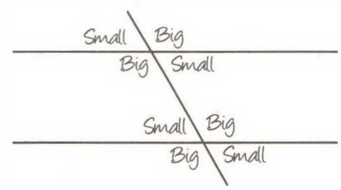


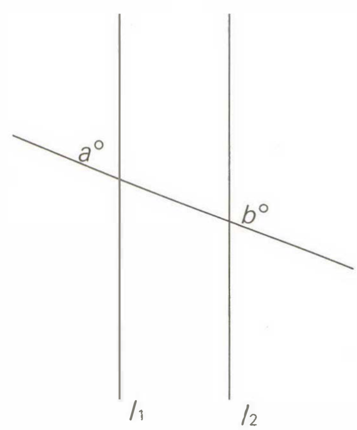
On the GRE, the measure of only one of the vertical angles is typically shown. But usually you'll need to use the other angle to solve the problem.

Parallel Lines

Parallel lines are lines that never intersect. When a pair of parallel lines is intersected by a third, two types of angles are formed: big angles and small angles. Any big angle is equal to any big angle, and any small angle is equal to any small angle. The sum of any big angle and any small angle will always equal 180. When

ETS tells you that two lines are parallel, this is what is being tested. The symbol for parallel lines and the word *parallel* are both clues that tell you what to look for in the problem. The minute you see either of them, immediately identify your big and small angles; they will probably come into play.





l_1 and l_2 are parallel

Quantity A	Quantity B
$a + b$	180

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Here's How to Crack It

Notice that you're told that these lines are parallel. Here's one very important point: You need to be told that. You can't assume that they are parallel just because they look like they are.

Okay, so as you just learned, only two angles are formed when two parallel lines are intersected by a third line: a big angle (greater than 90 degrees) and a small one (smaller than 90 degrees). Look at angle a. It looks smaller than 90, right? Now look at angle b. It looks bigger than 90, right? You also know that $a + b$ must add up to 180. The answer is (C).

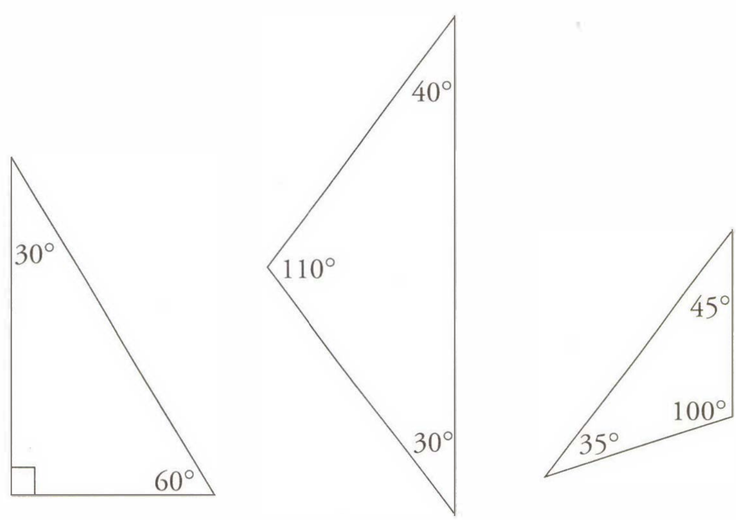
TRIANGLES

Triangles are perhaps ETS's favorite geometrical shape. Triangles have many properties, which make them great candidates for standardized test questions. Make sure you familiarize yourself with the following triangle facts.

Triangles are frequently tested on the GRE.

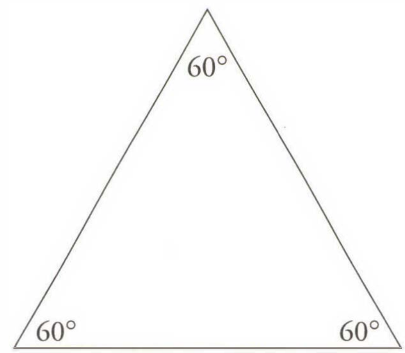
The Rule of 180°

Every triangle contains three angles that add up to 180 degrees. You must know this fact cold for the exam. This rule applies to every triangle, no matter what it looks like. Here are some examples:



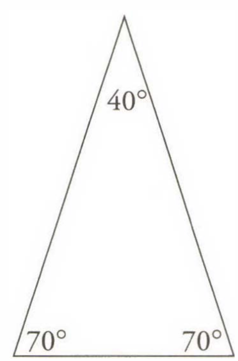
Equilateral Triangles

An **equilateral triangle** is a triangle in which all three sides are equal in length. Because all of the sides are equal in these triangles, all of the angles are equal. Each angle is 60 degrees because 180 divided by 3 is 60.



Isosceles Triangles

An **isosceles triangle** is a triangle in which two of the three sides are equal in length. This means that two of the angles are also equal.



Angle/Side Relationships in Triangles

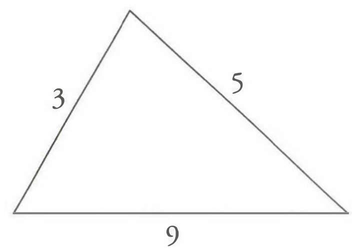
In any triangle, the longest side is opposite the largest interior angle; the shortest side is opposite the smallest interior angle. That's why the hypotenuse of a right triangle is its longest side—there couldn't be another angle in the triangle bigger than 90 degrees. Furthermore, equal sides are opposite equal angles.

Perimeter of a Triangle

The perimeter of a triangle is simply a measure of the distance around it. All you have to do to find the perimeter of a triangle is add up the lengths of the sides.

The Third Side Rule

Why is it impossible for the following triangle to exist? (Hint: It's not drawn to scale.)



This triangle could not exist because the length of any one side of a triangle is limited by the lengths of the other two sides. This can be summarized by the **third side rule:**

> The length of any one side of a triangle must be less than the sum of the other two sides and greater than the difference between the other two sides.

This rule is not tested frequently on the GRE, but when it is, it's usually the key to solving the problem. Here's what the rule means in application: Take the lengths of any two sides of a triangle. Add them together, then subtract one from the other. The length of the third side must lie between those two numbers.

Take the sides 3 and 5 from the triangle above. What's the longest the third side could measure? Just add and subtract. It could not be as long as 8 (5 + 3) and it could not be as short as 2 (5 – 3).

Therefore, the third side must lie between 2 and 8. It's important to remember that the third side cannot be equal to either 2 or 8. It must be greater than 2 and less than 8.

Try the following question:

A triangle has sides 4, 7, and x. Which of the following could be the perimeter of the triangle?

Indicate <u>all</u> such perimeters.

- ☐ 13
- ☐ 16
- ☐ 17
- ☐ 20
- ☐ 22

Here's How to Crack It

The perimeter of a triangle is equal to the sum of its three sides. So far, we have sides of 4 and 7, so our partial perimeter is $4 + 7 = 11$. What about the third side, x? The third-side rule tells us that the side could not be longer than $7 + 4 = 11$ or shorter than $7 - 4 = 3$. The third side must be greater than 3 and less than 11. Next we add the partial perimeter, 11, to both of these numbers to find the range of the perimeter. $11 + 3 = 14$ and $11 + 11 = 22$, so the perimeter must be greater than 14 and less than 22. Only choices (A) and (E) fall outside this range. For this question, we have to click on all of the answers that work, so the best answer is (B), (C), and (D).

Any time you see the word *area* or any other word that indicates that a formula is to be used, write the formula on your scratch paper and park the information you're given directly underneath.

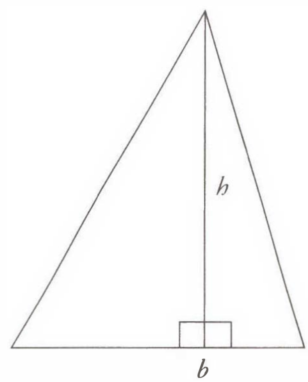
Area of a Triangle

The area of any triangle is equal to its height (or altitude) multiplied by its base, divided by 2, so

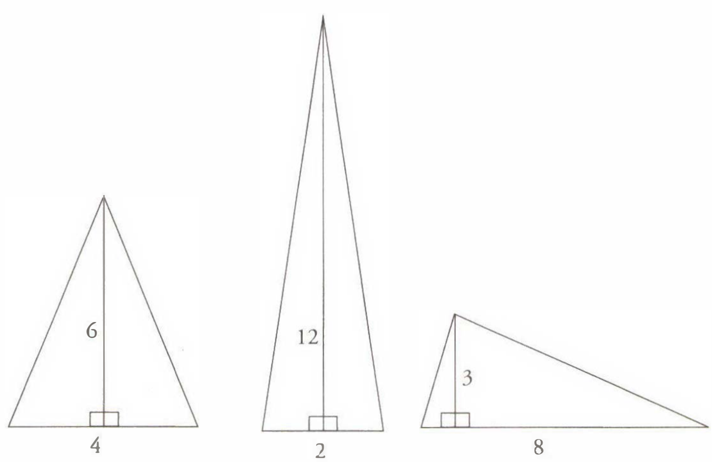
$$A = \frac{1}{2}bh$$

The height of a triangle is defined as the length of a perpendicular line drawn from the point of the triangle to its base.

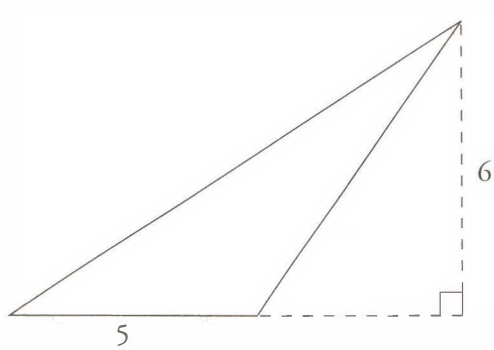
The height of a triangle must be perpendicular to the base.



This area formula works on any triangle.



area = 12 *area* = 12 *area* = 12

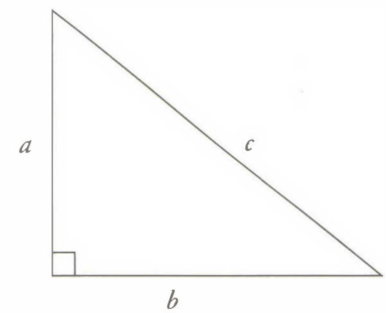


area = 15

ETS will sometimes try to intimidate you by using multiples of the common Pythagorean triples. For example, you might see a 10-24-26 triangle. That's just a 5-12-13 in disguise though.

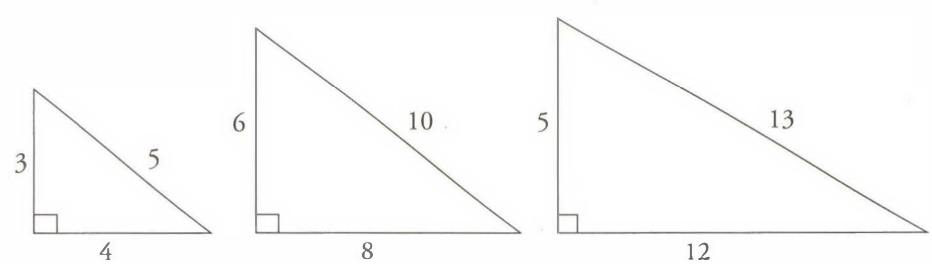
The Pythagorean Theorem

The Pythagorean theorem applies only to right triangles. This theorem states that in a right triangle, the square of the length of the hypotenuse (the longest side, remember?) is equal to the sum of the squares of the lengths of the two other sides. In other words, $c^2 = a^2 + b^2$, where c is the length of the hypotenuse and a and b are the lengths of the other sides. (The two sides that are not the hypotenuse are called the legs.)



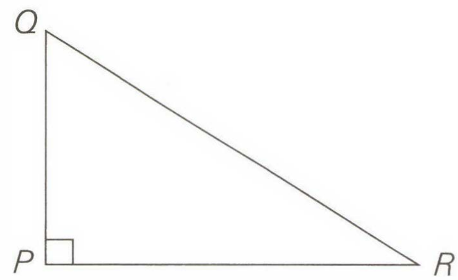
You can always use the Pythagorean theorem to calculate the third side of a right triangle.

Here are the most common right triangles:



Note that a triangle could have sides with actual lengths of 3, 4, and 5, or 3 : 4 : 5 could just be the ratio of the sides. If you double the ratio, you get a triangle with sides equal to 6, 8, and 10. If you triple it, you get a triangle with sides equal to 9, 12, and 15.

Let's try an example.



In the figure above, if the distance from point P to point Q is 6 miles and the distance from point Q to point R is 10 miles, what is the distance from point P to point R?

- 4
- 5
- 6
- 7
- 8

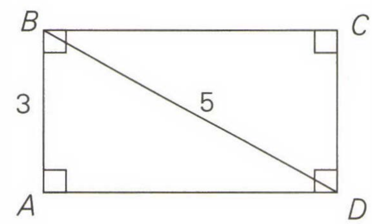
Write everything down on scratch paper! Don't do anything in your head.

Here's How to Crack It

Once you've sensitized yourself to the standard right triangles, this problem couldn't be easier. When you see a right triangle, be suspicious. One leg is 6. The hypotenuse is 10. The triangle has a ratio of 3 : 4 : 5. Therefore, the third side (the other leg) must be 8.

The Pythagorean theorem will sometimes help you solve problems that involve squares or rectangles. For example, every rectangle or square can be divided into two right triangles. This means that if you know the length and width of any rectangle or square, you also know the length of the diagonal—it's the shared hypotenuse of the hidden right triangles.

Here's an example:



In the rectangle above, what is the area of triangle ABD ?

Click on the answer box and type in a number. Backspace to erase.

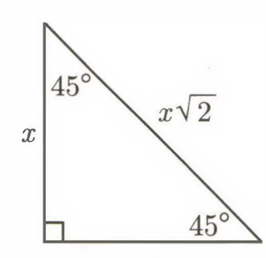
Here's How to Crack It

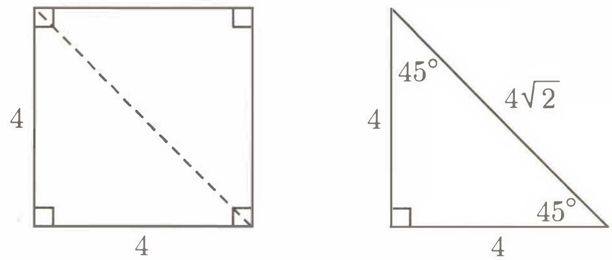
We were told that this is a rectangle (remember that you can never assume!), which means that triangle ABD is a right triangle. Not only that, but it's a 3 : 4 : 5 right triangle (with a side of 3 and a hypotenuse of 5, it must be), with side $AD = 4$. So, the area of triangle ABD is $\frac{1}{2}$ the base (3) times the height (4). That's $\frac{1}{2}$ of 12, otherwise known as 6. You could enter that value into the box.

Right Isosceles Triangles

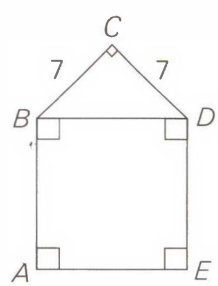
You always know the length of the diagonal of a square because it is one side of the square times $\sqrt{2}$.

If you take a square and cut it in half along its diagonal, you will create a right isosceles triangle. The two sides of the square stay the same. The 90-degree angle will stay the same, and the other two angles that were 90 degrees each get cut in half and are now 45 degrees. The ratio of sides in a right isosceles triangle is $x : x : x\sqrt{2}$. This is significant for two reasons. First, if you see a problem with a right triangle and there is a $\sqrt{2}$ anywhere in the problem, you know what to look for. Second, you always know the length of the diagonal of a square because it is one side times the square root of two.





Let's try an example involving a special right triangle.



In the figure above, what is the area of square *ABDE* ?

- ○ $28\sqrt{2}$
- ○ 49
- ○ $49\sqrt{2}$
- ○ 98
- ○ $98\sqrt{2}$

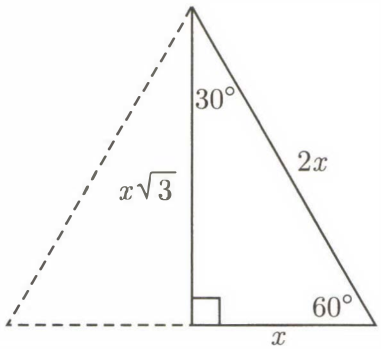
Here's How to Crack It

In order to figure out the area of square *ABDE*, we need to know the length of one of its sides. We can get the length of *BD* by using the isosceles right triangle attached to it. *BD* is the hypotenuse, which means its length is $7\sqrt{2}$. To get the area of the square we have to square the length of the side we know, or $\left(7\sqrt{2}\right)\left(7\sqrt{2}\right)$ = (49)(2) = 98. That's choice (D).

You can always calculate the area of an equilateral triangle because you know that the height is one half of one side times $\sqrt{3}$.

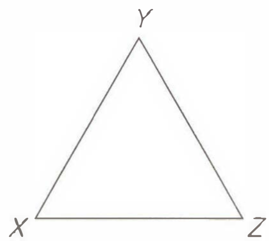
30 : 60 : 90 Triangles

If you take an equilateral triangle and draw in the height, you end up cutting it in half and creating a right triangle. The hypotenuse of the right triangle has not changed; it's just one side of the equilateral triangle. One of the 60 degree angles stays the same as well. The angle where the height meets the base is 90 degrees, naturally, and the side that was the base of the equilateral triangle has been cut in half. The smallest angle, at the top, opposite the smallest side, is 30 degrees. The ratio of sides on a 30 : 60 : 90 triangle is $x : x\sqrt{3} : 2x$. Here's what it looks like:



This is significant for two reasons. The first is that if you see a problem with a right triangle and one side is double the other or there is a $\sqrt{3}$ anywhere in the problem, you know what to look for. The second is that you always know the area of an equilateral triangle because you always know the height. It is one half of one side times the square root of three.

Here's one more:



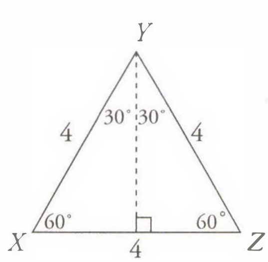
Triangle XYZ in the figure above is an equilateral triangle. If the perimeter of the triangle is 12, what is its area?

- ○ $2\sqrt{3}$
- ○ $4\sqrt{3}$
- ○ 8
- ○ 12
- ○ $8\sqrt{3}$

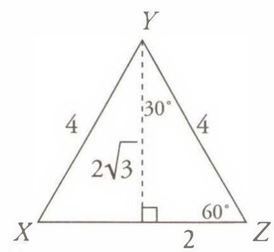
If you see $\sqrt{2}$ or $\sqrt{3}$ in the answer choices of the problem it's a tip-off that the problem is testing special right triangles.

Here's How to Crack It

Here we have an equilateral triangle with a perimeter of 12, which means that each side has a length of 4 and each angle is 60 degrees. Remember that in order to find the area of a triangle, we use the triangle area formula: $A = \frac{1}{2}bh$, but first we need to know the base and the height of the triangle. The base is 4, which now gives us $A = \frac{1}{2}4h$, and now the only thing we need is the height. Remember: The height always has to be perpendicular to the base. Draw a vertical line that splits the equilateral triangle in half. The top angle is also split in half, so now we have this:



What we've done is create two 30 : 60 : 90 right triangles, and we're going to use one of these right triangles to find the height. Let's use the one on the right. We know that the hypotenuse in a 30 : 60 : 90 right triangle is always twice the length of the short side. Here we have a hypotenuse (YZ) of 4, so our short side has to be 2. The long side of a 30 : 60 : 90 right triangle is always equal to the short side multiplied by the square root of 3. So if our short side is 2, then our long side must be $2\sqrt{3}$. That's the height.



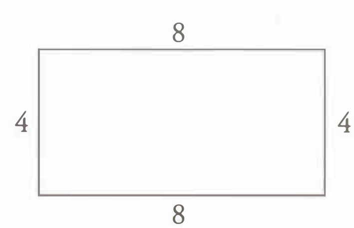
Finally, we return to our area formula. Now we have $A = \frac{1}{2} \times 4 \times 2\sqrt{3}$. Multiply it out and you get $A = 4\sqrt{3}$. The answer is (B).

FOUR-SIDED FIGURES

The four angles inside any figure that has four sides add up to 360 degrees. That includes rectangles, squares, and parallelograms. Parallelograms are four-sided figures made out of two sets of parallel lines whose area can be found with the formula $A = bh$, where h is the height of a line drawn perpendicular to the base.

Perimeter of a Rectangle

The perimeter of a rectangle is just the sum of the lengths of its four sides.



$$perimeter = 4 + 8 + 4 + 8$$

The area of a rectangle is equal to its length times its width. For example, the area of the rectangle above is 32 (or 8×4).

Area of a Rectangle

The area of a rectangle is equal to its length times its width. For example, the area of the rectangle above is 32 (or 8×4).

Squares

A square has four equal sides. The perimeter of a square is, therefore, 4 times the length of any side. The area of a square is equal to the length of any side times itself, or in other words, the length of any side, squared. The diagonal of a square splits it into two 45 : 45 : 90, or isosceles, right triangles.

The World of Pi

You may remember being taught that the value of pi (π) is 3.14, or even 3.14159. On the GRE, π = 3ish is a close enough approximation. You don't need to be any more precise than that when doing GRE problems.

What you might not recall about pi is that pi (π) is the ratio between the circumference of a circle and its diameter. When we say that π is a little bigger than 3, we're saying that every circle is about three times as far around as it is across.

CIRCLES

Circles are a popular test topic for ETS. There are a few properties that the GRE likes to test over and over again and problems with circles also always seem to use that funny little symbol π. Here's all you need to know about circles.

Radius and Diameter

The **radius** of a circle is any line that extends from the center of the circle to the edge of the circle. If the line extends from one edge of a circle to the other and goes through the circle's center, it's the circle's **diameter**. Therefore, the diameter of a circle is twice as long as its radius.

The radius is always the key to circle problems.

Circumference of a Circle

The **circumference** of a circle is like the perimeter of a triangle: It's the distance around the outside. The formula for finding the circumference of a circle is 2 times π times the radius, or π times the diameter.

Circumference is just a fancy way of saying perimeter.

$$\text{circumference} = 2\pi r \text{ or } \pi d$$

If the diameter of a circle is 4, then its circumference is 4π, or roughly 12+. If the diameter of a circle is 10, then its circumference is 10π, or a little more than 30.

An **arc** is a section of the outside, or circumference, of a circle. An angle formed by two radii is called a **central angle** (it comes out to the edge from the center of the circle). There are 360 degrees in a circle, so if there is an arc formed by, say, a 60-degree central angle, and 60 is $\frac{1}{6}$ of 360, then the arc formed by this 60-degree central angle will be $\frac{1}{6}$ of the circumference of the circle.

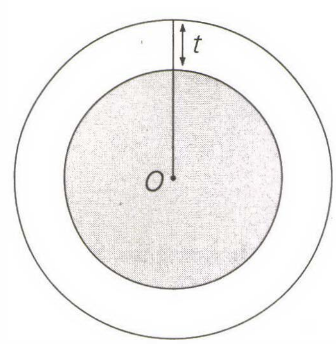
AREA OF A CIRCLE

When working with π, leave it as π in your calculations. Also, leave $\sqrt{3}$ as $\sqrt{3}$. The answer will have them that way.

The area of a circle is equal to π times the square of its radius.

$$\text{area} = \pi r^2$$

Let's try an example of a circle question.



Note: Figure not drawn to scale.

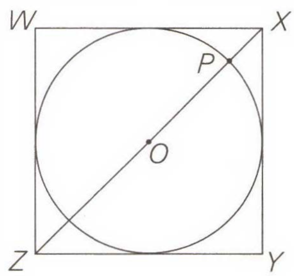
In the wheel above, with center O, the area of the entire wheel is 169π. If the area of the shaded hubcap is 144π, then $t =$

Click on the answer box and type in a number. Backspace to erase.

Here's How to Crack It

We have to figure out what t is, and it's going to be the length of the radius of the entire wheel minus the length of the radius of the hubcap. If the area of the entire wheel is 169π, the radius is $\sqrt{169}$, or 13. If the area of the hubcap is 144π, the radius is $\sqrt{144}$, or 12. $13 - 12 = 1$. Enter this value into the box.

Let's try another one.



In the figure above, a circle with the center O is inscribed in square $WXYZ$. If the circle has radius 3, then PZ =

- 6
- $3\sqrt{2}$
- $6 + \sqrt{2}$
- $3 + \sqrt{3}$
- $3\sqrt{2} + 3$

Ballparking answers will help you eliminate choices.

Here's How to Crack It

Inscribed means that the edges of the shapes are touching. The radius of the circle is 3, which means that PO is 3. If Z were at the other end of the diameter from P, this problem would be easy and the answer would be 6, right? But Z is beyond the edge of the circle, which means that PZ is a little more than 6. Let's stop there for a minute and glance at the answer choices. We can eliminate anything that's "out of the ballpark"—in other words, any answer choice that's less than 6, equal to 6 itself, or a lot more than 6. Remember when we told you to memorize a few of those square roots?

Let's use them:

(A) Exactly 6? Nope.

(B) That's 1.4 × 3, which is 4.2. Too small.

(C) That's 6 + 1.4, or 7.4. Not bad. Let's leave that one in.

(D) That's 3 + 1.7, or 4.7. Too small.

(E) That's (3 × 1.4) + 3, which is 4.2 + 3, or 7.2. Not bad. Let's leave that one in, too.

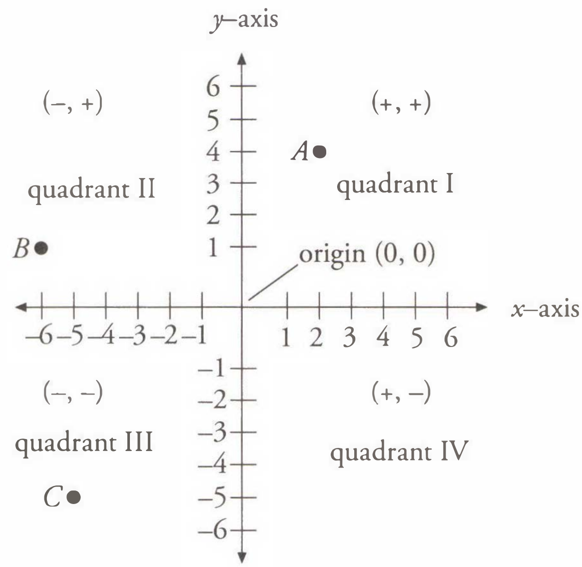
So we eliminated three choices with Ballparking. We're left with (C) and (E). You could take a guess here if you had to, but let's do a little more geometry to find the correct answer.

Because this circle is inscribed in the square, the diameter of the circle is the same as a side of the square. We already know that the diameter of the circle is 6, so that means that ZY, and indeed all the sides of the square, are also 6. Now, if ZY is 6, and XY is 6, what's XZ, the diagonal of the square? Well, XZ is also the hypotenuse of the isosceles right triangle XYZ. The hypotenuse of a right triangle with two sides of 6 is $6\sqrt{2}$. That's approximately 6 × 1.4, or 8.4.

The question is asking for PZ, which is a little less than XZ. It's somewhere between 6 and 8.4. The pieces that aren't part of the diameter of the circle are equal to 8.4 – 6, or 2.4. Divide that in half to get 1.2, which is the distance from the edge of the circle to Z. That means that PZ is 6 + 1.2, or 7.2. Check your remaining answers: Choice (C) is 7.4, and choice (E) is 7.2. Bingo! The answer is (E).

THE COORDINATE SYSTEM

On a coordinate system, the horizontal line is called the **x-axis** and the vertical line is called the **y-axis**. The four areas formed by the intersection of these axes are called **quadrants**. The point where the axes intersect is called the **origin**. This is what it looks like:

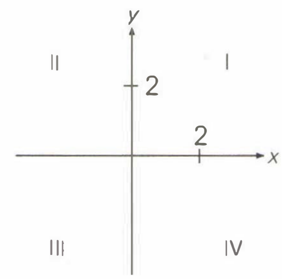


To express any point in the coordinate system, you first give the horizontal value, then the vertical value, or (x, y). In the diagram above, point A can be described by the coordinates (2, 4). That is, the point is two spaces to the right of the origin and four spaces above the origin. Point B can be described by the coordinates (–6, 1). That is, it is six spaces to the left and one space above the origin. What are the coordinates of point C? Right, it's (–5, –5).

Coordinate geometry questions often test basic shapes such as triangles and squares.

ALWAYS write A, B, C, D, E on your scratch paper to represent the answer choices (or A, B, C, D if it's a quant comp question.)

Here's a GRE example:



Points $(x, 5)$ and $(-6, y)$, not shown in the figure above, are in quadrants I and III, respectively. If $xy \neq 0$, in which quadrant is point (x, y) ?

- ○ IV
- ○ III
- ○ II
- ○ I
- ○ It cannot be determined from the information given.

Here's How to Crack It

If point $(x, 5)$ is in quadrant I, that means x is positive. If point y is in quadrant III, then y is negative. The quadrant that would contain coordinate points with a positive x and a negative y is quadrant IV. That's answer choice (A).

Slope

Trickier questions involving the coordinate system might give you the equation for a line on the grid, which will involve something called the slope of the line. The equation of a line is

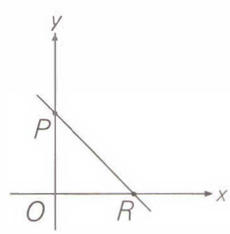
$$y = mx + b$$

In this equation x and y are both points on the line, b stands for the y-intercept, or the point at which the line crosses the y-axis, and m is the slope of the line.

Slope is defined as the vertical change divided by the horizontal change, often called "the rise over the run" or the "change in y over the change in x."

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

Sometimes on the GRE, m is written instead as a, as in $y = ax + b$. You'll see all this in action in a moment.



The line $y = -\frac{8}{7}x + 1$ is graphed on the rectangular coordinate axes.

Quantity A	Quantity B
OR	OP

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Coordinate Geometry

If you are at the high end of the scoring range, you are likely to see more coordinate geometry problems than you have in the past. Whenever you are dealing with coordinate geometry, you should keep a few basic rules in mind.

To watch a short video on the key concepts in coordinate geometry, register your book at PrincetonReview.com/cracking.

Here's How to Crack It

The y-intercept, or b, in this case is 1. That means the line crosses the y-axis at 1. So the coordinates of point P are (0, 1). Now we have to figure out what the coordinates of point R are. We know the y-coordinate is 0, so let's stick that into the equation (the slope and the y-intercept are constant; they don't change).

$$y = mx + b$$

$$0 = -\frac{8}{7}x + 1$$

Now let's solve for x.

$$0 = -\frac{8}{7}x + 1$$

$$0 - 1 = -\frac{8}{7}x + 1 - 1$$

$$-1 = -\frac{8}{7}x$$

$$\left(-\frac{7}{8}\right)(-1) = \left(-\frac{7}{8}\right)\left(-\frac{8}{7}\right)x$$

$$\frac{7}{8} = x$$

So the coordinates of point R are $(\frac{7}{8}, 0)$. That means OR, in Quantity A, is equal to $\frac{7}{8}$, and OP, in Quantity B, is equal to 1. The answer is (B).

Another approach to this question would be to focus on the meaning of slope. Because the slope is $-\frac{8}{7}$, that means the vertical change is 8 and the horizontal change is 7. In other words, you count up 8 and over 7. Clearly the "up" is more than the "over"; thus OP is more than OR.

Incidentally, if you're curious about the difference between a positive and negative slope, any line that rises from left to right has a positive slope. Any line that falls from left to right has a negative slope. (A horizontal line has a slope of 0, and a vertical line is said to have "no slope.")

VOLUME

You can find the volume of a three-dimensional figure by multiplying the area of a two-dimensional figure by its height (or depth). For example, to find the volume of a rectangular solid, you would take the area of a rectangle and multiply it by the depth. The formula is lwh (length × width × height). To find the volume of a circular cylinder, take the area of a circle and multiply by the height. The formula is πr^2 times the height (or $\pi r^2 h$).

DIAGONALS IN THREE DIMENSIONS

There's a special formula that you can use if you are ever asked to find the length of a diagonal (the longest distance between any two corners) inside a three-dimensional rectangular box. It is $a^2 + b^2 + c^2 = d^2$, where a, b, and c are the dimensions of the figure (kind of looks like the Pythagorean theorem, huh?).

Questions that ask about diagonals are really about the Pythagorean theorem.

Take a look:

What is the length of the longest distance between any two corners in a rectangular box with dimensions 3 inches by 4 inches by 5 inches?

- ○ 5
- ○ 12
- ○ $5\sqrt{2}$
- ○ $12\sqrt{2}$
- ○ 50

Here's How to Crack It

Let's use our formula, $a^2 + b^2 + c^2 = d^2$. The dimensions of the box are 3, 4, and 5.

$$
\begin{aligned}
3^2 + 4^2 + 5^2 &= d^2 \\
9 + 16 + 25 &= d^2 \\
50 &= d^2 \\
\sqrt{50} &= d \\
\sqrt{25 \times 2} &= d \\
\sqrt{25} \times \sqrt{2} &= d \\
5\sqrt{2} &= d
\end{aligned}
$$

That's choice (C).

Don't confuse surface area with volume.

SURFACE AREA

The surface area of a rectangular box is equal to the sum of the areas of all of its sides. In other words, if you had a box whose dimensions were 2 × 3 × 4, there would be two sides that are 2 by 3 (this surface would have an area of 6), two sides that are 3 by 4 (area of 12), and two sides that are 2 by 4 (area of 8). So, the total surface area would be 6 + 6 + 12 + 12 + 8 + 8, which is 52.

Key Formulas and Rules

Here is a review of the key rules and formulas to know for the GRE Math section.

Lines and angles

- All straight lines have 180 degrees.
- A right angle measures 90 degrees.
- Vertical angles are equal.
- Parallel lines cut by a third lines have two angles, big angles and small angles. All of the big angles are equal and all of the small angles are equal. The sum of a big angle and a small angle is 180 degrees.

Triangles

- All triangles have 180 degrees.
- The angles and sides of a triangle are in proportion—the largest angle is opposite the largest side and the smallest side is opposite the smallest angle.
- The Pythagorean theorem is $c^2 = a^2 + b^2$ where c is the length of the hypotenuse.
- The area formula for a triangle is $A = \frac{bh}{2}$.

Quadrilaterals

- All quadrilaterals have 360 degrees.
- The area formula for a squares and rectangles is bh.

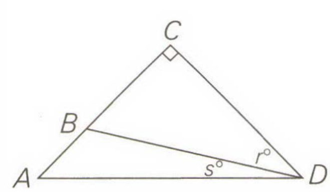
Circles

- All circles have 360 degrees.
- The radius is the distance from the center of the circle to any point on the edge.
- The area of a circle is πr^2.
- The perimeter of a circle is $2\pi r$.

PLUGGING IN ON GEOMETRY PROBLEMS

Remember: Whenever you have a question that has answer choices, like a regular multiple choice or a multiple choice, multiple answer question that has variables in the answer choices, Plug In. On geometry problems, you can Plug In values for angles or lengths as long as the values you Plug In don't contradict either the wording of the problem or the laws of geometry (you can't have the interior angles of a triangle add up to anything but 180, for instance).

Here's an example:



In the drawing above, if $AC = CD$, then $r =$

- ○ $45 - s$
- ○ $90 - s$
- ○ s
- ○ $45 + s$
- ○ $60 + s$

Here's How to Crack It

See the variables in the answer choices? Let's Plug In. First of all, we're told that *AC* and *CD* are equal, which means that *ACD* is an isosceles right triangle. So angles *A* and *D* both have to be 45 degrees. Now it's Plugging In time. The smaller angles, r and s, must add up to 45 degrees, so let's make $r = 40$ degrees and $s = 5$ degrees. The question asks for the value of r, which is 40, so that's our target answer. Now eliminate answer choices by Plugging In 5 for s .

Don't forget to Plug In on geometry questions. Just pick numbers according to the rules of geometry.

(A) $45 - 5 = 40$. Bingo! Check the other choices to be sure.

(B) $90 - 5 = 85$. Nope.

(C) 5. Nope.

(D) $45 + 5 = 50$. Eliminate it.

(E) $60 + 5 = 65$. No way.

By the way, we knew that the correct answer couldn't be greater than 45 degrees, because that's the measure of the entire angle *D*, so you could have eliminated (D) and (E) right away.

DRAW IT YOURSELF

When ETS doesn't include a drawing with a geometry problem, it usually means that the drawing, if supplied, would make ETS's answer obvious. In cases like this, you should just draw it yourself. Here's an example:

Quantity A	Quantity B
The diameter of a circle with area 49π	14

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the information given.

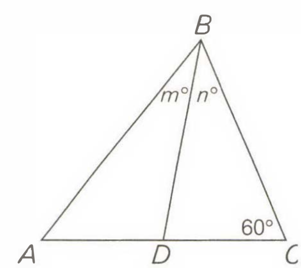
Here's How to Crack It

Visualize the figure. If the area is 49π, what's the radius? Right: 7. And if the radius is 7, what's the diameter? Right: 14. The answer is (C).

Redraw

On tricky quant comp questions, you may need to draw the figure once, eliminate two answer choices, and then draw it another way to try to disprove your first answer and to see if the answer is (D). Here's an example of a problem that might require you to do this:

For quant comp geometry questions, draw, eliminate, and REDRAW; it's like Plugging In twice.



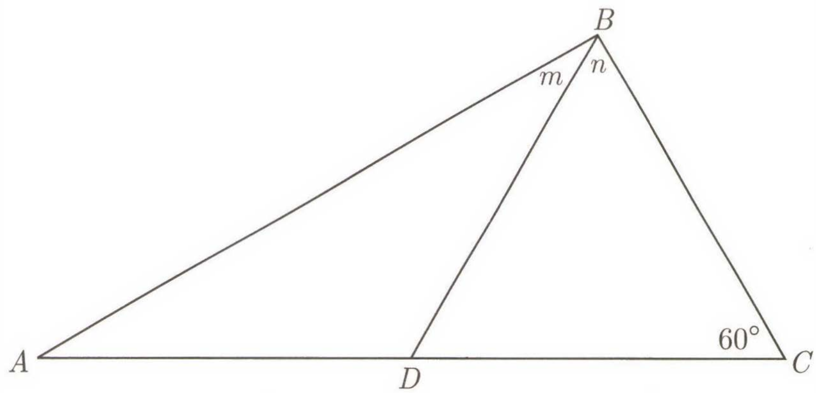
D is the midpoint of AC.

Quantity A	Quantity B
m	n

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Here's How to Crack It

Are you sure that the triangle looks exactly like this? Nope. We know only what we are told—that the lengths of AD and DC are equal; from this figure, it looks like angles m and n are also equal. Because this means that it's possible for them to be, we can eliminate choices (A) and (B). But let's redraw the figure to try to disprove our first answer.



Try drawing the triangle as stretched out as possible. Notice that n is now clearly greater than m, so you can eliminate (C), and the answer is (D).

Geometry Drill

Think you've mastered these concepts? Try your hand at the following problems and check your work after you've finished. You can find the answers in Part V.

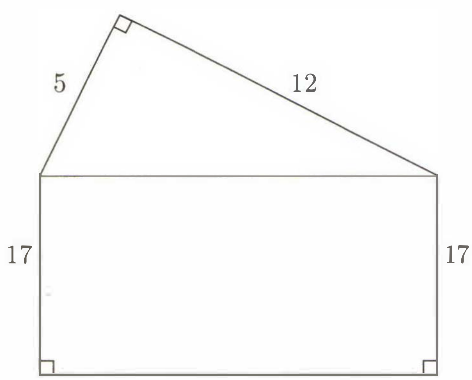
1 of 15

Which of the following could be the degree measures of two angles in a right triangle?

Indicate <u>all</u> such angles.

- ☐ $20°$ and $70°$
- ☐ $30°$ and $60°$
- ☐ $45°$ and $45°$
- ☐ $55°$ and $55°$
- ☐ $75°$ and $75°$

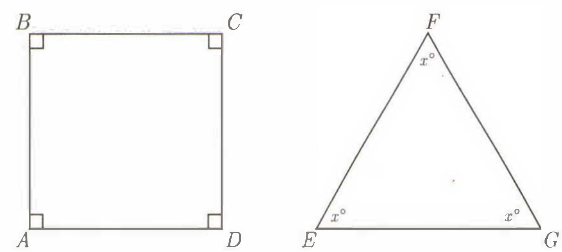
2 of 15



What is the perimeter of the figure above?

- ○ 51
- ○ 64
- ○ 68
- ○ 77
- ○ 91

3 of 15

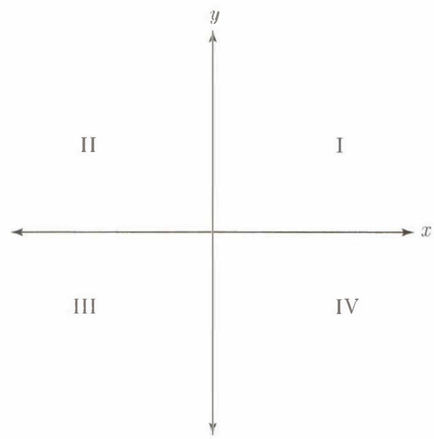


$$AB = BC = EG$$
$$FG = 8$$

<u>**Quantity A**</u>	<u>**Quantity B**</u>
The area of square $ABCD$	32

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

4 of 15



$(a, 6)$ is a point (not shown) in quadrant I.

$(-6, b)$ is a point (not shown) in quadrant II.

Quantity A	Quantity B
a	b

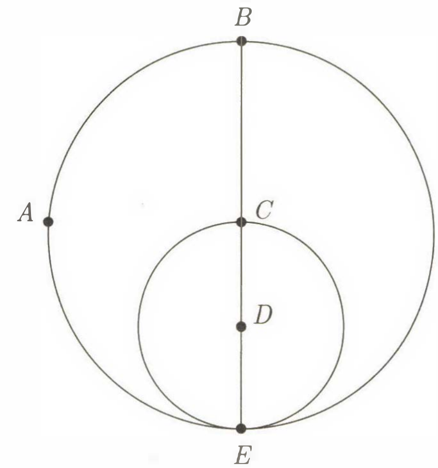
- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

5 of 15

A piece of twine of length t is cut into two pieces. The length of the longer piece is 2 yards greater than 3 times the length of the shorter piece. Which of the following is the length, in yards, of the longer piece?

- ○ $\dfrac{t+3}{3}$
- ○ $\dfrac{3t+2}{3}$
- ○ $\dfrac{t-2}{4}$
- ○ $\dfrac{3t+4}{4}$
- ○ $\dfrac{3t+2}{4}$

6 of 15



A circle with center D is drawn inside a circle with center C, as shown. If $CD = 3$, what is the area of semicircle EAB ?

- ○ $\dfrac{9}{2}\pi$
- ○ 9π
- ○ 12π
- ○ 18π
- ○ 36π

7 of 15

For the final exam in a scuba diving certification course, Karl has to navigate underwater from one point in a lake to another. Karl began the test 2 meters directly beneath the boat and swam due south for 7 meters. He then turned due east and swam for x meters, at which point he swam to the surface. When he surfaced, he was 25 meters from the boat. What is the value of x ?

[ ] meters

Click on the answer box and type in a number. Backspace to erase.

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Quantity A	Quantity B
The circumference of a circular region with radius r	The perimeter of a square with side r

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.

9 of 15

Triangle ABC is contained within a circle with center C. Points A and B lie on the circle. If the area of circle C is 25π, and the measure of angle ACB is $60°$, which of the following are possible lengths for the legs of triangle ABC?

Indicate <u>all</u> such lengths.

- ☐ 3
- ☐ 4
- ☐ 5
- ☐ 6
- ☐ 7

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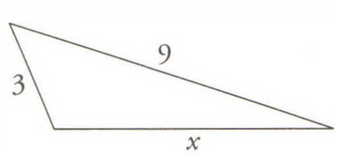

Quantity A	Quantity B
x	5.9

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.

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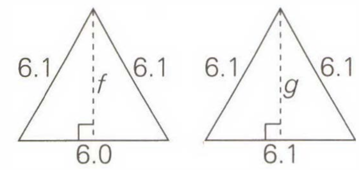

Quantity A	Quantity B
f	g

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.

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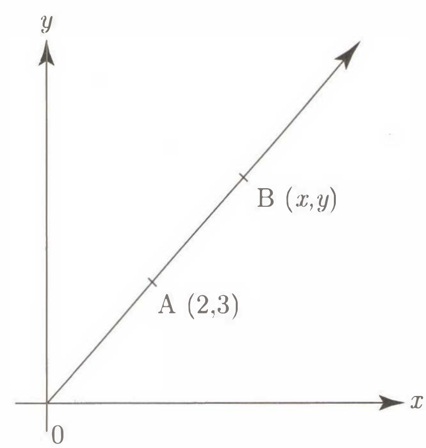
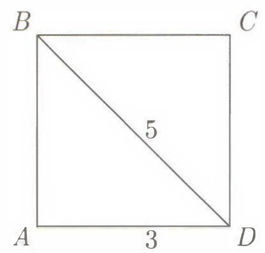


Given points $A(2, 3)$ and $B(x, y)$ in the rectangular coordinate system above, if $y = 4.2$, then $x =$

- ◯ 2.6
- ◯ 2.8
- ◯ 2.9
- ◯ 3.0
- ◯ 3.2

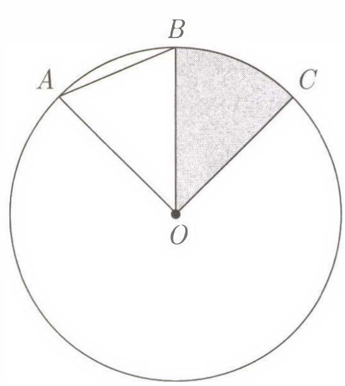
13 of 15



In rectangle $ABCD$ above, which of the following is the area of the triangle ABD ?

- ○ 6
- ○ 7.5
- ○ 10
- ○ 12
- ○ 15

14 of 15



The circle above has a center O.
$\angle AOB = \angle BOC$

Quantity A	**Quantity B**
The area of triangle AOB	The area of the shaded region

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

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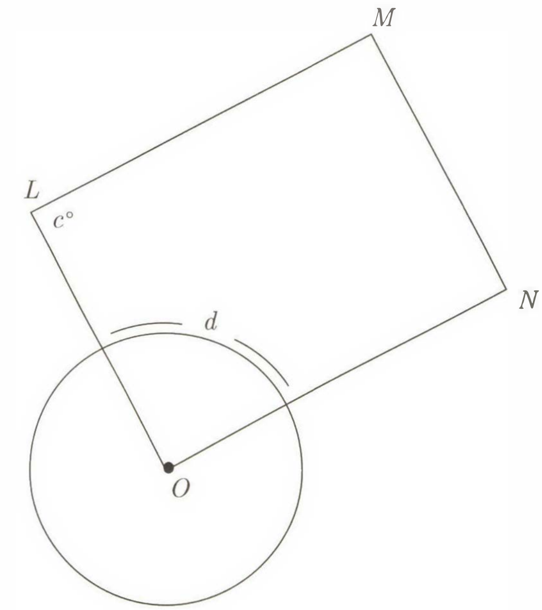


The circumference of the circle with center O is 15π. $LMNO$ is a parallelogram and $c = 108$. What is the value of d ?

- ○ 15π
- ○ 9π
- ○ 3π
- ○ 2π
- ○ It cannot be determined from the information given.

Summary

- There may only be a handful of geometry questions on the GRE, but you'll be expected to know a fair number of rules and formulas.
- Line and angle problems typically test your knowledge of vertical angles, parallel lines, right angles, and straight angles.
- Triangles are a popular geometry topic on the GRE. Make sure you know your triangle basics, including the total degrees of a triangle, the relationship between the angles and sides of a triangle, and the third side rule.
- Right triangle problems frequently test the Pythagorean theorem.
- Be aware of the two special right triangles that ETS likes to torture test takers with: the 45 : 45 : 90 triangle and 30 : 60 : 90 triangle.
- Know the area formulas for triangles, rectangles, squares, and circles.
- Problems involving the coordinate plane frequently test common geometry concepts such as the area of a triangle or a square. Other plane geometry questions will test you on slope and the equation of a line.
- Slope is defined as rise over run. Find it by finding the change in y-coordinates (the rise) and the change in x-coordinates (the run).
- The equation of a line is $y = mx + b$, where x and y are the coordinates of any point on the line, m is the slope and b is the y-intercept, the point at which the line crosses the y-axis.
- Don't forget to Plug In on geometry problems!

Chapter 12
Math Et Cetera

There are a few more math topics that may appear on the GRE that don't fit nicely into the preceding chapters. This chapter looks at some of these leftover topics, including probability, permutations and combinations, and factorials. The topics in this chapter are not essential to your GRE Math score, because these areas are not tested as frequently as the topics detailed earlier. However, if you feel confident with the previous math topics, and you're looking to maximize your GRE Math score, this chapter will show you all you need to know to tackle these more obscure GRE problems.

These topics show up rarely on the GRE, but if you're going for a very high score, they are useful to know.

OTHER MATH TOPICS

The bulk of the GRE Math section tests your knowledge of fundamentals, basic algebra, and geometry. However, there are a few other topics that may appear. These "et cetera" concepts usually show up only once or twice per test (although at higher scoring levels they may appear more frequently) and often cause anxiety among test takers. Many test takers worry excessively about probability problems, for example, even though knowledge of more familiar topics such as fractions and percents will be far more important in determining your GRE math score. So tackle these problems only after you've mastered the rest. If you find these concepts more difficult, don't worry—they won't make or break your GRE score.

PROBABILITY

If you flip a coin, what's the probability that it will land heads up? The probability is equal to one out of two, or $\frac{1}{2}$. What is the probability that it won't land heads up? Again, one out of two, or $\frac{1}{2}$. If you flip a coin nine times, what's the probability that the coin will land on heads on the tenth flip? Still 1 out of 2, or $\frac{1}{2}$. Previous flips do not affect the outcome of the current coin flip.

There's no need to be intimidated by probability questions. If you can work with fractions, you can work with probability questions!

You can think of probability as just another type of fraction. **Probabilities** express a special relationship, namely the chance of a certain outcome occurring. In a probability fraction, the denominator is the total number of possible outcomes that may occur, while the numerator is the number of outcomes that would satisfy the criteria. For example, if you have 10 shirts and 3 of them are black, the probability of selecting a black shirt from your closet without looking is $\frac{3}{10}$.

Think of probability in terms of fractions:

- If it is impossible for something to happen—if no outcomes satisfy the criteria—then the numerator of the probability fraction is 0 and the probability is equal to 0.
- If something is certain to happen—if all possible outcomes satisfy the criteria—then the numerator and denominator of the fraction are equal and the probability is equal to 1.
- If it is possible for something to occur, but it will not definitely occur, then the probability of it occurring is between 0 and 1.

$$\text{probablity} = \frac{\text{number of possible outcomes that satisfy the condition}}{\text{number of total possible outcomes}}$$

Let's see how it works.

At a meeting of 375 members of a neighborhood association, $\frac{1}{5}$ of the participants have lived in the community for less than 5 years and $\frac{2}{3}$ of the attendees have lived in the neighborhood for at least 10 years. If a member of the meeting is selected at random, what is the probability that the person has lived in the neighborhood for at least 5 years but less than 10 years?

- ○ $\frac{2}{15}$
- ○ $\frac{3}{10}$
- ○ $\frac{4}{15}$
- ○ $\frac{1}{2}$
- ○ $\frac{8}{15}$

Here's How to Crack It

In order to solve this problem, we need to put together our probability fraction. The denominator of our fraction is going to be 375, the total number of people from which we are selecting. Next we need to figure out how many attendees satisfy the criteria of having lived in the neighborhood for more than 5 years but fewer than 10 years.

What number goes on the bottom of the probability fraction?

First, we know that $\frac{1}{5}$ of the participants have lived in the neighborhood for less than 5 years. $\frac{1}{5}$ of 375 is 75 people, so we can take them out of the running. Also, $\frac{2}{3}$ of the attendees have lived in the neighborhood for at least 10 years. $\frac{2}{3}$ of 375 (be careful not to use 300 as the total!) is 250, so we can also remove them from consideration. Thus, if 75 people have lived in the neighborhood for less than 5 years and 250 have lived for at least 10, the remaining people are the ones we want. 250 + 75 is 325, so that leaves us with 50 people who satisfy the criteria. We need to make 50 the numerator of our fraction, which gives us $\frac{50}{375}$. This reduces to $\frac{2}{15}$, and answer choice (A) is the best answer.

Two Important Laws of Probability

Probability of A and B
= Probability of A
× Probability of B

When you want to find the probability of a series of events in a row, you multiply the probabilities of the individual events. What is the probability of getting two heads in a row if you flip a coin twice? The probability of getting a head on the first flip is $\frac{1}{2}$. The probability is also $\frac{1}{2}$ that you'll get a head on the second flip, so the combined probability of two heads is $\frac{1}{2} \times \frac{1}{2}$, which equals $\frac{1}{4}$. Another way to look at it is that there are four possible outcomes: HH, TT, HT, TH. Only one of those outcomes consists of two heads in a row. Thus, $\frac{1}{4}$ of the outcomes consist of two heads in a row. Sometimes the number of outcomes is small enough that you can list them and calculate the probability that way.

Probability of A or B
= Probability of A
+ Probability of B

Occasionally, instead of finding the probability of one event AND another event happening, you'll be asked to find the probability of either one event OR another event happening. In this situation, instead of multiplying the probabilities, you add them. Let's say you have a normal deck of 52 cards. If you select a card at random, what's the probability that you select a 7 or a 4? The probability of selecting a 7 is $\frac{4}{52}$, which reduces to $\frac{1}{13}$. The probability of selecting a 4 is the same; $\frac{1}{13}$. Therefore the probability of selecting a 7 or a 4 is $\frac{1}{13} + \frac{1}{13} = \frac{2}{13}$.

Let's look at a problem:

Julie is going to roll a pair of six-sided dice. What is the probability that she rolls either a 3 and a 4, OR a 5 and a prime number?

Click on each box and type in a number.
Backspace to erase.

Here's How to Crack It

Let's start with the first possibility. The probability of rolling a 3 is $\frac{1}{6}$, and the probability of rolling a 4 is $\frac{1}{6}$. So the probability of rolling a 3 and then a 4 is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. Now let's look at the second possibility. The probability of rolling a 5 is $\frac{1}{6}$ and the probability of rolling a prime number is $\frac{1}{2}$. (There are six outcomes when you roll a die and three of them are prime: 2, 3, and 5. So the probability of rolling a prime number is $\frac{3}{6}$, which reduces to $\frac{1}{2}$.) Therefore, the probability of rolling a 5 and then a prime number is $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$. So now we know the probability of rolling a 3 and then a 4 is $\frac{1}{36}$, and we know the probability of rolling a 5 and a prime number is $\frac{1}{12}$. To find the probability of one of these things OR the other happening, we add the individual probabilities. So $\frac{1}{12} + \frac{1}{36} = \frac{4}{36}$ which reduces to $\frac{1}{9}$.

One last important thing you should know about probabilities is that the probability of an event happening and the probability of an event not happening must add up to 1. For example, if the probability of snow falling on one night is $\frac{2}{3}$, then the probability of no snow falling must be $\frac{1}{3}$. If the probability that it will rain is 80%, then the probability that it won't rain must be 20%. The reason this is useful is that, on some GRE probability problems, it will be easier to find the probability that an event doesn't occur; once you have that, just subtract from 1 to find the answer.

Since probabilities are just fractions, they can also be expressed as percents.

Let's look at the following example.

Dipak has a 25% chance of winning each hand of blackjack he plays. If he has \$150 and bets \$50 a hand, what is the probability that he will still have money after the third hand?

- $\frac{1}{64}$
- $\frac{3}{16}$
- $\frac{27}{64}$
- $\frac{37}{64}$
- $\frac{3}{4}$

Here's How to Crack It

If Dipak still has money after the third hand, then he must have won at least one of the hands, and possibly more than one. However, directly calculating the probability that he wins at least one hand is tricky because there are so many ways it could happen (for example, he could lose-lose-win, or W-W-L or W-L-W or L-W-L, and so on). So think about it this way: The question asks for the probability that he will win at least one hand. What if he doesn't? That would mean that he doesn't win any hands at all. If we calculate the probability that he loses every hand, we can then subtract that from 1 and find the corresponding probability that he wins at least one hand. Since Dipak has a 25% chance of winning each hand, this means that he has a 75% chance of losing it, or $\frac{3}{4}$ (the answers are in fractions, so it's best to work with fractions). To find the probability that he loses all three hands, simply multiply the probabilities of his losing each individual hand. $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$ so there is a $\frac{27}{64}$ probability that he will lose all three hands. Subtracting this from 1 gives you the answer you're looking for. $1 - \frac{27}{64} = \frac{37}{64}$. The answer is (D).

Given events A and B, the probability of

- A and B = (Probability of A) × (Probability of B)
- A or B = Probability of A + Probability of B

Given event A

- A + *not* A = 1

FACTORIALS

The factorial of a number is equal to that number times every positive whole number smaller than that number, down to 1. For example, the factorial of 6 is equal to $6 \times 5 \times 4 \times 3 \times 2 \times 1$, which equals 720. The symbol for a factorial is ! so 4! doesn't mean we're really excited about the number 4, it means $4 \times 3 \times 2 \times 1$, which is equal to 24. (0! is equal to 1, by the way.) When factorials show up in GRE problems, always look for a shortcut like canceling or factoring. The point of a factorial problem is not to make you do a lot of multiplication. Let's try one.

Quantity A	Quantity B
$\frac{12!}{11!}$	$\frac{4!}{2!}$

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.

Probability
Most people struggle with problems related to probability because it is generally a tough subject. The way it is tested on the GRE, however, is pretty straightforward. Probability problems don't come up that often, but if they do, there are a few key concepts you need to know.

To watch a short video on the key concepts of probability, register your book at PrincetonReview.com/cracking.

Here's How to Crack It

Let's tackle Quantity A. We definitely don't want to multiply out the factorials since that would be pretty time-consuming: 12! and 11! are both huge numbers. Instead let's look at what they have in common. What we're really talking about here is $\frac{12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}$. Now it's clear that both factorials share everything from 11 on down to 1. The entire bottom of the fraction will cancel and the only thing left on top will be 12, so the value of Quantity A is 12. For Quantity B, we can also write out the factorials and get $\frac{4\times3\times2\times1}{2\times1}$.

The 2 and the 1 in the bottom cancel, and the only thing left on top will be 4×3, which is equal to 12. The two quantities are equal and the answer is (C).

PERMUTATIONS AND COMBINATIONS

Permutation problems often ask for arrangements, orders, schedules, or lists.

The basic definition of a **permutation** is an arrangement of things in a particular order. Suppose you were asked to figure out how many different ways you could arrange five statues on a shelf. All you have to do is multiply $5 \times 4 \times 3 \times 2 \times 1$, or 120. (Yes, this is another application of factorials.) You have five possible statues that could fill the first slot on the shelf, then, once the first slot is filled, there are four remaining statues that could fill the second slot, three that could fill the third slot, and so on, down to one.

Now suppose that there are five people running in a race. The winner of the race will get a gold medal, the person who comes in second will get a silver medal, and the person who comes in third will get a bronze medal. You're asked to figure out how many different orders of gold-silver-bronze winners there can be. (Notice that this is a permutation because the order definitely matters.)

First, ask yourself how many of these runners can come in first? Five. Once one of them comes in first, she's out of the picture, so how many can then come in second? Four. Once one of them comes in second, she's out of the picture, so how many of them can come in third? Three. And now you're done because all three slots have been filled. The answer is $5 \times 4 \times 3$, which is 60.

To solve a permutation

- Figure out how many slots you have.
- Write down the number of options for each slot.
- Multiply them.

The difference between a permutation and a combination is that in a combination, the order is irrelevant. **A combination** is just a group, and the order of elements within the group doesn't matter. For example, suppose you were asked to go to the store and bring home three different types of ice cream. Now suppose that when you got to the store, there were five flavors in the freezer—chocolate, vanilla, strawberry, butter pecan, and mocha. How many combinations of three ice cream flavors could you bring home? Notice that the order doesn't matter, because bringing home chocolate, strawberry, and vanilla is the same thing as bringing home strawberry, vanilla, and chocolate. One way to solve this is the brute force method; in other words, write out every combination.

Combination problems usually ask for groups, teams, or committees.

VCS VCB VCM VSB VSM VBM CSB CSM CBM SBM

Does the order matter?

That's 10 combinations, but there's a quicker way to do it. Start by filling in the three slots as you would with a permutation (there are three slots because you're supposed to bring home three different types of ice cream). Five flavors could be in the first slot, four could be in the second, and three could be in the third. So far, that's $5 \times 4 \times 3$. But remember, this takes into account all the different orders that three flavors can be arranged in. We don't want that, because the order doesn't matter in a combination. So we have to divide $5 \times 4 \times 3$ by the number of ways of arranging three things. In how many ways can three things be arranged? That's 3!, $3 \times 2 \times 1$, which is 6. Thus we end up with $\frac{5\times4\times3}{3\times2\times1}$, which is equal to $\frac{60}{6}$, or 10. Bingo.

To solve a combination

- Figure out how many slots you have.
- Fill in the slots as you would a permutation.
- Divide by the factorial of the number of slots.

The denominator of the fraction will always cancel out completely, so you can cancel first before you multiply.

Always cross off wrong answer choices on your scratch paper.

Here's an example:

Brooke wants to hang three paintings in a row on her wall. She has six paintings to choose from. How many arrangements of paintings on the wall can she create?

- ○ 6
- ○ 30
- ○ 90
- ○ 120
- ○ 720

Here's How to Crack It

The first thing you need to do is determine whether the order matters. In this case it does, because we're arranging the paintings on the wall. Putting the Monet on the left and the Van Gogh in the middle isn't the same arrangement as putting the Van Gogh on the left and the Monet in the middle. This is a permutation question. We have three slots to fill because we're arranging three paintings. There are 6 paintings that could fill the first slot, 5 paintings that could fill the second slot, and 4 paintings that could fill the third slot. So we have $6 \times 5 \times 4$, which equals 120. Thus, the correct answer is (D).

Here's another example:

A pizza may be ordered with any of eight possible toppings.

Quantity A	Quantity B
The number of different ways to order a pizza with three different toppings	The number of different ways to order a pizza with five different toppings

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Here's How to Crack It

First, note that for both quantities we're dealing with a combination, because the order of toppings doesn't matter. A pizza with mushrooms and pepperoni is the same thing as a pizza with pepperoni and mushrooms. Let's figure out Quantity A first.

We have eight toppings and we're picking three of them. That means we have three slots to fill. There are 8 toppings that could fill the first slot, 7 that could fill the second slot, and 6 that could fill the third, so we have $8 \times 7 \times 6$. Since this is a combination, we have to divide by the factorial of the number of slots. In this case we have three slots, so we have to divide by 3!, or $3 \times 2 \times 1$. So our problem looks like this: $\frac{8 \times 7 \times 6}{3 \times 2 \times 1}$. To make the multiplication easier, let's cancel first. The 6 on top will cancel with the 3×2 on the bottom, leaving us with $\frac{8 \times 7}{1}$, which is 56. Thus, there are 56 ways to order a three-topping pizza with eight toppings to choose from. Now let's look at Quantity B.

We still have eight toppings, but this time we're picking five of them so we have five slots to fill. There are 8 toppings that could fill the first slot, 7 that could fill the second slot, 6 that could fill the third, 5 that could fill the fourth, and 4 that could fill the fifth. That's $8 \times 7 \times 6 \times 5 \times 4$, but we still have to divide by the factorial of the number of slots. We have five slots, so that means we need to divide by 5!, or $5 \times 4 \times 3 \times 2 \times 1$. Thus we have $\frac{8 \times 7 \times 6 \times 5 \times 4}{5 \times 4 \times 3 \times 2 \times 1}$. We definitely want to cancel first here, rather than doing all that multiplication. The 5 on top will cancel with the 5 on the bottom. Likewise, the 4 on top will cancel with the 4 on the bottom. The 6 on top will cancel with the 3×2 on the bottom, leaving us again with $\frac{8 \times 7}{1}$, which is 56. Therefore, there are also 56 ways to order a five-topping pizza with eight toppings to choose from. The two quantities are equal, and the answer is (C).

Let's try one more:

Nicole needs to form a committee of 3 from a group of 8 research attorneys to study possible changes to the Superior Court. If two of the attorneys are too inexperienced to serve together on the committee, how many different arrangements of committees can Nicole form?

○ 20

○ 30

○ 50

○ 56

○ 336

Here's How to Crack It

This problem is a little more complicated than an ordinary combination problem, because an extra condition has been placed on the committee. Without that condition, this would be a fairly ordinary combination problem, and we'd simply calculate how many groups of three can be created with eight people to choose from.

There's more than one way to approach this problem. First, you should realize that there are two ways that we could form this committee. We could have three experienced attorneys, or we could have two experienced attorneys and one inexperienced attorney. If we find the number of ways to create each of those two possibilities, we can add them together and have our answer. It's fairly straightforward to calculate the number of ways to have three experienced attorneys on a committee: There are three slots to fill, and we have 6 options for the first slot, 5 for the second, and 4 for the third. Here the order doesn't matter, so we divide by 3! to get $\frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20$. Thus there are 20 ways to create the committee using three experienced attorneys. What about creating a committee that has two experienced attorneys and one inexperienced attorney? We have 6 options for the first experienced attorney and 5 options for the second. Order doesn't matter so we divide by 2!. So far we have $\frac{6\times5}{2\times1}$. Next we have 2 options for the inexperienced attorney, so now we have to multiply by 2, and our calculation is $\frac{6\times5}{2\times1}\times\frac{2}{1} = 30$. As you can see, there are 30 ways to create the committee using two experienced attorneys and one inexperienced attorney. Adding 20 and 30 gives us 50 total committees, and the answer is (C).

Here's another way that you could solve the problem. If there were no conditions placed on the committee, we could just calculate $\frac{8\times7\times6}{3\times2\times1}$, which would give us 56 committees. But we know some of those committees are not allowed; any committee that has the two inexperienced attorneys on it isn't allowed. How many of these types of committees are there? Let's call the inexperienced attorneys A and B. An unacceptable committee would be A B __, in which the last slot could be filled by any of the experienced attorneys. Since there are 6 experienced attorneys, there are 6 unacceptable committees. Subtracting them from 56 gives us 50 acceptable committees. Hey, the answer's still (C)!

FUNCTIONS AND FUNNY-LOOKING SYMBOLS

The GRE contains "function" problems, but they aren't like the functions that you may have learned in high school. GRE functions use funny-looking symbols, such as @, *, and #. Each symbol represents an arithmetic operation or a series of arithmetic operations. All you have to do is follow directions in the problem. Here's an example:

With funny-looking symbols, simply follow the directions.

For any non-negative integer x, let $x^* = x - 1$

Quantity A	Quantity B
$\frac{15^*}{3^*}$	$\left(\frac{15}{3}\right)^*$

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the information given.

Here's How to Crack It

Just follow the directions—15* = 15 – 1, or 14, and 3* = 3 – 1, or 2. So we get $\frac{14}{2}$, or 7, in Quantity A. Don't forget PEMDAS for Quantity B. First, $\frac{15}{3}$ is 5. Then, 5* = 5 – 1, or 4. So because Quantity A is 7 and Quantity B is 4, the answer is (A). Function questions aren't scary if you follow the directions. Be sure to write everything down on your scratch paper. By the way, these funny-looking symbols don't necessarily indicate exponents, but you'll always be told what they mean.

GROUPS

You might see one group problem on the GRE.

Group problems, although not too common on the GRE, can be troublesome if you don't know how to set them up. When confronted by a group problem, use the group equation

$$T = G_1 + G_2 - B + N$$

In the equation, T represents the Total, G_1 is one group, G_2 is the second group, B is for the members in both groups and N is for the members in neither group. Here's an example of a typical group problem.

A biologist studying breeding groups noted that of 225 birds tagged for the study, 85 birds made nests in pine trees, 175 made nests in oak trees, and 40 birds did not build nests in either type of tree. How many birds built nests in both types of trees?

- ○ 45
- ○ 60
- ○ 75
- ○ 80
- ○ 125

Here's How to Crack It

Let's use the group equation. The total is 225, one group consists of 85 birds, the other group has 175 birds in it, and we know that 40 birds built nests in neither type of tree. Our equation would look like this:

$$225 = 85 + 175 - B + 40$$

All we have to do is solve for B. Simplifying the equation gives us 225 = 300 – B, so B must equal 75. Choice (C) is our answer.

Et Cetera Drill

Here are some math questions to practice on. Remember to check your answers when you finish. You can find the answers in Part V.

1 of 10

15 marbles are placed in a bowl; some are red, and some are blue. If the number of red marbles is 1 more than the number of blue marbles, what is the probability that a marble taken from the bowl is blue?

- $\frac{1}{15}$
- $\frac{2}{15}$
- $\frac{7}{15}$
- $\frac{1}{2}$
- $\frac{8}{15}$

2 of 10

If ¥$(x) = 10x - 1$, what is ¥(5) – ¥(3) ?

- 15
- 18
- 19
- 20
- 46

3 of 10

Quantity A	Quantity B
The largest odd factor of 78	The largest prime factor of 78

- Quantity A is greater.
- Quantity B is greater.
- The two quantities are equal.
- The relationship cannot be determined from the information given.

4 of 10

At a recent dog show, there were 5 finalists. One of the finalists was awarded "Best in Show" and another finalist was awarded "Honorable Mention." In how many ways could the two awards be given out?

Click on the answer box and type in a number. Backspace to erase.

5 of 10

Company X spends \$40,000 per year on advertising for product A and \$30,000 per year on advertising for product B. The company spends \$15,000 on advertisements that advertise both product A and B as a system. The company spends \$90,000 total on advertising for all of its products.

Quantity A	Quantity B
The total amount the company spends advertising products other than products A and B.	\$20,000

- Quantity A is greater.
- Quantity B is greater.
- The two quantities are equal.
- The relationship cannot be determined from the information given.

6 of 10

Lee randomly selects a 2-digit prime number less than 50. What is the probability that the tens digit is greater than the units digit?

- $\frac{3}{14}$
- $\frac{3}{11}$
- $\frac{3}{8}$
- $\frac{1}{2}$
- $\frac{8}{11}$

7 of 10

An elected official wants to take five members of his staff to an undisclosed secure location. What is the minimum number of staff members the elected official must have in order to have at least 20 different groups from which to choose?

- 7
- 8
- 9
- 10
- 11

8 of 10

For all real numbers x and y, if $x \# y = x(x - y)$, then $x \# (x \# y) =$

- $x^2 - xy$
- $x^2 - 2xy$
- $x^3 - x^2 - xy$
- $x^3 - (xy)^2$
- $x^2 - x^3 + x^2y$

9 of 10

A jar contains 12 marbles. Each is either yellow or green and there are twice as many yellow marbles as green marbles. If two marbles are to be selected from the jar at random, what is the probability that exactly one of each color is selected?

- $\frac{8}{33}$
- $\frac{16}{33}$
- $\frac{1}{2}$
- $\frac{17}{33}$
- $\frac{25}{33}$

10 of 10

A set of 10 points lies in a plane such that no three points are collinear.

Quantity A	Quantity B
The number of distinct triangles that can be created from the set	The number of distinct quadrilaterals that can be created from the set

- Quantity A is greater.
- Quantity B is greater.
- The two quantities are equal.
- The relationship cannot be determined from the information given.

Comprehensive Math Drill

Let's do a drill involving all of the math topics we've covered throughout the book. Remember to check your answers when you finish. You can find the answers in Part V.

1 of 20

$$\frac{0.05}{0.6} = \frac{x}{.18}$$

Quantity A	Quantity B
x	.015

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

2 of 20

$$x \neq 0$$

Quantity A	Quantity B
$\frac{x}{10}$	$\frac{\left(\frac{x}{5}\right)}{2}$

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

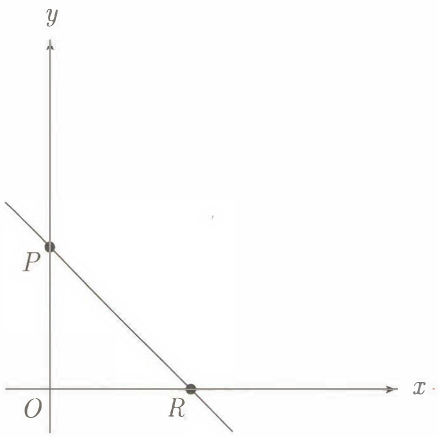
3 of 20

The test scores for a class have a normal distribution, a mean of 50, and a standard deviation of 4.

Quantity A	Quantity B
Percentage of scores at or above 58	Percentage of scores at or below 42

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

4 of 20



The line $y = -\frac{5}{6}x + 1$ is graphed on the rectangular coordinate axes.

Quantity A	Quantity B
OR	OP

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

5 of 20

At a dog show, there are 20 judges and 10 dogs in the final round.

Quantity A	Quantity B
The number of distinct pairs of judges	The number of possible rankings of dogs from first to third place

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

6 of 20

$k > 0$

$l > 1$

Quantity A	Quantity B
$\dfrac{1}{\frac{1}{k} + \frac{1}{l}}$	$\dfrac{kl}{\frac{1}{k} + \frac{1}{l}}$

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

7 of 20

Quantity A	Quantity B
$\sqrt{3} + \sqrt{4}$	$\sqrt{7}$

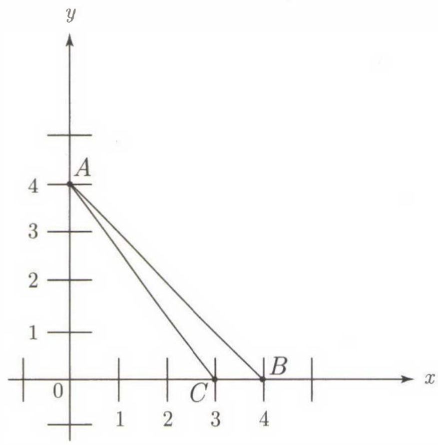
- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

8 of 20

Joe has \$200. If he buys a CD player for \$150, what is the greatest number of CDs he can buy with the remaining money if CDs cost \$12 each?

Click on the answer box and type in a number. Backspace to erase.

9 of 20



What is the area of triangle ABC in the figure above?

- ○ 2
- ○ 4
- ○ $4\sqrt{2}$
- ○ 7
- ○ 8

10 of 20

Which of the following could equal the result when $10(3^2 - 2)$ is divided by a positive integer?

Indicate all such values.

- ☐ 140
- ☐ 70
- ☐ 35
- ☐ 10
- ☐ 0

11 of 20

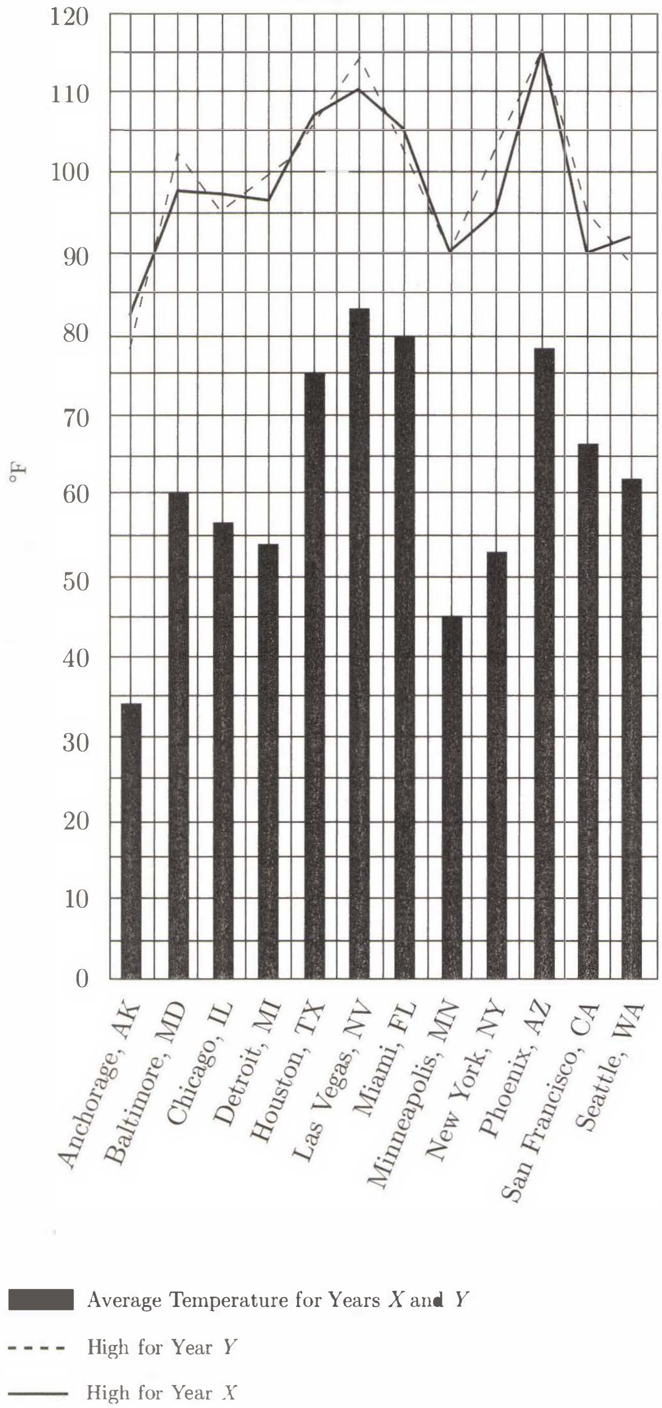
Roberta drove 50 miles in 2 hours. Her rate in miles per hour is equivalent to which of the following proportions?

Indicate all such proportions.

- ☐ 5 to 20
- ☐ 100 to 4
- ☐ 400 to 16
- ☐ 20 to 500

Questions 12 through 14 refer to the following graph.

TEMPERATURES OF U.S. CITIES IN YEARS X AND Y



12 of 20

For how many of the cities shown was the highest temperature in Year Y greater than or equal to the highest temperature in Year X ?

- 4
- 5
- 7
- 8
- 12

13 of 20

What is the approximate percent increase from the lowest average temperature for Years X and Y to the highest average temperature?

- 60%
- 82%
- 140%
- 188%
- 213%

14 of 20

If the average temperature for Years X and Y in Baltimore is equal to the average of that city's high and low temperatures for each of those years, then what is the average of the low temperatures for Baltimore in Years X and Y ?

- –9° F
- 11° F
- 20° F
- 44° F
- It cannot be determined from the information given.

15 of 20

If $|2x - 3| + 2 > 7$, which of the following could be the value of x ?

Indicate all such values.

- ☐ –4
- ☐ –3
- ☐ –2
- ☐ –1
- ☐ 0
- ☐ 1
- ☐ 2
- ☐ 3

16 of 20

If x, y, and z are consecutive odd integers where $x < y < z$ and $x + y + z < z$, then whch of the following could be the value of x ?

Indicate all such values.

- ☐ –3
- ☐ –1
- ☐ 0
- ☐ 1
- ☐ 3

17 of 20

If $4^x = 1{,}024$, then $(4^{x+1})(5^{x-1}) =$

- ○ 10^6
- ○ $(5^4)(10^5)$
- ○ $(4^4)(10^5)$
- ○ $(5^4)(10^4)$
- ○ $(4^4)(10^4)$

18 of 20

What is the greatest distance between two vertices of a rectangular solid with a height of 5, a length of 12, and a volume of 780?

- ○ 12
- ○ $12\sqrt{2}$
- ○ 13
- ○ $13\sqrt{2}$
- ○ $13\sqrt{3}$

19 of 20

Six children, three boys and three girls, sit in a row on a park bench. How many arrangements of children are possible if no boy can sit on either end of the bench?

Indicate all such values.

- ☐ 46,656
- ☐ 38,880
- ☐ 1,256
- ☐ 144
- ☐ 38

20 of 20

If 16 is the average of p, 24, and q, what is $16(p + q)$?

- ○ 180
- ○ 192
- ○ 384
- ○ 524
- ○ 768

Summary

- Topics such as probability, permutations and combinations, factorials, and functions represent only a small percentage of the math topics tested on the GRE. Make sure you've mastered all the more important topics before attempting these.
- Probability is expressed as a fraction. The denominator of the fraction represents the total number of possible outcomes, while the numerator stands for the desired outcomes.
- If a probability question asks for the chance of event A or event B, find the probability of each event and add them together. If the question asks for the probability of event A and event B, multiply the individual probabilities.
- The key to factorial problems is to look for ways to cancel or factor out terms.
- Permutations and combinations are related concepts. A permutation tells you how many arrangements or orderings of things are possible. A combination tells you how many groupings of things are possible.
- Function problems use funny looking symbols as shorthand for the operations to perform on a certain number.
- The group equation is Total = Group_1 + Group_2 – Members of Both Groups + Members of Neither Group.

Part V
Answers and Explanations to Drills and Practice Sets

Group 3 Exercises: Matching (Page 150)

1. D	6. A	11. E
2. G	7. C	12. B
3. K	8. N	13. J
4. I	9. H	14. L
5. M	10. F	

Group 4 Exercises: Matching (Page 153)

1. I	6. B	11. M
2. L	7. J	12. D
3. N	8. A	13. H
4. C	9. G	14. F
5. K	10. E	

CHAPTER 9: NUMBERS AND EQUATIONS

Numbers and Equations Drill (Page 215)

1. C, D, and F

   To solve this problem, try writing out the possibilities. The smallest prime number is 2. $(2 \times 2) + 3 = 7$; so answer choice (C) is correct. Answer choice (A) is incorrect because 1 is not a prime number. The next prime number is 3: $(3 \times 3) + 5 = 14$, so answer choice (D) is correct. The next prime number is 5: $(5 \times 5) + 7 = 32$, which is not an answer choice. The next prime number is 7: $(7 \times 7) + 11 = 60$, so answer choice (F) is correct. The next prime number is 11: $(11 \times 11) + 13 = 134$, which is much larger than the answer choice possibilities. All answer choices have been found.

2. B

   First, put the equation in standard form: $x^2 + 8x + 7 = 0$. Now factor: $(x + 7)(x + 1) = 0$. Solve: $x = -7$ or -1. Both of the possible values for x are negative, so Quantity B is always greater than Quantity A.

3. 27

Because $9 = 3^2$; the original equation becomes $3^3 \times (3^2)^{12} = 3^x$; or, $3^3 \times 3^{24} = 3^x$; or, $3^{3+24} = 3^x$. Therefore, $x = 27$.

4. E

Because there are variables in the answers, Plug In. Let's make $x = 10$, $y = 7$, and $c = 3$. Then $A = 2 \times 10 - (7 - 2 \times 3)$. Order of operations dictates that you solve the numbers in the parentheses before subtracting. $A = 20 - (7 - 6)$. Therefore, $A = 19$. $B = (2 \times 10 - 7) - 2 \times 3$. Again, solve numbers in the parentheses before subtracting. $B = (20 - 7) - 6$. Therefore, $B = 7$. Be careful, the question is asking for $A - B = 19 - 7 = 12$. Plug $y = 7$ and $c = 3$ into the answers. Only answer choice (E) will give you your target, 12. If you chose answer choice (C), you subtracted before you simplified the numbers in the parentheses.

5. A

You have the relationship among can prices, but no actual numbers, so try plugging in some numbers for can prices. The calculations will be easy if you make the large can cost $5 \times 7 = \$35$, which means that the medium can costs $35 \div 5 = \$7$, and the small can costs $35 \div 7 = \$5$. The amount of money that would buy 200 medium cans is $200 \times \$7 = \$1{,}400$. Because the customer buys the same number of small and large cans, she spends $40 on each small-and-large can combination. Divide $1,400 by $40 to get the number of sets she buys ($\$1{,}400 \div \$40 = 35$). She buys 35 sets of small and large cans, which means that she buys 35 small cans, choice (A).

6. 25

Stack and add the first two inequalities. Multiple the second inequality by –1 to make the signs point in the same direction.

$6k - 5l > 27$
$\underline{2k - 3l > 13}$
$8k - 8l > 40$

Divide by 8 to get $k - l > 5$. Multiply by 5 to get final answer of 25. $5k - 5l > 25$.

7. B

Translate the equation. $3a = 6b - 4$. Because the question asks for the value of $a - 2b$, go ahead and rearrange your equation so the a and b are on the same side. $3a - 6b = -4$. Next, divide both sides by 3. $\frac{3a - 6b}{3} = -\frac{4}{3}$.

8. D

You are given no information about the prices of the books, so you cannot assume that the books have the same price. You can't determine the number of books in Quantity A or Quantity B.

9. A and E

To begin, find the factors of 91: 1 and 91 or 7 and 13. Remember that the product of two negative numbers is positive, so the integers could also be negative factors. The question asks for the sum of the two integers. Answer choice (A) is the sum of –91 and –1. Answer choice (E) is the sum of 7 and 13.

10. A, B, and E

You have variables in the question and variables in the answers, so Plug In. If $x = 6$, then $a = 2$, and if $y = 36$, then $b = 4$. $2(x + y)$ equals 84, so that is your target number. Check your answer choices. A = 48, B = 60, C = 84, D = 84, and E = 96. Since we're looking for the ones that *don't* equal 84, the correct answers are A, B, and E.

CHAPTER 10: REAL WORLD MATH

Real World Math Drill (Page 258)

1. A

Plug In for r. If $r = 2$, we can now solve for s. $3(2 + s) = 7$, $2 + s = \frac{7}{3}$, $s = \frac{7}{3} - 2$. Convert the 2 to a fraction and get $s = \frac{7}{3} - \frac{6}{3} = \frac{1}{3}$. Go through the answer choices, Plugging In 2 for r. Choice (A) yields the target of $\frac{1}{3}$.

2. $\mathbf{\frac{1}{6}}$

Plugging In your own number is a good way to tackle this. The fractions used in the problem are $\frac{1}{3}$ and $\frac{1}{2}$, and multiplying the denominators will produce a good number with which to work. Sadie started with 6 paintings and gave away one third of them: $6 \times \frac{1}{3} = 2$. She has 4 paintings left. She then sold another half of the original 6: $6 \times \frac{1}{2} = 3$. So, she has 1 painting left, or $\frac{1}{6}$ of the total.

3. B, C, and D

A \$20 scarf can be discounted as much as 50 percent, and $\$20 \times \frac{50}{100} = \$10$, so the minimum sale price of a scarf is $\$20 - \$10 = \$10$. The smallest discount is 25 percent, and $\$20 \times \frac{25}{100} = \$5$, so the maximum sale price of a scarf is $\$20 - \$5 = \$15$. You have determined the range of possible sale prices for scarves is \$10 to \$15. Now, you need to eliminate answers that fall outside of that range: Choice (A) is too small, and choice (E) is too large.

4. C

To find the value in each column, translate the words into arithmetic. Rewrite Quantity A as $\frac{12}{100} \times 35 = \frac{12 \times 35}{100}$ and Quantity B as $\frac{35}{100} \times 12 = \frac{35 \times 12}{100}$. The expression in Quantity A is the same as the expression in Quantity B.

5. A

Use the bowtie to compare the quantities: Multiply opposing numerators and denominators, and compare the resulting products. Think of Quantity B as $\frac{10}{1}$. Multiply 2.6 × 1 to get 2.6 on the Quantity A side. Multiply 0.259 × 10 to get 2.59 on the Quantity B side. Because 2.6 is greater than 2.59, Quantity A is greater.

6. D

The population rankings for Year *X* are as follows: (1) Massachusetts, (2) Connecticut, (3) Maine, (4) Rhode Island, (5) New Hampshire, (6) Vermont. The rankings for Year *Y* are as follows: (1) Massachusetts; (2) Connecticut; (3) Rhode Island; (4) New Hampshire; (5) Maine; (6) Vermont. Maine, Rhode Island, and New Hampshire have different rankings from Year *X* to Year *Y*.

7. E

In Year *Y*, Vermont's population is 3 percent of 25 million (or 0.75 million), and Massachusetts' population is 37 percent of 25 million (or approximately 9 million). 9 million is what percent of 0.75 million? Now translate: 9 million = $\frac{x}{100}$ × 0.75 million: $x = 1{,}200$.

8. D

In Year *X* the population of Rhode Island was 10 percent of 15 million, or 1.5 million. In Year *Y* the population of Rhode Island was 15 percent of 25 million, or 3.75 million. The increase was 2.25 million, or 2,250,000.

9. B

The percent change of Connecticut's percent of total New England population from Year *X* (24 percent) to Year *Y* (27 percent) is $\frac{3}{24}$ = 12.5 percent. The percent change of Massachusetts's percentage of total New England population from Year *X* (40 percent) to Year *Y* (37 percent) is $\frac{3}{40}$ = 7.5 percent. The approximate difference is 5.

10. D

You are given that towns *A* and *B* each have 3,000 supporters of the referendum and that *B* and *D* have an average of 3,500 supporters. Using the average circle you find out that *D* has 4,000 supporters. You know nothing about *C*. Because *C* and *D* are the two southern–most towns, we cannot tell what their average is. For example, if *C* had zero supporters, the average of *C* and *D* would be 2,000, which is less than Quantity B. If *C* had 4,000 supporters, the average of *C* and *D* would be 4,000, which is greater than Quantity B.

11. E

Plug In The Answers, starting in the middle with choice (C). If each A employee was given $740, each *C* employee was given half of that, or $370. Each *B* employee received one-and-a-half times the *C* raise, so 1.5 × $370 = $555. Now calculate the total money spent on raises. 50 *A* employees got $740 each, for a total of 50 × $740 = $37,000. 100 *B* employees got $555 each, for a total of 100 × $555 = $55,500. 150 *C* employees got $370 each: 150 × $370 = 55,500. These add up to a total of $148,000, but the problem says that the total raise amount is $500,000. You need a much bigger answer. Rule out choices (A), (B), and (C). Try skipping directly to (E). If the *A* workers got $2,500, the *C* workers got $1,250, and the *B* workers got $1,875. 50 × $2,500 = $125,000; 100 × $1,875 = $187,500; and 150 × $1,250 = $187,500. Because these numbers add up to $500,000, choice (E) is correct.

12. **19**

Plug In \$100 for the price the retailer pays for the item. Therefore, the original selling price is \$140, or 40 percent more than the retail price. To find the reduced selling price, subtract 15 percent of \$140 from \$140 to get \$119. The retailer's profit (selling price – cost) is \$19. Translating the last line of the question, we get $\$19 = (x \div 100) \times 100$, or 19 percent.

13. B

Median means middle. In other words, if you put all the ninth graders in order by score, the middle student would have the median score. Thinking in terms of percentiles, the 50th percentile is the middle, so on the ninth grade pie chart, whatever score includes the 50th percentile when you put the scores in order is the median score. According to the chart, 16 percent of the ninth graders scored below 65, and 37 percent scored between 65 and 69 points. 16 percent + 37 percent = 53 percent. The 50th percentile, then, falls within the group that received 65–69, so 65–69 is the median score.

14. A

In 1975 there were 1,350 + 950 + 625 + 500, or ≈ 3,400 students in grades 9 through 12. 3,400 is 35 percent of School District X, so $3{,}400 = \frac{35}{100} \cdot x$, $x \approx 9{,}700$, so there were 9,700 students.

15. E

There were 1,200 ninth graders in 1993. 25 percent of them, or 300, scored in the 70–79 point range. 14 percent, or 168, scored in the 80–89 point range. The difference between 300 and 168 is 132. (E) is the closest choice.

16. A

Use the bowtie method to subtract the numbers in Quantity A. This gives you $\frac{50-27}{25} = \frac{23}{25}$. If you bowtie the fractions in Quantity B, you get $\frac{375+60}{625} = \frac{435}{625}$. To compare the two quantities, divide the fraction in Quantity B by $\frac{25}{25}$. This gives you a value of $\frac{17.4}{25}$ for Quantity B, which is smaller than Quantity A.

17. D

Use several ratio boxes on this problem. Because X has 2 parts of a and 3 parts of b, there are 5 parts total for X, while Y has 1 + 2 = 3 parts total. Convert these ratios so that they have the same total, which will allow you to compare them. Multiply X by 3 and Y by 5 so that each have 15 total. The new X is 6 parts a and 9 parts b, and the new Y is 5 parts a and 10 parts b. For solution Z there are 2 parts X, so 3 × 6 = 18 parts a and 3 × 9 = 27 parts b. There are 11 parts of Y in Z, so there are 11 × 5 = 55 parts a and 11 × 10 = 110 parts b. Thus, solution Z has 18 + 55 = 73 parts a and 27 + 110 = 137 parts b, and 73 + 137 = 210 total in the ratio. Because the actual total is 630, which is 210 × 3, there must be 73 × 3 = 219 parts of a in the final solution of Z.

18. 25

The library has 160 books on Sunday. Monday's total is 160 – 40, or 120. Tuesday is 120 + ($\frac{1}{2}$ × 40), or 140. Wednesday is 140 + ($\frac{1}{2}$ × 20) – 20, or 130. Thursday is 130 + 80 + ($\frac{1}{6}$ × 30), or 215. Friday and Saturday see 65 more books leave, so the total for the end of Saturday is 215 – 65 = 150. Note that the question asks for Monday, not the first Sunday. The percent change from Monday to Saturday is $\frac{(150-120)}{120}$ × 100, or 25 percent.

19. B and D

Use the Average Pie to find that Jill's mean of 3.75 for 8 evaluations gives her a current total of 3.75 × 8 = 30 points. Use the Average Pie to find that if she needs an average of 4.0 for 12 scores, she needs 4.0 × 12 = 48 total points. Jill still needs 48 – 30 = 18 points. Her four remaining scores must total 18 or greater. Only answers (B) and (D) have a total of at least 18.

CHAPTER 11: GEOMETRY

Geometry Drill (Page 294)

1. A, B, and C

You need to check if the two angles in each answer choice can be part of a right triangle. A right triangle has a 90-degree angle, and because the sum of all the angles of a triangle is 180 degrees, the sum of the other two angles must equal 180 – 90 = 90 degrees. In answer choice (A), 20 + 70 = 90 degrees, so these could be the other two angles in a right triangle. Answer choices (B) and (C) also add up to 90 degrees, and so they are correct as well. In choices (D) and (E), the two angles have a sum greater than 90 degrees, so they are incorrect.

2. B

To find the perimeter of the figure, you need to add up all of its external sides. As written, you're missing the measure of one side of the rectangle. Because the side of the rectangle is equal to the hypotenuse of the right triangle, use the triangle to find the missing side. To find the hypotenuse of the right triangle recognize the common right triangle (5 : 12 : 13), or use the Pythagorean Theorem ($5^2 + 12^2 = x^2$). The missing side of the rectangle is 13. Therefore, the perimeter equals 5 + 12 + 17 + 13 + 17 = 64. Answer choice (A) is the perimeter without the missing side of the rectangle. If you chose answer choice (D), you included an interior side of the rectangle.

3. A

We know that the triangle *EFG* is equilateral because all three angles are equal. That means all of its sides equal 8. From the first equation, we know that the sides of the square also equal 8. The area of the square is $s \times s = 8 \times 8 = 64$, which is larger than Quantity B.

4. D

Draw it on your scratch work, and plot the points. Both a and b must be positive, but their values could be equal or unequal. Quadrant I has (+, +) coordinates, Quadrant II has (–, +) coordinates, Quadrant III has (–, –) coordinates, and Quadrant IV has (+, –) coordinates.

5. E

There are variables in the answers, so Plug In. If the shorter piece is 2 yards long, then the longer piece is $3(2) + 2 = 8$ yards and t must be $2 + 8 = 10$. The target answer, the length of the longer piece, is 8. Plug In 10 for t into all of the answers. Answer choice (E) is the only answer choice that matches your target of 8.

6. D

If CD, the radius of the smaller circle, is 3, then the diameter of the smaller circle is 6. The diameter of the smaller circle is equal to the radius of the larger circle because the smaller circle touches the center and the edge of the larger circle. The formula for the area of a circle is πr^2, so the area of the larger circle is 36π. To find the area of the semicircle, divide by 2 to find 18π.

7. 24

Because Karl's turn from due south to due east forms a right angle, you can use the Pythagorean theorem, in which the hypotenuse is 25, one leg is 7, and the other leg is x. Therefore, you have $7^2 + x^2 = 25^2$. Solving for x, you get $49 + x^2 = 625$, or $x = 24$.

8. A

Circumference of a circle is $2\pi r$, which is greater than 6 times the radius. Perimeter of a square is 4 times the length of a side, or $4r$. Try plugging in values for r, and you will see that Quantity A is always greater than Quantity B.

9. C

The area of the circle is 25π, so the radius of the circle is 5. This means that both AC and BC have length 5, and angles A and B are equal to each other. Because angle C is 60° and the total angle measure of a triangle is 180°, the sum of angle A and B must be 120°. Thus, each angle in triangle ABC is 60°, making this is an equilateral triangle. An equilateral triangle has equal sides and equal angles, so the only possible length of the triangle legs is 5.

10. A

Remember the third side rule. The third side of a triangle must be less than the sum of the other two sides of a triangle, but greater than the difference. That gives us a clear range for x. It must be greater than 6 but less than 12. Quantity A, therefore, is greater than Quantity B; the answer is (A).

11. A

One trick to interpreting geometry problems is to exaggerate whatever is going on in your picture. You can see that the two triangles are almost the same, except that the base length in the triangle to the right is slightly larger. Well, what happens as you keep stretching out that base length? The triangle starts to collapse and its height gets smaller and smaller. Thus, height f must be greater than height g. This technique works quite well in a number of GRE quant comp geometry problems!

12. B

In order to find the x-coordinate of a point on a line, you must first find the slope of the line. Notice that along with points A and B, the origin is also a point on the line in the figure. Using the coordinates of (0, 0) and A (2, 3), the slope is $\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{3}{2}$. Because the slope of a line stays constant, you can use the value you just found to solve for the missing x-coordinate of point B. Using points A (2, 3) and B (x, 4.2), solve $\frac{(4.2 - 3)}{(x - 2)} = \frac{3}{2}$. Cross-multiply to find that $3x - 2 = 2.4$, so $x = 2.8$ or choice (B).

13. A

Use the 3 : 4 : 5 ratio or the Pythagorean theorem to determine that the length of AB is 4. Because the area of a triangle equals $\frac{1}{2} \times$ base $\times$ height, triangle ABD has an area of $\frac{1}{2} \times 3 \times 4$, or 6. Be wary of answer choice (D), which is the area of the rectangle.

14. B

Because the two angles have the same measure, the wedges of the circle they mark off will have the same area. The triangle is smaller than the wedge, so Quantity B is greater than Quantity A.

15. C

Because $LMNO$ is a parallelogram and $c = 108$, angle LON must be $180 - 108 = 72$. Angle LON is the same fraction of the entire circle (360 degrees) that arc d is of the entire circumference, $\frac{72}{360} = \frac{1}{5}$. Thus, arc d must be $\frac{1}{5}$ of the circumference. So, $\frac{1}{5} \times 15\pi = 3\pi$. If you were stuck on this problem, you could have estimated that d looks to be about a fourth or fifth of the circle's circumference. Thus, eliminate answers (A) and (B).

CHAPTER 12: MATH ET CETERA

Et Cetera Drill (Page 314)

1. C

If there is one more red marble than blue, there must be 7 blue marbles and 8 red ones, for a total of 15. The probability of choosing a blue marble is $\frac{\#\text{ of blue marbles}}{\text{Total }\#\text{ of marbles}}$, or $\frac{7}{15}$. If you selected choice (E), you probably computed the probability of drawing a red marble rather than the probability of drawing a blue one.

2. D

Plug the values into the function. First, find ¥(5): (5 × 10 – 1) = 49. Next, find ¥(3) = (3 × 10 – 1) = 29. Now subtract them: ¥(5) – ¥(3) = 49 – 29 = 20.

3. A

Find all the factors of 78. 78 = 1 × 78 = 2 × 39 = 3 × 26 = 6 × 13. The largest odd factor is 39; the largest prime factor is 13. Quantity A is greater than Quantity B.

4. **20**

All 5 finalists could be awarded "Best in Show." There are 4 choices left for "Honorable Mention," because a different dog must be chosen. Therefore, the total number of possibilities is 5 × 4, or 20.

5. A

Use the group equation: Group 1 + Group 2 – Both + Neither = Total. So, \$40,000 + \$30,000 – \$15,000 + Neither = \$90,000. Thus, \$55,000 + Neither = \$90,000. So, the company spends \$35,000 on other products. Quantity A is greater than Quantity B.

6. B

List the two-digit prime numbers less than 50: 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47. The numbers in which the tens digit is greater than the units digit are 31, 41, and 43. Because 3 out of the 11 possibilities meet the requirement, choice (B) is correct.

7. A

Plug In the answer choices, starting with (C). With 9 staff members, the elected official has $\frac{9!}{5!4!}$ (alternatively, you may have set this up as $\frac{9 \times 8}{1 \times 2}$). This works out to 36, which is too large. Try plugging in answer choice (A). With 7 staff members, the elected official has $\frac{7!}{5!2!}$ = 21 different groups of 5 from which to choose (again, you may have set this up the alternative way as $\frac{7 \times 6}{1 \times 2}$).

8. E

Plug In: Make $x = 2$ and $y = 3$. Now $x \# y = 2\,(2 - 3) = -2$. Watch out for traps: Answer choices (A) and (C) will give you –2, but because the question asks for $x \# (x \# y)$, you need to perform the operation again. $2 \# (-2) = 2\,[2 - (-2)] = 2\,(4) = 8$. Now put $x = 2$ and $y = 3$ into the answer choices to find a match for your target answer, 8. Be sure to eliminate choices (A), (B), (C), and (D) as soon as you realize they are negative. The only answer that matches is choice (E).

9. B

Use a ratio box to find that if there are twice as many yellow as green and 12 total, then there are 8 yellows and 4 greens. Two situations would fit the requirements of the problem: Pull out a yellow and then green, or pull out a green and then yellow. So, find the probability of each of these situations; then add these two probabilities together.

The probability of yellow and then green is $\frac{8}{12} \times \frac{4}{11} = \frac{8}{33}$. The probability of green and then yellow is $\frac{4}{12} \times \frac{8}{11} = \frac{8}{33}$.

Add these two probabilities to find $\frac{8}{33} + \frac{8}{33} = \frac{16}{33}$.

10. B

You could try to draw this all out, but it would probably be quite a headache. For Quantity A, if you're creating triangles, you're really choosing three points from the set of 10. This is a combination problem—order doesn't matter, because triangle ABC would be the same as triangle BCA. You could use the formula $\frac{10!}{3!(10-3)!} = \frac{10!}{3!(7!)} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1}$ = 120. For Quantity B, note that quadrilaterals are any four-sided figures, so you're just choosing 4 points from 10. You could use the formula for combinations: $\frac{10!}{4!(10-4!)} = \frac{10!}{4!(6!)} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}$ = 210.

Comprehensive Math Drill (Page 316)

1. C

Cross multiply. $\frac{0.05}{0.6} = \frac{x}{0.18}$, so $0.6x = (0.05)(0.18)$, and $x = \frac{(0.05)(0.18)}{0.6} = (0.05)(0.3) = 0.015$. Quantity A is equal to Quantity B.

2. C

Simplify the expression in Quantity B: $\frac{\frac{x}{5}}{2} = \frac{x}{5} \div \frac{2}{1} = \frac{x}{5} \times \frac{1}{2} = \frac{x}{10}$. The expressions in Quantity A and in Quantity B are the same.

3. C

Remember that the percentages for standard deviations are 34 percent, 14 percent, 2 percent in both directions from the mean. If the mean is 50, then 34 percent score between 50 and 54, 14 percent score between 54 and 58, and 2 percent score above 58. The same idea applies in the other direction: If the mean is 50, then 34 percent score between 50 and 46, 14 percent score between 46 and 42, and 2 percent score below 42. So, both quantities are equal to 2 percent.

4. A

The equation $y = mx + b$ describes a line where m is the slope and b is the y-intercept—the place where the line crosses the y-axis. Hence, the y-intercept of our line, or P, is (0, 1), which means the length of OP is 1. Because R is on the x-axis, the y-coordinate must be 0, and we can use the line equation to solve for x: $0 = -\frac{5}{6}x + 1$, so $-1 = -\frac{5}{6}x$, and $x = \frac{6}{5}$. That means OR = $\frac{6}{5}$, and Quantity A is greater. Because this is a Quant Comp, though, we can actually compare the quantities without solving them. If you recognize from the line equation that our slope is $-\frac{5}{6}$, and you remember that slope is defined as $\frac{rise}{run}$, you might also recognize that Quantity A, OR, is our run, and Quantity B, OP, is our rise. Disregarding the negative sign—distance is always an absolute value, and therefore positive—we can see that our rise is less than our run, and Quantity A is greater.

5. B

For Quantity A, "pairs" tells you that you're picking two and that order does not matter: This is a combination. You could use the formula $\frac{20!}{2!(20-2)!} = \frac{20!}{2!(18)!} = \frac{20 \times 19}{2}$ = 190. For Quantity B, the "rankings" tells you that order matters: This is a permutation. So, you could use the formula $\frac{10!}{(10-3)!} = \frac{10!}{7!}$ = 10 × 9 × 8 = 720.

6. D

The denominator is the same for both expressions, so we only need to compare numerators to determine which fraction is greater. Plug In to see whether kl is greater than or less than 1. Let $k = 0.5$ and $l = 1.5$. Therefore, $kl = 0.75$. Eliminate answer choices (B) and (C). Now let $k = 10$ and $l = 10$, $kl = 100$. Eliminate answer choice (A).

7. A

Approximate your values. In Quantity A, $\sqrt{3} + \sqrt{4} \cong 1.7 + 2 = 3.7$. In Quantity B, $\sqrt{7}$ is less than 3, so Quantity A is greater.

8. 4

If Joe starts with \$200 and spends \$150 on a CD player, he has only \$200 – \$150 = \$50 left. Each CD is \$12, so divide \$50 by \$12. It goes in 4 times with \$2 left over. Don't round! Joe can buy only 4.

9. A

For triangle ABC, the base is the difference between C and B, 1. Finding the height is a little more difficult. The height of a triangle is any perpendicular line dropped from the highest point to the level of the base. The height does not need to touch segment CB as long as it extends from A to the level of CB. For this triangle, distance from A to the origin is the height, 4. Plug In the base and height: Area = $\frac{1}{2} \times 1 \times 4 = 2$.

10. B, C, and D

To solve this problem, first use PEMDAS: $10(3^2 - 2) = 10(9 - 2) = 10(7) = 70$. The question states that 70 is divided by a positive integer, so try dividing 70 by different integers. Choices (B), (C) and (D) could work because you can divide 70 by 1, 2, and 7 respectively. Choice (A) does not work because to get 140, you would have to divide 70 by $\frac{1}{2}$, which is not an integer. Also, 0 is neither positive nor negative, and you cannot divide a number by 0, so choice (E) could not work either.

11. B and C

Roberta's rate is 50 miles in 2 hours. Notice that the first number in this proportion is greater than the second. Use that to eliminate choices (A) and (D). For choice (B), $\frac{100}{4} = \frac{50}{2}$, so this is the same as the original proportion. For choice (C), $\frac{400}{16} = \frac{50}{2}$, so this is also the same as the original proportion.

12. C

There were seven cities with temperatures in Year Y higher than or equal to those in Year X: Baltimore, Detroit, Las Vegas, Minneapolis, New York, Phoenix, and San Francisco.

13. C

The lowest average temperature was 34° F in Anchorage, and the highest was 83° F in Las Vegas. Percent change = $\frac{\text{difference}}{\text{original}} = \frac{49}{34} \approx 144$ percent.

14. C

You're averaging the highs and lows for Years X and Y, so the number of things is 4. The bar shows the average of Years X and Y, which reads 60. Multiply 60 by 4 to get the total, 240. Get the average high temperatures for Years X and Y from the straight and dotted lines on the chart. They're about 103 degrees and 97 degrees. The total is 240 = 103 + 97 + low Year X + low Year Y. If you subtract the highs from the total, you're left with 40 degrees as the total for the lows. Because you want the average of the lows, divide this total by 2. The closest answer is 20°.

15. A, B, and C

First, simplify the inequality by subtracting 2 from both sides: $|2x - 3| > 5$. Now plug each answer choice into the inequality to see which value of x makes the inequality true. The correct values are those in choices (A), (B), and (C).

16. A

The question states that x is an odd integer, so eliminate choice (C) because 0 is not odd. Simplify $x + y + z < z$ by subtracting z from each side: $x + y < 0$. Because x is less than y , x must be negative so that when added to y, the answer will be less than zero. Therefore, eliminate choices (D) and (E). Now Plug In the remaining answers to see which value of x will work in the inequality. Choice (A) is the only choice that works.

17. E

First, solve for x by multiplying 4 by itself until you get 1024. This means that x equals 5. If you substitute 5 for x in the second equation, the equation reads, $4^6 \times 5^4$. Because the answers are expressed in terms of 4^n, 5^n, and 10^n, expand out $4^6 \times 5^4$ to get $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 5 \times 5 \times 5 \times 5$. Now try to express it using 10^n. We need to factor two of the fours and rewrite this as $4 \times 4 \times 4 \times 4 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$. Now, convert this back into exponents to get $4^4 \times 2^4 \times 5^4$, or $4^4 \times 10^4$.

18. D

First, use the volume formula to find the width: $V = l \times w \times h$. So, $780 = 12 \times w \times 5$. Thus, the width is 13. Next, draw the figure. Notice that the greatest distance is from one corner to the opposite corner, such as from the front left bottom corner diagonally to the rear right top corner. You can use the formula for diagonal of a rectangular solid, $a^2 + b^2 + c^2 = d^2$, in which a, b, and c are the dimensions of the rectangular solid and d is the diagonal, and love that you have a calculator. Thus, $(5)^2 + (12)^2 + (13)^2 = d^2$. So, $25 + 144 + 169 = d^2$, and thus $d = \sqrt{338}$ or $13\sqrt{2}$.

19. D

There are six spots to fill. Because no boys can sit on the end of the bench, 3 girls are available to fill one spot at one end of the bench. Once one girl has been chosen to fill that spot, there are 2 girls available to fill the spot on the other end of the bench. Then, there are 4 children (boys and girls) available to fill the other four spots. Because $3 \times 2 \times 4 \times 3 \times 2 \times 1 = 144$, choice (D) is correct.

20. C

Use the average pie. If 16 is the average of 3 numbers, their total is 48. You know that one of the numbers is 24, so $p + q + 24 = 48$. Thus, $(p + q) = 24$. You need to find $16(p + q)$, so find 16(24), which equals 384.