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PART THREE

Quantitative Reasoning

CHAPTER 9

Introduction to Quantitative Reasoning

OVERVIEW

The Quantitative Reasoning section of the GRE is designed to place most of its emphasis on your ability to reason quantitatively—to read a math problem, understand what it's asking, and solve it. The mathematical concepts tested on the GRE are similar to those tested on the SAT. You will see questions related to arithmetic, algebra, geometry, and data interpretation. There is no trigonometry or calculus on the GRE. The emphasis in the Quantitative Reasoning section is on your ability to reason, using your knowledge of the various topics. The goal is to make the test an accurate indicator of your ability to apply given information, think logically, and draw conclusions. These are skills you will need at the graduate level of study.

In this section of the book, we'll take you through all the types of Quantitative Reasoning questions you'll see on the GRE and give you the strategies you need to answer them quickly and correctly. Also, all of the mathematical concepts you'll encounter on the test are included in the "Math Reference" Appendix at the back of this book. Think of the examples there as building blocks for the questions you will see on the test.

QUANTITATIVE REASONING QUESTION TYPES

The GRE contains two Quantitative Reasoning sections with 20 questions each. Each section will last 35 minutes and be composed of a selection of the following question types:

- Quantitative Comparison
- Problem Solving
- Data Interpretation

The Quantitative Reasoning portion of the GRE draws heavily upon your ability to combine your knowledge of mathematical concepts with your reasoning powers. Specifically, it evaluates your ability to do the following:

- Compare quantities using reasoning
- Solve word problems
- Interpret data presented in charts and graphs

Within each Quantitative Reasoning section on the GRE, you will see an assortment of question types.

PACING STRATEGY

As a multi-stage test, the GRE allows you to move freely backward and forward within each section, which can be a big advantage on Test Day. If you get stuck on a particular question, you can mark it and come back to it later when you have time. You only score points for correct answers, so you don't want to get bogged down on one problem and lose time you could have used to answer several other questions correctly.

You will have 35 minutes to work on each Quantitative Reasoning section. The 20 questions in each section will be an assortment of Quantitative Comparison, Problem Solving, and Data Interpretation items. However, these types are not distributed equally. The chart below shows how many questions you can expect of each question type, as well as the average amount of time you should spend per question type.

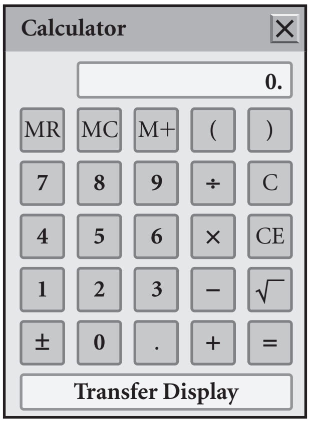
	Quantitative Comparison	Problem Solving	Data Interpretation
Number of Questions	approx. 7–8	approx. 9–10	approx. 3
Time per Question	1.5 minutes	1.5–2 minutes	2 minutes

Try to keep these time estimates in mind as you prepare for the test. If you use them as you practice, you will be comfortable keeping to the same amounts of time on Test Day. Additionally, you will be prepared to use the Mark and Review buttons to your advantage while taking the actual test.

To Calculate or Not

An onscreen calculator will be available during the GRE. Numbers can be entered either by clicking on the numbers on the calculator with your mouse or by entering numbers from the keyboard. There are several points to consider about using the calculator on Test Day. A calculator can be a time-saver, and time is immensely important on a standardized test. But while calculators can speed up computations, they can also foster dependence, making it hard for you to spot the shortcuts in GRE questions. Using the calculator for a long, involved computation to answer a question will gobble up your allotted time for that question—and perhaps for several more. You may even make a mistake in your computation, leading to an incorrect answer. Remember, this is a *reasoning* test. The quantitative questions on the GRE are not designed to require lengthy computations.

If that is the case, why is a calculator provided? A calculator can be an asset for the occasional computation that a few questions require. It may prevent an error caused by a freehand calculation. The onscreen calculator provided is a simple four-function calculator. An image of the calculator is provided below, showing the function keys, including the square root key and change-of-sign key.



By not relying on the calculator, you will be free to focus on interpreting numbers and data and using your critical thinking skills. This is the intention of the writers of the test. For example, Problem Solving questions will likely involve more algebra than calculating, and Quantitative Comparison questions will require more reasoning than calculating.

NAVIGATING THE QUANTITATIVE REASONING SECTION OF THIS BOOK

The chapter immediately following this one concerns Math Foundations and Content Review and will review the classic math concepts and topics that you may encounter on the GRE. This section of the book also includes individual chapters on Quantitative Comparison, Problem Solving, and Data Interpretation questions. Each chapter includes an introduction to the relevant question types and then a review with strategies you can follow to answer those questions quickly and correctly. In addition, you'll find a practice set of questions with answers and explanations for each of the question types you'll encounter on the GRE.

Finally, at the end of this section, you'll find the Quantitative Reasoning Practice Sets, three sets of 20 Quantitative Reasoning questions with answers and explanations. Use the Practice Sets to test your skills and pinpoint areas for more focused study. When you are finished with this section of the book, you should be thoroughly prepared for any question you might encounter on the Quantitative Reasoning section of the GRE.

CHAPTER 10

Math Foundations and Content Review

ARITHMETIC

Terms

Consecutive numbers: Numbers of a certain type, following one another without interruption. Numbers may be consecutive in ascending or descending order. The GRE prefers to test consecutive integers (e.g., $-2, -1, 0, 1, 2, 3\ldots$), but you may encounter other types of consecutive numbers. For example:

$-4, -2, 0, 2, 4, 6\ldots$ is a series of consecutive even numbers.

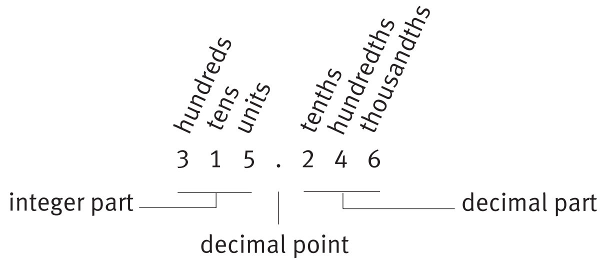
$-3, 0, 3, 6, 9\ldots$ is a series of consecutive multiples of 3.

$2, 3, 5, 7, 11\ldots$ is a series of consecutive prime numbers.

Cube: A number raised to the 3rd power. For example $4^3 = (4)(4)(4) = 64$, showing that 64 is the cube of 4.

Decimal: A fraction written in decimal system format. For example, 0.6 is a decimal. To convert a fraction to a decimal, divide the numerator by the denominator. For instance, $\frac{5}{8} = 5 \div 8 = 0.625$.

Decimal system: A numbering system based on the powers of 10. The decimal system is the only numbering system used on the GRE. Each figure, or digit, in a decimal number occupies a particular position, from which it derives its place value.



Denominator: The quantity in the bottom of a fraction, representing the whole.

Difference: The result of subtraction.

Digit: One of the numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9. A number can have several digits. For example, the number 542 has three digits: a 5, a 4, and a 2. The number 321,321,000 has nine digits but only four distinct (different) digits: 3, 2, 1, and 0.

Distinct: Different from each other. For example, 12 has three prime factors (2, 2, and 3) but only 2 distinct factors (2 and 3).

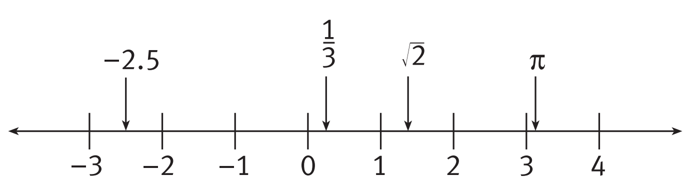
Element: One of the members of a set.

Exponent: The number that denotes the power to which another number or variable is raised. The exponent is typically written as a superscript to a number. For example, 5^3 equals (5)(5)(5). The exponent is also occasionally referred to as a "power." For example, 5^3 can be described as "5 to the 3rd power." The product, 125, is "the 3rd power of 5." Exponents may be positive or negative integers or fractions, and they may include variables.

Fraction: The division of a part by a whole. $\frac{\text{Part}}{\text{Whole}} = \text{Fraction}$. For example, $\frac{3}{5}$ is a fraction.

Integer: A number without fractional or decimal parts, including positive and negative whole numbers and zero. All integers are multiples of 1. The following are examples of integers: $-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$.

Number Line: A straight line, extending infinitely in either direction, on which numbers are represented as points. The number line below shows the integers from -3 to 4. Decimals and fractions can also be depicted on a number line, as can irrational numbers, such as $\sqrt{2}$.



The values of numbers get larger as you move to the right along the number line. Numbers to the right of zero are *positive*; numbers to the left of zero are *negative*.

Zero is neither positive nor negative. Any positive number is larger than any negative number. For example, $-300 < 4$.

Numerator: The quantity in the top of a fraction, representing the part.

Operation: A function or process performed on one or more numbers. The four basic arithmetic operations are addition, subtraction, multiplication, and division.

Part: A specified number of the equal sections that compose a whole.

Product: The result of multiplication.

Sequence: Lists that have an infinite number of terms, in order. The terms of a sequence are often indicated by a letter with a subscript indicating the position of the number in the sequence. For instance, a_3 denotes the third number in a sequence, while a_n indicates the nth term in a sequence.

Set: A well-defined collection of items, typically numbers, objects, or events. The bracket symbols { } are normally used to define sets of numbers. For example, {2, 4, 6, 8} is a set of numbers.

Square: The product of a number multiplied by itself. A squared number has been raised to the 2nd power. For example, $4^2 = (4)(4) = 16$, and 16 is the square of 4.

Sum: The result of addition.

Whole: A quantity that is regarded as a complete unit.

Symbols

$=$	is equal to
$\neq$	is not equal to
$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to
$\div$	divided by
π	pi (the ratio of the circumference of a circle to the diameter)
$\pm$	plus or minus
$\sqrt{\ }$	square root
$\angle$	angle

Rules of Operation

There are certain mathematical laws governing the results of the four basic operations: addition, subtraction, multiplication, and division. Although you won't need to know the names of these laws for the GRE, you'll benefit from understanding them.

PEMDAS

A string of operations must be performed in proper order. The acronym PEMDAS stands for the correct order of operations:

Parentheses

Exponents

Multiplication
Division
simultaneously from left to right

Addition
Subtraction
simultaneously from left to right

If you have trouble remembering PEMDAS, you can think of the mnemonic "Please Excuse My Dear Aunt Sally."

Example:

$66\,(3-2)\div 11$

If you were to perform all the operations sequentially from left to right, without using PEMDAS, you would arrive at an answer of $\frac{196}{11}$. But if you perform the operation within the parentheses first, you get $66(1)\div 11 = 66\div 11 = 6$, which is the correct answer.

Example:

$$30-5(4)+\frac{(7-3)^2}{8}$$
$$=30-5(4)+\frac{4^2}{8}$$
$$=30-5(4)+\frac{16}{8}$$
$$=30-20+2$$
$$=10+2$$
$$=12$$

Commutative Laws of Addition and Multiplication

Addition and multiplication are both commutative; switching the order of any two numbers being added or multiplied together does not affect the result.

Example:

$$\begin{aligned} 5 + 8 &= 8 + 5 \\ (2)(3)(6) &= (6)(3)(2) \\ a + b &= b + a \\ ab &= ba \end{aligned}$$

Division and subtraction are not commutative; switching the order of the numbers changes the result. For instance, $3 - 2 \neq 2 - 3$; the left side yields a difference of 1, while the right side yields a difference of -1. Similarly, $\frac{6}{2} \neq \frac{2}{6}$; the left side equals 3, while the right side equals $\frac{1}{3}$.

Associative Laws of Addition and Multiplication

Addition and multiplication are also associative; regrouping the numbers does not affect the result.

Example:

$$\begin{aligned} (3 + 5) + 8 &= 3 + (5 + 8) \\ 8 + 8 &= 3 + 13 \\ 16 &= 16 \end{aligned} \qquad \begin{aligned} (a + b) + c &= a + (b + c) \\ (ab)c &= a(bc) \end{aligned}$$

The Distributive Law

The distributive law of multiplication allows you to "distribute" a factor over numbers that are added or subtracted. You do this by multiplying that factor by each number in the group.

Example:

$$\begin{aligned} 4(3 + 7) &= (4)(3) + (4)(7) \\ 4(10) &= 12 + 28 \\ 40 &= 40 \end{aligned} \qquad a(b + c) = ab + ac$$

The law works for the numerator in division as well.

$$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$$

However, when the sum or difference is in the denominator—that is, when you're dividing by a sum or difference—no distribution is possible.

$\frac{9}{4+5}$ is *not* equal to $\frac{9}{4} + \frac{9}{5}$.

NUMBER PROPERTIES

Adding and Subtracting

Numbers can be treated as though they have two parts: a positive or negative sign and a number. Numbers without any sign are understood to be positive.

To add two numbers that have the same sign, add the number parts and keep the sign. Example: To add $(-6) + (-3)$, add 6 and 3 and then attach the negative sign from the original numbers to the sum: $(-6) + (-3) = -9$.

To add two numbers that have different signs, find the difference between the number parts and keep the sign of the number whose number part is larger. Example: To add $(-7) + (+4)$, subtract 4 from 7 to get 3. Because $7 > 4$ (the number part of -7 is greater than the number part of 4), the final sum will be negative: $(-7) + (+4) = -3$.

Subtraction is the opposite of addition. You can rephrase any subtraction problem as an addition problem by changing the operation sign from a minus to a plus and switching the sign on the second number. Example: $8 - 5 = 8 + (-5)$. There's no real advantage to rephrasing if you are subtracting a smaller positive number from a larger positive number. But the concept comes in very handy when you are subtracting a negative number from any other number, a positive number from a negative number or a larger positive number from a smaller positive number.

To subtract a negative number, rephrase as an addition problem and follow the rules for addition of signed numbers. For instance, $9 - (-10) = 9 + 10 = 19$.

To subtract a positive number from a negative number or from a smaller positive number, change the sign of the number that you are subtracting from positive to negative and follow the rules for addition of signed numbers. For example, $(-4) - 1 = (-4) + (-1) = -5$.

Multiplication and Division of Positive and Negative Numbers

Multiplying or dividing two numbers with the same sign gives a positive result.

Examples:

$$\begin{aligned} (-4)(-7) &= +28 \\ (-50) \div (-5) &= +10 \end{aligned}$$

Multiplying or dividing two numbers with different signs gives a negative result.

Examples:

$$(-2)(+3) = -6$$
$$8 \div (-4) = -2$$

Absolute Value

The absolute value of a number is the value of a number without its sign. It is written as two vertical lines, one on either side of the number and its sign.

Example:

$$|-3| = |+3| = 3$$

The absolute value of a number can be thought of as the number's distance from zero on the number line. Since both 3 and -3 are 3 units from 0, each has an absolute value of 3. If you are told that $|x| = 5$, x could equal 5 or -5.

Properties of Zero

Adding zero to or subtracting zero from a number does not change the number.

$$x + 0 = x$$
$$0 + x = x$$
$$x - 0 = x$$

Examples:

$$5 + 0 = 5$$
$$0 + (-3) = -3$$
$$4 - 0 = 4$$

Notice, however, that subtracting a number from zero changes the number's sign. It's easy to see why if you rephrase the problem as an addition problem.

Example:

Subtract 5 from 0.

$0 - 5 = -5$. That's because $0 - 5 = 0 + (-5)$, and according to the rules for addition with signed numbers, $0 + (-5) = -5$.

The product of zero and any number is zero.

Examples:

$$(0)(z) = 0$$
$$(z)(0) = 0$$
$$(0)(12) = 0$$

Division by zero is undefined. For GRE purposes, that translates to "It can't be done." Since fractions are essentially division (that is, $\frac{1}{4}$ means $1 \div 4$), any fraction with zero in the denominator is also undefined. So when you are given a fraction that has an algebraic expression in the denominator, be sure that the expression cannot equal zero.

Properties of 1 and –1

Multiplying or dividing a number by 1 does not change the number.

$$\begin{aligned}(a)(1) &= a\\(1)(a) &= a\\a \div 1 &= a\end{aligned}$$

Examples:

$$\begin{aligned}(4)(1) &= 4\\(1)(-5) &= -5\\(-7) \div 1 &= -7\end{aligned}$$

Multiplying or dividing a nonzero number by -1 changes the sign of the number.

$$\begin{aligned}(a)(-1) &= -a\\(-1)(a) &= -a\\a \div (-1) &= -a\end{aligned}$$

Examples:

$$\begin{aligned}(6)(-1) &= -6\\(-3)(-1) &= 3\\(-8) \div (-1) &= 8\end{aligned}$$

Factors, Multiples, and Remainders

Multiples and Divisibility

A *multiple* is the product of a specified number and an integer. For example, 3, 12, and 90 are all multiples of 3: $3 = (3)(1)$; $12 = (3)(4)$; and $90 = (3)(30)$. The number 4 is not a multiple of 3, because there is no integer that can be multiplied by 3 and yield 4.

Multiples do not have to be of integers, but all multiples must be the product of a specific number and an integer. For instance, 2.4, 12, and 132 are all multiples of 1.2: $2.4 = (1.2)(2)$; $12 = (1.2)(10)$; and $132 = (1.2)(110)$.

The concepts of multiples and factors are tied together by the idea of *divisibility*. A number is said to be evenly divisible by another number if the result of the division is an integer with no remainder. A number that is evenly divisible by a second number is also a multiple of the second number.

For example, $52 \div 4 = 13$, which is an integer. So 52 is evenly divisible by 4, and it's also a multiple of 4.

On some GRE math problems, you will find yourself trying to assess whether one number is evenly divisible by another. You can use several simple rules to save time.

- An integer is divisible by 2 if its last digit is divisible by 2.
- An integer is divisible by 3 if its digits add up to a multiple of 3.
- An integer is divisible by 4 if its last two digits are a multiple of 4.
- An integer is divisible by 5 if its last digit is 0 or 5.
- An integer is divisible by 6 if it is divisible by both 2 and 3.
- An integer is divisible by 9 if its digits add up to a multiple of 9.

Example:

6,930 is a multiple of 2, since 0 is even.

. . . a multiple of 3, since $6 + 9 + 3 + 0 = 18$, which is a multiple of 3.

. . . not a multiple of 4, since 30 is not a multiple of 4.

. . . a multiple of 5, since it ends in zero.

. . . a multiple of 6, since it is a multiple of both 2 and 3.

. . . a multiple of 9, since $6 + 9 + 3 + 0 = 18$, which is a multiple of 9.

Properties of Odd/Even Numbers

Even numbers are integers that are evenly divisible by 2; *odd* numbers are integers that are not evenly divisible by 2. Integers whose last digit is 0, 2, 4, 6, or 8 are even; integers whose last digit is 1, 3, 5, 7, or 9 are odd. The terms *odd* and *even* apply only to integers, but they may be used for either positive or negative integers. 0 is considered even.

Rules for Odds and Evens

$$\begin{aligned}
\text{Odd} + \text{Odd} &= \text{Even}\\
\text{Even} + \text{Even} &= \text{Even}\\
\text{Odd} + \text{Even} &= \text{Odd}\\
\text{Odd} \times \text{Odd} &= \text{Odd}\\
\text{Even} \times \text{Even} &= \text{Even}\\
\text{Odd} \times \text{Even} &= \text{Even}
\end{aligned}$$

Note that multiplying any even number by *any* integer always produces another even number.

It may be easier to use the Picking Numbers strategy in problems that ask you to decide whether some unknown will be odd or even.

Example:

Is the sum of two odd numbers odd or even?

Pick any two odd numbers, for example, 3 and 5: $3 + 5 = 8$. Since the sum of the two odd numbers that you picked is an even number, 8, it's safe to say that the sum of any two odd numbers is even.

Picking Numbers will work in any odds/evens problem, no matter how complicated. The only time you have to be careful is when division is involved, especially if the problem is in Quantitative Comparison format; different numbers may yield different results.

Example:

Integer x is evenly divisible by 2. Is $\frac{x}{2}$ even?

By definition, any multiple of 2 is even, so integer x is even. And $\frac{x}{2}$ must be an integer. But is $\frac{x}{2}$ even or odd? In this case, picking two different even numbers for x can yield two different results. If you let $x = 4$, then $\frac{x}{2} = \frac{4}{2} = 2$, which is even. But if you let $x = 6$, then $\frac{x}{2} = \frac{6}{2} = 3$, which is odd. So $\frac{x}{2}$ could be even or odd—and you wouldn't know that if you picked only one number.

Factors and Primes

The *factors*, or *divisors*, of an integer are the positive integers by which it is evenly divisible (leaving no remainder).

Example:

What are the factors of 36?

36 has nine factors: 1, 2, 3, 4, 6, 9, 12, 18, and 36. We can group these factors in pairs: $(1)(36) = (2)(18) = (3)(12) = (4)(9) = (6)(6)$.

The *greatest common factor*, or greatest common divisor, of a pair of integers is the largest factor that they share.

To find the greatest common factor (GCF), break down both integers into their prime factorizations and multiply all the prime factors they have in common: $36 = (2)(2)(3)(3)$, and $48 = (2)(2)(2)(2)(3)$. What they have in common is two 2s and one 3, so the GCF is $(2)(2)(3) = 12$.

A *prime number* is an integer greater than 1 that has only two factors: itself and 1. The number 1 is not considered a prime, because it is divisible only by itself. The number 2 is the smallest prime number and the only even prime. (Any other even number must have 2 as a factor and therefore cannot be prime.)

Prime Factors

The *prime factorization* of a number is the expression of the number as the product of its prime factors (the factors that are prime numbers).

There are two common ways to determine a number's prime factorization. The rules given above for determining divisibility by certain numbers come in handy in both methods.

Method #1: Work your way up through the prime numbers, starting with 2. (You'll save time in this process, especially when you're starting with a large number, by knowing the first ten prime numbers by heart: 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.)

Example:

What is the prime factorization of 210?

$$210 = (2)(105)$$

Since 105 is odd, it can't contain another factor of 2. The next smallest prime number is 3. The digits of 105 add up to 6, which is a multiple of 3, so 3 is a factor of 105.

$$210 = (2)(3)(35)$$

The digits of 35 add up to 8, which is not a multiple of 3. But 35 ends in 5, so it is a multiple of the next largest prime number, 5.

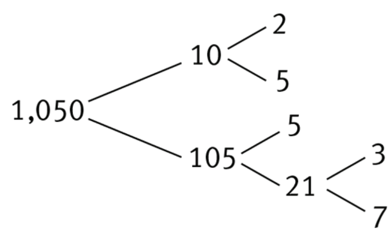
$$210 = (2)(3)(5)(7)$$

Since 7 is a prime number, this equation expresses the complete prime factorization of 210.

Method #2: Figure out one pair of factors and then determine their factors, continuing the process until you're left with only prime numbers. Those primes will be the prime factorization.

Example:

What is the prime factorization of 1,050?



The distinct prime factors of 1,050 are therefore 2, 5, 3, and 7, with the prime number 5 occurring twice in the prime factorization. We usually write out the prime factorization by putting the prime numbers in increasing order. Here, that would be (2)(3)(5)(5)(7). The prime factorization can also be expressed in exponential form: $(2)(3)(5^2)(7)$.

The Least Common Multiple

The *least common multiple* of two or more integers is the smallest number that is a multiple of each of the integers. Here's one quick way to find it:

(1) Determine the prime factorization of each integer.

(2) Write out each prime number the maximum number of times that it appears in any one of the prime factorizations.

(3) Multiply those prime numbers together to get the least common multiple of the original integers.

Example:

What is the least common multiple of 6 and 8?

Start by finding the prime factors of 6 and 8.

$$\begin{aligned} 6 &= (2)(3) \\ 8 &= (2)(2)(2) \end{aligned}$$

The factor 2 appears three times in the prime factorization of 8, while 3 appears as only a single factor of 6. So the least common multiple of 6 and 8 will be (2)(2)(2)(3), or 24.

Note that the least common multiple of two integers is smaller than their product if they have any factors in common. For instance, the product of 6 and 8 is 48, but their least common multiple is only 24.

In addition to answering questions using the term *least common multiple*, you'll find the concept useful whenever you're adding or subtracting fractions with different denominators.

Remainders

The *remainder* is what is "left over" in a division problem. A remainder is always smaller than the number you are dividing by. For instance, 17 divided by 3 is 5, with a remainder of 2. Likewise, 12 divided by 6 is 2, with a remainder of 0 (since 12 is evenly divisible by 6).

GRE writers often disguise remainder problems. For instance, a problem might state that the slats of a fence are painted in three colors, which appear in a fixed order, such as red, yellow, blue, red, yellow, blue.... You would then be asked something like, "If the first slat is red, what color is the 301st slat?" Since 3 goes into 300 evenly, the whole pattern must finish on the 300th slat and start all over again on the 301st. Therefore, the 301st would be red.

Exponents and Roots

Rules of Operations with Exponents

To multiply two powers with the same base, keep the base and add the exponents together.

Example:

$$2^2 \times 2^3 = (2 \times 2)(2 \times 2 \times 2) = 2^5$$

or

$$2^2 \times 2^3 = 2^{2+3} = 2^5$$

To divide two powers with the same base, keep the base and subtract the exponent of the denominator from the exponent of the numerator.

Example:

$$4^5 \div 4^2 = \frac{(4)(4)(4)(4)(4)}{(4)(4)} = 4^3$$

or

$$4^5 \div 4^2 = 4^{5-2} = 4^3$$

To raise a power to another power, multiply the exponents.

Example:

$(3^2)^4 \quad = \quad (3 \times 3)^4$

or

$(3^2)^4 \quad = \quad (3 \times 3)(3 \times 3)(3 \times 3)(3 \times 3)$

or

$(3^2)^4 \quad = \quad 3^{2 \times 4} = 3^8$

To multiply two powers with different bases but the same power, multiply the bases together and raise to the power.

Example:

$(3^2)\ (5^2) = (3 \times 3)(5 \times 5) = (3 \times 5)(3 \times 5) = (3 \times 5)^2 = 15^2$

A base with a negative exponent indicates the reciprocal of that base to the positive value of the exponent.

Example:

$$5^{-3} = \frac{1}{5^3} = \frac{1}{125}$$

Raising any non-zero number to an exponent of zero equals 1.

Examples:

$5^0 = 1$

$161^0 = 1$

$(-6)^0 = 1$

Commonly Tested Properties of Powers

Many Quantitative Comparison problems test your understanding of what happens when negative numbers and fractions are raised to a power.

Raising a fraction between zero and one to a power produces a smaller result.

Example:

$$\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$$

Raising a negative number to an even power produces a positive result.

Example:

$(-2)^2 = 4$

Raising a negative number to an odd power gives a negative result.

Example:

$(-2)^3 = -8$

Raising an even number to any exponent gives an even number. Raising an odd number to any exponent gives an odd number.

Examples:

$8^5 = 32{,}768$, an even number

$5^8 = 390{,}625$, an odd number

Powers of 10

When 10 is raised to an exponent that is a positive integer, that exponent tells how many zeros the number would contain if it were written out.

Example:

Write 10^6 in ordinary notation.

The exponent 6 indicates that you will need six zeros after the 1: 1,000,000. That's because 10^6 means six factors of 10, that is, (10)(10)(10)(10)(10)(10).

To multiply a number by a power of 10, move the decimal point the same number of places to the right as the value of the exponent (or as the number of zeros in that power of 10).

Example:

Multiply 0.029 by 10^3

The exponent is 3, so move the decimal point three places to the right.

$$(0.029)10^3 = 0029. = 29$$

If you had been told to multiply 0.029 by 1,000, you could have counted the number of zeros in 1,000 and done exactly the same thing.

Sometimes you'll have to add zeros as placeholders.

Example:

Multiply 0.029 by 10^6.

Add zeros until you can move the decimal point six places to the right:

$$0.029 \times 10^6 = 0029000. = 29{,}000$$

To divide by a power of 10, move the decimal point the corresponding number of places to the left, inserting zeros as placeholders if necessary.

Example:

Divide 416.03 by 10,000

There are four zeros in 10,000, but only three places to the left of the decimal point. You'll have to insert another zero:

$$416.03 \div 10{,}000 = .041603 = 0.041603$$

By convention, one zero is usually written to the left of the decimal point on the GRE. It's a placeholder and doesn't change the value of the number.

Scientific Notation

Very large numbers (and very small decimals) take up a lot of space and are difficult to work with. So, in some scientific texts, they are expressed in a shorter, more convenient form called *scientific notation*.

For example, 123,000,000,000 would be written in scientific notation as 1.23×10^{11}, and 0.000000003 would be written as 3×10^{-9}. (If you're already familiar with the concept of negative exponents, you'll know that multiplying by 10^{-9} is equivalent to dividing by 10^9.)

To express a number in scientific notation, rewrite it as a product of two factors. The first factor must be greater than or equal to 1 but less than 10. The second factor must be a power of 10.

To translate a number from scientific notation to ordinary notation, use the rules for multiplying and dividing by powers of 10.

Example:

$5.6 \times 10^6 = 5{,}600{,}000$, or 5.6 million

Rules of Operations with Roots and Radicals

A *square root* of any non-negative number *x* is a number that, when multiplied by itself, yields *x*. Every positive number has two square roots, one positive and one negative. For instance, the positive square root of 25 is 5, because $5^2 = 25$. The negative square root of 25 is -5, because $(-5)^2$ also equals 25.

By convention, the radical symbol $\sqrt{\ }$ stands for the positive square root only. Therefore, $\sqrt{9} = 3$ only, even though both 3^2 and $(-3)^2$ equal 9.

When applying the four basic arithmetic operations, radicals (roots written with the radical symbol) are treated in much the same way as variables.

Addition and Subtraction of Radicals

Only like radicals can be added to or subtracted from one another.

Example:

$$2\sqrt{3} + 4\sqrt{2} - \sqrt{2} - 3\sqrt{3} =$$
$$(4\sqrt{2} - \sqrt{2}) + (2\sqrt{3} - 3\sqrt{3}) =$$
$$3\sqrt{2} + (-\sqrt{3}) =$$
$$3\sqrt{2} - \sqrt{3}$$

This expression cannot be simplified any further.

Multiplication and Division of Radicals

To multiply or divide one radical by another, multiply or divide the numbers outside the radical signs, then the numbers inside the radical signs.

Example:

$$(6\sqrt{3})2\sqrt{5} = (6)(2)(\sqrt{3})(\sqrt{5}) = 12\sqrt{15}$$

Example:

$$12\sqrt{15} \div 2\sqrt{5} = \left(\frac{12}{2}\right)\left(\frac{\sqrt{15}}{\sqrt{5}}\right) = 6\sqrt{\frac{15}{5}} = 6\sqrt{3}$$

Simplifying Radicals

If the number inside the radical is a multiple of a perfect square, the expression can be simplified by factoring out the perfect square.

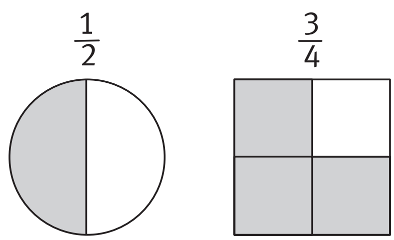
Example:

$$\sqrt{72} = (\sqrt{36})\sqrt{2} = 6\sqrt{2}$$

PROPORTIONS AND MATH FORMULAS

Fractions

The simplest way to understand the meaning of a fraction is to picture the denominator as the number of equal parts into which a whole unit is divided. The numerator represents a certain number of those equal parts.



On the left, the shaded portion is one of two equal parts that make up the whole. On the right, the shaded portion is three of four equal parts that make up the whole.

The fraction bar is interchangeable with a division sign. You can divide the numerator of a fraction by the denominator to get an equivalent decimal. However, the numerator and denominator must each be treated as a single quantity.

Example:

Evaluate $\frac{5 + 2}{7 - 3}$

You can't just rewrite the fraction as $5 + 2 \div 7 - 3$, because the numerator and the denominator are each considered distinct quantities. Instead, you would rewrite the fraction as $(5 + 2) \div (7 - 3)$. The order of operations (remember PEMDAS?) tells us that operations in parentheses must be performed first. That gives you $7 \div 4$. Your final answer would be $\frac{7}{4}$, $1\frac{3}{4}$, or 1.75, depending on the form of the answer choices.

Equivalent Fractions

Since multiplying or dividing a number by 1 does not change the number, multiplying the numerator and denominator of a fraction by the same nonzero number doesn't change the value of the fraction—it's the same as multiplying the entire fraction by 1.

Example:

Change $\frac{1}{2}$ into an equivalent fraction with a denominator of 4.

To change the denominator from 2 to 4, you'll have to multiply it by 2. But to keep the value of the fraction the same, you'll also have to multiply the numerator by 2.

$$\frac{1}{2} = \frac{1}{2}\left(\frac{2}{2}\right) = \frac{2}{4}$$

Similarly, dividing the numerator and denominator by the same nonzero number leaves the value of the fraction unchanged.

Example:

Change $\frac{16}{20}$ into an equivalent fraction with a denominator of 10.

To change the denominator from 20 to 10, you'll have to divide it by 2. But to keep the value of the fraction the same, you'll have to divide the numerator by the same number.

$$\frac{16}{20} = \frac{16 \div 2}{20 \div 2} = \frac{8}{10}$$

Reducing (Canceling)

Most fractions on the GRE are in lowest terms. That means that the numerator and denominator have no common factor greater than 1.

For example, the final answer of $\frac{8}{10}$ that we obtained in the previous example was not in lowest terms, because both 8 and 10 are divisible by 2. In contrast, the fraction $\frac{7}{10}$ is in lowest terms, because there is no factor greater than 1 that 7 and 10 have in common. To convert a fraction to its lowest terms, we use a method called reducing, or canceling. To reduce, simply divide any common factors out of both the numerator and the denominator.

Example:

Reduce $\frac{15}{35}$ to lowest terms.

$\frac{15}{35} = \frac{15 \div 5}{35 \div 5} = \frac{3}{7}$ (because a 5 cancels out, top and bottom)

The fastest way to reduce a fraction that has very large numbers in both the numerator and denominator is to find the greatest common factor and divide it out of both the top and the bottom.

Example:

Reduce $\frac{1{,}040}{1{,}080}$ to lowest terms.

$$\frac{1{,}040}{1{,}080} = \frac{104}{108} = \frac{52}{54} = \frac{26}{27}$$

Adding and Subtracting Fractions

You cannot add or subtract fractions unless they have the same denominator. If they don't, you'll have to convert each fraction to an equivalent fraction with the least common denominator. Then add or subtract the numerators (not the denominators!) and, if necessary, reduce the resulting fraction to its lowest terms.

Given two fractions with different denominators, the least common denominator is the least common multiple of the two denominators, that is, the smallest number that is evenly divisible by both denominators.

Example:

What is the least common denominator of $\frac{2}{15}$ and $\frac{3}{10}$?

The least common denominator of the two fractions will be the least common multiple of 15 and 10.

Because $15 = (5)(3)$ and $10 = (5)(2)$, the least common multiple of the two numbers is $(5)(3)(2)$, or 30. That makes 30 the least common denominator of $\frac{2}{15}$ and $\frac{3}{10}$.

Example:

$\frac{2}{15} + \frac{3}{10} = ?$

As we saw in the previous example, the least common denominator of the two fractions is 30. Change each fraction to an equivalent fraction with a denominator of 30.

$$\frac{2}{15}\left(\frac{2}{2}\right) = \frac{4}{30}$$

$$\frac{3}{10}\left(\frac{3}{3}\right) = \frac{9}{30}$$

Then add:

$$\frac{4}{30} + \frac{9}{30} = \frac{13}{30}$$

Since 13 and 30 have no common factor greater than 1, $\frac{13}{30}$ is in lowest terms. You can't reduce it further.

Multiplying Fractions

To multiply fractions, multiply the numerators and multiply the denominators.

$$\frac{5}{7}\left(\frac{3}{4}\right) = \frac{15}{28}$$

Multiplying numerator by numerator and denominator by denominator is simple. But it's easy to make careless errors if you have to multiply a string of fractions or work with large numbers. You can minimize those errors by reducing before you multiply.

Example:

Multiply $\left(\frac{10}{9}\right)\left(\frac{3}{4}\right)\left(\frac{8}{15}\right)$.

First, cancel a 5 out of the 10 and the 15, a 3 out of the 3 and the 9, and a 4 out of the 8 and the 4:

$$\left(\frac{\cancel{10}^{2}}{\cancel{9}_{3}}\right)\left(\frac{\cancel{3}^{1}}{\cancel{4}_{1}}\right)\left(\frac{\cancel{8}^{2}}{\cancel{15}_{3}}\right)$$

Then multiply numerators together and denominators together:

$$\left(\frac{2}{3}\right)\left(\frac{1}{1}\right)\left(\frac{2}{3}\right) = \frac{4}{9}$$

Reciprocals

To get the reciprocal of a common fraction, turn the fraction upside-down so that the numerator becomes the denominator, and vice versa. If a fraction has a numerator of 1, the fraction's reciprocal will be equivalent to an integer.

Example:

What is the reciprocal of $\frac{1}{25}$?

Inverting the fraction gives you the reciprocal, $\frac{25}{1}$. But dividing a number by 1 doesn't change the value of the number.

Since $\frac{25}{1}$ equals 25, the reciprocal of $\frac{1}{25}$ equals 25.

Dividing Common Fractions

To divide fractions, multiply by the reciprocal of the number or fraction that follows the division sign.

$$\frac{1}{2} \div \frac{3}{5} = \frac{1}{2}\left(\frac{5}{3}\right) = \frac{5}{6}$$

(The operation of division produces the same result as multiplication by the inverse.)

Example:

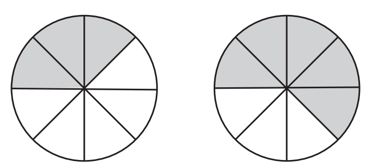
$$\frac{4}{3} \div \frac{4}{9} = \frac{4}{3}\left(\frac{9}{4}\right) = \frac{36}{12} = 3$$

Comparing Positive Fractions

Given two positive fractions with the same denominator, the fraction with the larger numerator will have the larger value.

Example:

Which is greater, $\frac{3}{8}$ or $\frac{5}{8}$?



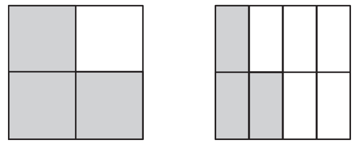
$$\frac{3}{8} < \frac{5}{8}$$

But if you're given two positive fractions with the same numerator but different denominators, the fraction with the smaller denominator will have the larger value.

Example:

Which is greater, $\frac{3}{4}$ or $\frac{3}{8}$?

The diagrams below show two wholes of equal size. The one on the left is divided into 4 equal parts, 3 of which are shaded. The one on the right is divided into 8 equal parts, 3 of which are shaded.



$\frac{3}{4}$ is clearly greater than $\frac{3}{8}$

If neither the numerators nor the denominators are the same, you have three options. You can turn both fractions into their decimal equivalents. Or you can express both fractions in terms of some common denominator and then see which new equivalent fraction has the largest numerator. Or you can cross multiply the numerator of each fraction by the denominator of the other. The greater result will wind up next to the greater fraction.

Example:

Which is greater, $\frac{5}{6}$ or $\frac{7}{9}$?

$$45\;\frac{5}{6} \times \frac{7}{9}\;42$$

Since $45 > 42$, $\frac{5}{6} > \frac{7}{9}$.

Mixed Numbers and Improper Fractions

A *mixed number* consists of an integer and a fraction.

An *improper fraction* is a fraction whose numerator is greater than its denominator. To convert an improper fraction to a mixed number, divide the numerator by the denominator. The number of "whole" times that the denominator goes into the numerator will be the integer portion of the improper fraction; the remainder will be the numerator of the fractional portion.

Example:

Convert $\frac{23}{4}$ to a mixed number.

Dividing 23 by 4 gives you 5 with a remainder of 3, so $\frac{23}{4} = 5\frac{3}{4}$.

To change a mixed number to a fraction, multiply the integer portion of the mixed number by the denominator and add the numerator. This new number is your numerator. The denominator will not change.

Example:

Convert $2\frac{3}{7}$ to a fraction.

$$2\frac{3}{7} = \frac{7(2) + 3}{7} = \frac{17}{7}$$

Properties of Fractions Between –1 and +1

The reciprocal of a fraction between 0 and 1 is greater than both the original fraction and 1.

Example:

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, which is greater than both 1 and $\frac{2}{3}$.

The reciprocal of a fraction between -1 and 0 is less than both the original fraction and -1.

Example:

The reciprocal of $-\frac{2}{3}$ is $-\frac{3}{2}$, or $-1\frac{1}{2}$, which is less than both -1 and $-\frac{2}{3}$.

The square of a fraction between 0 and 1 is less than the original fraction.

Example:

$$\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$$

But the square of any fraction between 0 and -1 is greater than the original fraction, because multiplying two negative numbers gives you a positive product and any positive number is greater than any negative number.

Example:

$$\left(-\frac{1}{2}\right)^2 = \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) = \frac{1}{4}$$

Multiplying any positive number by a fraction between 0 and 1 gives a product smaller than the original number.

Example:

$$6\left(\frac{1}{4}\right) = \frac{6}{4} = \frac{3}{2}$$

Multiplying any negative number by a fraction between 0 and 1 gives a product greater than the original number.

Example:

$$(-3)\left(\frac{1}{2}\right) = -\frac{3}{2}$$

Decimals

Converting Decimals

It's easy to convert decimals to common fractions, and vice versa. Any decimal fraction is equivalent to some common fraction with a power of 10 in the denominator.

To convert a decimal between 0 and 1 to a fraction, determine the place value of the last nonzero digit and set that value as the denominator. Then use all the digits of the decimal number as the numerator, ignoring the decimal point. Finally, if necessary, reduce the fraction to its lowest terms.

Example:

Convert 0.875 to a fraction in lowest terms.

The last nonzero digit is the 5, which is in the thousandths place. So the denominator of the common fraction will be 1,000. The numerator will be 875: $\frac{875}{1{,}000}$.

(You can ignore the zero to the left of the decimal point, since there are no nonzero digits to its left; it's just a "placeholder.")

Both 875 and 1,000 contain a factor of 25. Canceling it out leaves you with $\frac{35}{40}$.

Reducing that further by a factor of 5 gives you $\frac{7}{8}$, which is in lowest terms.

To convert a fraction to a decimal, simply divide the numerator by the denominator.

Example:

What is the decimal equivalent of $\frac{4}{5}$?

$$4 \div 5 = 0.8$$

Comparing Decimals

Knowing place values allows you to assess the relative values of decimals.

Example:

Which is greater, 0.254 or 0.3?

Of course, 254 is greater than 3. But $0.3 = \frac{3}{10}$, which is equivalent to $\frac{300}{1{,}000}$, while 0.254 is equivalent to only $\frac{254}{1{,}000}$. Since $\frac{300}{1{,}000} > \frac{254}{1{,}000}$, 0.3 is greater than 0.254.

Here's the simplest way to compare decimals: add zeros after the last digit to the right of the decimal point in each decimal fraction until all the decimals you're comparing have the same number of digits. Essentially, what you're doing is giving all the fractions the same denominator so that you can just compare their numerators.

Example:

Arrange in order from smallest to largest: 0.7, 0.77, 0.07, 0.707, and 0.077.

The numbers 0.707 and 0.077 end at the third place to the right of the decimal point—the thousandths place. Add zeros after the last digit to the right of the decimal point in each of the other fractions until you reach the thousandths place:

$$0.7 = 0.700 = \frac{700}{1{,}000}$$

$$0.77 = 0.770 = \frac{770}{1{,}000}$$

$$0.07 = 0.070 = \frac{70}{1{,}000}$$

$$0.707 = \frac{707}{1{,}000}$$

$$0.077 = \frac{77}{1{,}000}$$

$$\frac{70}{1{,}000} < \frac{77}{1{,}000} < \frac{700}{1{,}000} < \frac{707}{1{,}000} < \frac{770}{1{,}000}$$

Therefore, $0.07 < 0.077 < 0.7 < 0.707 < 0.77$.

Estimation and Rounding on the GRE

You should be familiar and comfortable with the practice of "rounding off" numbers. To round off a number to a particular place, look at the digit immediately to the right of that place. If the digit is 0, 1, 2, 3, or 4, don't change the digit that is in the place to which you are rounding. If it is 5, 6, 7, 8, or 9, change the digit in the place to which you are rounding to the next higher digit. Replace all digits to the right of the place to which you are rounding with zeros.

For example, to round off 235 to the tens place, look at the units place. Since it is occupied by a 5, you'll round the 3 in the tens place up to a 4, giving you 240. If you had been rounding off 234, you would have rounded down to the existing 3 in the tens place; that would have given you 230.

Example:

Round off 675,978 to the hundreds place.

The 7 in the tens place means that you will have to round the hundreds place up. Since there is a 9 in the hundreds place, you'll have to change the thousands place as well. Rounding 675,978 to the hundreds place gives you 676,000.

Rounding off large numbers before calculation will allow you to quickly estimate the correct answer.

Estimating can save you valuable time on many GRE problems. But before you estimate, check the answer choices to see how close they are. If they are relatively close together, you'll have to be more accurate than if they are farther apart.

Percents

The word *percent* means "hundredths," and the percent sign, %, means $\frac{1}{100}$. For example, 25% means $25\left(\frac{1}{100}\right) = \frac{25}{100}$. (Like the division sign, the percent sign evolved from the fractional relationship; the slanted bar in a percent sign represents a fraction bar.)

Percents measure a part-to-whole relationship with an assumed whole equal to 100. The percent relationship can be expressed as $\frac{\text{Part}}{\text{Whole}}(100\%)$. For example, if $\frac{1}{4}$ of a rectangle is shaded, the percent of the rectangle that is shaded is $\frac{1}{4}(100\%) = 25\%$.

Like fractions, percents express the relationship between a specified part and a whole; more specifically, percents express a relationship of a part out of 100. Thus, 25%, $\frac{25}{100}$, and 0.25 are simply different names for the same part-whole relationship.

Translating English to Math in Part-Whole Problems

On the GRE, many fractions and percents appear in word problems. You'll solve the problems by plugging the numbers you're given into some variation of one of the three basic formulas:

$$\frac{\text{Part}}{\text{Whole}} = \textit{Fraction}$$
$$\frac{\text{Part}}{\text{Whole}} = \textit{Decimal}$$
$$\frac{\text{Part}}{\text{Whole}}(100) = \textit{Percent}$$

To avoid careless errors, look for the key words *is* and *of*. *Is* (or *are*) often introduces the part, while *of* almost invariably introduces the whole.

Properties of 100%

Since the percent sign means $\frac{1}{100}$, 100% means $\frac{100}{100}$, or one whole. The key to solving some GRE percent problems is to recognize that all the parts add up to one whole: 100%.

Example:

All 1,000 registered voters in Smithtown are Democrats, Republicans, or independents. If 75% of the registered voters are Democrats and 5% are independents, how many are Republicans?

We calculate that 75% + 5%, or 80% of the 1,000 registered voters, are either Democrats or independents. The three political affiliations together must account for 100% of the voters; thus, the percentage of Republicans must be 100% − 80%, or 20%. Therefore, the number of Republicans must be 20% of 1,000, which is 20% (1,000), or 200.

Multiplying or dividing a number by 100% is just like multiplying or dividing by 1; it doesn't change the value of the original number.

Converting Percents

To change a fraction to its percent equivalent, multiply by 100%.

Example:

What is the percent equivalent of $\frac{5}{8}$?

$$\frac{5}{8}(100\%) = \frac{500}{8}\% = 62\frac{1}{2}\%$$

To change a decimal fraction to a percent, you can use the rules for multiplying by powers of 10. Move the decimal point two places to the right and insert a percent sign.

Example:

What is the percent equivalent of 0.17?

$$0.17 = 0.17\ (100\%) = 17\%$$

To change a percent to its fractional equivalent, divide by 100%.

Example:

What is the common fraction equivalent of 32%?

$$32\% = \frac{32\%}{100\%} = \frac{8}{25}$$

To convert a percent to its decimal equivalent, use the rules for dividing by powers of 10—just move the decimal point two places to the left.

Example:

What is the decimal equivalent of 32%?

$$32\% = \frac{32\%}{100\%} = \frac{32}{100} = 0.32$$

When you divide a percent by another percent, the percent sign "drops out," just as you would cancel out a common factor.

Example:

$$\frac{100\%}{5\%} = \frac{100}{5} = 20$$

Translation: There are 20 groups of 5% in 100%.

But when you divide a percent by a regular number (not by another percent), the percent sign remains.

Example:

$$\frac{100\%}{5} = 20\%$$

Translation: One-fifth of 100% is 20%.

Common Percent Equivalents

As you can see, changing percents to fractions, or vice versa, is pretty straightforward. But it does take a second or two that you might spend more profitably doing other computations or setting up another GRE math problem. Familiarity with the following common equivalents will save you time.

$\frac{1}{20} = 5\%$

$\frac{1}{12} = 8\frac{1}{3}\%$

$\frac{1}{10} = 10\%$

$\frac{1}{8} = 12\frac{1}{2}\%$

$\frac{1}{6} = 16\frac{2}{3}\%$

$\frac{1}{5} = 20\%$

$\frac{1}{4} = 25\%$

$\frac{3}{10} = 30\%$

$\frac{1}{3} = 33\frac{1}{3}\%$

$\frac{3}{8} = 37\frac{1}{2}\%$

$\frac{2}{5} = 40\%$

$\frac{1}{2} = 50\%$

$\frac{3}{5} = 60\%$

$\frac{5}{8} = 62\frac{1}{2}\%$

$\frac{2}{3} = 66\frac{2}{3}\%$

$\frac{7}{10} = 70\%$

$\frac{3}{4} = 75\%$

$\frac{4}{5} = 80\%$

$\frac{5}{6} = 83\frac{1}{3}\%$

$\frac{7}{8} = 87\frac{1}{2}\%$

$\frac{9}{10} = 90\%$

$\frac{11}{12} = 91\frac{2}{3}\%$

Using the Percent Formula to Solve Percent Problems

You can solve most percent problems by plugging the given data into the percent formula:

$$\frac{\text{Part}}{\text{Whole}}(100\%) = \text{Percent}$$

Most percent problems give you two of the three variables and ask for the third.

Example:

Ben spends \$30 of his annual gardening budget on seed. If his total annual gardening budget is \$150, what percentage of his budget does he spend on seed?

This problem specifies the whole (\$150) and the part (\$30) and asks for the percentage. Plugging those numbers into the percent formula gives you this:

$$\text{Percent} = \frac{30}{150}(100\%) = \frac{1}{5}(100\%) = 20\%$$

Ben spends 20% of his annual gardening budget on seed.

Percent Increase and Decrease

When the GRE tests percent increase or decrease, use the formula:

$$\text{Percent increase} = \frac{\text{Increase }(100\%)}{\text{Original}}$$

or

$$\text{Percent decrease} = \frac{\text{Decrease }(100\%)}{\text{Original}}$$

To find the increase or decrease, just take the difference between the original and the new. Note that the “original” is the base from which change occurs. It may or may not be the first number mentioned in the problem.

Example:

Two years ago, 450 seniors graduated from Inman High School. Last year, 600 seniors graduated. By what percentage did the number of graduating seniors increase?

The original is the figure from the earlier time (two years ago): 450. The increase is $600 - 450$, or 150. So the percentage increase is $\frac{150}{450}(100\%) = 33\frac{1}{3}\%$.

Example:

If the price of a \$120 dress is increased by 25%, what is the new selling price?

To find the new whole, you'll first have to find the amount of increase. The original whole is \$120, and the percent increase is 25%. Plugging in, we find that:

$$\begin{aligned}
\frac{\text{Increase}}{120}(100\%) &= 25\% \\
\frac{\text{Increase}}{120} &= \frac{25}{100} \\
\frac{\text{Increase}}{120} &= \frac{1}{4} \\
\text{Increase} &= \frac{120}{4} \\
\text{Increase} &= 30
\end{aligned}$$

The amount of increase is \$30, so the new selling price is \$120 + \$30, or \$150.

Multi-Step Percent Problems

On some difficult problems, you'll be asked to find more than one percent or to find a percent of a percent. Be careful: You can't add percents of different wholes.

Example:

The price of an antique is reduced by 20 percent, and then this price is reduced by 10 percent. If the antique originally cost \$200, what is its final price?

The most common mistake in this kind of problem is to reduce the original price by a total of 20% + 10%, or 30%. That would make the final price 70 percent of the original, or 70% (\$200) = \$140. This is not the correct answer. In this example, the second (10%) price reduction is taken off of the first sale price—the new whole, not the original whole.

To get the correct answer, first find the new whole. You can find it by calculating either \$200 − (20% of \$200) or 80% (\$200). Either way, you will find that the first sale price is \$160. That price then has to be reduced by 10%. Either calculate \$160 − (10% (\$160)) or 90% (\$160). In either case, the final price of the antique is \$144.

Picking Numbers with Percents

Certain types of percent problems lend themselves readily to the alternative technique of Picking Numbers. These include problems in which no actual values are mentioned, just percents. If you assign values to the percents you are working with, you'll find the problem less abstract.

You should almost always pick 100 in percent problems, because it's relatively easy to find percentages of 100.

Example:

The price of a share of company A's stock fell by 20 percent two weeks ago and by another 25 percent last week to its current price. By what percent of the current price does the share price need to rise in order to return to its original price?

(A) 45%
(B) 55%
(C) $66\frac{2}{3}\%$
(D) 75%
(E) 82%

Pick a value for the original price of the stock. Since this is a percent question, picking \$100 will make the math easy. The first change in the price of the stock was by 20% of \$100, or \$20, making the new price $\$100 - \$20 = \$80$.

The price then fell by another 25%. You know that 25% is the same as $\frac{1}{4}$, and $\frac{1}{4}$ of \$80 is \$20. Therefore, the current price is $\$80 - \$20 = \$60$. To return to its original price, the stock needs to rise from \$60 to \$100, that is, by $\$100 - \$60 = \$40$. Then \$40 is what percent of the current price, \$60?

$$\frac{40}{60}(100\%) = \frac{2}{3}(100\%) = 66\frac{2}{3}\%$$

Percent Word Problems

Percent problems are often presented as word problems. We have already seen how to identify the percent, the part, and the whole in simple percent word problems. Here are some other terms that you are likely to encounter in more complicated percent word problems:

Profit made on an item is the seller's price minus the cost to the seller. If a seller buys an item for \$10 and sells it for \$12, he has made \$2 profit. The percent of the selling price that is profit is as follows:

$$\frac{\text{Profit}}{\text{Original selling price}}(100\%) = \frac{\$2}{\$12}(100\%) = 16\frac{2}{3}\%$$

A *discount* on an item is the original price minus the reduced price. If an item that usually sells for \$20 is sold for \$15, the discount is \$5. A discount is often represented as a percentage of the original price. In this case, the

$$\text{Percentage discount} = \frac{\text{Discount}}{\text{Original price}}(100\%) = \frac{\$5}{\$20} = 25\%$$

The *sale price* is the final price after discount or decrease.

Occasionally, percent problems will involve *interest*. Interest is given as a percent per unit of time, such as 5% per month. The sum of money invested is the *principal*. The most common type of interest you will see is *simple interest*. In simple interest, the interest payments received are kept separate from the principal.

Example:

If an investor invests \$100 at 20% simple annual interest, how much does she have at the end of three years?

The principal of \$100 yields 20% interest every year. Because 20% of \$100 is \$20, after three years the investor will have three years of interest, or \$60, plus the principal, for a total of \$160.

In *compound interest*, the money earned as interest is reinvested. The principal grows after every interest payment received.

Example:

If an investor invests \$100 at 20% compounded annually, how much does he have at the end of 3 years?

The first year the investor earns 20% of $\$100 = \$20$. So, after one year, he has $\$100 + \$20 = \$120$.

The second year the investor earns 20% of $\$120 = \$24$. So, after two years, he has $\$120 + \$24 = \$144$.

The third year the investor earns 20% of $\$144 = \$28.80$. So, after three years, he has $\$144 + \$28.80 = \$172.80$.

Ratios

A *ratio* is the proportional relationship between two quantities. The ratio, or relationship, between two numbers (for example, 2 and 3) may be expressed with a colon between the two numbers (2:3), in words ("the ratio of 2 to 3"), or as a fraction $\frac{2}{3}$.

To translate a ratio in words to numbers separated by a colon, replace *to* with a colon.

To translate a ratio in words to a fractional ratio, use whatever follows the word *of* as the numerator and whatever follows the word *to* as the denominator. For example, if we had to express the ratio *of* glazed doughnuts *to* chocolate doughnuts in a box of doughnuts that contained 5 glazed and 7 chocolate doughnuts, we would do so as $\frac{5}{7}$.

Note that the fraction $\frac{5}{7}$ does not mean that $\frac{5}{7}$ of all the doughnuts are glazed doughnuts. There are $5 + 7$, or 12 doughnuts altogether, so of the doughnuts, $\frac{5}{12}$ are glazed. The $\frac{5}{7}$ ratio merely indicates the proportion of glazed to chocolate doughnuts. For every five glazed doughnuts, there are seven chocolate doughnuts.

Treating ratios as fractions can make computation easier. Like fractions, ratios often require division. And, like fractions, ratios ultimately should be reduced to lowest terms.

Example:

Joe is 16 years old, and Mary is 12 years old. Express the ratio of Joe's age to Mary's age in lowest terms.

The ratio of Joe's age to Mary's age is $\frac{16}{12} = \frac{4}{3}$, or 4:3.

Part:Whole Ratios

In a part:whole ratio, the "whole" is the entire set (for instance, all the workers in a factory), while the "part" is a certain subset of the whole (for instance, all the female workers in the factory).

In GRE ratio question stems, the word *fraction* generally indicates a part:whole ratio. "What fraction of the workers are female?" means "What is the ratio of the number of female workers to the total number of workers?"

Example:

The sophomore class at Milford Academy consists of 15 boys and 20 girls. What fraction of the sophomore class is female?

The following three statements are equivalent:

1. $\frac{4}{7}$ of the sophomores are female.
2. Four out of every seven sophomores are female.
3. The ratio of female sophomores to total sophomores is 4:7.

Ratio vs. Actual Number

Ratios are usually reduced to their simplest form (that is, to lowest terms). If the ratio of men to women in a room is 5:3, you cannot necessarily infer that there are exactly five men and three women.

If you knew the total number of people in the room, in addition to the male-to-female ratio, you could determine the number of men and the number of women in the room. For example, suppose you know that there are 32 people in the room. If the male-to-female ratio is 5 to 3, then the ratio of males to the total is 5:$(5 + 3)$, which is 5:8. You can set up an equation as $\frac{5}{8} = \frac{\text{\# of males in room}}{32}$. Solving, you will find that the number of males in the room is 20.

Example:

The ratio of domestic sales revenues to foreign sales revenues of a certain product is 3:5. What fraction of the total sales revenues comes from domestic sales?

At first, this question may look more complicated than the previous example. You have to convert from a part:part ratio to a part:whole ratio (the ratio of domestic sales revenues to total sales revenues). And you're not given actual dollar figures for domestic or foreign sales. But since all sales are either foreign or domestic, "total sales revenues" must be the sum of the revenues from domestic and foreign sales. You can convert the given ratio to a part:whole ratio because the sum of the parts equals the whole.

Although it's impossible to determine dollar amounts for the domestic, foreign, or total sales revenues from the given information, the 3:5 ratio tells you that of every \$8 in sales revenues, \$3 comes from domestic sales and \$5 from foreign sales. Therefore, the ratio of domestic sales revenues to total sales revenues is 3:8, or $\frac{3}{8}$.

You can convert a part:part ratio to a part:whole ratio (or vice versa) only if there are no missing parts and no overlap among the parts—that is, if the whole is equal to the sum of the parts.

Example:

In a certain bag, the ratio of the number of red marbles to the number of blue marbles is 3:5. If there are only red and blue marbles in the bag, what is the ratio of the number of red marbles to the total number of marbles?

In this case, you can convert a part-to-part ratio (red marbles to blue marbles) to a part-to-whole ratio (red marbles to all marbles) because you know there are only red and blue marbles in the bag. The ratio of red marbles to the total number of marbles is 3:8.

Example:

Of the 25 people in Fran's apartment building, there are 9 residents who use the roof only for tanning and 8 residents who use the roof only for gardening. The roof is only used by tanners and gardeners.

Quantity A	Quantity B
The ratio of people who use the roof to total residents	17:25

In this question, we do not know if there is any overlap between tanners and gardeners. How many, if any, residents do both activities? Since we don't know, the relationship cannot be determined from the information given.

Ratios of More Than Two Terms

Most of the ratios that you'll see on the GRE have two terms. But it is possible to set up ratios with more than two terms. These ratios express more relationships, and therefore convey more information, than do two-term ratios. However, most of the principles discussed so far with respect to two-term ratios are just as applicable to ratios of more than two terms.

Example:

The ratio of x to y is 5:4. The ratio of y to z is 1:2. What is the ratio of x to z?

We want the y's in the two ratios to equal each other, because then we can combine the x:y ratio and the y:z ratio to form the x:y:z ratio that we need to answer this question. To make the y's equal, we can multiply the second ratio by 4. When we do so, we must perform the multiplication on both components of the ratio. Since a ratio is a constant proportion, it can be multiplied or divided by any number without losing its meaning, as long as the multiplication and division are applied to all the components of the ratio. In this case, we find that the new ratio for y to z is 4:8. We can combine this with the first ratio to find a new x to y to z ratio of 5:4:8. Therefore, the ratio of x to z is 5:8.

Rates

A *rate* is a special type of ratio. Instead of relating a part to the whole or to another part, a rate relates one kind of quantity to a completely different kind. When we talk about rates, we usually use the word *per*, as in "miles per hour," "cost per item," etc. Since *per* means "for one" or "for each," we express the rates as ratios reduced to a denominator of 1.

Speed

The most commonly tested rate on the GRE is speed. This is usually expressed in miles or kilometers per hour. The relationship between speed, distance, and time is given by the formula $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$, which can be rewritten two ways: $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$ and Distance = (Speed)(Time).

Anytime you can find two out of the three elements in this equation, you can find the third.

For example, if a car travels 300 miles in 5 hours, it has averaged $\frac{300\text{ miles}}{5\text{ hours}} = 60\text{ miles per hour}$. (Note that speeds are usually expressed as averages because they are not necessarily constant. In this example, the car moved at an "average speed" of 60 miles per hour, but probably not at a constant speed of 60 miles per hour.)

Likewise, a rearranged version of the formula can be used to solve for missing speed or time.

Example:

How far do you drive if you travel for 5 hours at 60 miles per hour?

$$\begin{aligned}\text{Distance} &= (\text{Speed})(\text{Time})\\ \text{Distance} &= (60\text{ mph})(5\text{ hours})\\ \text{Distance} &= 300\text{ miles}\end{aligned}$$

Example:

How much time does it take to drive 300 miles at 60 miles per hour?

$$\begin{aligned}\text{Time} &= \frac{\text{Distance}}{\text{Speed}}\\ \text{Time} &= \frac{300\text{ miles}}{60\text{ mph}}\\ \text{Time} &= 5\text{ hours}\end{aligned}$$

Other Rates

Speed is not the only rate that appears on the GRE. For instance, you might get a word problem involving liters per minute or cost per unit. All rate problems, however, can be solved using the speed formula and its variants by conceiving of "speed" as "rate" and "distance" as "quantity."

Example:

How many hours will it take to fill a 500-liter tank at a rate of 2 liters per minute?

Plug the numbers into our rate formula:

$$\begin{aligned}\text{Time} &= \frac{\text{Quantity}}{\text{Rate}}\\ \text{Time} &= \frac{500\,\text{liters}}{2\,\text{liters per minute}}\\ \text{Time} &= 250\,\text{minutes}\end{aligned}$$

Now convert 250 minutes to hours: 250 minutes $\div$ 60 minutes per hour $=$ $4\frac{1}{6}$ hours to fill the tank. (As you can see from this problem, GRE Problem Solving questions test your ability to convert minutes into hours and vice versa. Pay close attention to what units the answer choice must use.)

In some cases, you should use proportions to answer rate questions.

Example:

If 350 widgets cost \$20, how much will 1,400 widgets cost at the same rate?

Set up a proportion:

$$\frac{\text{Number of widgets}}{\text{Cost}} = \frac{350\,\text{widgets}}{\$20} = \frac{1{,}400\,\text{widgets}}{\$x}$$

Solving, you will find that $x = 80$.

So, 1,400 widgets will cost \$80 at that rate.

Combined Rate Problems

Rates can be added.

Example:

Nelson can mow 200 square meters of lawn per hour. John can mow 100 square meters of lawn per hour. Working simultaneously but independently, how many hours will it take Nelson and John to mow 1,800 square meters of lawn?

Add Nelson's rate to John's rate to find the combined rate.

200 meters per hour $+$ 100 meters per hour $=$ 300 meters per hour.

Divide the total lawn area, 1,800 square meters, by the combined rate, 300 square meters per hour, to find the number of required hours, 6.

Work Problems (Given Hours per Unit of Work)

The work formula can be used to find out how long it takes a number of people working together to complete a task. Let's say we have three people. The first takes a units of time to complete the job, the second b units of time to complete the job, and the third c units of time. If the time it takes all three working together to complete the job is T, then $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{T}$.

Example:

John can weed the garden in 3 hours. If Mary can weed the garden in 2 hours, how long will it take them to weed the garden at this rate, working independently?

Set John's time per unit of work as a and Mary's time per unit of work as b. (There is no need for the variable c, since there are only two people.) Plugging in, you find that

$$\begin{aligned} \frac{1}{3} + \frac{1}{2} &= \frac{1}{T} \\ \frac{2}{6} + \frac{3}{6} &= \frac{1}{T} \\ \frac{5}{6} &= \frac{1}{T} \\ T &= \frac{6}{5} \text{ hours} \end{aligned}$$

Work Formula for Two

When there are only two people or machines in a combined work problem, we can use a simplified work formula.

$$\begin{aligned} \frac{1}{a} + \frac{1}{b} &= \frac{1}{T} \\ (ab)\left(\frac{1}{a} + \frac{1}{b}\right) &= \left(\frac{1}{T}\right)(ab) \\ \frac{ab}{a} + \frac{ab}{b} &= \frac{ab}{T} \\ b + a &= \frac{ab}{T} \\ T(b + a) &= \left(\frac{ab}{T}\right)T \\ T(b + a) &= ab \\ T &= \frac{ab}{a + b} \end{aligned}$$

Here, a = the amount of time it takes person a to complete the job, and b = the amount of time it takes person b to complete the job.

Example:

Let's use the same example from above: John takes 3 hours to weed the garden, and Mary takes 2 hours to weed the same garden. How long will it take them to weed the garden together?

$$\text{Work formula} = \frac{a \times b}{a + b} = \frac{3 \times 2}{3 + 2} = \frac{6}{5}\text{ hours}$$

AVERAGES

The *average* of a group of numbers is defined as the sum of the terms divided by the number of terms.

$$\text{Average} = \frac{\text{Sum of terms}}{\text{Number of terms}}$$

This equation can be rewritten two ways:

$$\begin{aligned}\text{Number of terms} &= \frac{\text{Sum of terms}}{\text{Average}}\\ \text{Sum of terms} &= (\text{Number of terms})(\text{Average})\end{aligned}$$

Thus, any time you have two out of the three values (average, sum of terms, number of terms), you can find the third.

Example:

Henry buys three items costing \$2.00, \$1.75, and \$1.05. What is the average price (arithmetic mean) of the three items? (Don't let the phrase *arithmetic mean* throw you; it's just another term for *average*.)

$$\begin{aligned}\text{Average} &= \frac{\text{Sum of terms}}{\text{Number of terms}}\\ \text{Average} &= \frac{\$2.0 + \$1.75 + \$1.05}{3}\\ \text{Average} &= \frac{\$4.80}{3}\\ \text{Average} &= \$1.60\end{aligned}$$

Example:

June pays an average price of \$14.50 for 6 articles of clothing. What is the total price of all 6 articles?

$$\begin{aligned}\text{Sum of terms} &= (\text{Average})\ (\text{Number of terms})\\ \text{Sum of terms} &= (\$14.50)\ (6)\\ \text{Sum of terms} &= \$87.00\end{aligned}$$

Example:

The total weight of the licorice sticks in a jar is 30 ounces. If the average weight of each licorice stick is 2 ounces, how many licorice sticks are there in the jar?

$$\begin{aligned}\text{Number of terms} &= \frac{\text{Sum of terms}}{\text{Average}}\\ \text{Number of terms} &= \frac{30\ \text{ounces}}{2\ \text{ounces}}\\ \text{Number of terms} &= 15\end{aligned}$$

Using the Average to Find a Missing Number

If you're given the average, the total number of terms, and all but one of the actual numbers, you can find the missing number.

Example:

The average annual rainfall in Boynton for 1976–1979 was 26 inches per year. Boynton received 24 inches of rain in 1976, 30 inches in 1977, and 19 inches in 1978. How many inches of rainfall did Boynton receive in 1979?

You know that total rainfall equals $24 + 30 + 19 +$ (number of inches of rain in 1979).

You know that the average rainfall was 26 inches per year.

You know that there were 4 years.

So, plug these numbers into any of the three expressions of the average formula to find that Sum of terms = (Average)(Number of terms):

$$\begin{aligned} 24 + 30 + 19 + \text{inches in 1979} &= (26)(4) \\ 73 + \text{inches in 1979} &= (26)(4) \\ 73 + \text{inches in 1979} &= 104 \\ \text{inches in 1979} &= 31 \end{aligned}$$

Another Way to Find a Missing Number: The Concept of "Balanced Value"

Another way to find a missing number is to understand that the *sum of the differences between each term and the mean of the set must equal zero*. Plugging in the numbers from the previous problem, for example, we find that:

$$\begin{aligned} (24 - 26) + (30 - 26) + (19 - 26) + (\text{inches in 1979} - 26) &= 0 \\ (-2) + (4) + (-7) + (\text{inches in 1979} - 26) &= 0 \\ -5 + (\text{inches in 1979} - 26) &= 0 \\ \text{inches in 1979} &= 31 \end{aligned}$$

It may be easier to comprehend why this is true by visualizing a balancing, or weighting, process. The combined distance of the numbers above the average from the mean must be balanced with the combined distance of the numbers below the average from the mean.

Example:

The average of 63, 64, 85, and x is 80. What is the value of x?

Think of each value in terms of its position relative to the average, 80.

63 is 17 less than 80.

64 is 16 less than 80.

85 is 5 greater than 80.

So these three terms are a total of $17 + 16 - 5$, or 28, less than the average. Therefore, x must be 28 greater than the average to restore the balance at 80. So $x = 28 + 80 = 108$.

Average of Consecutive, Evenly Spaced Numbers

When consecutive numbers are evenly spaced, the average is the middle value. For example, the average of consecutive integers 6, 7, and 8 is 7.

If there is an even number of evenly spaced numbers, there is no single middle value. In that case, the average is midway between (that is, the average of) the middle two values. For example, the average of 5, 10, 15, and 20 is 12.5, midway between the middle values 10 and 15.

Note that not all consecutive numbers are evenly spaced. For instance, consecutive prime numbers arranged in increasing order are not evenly spaced. But you can use the handy technique of finding the middle value whenever you have consecutive integers, consecutive odd or even numbers, consecutive multiples of an integer, or any other consecutive numbers that are evenly spaced.

Combining Averages

When there is an equal number of terms in each set, and *only when there is an equal number of terms in each set*, you can average averages.

For example, suppose there are two bowlers and you must find their average score per game. One has an average score per game of 100, and the other has an average score per game of 200. If both bowlers bowled the same number of games, you can average their averages to find their combined average. Suppose they both bowled 4 games. Their combined average will be equally influenced by both bowlers. Hence, their combined average will be the average of 100 and 200. You can find this quickly by remembering that the quantity above the average and the quantity below the average must be equal. Therefore, the average will be halfway between 100 and 200, which is 150. Or, we could solve using our average formula:

$$\text{Average} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{4(100) + 4(200)}{8} = 150$$

However, if the bowler with the average score of 100 had bowled 4 games and the bowler with the 200 average had bowled 16 games, the combined average would be weighted further toward 200 than toward 100 to reflect the greater influence of the 200 bowler than the 100 bowler upon the total. This is known as a *weighted average*.

Again, you can solve this by using the concept of a balanced average or by using the average formula.

Since the bowler bowling an average score of 200 bowled $\frac{4}{5}$ of the games, the combined average will be $\frac{4}{5}$ of the distance along the number line between 100 and 200, which is 180. Or, you can plug numbers into an average formula to find the following:

$$\begin{aligned}
\text{Average} &= \frac{\text{Sum of terms}}{\text{Number of terms}} \\
\text{Average} &= \frac{4(100) + 16(200)}{20} \\
\text{Average} &= \frac{400 + 3{,}200}{20} \\
\text{Average} &= 180
\end{aligned}$$

ALGEBRA

Algebraic Terms

Variable: A letter or symbol representing an unknown quantity.

Constant (term): A number not multiplied by any variable(s).

Term: A numerical constant; also, the product of a numerical constant and one or more variables.

Coefficient: The numerical constant by which one or more variables are multiplied. The coefficient of $3x^2$ is 3. A variable (or product of variables) without a numerical coefficient, such as z or xy^3, is understood to have a coefficient of 1.

Algebraic expression: An expression containing one or more variables, one or more constants, and possibly one or more operation symbols. In the case of the expression x, there is an implied coefficient of 1. An expression does not contain an equal sign. x, $3x^2 + 2x$, and $\dfrac{7x + 1}{3x^2 - 14}$ are all algebraic expressions.

Monomial: An algebraic expression with only one term. To *multiply monomials*, multiply the coefficients and the variables separately: $2a \times 3a = (2 \times 3)(a \times a) = 6a^2$.

Polynomial: The general name for an algebraic expression with more than one term. An algebraic expression with two terms is called a **binomial**.

Algebraic equation: Two algebraic expressions separated by an equal sign or one algebraic expression separated from a number by an equal sign.

Basic Operations

Combining Like Terms

The process of simplifying an expression by adding together or subtracting terms that have the same variable factors is called *combining like terms*.

Example:

Simplify the expression $2x - 5y - x + 7y$.

$2x - 5y - x + 7y = (2x - x) + (7y - 5y) = x + 2y$

Notice that the commutative, associative, and distributive laws that govern arithmetic operations with ordinary numbers also apply to algebraic terms and polynomials.

Adding and Subtracting Polynomials

To *add or subtract polynomials*, combine like terms.

$$(3x^2 + 5x + 7) - (x^2 + 12) = (3x^2 - x^2) + 5x + (7 - 12) = 2x^2 + 5x - 5$$

Factoring Algebraic Expressions

Factoring a polynomial means expressing it as a product of two or more simpler expressions. Common factors can be factored out by using the distributive law.

Example:

Factor the expression $2a + 6ac$.

The greatest common factor of $2a + 6ac$ is $2a$. Using the distributive law, you can factor out $2a$ so that the expression becomes $2a(1 + 3c)$.

Example:

All three terms in the polynomial $3x^3 + 12x^2 - 6x$ contain a factor of $3x$. Pulling out the common factor yields $3x(x^2 + 4x - 2)$.

Advanced Operations

Substitution

Substitution, a process of plugging values into equations, is used to evaluate an algebraic expression or to express it in terms of other variables.

Replace every variable in the expression with the number or quantity you are told is its equivalent. Then carry out the designated operations, remembering to follow the order of operations (PEMDAS).

Example:

Express $\frac{a - b^2}{b - a}$ in terms of x if $a = 2x$ and $b = 3$.

Replace every a with $2x$ and every b with 3:

$$\frac{a - b^2}{b - a} = \frac{2x - 9}{3 - 2x}$$

Without more information, you can't simplify or evaluate this expression further.

Solving Equations

When you manipulate any equation, *always do the same thing on both sides of the equal sign*. Otherwise, the two sides of the equation will no longer be equal.

To solve an algebraic equation without exponents for a particular variable, you have to manipulate the equation until that variable is on one side of the equal sign with all numbers or other variables on the other side. You can perform addition, subtraction, or multiplication; you can also perform division, as long as the quantity by which you are dividing does not equal zero.

Typically, at each step of the process, you'll try to isolate the variable by using the reverse of whatever operation has been applied to the variable. For example, in solving the equation $n + 6 = 10$ for n, you have to get rid of the 6 that has been added to the n. You do that by subtracting 6 from both sides of the equation: $n + 6 - 6 = 10 - 6$, so $n = 4$.

Example:

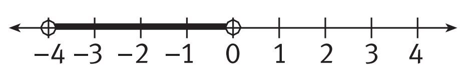
If $4x - 7 = 2x + 5$, what is the value of x ?

Start by adding 7 to both sides. This gives us $4x = 2x + 12$. Now subtract $2x$ from both sides. This gives us $2x = 12$. Finally, let's divide both sides by 2. This gives us $x = 6$.

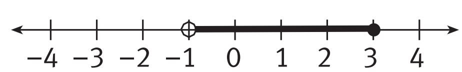
Inequalities

There are two differences between solving an *inequality* (such as $2x < 5$) and solving an *equation* (such as $2x - 5 = 0$).

First, the solution to an inequality is almost always a range of possible values, rather than a single value. You can see the range most clearly by expressing it visually on a number line.



The shaded portion of the number line above shows the set of all numbers between -4 and 0 excluding the endpoints -4 and 0; this range would be expressed algebraically by the inequality $-4 < x < 0$.



The shaded portion of the number line above shows the set of all numbers greater than -1, up to and including 3; this range would be expressed algebraically by the inequality $-1 < x \leq 3$.

The other difference when solving an inequality—and the only thing you really have to remember—is that **if you multiply or divide the inequality by a negative number, you have to reverse the direction of the inequality**. For example, when you multiply both sides of the inequality $-3x < 2$ by -1, you get $3x > -2$.

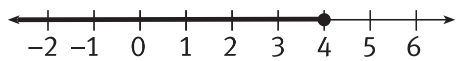
Example:

Solve for x: $3 - \frac{x}{4} \geq 2$

Multiply both sides of the inequality by 4: $12 - x \geq 8$

Subtract 12 from both sides: $-x \geq -4$

Multiply (or divide) both sides by -1 and change the direction of the inequality sign: $x \leq 4$.



As you can see from the number line, the range of values that satisfies this inequality includes 4 and all numbers less than 4.

Solving for One Unknown in Terms of Another

In general, in order to solve for the value of an unknown, you need as many distinct equations as you have variables. If there are two variables, for instance, you need two distinct equations.

However, some GRE problems do not require you to solve for the numerical value of an unknown. Instead, you are asked to solve for one variable in terms of the other(s). To do so, isolate the desired variable on one side of the equation and move all the constants and other variables to the other side.

Example:

In the formula $z = \frac{xy}{a + yb}$, solve for y in terms of x, z, a, and b.

Clear the denominator by multiplying both sides by $a + yb$: $(a + yb)z = xy$

Remove parentheses by distributing: $az + ybz = xy$

Put all terms containing y on one side and all other terms on the other side: $az = xy - ybz$

Factor out the common factor, y: $az = y(x - bz)$

Divide by the coefficient of y to get y alone: $\frac{az}{x - bz} = y$

Simultaneous Equations

We've already discovered that you need as many different equations as you have variables to solve for the actual value of a variable. When a single equation contains more than one variable, you can only solve for one variable in terms of the others.

This has important implications for Quantitative Comparisons. To have enough information to compare the two quantities, you usually must have at least as many equations as you have variables.

On the GRE, you will often have to solve two simultaneous equations, that is, equations that give you different information about the same two variables. There are two methods for solving simultaneous equations.

Method 1—Substitution

Step 1: Solve one equation for one variable in terms of the second.

Step 2: Substitute the result back into the other equation and solve.

Example:

If $x - 15 = 2y$ and $6y + 2x = -10$, what is the value of y ?

Solve the first equation for x by adding 15 to both sides.

$$x = 2y + 15$$

Substitute $2y + 15$ for x in the second equation:

$$\begin{aligned} 6y + 2(2y + 15) &= -10 \\ 6y + 4y + 30 &= -10 \\ 10y &= -40 \\ y &= -4 \end{aligned}$$

Method 2—Adding to Cancel

Combine the equations in such a way that one of the variables cancels out. To solve the two equations $4x + 3y = 8$ and $x + y = 3$, multiply both sides of the second equation by -3 to get $-3x - 3y = -9$. Now add the two equations; the $3y$ and the $-3y$ cancel out, leaving: $x = -1$.

Before you use either method, make sure you really do have two distinct equations. For example, $2x + 3y = 8$ and $4x + 6y = 16$ are really the same equation in different forms; multiply the first equation by 2, and you'll get the second.

Whichever method you use, you can check the result by plugging both values back into both equations and making sure they fit.

Example:

If $m = 4n - 10$ and $3m + 2n = 26$, find the values of m and n.

Since the first equation already expresses m in terms of n, this problem is best approached by substitution.

Substitute $4n - 10$ for m into $3m + 2n = 26$, and solve for n.

$$\begin{aligned} 3(4n - 10) + 2n &= 26 \\ 12n - 30 + 2n &= 26 \\ 14n &= 56 \\ n &= 4 \end{aligned}$$

Now solve either equation for m by plugging in 4 for n.

$m = 4n - 10$

$m = 4(4) - 10$

$m = 16 - 10$

$m = 6$

So $m = 6$ and $n = 4$.

Example:

If $3x + 3y = 18$ and $x - y = 10$, find the values of x and y.

You could solve this problem by the substitution method. But look what happens if you multiply the second equation by 3 and add it to the first:

$$\begin{array}{rcl} 3x + 3y & = & 18 \\ +(3x - 3y & = & 30) \\ \hline 6x & = & 48 \end{array}$$

If $6x = 48$, then $x = 8$. Now you can just plug 8 into either equation in place of x and solve for y. Your calculations will be simpler if you use the second equation: $8 - y = 10$; $-y = 2$; $y = -2$.

Example:

The GRE will sometimes reward you with a shortcut to finding combined value using multiple variables.

If $5x + 5y = 20$, what is the value of $x + y$?

We don't need the value of either variable by itself, but their sum. If we divided both sides by 5, we could find the value of $x + y$.

$5x + 5y = 20$

$5(x + y) = 20$

$x + y = 4$

Example:

If $3x - 5y = 10$ and $6y - 2x = 20$, what is the value of $x + y$?

By aligning the two equations with the same-variable order, you can see a shortcut to adding the two together to find the solution.

$$3x - 5y = 10$$
$$-2x + 6y = 20$$
$$x + y = 30$$

While we don't know the individual values for x or y, we don't need to know them.

Symbolism

Don't panic if you see strange symbols like ★, ✧, and ♦ in a GRE problem.

Problems of this type usually require nothing more than substitution. Read the question stem carefully for a definition of the symbols and for any examples of how to use them. Then, just follow the given model, substituting the numbers that are in the question stem.

Example:

An operation symbolized by ✯ is defined by the equation x ✯ $y = x - \frac{1}{y}$. What is the value of 2 ✯ 7?

The ✯ symbol is defined as a two-stage operation performed on two quantities, which are symbolized in the equation as x and y. The two steps are (1) find the reciprocal of the second quantity and (2) subtract the reciprocal from the first quantity. To find the value of 2 ✯ 7, substitute the numbers 2 and 7 into the equation, replacing the x (the first quantity given in the equation) with the 2 (the first number given) and the y (the second quantity given in the equation) with the 7 (the second number given). The reciprocal of 7 is $\frac{1}{7}$, and subtracting $\frac{1}{7}$ from 2 gives you the following:

$$2 - \frac{1}{7} = \frac{14}{7} - \frac{1}{7} = \frac{13}{7}$$

When a symbolism problem involves only one quantity, the operations are usually a little more complicated. Nonetheless, you can follow the same steps to find the correct answer.

Example:

Let $x^\star$ be defined by the equation: $x^\star = \frac{x^2}{1-x^2}$. Evaluate $\left(\frac{1}{2}\right)^\star$.

$$\left(\frac{1}{2}\right)^\star = \frac{\left(\frac{1}{2}\right)^2}{1-\left(\frac{1}{2}\right)^2} = \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{4} \times \frac{4}{3} = \frac{1}{3}$$

Every once in a while, you'll see a symbolism problem that doesn't even include an equation. The definitions in this type of problem usually test your understanding of number properties.

Example:

✽x is defined as the largest even number that is less than the negative square root of x. What is the value of ✽81?

Ⓐ −82
Ⓑ −80
Ⓒ −10
Ⓓ −8
Ⓔ 8

Plug in 81 for x and work backward logically. The negative square root of 81 is −9 because $(-9)(-9) = 81$. The largest even number that is less than −9 is −10. (The number part of −8 is smaller than the number part of −9; however, you're dealing with negative numbers, so you have to look for the even number that would be just to the *left* of −9 along the number line.) Thus, the correct answer choice is **(C)** −10.

Sequences

Sequences are lists of numbers. The value of a number in a sequence is related to its position in the list. Sequences are often represented on the GRE as follows:

$$s_1, s_2, s_3, \ldots s_n, \ldots$$

The subscript part of each number gives you the position of each element in the series. s_1 is the first number in the list, s_2 is the second number in the list, and so on.

You will be given a formula that defines each element. For example, if you are told that $s_n = 2n + 1$, then the sequence would be $(2 \times 1) + 1$, $(2 \times 2) + 1$, $(2 \times 3) + 1, \ldots$, or 3, 5, 7,…

Polynomials and Quadratics

The FOIL Method

When two binomials are multiplied, each term is multiplied by each term in the other binomial. This process is often called the *FOIL method*, because it involves adding the products of the First, Outer, Inner, and Last terms. Using the FOIL method to multiply out $(x + 5)(x - 2)$, the product of the first terms is x^2, the product of the outer terms is $-2x$, the product of the inner terms is $5x$, and the product of the last terms is −10. Adding, the answer is $x^2 + 3x - 10$.

Factoring the Product of Binomials

Many of the polynomials that you'll see on the GRE can be factored into a product of two binomials by using the FOIL method backward.

Example:

Factor the polynomial $x^2 - 3x + 2$.

You can factor this into two binomials, each containing an x term. Start by writing down what you know:

$$x^2 - 3x + 2 = (x\ \ )(x\ \ )$$

You'll need to fill in the missing term in each binomial factor. The product of the two missing terms will be the last term in the original polynomial: 2. The sum of the two missing terms will be the coefficient of the second term of the polynomial: -3. Find the factors of 2 that add up to -3. Since $(-1) + (-2) = -3$, you can fill the empty spaces with -1 and -2.

Thus, $x^2 - 3x + 2 = (x - 1)(x - 2)$.

Note: Whenever you factor a polynomial, you can check your answer by using FOIL to multiply the factors and obtain the original polynomial.

Factoring the Difference of Two Squares

A common factorable expression on the GRE is the difference of two squares (for example, $a^2 - b^2$). Once you recognize a polynomial as the difference of two squares, you'll be able to factor it automatically, since any polynomial of the form $a^2 - b^2$ can be factored into a product of the form $(a + b)(a - b)$.

Example:

Factor the expression $9x^2 - 1$.

$9x^2 = (3x)^2$ and $1 = 1^2$, so $9x^2 - 1$ is the difference of two squares.

Therefore, $9x^2 - 1 = (3x + 1)(3x - 1)$.

Factoring Polynomials of the Form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$

Any polynomial of this form is the square of a binomial expression, as you can see by using the FOIL method to multiply $(a + b)(a + b)$ or $(a - b)(a - b)$.

To factor a polynomial of this form, check the sign in front of the $2ab$ term. If it's a *plus* sign, the polynomial is equal to $(a + b)^2$. If it's a *minus* sign, the polynomial is equal to $(a - b)^2$.

Example:

Factor the polynomial $x^2 + 6x + 9$.

x^2 and 9 are both perfect squares, and $6x$ is $2(3x)$, which is twice the product of x and 3, so this polynomial is of the form $a^2 + 2ab + b^2$ with $a = x$ and $b = 3$. Since there is a plus sign in front of the $6x$, $x^2 + 6x + 9 = (a + 3)^2$.

Quadratic Equations

A *quadratic equation* is an equation of the form $ax^2 + bx + c = 0$. Many quadratic equations have two solutions. In other words, the equation will be true for two different values of x.

When you see a quadratic equation on the GRE, you'll generally be able to solve it by factoring the algebraic expression, setting each of the factors equal to zero, and solving the resulting equations.

Example:

$x^2 - 3x + 2 = 0$. Solve for x.

To find the solutions, or roots, start by factoring $x^2 - 3x + 2 = 0$ into $(x - 2)(x - 1) = 0$.

The product of two quantities equals zero only if one (or both) of the quantities equals zero. So if you set each of the factors equal to zero, you will be able to solve the resulting equations for the solutions of the original quadratic equation. Setting the two binomials equal to zero gives you this:

$$x - 2 = 0 \text{ or } x - 1 = 0$$

That means that x can equal 2 or 1. As a check, you can plug each of those values in turn into $x^2 - 3x + 2 = 0$, and you'll see that either value makes the equation work.

Alternative Strategies for Multiple-Choice Algebra

Backsolving

On GRE Problem Solving questions, you may find it easier to attack algebra problems by Backsolving. To Backsolve, substitute each answer choice into the equation until you find the one that satisfies the equation.

Example:

If $x^2 + 10x + 25 = 0$, what is the value of x?

(A) 25
(B) 10
(C) 5
(D) –5
(E) –10

The textbook approach to solving this problem would be to recognize the polynomial expression as the square of the binomial $(x + 5)$ and set $x + 5 = 0$. That's the fastest way to arrive at the correct answer of –5.

But you could also plug each answer choice into the equation until you found the one that makes the equation true. Backsolving can be pretty quick if the correct answer is the first choice you plug in, but here, you have to get all the way down to choice **(D)** before you find that $(-5)^2 + 10(-5) + 25 = 0$.

Example:

If $\frac{5x}{3} + 9 = \frac{x}{6} + 18$, $x =$

(A) 12
(B) 8
(C) 6
(D) 5
(E) 4

To avoid having to try all five answer choices, look at the equation and decide which choice(s), if plugged in for x, would make your calculations easiest. Since x is in the numerators of the two fractions in this equation and the denominators are 3 and 6, try plugging in a choice that is divisible by both 3 and 6. Choices **(A)** and **(C)** are divisible by both numbers, so start with one of them.

Choice **(A)**:

$$20 + 9 = 2 + 18$$
$$29 \neq 20$$

This is not true, so x cannot equal 12.

Choice **(C)**:

$$10 + 9 = 1 + 18$$
$$19 = 19$$

This is correct, so *x* must equal 6. Therefore, choice **(C)** is correct.

Backsolving may not be the fastest method for a multiple-choice algebra problem, but it's useful if you don't think you'll be able to solve the problem in the conventional way.

Picking Numbers

On other types of multiple-choice algebra problems, especially where the answer choices consist of variables or algebraic expressions, you may want to Pick Numbers to make the problem less abstract. Evaluate the answer choices and the information in the question stem by picking a number and substituting it for the variable wherever the variable appears.

Example:

If $a > 1$, the ratio of $2a + 6$ to $a^2 + 2a - 3$ is

(A) $2a$

(B) $a + 3$

(C) $\frac{2}{a - 1}$

(D) $\frac{2a}{3(3 - a)}$

(E) $\frac{a - 1}{2}$

You can simplify the process by replacing the variable *a* with a number in each algebraic expression. Since *a* has to be greater than 1, why not pick 2? Then the expression $2a + 6$ becomes $2(2) + 6$, or 10. The expression $a^2 + 2a - 3$ becomes $2^2 + 2(2) - 3 = 4 + 4 - 3 = 5$.

So now the question reads, "The ratio of 10 to 5 is what?" That's easy enough to answer: 10:5 is the same as $\frac{10}{5}$, or 2. Now you can just eliminate any answer choice that doesn't give a result of 2 when you substitute 2 for *a*. Choice **(A)** gives you 2(2), or 4, so discard it. Choice **(B)** results in 5—also not what you want. Choice **(C)** yields $\frac{2}{1}$ or 2. That looks good, but you can't stop here.

If another answer choice gives you a result of 2, you will have to pick another number for a and reevaluate the expressions in the question stem and the choices that worked when you let $a = 2$.

Choice **(D)** gives you $\frac{2(2)}{3(3-2)}$ or $\frac{4}{3}$, so eliminate choice **(D)**.

Choice **(E)** gives you $\frac{2-1}{2}$ or $\frac{1}{2}$, so discard choice **(E)**.

Fortunately, in this case, only choice **(C)** works out equal to 2, so it is the correct answer. But remember: When Picking Numbers, always check every answer choice to make sure you haven't chosen a number that works for more than one answer choice.

Using Picking Numbers to Solve for One Unknown in Terms of Another

It is also possible to solve for one unknown in terms of another by Picking Numbers. If the first number you pick doesn't lead to a single correct answer, be prepared to either pick a new number (and spend more time on the problem) or settle for guessing strategically among the answers that you haven't eliminated.

Example:

If $\frac{x^2 - 16}{x^2 + 6x + 8} = y$ and $x > -2$, which of the following is an expression for x in terms of y?

(A) $\frac{1+y}{2-y}$

(B) $\frac{2y+4}{1-y}$

(C) $\frac{4y-4}{y+1}$

(D) $\frac{2y-4}{2+y}$

(E) $\frac{y+4}{y+1}$

Pick a value for x that will simplify your calculations. If you let x equal 4, then $x^2 - 16 = 4^2 - 16 = 0$, and so the entire fraction on the left side of the equation is equal to zero.

Now, substitute 0 for y in each answer choice in turn. Each choice is an expression for x in terms of y, and since $y = 0$ when $x = 4$, the correct answer will have to give a value of 4 when $y = 0$. Just remember to evaluate all the answer choices, because you might find more than one that gives a result of 4.

Substituting 0 for y in choices **(A)**, **(C)**, and **(D)** yields $\frac{1}{2}$, $-\frac{4}{1}$, and $-\frac{4}{2}$, respectively, so none of those choices can be right. But both **(B)** and **(E)** give results of 4 when you make the substitution; choosing between them will require picking another number.

Again, pick a number that will make calculations easy. If $x = 0$, then $y =$

$$\frac{x^2 - 16}{x^2 + 6x + 8} = \frac{0 - 16}{0 + 0 + 8} = \frac{-16}{8} = -2$$

Therefore, $y = -2$ when $x = 0$. You don't have to try the new value of y in all the answer choices, just in **(B)** and **(E)**. When you substitute –2 for y in choice **(B)**, you get 0. That's what you're looking for, but again, you have to make sure it doesn't work in choice **(E)**. Plugging –2 in for y in **(E)** yields –2 for x, so **(B)** is correct.

STATISTICS

Median, Mode, and Range

Median: The middle term in a group of terms that are arranged in numerical order. To find the median of a group of terms, first arrange the terms in numerical order. If there is an odd number of terms in the group, then the median is the middle term.

Example:

Bob's test scores in Spanish are 84, 81, 88, 70, and 87. What is his median score?

In increasing order, his scores are 70, 81, 84, 87, and 88. The median test score is the middle one: 84.

If there is an even number of terms in the group, the median is the average of the two middle terms.

Example:

John's test scores in biology are 92, 98, 82, 94, 85, and 97. What is his median score?

In numerical order, his scores are 82, 85, 92, 94, 97, and 98. The median test score is the average of the two middle terms, or $\frac{92 + 94}{2} = 93$.

The median of a group of numbers is often different from its average.

Example:

Caitlin's test scores in math are 92, 96, 90, 85, and 82. Find the difference between Caitlin's median score and the average (arithmetic mean) of her scores.

In ascending order, Caitlin's scores are 82, 85, 90, 92, and 96. The median score is the middle one: 90. Her average score is

$$\frac{82 + 85 + 90 + 92 + 96}{5} = \frac{445}{5} = 89$$

As you can see, Caitlin's median score and her average score are not the same. The difference between them is $90 - 89$, or 1.

Mode: The term that appears most frequently in a set.

Example:

The daily temperatures in city Q for one week were 25°, 33°, 26°, 25°, 27°, 31°, and 22°. What was the mode of the daily temperatures in city Q for that week?

Each of the temperatures occurs once on the list, except for 25°, which occurs twice. Since 25° appears more frequently than any other temperature, it is the mode.

A set may have more than one mode if two or more terms appear an equal number of times within the set and each appears more times than any other term.

Example:

The table below represents the score distribution for a class of 20 students on a recent chemistry test. Which score, or scores, are the mode?

Score	# of Students Receiving That Score
100	2
91	1
87	5
86	2
85	1
84	5
80	1
78	2
56	1

The largest number in the second column is 5, which occurs twice. Therefore, there were two mode scores on this test: 87 and 84. Equal numbers of students received those scores, and more students received those scores than any other score.

If every element in the set occurs an equal number of times, then the set has no mode.

Combination

A *combination* question asks you how many unordered subgroups can be formed from a larger group.

Some combination questions on the GRE can be solved without any computation just by counting or listing possible combinations.

Example:

Allen, Betty, and Claire must wash the dishes. They decide to work in shifts of two people. How many shifts will it take before all possible combinations have been used?

It is possible, and not time-consuming, to solve this problem by writing a list. Call Allen "*A*," Betty "*B*," and Claire "*C*." There are three (*AB*, *AC*, *BC*) possible combinations.

The Combination Formula

Some combination questions use numbers that make quick, noncomputational solving difficult. In these cases, use the combination formula $\frac{n!}{k!(n-k)!}$, where n is the number of items in the group as a whole and k is the number of items in each subgroup formed. The ! symbol means factorial (for example, $5! = (5)(4)(3)(2)(1) = 120$).

Example:

The 4 finalists in a spelling contest win commemorative plaques. If there are 7 entrants in the spelling contest, how many possible groups of winners are there?

Plug the numbers into the combination formula, such that n is 7 (the number in the large group) and k is 4 (the number of people in each subgroup formed).

$$\frac{7!}{4!(7-4)!}$$

$$\frac{7!}{4!3!}$$

At this stage, it is helpful to reduce these terms. Since 7 factorial contains all the factors of 4 factorial, we can write 7! as (7)(6)(5)(4!) and then cancel the 4! in the numerator and denominator.

$$\frac{(7)(6)(5)}{(3)(2)(1)} = ?$$

We can reduce further by crossing off the 6 in the numerator and the (3)(2) in the denominator.

$$\frac{(7)(5)}{1} = 35$$

There are 35 potential groups of spelling contest finalists.

When you are asked to find potential combinations from multiple groups, multiply the potential combinations from each group.

Example:

How many groups can be formed consisting of 2 people from room A and 3 people from room B if there are 5 people in room A and 6 people in room B?

Insert the appropriate numbers into the combination formula for each room and then multiply the results. For room A, the number of combinations of 2 in a set of 5 is as follows:

$$\frac{n!}{k!(n-k)!} = \frac{5!}{2!3!} = \frac{(5)(4)(3)(2)(1)}{(2)(1)(3)(2)(1)}$$

Reducing this you get $\frac{(5)(4)}{(2)} = 10.$ For room B, the number of combinations of 3 in a set of 6 is as follows:

$$\frac{n!}{k!(n-k)!} = \frac{6!}{3!3!} = \frac{(6)(5)(4)(3)(2)(1)}{(3)(2)(1)(3)(2)(1)}$$

Reducing this, you get $\frac{(6)(5)(4)}{(3)(2)} = 20.$

Multiply these to find that there are $(10)(20) = 200$ possible groups consisting of 2 people from room A and 3 people from room B.

Sometimes the GRE will ask you to find the number of possible subgroups when choosing one item from a set. In this case, the number of possible subgroups will always equal the number of items in the set.

Example:

Restaurant A has 5 appetizers, 20 main courses, and 4 desserts. If a meal consists of 1 appetizer, 1 main course, and 1 dessert, how many different meals can be ordered at restaurant A?

The number of possible outcomes from each set is the number of items in the set. So there are 5 possible appetizers, 20 possible main courses, and 4 possible desserts. The number of different meals that can be ordered is $(5)(20)(4) = 400$.

Permutation

Within any group of items or people, there are multiple arrangements, or *permutations*, possible. For instance, within a group of three items (for example: *A*, *B*, *C*), there are six permutations (*ABC*, *ACB*, *BAC*, *BCA*, *CAB*, and *CBA*).

Permutations differ from combinations in that permutations are ordered. By definition, each combination larger than 1 has multiple permutations. On the GRE, a question asking "How many ways/arrangements/orders/schedules are possible?" generally indicates a permutation problem.

To find permutations, think of each place that needs to be filled in a particular arrangement as a blank space. The first place can be filled with any of the items in the larger group. The second place can be filled with any of the items in the larger group except for the one used to fill the first place. The third place can be filled with any of the items in the group except for the two used to fill the first two places, etc.

Example:

In a spelling contest, the winner will receive a gold medal, the second-place finisher will receive a silver medal, the third-place finisher will receive a bronze medal, and the fourth-place finisher will receive a blue ribbon. If there are 7 entrants in the contest, how many different arrangements of award winners are there?

The gold medal can be won by any of 7 people. The silver medal can be won by any of the remaining 6 people. The bronze medal can be won by any of the remaining 5 people. And the blue ribbon can be won by any of the remaining 4 people. Thus, the number of possible arrangements is $(7)(6)(5)(4) = 840$.

Probability

Probability is the numerical representation of the likelihood of an event or combination of events. This is expressed as a ratio of the number of desired outcomes to the total number of possible outcomes. Probability is usually expressed as a fraction (for example, "The probability of event A occurring is $\frac{1}{3}$), but it can also be expressed in words ("The probability of event *A* occurring is 1 in 3"). The probability of any event occurring cannot exceed 1 (a probability of 1 represents a 100% chance of an event occurring), and it cannot be less than 0 (a probability of 0 represents a 0% chance of an event occurring).

Example:

If you flip a fair coin, what is the probability that it will fall with the "heads" side facing up?

The probability of the coin landing heads up is $\frac{1}{2}$, since there is one outcome you are interested in (landing heads up) and two possible outcomes (heads up or tails up).

Example:

What is the probability of rolling a 5 or a 6 on a six-sided die numbered 1 through 6?

The probability of rolling a 5 or a 6 on a six-sided die numbered 1 through 6 is $\frac{2}{6} = \frac{1}{3}$, since there are 2 desired outcomes (rolling a 5 or a 6) and 6 possible outcomes (rolling a 1, 2, 3, 4, 5, or 6).

The sum of all possible outcomes, desired or otherwise, must equal 1. In other words, if there is a 25% chance that event *A* will occur, then there is a 75% chance that it will not occur. So, to find the probability that an event *will not* occur, subtract the probability that it *will* occur from 1. In the previous example, the probability of not throwing a 5 or a 6 on the die is $1 - \frac{1}{3} = \frac{2}{3}$.

When events are independent, that is, the events do not depend on the other event or events, the probability that several events will all occur is the product of the probability of each event occurring individually.

Example:

A fair coin is flipped twice. What is the probability of its landing with the heads side facing up on both flips?

Multiply the probability for each flip: $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$.

Probability of Dependent Events

In some situations, the probability of a later event occurring varies according to the results of an earlier event. In this case, the probability fraction for the later event must be adjusted accordingly.

Example:

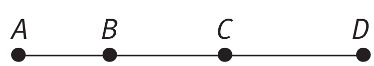
A bag contains 10 marbles, 4 of which are blue and 6 of which are red. If 2 marbles are removed without replacement, what is the probability that both marbles removed are red?

The probability that the first marble removed will be red is $\frac{6}{10} = \frac{3}{5}$. The probability that the second marble removed will be red will not be the same, however. There will be fewer marbles overall, so the denominator will be one less. There will also be one fewer red marble. (Note that since we are asking about the odds of picking two red marbles, we are only interested in choosing a second marble if the first was red. Don't concern yourself with situations in which a blue marble is chosen first.) If the first marble removed is red, the probability that the second marble removed will also be red is $\frac{5}{9}$. So the probability that both marbles removed will be red is $\left(\frac{3}{5}\right)\left(\frac{5}{9}\right) = \frac{15}{45} = \frac{1}{3}$.

GEOMETRY

Lines and Angles

A **line** is a one-dimensional geometrical abstraction—infinitely long, with no width. A straight line is the shortest distance between any two points. There is exactly one straight line that passes through any two points.

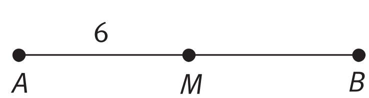


Example:

In the figure above, $AC = 9$, $BD = 11$, and $AD = 15$. What is the length of BC?

When points are in a line and the order is known, you can add or subtract lengths. Since $AC = 9$ and $AD = 15$, $CD = AD - AC = 15 - 9 = 6$. Now, since $BD = 11$ and $CD = 6$, $BC = BD - CD = 11 - 6 = 5$.

A **line segment** is a section of a straight line of finite length, with two endpoints. A line segment is named for its endpoints, as in segment AB.



Example:

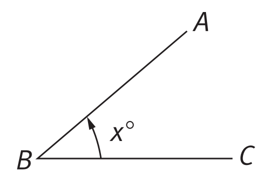
In the figure above, A and B are the endpoints of the line segment AB, and M is the midpoint ($AM = MB$). What is the length of AB?

Since *AM* is 6, *MB* is also 6, and so *AB* is $6 + 6$, or 12.

Two lines are **parallel** if they lie in the same plane and never intersect regardless of how far they are extended. If line ℓ_1 is parallel to line ℓ_2, we write $\ell_1 \parallel \ell_2$. If two lines are both parallel to a third line, then they are parallel to each other as well.

A **vertex** is the point at which two lines or line segments intersect to form an **angle**. Angles are measured in **degrees** (°).

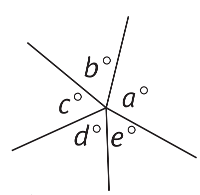
Angles may be named according to their vertices. Sometimes, especially when two or more angles share a common vertex, an angle is named according to three points: a point along one of the lines or line segments that form the angle, the vertex point, and another point along the other line or line segment. A diagram will sometimes show a letter inside the angle; this letter may also be used to name the angle.



The angle shown in the diagram above could be called $\angle x$, $\angle ABC$, or $\angle B$. (We use a lowercase *x* because *x* is not a point.)

Sum of Angles Around a Point

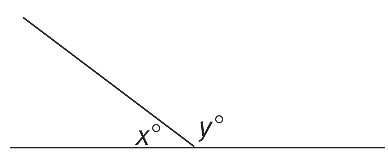
The sum of the measures of the angles around a point is 360°.



$$a + b + c + d + e = 360$$

Sum of Angles Along a Straight Line

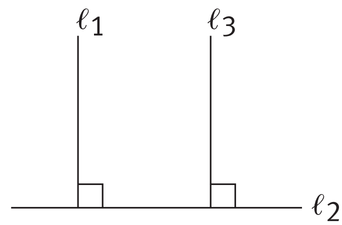
The sum of the measures of the angles on one side of a straight line is 180°. Two angles are *supplementary* to each other if their measures sum to 180°.



$$x + y = 180$$

Perpendicularity and Right Angles

Two lines are *perpendicular* if they intersect at a 90° angle (a right angle). If line ℓ_1 is perpendicular to line ℓ_2, we write $\ell_1 \perp \ell_2$. If lines ℓ_1, ℓ_2, and ℓ_3 all lie in the same plane, and if $\ell_1 \perp \ell_2$ and $\ell_2 \perp \ell_3$, then $\ell_1 \parallel \ell_3$, as shown in the diagram below.



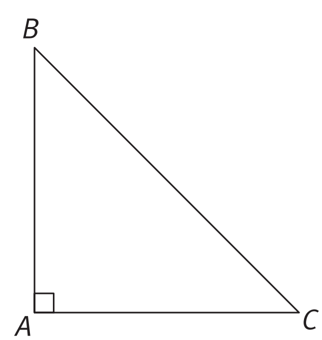
$\ell_1 \perp \ell_2$
$\ell_2 \perp \ell_3$
$\ell_1 \parallel \ell_3$

To find the shortest distance from a point to a line, draw a line segment from the point to the line such that the line segment is perpendicular to the line. Then, measure the length of that segment.

Example:

$\angle A$ of triangle *ABC* is a right angle. Is side *BC* longer or shorter than side *AB*?

This question seems very abstract, until you draw a diagram of a right triangle, labeling the vertex with the 90° angle as point *A*.

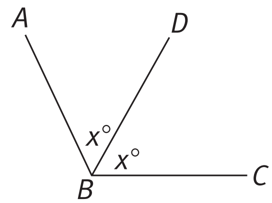


Line segment *AB* has to be the shortest route between point *B* and side *AC*, since side *AB* is perpendicular to side *AC*. If *AB* is the shortest line segment that can join point *B* to side *AC*, *BC* must be longer than *AB*. **Note:** The side opposite the 90° angle, called the *hypotenuse*, is always the longest side of a right triangle.

Two angles are *complementary* to each other if their measures sum to 90°. An *acute angle* measures less than 90°, and an *obtuse angle* measures between 90° and 180°. Two angles are *supplementary* if their measures sum to 180°.

Angle Bisectors

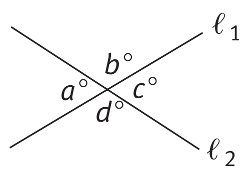
A line or line segment *bisects* an angle if it splits the angle into two smaller, equal angles. Line segment *BD* below bisects $\angle ABC$, and $\angle ABD$ has the same measure as $\angle DBC$. The two smaller angles are each half the size of $\angle ABC$.



BD bisects $\angle ABC$
$\angle ABD + \angle DBC = \angle ABC$

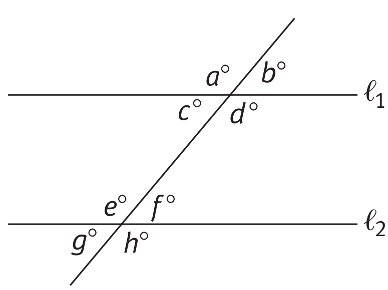
Adjacent and Vertical Angles

Two intersecting lines form four angles. The angles that are adjacent (next) to each other are *supplementary* because they lie along a straight line. The two angles that are not adjacent to each other are *opposite*, or *vertical*. Opposite angles are equal in measure because each of them is supplementary to the same adjacent angle.



In the diagram above, ℓ_1 intersects ℓ_2 to form angles *a*, *b*, *c*, and *d*. Angles *a* and *c* are opposite, as are angles *b* and *d*. So the measures of angles *a* and *c* are equal to each other, and the measures of angles *b* and *d* are equal to each other. And each angle is supplementary to each of its two adjacent angles.

Angles Around Parallel Lines Intersected by a Transversal



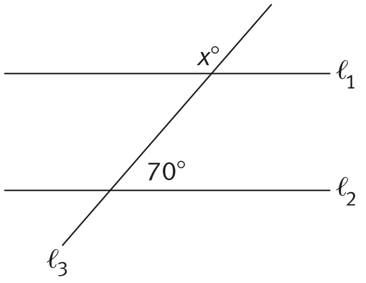
$\ell_1 \parallel \ell_2$

A line that intersects two parallel lines is called a *transversal*. Each of the parallel lines intersects the third line at the same angle. In the figure above, $a = e$.

Since a and e are equal, and since $a = d$ and $e = h$ (because they are opposite angles), $a = d = e = h$. By similar reasoning, $b = c = f = g$.

In short, when two (or more) parallel lines are cut by a transversal, all acute angles formed are equal, all obtuse angles formed are equal, and any acute angle formed is supplementary to any obtuse angle formed.

Example:



In the diagram above, line ℓ_1 is parallel to line ℓ_2. What is the value of x?

The angle marked $x°$ and the angle adjacent and to the left of the 70° angle on line ℓ_2 are corresponding angles. Therefore, the angle marked $x°$ must be supplementary to the 70° angle. If $70° + x° = 180°$, x must equal 110.

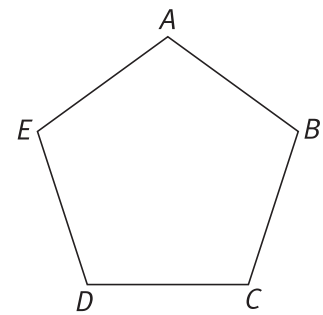
Polygons

Important Terms

Polygon: A closed figure whose sides are straight line segments. Families or classes of polygons are named according to their number of sides. A triangle has three sides, a quadrilateral has four sides, a pentagon has five sides, and a hexagon has six sides. Triangles and quadrilaterals are by far the most important polygons on the GRE; other polygons appear only occasionally.

Perimeter: The distance around a polygon; the sum of the lengths of its sides.

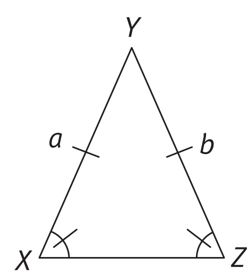
Vertex of a polygon: A point where two sides intersect (plural: *vertices*). Polygons are named by assigning each vertex a letter and listing them in order, as in pentagon *ABCDE* below.



Diagonal of a polygon: A line segment connecting any two nonadjacent vertices.

Regular polygon: A polygon with sides of equal length and interior angles of equal measure.

Small slash marks can provide important information in diagrams of polygons. Sides with the same number of slash marks are equal in length, while angles with the same number of slash marks through circular arcs have the same measure. In the triangle below, for example, $a = b$, and angles X and Z are equal in measure.

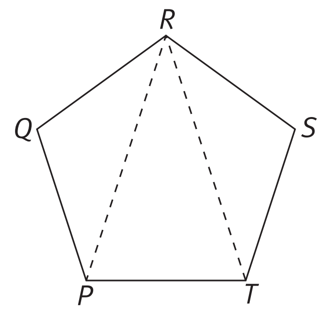


You can figure out the sum of the interior angles of a polygon by dividing the polygon into triangles. Draw diagonals from any vertex to all the nonadjacent vertices. Then, multiply the number of triangles by 180° to get the sum of the interior angles of the polygon. This works because the sum of the interior angles of any triangle is always 180°.

Example:

What is the sum of the interior angles of a pentagon ?

Draw a pentagon (a five-sided polygon) and divide it into triangles, as discussed above.



No matter how you've drawn the pentagon, you'll be able to form three triangles. Therefore, the sum of the interior angles of a pentagon is $3 \times 180° = 540°$.

Triangles

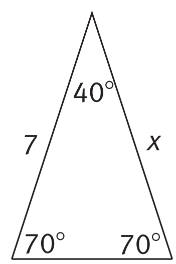
Important Terms

Triangle: A polygon with three straight sides and three interior angles.

Right triangle: A triangle with one interior angle of 90° (a right angle).

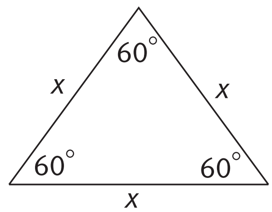
Hypotenuse: The longest side of a right triangle. The hypotenuse is always opposite the right angle.

Isosceles triangle: A triangle with two equal sides, which are opposite two equal angles. In the figure below, the sides opposite the two 70° angles are equal, so $x = 7$.

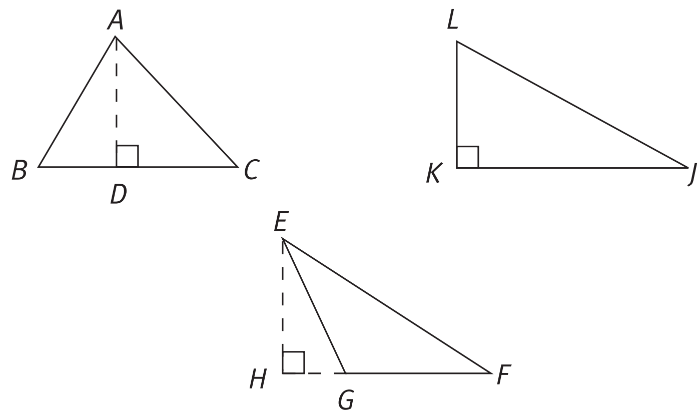


Legs: The two equal sides of an isosceles triangle or the two shorter sides of a right triangle (the ones forming the right angle). **Note:** The third, unequal side of an isosceles triangle is called the *base*.

Equilateral triangle: A triangle whose three sides are all equal in length and whose three interior angles each measure 60°.



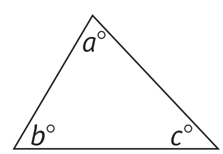
The **altitude,** or **height,** of a triangle is the perpendicular distance from a vertex to the side opposite the vertex. The altitude may fall inside or outside the triangle, or it may coincide with one of the sides.



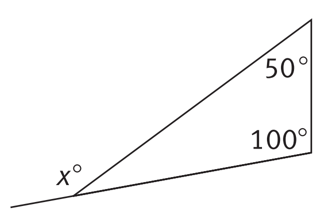
In the diagrams above, AD, EH, *and* LK *are altitudes.*

Interior and Exterior Angles of a Triangle

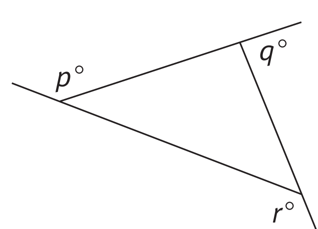
The sum of the interior angles of any triangle is 180°. Therefore, in the figure below, $a + b + c = 180$.



An *exterior angle of a triangle* is equal to the sum of the remote interior angles. The exterior angle labeled $x°$ is equal to the sum of the remote angles: $x = 50 + 100 = 150$.



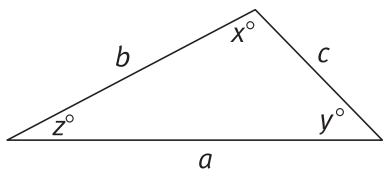
The three exterior angles of any triangle add up to 360°.



In the figure above, $p + q + r = 360$.

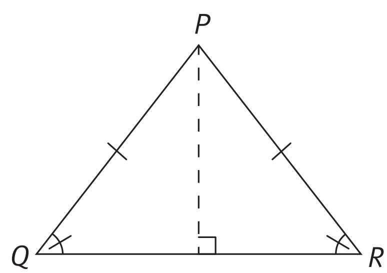
Sides and Angles

The sum of the lengths of any two sides of a triangle is greater than the length of the third side. In the triangle below, $b + c > a$, $a + b > c$, and $a + c > b$.



If the lengths of two sides of a triangle are unequal, the greater angle lies opposite the longer side, and vice versa. In the figure above, if $x > y > z$, then $a > b > c$.

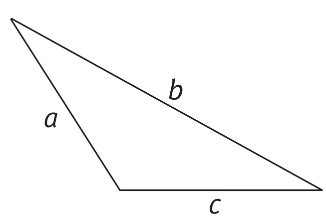
Since the two legs of an isosceles triangle have the same length, the two angles opposite the legs must have the same measure. In the figure below, $PQ = PR$, and $\angle Q = \angle R$.



Perimeter and Area of Triangles

There is no special formula for the perimeter of a triangle; it is just the sum of the lengths of the sides.

Example:



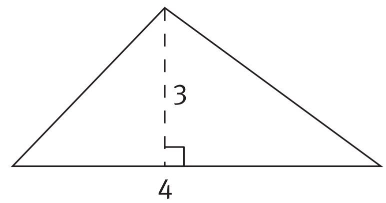
If $b = 2a$ and $c = \frac{b}{2}$, find the perimeter of the triangle above in terms of a.

Perimeter $= a + b + c = a + 2a + \frac{2a}{2} = 3a + \frac{2a}{2} = 3a + a = 4a$.

Incidentally, this is really an isosceles triangle, since $c = \frac{b}{2} = \frac{2a}{2} = a$.

The area of a triangle is $\left(\frac{1}{2}\right)$(Base)(Height).

Example:

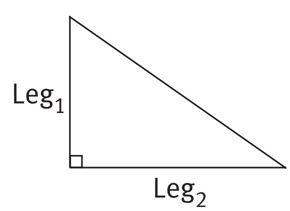


In the diagram above, the base has length 4, and the altitude has length 3. What is the area of the triangle?

$$\begin{aligned} \text{Area} &= \frac{1}{2}bh \\ &= \frac{bh}{2} \\ &= \frac{4 \times 3}{2} \\ &= 6 \end{aligned}$$

Since the lengths of the base and altitude were not given in specific units, such as centimeters or feet, the area of the triangle is simply said to be 6 square units.

The area of a right triangle is easy to find. Think of one leg as the base and the other as the height. Then the area is one-half the product of the legs, or $\frac{1}{2} \times \text{Leg}_1 \times \text{Leg}_2$.

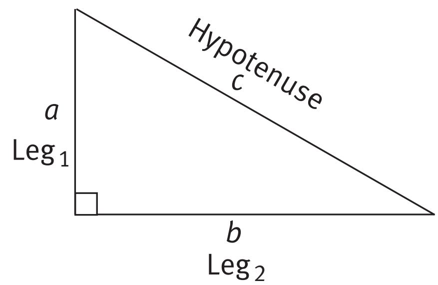


Right Triangles

The right angle is always the largest angle in a right triangle; therefore, the hypotenuse, which lies opposite the right angle, is always the longest side.

Pythagorean Theorem

The *Pythagorean theorem*, which holds for all right triangles and for no other triangles, states that the square of the hypotenuse is equal to the sum of the squares of the legs.



$$(\text{Leg}_1)^2 + (\text{Leg}_2)^2 = (\text{Hypotenuse})^2$$
or $a^2 + b^2 = c^2$

The Pythagorean theorem is very useful whenever you're given the lengths of any two sides of a right triangle; as long as you know whether the remaining side is a leg or the hypotenuse, you can find its length by using the Pythagorean theorem.

Example:

What is the length of the hypotenuse of a right triangle with legs of lengths 9 and 10?

$$\begin{aligned}(\text{Hypotenuse})^2 &= (\text{Leg}_1)^2 + (\text{Leg}_2)^2\\ &= 9^2 + 10^2\\ &= 81 + 100\\ &= 181\end{aligned}$$

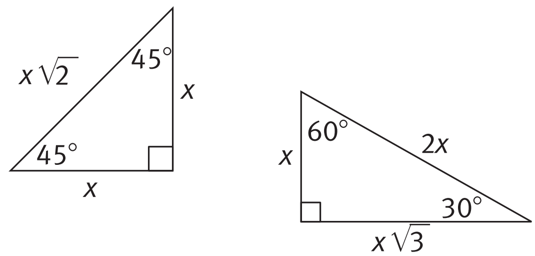
If the square of the hypotenuse equals 181, then the hypotenuse itself must be the square root of 181, or $\sqrt{181}$.

Pythagorean Triples

Certain ratios of integers always satisfy the Pythagorean theorem. You might like to think of them as "Pythagorean triples." One such ratio is 3, 4, and 5. A right triangle with legs of lengths 3 and 4 and a hypotenuse of length 5 is probably the most common kind of right triangle on the GRE. Whenever you see a right triangle with legs of 3 and 4, with a leg of 3 and a hypotenuse of 5, or with a leg of 4 and a hypotenuse of 5, you immediately know the length of the remaining side. In addition, any multiple of these lengths makes another Pythagorean triple; for instance, $6^2 + 8^2 = 10^2$, so a triangle with sides of lengths 6, 8, and 10 is also a right triangle.

The other triple that commonly appears on the GRE is 5, 12, and 13.

Special Right Triangles



There are two more special kinds of right triangles for which you won't have to use the Pythagorean theorem to find the lengths of the sides. There are special ratios between the lengths of the sides in isosceles right triangles (45°/45°/90° right triangles) and 30°/60°/90° right triangles (right triangles with acute angles of 30° and 60°). As you can see in the first drawing above, the sides of an isosceles right triangle are in a ratio of $x:x:x\sqrt{2}$, with the $x\sqrt{2}$ in the ratio representing the hypotenuse. The sides of a 30°/60°/90° right triangle are in a ratio of $x:x\sqrt{3}:2x$, where $2x$ represents the hypotenuse and x represents the side opposite the 30° angle. (Remember: The longest side has to be opposite the greatest angle.)

Example:

What is the length of the hypotenuse of an isosceles right triangle with legs of length 4?

You can use the Pythagorean theorem to find the hypotenuse, but it's quicker to use the special right triangle ratios. In an isosceles right triangle, the ratio of a leg to the hypotenuse is $x:x\sqrt{2}$. Since the length of a leg is 4, the length of the hypotenuse must be $4\sqrt{2}$.

Triangles and Quantitative Comparison

All Quantitative Comparison questions require you to judge whether enough information has been given to make a comparison. In geometry, making this judgment is often a matter of knowing the correct definition or formula. For triangles, keep in mind the following:

- If you know two angles, you know the third.
- To find the area, you need the base and the height.
- In a right triangle, if you have two sides, you can find the third. And if you have two sides, you can find the area.
- In isosceles right triangles and 30°/60°/90° triangles, if you know one side, you can find everything.

Be careful, though! Be sure you know as much as you think you do.

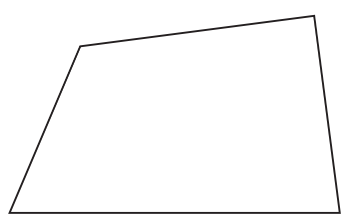
Example:

Quantity A	Quantity B
Area of right triangle *ABC*, where $\overline{AB} = 5$ *and* $\overline{BC} = 4$	6

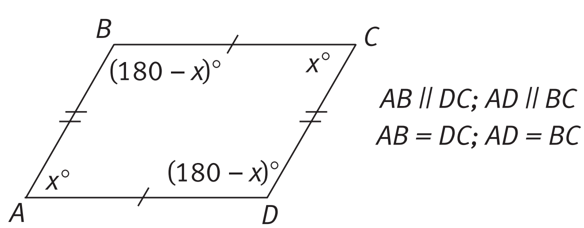
You may think at first that *ABC* must be a 3:4:5 right triangle. Not so fast! We're given two sides, but we don't know which sides they are. If *AB* is the hypotenuse, then it is a 3:4:5 triangle and the area is $\frac{1}{2}(3 \times 4) = 6,$ but it's also possible that *AC*, the missing side, is the hypotenuse. In that case, the area would be $\frac{1}{2}(4 \times 5) = 10.$ Because Quantity A can either be equal to Quantity B or can be larger than Quantity B, their relationship cannot be determined from the information given.

Quadrilaterals

A **quadrilateral** is a four-sided polygon. Regardless of a quadrilateral's shape, the four interior angles sum to 360°.

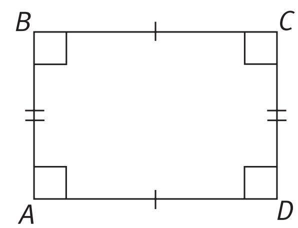


A **parallelogram** is a quadrilateral with two pairs of parallel sides. Opposite sides are equal in length; opposite angles are equal in measure; angles that are not opposite are supplementary to each other (measure of $\angle A$ + measure of $\angle D = 180°$ in the figure below).



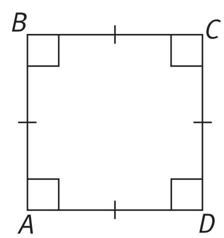
measure of $\angle A$ = measure of $\angle C$;
measure of $\angle B$ = measure of $\angle D$

A **rectangle** is a parallelogram with four right angles. Opposite sides are equal; diagonals are equal.



$AB = DC$
$AD = BC$
$AC = BD$

A **square** is a rectangle with equal sides.



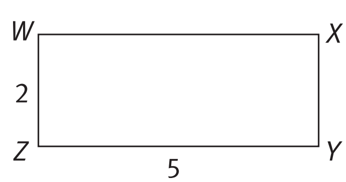
$AB = BC = CD = DA$

Perimeters of Quadrilaterals

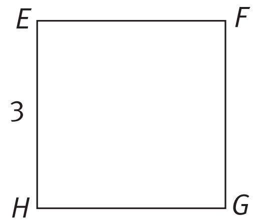
To find the perimeter of any polygon, you can simply add the lengths of its sides. However, the properties of rectangles and squares lead to simple formulas that may speed up your calculations.

Because the opposite sides are equal, the *perimeter of a rectangle* is twice the sum of the length and the width: Perimeter $=$ 2(Length $+$ Width)

The perimeter of a 5 by 2 rectangle is $2(5 + 2) = 14$.

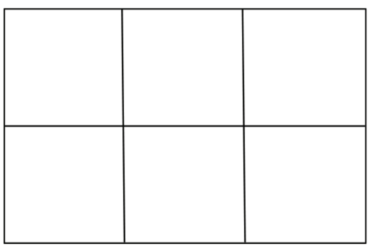


The *perimeter of a square* is equal to the sum of the lengths of the 4 sides. Because all 4 sides are the same length, Perimeter $=$ 4 (Side). If the length of one side of a square is 3, the perimeter is $4 \times 3 = 12$.



Areas of Quadrilaterals

Area formulas always involve multiplication, and the results are always stated in "square" units. You can see why if you look at the drawing below:



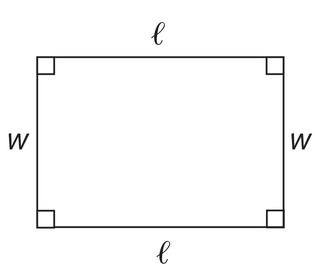
The rectangle is composed of six squares, all equal in size. Let's say that the side of a single small square is 1 unit. Then, we would say that a single square measures "1 by 1." That translates into math as 1×1, or 1^2—in other words, "one square unit."

As you can see from the drawing, there are 6 such square units in the rectangle. That's its area: 6 square units. But you could also find the area by multiplying the number of squares in a row by the number of squares in a column: 3×2, or 6. And since we've defined the length of the side of a square as 1 unit, that's also equivalent to multiplying the length of a horizontal side by the length of a vertical side: again, $3 \times 2 = 6$.

Formulas for Area

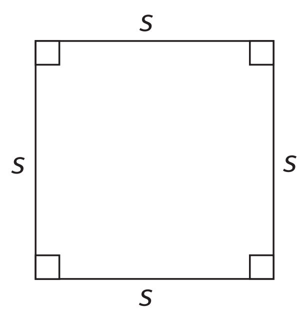
To find the area of a rectangle, multiply the **length** by the **width**.

$$\text{Area of rectangle} = \ell w$$



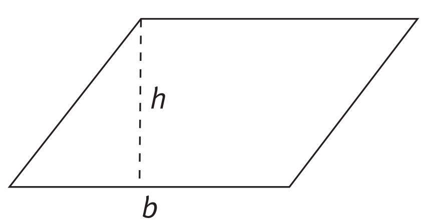
Since the length and width of a square are equal, the area formula for a square just uses the length of a **side:**

$$\text{Area of square} = (\text{Side})^2 = s^2$$



If you're working with a parallelogram, designate one side as the **base**. Then, draw a line segment from one of the vertices opposite the base down to the base so that it intersects the base at a right angle. That line segment will be called the **height**. To find the area of the parallelogram, multiply the length of the base by the length of the height:

$$\text{Area of parallelogram} = (\text{Base})(\text{Height}), \text{ or } A = bh$$



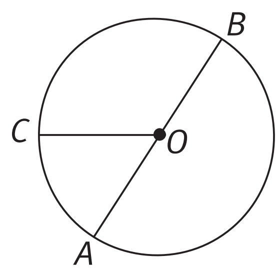
Remember the following:

- In a parallelogram, if you know two adjacent sides, you know all of them; and if you know two adjacent angles, you know all of them.
- In a rectangle, if you know two adjacent sides, you know the area.
- In a square, if you're given virtually any measurement (area, length of a side, length of a diagonal), you can figure out the other measurements.

Circles

Important Terms

Circle: The set of all points in a plane at the same distance from a certain point. This point is called the center of the circle. A circle is labeled by its center point; circle O means the circle with center point O.

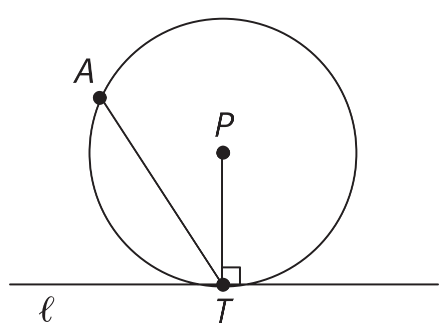


Diameter: A line segment that connects two points on the circle and passes through the center of the circle. *AB* is a diameter of circle *O* above.

Radius: A line segment that connects the center of the circle with any point on the circle (plural: *radii*). The radius of a circle is one-half the length of the diameter. In circle *O* above, *OA*, *OB*, and *OC* are radii.

Central angle: An angle formed by two radii. In circle *O* above, *AOC* is a central angle. *COB* and *BOA* are also central angles. (The measure of *BOA* happens to be 180°.) The total degree measure of a circle is 360°.

Chord: A line segment that joins two points on the circle. The longest chord of a circle is its diameter. *AT* is a chord of circle *P* below.



Tangent: A line that touches only one point on the circumference of a circle. A line drawn tangent to a circle is perpendicular to the radius at the point of tangency. In the diagram above, line ℓ is tangent to circle *P* at point *T*.

Circumference and Arc Length

The distance around a polygon is called its **perimeter**; the distance around a circle is called its **circumference**.

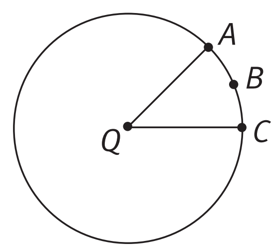
The ratio of the circumference of any circle to its diameter is a constant, called **pi** (π). For GRE purposes, the value of π is usually approximated as 3.14.

Since π equals the ratio of the circumference, C, to the diameter, d, we can say that $p = \dfrac{\text{Circumference}}{\text{Diameter}} = \dfrac{C}{d}$.

The formula for the circumference of a circle is $C = \pi d$.

The circumference formula can also be stated in terms of the radius, r. Since the diameter is twice the length of the radius, that is, $d = 2r$, then $C = 2\pi r$.

An **arc** is a section of the circumference of a circle. Any arc can be thought of as the portion of a circle cut off by a particular central angle. For example, in circle Q, arc ABC is the portion of the circle that is cut off by central angle AQC. Since arcs are associated with central angles, they can be measured in degrees. The degree measure of an arc is equal to that of the central angle that cuts it off. So in circle Q, arc ABC and central angle AQC would have the same degree measure.

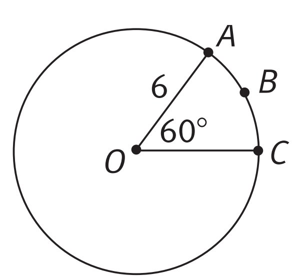


An arc that is exactly half the circumference of its circle is called a **semicircle**.

The length of an arc is the same fraction of a circle's circumference as its degree measure is of 360° (the degree measure of a whole circle). For an arc with a central angle measuring n°:

$$\begin{aligned}\text{Arc length} &= \frac{n}{360}(\text{Circumference}) \\ &= \frac{n}{360} \times 2\pi r\end{aligned}$$

Example:



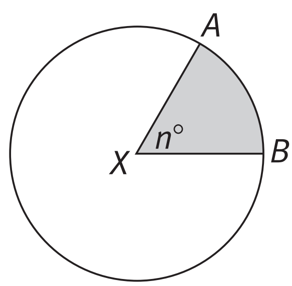
What is the length of arc ABC of circle O above?

$C = 2\pi r$; therefore, if $r = 6$, $C = 2 \times \pi \times 6 = 12\pi$. Since AOC measures 60°, arc ABC is $\frac{60}{360}$, or $\frac{1}{6}$ of the circumference. Thus, the length of arc ABC is $\frac{1}{6} \times 12\pi$, or 2π.

Area and Sector Area Formulas

The area of a circle is πr^2.

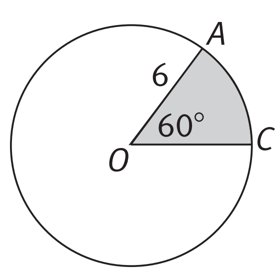
A **sector** is a portion of a circle's area that is bounded by two radii and an arc. The shaded area of circle *X* is sector *AXB*.



Like arcs, sectors are associated with central angles. And the process and formula used to find the area of a sector are similar to those used to determine arc length. First, find the degree measure of the sector's central angle and figure out what fraction that degree measure is of 360°. Then, multiply the area of the whole circle by that fraction. In a sector whose central angle measures n°:

$$\text{Area of sector} = \frac{n}{360}(\text{Area of circle}) = \frac{n}{360}\pi r^2$$

Example:



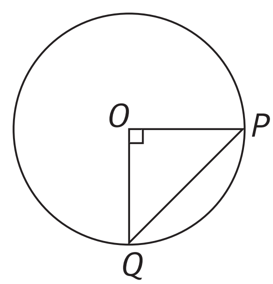
In circle *O* above, what is the area of sector *AOC*?

Since $\angle AOC$ measures 60°, a 60° "slice" of the circle is $\frac{60°}{360°}$, or $\frac{1}{6}$ of the total area of the circle. Therefore, the area of the sector is $\frac{1}{6}\pi r^2 = \frac{1}{6}(36\pi) = 6\pi$.

Circles and Data Sufficiency

A circle is a regular shape whose area and perimeter can be determined through the use of formulas. If you're given virtually any measurement (radius, diameter, circumference, area), you can determine all the other measurements.

Example:



If the length of chord $PQ = 4\sqrt{2}$, what is the circumference of the circle with center O?

Ⓐ 4
Ⓐ 8
Ⓐ 4π
Ⓐ 8π
Ⓐ $8\pi\sqrt{2}$

To find the circumference, we need the radius, which is either OP or OQ in this circle. We are given the length of PQ. PQ is a chord of the circle (it connects two points on the circle), but it's also the hypotenuse of right triangle OPQ. Do we know anything else about that triangle? Since OP and OQ are both radii of the circle, they must have the same length, so the triangle is an isosceles right triangle. Using the ratio of the lengths of sides of a 45:45:90 right triangle, with PQ as the hypotenuse, the length of each radius is 4, making the circumference $2\pi r$ or 8π, answer choice (**D**).

Coordinate Geometry

In coordinate geometry, the locations of points in a plane are indicated by ordered pairs of real numbers.

Important Terms and Concepts

Plane: A flat surface that extends indefinitely in any direction.

***x*-axis and *y*-axis:** The horizontal (x) and vertical (y) lines that intersect perpendicularly to indicate location on a coordinate plane. Each axis is a number line.

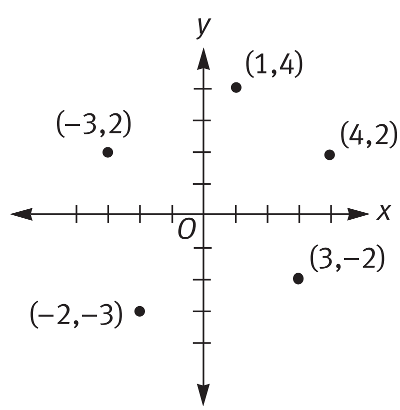
Ordered pair: Two numbers or quantities separated by a comma and enclosed in parentheses. An example would be (8,7). All the ordered pairs that you'll see in GRE coordinate geometry problems will be in the form (x,y), where the first quantity, x, tells you how far the point is to the left or right of the y-axis, and the second quantity, y, tells you how far the point is above or below the x-axis.

Coordinates: The numbers that designate distance from an axis in coordinate geometry. The first number is the x-coordinate; the second is the y-coordinate. In the ordered pair (8,7), 8 is the x-coordinate and 7 is the y-coordinate.

Origin: The point where the x- and y-axes intersect; its coordinates are (0,0).

Plotting Points

Here's what a coordinate plane looks like:

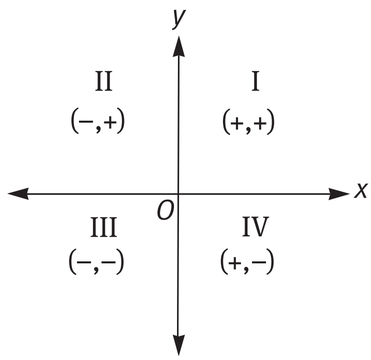


Any point in a coordinate plane can be identified by an ordered pair consisting of its x-coordinate and its y-coordinate. Every point that lies on the x-axis has a y-coordinate of 0, and every point that lies on the y-axis has an x-coordinate of 0.

When you start at the origin and move:

to the right	x is positive
to the left	x is negative
up	y is positive
down	y is negative

Therefore, the coordinate plane can be divided into four quadrants, as shown below.

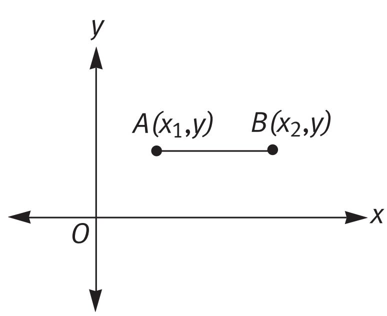


Distances on the Coordinate Plane

The distance between two points is equal to the length of the straight-line segment that has those two points as endpoints.

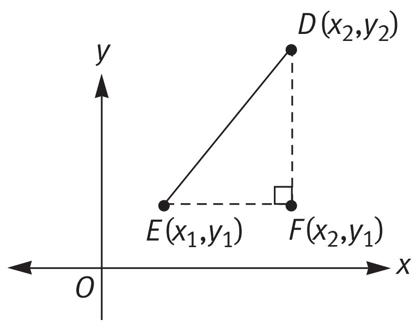
If a line segment is parallel to the *x*-axis, the *y*-coordinate of every point on the line segment will be the same. Similarly, if a line segment is parallel to the *y*-axis, the *x*-coordinate of every point on the line segment will be the same.

Therefore, to find the length of a line segment parallel to one of the axes, all you have to do is find the difference between the endpoint coordinates that do change. In the diagram below, the length of *AB* equals $x_2 - x_1$.



You can find the length of a line segment that is not parallel to one of the axes by treating the line segment as the hypotenuse of a right triangle. Simply draw in the legs of the triangle parallel to the two axes. The length of each leg will be the difference between the *x*- or *y*-coordinates of its endpoints. Once you've found the lengths of the legs, you can use the Pythagorean theorem to find the length of the hypotenuse (the original line segment).

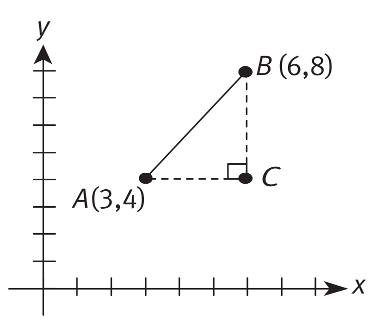
In the diagram below, $(DE)^2 = (EF)^2 + (DF)^2$.



Example:

If the coordinates of point A are (3,4) and the coordinates of point B are (6,8), what is the distance between points A and B?

You don't have to draw a diagram to use the method just described, but drawing one may help you to visualize the problem. Plot points A and B and draw in line segment AB. The length of AB is the distance between the two points. Now draw a right triangle, with AB as its hypotenuse. The missing vertex will be the intersection of a line segment drawn through point A parallel to the x-axis and a line segment drawn through point B parallel to the y-axis. Label the point of intersection C. Since the x- and y-axes are perpendicular to each other, AC and BC will also be perpendicular to each other.



Point C will also have the same x-coordinate as point B and the same y-coordinate as point A. That means that point C has coordinates (6,4).

To use the Pythagorean theorem, you'll need the lengths of AC and BC. The distance between points A and C is simply the difference between their x-coordinates, while the distance between points B and C is the difference between their y-coordinates. So $AC = 6 - 3 = 3$, and $BC = 8 - 4 = 4$. If you recognize these as the legs of a 3:4:5 right triangle, you'll know immediately that the distance between points A and B must be 5. Otherwise, you'll have to use the Pythagorean theorem to come to the same conclusion.

Equations of Lines

Straight lines can be described by linear equations.

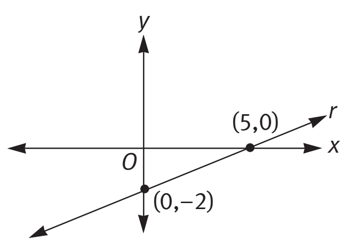
Commonly:

$$y = mx + b,$$

where m is the slope $\left(\frac{\Delta y}{\Delta x}\right)$ and b is the point where the line intercepts the y-axis, that is, the value of y where $x = 0$.

Lines that are parallel to the x-axis have a slope of zero and therefore have the equation $y = b$. Lines that are parallel to the y-axis have the equation $x = a$, where a is the x-intercept of that line.

If you're comfortable with linear equations, you'll sometimes want to use them to find the slope of a line or the coordinates of a point on a line. However, many such questions can be answered without determining or manipulating equations. Check the answer choices to see if you can eliminate any by common sense.



Example:

Line r is a straight line as shown above. Which of the following points lies on line r?

Ⓐ (6,6)
Ⓑ (7,3)
Ⓒ (8,2)
Ⓓ (9,3)
Ⓔ (10,2)

Line r intercepts the y-axis at (0,–2), so you can plug –2 in for b in the slope-intercept form of a linear equation. Line r has a rise (Δy) of 2 and a run (Δx) of 5, so its slope is $\frac{2}{5}$. That makes the slope-intercept form $y = \frac{2}{5}x - 2$.

The easiest way to proceed from here is to substitute the coordinates of each answer choice into the equation in place of x and y; only the coordinates that satisfy the equation can lie on the line. Choice **(E)** is the best answer to start with, because 10 is the only x-coordinate that will not create a fraction on the right side of the equal sign. Plugging in (10,2) for x and y in the slope-intercept equation gives you $2 = \frac{2}{5}(10) - 2$, which simplifies to $2 = 4 - 2$.

That's true, so the correct answer choice is **(E)**.

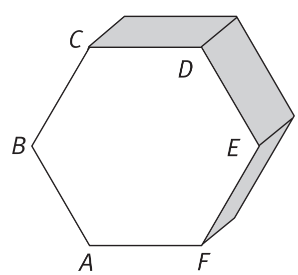
Solids

Important Terms

Solid: A three-dimensional figure. The dimensions are usually called length, width, and height (ℓ, w, and h) or height, width, and depth (h, w, and d). There are only two types of solids that appear with any frequency on the GRE: rectangular solids (including cubes) and cylinders.

Uniform solid: A solid that could be cut into congruent cross sections (parallel "slices" of equal size and shape) along a given axis. Solids you see on the GRE will almost certainly be uniform solids.

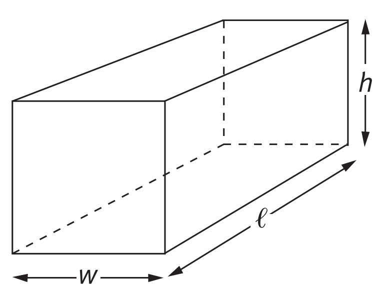
Face: The surface of a solid that lies in a particular plane. Hexagon *ABCDEF* is one face of the solid pictured below.



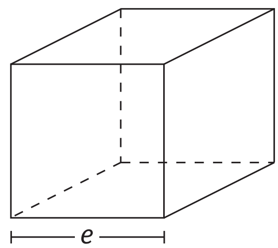
Edge: A line segment that connects adjacent faces of a solid. The sides of hexagon *ABCDEF* are also edges of the solid pictured above.

Base: The "bottom" face of a solid as oriented in any given diagram.

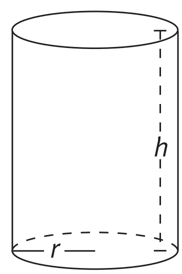
Rectangular solid: A solid with six rectangular faces. All edges meet at right angles. Examples of rectangular solids are cereal boxes, bricks, etc.



Cube: A special rectangular solid in which all edges are of equal length, e, and therefore all faces are squares. Sugar cubes and dice without rounded corners are examples of cubes.



Cylinder: A uniform solid whose horizontal cross section is a circle—for example, a soup can or a pipe that is closed at both ends. A cylinder's measurements are generally given in terms of its radius, r, and its height, h.



Lateral surface of a cylinder: The "pipe" surface, as opposed to the circular "ends." The lateral surface of a cylinder is unlike most other surfaces of solids that you'll see on the GRE, first because it does not lie in a plane and second because it forms a closed loop. Think of it as the label around a soup can. If you could remove it from the can in one piece, you would have an open tube. If you then cut the label and unrolled it, it would form a rectangle with a length equal to the circumference of the circular base of the can and a height equal to that of the can.

Formulas for Volume and Surface Area

Volume of a rectangular solid = (Area of base) (Height) = (Length × Width) (Height) = lwh

Surface area of a rectangular solid = Sum of areas of faces = $2lw + 2lh + 2hw$

Since a cube is a rectangular solid for which $l = w = h$, the formula for its volume can be stated in terms of any edge:

- Volume of a cube = lwh = (Edge)(Edge)(Edge) = e^3
- Surface area of a cube = Sum of areas of faces = $6e^2$

To find the volume or surface area of a cylinder, you'll need two pieces of information: the height of the cylinder and the radius of the base.

- Volume of a cylinder = (Area of base)(Height) = $\pi r^2 h$
- Lateral surface area of a cylinder = (Circumference of base)(Height) = $2\pi rh$
- Total surface area of a cylinder = Areas of circular ends + Lateral surface area = $2\pi r^2 + 2\pi rh$

Multiple Figures

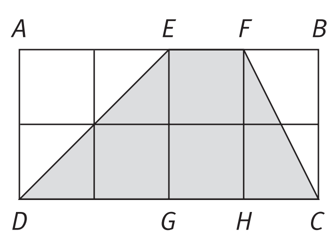
Some GRE geometry problems involve combinations of different types of figures. Besides the basic rules and formulas that you would use on normal geometry problems, you'll need an intuitive understanding of how various geometrical concepts relate to each other to answer these "multiple figures" questions correctly. For example, you may have to revisualize the side of a rectangle as the hypotenuse of a neighboring right triangle or as the diameter of a circumscribed circle. Keep looking for the relationships between the different figures until you find one that leads you to the answer.

Area of Shaded Regions

A common multiple-figures question involves a diagram of a geometrical figure that has been broken up into different, irregularly shaped areas, often with one region shaded. You'll usually be asked to find the area of the shaded (or unshaded) portion of the diagram. Your best bet will be to take one of the following two approaches:

- Break the area into smaller pieces whose separate areas you can find; add those areas together.
- Find the area of the whole figure; find the area of the region(s) that you're *not* looking for; subtract the latter from the former.

Example:



Rectangle *ABCD* above has an area of 72 and is composed of 8 equal squares. What is the area of the shaded region?

The first thing you have to realize is that, for the 8 equal squares to form a total area of 72, each square must have an area of $72 \div 8$, or 9. Since the area of a square equals the square of the length of a side, each side of a square in the diagram must have a length of $\sqrt{9}$ or 3.

At this point, you choose your approach. Either one will work:

Approach 1:

Break up the shaded area into right triangle *DEG*, rectangle *EFHG*, and right triangle *FHC*. The area of triangle *DEG* is $\frac{1}{2}(6)(6) = 18$. The area of rectangle *EFHG* is (3)(6), or 18. The area of triangle *FHC* is $\frac{1}{2}(3)(6)$, or 9. The total shaded area is 18 + 18 + 9, or 45.

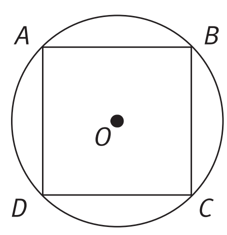
Approach 2:

The area of unshaded right triangle *AED* is $\frac{1}{2}(6)(6)$, or 18. The area of unshaded right triangle *FBC* is $\frac{1}{2}(3)(6)$, or 9. Therefore, the total unshaded area is 18 + 9 = 27. Subtract the total unshaded area from the total area of rectangle *ABCD*: 72 – 27 = 45.

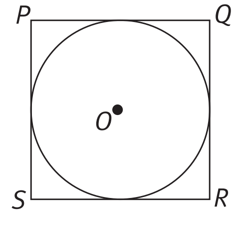
Inscribed/Circumscribed Figures

A polygon is inscribed in a circle if all the vertices of the polygon lie on the circle. A polygon is circumscribed about a circle if all the sides of the polygon are tangent to the circle.

Square *ABCD* is inscribed in circle *O*. We can also say that circle *O* is circumscribed about square *ABCD*.

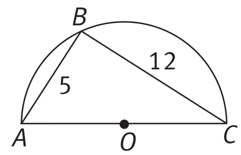


Square *PQRS* is circumscribed about circle *O*. We can also say that circle *O* is inscribed in square *PQRS*.



When a triangle is inscribed in a semicircle in such a way that one side of the triangle coincides with the diameter of the semicircle, the triangle is a right triangle.

Example:



What is the diameter of semicircle O above?

AC is a diameter of semicircle O because it passes through center point O. So triangle ABC fits the description given above of a right triangle. Moreover, triangle ABC is a special 5:12:13 right triangle with a hypotenuse of 13. Therefore, the length of diameter AC is 13.

OTHER TOPICS

Dealing with Word Problems

The key to solving word problems is translation: turning English into math. Rather than having an equation set up for you, *you* have to decide what arithmetic or algebraic operations to perform on which numbers.

For example, suppose the core of a problem involves working with the equation $3j = s - 4$.

In a word problem, this might be presented as "If John had three times as many macaroons as he has now, he would have four fewer macaroons than Susan would."

Your job is to translate the problem from English into math. A phrase like "three times as many as John has" can be translated as $3j$; the phrase "four fewer than Susan" can be translated as "$s - 4$."

Many people dislike word problems. But on the GRE, the math involved is often easier than in other math problems. Once you've translated the language, most word problems boil down to rather simple mathematical concepts and processes—probably because the testmakers figure that the extra step of translation makes the problem difficult enough.

Here's a general approach to any word problem:

1. Read through the whole question once, without lingering over details, to get a sense of the overall problem.
2. Identify and label the variables or unknowns in a way that makes it easy to remember what they stand for.
3. Translate the problem into one or more equations, sentence by sentence. Be careful of the order in which you translate the terms. For example, consider the phrase "5 less than $4x$ equals 9." The *correct* way to translate it is "$4x - 5 = 9$."

But many students make the mistake of writing the terms in the order in which they appear in words: "$5 - 4x = 9$."

4. Solve the equation(s).
5. Check your work, if time permits.

Translation Table

This table contains common phrases used in GRE math problems. The left column lists words and phrases that occur frequently; the right column lists the corresponding algebraic symbols.

equals, is, was, will be, has, costs, adds up to, is the same as	$=$
times, of, multiplied by, product of, twice, double, half, triple	$\times$
divided by, per, out of, each, ratio of _ to _	$\div$
plus, added to, sum, combined, and, total	$+$
minus, subtracted from, less than, decreased by, difference between	$-$
what, how much, how many, a number	variable (x, n, etc.)

Example:

Beatrice has three dollars more than twice the number of dollars Allan has.

Translate into $B = 3 + 2A$.

For Word Problems:

Add...

- when you are given the amounts of individual quantities and asked to find the total.

Example:

If the sales tax on a \$12.00 lunch is \$1.20, what is the total amount of the check?

$$\$12.00 + \$1.20 = \$13.20$$

- when you are given an original amount and an increase and are then asked to find the new amount.

Example:

The bus fare used to be 55 cents. If the fare increased by 35 cents, what is the new fare?

$$55 \text{ cents} + 35 \text{ cents} = 90 \text{ cents}$$

Subtract...

- when you are given the total and one part of the total and you want to find the remaining part or parts.

Example:

If 32 out of 50 children are girls, what is the number of boys?

$$50 \text{ children} - 32 \text{ girls} = 18 \text{ boys}$$

- when you are given two numbers and asked *how much more* or *how much less* one number is than the other. The amount is called the **difference**.

Example:

How much larger than 30 is 38?

$$38 \text{ (larger)} - 30 \text{ (smaller)} = 8$$

Multiply...

- when you are given an amount for one item and asked for the total amount of *many* of these items.

Example:

If 1 book costs \$6.50, what is the cost of 12 copies of the same book?

$$12(\$6.50) = \$78.00$$

Divide...

- when you are given a total amount for *many* items and asked for the amount for *one* item.

Example:

If 5 pounds of apples cost \$6.75, what is the price of 1 pound of apples?

$$\$6.75 \div 5 = \$1.35$$

- when you are given the size of one group and the total size for many such identical groups and are asked how many of the small groups fit into the larger one.

Example:

How many groups of 30 students can be formed from a total of 240 students?

$$240 \div 30 = 8 \text{ groups of 30 students}$$

SPECIAL WORD PROBLEMS TIP #1
Don't try to combine several sentences into one equation; each sentence usually translates into a separate equation.

SPECIAL WORD PROBLEMS TIP #2
Pay attention to what the question asks for and make a note to yourself if it is not one of the unknowns in the equation(s). Otherwise, you may stop working on the problem too early.

Logic Problems

You won't always have to set up an equation to solve a word problem. Some of the word problems you'll encounter on the GRE won't fall into recognizable textbook categories. Many of these problems are designed to test your analytical and deductive logic. You can solve them with common sense and a little basic arithmetic. Ask yourself how it would be helpful to arrange the information, such as by drawing a diagram or making a table.

In these problems, the issue is not so much translating English into math as simply using your head. The problem may call for nonmath skills, including the ability to organize and keep track of different possibilities, the ability to visualize something (for instance, the reverse side of a symmetrical shape), the ability to think of the exception that changes the answer to a problem, or the ability to deal with overlapping groups.

Example:

If ! and ∫ are digits and $(!!)(\int\int) = 60\int$, what is the value of ∫ ?

Since each of the symbols represents a digit from 0–9, we know that the product of the multiplication equals a value from 600 to 609. We know that the two quantities multiplied each consist of a two-digit integer in which both digits are the same. So list the relevant two-digit integers (00, 11, 22, 33, 44, 55, 66, 77, 88, and 99) and see which two of them can be multiplied evenly into the 600 to 609 range. Only (11)(55) satisfies this requirement. The ∫ symbol equals 5.

Tables, Graphs, and Charts

Some questions, especially in Data Interpretation, combine numbers and text with visual formats. Different formats are suitable for organizing different types of information. The formats that appear most frequently on GRE math questions are tables, bar graphs, line graphs, and pie charts.

Questions involving tables, graphs, and charts may *look* different from other GRE math questions, but the ideas and principles are the same. The problems are unusual only in the way that they present information, not in what they ask you to do with that information.

Tables

The most basic way to organize information is to create a table. Tables are in some ways the most accurate graphic presentation format—the only way you can misunderstand a number is to read it from the wrong row or column—but they don't allow the reader to spot trends or extremes very readily.

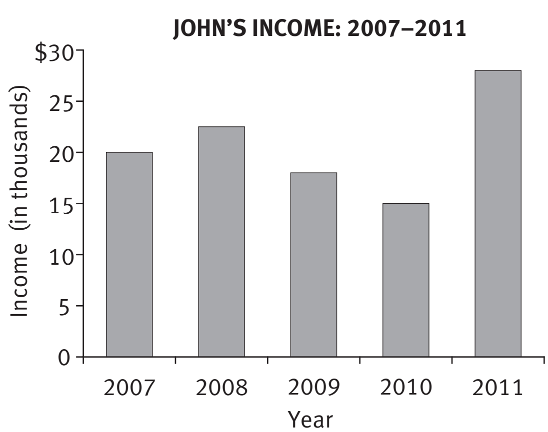
Here's an example of a very simple table.

JOHN'S INCOME: 2007–2011	
Year	Income
2007	$20,000
2008	$22,000
2009	$18,000
2010	$15,000
2011	$28,000

An easy question might ask for John's income in a particular year or for the difference in his income between two years. To find the difference, you would simply look up the amount for both years and subtract the smaller income from the larger income. A harder question might ask for John's average annual income over the five-year period shown; to determine the average, you would have to find the sum of the five annual incomes and divide it by 5.

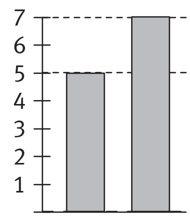
Bar Graphs

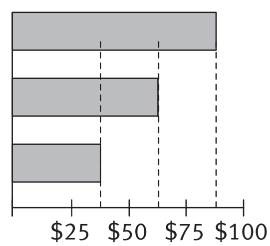
Here's the same information that you saw previously in a table. This time, it's presented as a bar graph.



Bar graphs can be used to visually show information that would otherwise appear as numbers in a table. Bar graphs are somewhat less accurate than tables, but that's not necessarily a bad attribute, especially on the GRE, where estimating often saves time on calculations.

What's handy about a bar graph is that you can see which values are larger or smaller without reading actual numbers. Just a glance at this graph shows that John's 2011 income was almost double his 2010 income. Numbers are represented on a bar graph by the heights or lengths of the bars. To find the height of a vertical bar, look for the point where a line drawn across the top of the bar parallel to the horizontal axis would intersect the vertical axis. To find the length of a horizontal bar, look for the point where a line drawn across the end of the bar parallel to the vertical axis would intersect the horizontal axis.

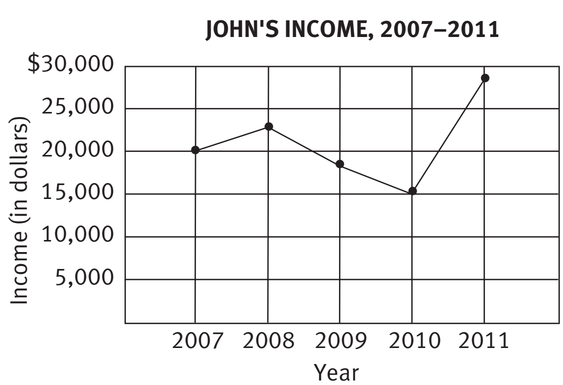




If the height or length of the bar falls in between two numbers on the axis, you will have to estimate.

Line Graphs

Line graphs follow the same general principle as bar graphs, except that instead of using the lengths of bars to represent numbers, they use points connected by lines. The lines further emphasize the relative values of the numbers.



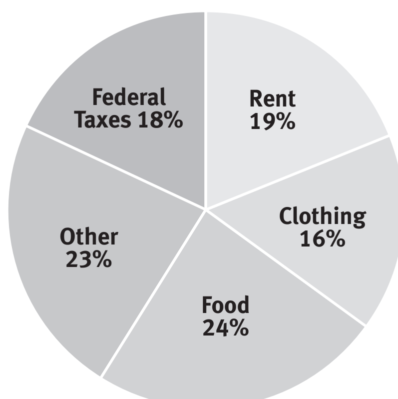
To read John's income for any particular year from this line graph, determine where a line drawn from the appropriate point would intersect the vertical axis.

Pie Charts

Pie charts show how things are distributed. The fraction of a circle occupied by each piece of the "pie" indicates what fraction of the whole that piece represents. In most pie charts, the percentage of the pie occupied by each "slice" will be shown on the slice itself or, for very narrow slices, outside the circle with an arrow or a line pointing to the appropriate slice.

The total size of the whole pie is usually given at the top or bottom of the graph, either as "TOTAL = xxx" or as "100% = xxx." To find the approximate amount represented by a particular piece of the pie, just multiply the whole by the appropriate percent.

JOHN'S EXPENDITURES, 2007
TOTAL = $20,000



For instance, to find the total tax that John paid to the federal government in 2007, look at the slice of this chart labeled "Federal Tax." It represents 18% of John's 2007 expenditures. Since his total 2007 expenditures were $20,000, he paid 0.18($20,000) = $3,600 in federal taxes in 2007.

One important note about pie charts: If you're not given the whole and you don't know both the percentage and the actual number that at least one slice represents, you won't be able to find the whole. Pie charts are ideal for presenting the kind of information that ratio problems present in words.

CHAPTER 11

Quantitative Comparison

INTRODUCTION TO QUANTITATIVE COMPARISON

In each Quantitative Comparison question, you'll see two mathematical expressions. One is Quantity A, and the other is Quantity B. You will be asked to compare them. Some questions include additional centered information. This centered information applies to both quantities and is essential to making the comparison. Since this type of question is about the relationship between the two quantities, you usually won't need to calculate a specific value for either quantity. Therefore, you do not want to rely on the onscreen calculator to answer these questions.

The directions for a Quantitative Comparison question will look like this:

Directions: Select the correct answer.

Sample Question

Exit Section | Review | Mark | Help | Back | Next

$$h > 1$$

Quantity A	Quantity B
The number of minutes in h hours	$\frac{60}{h}$

- Quantity A is greater.
- Quantity B is greater.
- The two quantities are equal.
- The relationship cannot be determined from the information given.

Click to select your choice.

THE KAPLAN METHOD FOR QUANTITATIVE COMPARISON

STEP 1 Analyze the centered information and quantities.

STEP 2 Approach strategically.

How the Kaplan Method for Quantitative Comparison Works

Now let's discuss how the Kaplan Method for Quantitative Comparison works.

STEP 1

Analyze the centered information and the quantities.

Notice whether the quantities contain numbers, variables, or both. If there is centered information, decide how it affects the information given in the quantities. Note that a variable has the same value each time it appears within a question.

STEP 2

Approach strategically.

Think about a strategy you could use to compare the quantities now that you've determined the information you have and the information you need. There are a variety of approaches to solving a Quantitative Comparison question, and the practice examples will take you through several of these.

How to Apply the Kaplan Method for Quantitative Comparison

Now let's apply the Kaplan Method to a Quantitative Comparison question:

Quantity A	Quantity B
$\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}$	$\dfrac{1}{\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}}$

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

STEP 1

Analyze the centered information and the columns.

This problem would be a nightmare to calculate under timed conditions. But the only thing you need to figure out is whether one quantity is greater than the other. One thing you might notice is that choice **(D)** is not an option here. Because both quantities contain only numbers, there is a definite value for each quantity, and a relationship can be determined. Answer choice **(D)** is never correct when the quantities contain only numbers.

Note that the quantity on the left is the same as the quantity in the denominator of the fraction on the right. You can think about this problem as a comparison of x and $\frac{1}{x}$ (or the reciprocal of x), where x has a definite value. Your job now is to figure out just how to compare them.

STEP 2

Approach strategically.

Before you start to do a long calculation, think about what you already know. While you may not know the sum of the four fractions, you do know two things: $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$, and $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{7}$ are each less than $\frac{1}{4}$. Because the reciprocal of any number between 0 and 1 is greater than 1, and Quantity A is a positive number less than 1, its reciprocal in Quantity B is greater than 1. So choice **(B)** is correct. Quantitative Comparisons rarely, if ever, ask for exact values, so don't waste time calculating them.

Now let's apply the Kaplan Method to a second Quantitative Comparison question:

$$w > x > 0 > y > z$$

Quantity A	Quantity B
$w + y$	$x + z$

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

STEP 1

Analyze the centered information and the quantities.

In this problem, there are four variables: *w*, *x*, *y*, and *z*. You are asked to compare the values of the sums of pairs of variables. You know the relative values of the different variables, but you don't know the actual amounts. You do know that two of the variables (*w* and *x*) must be positive and two of the variables (*y* and *z*) must be negative numbers.

STEP 2

Approach strategically.

In this case, think about the different sums as pieces of the whole. If every "piece" in one quantity is greater than a corresponding "piece" in the other quantity, and if the only operation involved is addition, then the quantity with the greater individual values will have the greater total value. From the given information, we know the following:

- $w > x$
- $y > z$

The first term, *w*, in Quantity A is greater than the first term, *x*, in Quantity B. Similarly, the second term, *y*, in Quantity A is greater than the second term, *z*, in Quantity B. Because each piece in Quantity A is greater than the corresponding piece in Quantity B, Quantity A must be greater; the answer is **(A)**.

Now let's apply the Kaplan Method to a third Quantitative Comparison question:

The diameter of circle O is d, and the area is a.

Quantity A	Quantity B
$\frac{\pi d^2}{2}$	a

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

STEP 1

Analyze the centered information and the quantities.

In this problem, you are given additional information: the sentence that tells you the diameter of circle *O* is *d* and the area is *a*. This is important information because it gives you a key to unlocking this question. Given that information, you can tell that you are comparing the area, *a*, of circle *O* and a quantity that includes the diameter of the same circle. If you're thinking about the formula for calculating area given the diameter, you're thinking right!

STEP 2

Approach strategically.

Make Quantity B look more like Quantity A by rewriting a, the area of the circle, in terms of the diameter, d. The area of any circle equals πr^2, where r is the radius. Because the radius is half the diameter, you can substitute $\frac{d}{2}$ for r in the area formula to get $a = \pi r^2 = \pi\left(\frac{d}{2}\right)^2$ in Quantity B. Simplifying, you get $\frac{\pi d^2}{4}$.

Because both quantities contain π, we could compare $\frac{d^2}{2}$ to $\frac{d^2}{4}$. But let's take it one step further. You know that d is a distance and must be a positive number. That makes it possible to divide both quantities, $\frac{d^2}{2}$ and $\frac{d^2}{4}$, by d^2 and then just compare $\frac{1}{2}$ to $\frac{1}{4}$. This makes it easy to see that Quantity A is always greater because $\frac{1}{2} > \frac{1}{4}$. Choice **(A)** is correct.

KAPLAN'S ADDITIONAL TIPS FOR QUANTITATIVE COMPARISON QUESTIONS

Memorize the answer choices

It is a good idea to memorize what the Quantitative Comparison answer choices mean. This is not as difficult as it sounds. The choices are always the same. The wording and the order never vary. As you work through the practice problems, the choices will become second nature to you, and you will get used to reacting to the questions without reading the four answer choices, thus saving you lots of time on Test Day.

When there is at least one variable in a problem, try to demonstrate two different relationships between quantities

Here's why demonstrating two different relationships between the quantities is an important strategy: if you can demonstrate two different relationships, then choice **(D)** is correct. There is no need to examine the question further.

But how can this demonstration be done efficiently? A good suggestion is to look at the expression(s) containing a variable and notice the possible values of the variable given the mathematical operation involved. For example, if x can be any real number and you need to compare $(x + 1)^2$ to $(x + 1)$, pick a value for x that will make $(x + 1)$ a fraction between 0 and 1 and then pick a value for x that will make $(x + 1)$ greater than 1. By choosing values for x in this way, you are basing your number choices on mathematical properties you already know: a positive fraction less than 1 becomes smaller when squared, but a number greater than 1 grows larger when squared.

Compare quantities piece by piece

Compare the value of each "piece" in each quantity. If every "piece" in one quantity is greater than a corresponding "piece" in the other quantity, and the operation involved is either addition or multiplication, then the quantity with the greater individual values will have the greater total value.

Make one quantity look like the other

When the Quantities A and B are expressed differently, you can often make the comparison easier by changing the format of one quantity so that it looks like the other. This is a great approach when the quantities look so different that you can't compare them directly.

Do the same thing to both quantities

If the quantities you are given seem too complex to compare immediately, look closely to see if there is an addition, subtraction, multiplication, or division operation you can perform on both quantities to make them simpler—provided you do not multiply or divide by zero or a negative number. For example, suppose you have the task of comparing $1 + \frac{w}{1+w}$ to $1 + \frac{1}{1+w}$, where w is greater than 0. To get to the heart of the comparison, subtract 1 from both quantities and you have $\frac{w}{1+w}$ compared to $\frac{1}{1+w}$. To simplify even further, multiply both quantities by $(1 + w)$, and then you can compare w to 1—much simpler.

Don't be tricked by misleading information

To avoid Quantitative Comparison traps, stay alert and don't assume anything. If you are using a diagram to answer a question, use only information that is given or information that you know must be true based on properties or theorems. For instance, don't assume angles are equal or lines are parallel unless it is stated or can be deduced from other information given.

A common mistake is to assume that variables represent only positive integers. As you saw when using the Picking Numbers strategy, fractions or negative numbers often show a different relationship between the quantities.

Don't forget to consider other possibilities

If an answer looks obvious, it may very well be a trap. Consider this situation: a question requires you to think of two integers whose product is 6. If you jump to the conclusion that 2 and 3 are the integers, you will miss several other possibilities.

Not only are 1 and 6 possibilities, but there are also pairs of negative integers to consider: -2 and -3, -1 and -6.

Don't fall for look-alikes

Even if two expressions look similar, they may be mathematically different. Be especially careful with expressions involving parentheses or radicals. If you were asked to compare $\sqrt{5x} + \sqrt{5x}$ to $\sqrt{10x}$, you would not want to fall into the trap of saying the two expressions were equal. Although time is an important factor in taking the GRE, don't rush to the extent that you do not apply your skills correctly. In this case, $\sqrt{5x} + \sqrt{5x} = 2\sqrt{5x}$, which is not the same as $\sqrt{10x}$ unless $x = 0$.

QUANTITATIVE COMPARISON PRACTICE SET

Try the following Quantitative Comparison questions using the Kaplan Method for Quantitative Comparison. If you're up to the challenge, time yourself; on Test Day, you'll want to spend only 1.5 minutes on each question.

1.

Quantity A	Quantity B
$x^2 + 2x - 2$	$x^2 + 2x - 1$

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

2. $x = 2y$; y is a positive integer.

Quantity A	Quantity B
4^{2y}	2^x

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

3. q, r, and s are positive numbers; $qrs > 12$.

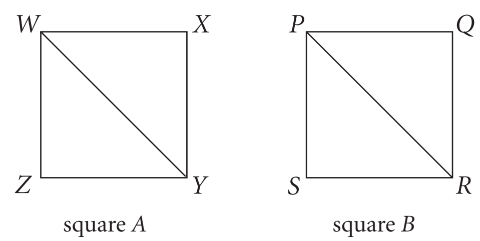
Quantity A	Quantity B
$\frac{qr}{5}$	$\frac{3}{s}$

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

4. In triangle XYZ not given, the measure of angle X equals the measure of angle Y.

Quantity A	Quantity B
The degree measure of angle Z	The degree measure of angle X plus the degree measure of angle Y

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.



5.

Quantity A	Quantity B
$\dfrac{\text{Perimeter of square } A}{\text{Perimeter of square } B}$	$\dfrac{\text{Length of } WY}{\text{Length of } PR}$

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

QUANTITATIVE COMPARISON PRACTICE SET ANSWERS AND EXPLANATIONS

1. B

Comparing the two quantities piece by piece, you find that the only difference is the third piece: -2 in Quantity A and -1 in Quantity B. You don't know the value of x, but whatever it is, x^2 in Quantity A must have the same value as x^2 in Quantity B, and $2x$ in Quantity A must have the same value as $2x$ in Quantity B. Because any quantity minus 2 must be less than that quantity minus 1, Quantity B is greater than Quantity A. The correct choice is **(B)**.

2. A

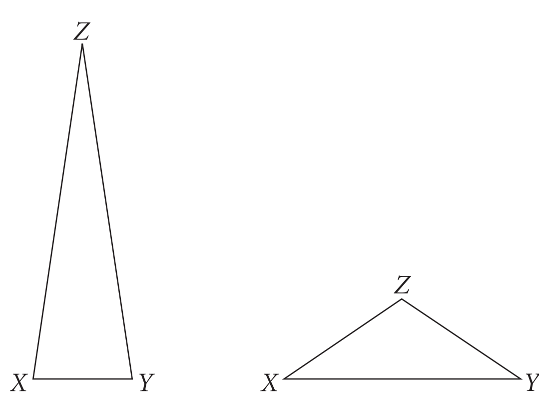
Replacing the exponent x in Quantity B with the equivalent value given in the centered information, you're comparing 4^{2y} with 2^{2y}. Because y is a positive integer, raising 4 to the exponent $2y$ will result in a greater value than raising 2 to the exponent $2y$. The correct choice is **(A)**.

3. D

Do the same thing to both quantities to make them look like the centered information. When you multiply both quantities by $5s$, you get qrs in Quantity A and 15 in Quantity B. Because qrs could be any integer greater than 12, qrs could be greater than, equal to, or less than 15. Choice **(D)** is correct.

4. D

Because angle $X =$ angle Y, at least two sides of the triangle are equal. You can draw two diagrams with X and Y as the base angles of a triangle. In one diagram, make the triangle tall and narrow so that angle X and angle Y are very large and angle Z is very small. In this case, Quantity B is greater. In the second diagram, make the triangle short and wide so that angle Z is much larger than angle X and angle Y. In this case, Quantity A is greater. Because more than one relationship between the quantities is possible, the correct answer is **(D)**.



5. C

You don't know the exact relationship between square *A* and square *B*, but it doesn't matter. The problem is actually just comparing the ratios of corresponding parts of two squares. The relationship between the specific side lengths of both squares will also exist between them for any other corresponding length. If a side of one square is twice the length of a side of the second square, the diagonal will also be twice as long. The ratio of the perimeters of the two squares is the same as the ratio of the diagonals.

You can make this abstract relationship concrete by Picking Numbers for the sides of the two squares. Say, for example, that each side of square *A* is 2 and each side of square *B* is 3. Then the ratio of the perimeters is 8:12 or 2:3, and the ratio of the diagonals is $2\sqrt{2}:3\sqrt{2}$ or $2:3$. Therefore, the quantities are equal. Choice **(C)** is correct.

CHAPTER 12

Problem Solving

INTRODUCTION TO PROBLEM SOLVING

Problem Solving can be broken up into several general mathematics categories: algebra, arithmetic, number properties, and geometry.

In a Problem Solving question, you may be asked to solve a pure math problem or a word problem involving a real-world situation. You will be asked to enter your answer into an onscreen box, select one answer, or select one or more options that correctly answer the problem.

The directions for a Problem Solving question requiring a single answer will look like this:

> **Directions:** Click to select your choice.

A Problem Solving question requiring you to select a single answer will look like this, with ovals next to each answer choice:

Sample Question

Exit Section | Review | Mark | Help | Back | Next

A health club charges $35 per month plus $2.50 for each aerobics class attended. How many aerobics classes were attended in a certain month if the total monthly charge was $52.50?

- ◯ 7
- ◯ 8
- ◯ 9
- ◯ 10
- ◯ 11

Click to select your choice.

The directions for a Problem Solving question requiring you to select one or more answers will look like this:

Directions: Click to select your choice(s).

If a Problem Solving question asks you to select your choice(s), at least one answer is correct, but as many as all the choices may be correct. You must select all of the correct choices (and none of the incorrect ones) for the question to be counted as correct.

A Problem Solving question requiring you to select one or more answers will look like this, with rectangles next to each answer choice:

Sample Question

Exit Section | Review | Mark | Help | Back | Next

If $0 < x < 1$, which of the following *must* be true?

Choose <u>all</u> possible answers.

- ☐ $2x < x$
- ☐ $2x < 1$
- ☐ $2x > 1$
- ☐ $x^2 < x$
- ☐ $x^2 < 1$

Click to select your choice(s).

The directions for a Problem Solving question requiring you to make a Numeric Entry will look like this:

> **Directions:** Click in the box and type your numeric answer. Backspace to erase.

Enter your answer as an integer or decimal if there is one box or as a fraction if there are two boxes.

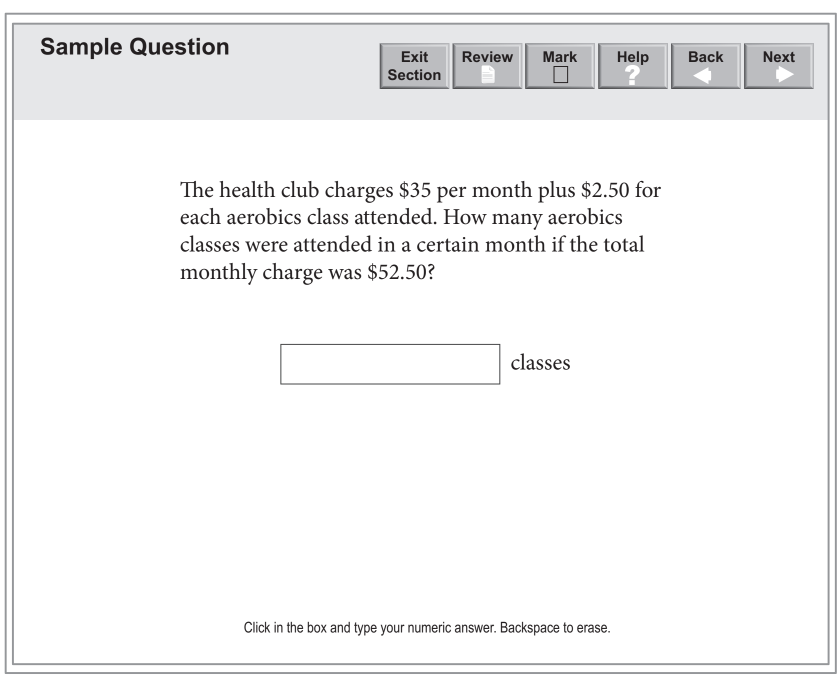
To enter an integer or decimal, type directly in the box or use the Transfer Display button on the calculator.

- Use the backspace key to erase.
- Use a hyphen to enter a negative sign; type a hyphen a second time to remove it. The digits will remain.
- Use a period for a decimal point.
- The Transfer Display button will enter your answer directly from the calculator.
- Equivalent forms of decimals are all correct. (*Example:* $0.14 = 0.140$)
- Enter the exact answer unless the question asks you to round your answer.

To enter a fraction, type the numerator and denominator in the appropriate boxes.

- Use a hyphen to enter a negative sign.
- The Transfer Display button does not work for fractions.
- Equivalent forms of fractions are all correct. (*Example:* $\frac{25}{15} = \frac{5}{3}$.) If numbers are large, reduce fractions to fit in boxes.

A Problem Solving question with Numeric Entry will look like this:



THE KAPLAN METHOD FOR PROBLEM SOLVING

STEP 1 **Analyze the question.**

STEP 2 **Identify the task.**

STEP 3 **Approach strategically.**

STEP 4 **Confirm your answer.**

How the Kaplan Method for Problem Solving Works

Now let's discuss how the Kaplan Method for Problem Solving works:

STEP 1

Analyze the question.

Look at what the question is asking and what area of math is being tested. Also note any particular trends in the answer choices (e.g., numbers/variables, integers/non-integers) and what information is being given. Unpack as much information as possible.

STEP 2

Identify the task.

Determine what question is being asked before solving the problem. Ask yourself, "What does the correct answer represent?" The GRE intentionally provides wrong answers for test takers who get the right answer to the wrong question.

STEP 3

Approach strategically.

Depending on the type of problem, you may use straightforward math—the textbook approach—to calculate your answer, or you may choose one of the following strategies: Picking Numbers, Backsolving, or Strategic Guessing.

When Picking Numbers to substitute for variables, choose numbers that are manageable and fit the description given in the problem. Backsolving is another form of Picking Numbers; you'll start with one of the answer choices and plug that choice back into the question. Lastly, Strategic Guessing can be a great time-saver on the GRE—being able to make a smart guess on a question is preferable to taking too much time and thus compromising your ability to answer other questions correctly.

STEP 4

Confirm your answer.

Check that your answer makes sense. Also check that you answered the question that was asked.

How to Apply the Kaplan Method for Problem Solving

Now let's apply the Kaplan Method to a Problem Solving question:

> In a bag of candy, 7 of the candies are cherry flavored, 8 are lemon, and 5 are grape. If a candy is chosen randomly from the bag, what is the probability that the candy is *not* lemon?

$$\frac{\boxed{}}{\boxed{}}$$

STEP 1

Analyze the question.

You are given the number of candies in a bag and asked to identify the probability that a randomly selected candy is not lemon flavored. You will have to type your answer into the box.

STEP 2

Identify the task.

The probability of an event is defined as $\frac{\text{Number of desired outcomes}}{\text{Number of possible outcomes}}$. You will need to find the number of desired outcomes (those in which you don't choose a lemon candy) and the total number of possible outcomes.

STEP 3

Approach strategically.

There are 20 candies in the bag, so there are 20 possible outcomes. Of all the candies, 12 are not lemon, so there are 12 desired outcomes. So, the probability of *not* lemon is $\frac{12}{20}$. You should avoid reducing fractions for Numeric Entry questions, since all equivalent forms will be counted as correct. Save your time for other questions and limit your risk of committing an error in calculation.

STEP 4

Confirm your answer.

Although it might be fun to get a bag of candies and check your answer in a real-world way, it's not practical, especially on Test Day. A more practical check would be to find the probability of choosing a lemon candy at random to be certain that $P(\text{lemon}) = 1 - P(\text{not lemon})$. There are 8 lemon candies out of 20, so this check can be done easily.

$$
\begin{aligned}
P(\text{lemon}) &\stackrel{?}{=} 1 - P\,(\text{not lemon}) \\
\frac{8}{20} &\stackrel{?}{=} 1 - \frac{12}{20} \\
\frac{8}{20} &\stackrel{?}{=} \frac{20}{20} - \frac{12}{20} \\
\frac{8}{20} &= \frac{8}{20}
\end{aligned}
$$

This check is a way to confirm that the correct numbers have been used in the problem and the correct answer has been found.

Now let's apply the Kaplan Method to a second Problem Solving question:

> When n is divided by 14, the remainder is 10. What is the remainder when n is divided by 7?
>
> (A) 2
> (B) 3
> (C) 4
> (D) 5
> (E) 6

STEP 1

Analyze the question.

In this question, you are asked to compare the relationship between the numbers 14 and 7 used as divisors.

STEP 2

Identify the task.

The task is to use the fact that division of a number, n, by 14 yields a remainder of 10 to identify the remainder when the same number is divided by 7.

STEP 3

Approach strategically.

A good strategy for this question is to pick a number for n that satisfies the condition for division by 14 and then see what happens when it is divided by 7.

Any number divided by itself will give a remainder of zero. So if we need a remainder of 10, we want a number that is 10 more than the number we are dividing by. Be careful; you may be thinking of choosing $14 \div 7 = 2$ or $10 \div 2 = 5$. But these are both trap answer choices because the question also involves using a remainder. Therefore, 24 is a great number to pick here, because when we try 24:

$24 \div 14 = 1$ Remainder 10

Now that we've confirmed that 24 works, we answer the question that's being asked. Divide 24 by 7:

$24 \div 7 = 3$ Remainder 3

Answer choice **(B)** is the correct answer.

STEP 4

Confirm your answer.

You can quickly double-check your work, or you can try another number for *n* that results in a remainder of 10 when divided by 14:

$38 \div 14 = 2$ Remainder 10, and $38 \div 7 = 5$ Remainder 3

So the remainder is 3 in each case. The correct answer is **(B)**.

Now let's apply the Kaplan Method to a third Problem Solving question:

The line $4x + 6y = 24$ passes through which of the following points?

Indicate <u>all</u> possible answers.

- A (0,4)
- B (2,3)
- C (3,2)
- D (5,4)
- E (9,–1)

STEP 1

Analyze the question.

This question is about a line on the coordinate plane. The equation is a function that represents a line. The numbers in the parentheses in the answer choices represent points (x,y) that are mentioned in the equation.

STEP 2

Identify the task.

Your job is to identify which of the given points lie on the line. A line passes through a point if the coordinates of the point make the equation of the line true, so this is the same as saying that you need to find out which point(s), when plugged into the equation, make the equation true.

STEP 3

Approach strategically.

You need to find all correct answers, so test all of them. Substitute the first coordinate for x and the second coordinate for y.

(A) Test (0,4): $4x + 6y = 24 \rightarrow 4(0) + 6(4) = 0 + 24 = 24$. This works.
(B) Test (2,3): $4x + 6y = 24 \rightarrow 4(2) + 6(3) = 8 + 18 \neq 24$. Eliminate.
(C) Test (3,2): $4x + 6y = 24 \rightarrow 4(3) + 6(2) = 12 + 12 = 24$. This works.
(D) Test (5,4): $4x + 6y = 24 \rightarrow 4(5) + 6(4) = 20 + 24 \neq 24$. Eliminate.
(E) Test (9,–1): $4x + 6y = 24 \rightarrow 4(9) + 6(-1) = 36 - 6 \neq 24$. Eliminate.

So choices **(A)** and **(C)** are correct.

STEP 4

Confirm your answer.

Double-check your work to make sure you haven't made any careless errors, such as mistakenly plugging in a value for x when dealing with the variable y.

KAPLAN'S ADDITIONAL TIPS FOR PROBLEM SOLVING

Choose an efficient strategy.

The GRE is not a traditional math test that requires that you show your work in order to get credit, testing the process as well as the answer. The GRE tests only the answer—not how you found it. Because time is often your biggest concern on the GRE, the best way to each solution is often the quickest way, and the quickest way is often not straightforward math. Through practice, you'll become familiar with approaching each question in a more strategic way.

Rely on kaplan math strategies.

Using Kaplan strategies is a way to use reasoning in conjunction with mathematics to answer a question quickly. There may also be cases in which you can combine approaches: for example, using straightforward math to simplify an equation, then picking manageable numbers for the variables to solve that equation.

Picking numbers.

Problems that seem difficult can be good candidates for the Picking Numbers strategy. They include problems where either the question or the answer choices have variables, the problem tests a number property you don't recall, or the problem and the answer choices deal with percents or fractions without using actual values.

Backsolving.

Backsolving is a similar strategy to Picking Numbers, except that you'll use one of the five answer choices as the number to pick. After all, the testmaker gives you the correct answer; it's just mixed in with the wrong answers. Remember, numerical answer choices are always in ascending or descending order. Use that information to your advantage when using Backsolving. Start with either **(B)** or **(D)** first, because you'll have a 40 percent chance of finding the correct answer based on your first round of calculations. If you don't happen to pick the correct answer the first time, reason whether the number you started with was too large or too small. If you test choice **(B)** when the answer choices are in ascending order and **(B)** turns out to be too large, then **(A)** is the correct answer. If **(B)** is too small, then test choice **(D)**. If **(D)** is too large, then **(C)** is the correct answer. If **(D)** is too small, then **(E)** is correct. The opposite would be true if the choices were in descending order. Backsolving allows you to find the correct answer without ever needing to test more than two of the answer choices.

Use strategic guessing.

This is a good strategy if you can eliminate choices by applying number property rules or by estimating because gaps between answer choices are wide.

If some of the choices are out of the realm of possibility, eliminate them and move on.

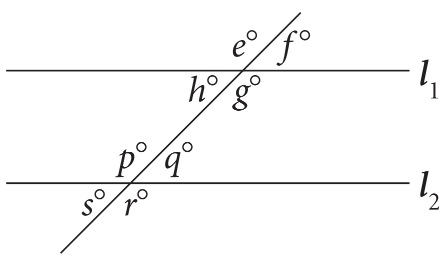
PROBLEM SOLVING PRACTICE SET

Try the following Problem Solving questions using the Kaplan Method for Problem Solving. If you're up to the challenge, time yourself; on Test Day, you'll want to spend only about 2 minutes on each question.

1. If $r = 3s$, $s = 5t$, $t = 2u$, and $u \neq 0$, what is the value of $\frac{rst}{u^3}$?

   Ⓐ 30
   Ⓑ 60
   Ⓒ 150
   Ⓓ 300
   Ⓔ 600

2. In the diagram, l_1 is parallel to l_2. The measure of angle q is 40 degrees. What is the sum of the measures of the acute angles shown in the diagram?



Note: Figure not drawn to scale.

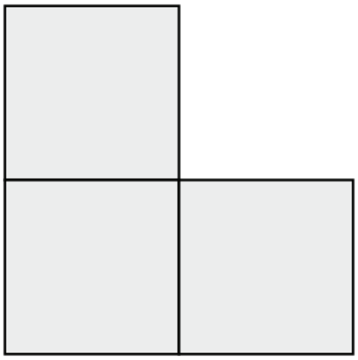
[ ] degrees

3. At Central Park Zoo, the ratio of sea lions to penguins is 4:11. If there are 84 more penguins than sea lions, how many sea lions are there?

   Ⓐ 24
   Ⓑ 36
   Ⓒ 48
   Ⓓ 72
   Ⓔ 121

4. Which of the following are prime numbers between $\frac{5}{2}$ and $\frac{43}{5}$? Indicate <u>all</u> possible answers.

   [A] 3
   [B] 4
   [C] 5
   [D] 7
   [E] 9



5. The figure above is made up of 3 squares. If the perimeter of the figure is 40 units, what is the area of the figure in square units?

   (A) 50
   (B) 75
   (C) 120
   (D) 150
   (E) 200

PROBLEM SOLVING PRACTICE SET ANSWERS AND EXPLANATIONS

1. E

The other variables all build upon u, so use the Picking Numbers strategy: pick a small number for u and find the values for r, s, and t. For instance, if $u = 1$, then $t = 2u$, so $t = 2$; $s = 5t$, so $s = 10$; and $r = 3s$, so $r = 30$.

So, $\frac{rst}{u^3} = \frac{30 \times 10 \times 2}{1 \times 1 \times 1} = 600$. The correct answer is (E).

2. 160

In the diagram, there are four acute angles and four obtuse angles created when the parallel lines are cut by the transversal. If angle q has a measure of 40°, then angles s, h, and f each also has a measure of 40°. Therefore, the sum of their degree measures is **160**.

3. C

You need to find the number of sea lions, and there are fewer sea lions than penguins, so starting small is a good idea. You can use the Backsolving strategy; start with choice **(B)**, 36. If there are 36 sea lions, then there are $36 + 84 = 120$ penguins, and the ratio of sea lions to penguins is $\frac{36}{120} = \frac{3}{10}$. This ratio is less than $\frac{4}{11}$, so your answer must be larger. If you try **(D)**, there are 72 sea lions and there are $72 + 84 = 156$ penguins, and the ratio of sea lions to penguins is $\frac{72}{156} = \frac{6}{13}$. Since this ratio is too large, the correct answer must be **(C)**.

4. A, C, D

You need to find a range of values between two improper fractions. First, change the improper fractions to mixed numbers: $\frac{5}{2} = 2\frac{1}{2}$ and $\frac{43}{5} = 8\frac{3}{5}$. Now, a prime number is a positive integer with only two distinct factors, 1 and itself. The prime numbers in the answer choices are 3, 5, and 7, and they are all between $2\frac{1}{2}$ and $8\frac{3}{5}$. So the correct answers are **(A)**, **(C)**, and **(D)**.

5. B

There are 8 side lengths of the squares that make up the perimeter, which you are told is 40. So, each side of each square must be 5 units. The area of each square can be found by squaring one side, so each square has an area of 25 square units. Since there are three squares, the total area of the figure is 75 square units. The correct answer is **(B)**.

CHAPTER 13

Data Interpretation

INTRODUCTION TO DATA INTERPRETATION QUESTIONS

Data Interpretation questions are based on information located in tables or graphs, and they are often statistics oriented. The data may be located in one table or graph, but you might also need to extract data from two or more tables or graphs. There will be a set of questions for you to answer based on each data presentation.

You may be asked to choose one or more answers from a set of answer choices or to enter your answer in a Numeric Entry field.

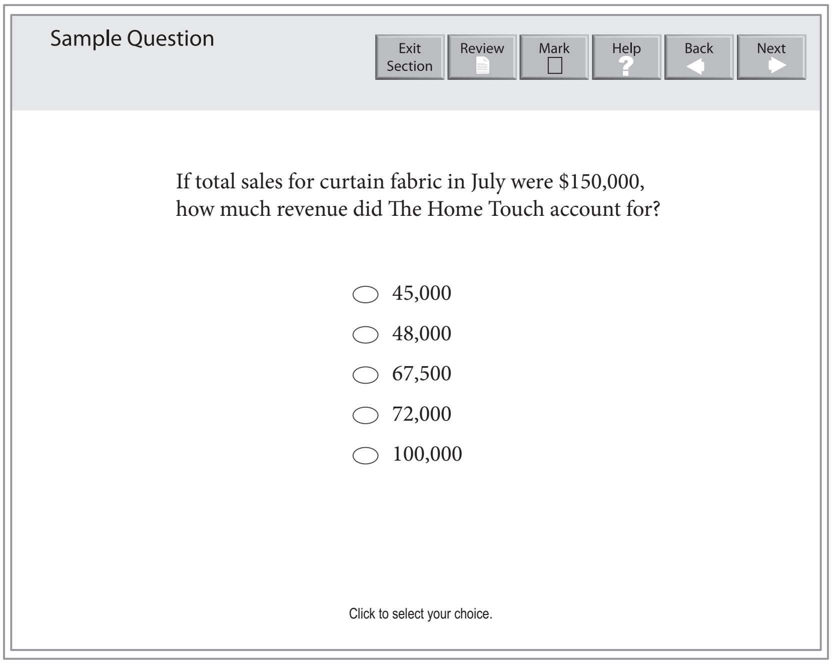
The directions for Data Interpretation questions will look like this:

Questions 15–17 are based on the following table.

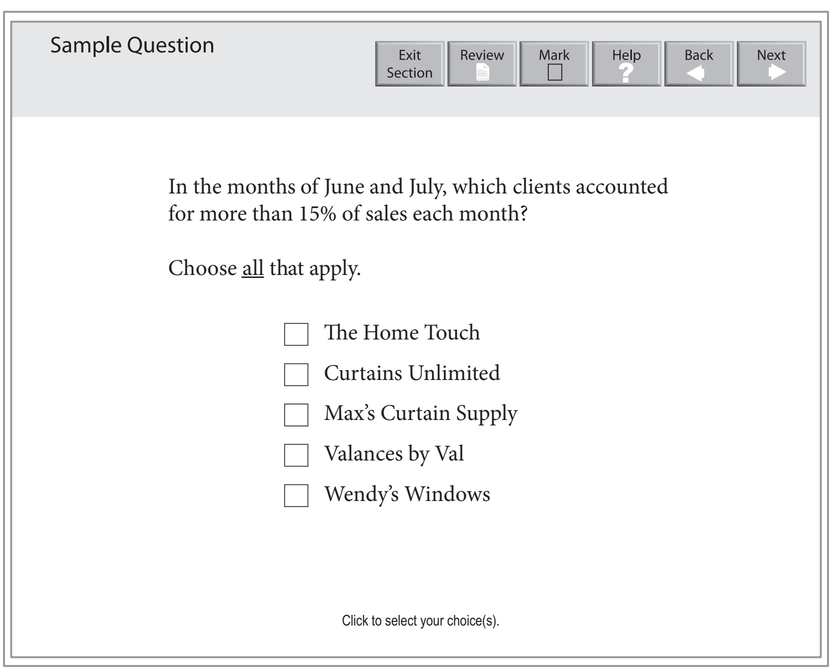
PERCENT OF SALES PER CLIENT FOR CURTAIN FABRIC OVER THREE MONTHS

	May	June	July
The Home Touch	45%	25%	48%
Curtains Unlimited	30%	23%	23%
Max's Curtain Supply	9%	23%	17%
Valances by Val	13%	20%	8%
Wendy's Windows	3%	9%	4%

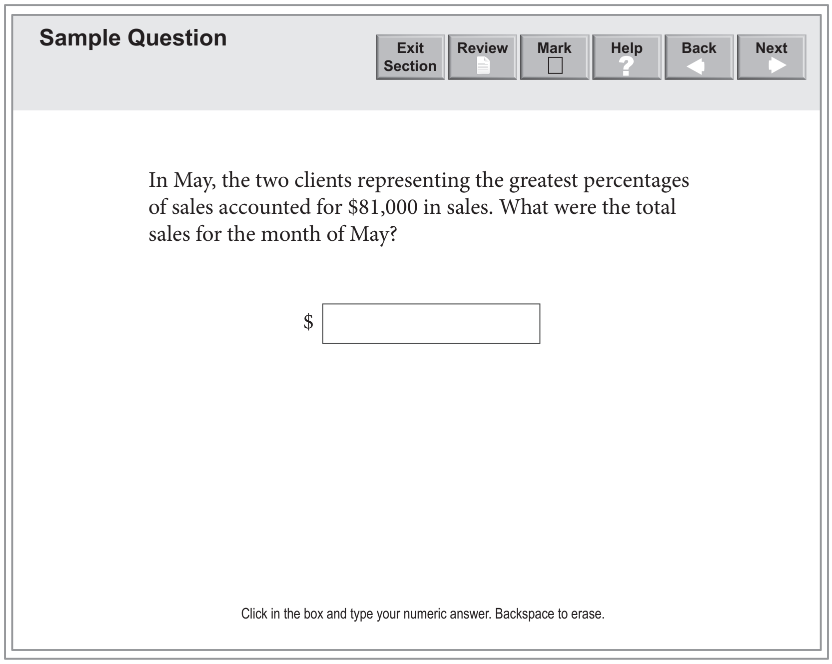
A Data Interpretation question that requires you to choose exactly one correct answer will look like this:



A Data Interpretation question that requires you to select all the answer choices that apply will look like this:



A Data Interpretation question that requires you to enter your numeric answer in a box will look like this:



THE KAPLAN METHOD FOR DATA INTERPRETATION

STEP 1 Analyze the tables and graphs.

STEP 2 Approach strategically.

How the Kaplan Method for Data Interpretation Works

Now let's discuss how the Kaplan Method for Data Interpretation works.

STEP 1

Analyze the tables and graphs.

Tables, graphs, and charts often come in pairs that are linked in some way (for example, a manufacturer's total revenue and its revenue by product line). Familiarize yourself with the information in both graphs (or tables) and with how the two are related before attacking the questions. Scan the figures for these components:

- **Title**. Read the charts' titles to ensure you can get to the right chart or graph quickly.
- **Scale**. Check the units of measurement. Does the graph measure miles per minute or hour? Missing the units can drastically change your answer.
- **Notes**. Read any accompanying notes—the GRE will typically give you information only if it is helpful or even critical to getting the correct answer.
- **Key**. If there are multiple bars or lines on a graph, make sure you understand the key so you can match up the correct quantities with the correct items.

STEP 2

Approach strategically.

Data Interpretation questions are designed to test your understanding of fractions and percents and your attention to detail. Taking a split second to make sure you answered the right question can make the difference between a correct answer and the "right" answer to the wrong question.

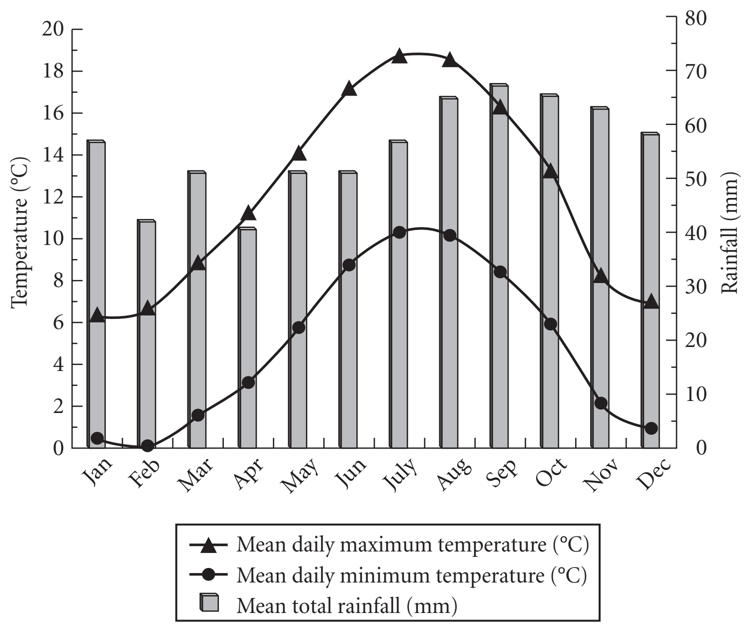
Questions tend to become more complex as you move through a set. For instance, if a question set contains two graphs, the first question likely refers to just one graph. A later question will most often combine data from both graphs. If you don't use both graphs for this later question, the chances are good you have missed something.

No matter how difficult graph questions appear at first glance, you can usually simplify single-answer multiple-choice questions by taking advantage of their answer-choice format. By approximating the answer rather than calculating it wherever possible, you can quickly identify the right one. As we saw with Problem Solving, estimation can be one of the fastest ways to identify the correct answer in math problems. Data Interpretation questions benefit from this strategy, as they tend to be the most time-consuming questions to answer.

How to Apply the Kaplan Method for Data Interpretation

Now let's apply the Kaplan Method to a Data Interpretation question:

CLIMOGRAPH OF CITY S



The Tourism Board of City S uses the information provided in the climograph to market the city as a tourist destination. One criterion is that the average monthly rainfall be less than 60 millimeters. What fraction of the months meet this criterion?

STEP 1

Analyze the tables and graphs.

Take the analysis of the graph step-by-step. Start with the title of the graph to verify that the data given are for City S. Then take note of the scale for each type of information—degrees Celsius for temperature and millimeters for rainfall. There are data for each month of the year, which means you will not have to convert the units to answer the question that's being asked.

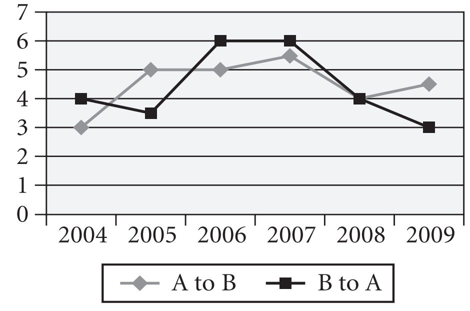
STEP 2

Approach strategically.

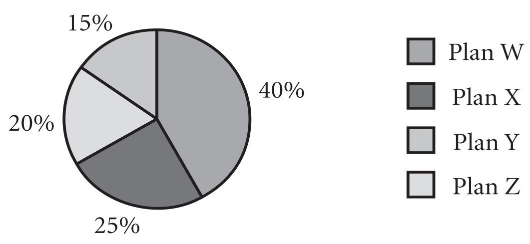
The question asks only about rainfall; those data are given by the bars on the graph. According to the bars, rainfall is greater than 60 mm in August, September, October, and November. That's 4 of 12 months that *do not* meet the criteria, so 8 of 12 months *do* meet it. You may enter the fraction $\frac{8}{12}$ directly into the boxes, and your answer will be accepted. It is *not* required that you reduce it.

Now let's apply the Kaplan Method to a second Data Interpretation question:

CUSTOMERS WHO SWITCHED SERVICE PROVIDERS (IN MILLIONS OF CUSTOMERS)



COMPANY A PROFIT 2008



In 2008, Company A had a total profit of $220 million. If half of the customers who switched to Company A were responsible for half of the profit for Plan X, how much did these customers contribute per person toward Company A's profit for the year?

- A $1.10
- B $13.75
- C $20.25
- D $27.50
- E $55.00

STEP 1

Analyze the tables and graphs.

This question has information about numbers of customers switching service providers for various years. It also has information about one company's profit for the year 2008, so the data in the two graphs will be linked by the year 2008.

STEP 2

Approach strategically.

Approach the question methodically, starting with identifying the number of customers who switched to Company A. The line chart indicates that 4 million customers switched to Company A. This is the only information needed from the top graph.

The pie chart shows the breakdown of profit from the various plans offered and indicates that 25 percent of the profit came from Plan X.

The other information you need to get to the correct answer is given in the question stem:

- Profit of $220 million.
- Half of the customers who switched were responsible for half of Plan X's profits.

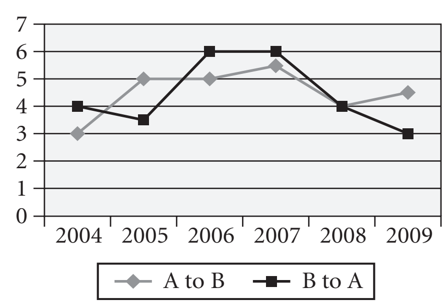
Now that your information is organized, all you need to do is the calculation. Plan X accounts for 25 percent of $220 million = $55 million. Half of $55 million is $27.5 million.

If 4 million people switched, then half of the people who switched would be 2 million.

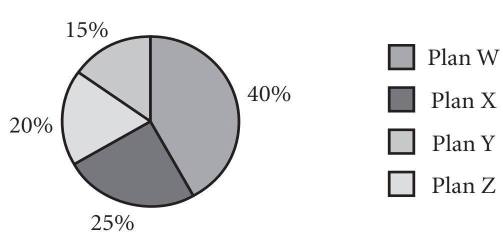
The last step is to divide $27.5 by 2 (you can drop the zeroes in the millions because they will cancel out): $27.5 ÷ 2 = $13.75. The correct choice is **(B)**.

Now let's apply the Kaplan Method to a third Data Interpretation question:

CUSTOMERS WHO SWITCHED SERVICE PROVIDERS
(IN MILLIONS OF CUSTOMERS)



COMPANY A PROFIT 2008



The management of Company B is most interested in the data for the years in which there were at least one million *more* customers who switched from

Company A to Company B than switched from Company B to Company A. In which years did this happen?

Choose <u>all</u> that apply.

- [A] 2005
- [B] 2006
- [C] 2007
- [D] 2008
- [E] 2009

STEP 1

Analyze the tables and graphs.

This question asks for a comparison of facts between Company A and Company B. Take time to verify which line in the top graph represents customers switching to Company A and which line represents customers switching to Company B. Confirm that the title states that the data are given in millions and then look at the scale on the line graph.

STEP 2

Approach strategically.

After examining the line graph carefully, you are ready to gather the information needed to answer the question. The years that satisfy the requirement are those years for which the line representing A to B is at least one full horizontal row above the line representing B to A. Read the graph carefully because you must identify all the correct choices to get credit for a correct answer.

When you are clear what to look for on the graph, start from the left and identify the years 2005 and 2009 as those in which at least one million more customers switched from A to B than switched from B to A. These are choices **(A)** and **(E)**.

KAPLAN'S ADDITIONAL TIPS FOR DATA INTERPRETATION QUESTIONS

Slow Down

There's always a lot going on in Data Interpretation problems—both in the charts and in the questions themselves. If you slow down the first time through, you can avoid calculation errors and having to reread the questions and charts.

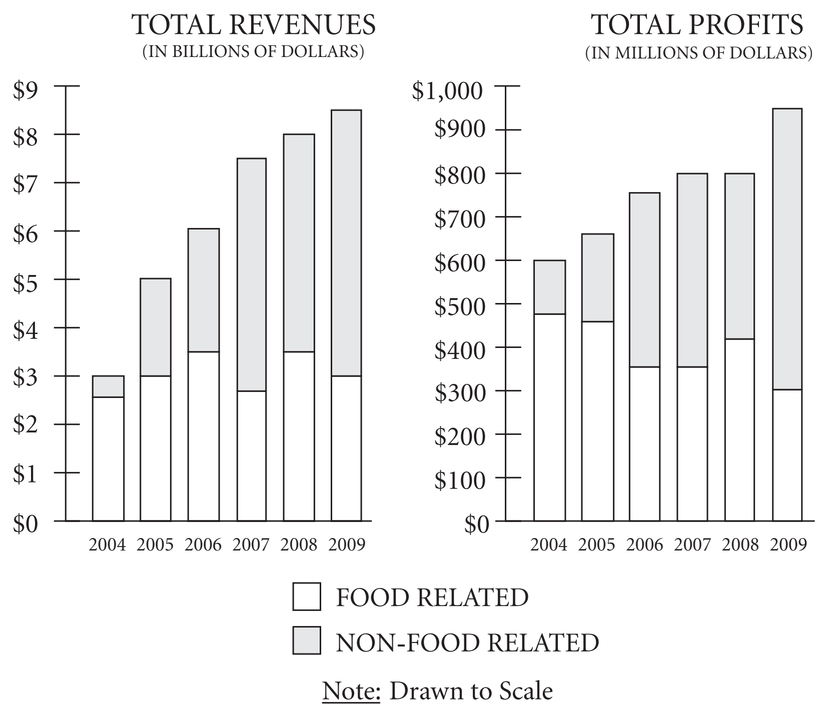
Pace Yourself Wisely

To ensure that you score as many points on the exam as possible, use the allotted time for a section wisely. Remember that each question type has the same value. If you must miss a few questions in a section, make them the ones that would take you the longest to answer, not the ones at the end of the section that you could have answered correctly but simply didn't get to. Data Interpretation questions are generally some of the more time-consuming ones to answer, and if answering them isn't one of your strong suits, save them for the end.

DATA INTERPRETATION PRACTICE SET

Try the following Data Interpretation questions using the Kaplan Method for Data Interpretation. If you're up to the challenge, time yourself; on Test Day, you'll want to spend only about 2 minutes on each question.

Questions 1–5 are based on the following graphs.



PERCENT OF REVENUES FROM FOOD-RELATED OPERATIONS IN 2009 BY CATEGORY

1. Approximately how much did total revenues increase from 2004 to 2007?

   Ⓐ $0.5 billion
   Ⓑ $1.5 billion
   Ⓒ $4 billion
   Ⓓ $4.5 billion
   Ⓔ $5 billion

2. For the year in which profits from food-related operations increased over the previous year, total revenues were approximately:

   Ⓐ $3.5 billion
   Ⓑ $4.5 billion
   Ⓒ $5.7 billion
   Ⓓ $6 billion
   Ⓔ $8 billion

3. In 2008, total profits represented approximately what percent of total revenues?

   Ⓐ 50%
   Ⓑ 20%
   Ⓒ 10%
   Ⓓ 5%
   Ⓔ 1%

4. For the first year in which revenues from non-food-related operations surpassed $4.5 billion, total profits were approximately:

   Ⓐ $250 million
   Ⓑ $450 million
   Ⓒ $550 million
   Ⓓ $650 million
   Ⓔ $800 million

5. In 2009, how many millions of dollars were revenues from frozen food operations?

   [ ] millions of dollars

DATA INTERPRETATION PRACTICE SET ANSWERS AND EXPLANATIONS

1. D

This question asks about total revenues, so you should refer to the left bar graph. Each bar in the graph has two components, but you want to look at the total height of the bars for 2004 and 2007 because the question asks about total revenues. Total revenues for 2004 appear to be $3 billion, and for 2007 they appear to be about $7.5 billion. So the increase is roughly $7.5 billion – $3 billion = $4.5 billion. Answer choice **(D)** is correct.

2. E

You have to refer to both bar graphs to answer this question. First, refer to the right bar graph to find the lone year in which food-related profits increased over the previous year—the only year in which the unshaded portion of the bar increases in size is 2008. Now that you've zeroed in on the year, refer to the left bar graph to determine the total revenues for that year, which appear to be about $8 billion. Answer choice **(E)** is correct.

3. C

This is a percent question, so start with the bar graphs. You need the figures from both food-related and non-food-related sources, so look at the total height of the bars. From the right bar graph, the total profits for 2008 appear to be $800 million; from the left bar graph, total revenues for that year appear to be $8 billion (i.e., $8,000 million). Now, convert the part/whole into a percent:

$$\frac{800\,\text{million}}{8\,\text{billion}} = \frac{800\,\text{million}}{8{,}000\,\text{million}} = \frac{1}{10} = 10\%$$

4. E

First, find the year for which revenues from non-food-related operations surpassed $4.5 billion on the left bar graph. Finding the correct bar is made more difficult by the fact that you have to deal with the shaded portion, which is at the top of the bar, not at the bottom. Looking carefully, you should then see that 2007 is the year in question. The question asks for total *profits*, so once again refer to the right bar graph, and you'll see the profits for that year are around $800 million. This matches answer choice **(E)**.

5. 600

Finally, you have a question that refers to the pie chart. You are asked about revenues from frozen food operations, and the pie chart tells you that frozen foods represent 20 percent of all food-related revenues for 2009. To convert this into an amount, you need to locate the amount of food-related revenues for 2009. Once again, refer to the left bar graph, where you'll find that food-related revenues in 2009 were $3 billion, or $3,000 million. Then calculate that 20 percent of $3,000 million is **$600** million.

CHAPTER 14

Quantitative Reasoning Practice Sets

In this chapter, you will take three practice sections, composed of 20 questions each. A diagnostic tool is provided after each section to help you learn from your mistakes. Then you can continue to the next set with more awareness of the traps you may encounter.

REVIEW OF THE KAPLAN METHODS FOR QUANTITATIVE REASONING QUESTION TYPES

Before starting your practice sets, review the steps and strategies you have studied for answering each type of Quantitative Reasoning question quickly, efficiently, and correctly before starting your Practice Sets.

THE KAPLAN METHOD FOR QUANTITATIVE COMPARISON

STEP 1 Analyze the centered information and quantities.

STEP 2 Approach strategically.

THE KAPLAN METHOD FOR PROBLEM SOLVING

STEP 1 **Analyze the question.**

STEP 2 **Identify the task.**

STEP 3 **Approach strategically.**

STEP 4 **Confirm your answer.**

THE KAPLAN METHOD FOR DATA INTERPRETATION

STEP 1 **Analyze the tables and graphs.**

STEP 2 **Approach strategically.**

QUANTITATIVE REASONING PRACTICE SET 1

Numbers

All numbers are real numbers.

Figures

The position of points, lines, angles, and so on may be assumed to be in the order shown; all lengths and angle measures may be assumed to be positive.

Lines shown as straight may be assumed to be straight.

Figures lie in the plane of the paper unless otherwise stated.

Figures that accompany questions are intended to provide useful information. However, unless a note states that a figure has been drawn to scale, you should solve the problems by using your knowledge of mathematics, not by estimation or measurement.

Directions

Each of the following questions, 1–8, consists of two quantities, Quantity A and Quantity B. You are to compare the two quantities and choose

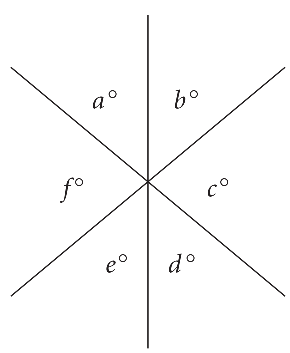
- (A) if Quantity A is greater
- (B) if Quantity B is greater
- (C) if the two quantities are equal
- (D) if the relationship cannot be determined from the information given

Common Information

In a question, information concerning one or both of the quantities to be compared is centered above the two quantities. A symbol that appears in both quantities represents the same thing in Quantity A as it does in Quantity B.

1. | Quantity A | Quantity B |
|---|---|
| The number of distinct ways to form an ordered line of 3 people by choosing from 6 people | The number of distinct ways to form an unordered group of 3 people by choosing from 10 people |

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.



2. | Quantity A | Quantity B |
|---|---|
| $a + c + e$ | $b + d + f$ |

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

$$7p + 3 = r$$
$$3p + 7 = s$$

3. | Quantity A | Quantity B |
|---|---|
| r | s |

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

The original cost of a shirt is x dollars.

4. | Quantity A | Quantity B |
|---|---|
| x | The cost of the shirt if the original cost is first increased by 10% and then decreased by 10% |

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

There were x dictionaries in a bookstore. After $\frac{1}{8}$ of them were purchased, 10 more dictionaries were shipped in, bringing the total number of dictionaries to 52.

5. Quantity A — Quantity B

Quantity A	Quantity B
x	50

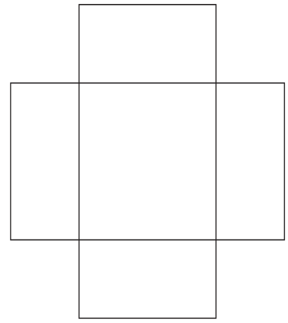
Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

There are n people in a room. One-third of them leave the room. Four people enter the room. There are now $\frac{5}{6}$ of the original number of people in the room.

6.

Quantity A	Quantity B
n	20

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.



Note: Figure not drawn to scale.

Two rectangles with dimensions 2 meters by 4 meters overlap to form the figure above. All the angles shown measure 90°.

7.

Quantity A	Quantity B
The perimeter of the figure, in meters	16

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

x is an integer.
$1 < x < 9$

8.

Quantity A	Quantity B
$(\sqrt{x} + \sqrt{x})^2$	$x + x\sqrt{x}$

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

9. If $\frac{x}{y} = \frac{2}{3}$ and $x + y = 15$, which of the following is greater than y?

Indicate <u>all</u> possible choices.

A $\sqrt{65}$
B $\sqrt{82}$
C $\sqrt{99}$
D $\sqrt{101}$
E $\sqrt{122}$

10. The product of two integers is 10. Which of the following could be the average (arithmetic mean) of the two numbers?

Indicate <u>all</u> possible choices.

A -5.5
B -3.5
C -1.5
D 1.5
E 3.5

11. Which of the following is greater than the sum of the distinct prime factors of 210?

Indicate <u>all</u> possible choices.

A 12
B 17
C 19
D 21
E 24

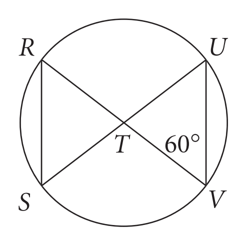
12. The average (arithmetic mean) bowling score of n bowlers is 160. The average of these n scores together with a score of 170 is 161. What is the number of bowlers, n?

[ ] bowlers

13. Set T consists of five integers: the first five odd prime numbers when counting upward from zero. This gives set T a standard deviation of approximately 3.71. Which of the following values, if added to the set T, would increase the standard deviation of set T?

(A) 11
(B) 9
(C) 7.8
(D) 4.15
(E) 3.7

14.



The circle shown has center T. The measure of angle TVU is 60°. If the circle has a radius of 3, what is the length of segment RS?

(A) 2
(B) $2\sqrt{2}$
(C) 3
(D) $3\sqrt{3}$
(E) $6\sqrt{2}$

15. What is the probability of rolling a total of 7 with a single roll of two fair six-sided dice, each with the distinct numbers 1–6 on each side?

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{2}{7}$

(D) $\frac{1}{3}$

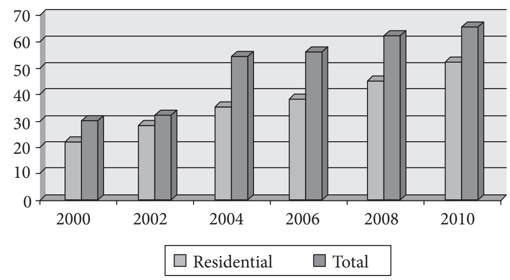
(E) $\frac{1}{2}$

16. If it takes three days for 10 workers to finish building one house, how many days will it take 15 workers to finish four houses?

[ ] days

Questions 17–20 are based on the following graph and table.

WATER USAGE BY YEAR, TOWN W
(IN BILLIONS OF GALLONS)



DAILY WATER USAGE STATISTICS

(*with efficient appliances and good maintenance*)

Use	Gallons per Capita
Showers	9
Clothes washers	10
Toilets	8
Leaks	4
Faucets	11
Other	4

17. Which best describes the range (in billions of gallons) for residential water consumption from 2000 to 2010, inclusive?

(A) 10
(B) 20
(C) 30
(D) 40
(E) 50

18. In the year in which total usage exceeded residential usage by the least number of gallons, approximately what percent of total usage was residential usage?

(A) 68%
(B) 75%
(C) 88%
(D) 95%
(E) 98%

19. In 2004, only 10,000 residents of town W lived in homes with efficient appliances and good maintenance. How many gallons per day were used by these residents for the three daily household purposes requiring the most water?

(A) 110,000
(B) 160,000
(C) 270,000
(D) 300,000
(E) 460,000

20. Households with efficient appliances and good maintenance can reduce water consumption by about 35%. If half of the residential consumption in town W in 2010 was by households with efficient appliances and good maintenance, how many gallons of water (in billions) were saved that year?

(A) 5
(B) 14
(C) 40
(D) 52
(E) 65

QUANTITATIVE REASONING PRACTICE SET 1 ANSWER KEY

1. C
2. C
3. D
4. A
5. B
6. A
7. C
8. A
9. B, C, D, E
10. A, B, E
11. C, D, E
12. 9
13. E
14. C
15. B
16. 8
17. C
18. C
19. D
20. B

Diagnose Your Results

Diagnostic Tool

Tally up your score and write your results below.

Total

Total Correct: ________ out of 20 correct

By Question Type

Quantitative Comparison (questions 1–8) ________ out of 8 correct

Problem Solving (questions 9–16) ________ out of 8 correct

Data Interpretation (questions 17–20) ________ out of 4 correct

Look back at the questions you got wrong and think about your experience answering them.

STEP 1

Find the roadblocks.

If you struggled to answer some questions, then to improve your score, you need to pinpoint exactly what "roadblocks" tripped you up. To do that, ask yourself the following two questions:

Am I weak in the skills being tested?

This will be very easy for you to judge. Maybe you've forgotten how to figure out the area of a triangle or what PEMDAS stands for. If you know you need to brush up on your math skills, try the *Kaplan GRE Math Workbook*, which contains a focused review of all the fundamental math concepts tested on the GRE, as well as practice exercises to build speed and accuracy.

Did the question types throw me off?

Then you need to become more comfortable with them! Quantitative Comparisons have a unique format, and Data Interpretation can be daunting with its charts, graphs, and tables. If you struggled, go back to the beginning of this chapter and review the Kaplan principles and methods for the question types you found challenging. Make sure you understand the principles and how to apply the methods. These strategies will help you improve your speed and efficiency on Test Day. Remember, it's not a math test; it's a critical reasoning test.

Also, get as much practice as you can so that you grow more at ease with the question formats. For even more practice, try the *Kaplan GRE Math Workbook,* which includes practice sets for each question type.

STEP 2

Find the blind spots.

Did you answer some questions quickly and confidently but get them wrong anyway?

When you come across wrong answers like these, you need to figure out what you thought you were doing right, what it turns out you were doing wrong, and why that happened. The best way to do that is to **read the answer explanations!**

The explanations give you a detailed breakdown of why the correct answer is correct and why all the other answers choices are incorrect. This helps to reinforce the Kaplan principles and methods for each question type and helps you figure out what blindsided you so it doesn't happen again. Also, just as with your "roadblocks," try to get in as much practice as you can.

STEP 3

Reinforce your strengths.

Now read through all the answer explanations for the ones you got right. You should check every answer because if you guessed correctly without actually knowing how to get the right answer, reading the explanations helps you make sure that whatever needs fixing gets fixed. Again, this helps to reinforce the Kaplan principles and methods for each question type, which in turn helps you work more efficiently so you can get the score you want. Keep your skills sharp with more practice.

As soon as you are comfortable with all the GRE question types and Kaplan methods, complete a full-length practice test under timed conditions. Practice tests serve as milestones; they help you to chart your progress! So don't save them all for the final weeks before your Test Day. For even more practice, you can also try the Kaplan GRE Quiz Bank. You get more than 2,500 questions that you can access 24/7 from any Internet browser, and each question comes with a comprehensive explanation. You can even customize your quizzes based on question type, content, and difficulty level. Take quizzes in Timed Mode to test your stamina or in Tutor Mode to see explanations as you work. Best of all, you also get detailed reports to track your progress.

Visit **kaptest.com/GRE** for more details on our Quiz Bank and for more information on our other online and classroom-based options.

QUANTITATIVE REASONING PRACTICE SET 1 ANSWERS AND EXPLANATIONS

1. C

Quantity A is a permutation because order matters. The number of ways 3 people chosen from a group of 6 can be arranged in a line, where order matters, is $6 \times 5 \times 4 = 120$. Quantity B is a combination because order does not matter. The number of ways 3 people can be selected from a group of 10, where order does not matter, is

$$_{10}C_3 = \frac{10!}{3!(10-3)!} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = \frac{720}{6} = 120$$

The two quantities are equal.

2. C

There are three sets of vertical angles in this diagram: (a, d), (b, e), and (c, f). In Quantity A, you can substitute b for e because they are vertical angles and therefore equal; this leaves the sum $a + b + c$ in Quantity A. Because these are the three angles on one side of a straight line, they sum to 180°. Similarly, after substituting e for b in Quantity B, $b + d + f$ is the same thing as $d + e + f$, or also 180°. The two quantities are equal.

3. D

Pick a value for p and see what effect it has on r and s. If $p = 1$, $r = (7 \times 1) + 3 = 10$, and $s = (3 \times 1) + 7 = 10$, and the two quantities are equal. But if $p = 0$, $r = (7 \times 0) + 3 = 3$, and $s = (3 \times 0) + 7 = 7$, and Quantity A is less than Quantity B. Because there are at least two different possible relationships, the answer is **(D).**

4. A

Use the Picking Numbers strategy to answer this question. Suppose the original selling price of the shirt, x, is \$100. After a 10% increase in price, the shirt would sell for 110% of \$100, which is \$110. If there is a 10% decrease next, the shirt would sell for 90% of the current price. That would be 90% of \$110: $0.9 \times \$110 = \$99$. This price is less than the original amount, x, so Quantity A is greater.

5. B

Try to set the quantities equal. If x is 50, then the bookstore started out with 50 dictionaries. Then $\frac{1}{8}$ of them were purchased. You can see already that the quantities can't be equal, because $\frac{1}{8}$ of 50 won't yield an integer. But go ahead and see whether the answer is **(A)** or **(B)**. Because $\frac{1}{8}$ of 50 is close to 6, after these dictionaries were purchased, the store would have been left with about $50 - 6$ or 44 dictionaries. Then it received 10 more, giving a total of about 54 dictionaries. But this is more than the store actually ended up with; it only had 52. Therefore, it must have started with *fewer* than 50 dictionaries, and Quantity B is greater. (The last thing you care about is how many dictionaries it really had.)

6. A

There are n people in a room. One-third of them leave the room. So, there are $n - \frac{1}{3}n$ people in the room. Four people enter the room, so you have $n - \frac{1}{3}n + 4$ people. There are now $\frac{5}{6}$ of the original number of people in the room, therefore $n - \frac{1}{3}n + 4 = \frac{5}{6}n$. Now solve for n.

$$\begin{aligned} n - \frac{1}{3}n + 4 &= \frac{5}{6}n \\ \frac{2}{3}n + 4 &= \frac{5}{6}n \\ 4 &= \frac{5}{6}n - \frac{2}{3}n \\ 4 &= \frac{5}{6}n - \frac{4}{6}n \\ 4 &= \frac{1}{6}n \\ 24 &= n \end{aligned}$$

So, $n = 24$ and Quantity A is larger.

7. C

You may have thought this was a choice **(D)** question; after all, you don't know exactly where the boards overlap, whether in the middle of each board, as pictured, or near the end of one of the boards. But that doesn't matter; all you need to know is that they overlap and that all the angles are right angles. If the boards did not overlap, it would be easy to find the perimeter: $2 + 2 + 4 + 4 = 12$ for each board, or 24 for both boards. Now, because the boards do overlap, the perimeter of the figure will be smaller than that, but how much smaller? It will be smaller by the amount of that "lost perimeter" in the middle; the perimeter of the square where the boards overlap. (You know it's a square since all the angles are right angles.) The length of a side of that square is the shorter dimension of each of the boards: 2. Therefore, the perimeter of the square is 4×2 or 8. The perimeter of the figure, then, is $24 - 8$ or 16. The two quantities are equal.

8. A

Start by simplifying the quantity in Quantity A: $\left(\sqrt{x}+\sqrt{x}\right)^2$ is the same as $\left(2\sqrt{x}\right)^2$, which is $4x$. Subtract x from both quantities, and you're left with $3x$ in Quantity A and $x\sqrt{x}$ in Quantity B. Now divide both sides by x, and you're left with 3 in Quantity A and $\sqrt{x}$ in Quantity B. Square both quantities, and you get 9 in Quantity A and x in Quantity B. Since x is an integer between 1 and 9, exclusive, Quantity A is larger. If the algebra seems too abstract, go ahead and use the Picking Numbers strategy. If x equals 4, then Quantity A equals $(2 + 2)^2 = 16$, and Quantity B equals $4 + 8 = 12$.

9. B, C, D, E

If $\frac{x}{y} = \frac{2}{3}$, then $3x = 2y$ and $y = \frac{3x}{2}$. Substitute $y = \frac{3x}{2}$ into the equation $x + y = 15$: $x + \frac{3x}{2} = 15$, $2x + 3x = 30$, $5x = 30$, $x = 6$. Then, $y = \frac{3x}{2} = \frac{3(6)}{2} = 9$ and $y^2 = 81$. So any answer with greater than 81 under the radical will be greater than y. Therefore, the correct choices are **(B)**, **(C)**, **(D)**, and **(E)**.

10. A, B, E

The best place to start here is with pairs of positive integers that have a product of 10. The numbers 5 and 2 have a product of 10, as do 10 and 1. But remember that integers may be negative, so –1 and –10 are possible, as well as –2 and –5. The mean of –1 and –10 is –5.5; the mean of –2 and –5 is –3.5. The mean of 2 and 5 is 3.5. The correct answers are **(A)**, **(B)**, and **(E)**.

11. C, D, E

The prime factorization of 210 is $2 \times 3 \times 5 \times 7$. The sum of the prime factors is $2 + 3 + 5 + 7 = 17$. So, the correct choices are **(C)**, **(D)**, and **(E)**.

12. 9

Use the definition of *average* to write the sum of the first n bowlers' scores: $\frac{\text{sum of scores}}{n} = \text{average}$; therefore, $n \times \text{average} = \text{sum of scores}$. Substitute the values given in the question, and you have $160n = \text{sum of scores}$ for the initial set of bowlers. Now write the formula for the average again, using the additional score of 170. Now there are $n + 1$ bowlers.

$$\frac{\text{sum of scores}}{n} = \text{average}$$

$$\frac{160n + 170}{n + 1} = 161$$

Cross multiply and use algebra to solve for n.

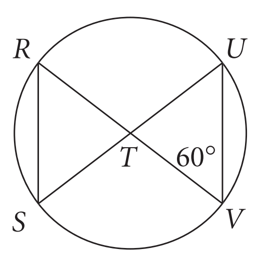
$$\begin{aligned} 160n + 170 &= 161(n + 1) \\ 160n + 170 &= 161n + 161 \\ 170 - 161 &= 161n - 160n \\ 9 &= n \end{aligned}$$

There were **9** bowlers in the original group.

13. E

First, identify the numbers in set T: 3, 5, 7, 11, 13. The average of the numbers in set T is $\frac{3 + 5 + 7 + 11 + 13}{5} = \frac{39}{5} = 7.8$. Its standard deviation is given in the question stem as 3.71. In order to increase the standard deviation of a set of numbers, you must add a value that is more than one standard deviation away from the mean. One standard deviation below the mean for set T would be $7.8 - 3.71 = 4.09$, and one standard deviation *above* the mean would be $7.8 + 3.71 = 11.51$. Any value outside this range $4.09 \leq x \leq 11.51$ would increase set T's standard deviation, since it would make the set more "spread out" from the mean than it currently is. The only choice that does that is choice **(E)**.

14. C



Solving this problem involves several steps, but none is too complicated. The circle has its center at point T. Start with the triangle on the right whose vertices are at T and two points on the circumference of the circle. This makes two of its sides radii of the circle, which we're told each have a length of 3. Because all radii must have equal length, this makes the triangle an isosceles triangle. In addition, you're told one of the base angles of this triangle has measure 60°. Thus, the other base angle must also have measure 60° (since the base angles in an isosceles triangle have equal measure). The sum of the two base angles is 120°, leaving 180° − 120° or 60° for the other angle, the one at point T (making $\triangle TUV$ an equilateral triangle with sides of 3).

Now, angle *RTS* is opposite this 60° angle, so its measure must also be 60°. Therefore, ΔRST is another equilateral triangle, and its sides are 3. Therefore, the length of *RS* is 3, choice **(C)**.

15. B

The probability formula is

$$\text{Probability} = \frac{\text{Number of desired outcomes}}{\text{Number of possible outcomes}}$$

When one die is rolled, there are six possible outcomes. When two dice are rolled, the number of possible outcomes is 6×6, or 36. Getting a total value of 7 can be achieved in the following ways: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), and (6, 1). There are six possible ways.

So the probability of rolling a total of 7 is $\frac{6}{36}$, which can be reduced to $\frac{1}{6}$, choice **(B)**.

16. 8

In the first scenario, each day $\frac{1 \text{ house}}{3 \text{ days}} = \frac{1}{3}$ of the house will be built. Because there are 10 workers, each person can build $\frac{1}{30}$ of a house each day. In the second scenario, there are 15 workers, so that means $15 \times \frac{1}{30} = \frac{1}{2}$ a house can be built each day. Four houses could, therefore, be built in 8 days:

$$\frac{4 \text{ houses}}{\frac{1}{2} \text{ house/day}} = 4 \times 2 = 8 \text{ days}$$

17. C

The residential usage (in billions) in 2000 was about 22; the usage was about 52 in 2010. The range is the difference because the residential usage increased over the time period. Therefore, $52 - 22 = 30$, and the range is about 30 billion gallons. The correct answer is **(C)**.

18. C

The two amounts were closest to each other in 2002. The residential amount appears to be about 28; the total appears to be about 32: $28 \div 32 = 0.875$. Choice **(C)** is the closest.

19. D

The three usages with the greatest amounts per person are faucets, washers, and showers, totaling 30 gallons per day. Multiply by 10,000 to get 300,000, choice **(D)**.

20. B

The residential consumption (in billions) in 2010 was approximately 52. Take half of that amount, 26, to represent the amount of water used by households with efficient appliances and plumbing. Let *W* represent the amount of water these households would have used otherwise.

Set up a percent equation to solve for *W*. Remember, the savings were 35%, so subtract 35 from 100 to find the percent that would have been used.

$$\begin{aligned} 26 &= (100\% - 35\%) \times W \\ 26 &= 65\% \times W \\ 26 &= 0.65 \times W \\ \frac{26}{0.65} &= W \end{aligned}$$

The savings in billions of gallons was $40 - 26 = 14$. The correct answer is **(B)**.

QUANTITATIVE REASONING PRACTICE SET 2

Numbers

All numbers are real numbers.

Figures

The position of points, lines, angles, and so on may be assumed to be in the order shown; all lengths and angle measures may be assumed to be positive.

Lines shown as straight may be assumed to be straight.

Figures lie in the plane of the paper unless otherwise stated.

Figures that accompany questions are intended to provide useful information. However, unless a note states that a figure has been drawn to scale, you should solve the problems by using your knowledge of mathematics, not by estimation or measurement.

Directions

Each of the following questions, 1–10, consists of two quantities, Quantity A and Quantity B. You are to compare the two quantities and choose

- (A) if Quantity A is greater
- (B) if Quantity B is greater
- (C) if the two quantities are equal
- (D) if the relationship cannot be determined from the information given

Common Information

In a question, information concerning one or both of the quantities to be compared is centered above the two quantities. A symbol that appears in both quantities represents the same thing in Quantity A as it does in Quantity B.

1.

Quantity A	Quantity B
The average (arithmetic mean) of 100, 101, and 103	The median of 100, 101, and 103

- (A) Quantity A is greater.
- (B) Quantity B is greater.
- (C) The two quantities are equal.
- (D) The relationship cannot be determined from the information given.

A and *B* are points on the circumference of the circle with center *O* (not shown). The length of chord *AB* is 15.

2.

Quantity A	Quantity B
Circumference of circle *O*	12π

- (A) Quantity A is greater.
- (B) Quantity B is greater.
- (C) The two quantities are equal.
- (D) The relationship cannot be determined from the information given.

$$x = \frac{4}{3}r^2h^2$$

$$x = 1$$

r and h are positive.

3.

Quantity A	Quantity B
h	$\frac{\sqrt{3}}{2r}$

- (A) Quantity A is greater.
- (B) Quantity B is greater.
- (C) The two quantities are equal.
- (D) The relationship cannot be determined from the information given.

ΔABC lies in the xy-plane with C at (0,0), B at (6,0), and A at (x,y), where x and y are positive. The area of ΔABC is 18 square units.

4.

Quantity A	Quantity B
y	6

- (A) Quantity A is greater.
- (B) Quantity B is greater.
- (C) The two quantities are equal.
- (D) The relationship cannot be determined from the information given.

For $x \neq y$, $x \,\Phi\, y = \dfrac{x+y}{x-y}$

$$p > 0 > q$$

5.

Quantity A	Quantity B
$p \,\Phi\, q$	$q \,\Phi\, p$

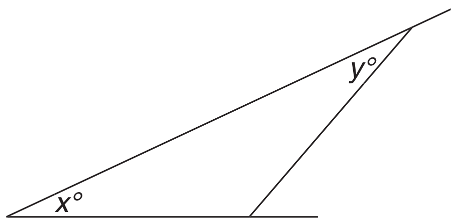
- (A) Quantity A is greater.
- (B) Quantity B is greater.
- (C) The two quantities are equal.
- (D) The relationship cannot be determined from the information given.

$x \neq 0$

6. Quantity A | Quantity B

$\frac{1}{x} + \frac{1}{x}$ | $\frac{1}{x} \times \frac{1}{x}$

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.



7. Quantity A | Quantity B

$x + y$ | $180°$

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

$$\begin{aligned} 4x - 5y &= 10 \\ -3x + 6y &= 22 \end{aligned}$$

8. Quantity A | Quantity B

33 | $x + y$

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

$6(10)^n > 60{,}006$

9. Quantity A | Quantity B

n | 6

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

In a four-digit positive integer y, the thousands digit is 2.5 times the tens digit.

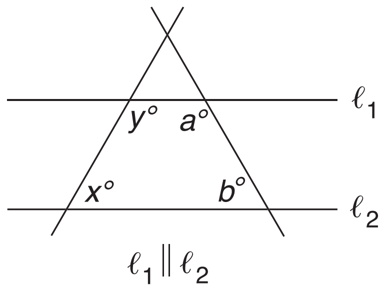
10. Quantity A | Quantity B

The tens digits of y | 4

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

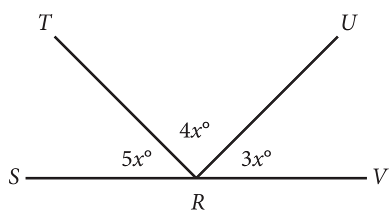
11. What is the average (arithmetic mean) of $2x + 3$, $5x - 4$, $6x - 6$, and $3x - 1$?

Ⓐ $2x + 4$
Ⓑ $3x - 2$
Ⓒ $3x + 2$
Ⓓ $4x - 2$
Ⓔ $4x + 2$



12. Which of the following statements must be true about the figure shown above?

Ⓐ $x = a$
Ⓑ $x = b$
Ⓒ $a = b$
Ⓓ $y = b$
Ⓔ $x + y = a + b$



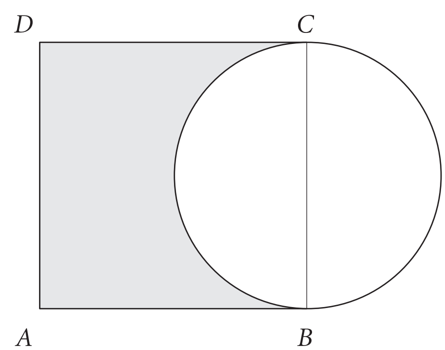
13. What is the degree measure of angle *SRU*?

Ⓐ 15
Ⓑ 45
Ⓒ 105
Ⓓ 135
Ⓔ 180

14. There are at least 200 apples in a grocery store. The ratio of the number of oranges to the number of apples is 9 to 10. How many oranges could there be in the store?

Indicate <u>all</u> possible choices.

[A] 171
[B] 180
[C] 216
[D] 252
[E] 315

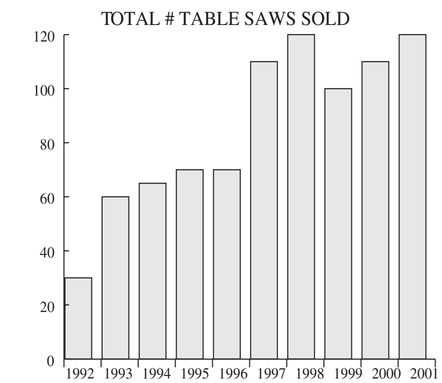


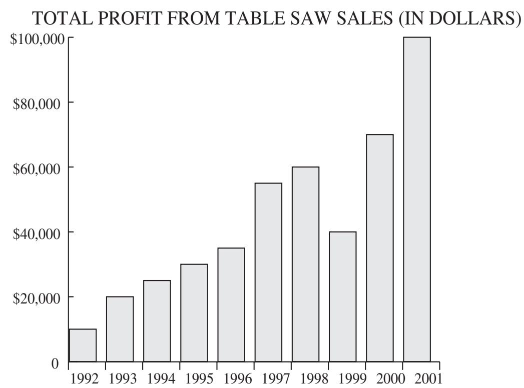
15. Square *ABCD* has a side length of 4. *BC* is the diameter of the circle. Which of the following is greater than or equal to the area of the shaded region, in square units?

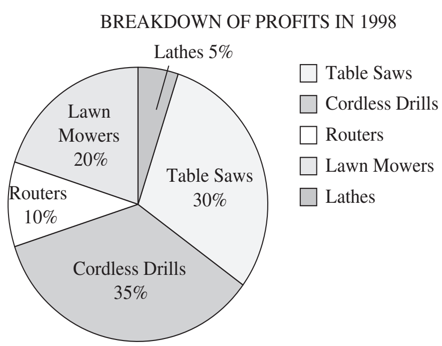
Indicate <u>all</u> possible choices.

[A] $16 - 16\pi$
[B] $16 - 4\pi$
[C] $16 - 2\pi$
[D] $16 + \pi$
[E] $16 + 4\pi$

Questions 16–20 are based on the following graphs.







16. In 1998, what were the total profits from all hardware tool sales?

$

17. Which year had the greatest percentage increase in number of table saws sold from the previous year?

Ⓐ 1993
Ⓑ 1995
Ⓒ 1997
Ⓓ 2000
Ⓔ 2001

18. Of the following, what is the closest to the percentage change in profits from table saws between 1998 and 1999?

Ⓐ A 50% increase
Ⓑ A 33% increase
Ⓒ A 17% decrease
Ⓓ A 33% decrease
Ⓔ A 50% decrease

19. If the total manufacturing cost of table saws in 1993 was $22,000, what was the price per saw?

$

20. In 1998, what were the approximate profits from sales of cordless drills?

Ⓐ $50,000
Ⓑ $70,000
Ⓒ $80,000
Ⓓ $90,000
Ⓔ $100,000

QUANTITATIVE REASONING PRACTICE SET 2 ANSWER KEY

1. A
2. A
3. C
4. C
5. D
6. D
7. B
8. A
9. D
10. B
11. D
12. E
13. D
14. B, C, D, E
15. C, D, E
16. 200,000
17. A
18. D
19. 700
20. B

Diagnose Your Results

Diagnostic Tool

Tally up your score and write your results below.

Total

Total Correct: ________ out of 20 correct

By Question Type

Quantitative Comparison (questions 1–10) ________ out of 10 correct

Problem Solving (questions 11–15) ________ out of 5 correct

Data Interpretation (questions 16–20) ________ out of 5 correct

Repeat the steps outlined in the Diagnose Your Results page that follows the Quantitative Reasoning Practice Set 1 answer key.

QUANTITATIVE REASONING PRACTICE SET 2 ANSWERS AND EXPLANATIONS

1. A

This question requires no computation but only a general understanding of how averages work and what the word "median" means. The *median* of a group of numbers is the "middle number"; it is the value above which half of the numbers in the group fall and below which the other half fall. If you have an even number of values, the median is the average of the two "middle" numbers; if you have an odd number of values, the median is one of the values. Here, in Quantity B, the median is 101. In Quantity A, if the numbers were 100, 101, and 102, then the average would also be 101, but because the third number, 103, is greater than 102, then the average must be greater than 101. Quantity A is greater than 101, and Quantity B equals 101; Quantity A is larger.

2. A

Start with the information you are given. You know that the length of the chord is 15. What does that mean? Well, because you don't know exactly where *A* and *B* are, it doesn't mean too much, but it does tell you that the distance between two points on the circle is 15. That tells you that the diameter must be at least 15. If the diameter were less than 15, then you couldn't have a chord that was equal to 15, because the diameter is always the longest chord in a circle. The diameter of the circle is 15 or greater, so the circumference must be at least 15π. That means that Quantity A must be larger than Quantity B.

3. C

The equation in the centered information looks complicated, but we'll take it one step at a time. Because Quantity A has only *h* in it, solve the equation for *h*, leaving *h* on one side of the equal sign and *r* on the other side. First, substitute the value for *x* into the equation; then solve for *h* in terms of *r*.

$x = \frac{4}{3}r^2h^2$	Substitute 1 for *x*.
$1 = \frac{4}{3}r^2h^2$	Divide both sides by $\frac{4}{3}$.
$\frac{3}{4} = r^2h^2$	Take the positive square root of both sides, using the information that *r* and *h* are positive.
$\frac{\sqrt{3}}{2} = rh$	Divide both sides by *r* to get *h* alone.
$h = \frac{\sqrt{3}}{2r}$	The two quantities are equal.

4. C

Draw an *xy*-plane and label the points given to help solve this problem. You know where points *B* and *C* are; they're on the *x*-axis. You don't know where *A* is, however, which may make you think that the answer is choice **(D)**. But you're given more information: you know that the triangle has an area of 18. The area of any triangle is one-half the product of the base and the height. Make side *BC* the base of the triangle; you know the coordinates of both points, so you can find their distance apart, which is the length of that side. *C* is at the origin, the point (0, 0); *B* is at the point (6, 0). The distance between them is the distance from 0 to 6 along the *x*-axis, or just 6. So that's the base. What about the height? Because you know that the area is 18, you can plug what you know into the area formula.

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\ 18 &= \frac{1}{2} \times 6 \times \text{height} \\ \text{height} &= \frac{18}{3} \\ \text{height} &= 6 \end{aligned}$$

That's the other dimension of the triangle. The height is the distance between the *x*-axis and point *A*. Now you know that *A* must be somewhere in the first quadrant, since both the *x*- and *y*-coordinates are positive. Don't worry about the *x*-coordinate of the point, because that's not what's being compared; you care only about the value of *y*. You know that the distance from the *x*-axis to the point is 6, because that's the height of the triangle, and that *y* must be positive. Therefore, the *y*-coordinate of the point must be 6. That's what the *y*-coordinate is: a measure of the point's vertical distance from the *x*-axis. (Note that if you hadn't been told that *y* was positive, there would be two possible values for *y*: 6 and -6. A point that's 6 units below the *x*-axis would also give a triangle with height 6.) You still don't know the *x*-coordinate of the point, and in fact you can't figure that out, but you don't care. You know that *y* is 6; therefore, the two quantities are equal.

5. D

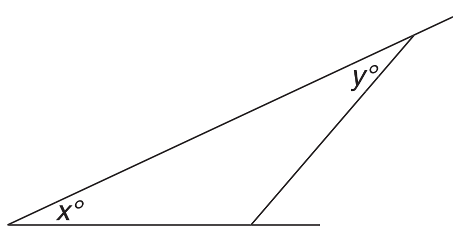
With symbolism problems like this, it sometimes helps to put the definition of the symbol into words. For this symbol, you can say something like "$x \Phi y$ means take the sum of the two numbers and divide that by the difference of the two numbers." One good way to do this problem is to Pick Numbers. You know that *p* is positive and *q* is negative. So suppose *p* is 1 and *q* is -1. Figure out what $p \Phi q$ is first. You start by taking the sum of the numbers, or $1 + (-1) = 0$. That's the numerator of the fraction, and you don't really need to go any further than that. Whatever their difference is, because the numerator is 0, the whole fraction must equal 0. (The difference can't be 0 also, since $p \neq q$.) So that's $p \Phi q$. Now what about $q \Phi p$? Well, that's going to have the same numerator as $p \Phi q$: 0. The only thing that changes when you reverse the order of the numbers is the denominator of the fraction. So $q \Phi p$ has a numerator of 0, and that fraction must equal 0 as well.

So you've found a case where the quantities are equal. Try another set of values and see whether the quantities are always equal. If $p = 1$ and $q = -2$, then the sum of the numbers is $1 + (-2)$ or -1. So that's the numerator of the fraction in each quantity. Now for the denominator of $p \Phi q$, you need $p - q = 1 - (-2) = 1 + 2 = 3$. Then the value of $p \Phi q$ is $\frac{-1}{3}$. The denominator of $q \Phi p$ is $q - p = -2 - 1 = -3$. In that case, the value of $q \Phi p$ is $\frac{-1}{-3}$ or $\frac{1}{3}$. The quantities are different; therefore, the answer is **(D)**.

6. D

Picking Numbers will help you solve this problem. For $x = 1, \frac{1}{x} + \frac{1}{x} = \frac{1}{1} + \frac{1}{1} = 2$ and $\frac{1}{x} \times \frac{1}{x} = \frac{1}{1} \times \frac{1}{1} = 1$, so Quantity A is larger. For $x = -1, \frac{1}{x} + \frac{1}{x} = \frac{1}{-1} + \frac{1}{-1} = -2$ and $\frac{1}{x} \times \frac{1}{x} = \frac{1}{-1} \times \frac{1}{-1} = 1$, so Quantity B is larger. The quantities are different; therefore, the answer is **(D).**

7. B



The sum of the three interior angles of a triangle is 180°. Because x and y are only two of the angles, their sum must be less than 180°. Quantity B is greater.

8. A

For the system of equations $4x - 5y = 10$ and $-3x + 6y = 22$, it is not necessary to solve for the values of x and y. Rather, you want to know about the sum of x and y. Notice what happens when you add the two equations.

$$\begin{array}{rcl} 4x - 5y & = & 10 \\ -3x + 6y & = & 22 \\ \hline x + y & = & 32 \end{array}$$

Because $x + y = 32$ and $33 > 32$, Quantity A is larger.

9. D

Divide both sides of the inequality by 6. You're left with $(10)^n > 10{,}001$. The number 10,001 can also be written as $10^4 + 1$, so you know that $(10)^n > 10^4 + 1$. Therefore, Quantity A, n, must be 5 or greater. Quantity B is 6. Because n could be less than, equal to, or greater than 6, you need more information.

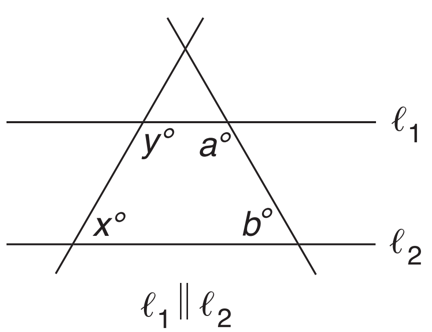
10. B

Try to set the quantities equal. Could the tens digit of y be 4? If it is, and the thousands digit is 2.5 times the units digit, then the tens digit must be … 10? That can't be right. A digit must be one of the integers 0–9; 10 isn't a digit. Therefore, 4 is too big to be the tens digit of y. You don't know what the tens digit of y is, but you know that it must be less than 4. Quantity B is greater than Quantity A.

11. D

To find the average, add the quantities together and divide by 4: $(2x + 3) + (5x - 4) + (6x - 6) + (3x - 1) = 16x - 8$ and $\frac{16x - 8}{4} = 4x - 2$. The correct choice is **(D).**

12. E



When a transversal cuts a pair of parallel lines, in this case ℓ_1 and ℓ_2, the angles are always supplementary and their sum is 180. So, the sum $(x + y)$ is equal to the sum $(a + b)$. The exact values of the individual angle measures cannot be determined from the figure. The answer is **(E).**

13. D

First, find the value of x, using the fact that there are 180° in a straight line. Set the sum of the angle measures equal to 180: $5x + 4x + 3x = 180$, $12x = 180$, and $x = 15$. Angle SRU equals $4x + 5x = 9x$, which is 135°. Choice **(D)** is correct.

14. B, C, D, E

You know that the ratio of oranges to apples is 9 to 10 and that there are at least 200 apples. The ratio tells you that there are more apples than oranges. At the minimum, there must be 180 oranges to satisfy the proportion $\frac{9}{10} = \frac{180}{200}$. There could be more than 200 apples, so any number of oranges greater than 180 for which the ratio 9:10 applies is also correct. All of the choices are multiples of 9, so the correct choices are **(B)**, **(C)**, **(D)**, and **(E)**.

15. C, D, E

The area of the shaded region is the area of the square minus the area of the portion of the circle that is inside the square. The area of a square is its side squared. The area of square $ABCD$ is $4^2 = 4 \times 4$, which is 16. Now find the area of the portion of the circle that is inside the square. Because the diameter of the circle is a side of the square, you know that exactly one-half of the circle's area is inside the square. Also, because the diameter of the circle is twice the radius, the radius of the circle is $\frac{4}{2}$ or 2. The area of a circle with a radius r is πr^2. The area of the complete circle in this question is $\pi(2)^2$, which is 4π. So half the area of this circle is 2π. Thus, the area of the shaded region is $16 - 2\pi$.

That means that $16 - 4\pi$ and $16 - 16\pi$ are less than $16 - 2\pi$, so they cannot be correct choices. However, the sum of 16 and any positive number is greater than 16 and also greater than $16 - 2\pi$. So, the correct choices are **(C)**, **(D)**, and **(E)**.

16. 200,000

From the second bar graph, the profits from table saws in 1998 were \$60,000. From the pie chart, table saws were 30% of the total profits. Let's call the total profits T dollars. Then 30% of T dollars is \$60,000. So $0.3T = 60{,}000$, and

$$T = \frac{60{,}000}{0.3} = \frac{60{,}000}{\frac{3}{10}} = \frac{10 \times 60{,}000}{3} = \frac{600{,}000}{3} = 200{,}000.$$

17. A

Use the first bar graph to analyze number of table saws sold. The year with the biggest percent increase over the previous year will be the year in which the increase is the biggest fraction of the amount from the previous year. Notice that in 1993, the increase from 1992 was approximately 60 − 30, or 30. This is approximately a 100% increase, and it is the greatest percent increase over the previous year among all the years from 1993 to 2001. There was a greater increase in number of table saws from 1996 to 1997 than from 1992 to 1993, about $110 - 70 = 40$. However, the *percent* increase from 1996 to 1997 is approximately $\frac{40}{70} \times 100\%,$ which is less than 100%, so choice **(A)** is correct.

18. D

In 1998, the profits from table saws were approximately \$60,000. In 1999, the profits from table saws were approximately \$40,000. From 1998 to 1999, there was a decrease in the profits from table saws. In general,

$$\text{Percent decrease} = \frac{\text{Original value} - \text{New value}}{\text{Original value}} \times 100\%$$

Here, the percent decrease is approximately

$$\frac{\$60{,}000 - \$40{,}000}{\$60{,}000} \times 100\% = \frac{\$20{,}000}{60{,}000} \times 100\% = \frac{1}{3} \times 100\% = 33\frac{1}{3}\%$$

A percent decrease of $33\frac{1}{3}\%$ is closest to **(D)**.

19. 700

In 1993, the profits were \$20,000. Using the formula Profit = Revenue − Cost, you can write Revenue = Cost + Profit. The cost was \$22,000. So the revenue was \$22,000 + \$20,000 = \$42,000. Because in 1993, 60 table saws were sold, each table saw was sold for $\frac{\$42{,}000}{60}$, which is **\$700**.

20. B

In 1998, the profits from table saws were about \$60,000, and this profit was 30% of the total profits. Let's call the total profits T dollars. Then 30% of T dollars is \$60,000. So $0.3T = 60{,}000$, and $T = \frac{60{,}000}{0.3} = 60{,}000 \times \frac{10}{3} = \frac{10 \times 60{,}000}{3} = \frac{600{,}00}{3} = 200{,}000.$

The total profits in 1998 were approximately \$200,000 (you may also have remembered this calculation from question 16). The profits from cordless drills were 35% of the total. So the profits from cordless drills were approximately 0.35(\$200,000), which is \$70,000 or answer choice **(B)**.

QUANTITATIVE REASONING PRACTICE SET 3

Numbers

All numbers are real numbers.

Figures

The position of points, lines, angles, and so on may be assumed to be in the order shown; all lengths and angle measures may be assumed to be positive.

Lines shown as straight may be assumed to be straight.

Figures lie in the plane of the paper unless otherwise stated.

Figures that accompany questions are intended to provide useful information. However, unless a note states that a figure has been drawn to scale, you should solve the problems by using your knowledge of mathematics, not by estimation or measurement.

Directions

Each of the following questions, 1–8, consists of two quantities, Quantity A and Quantity B. You are to compare the two quantities and choose

- (A) if Quantity A is greater
- (B) if Quantity B is greater
- (C) if the two quantities are equal
- (A) if the relationship cannot be determined from the information given

Common Information

In a question, information concerning one or both of the quantities to be compared is centered above the two quantities. A symbol that appears in both quantities represents the same thing in Quantity A as it does in Quantity B.

The diameter of a circle equals the diagonal of a square whose side length is 4.

1. | Quantity A | Quantity B |
|---|---|
| The circumference of the circle | $20\sqrt{2}$ |

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

$$x < y < z$$
$$0 < z$$

2. | Quantity A | Quantity B |
|---|---|
| x | 0 |

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

3. | Quantity A | Quantity B |
|---|---|
| The number of distinct positive integer factors of 96 | The number of distinct positive integer factors of 72 |

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

$$x > 0$$

4. | Quantity A | Quantity B |
|---|---|
| $\frac{x+1}{x}$ | $\frac{x}{x+1}$ |

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

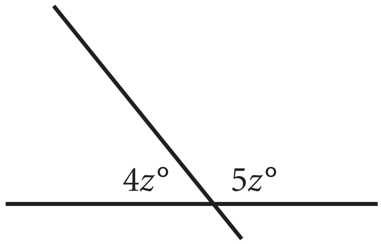
$$2^p = 4^q$$

5. | Quantity A | Quantity B |
|---|---|
| p | $2q$ |

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

6. | Quantity A | Quantity B |
|---|---|
| The number of seconds in 7 hours | The number of hours in 52 weeks |

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.



7. Quantity A | Quantity B

z | 20

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

$x > 2$

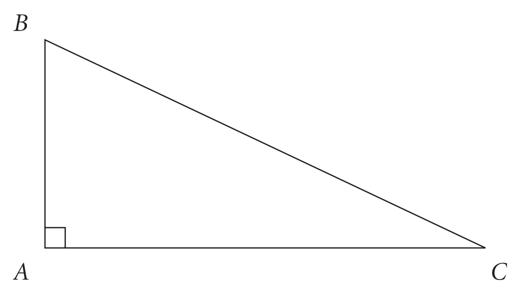
8. Quantity A | Quantity B

x^3 | $4x$

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

9. If $A \blacklozenge B = \dfrac{A + B}{B}$, and $C\clubsuit = C + 3$, what is the value of $(9\clubsuit)\blacklozenge 3$?

10. Rectangle A has a length of 12 inches and a width of 5 inches. Rectangle B has a length of 9 inches and a width of 10 inches. By what number must the area of rectangle A be multiplied in order to get the area of rectangle B?



11. In right triangle ABC above, side AB has a length of 5 units, while side BC has a length of 13 units. What is the area of ABC, in square units?

square units

12. If the average test score of four students is 85, which of the following scores could a fifth student receive such that the average of all five scores is greater than 84 and less than 86?

Indicate <u>all</u> such scores.

- [A] 88
- [B] 86
- [C] 85
- [D] 83
- [E] 80

13. Meg is twice as old as Rolf, but three years ago, she was two years older than Rolf is now. How old is Rolf now?

[ ] years old

14. The cost, in cents, of manufacturing x crayons is $570 + 0.5x$. The crayons sell for 10 cents each. What is the minimum number of crayons that need to be sold so that the revenue received recoups the manufacturing cost?

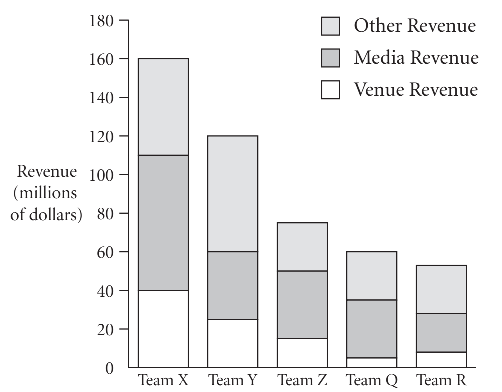
Ⓐ 50
Ⓑ 57
Ⓒ 60
Ⓓ 61
Ⓔ 95

15. If $xy \neq 0$, $\dfrac{1-x}{xy} =$

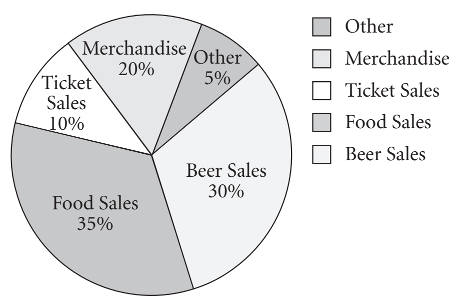
Ⓐ $\dfrac{1}{xy} - \dfrac{1}{y}$
Ⓑ $\dfrac{x}{y} - \dfrac{1}{x}$
Ⓒ $\dfrac{1}{xy} - 1$
Ⓓ $\dfrac{1}{xy} - \dfrac{x^2}{y}$
Ⓔ $\dfrac{1}{x} - \dfrac{1}{y}$

Questions 16–20 refer to the following graphs:

TEAM REVENUES FOR 1997



PERCENTAGES OF VENUE REVENUES FOR TEAM X, 1997



16. For the team with the median venue revenue in 1997, media revenue represented approximately what percent of that team's total revenue?

Ⓐ 25%
Ⓑ 30%
Ⓒ 45%
Ⓓ 70%
Ⓔ 85%

17. Of the following, which is greater than the amount of revenue, in millions of dollars, earned by Team X through food sales in 1997?

Indicate <u>all</u> such amounts.

- [A] 7
- [B] 10
- [C] 14
- [D] 18
- [E] 22

18. In 1997, which teams had media revenues of less than $25 million?

Indicate <u>all</u> such teams.

- [A] Team X
- [B] Team Y
- [C] Team Z
- [D] Team Q
- [E] Team R

19. If Team Y earned total revenues of at least $150 million in 1998, then Team Y's total revenue could have increased by what percent from 1997 to 1998?

Indicate <u>all</u> such percents.

- [A] 20%
- [B] 25%
- [C] 30%
- [D] 35%
- [E] 40%

20. The venue revenues for Team X from merchandise sales and ticket sales were approximately what percent of the venue revenues for Team X from food sales?

- (A) 43%
- (B) 53%
- (C) 67%
- (D) 71%
- (E) 86%

QUANTITATIVE REASONING PRACTICE SET 3 ANSWER KEY

1. B
2. D
3. C
4. A
5. C
6. A
7. C
8. A
9. 5
10. 1.5
11. 30
12. A, B, C, D
13. 5
14. C
15. A
16. C
17. D, E
18. E
19. B, C, D, E
20. E

Diagnose Your Results

Diagnostic Tool

Tally up your score and write your results below.

Total

Total Correct: ________ out of 20 correct

By Question Type

Quantitative Comparison (questions 1–8) ________ out of 8 correct

Problem Solving (questions 9–15) ________ out of 7 correct

Data Interpretation (questions 16–20) ________ out of 5 correct

Repeat the steps outlined in the Diagnose Your Results page that follows the Quantitative Reasoning Practice Set 1 answer key.

QUANTITATIVE REASONING PRACTICE SET 3 ANSWERS AND EXPLANATIONS

1. B

The diagonal of a square of side 4 is $4\sqrt{2}$. The circumference of a circle is π times the diameter. So, the circumference of this circle is $4\sqrt{2}\pi$. Now write Quantity B, $20\sqrt{2}$, as $4(5)\sqrt{2}$ and you can compare the quantities piece by piece. The factors of 4 and $\sqrt{2}$ are the same in both quantities, but π is less than 5. So, Quantity B is larger.

2. D

You could Pick Numbers here or else just use logic. You know that z is positive and that x and y are less than z. But does that mean that x or y must be negative? Not at all—they could be, but they could also be positive. For instance, suppose $x = 1$, $y = 2$, and $z = 3$. Then Quantity A would be larger. However, if $x = -1$, $y = 0$, and $z = 1$, then Quantity B would be larger. You need more information to determine the relationship between the quantities. The answer is **(D)**.

3. C

There are 12 positive integer factors of 96: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, and 96. There are 12 positive integer factors of 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, and 72. The two quantities are equal.

4. A

If $x > 0$, then $\frac{x+1}{x}$, which also equals $1 + \frac{1}{x}$, must be greater than 1. On the other hand, $\frac{x}{x+1}$ must be less than 1. This is because when $x > 0$, the numerator x is smaller than the denominator, so the ratio $\frac{x}{x+1} < 1$. Therefore, $\frac{x+1}{x} > \frac{x}{x+1}$ when $x > 0$, and Quantity A is greater.

5. C

For this question, notice the relationship between the bases, 2 and 4. When comparing exponents, it's easiest to work with equal bases.

You know that $4 = 2^2$. Therefore, $4^q = (2^2)^q = 2^{2q}$. Now you have $2^p = 2^{2q}$, so $p = 2q$. The quantities are equal, choice **(C)**.

6. A

Before you go to the trouble of multiplying the terms, let's see if there's a shortcut. For the GRE, make sure you know the common unit conversions for time. There are 60 seconds in a minute and 60 minutes in an hour, so there are $7 \times 60 \times 60$ seconds in 7 hours. There are 24 hours in a day and 7 days in a week, so there are $7 \times 24 \times 52$ hours in 52 weeks. Let's rewrite the quantities:

Quantity A	Quantity B
$7 \times 60 \times 60$	$7 \times 24 \times 52$

Taking away the common values gives you:

Quantity A	Quantity B
60×60	24×52

You still shouldn't do the math, however. The best strategy is to compare piece by piece, which shows that Quantity A is larger than Quantity B.

7. C

The sum of the measures of the angles on one side of a straight line is 180°. Therefore, $4z + 5z = 180$, so $9z = 180$. Divide both sides by 9 to find $z = 20$. **(C)** is the answer.

8. A

Since $x > 2$, you know $x > 0$ and you can divide both quantities by x without changing their relationship. Quantity A is then x^2 and Quantity B is 4. Since $x > 2$, the least value for x^2 is greater than $2^2 = 4$. Therefore, **(A)** is correct.

9. 5

Let's first find the value of $9\clubsuit$. Then we'll find the value of $(9\clubsuit)\blacklozenge 3$.

Since $C\clubsuit = C + 3, 9\clubsuit = 9 + 3 = 12.$
Therefore, $(9\clubsuit)\blacklozenge 3 = 12\blacklozenge 3.$
Since $A \blacklozenge B = \dfrac{A + B}{B}$, $12 \blacklozenge 3 = \dfrac{12 + 3}{3} = \dfrac{15}{3}$
Therefore, $(9\clubsuit)\blacklozenge 3 = 5.$

10. 1.5

The area of a rectangle is its length times its width.

The area of rectangle A is $12 \times 5 = 60$.

The area of rectangle B is $9 \times 10 = 90$.

So the area 60 of rectangle A must be multiplied by a number, which you can call x, to obtain the area 90 of rectangle B.

Then $60x = 90$. So $x = \frac{90}{60} = \frac{3}{2} = 1.5$.

11. 30

Here's a problem where it really pays to have learned the special right triangles. Because one leg of the right triangle is 5 and the hypotenuse is 13, you have a special right triangle, the 5:12:13 right triangle. So the length of AC is 12.

The area of a triangle is $\frac{1}{2}$ of the base times the height. The area of a right triangle is $\frac{1}{2} \times (\text{leg})_1 \times (\text{leg})_2$, because one leg can be considered to be the base and the other leg can be considered to be the height. So the area of triangle ABC is

$$\frac{1}{2} \times (AC) \times (AB) = \frac{1}{2} \times 12 \times 5 = 6 \times 5 = 30$$

The answer is **30**.

12. A, B, C, D

The average formula is as follows:

$$\text{Average} = \frac{\text{Sum of the terms}}{\text{Number of terms}}$$

Therefore,

$$\text{Sum of the terms} = \text{Average} \times \text{Number of terms}$$

The sum of the scores of the four students whose average was 85 is $85(4) = 340$. Let's call the fifth student's score x. If the new average is to be greater than 84 and less than 86 and the sum of the scores of all five students is $340 + x$, then $84 < \frac{340 + x}{5} < 86$. If you multiply all parts of the inequality by 5, you get $420 < 340 + x < 430$. Subtracting 340 from all parts of the inequality, you get $80 < x < 90$, making **(A)**, **(B)**, **(C)**, and **(D)** the correct choices.

13. 5

This question can be broken into two equations with two unknowns, Meg's age now (M) and Rolf 's age now (R). Equation (i) shows the relationship now; equation (ii) shows the relationship three years ago.

$$\text{(i) } M = 2 \times R \qquad \text{(ii) } M - 3 = R + 2$$

Substitute $2R$ for M in equation (ii) and solve for R:

$$\begin{aligned} M - 3 &= R + 2 \\ 2R - 3 &= R + 2 \\ 2R - R &= 2 + 3 \\ R &= 5 \end{aligned}$$

Rolf is **5** years old now.

14. C

The cost of manufacturing x crayons is $(570 + 0.5x)$ cents. Because each crayon sells for 10 cents, x crayons will sell for $10x$ cents. You want the smallest value of x such that $10x$ cents is at least $570 + 0.5x$ cents. So you must solve the equation $10x = 570 + 0.5x$ for the value of x that will recoup the investment.

$$\begin{aligned} 10x &= 570 + 0.5x \\ 9.5x &= 570 \\ x &= 60 \end{aligned}$$

The minimum number of crayons is 60, choice **(C)**.

Alternatively, you could have avoided setting up an algebraic equation by Backsolving, starting with either **(B)** or **(D)**.

15. A

You can write that $\frac{1 - x}{xy} = \frac{1}{xy} - \frac{x}{xy}$. Canceling a factor of x from the numerator and denominator of $\frac{x}{xy}$, you have $\frac{x}{xy} = \frac{1}{y}$.

So, $\frac{1 - x}{xy} = \frac{1}{xy} - \frac{x}{xy} = \frac{1}{xy} - \frac{1}{y}$. The answer is **(A)**.

16. C

Before you answer any graph question, begin by examining the graphs. Here you have two graphs, a segmented bar graph representing team revenue breakdowns for five teams and a pie chart showing the distribution of venue revenues for Team X.

You're now ready to attack the question, which asks you to find the team with the median venue revenue for 1997 and to determine what percent of that team's total revenue is media revenue. This question must refer to the first graph, and the first part of the question—finding the team with the median venue revenue—is straightforward. *Median* refers to the number in the middle. Looking at the white portions of the bars in the top graph, you see that Team Z has the median venue revenue. The fastest approach to the answer here (and throughout graph questions generally) is to approximate. The downside to bar graphs is that it's often very hard to get a read on the values. The upside is that if you approximate, often you don't have to read the values. Here you need to determine what percent of Team Z's bar is represented by media revenue (the segment in the middle—always be especially careful to isolate the correct piece of data). By approximating, you can see that the middle segment is about half of the entire bar. Thus the correct answer has to be close to 50%. The only answer choice that works is **(C)**, 45%.

17. D, E

The key to this question is that it involves both graphs. The question asks for the amount Team X earned through food sales, which takes you first to the pie chart, where you see that food sales accounted for 35% of the venue revenues for Team X. But to convert that to a dollar amount, you need a figure for the amount earned in venue revenues by Team X in 1997. According to the bar graph, this is somewhere around \$40 million. Now, take 35% of \$40 million: $0.35 \times 40 = 14$, so the answer is any amount greater than 14. The answers are **(D)** and **(E)**.

18. E

Look at the bar graph: Team Q had media revenues of $35 - 5 = 30$ million, and Teams X, Y, and Z had media revenues greater than those of team Q. Team R had media revenue of $30 - 10 = 20$ million. The only correct choice is **(E)**.

19. B, C, D, E

Percent change problems are extremely popular graph questions, and as long as you set them up correctly, they are a great opportunity. This question asks for the approximate percent increase in Team Y's total revenue from 1997 to 1998, so you need to figure out (roughly) the amount of increase, place that over the original (or smaller) amount, and then convert the fraction into a percent. You are given the total revenue for 1998 as at least \$150 million, so you need to locate the total revenue for 1997 from the bar graph. It looks to be approximately \$120 million, so the amount of increase is \$30 million (or more), and the original amount is \$120 million. Now let's apply the formula:

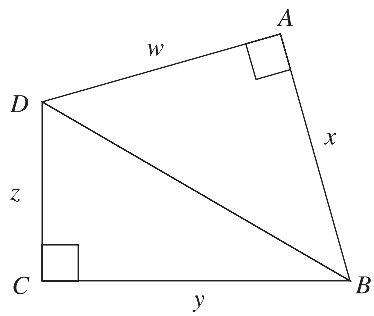
$$\begin{aligned}
\text{Percent increase} &= \frac{\$30 \text{ million}}{\$120 \text{ million}} \times 100\% \\
&= \frac{1}{4} \times 100\% \\
&= 25\%
\end{aligned}$$

So, any percent greater than or equal to 25% is the answer. The answers are **(B)**, **(C)**, **(D)**, and **(E)**.

20. E

Looking at the bar graph, you see from the lowest portion of the bar for Team X that venue revenues of Team X were approximately 40 million dollars (call it 40m, for short). From the pie chart, the venue revenues of Team X from merchandise sales were approximately 20% of 40 million dollars, the venue revenues from ticket sales were approximately 10%, and the venue revenues from food sales were approximately 35%. The venue revenues of Team X from merchandise, in dollars, were approximately $0.2(40\text{m}) = 8\text{m}$. The venue revenues of Team X from ticket sales, in dollars, were approximately $0.1(40\text{m}) = 4\text{m}$. So the venue revenues of Team X from merchandise sales and ticket sales, in dollars, were approximately $8\text{m} + 4\text{m} = 12\text{m}$. The venue revenues of Team X from food sales, in dollars, were approximately $0.35(40\text{m}) = 14\text{m}$. The percent that the venue revenues of Team X that were from merchandise sales and ticket sales, out of the venue revenues of Team X that were from food sales, is approximately $\frac{12m}{14m} \times 100\% = \frac{6}{7} \times 100\% \approx 85.7\%$

To the nearest percent, 85.7% is 86%. Choice **(E)** is correct.



$\triangle ABD$ and $\triangle CDB$ are right triangles.

1. | Quantity A | Quantity B |
|---|---|
| $w^2 + x^2$ | $y^2 + z^2$ |

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

2.

$$x + 4y = 6$$
$$x = 2y$$

Quantity A	Quantity B
x	y

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

3. In a certain accounting firm, each employee is either a manager, a technician, or an assistant. Twenty-five percent of all employees are managers. Of the remaining employees, one-third are assistants.

Quantity A	Quantity B
The number of managers	Half of the number of technicians

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

4. | Quantity A | Quantity B |
|---|---|
| $(a + 1)(b + 1)$ | $ab + 1$ |

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

5. In the two-digit number jk, the value of the digit j is twice the value of the digit k.

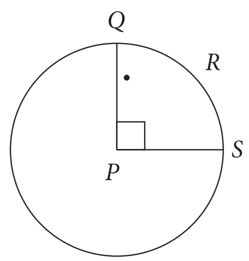
Quantity A	Quantity B
k	6

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

6. Henry purchased x apples, and Jack purchased 10 apples less than one-third of the number of apples Henry purchased.

Quantity A	Quantity B
The number of apples Jack purchased	$\frac{x-30}{3}$

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.



P is the center of a circle with diameter of 8.

7.

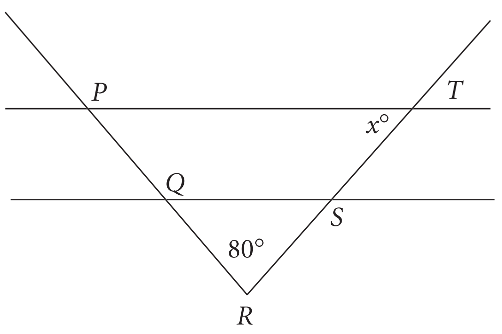
Quantity A	Quantity B
The length of arc *QRS*	2

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

8. $4 < x < 6$
$1 < y < 2$

Quantity A	Quantity B
The volume of a rectangular solid with a length of 5 feet, a width of 4 feet, and a height of x feet	The volume of a rectangular solid with a length of 10 feet, a width of 8 feet, and a height of y feet

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.



9. In the figure shown above, what is x?

Ⓐ 40
Ⓑ 50
Ⓒ 60
Ⓓ 70
Ⓔ 80

10. A producer must select a duo, consisting of one lead actor and one supporting actor, from six candidates. What is the number of possible duos the producer could select?

[ ] possible duos

11. Jane must select three different items for each dinner she will serve. The items are to be chosen from among five different vegetarian and four different meat selections. If at least one of the selections must be vegetarian, how many different dinners could Jane create?

(A) 30
(B) 40
(C) 60
(D) 70
(E) 80

12. A computer can perform 30 identical tasks in six hours. At that rate, what is the minimum number of computers that should be assigned to complete 80 tasks within three hours?

[ ] computers

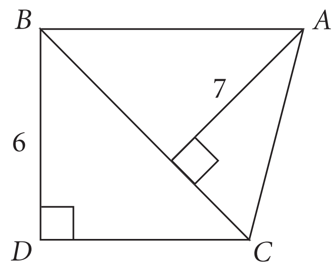
13. Given a positive integer c, how many integers are greater than c and less than $2c$?

(A) $\frac{c}{2}$
(B) c
(C) $c - 1$
(D) $c - 2$
(E) $c + 1$

14. If the ratio of $2a$ to b is 8 times the ratio of b to a, then $\frac{b}{a}$ could be which of the following?

Indicate <u>all</u> possible choices.

[A] -2
[B] $-\frac{1}{2}$
[C] $\frac{1}{4}$
[D] $\frac{1}{2}$
[E] 2



15. In the figure above, the area of ΔABC is 35. What is the length of DC?

[ ]

16. If $3^m = 81$, then $m^3 =$

Ⓐ 4
Ⓑ 9
Ⓒ 16
Ⓓ 64
Ⓔ 81

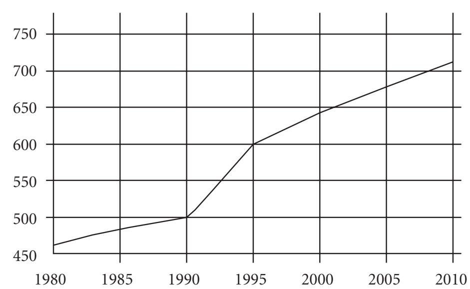
17. If $0 < x < 1$, which of the following must be true?

Indicate all possible choices.

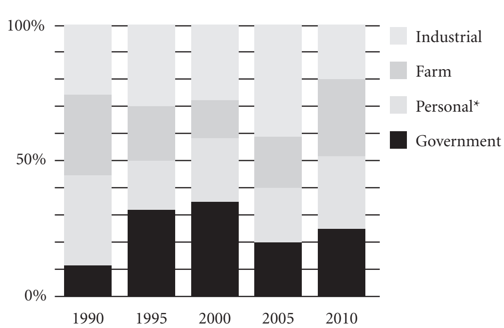
[A] $2x < x$
[B] $2x < 1$
[C] $2x > 1$
[D] $x^2 < x$
[E] $x^2 < 1$

Questions 18–20 are based on the following graphs.

ENERGY USE BY YEAR, COUNTRY Y, 1980–2010
(IN MILLIONS OF KILOWATT-HOURS)



ENERGY USE BY TYPE, COUNTRY Y



*Total personal use = population × per capita personal use

18. In 1995, how many of the categories shown had energy use greater than 150 million kilowatt-hours?

Ⓐ None
Ⓑ One
Ⓒ Two
Ⓓ Three
Ⓔ Four

19. If the population of Country Y in 2005 was 500 million, what was the per capita personal energy use in 2005? (in millions of kilowatt-hours)

Ⓐ 0.04
Ⓑ 0.14
Ⓒ 0.27
Ⓓ 0.37
Ⓔ 0.50

20. According to the graphs, total kilowatt-hours of energy for farm use increased between which of the following years?

Choose all that apply.

[A] 1990 and 1995
[B] 1995 and 2000
[C] 2000 and 2005
[D] 2005 and 2010

QUANTITATIVE REASONING 2

35 Minutes — 20 Questions

Directions: For each question, indicate the best answer, using the directions given.

You may use a calculator for all the questions in this section.

If a question has answer choices with **ovals**, then the correct answer is a single choice. If a question has answer choices with **squares**, then the correct answer consists of one or more answer choices. Read each question carefully.

Important Facts:

All numbers used are real numbers.

All figures lie in a plane unless otherwise noted.

Geometric figures, such as lines, circles, triangles, and quadrilaterals, **may or may not be** drawn to scale. That is, you should not assume that quantities such as lengths and angle measures are as they appear in a drawing. But you can assume that lines shown as straight are indeed straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. For questions involving drawn figures, base your answers on geometric reasoning, rather than on estimation, measurement, or comparison by sight.

Coordinate systems, such as *xy*-planes and number lines, **are** drawn to scale. Therefore, you may read, estimate, and compare quantities in these figures by sight or by measurement.

Graphical data presentations, such as bar graphs, line graphs, and pie charts, **are** drawn to scale. Therefore, you may read, estimate, and compare data values by sight or by measurement.

1. The perimeter of isosceles ΔABC is 40, and the length of side BC is 12.

Quantity A	Quantity B
The length of side AB	14

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

2. $f(x) = (x + 3)^2$

Quantity A	Quantity B
$f(0.5)$	9

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

3.

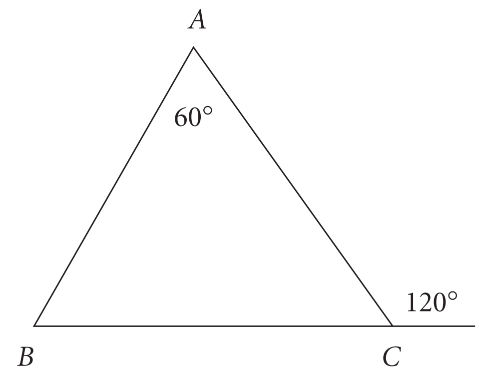
Quantity A	Quantity B
The number of miles traveled by a car that traveled for four hours at an average speed of 40 miles per hour	The number of miles traveled by a train that traveled for two and a half hours at an average speed of 70 miles per hour

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

4. A single cookie weighs between 5 and 15 grams. A single grape weighs exactly 1 gram.

Quantity A	Quantity B
The number of cookies in a bag that weighs 300 grams and contains only cookies	The number of grapes in a bag that weighs 50 grams and contains only grapes

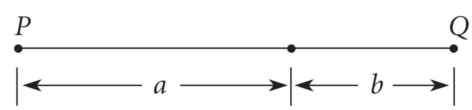
Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.



5.

Quantity A	Quantity B
The length of side AB	The length of side BC

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given



$$8a + 8b = 24$$

6.

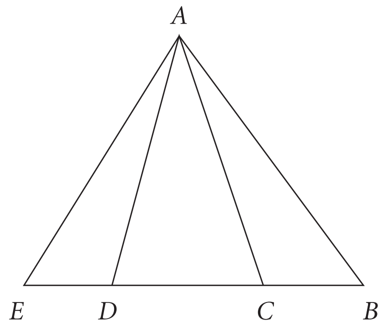
Quantity A	Quantity B
The length of segment PQ	2

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

7. $x < y$

Quantity A	Quantity B
$y - x$	$x - y$

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

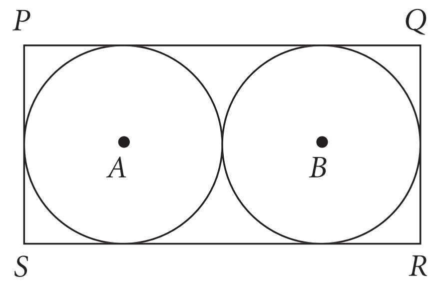


ΔABE is an equilateral triangle
The area of ΔACE = the area of ΔABD

8.

Quantity A	Quantity B
The length of side AD	The length of side AC

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.



9. The two circles with centers A and B have the same radius, r. If $r = 3$, what is the perimeter of rectangle $PQRS$?

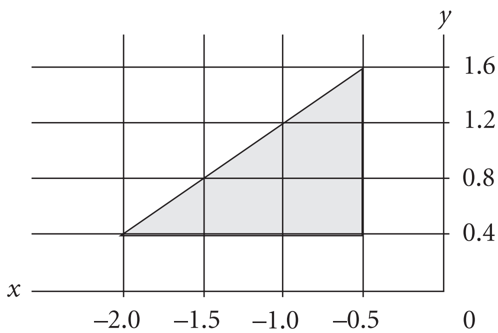
(A) 12
(B) 18
(C) 24
(D) 36
(E) 48

10. What is the least integer value of x for which $1-\left(\frac{1}{4}\right)^x$ is greater than 0?

(A) -2
(B) -1
(C) 0
(D) 1
(E) 2

11. If $\frac{p-q}{p}=\frac{2}{7}$, then $\frac{q}{p}=$

(A) $\frac{2}{5}$
(B) $\frac{5}{7}$
(C) 1
(D) $\frac{7}{5}$
(E) $\frac{7}{2}$



12. What is the area of the shaded region in the figure above?

[ ] square units

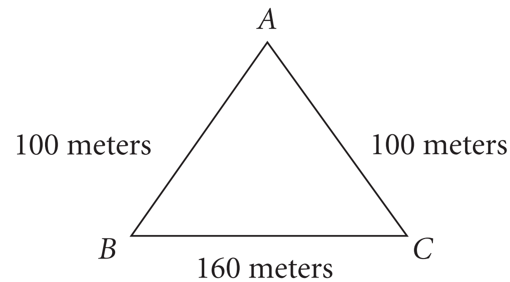
13. Which of the following is 850% greater than 8×10^3?

- (A) 8.5×10^3
- (B) 6.4×10^4
- (C) 6.8×10^4
- (D) 7.6×10^4
- (E) 1.6×10^5

14. Which of the following are divisible by exactly 4 distinct, positive integers?

Indicate all possible numbers.

- [A] 4
- [B] 6
- [C] 8
- [C] 12
- [D] 14



15 The figure above represents a triangular field. What is the minimum distance, in meters, that a person would have to walk to go from point A to a point on side BC?

[ ] meters

16. If the average of two numbers is $3y$ and one of the numbers is $y - z$, what is the other number, in terms of y and z?

- (A) $y + z$
- (B) $3y + z$
- (C) $4y - z$
- (D) $5y - z$
- (E) $5y + z$

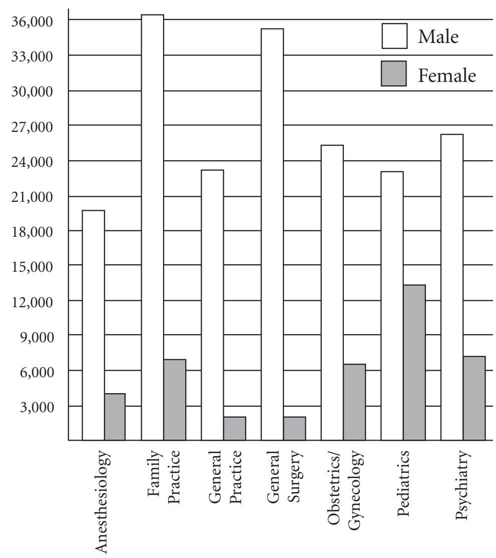
17. Which points lie on the graph of $y = \frac{x^2}{x+1}$?

Indicate all possible choices.

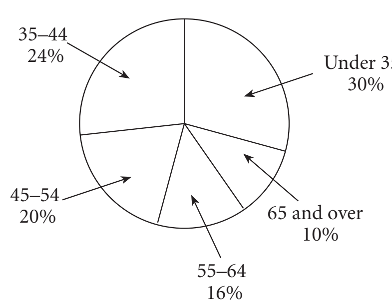
- [A] $(-3, -5)$
- [B] $(-2, -4)$
- [C] $(-1, -3)$
- [D] $\left(1, \frac{1}{2}\right)$
- [E] $\left(3, 2\frac{1}{2}\right)$

Questions 18–20 refer to the charts below.

U.S. PHYSICIANS IN SELECTED SPECIALTIES
BY GENDER, 1986



GENERAL SURGERY PHYSICIANS BY AGE, 1986



18. Which of the following physician specialties had the lowest ratio of males to females in 1986?

Ⓐ family practice
Ⓑ general surgery
Ⓒ obstetrics/gynecology
Ⓓ pediatrics
Ⓔ psychiatry

19. If the number of female general surgery physicians in the under-35 category represented 3.5 percent of all the general surgery physicians, approximately how many male general surgery physicians were under 35 years?

Ⓐ 9,200
Ⓑ 9,800
Ⓒ 10,750
Ⓓ 11,260
Ⓔ 11,980

20. Approximately what percent of all general practice physicians in 1986 were male?

Ⓐ 23%
Ⓑ 50%
Ⓒ 75%
Ⓓ 82%
Ⓔ 90%

Your Practice Test is now complete.

CHAPTER 21

Practice Test Answers

VERBAL REASONING 1 ANSWER KEY

1. C
2. B
3. A, D
4. B, D
5. A, E, H
6. A, E
7. C, E
8. A, D
9. B, C
10. D, F
11. A
12. D
13. C
14. D
15. A, B
16. E
17. E
18. A, B
19. C
20. D

QUANTITATIVE REASONING 1 ANSWER KEY

1. C
2. A
3. C
4. D
5. B
6. C
7. A
8. D
9. B
10. 30
11. E
12. 6
13. C
14. B, D
15. 8
16. D
17. D, E
18. C
19. C
20. C, D

VERBAL REASONING 2 ANSWER KEY

1. B
2. B, D, G
3. A, F, H
4. E
5. A, E, I
6. D
7. A, F
8. A, F
9. B, D
10. B, C
11. E
12. C
13. A, B
14. C
15. E
16. D
17. A
18. B
19. C
20. E

QUANTITATIVE REASONING 2 ANSWER KEY

1. D
2. A
3. B
4. D
5. C
6. A
7. A
8. C
9. D
10. D
11. B
12. 0.9
13. D
14. B, C, E
15. 60
16. E
17. B, D
18. D
19. B
20. E

Diagnostic Tool

Tally up your score and write the results below.

You can also use the online answer grid available in your Online Center to enter your answers to the multiple-choice sections of this test. If you do so, you will receive a score estimate and a detailed breakdown of your performance by question type and topic. (The score you will receive is only an estimate, however, since a paper-based test, by definition, cannot mimic the adaptive nature and scoring algorithm of the GRE multi-stage test. For practice taking real MSTs, use the tests in your Online Center.)

Total

Total Correct: ________ out of 80

By Section

Verbal Reasoning ________ out of 40
Quantitative Reasoning ________ out of 40

QUANTITATIVE REASONING 1 ANSWERS AND EXPLANATIONS

1. C

Right triangles *ABD* and *CDB* share a hypotenuse, segment *DB*. The shared hypotenuse should clue you to use the Pythagorean theorem. See that w and x are lengths of the legs of right triangle *ABD*; side *AD* has length w, side *AB* has length x. Also, y and z are lengths of the legs of right triangle *CDB*; side *CD* has length z, side *CB* has length y. Where a and b are lengths of the legs of a right triangle, and c is the length of the hypotenuse, $a^2 + b^2 = c^2$. So here $w^2 + x^2 =$ length BD^2; $y^2 + z^2$ also equals length BD^2. The quantities are equal, and the answer is **(C)**.

2. A

You have $x + 4y = 6$ and $x = 2y$, and you want to compare x and y. Let's start by finding y. Substitute $2y$ for x in the first equation and get $2y + 4y = 6$ or $6y = 6$. Divide both sides by 6 and get $y = 1$. If $y = 1$ and $x = 2y$, as the second equation states, x must equal 2. Because 2 is greater than 1, Quantity A is greater.

3. C

The problem doesn't say how many employees work at the firm, so let's pick a number. Since the problem involves percents, let's pick 100 as the total number of employees.

If there are 100 employees working at the firm, then one quarter of them, or 25, are managers. That leaves 75 employees, one-third of which, or 25, are assistants. Consequently, $100 - 25 - 25 = 50$ employees are left to be technicians. Now check the quantities. Quantity A, the number of managers, is 25. Quantity B, half the number of technicians, is half of 50, which is also 25. Pick **(C)** *the two quantities are equal*.

4. D

To make the quantities look as much alike as you can, use FOIL to multiply out Quantity A. You'll multiply $a \times b$, $1 \times b$, $1 \times a$, and 1×1 and get $ab + a + b + 1$. Quantity B also has $ab + 1$. Quantity A has the additional terms a and b. There is no information given about possible values for a or b. Because $a + b$ could be positive, negative, or zero, a relationship cannot be determined, and the answer is **(D)**.

You can also use Picking Numbers; let $a = 1$ and $b = 2$. Then Quantity A is $(1 + 1)(2 + 1) = 6$ and Quantity B is $(1 \times 2) + 1 = 3$. In this case, Quantity A is greater. But if you let $a = -1$ and $b = -2$, you have Quantity A = $(-1 + 1)(-2 + 1) = 0$ and Quantity B = $(-1 \times -2) + 1 = 3$. In this case, Quantity B is greater. You have demonstrated that a definite relationship cannot be determined, leading to answer choice **(D)**.

5. B

In the two-digit number *jk*, the value of digit *j* is twice the value of digit *k*. You have to compare the value of *k* in Quantity A with 6 in Quantity B. If you plug in 6 for *k*, it is not possible to enter "twice the value of the digit *k*" for the digit *j*. That is because *j* can only be a single digit; it cannot be 12. In other words, *k* has to be something less than 6, so the answer must be **(B)**. The value in Quantity B is greater.

6. C

Henry purchased *x* apples, and Jack purchased 10 apples less than one-third the number of apples Henry purchased. *One-third of* means the same as *one-third times*, and the number of apples Henry purchased is *x*. Thus, this boils down to $J = \frac{1}{3}x - 10$. You can plug this in for Quantity A. We have $\frac{1}{3}x - 10$ in Quantity A and $\frac{x-30}{3}$ in Quantity B. Now you can clear the fraction in Quantity B. Let's split Quantity B into two fractions: $\frac{x}{3} - \frac{30}{3}$. Leave the $\frac{x}{3}$ alone and cancel the factor of 3 from the numerator and denominator of $\frac{30}{3}$ and you're left with $\frac{x}{3} - 10$. What's $\frac{x}{3}$? It's one-third of *x*. So Quantity A equals $\frac{1}{3}x - 10$, while Quantity B also equals $\frac{1}{3}x - 10$, and the answer is **(C)**.

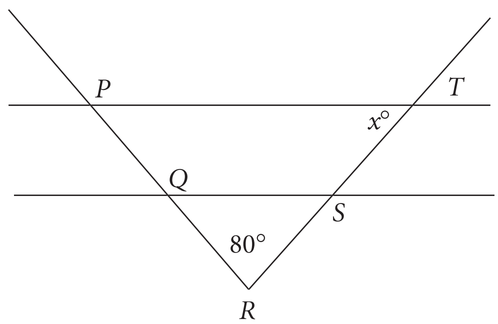
7. A

The figure shows a circle with diameter 8. The circumference of the circle is therefore 8π. Since the $90°$ central angle cuts a quarter of the circle, the length of arc *QRS* must likewise be a quarter of the circumference. Quantity A is therefore one-quarter of 8π, or 2π. Because π is a positive number, 2π must be greater than 2. Pick **(A)** *Quantity A is greater.*

8. D

You can suspect **(D)** because there is a range of possible values for the variables. In Quantity A, you have the volume of a rectangular solid with length 5 feet, width 4 feet, and height *x* feet. The formula is length times width times height, so the volume is 5 times 4 times *x*, or 20*x*. The volume of Quantity B is therefore 10 times 8 times *y*, or 80*y*. If $4 < x < 6$, then the range of values for Quantity A is $80 < V < 120$. If $1 < y < 2$, then the range of values for Quantity B is $80 < V < 160$. Since the two ranges overlap, it's possible that the two quantities are equal or that one is greater than the other. So, the correct answer is **(D)**.

9. B



$$PQ = ST$$
$$QR = RS$$

The goal is to find x, the measure of one of the angles formed by the intersection of ST and PT. Now angle QRS is labeled 80°. You also know PQ and ST have the same length and QR and RS have the same length. If you add PQ and QR, you get PR. If you add ST and RS, you get RT. If you add equals to equals, you get equals, so $PQ + QR$ must be the same as $ST + RS$, which means that PR and RT are the same. Thus, you have isosceles triangle PRT, and you're given one angle that has measure 80 and a second angle that has measure x. The angle measuring x is opposite equal side PR. That means the other angle must have the same measure. The sum of the interior angles in a triangle always equals 180°. Thus, $x + x + 80$ must equal 180, $2x = 100$, and $x = 50$. The answer is **(B).**

10. 30

This is a permutation problem because the order in which the duo is chosen matters. The producer has two slots to fill. For the lead role, there are 6 people to choose from. For the supporting role, there will be 5 people to choose from. So the number of possible duos is $6 \times 5 =$ **30**.

11. E

The question asks for the number of different dinners Jane could make. Since the order of the selections in the dinner doesn't matter, this is a combination problem. But it involves three possible combination types: Veg, Meat, Meat; Veg, Veg, Meat; or Veg, Veg, Veg. We must calculate the possibilities for each type of combination and then add the results to find the total number of different combinations possible.

Let V represent vegetarian and M represent meat.

Then with V, M, M, she has 5 choices for the vegetarian
(she must choose 1) $\times$ 4 choices for meat (she must choose 2).

For V, V, M, she will choose 2 from among 5 for the vegetarian and 1 among 4 for the meat.

If she goes with V, V, V, the all-vegetarian menu, she will choose a subgroup of 3 from among 5 vegetarian choices.

If n and k are positive integers where $n = k$, then the number of different subgroups consisting of k objects that can be selected from a group consisting of n different objects, denoted by ${}_nC_k$, is given by the formula

$${}_nC_k = \frac{n!}{k!(n-k)!}$$

Here the total number of different possible servings for a plate is $({}_5C_1)({}_4C_2) + ({}_5C_2)({}_4C_1) + ({}_5C_3)$.

Now ${}_5C_1$ represents choosing 1 type of vegetable selection from 5 different types, so ${}_5C_1 = 5$. (The formula also gives this result.) Now we use the formula to find the next two combinations:

$$
\begin{aligned}
{}_4C_2 &= \frac{4!}{2!(4-2)!} = \frac{4!}{2!\times 2!} = \frac{4\times 3\times 2\times 1}{2\times 1\times 2\times 1} = 6 \\
{}_5C_2 &= \frac{5!}{2!(5-2)!} = \frac{5!}{2!\times 3!} = \\
&\quad \frac{5\times \cancel{4}^{2}\times \cancel{3}\times \cancel{2}\times \cancel{1}}{\cancel{2}\times 1\times \cancel{3}\times \cancel{2}\times \cancel{1}} = \\
&\quad \frac{5\times 4\times 3\times 2\times 1}{2\times 1\times 3\times 2\times 1} = 10
\end{aligned}
$$

Here ${}_4C_1$ corresponds to choosing 1 type of meat selection from 4 different types, so ${}_4C_1 = 4$. Then we use the formula again:

$$
\begin{aligned}
{}_5C_3 &= \frac{5!}{3!(5-3)!} = \frac{5!}{3!\times 2!} = \\
&\quad \frac{5\times 4\times 3\times 2\times 1}{3\times 2\times 1\times 2\times 1} = 10
\end{aligned}
$$

So the number of different possible dinners of these three items is $5 \times 6 + 10 \times 4 + 10 = 80$, choice **(E)**.

12. 6

You could find the number of tasks per hour from one computer, but that would add extra steps, because you want to find out how many computers you need to do a certain number of tasks in three hours. Well, if the computer can do 30 tasks in six hours, it can do 15 tasks in three hours. So, two computers could complete 30 tasks in that time. Three computers could do 45; four could do 60; five could do 75; six could do 90. You can't get by with five computers because you have to get 80 tasks done, so you'll need **6** computers.

13. C

Picking Numbers is the best strategy, since there are variables in the question and the answer choices. If $c = 3$, then $2c = 6$. There are two integers between 3 and 6, so plug $c = 3$ into the answer choices to see which one is equal to 2.

(A) $\frac{3}{2} \neq 2$

(B) $3 \neq 2$

(C) $3 - 1 = 2$

(D) $3 - 2 \neq 2$

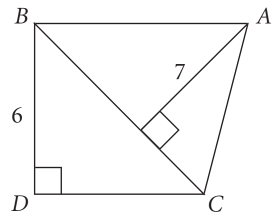
(E) $3 + 1 \neq 2$

The only answer choice that equals 2 when $c = 3$ is **(C)**, so **(C)** is correct.

14. B, D

You're asked to find what $\frac{b}{a}$ *could* be; that tells you there may be more than one possible value for $\frac{b}{a}$. You're told the ratio of $2a$ to b is 8 times the ratio of b to a. That's awkward to keep track of in English—it's a little easier to write fractions. The ratio of $2a$:b equals $8\left(\frac{b}{a}\right)$. So, $2\left(\frac{a}{b}\right) = 8\left(\frac{b}{a}\right)$, or $\frac{2a}{b} = \frac{8b}{a}$. Cross multiply to get $2a^2 = 8b^2$, or $a^2 = 4b^2$. Multiply each side of the equation by $\frac{1}{4a^2}$: $\frac{a^2}{4a^2} = \frac{4b^2}{4a^2}$. This is the same as $\frac{1}{4} = \frac{b^2}{a^2}$. Take the square root of both sides of the equation: $\pm\frac{1}{2} = \frac{b}{a}$. The ratio of b to a is $\frac{1}{2}$ or $-\frac{1}{2}$. So, **(B)** and **(D)** are the answers. This problem is also a great candidate for Backsolving, although since this question could have more than one correct answer, you would need to test all answer choices to see which ones work out.

15. 8



It is given that the area of triangle *ABC* is 35, and in the diagram, you're given a height for triangle *ABC*. If you use *BC* as the base of the triangle, the triangle's height is 7, so you can find the length of *BC*. The length *BC*, which is the base of triangle *ABC*, is also the hypotenuse of right triangle *BDC*. Given the hypotenuse and the length of leg *BD*, which is given in the diagram as 6, you'll be able to find the third leg of the triangle, side *DC*, which is what you're looking for.

Going back to triangle *ABC*, the area is 35 and the height is 7. The area of a triangle is $\frac{1}{2} \times \text{base} \times \text{height}$, so $\frac{1}{2} \times \text{base} \times \text{height}$ is 35. Therefore, $\frac{1}{2} \times 7 \times \text{length } BC$ is 35. That means $7 \times$ length *BC* is 70, so *BC* must have length 10. Now look at right triangle *BDC*. Here is a right triangle with one leg of length 6, the hypotenuse of length 10, and the third side unknown. That's one of the famous Pythagorean ratios—it's a 3:4:5 triangle. So *DC* must have length 2×4, or **8**.

16. D

First, find the value of *m*. You are told that 3^m is 81. Well, 81 is 9×9 and 9 is 3^2. So you have $3^2 \times 3^2 = 81$ or $3 \times 3 \times 3 \times 3 = 81$. There are four factors of 3 in 81, so *m* has the value 4. Now 4^3 is $4 \times 4 \times 4$ is 64. So **(D)** is correct. Note that **(A)** is a trap—that's the value of *m*, not m^3.

17. D, E

The problem states that *x* is between 0 and 1, so *x* must be a positive fraction (or decimal) less than 1. We can pick a number to get to the correct answer(s) here because both the question and the answer choices have variables. The decimal 0.5 is in the middle of the given range, so it's a good starting point.

(A) Incorrect. Doubling any positive value always produces a greater value, not a lesser value.

(B) Incorrect. $2 \times 0.5 = 1$; a smaller fraction could make this statement true, but our correct answer(s) must always be true.

(C) Incorrect. $2 \times 0.5 = 1$; a larger fraction could make this statement true, but our correct answer(s) must always be true.

(D) Correct. $0.5^2 = 0.25$; the square of any number between 0 and 1 (exclusive) will be less than the original number. This example illustrates that property.

(E) Correct. $0.25 < 1$; the square of any number between 0 and 1 (exclusive) will be less than 1. This is an example of that property. So, the correct answers are **(D)** and **(E)**.

18. C

To find how many categories had energy use greater than 150 million kilowatt-hours, you have to find out how many total kilowatt-hours were used in that year using the line graph. You see that 600 million kilowatt-hours were used in 1995. What is the relationship of 150 million kilowatt-hours to 600 million kilowatt-hours? It's 25% of 600 million kilowatt-hours, so you're looking for categories with more than 25% of the energy use for 1995. How many categories exceeded 25%? Just two, government and industrial. So your answer is **(C)**.

19. C

To find the per capita, or per person, personal energy use in Country Y in 2005, divide the personal energy use by the number of people. Since the question gives you the population—500 million people—you only need to find the personal energy use.

According to the top graph, total energy use in 2005 was about 675 million kilowatt-hours. According to the bottom graph, personal energy use was about 20% of the total, or $675 \times 0.20 = 135$ million kilowatt-hours. Divide this number by the population of Country Y in 2005: 135 million kilowatt-hours divided by 500 million people $= 0.27$ kilowatt-hours per person, or choice **(C)**.

20. C, D

Since this is an all-that-apply question, check each choice systematically.

Choice **(A)**: According to the bottom graph, energy for farm use decreased from 30% to 20% of the total between 1990 and 1995. Since 30 is 50% greater than 20, total energy use would have had to increase by at least 50% to compensate. The top graph shows that it did not, so choice **(A)** is incorrect.

Choice **(B)**: According to the bottom graph, energy for farm use decreased from 20% to 10% of the total between 1995 and 2000. Since 20 is double 10, total energy use would have had to double in order to compensate—that definitely did not happen! Choice **(B)** is out.

The bottom graph shows that energy for farm use, as a percentage of the total, increased from 2000 onward. Since total energy use increased every year, total farm energy use must have increased also. Thus, without any calculation, you know that choices **(C)** and **(D)** must be correct.

QUANTITATIVE REASONING 2 ANSWERS AND EXPLANATIONS

1. D

The perimeter of *ABC* is 40 and the length of *BC* is 12, and you want to compare the length of *AB* with 14. In an isosceles triangle, there are two sides with equal length, but you don't know whether side *BC* is one of those sides or not. If side *BC* is the unequal side, there are two unknown sides plus 12, and they have a sum of 40, the perimeter. The two remaining sides have a sum of 28, so each is 14. That would mean that *AB* and *AC* would have length 14. Then the answer would be **(C)**. If *BC* is one of the equal sides, however, there are two sides with length 12 and a third unknown side, and the sum is 40. Because $12 + 12$ is 24, the third side has length 16. *AB* could be one of the sides of length 12 or the side of length 16. There are three possible lengths for side *AB*—16, 14, and 12—so the answer is **(D)**.

2. A

Plug 0.5 in for x and solve. $f(0.5) = (0.5+3)^2 = 3.5^2 = 12.25$. This is greater than 9, so the answer is **(A)**.

3. B

In both quantities, use the basic formula: rate $\times$ time $=$ distance. In Quantity A, 40 mph $\times$ 4 hours traveled gives you 160 miles. In Quantity B, $70 \text{ mph} \times 2\frac{1}{2} \text{ hours} = 175 \text{ miles}$.

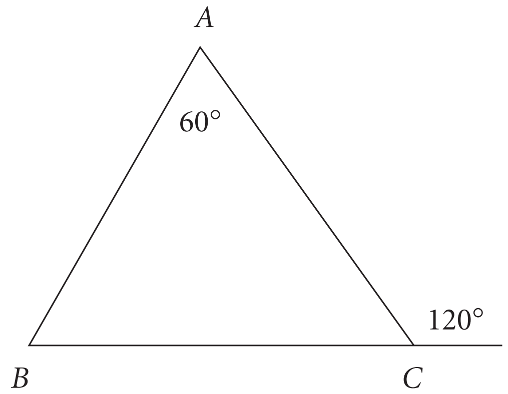
As 175 is greater than 160, the answer is **(B)**.

4. D

Quantity A cannot be precisely defined, but Quantity B can. The grape bag weighs 50 grams, and each grape weighs exactly 1 gram, so there must be 50 grapes in the bag. Quantity B is 50.

Now consider Quantity A. If every cookie in the bag is on the lighter end, weighing only 5 grams, then the number of cookies in the bag is 300 divided by 5, or 60. If, contrarily, the bag is full of the heaviest cookies in town, each weighing 15 grams, then there are 300 divided by 15, or 20, cookies in the bag. Thus, Quantity A is somewhere between 20 and 60, which means it could be less than, equal to, or greater than Quantity B. You have no idea which is the case, so pick **(D)**.

5. C



Here you have triangle *ABC*—base *BC* has been extended on one side and there is an exterior angle drawn in and labeled 120°. You want to compare side lengths *AB* and *BC*. In any triangle, the largest side will be opposite the largest angle, so you want to see which of these sides is opposite a larger angle. Angle *A* is labeled 60°, but is angle *C* less than, equal to, or greater than 60? Notice that the adjacent angle is 120°—the two together form a straight line, so their sum is 180°. And $180 - 120 = 60$, so angle *C* is a 60° angle. Since the angles are equal, the sides are equal, and the answer is **(C)**.

6. A

Notice the way the diagram is set up: $a + b$ is the same as *PQ*. The equation is $8a + 8b = 24$. Divide both sides by 8. You end up with $a + b = 3$. *PQ* is 3 and because 3 is greater than 2, the answer is **(A)**. Note that you did not have to solve for a or b individually.

7. A

All you know is that x is less than y, but even though you don't know their values, you know enough to determine a relationship. In Quantity A, you have $y - x$, the larger number minus the smaller number, so you must get a positive difference, even if both numbers are negative. In Quantity B, you have the smaller number minus the larger number—this time the difference is negative. So you can determine a relationship—you know the answer is **(A)**, Quantity A is always greater than Quantity B.

8. C

The area of a triangle equals $\frac{1}{2} \times \text{base} \times \text{height}$. Triangles *ACE* and *ABD* have the same height, because they have the same apex point *A*. The problem states that their areas are equal, so they must have the same base, too. Thus, $EC = DB$ and $ED = CB$. Since triangle *ABE* is equilateral, you also know that $AE = AB$. This means that sides *AD* and *AC* are equal as well. If they weren't, one of *ED* or *CB* would have to be longer than the other, and you already know they're equal. The answer is **(C)**.

9. D

If the radius of each circle is 3, then the diameter of each circle is 6. Then PS and $QR = 6$, and PQ and $SR = 12$. The perimeter of rectangle $PQRS = 6 + 12 + 6 + 12 = 36$. The answer is **(D)**.

10. D

In this question, you have a fraction as a base and must consider various values for x, the exponent. Consider what happens when $x = -1$. We know that $\left(\frac{1}{4}\right)^{-1} = \frac{1}{\left(\frac{1}{4}\right)^{1}} = 4.$

Putting that into the full equation, we get $1-\left(\frac{1}{4}\right)^{x} = 1-4 = -3.$ This is not greater than zero, and if $x = -2$, the result will be even lower, so choices **(A)** and **(B)** are out. Next, consider what happens when $x = 0$. Any base to the zero power equals 1; then $1-\left(\frac{1}{4}\right)^{x} = 1-1 = 0.$ You want the value of x that makes the expression greater than 0, so try $x = 1$: $1-\left(\frac{1}{4}\right)^{x} = 1-\left(\frac{1}{4}\right) = \frac{3}{4}$. The answer is **(D)**.

11. B

Begin with cross multiplication and use algebra to isolate $\frac{q}{p}$:

$\frac{p-q}{p} = \frac{2}{7}$	
$7(p - q) = 2p$	Cross multiply.
$7p - 7q = 2p$	Remove parentheses.
$5p = 7q$	Add $7q$; subtract $2p$ on both sides.
$\frac{5}{7} = \frac{q}{p}$	Divide both sides by $7p$.

Choice **(B)** is correct.

12. 0.9

The shaded region is a right triangle. So, use the numbers on the grid to calculate the base and height of the triangle. The length horizontally is $(-2.0) - (-0.5) = -2.0 + 0.5 = -1.5$. Distances are always positive, so use 1.5 as the base of the triangle. The height of the triangle is $1.6 - 0.4 = 1.2$. Use the equation for the area of a triangle:

$$A = \frac{1}{2}bh = \frac{1}{2} \times 1.5 \times 1.2 = 0.9$$

The area is **0.9**.

13. D

The question asks for the number that is 850% greater than 8×10^3. First, determine the value of 8×10^3. That number is 8,000. To 8,000, you need to add 850% of 8,000. Here's what the math looks like:

$8{,}000 + (850\% \times 8{,}000) = 8{,}000 + (8.5 \times 8{,}000) = 8{,}000 + 68{,}000 = 76{,}000$

In scientific notation, this is 7.6×10^4, choice **(D)**.

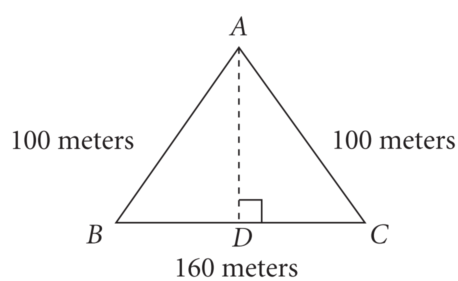
14. B, C, E

List the factors for each number to check for all correct choices.

Number	Factors	Number of Factors
4	1, 2, 4	3
6	1, 2, 3, 6	4
8	1, 2, 4, 8	4
12	1, 2, 3, 4, 6, 12	6
14	1, 2, 7, 14	4

So the correct choices are **(B)**, **(C)**, and **(E)**.

15. 60



You're trying to find the shortest distance in meters a person would walk to go from point *A* to a point on side *BC* of the triangular field represented in the diagram. To get the shortest distance from point *A* to side *BC*, draw a perpendicular line from point *A* to side *BC*. Call the new vertex point *D*. Now two smaller right triangles, *ADC* and *ADB* have been created.

From the diagram, length *BC* is 160 meters, *AB* is 100 meters, and *AC* is 100 meters. Each of the two right triangles formed has 100 meters as the length of its hypotenuse.

What does that tell you about triangle *ABC*? *AB* and *AC* have the same length, so this is an isosceles triangle. That means that when you drew in the perpendicular distance from *A* down to *D*, you split the isosceles triangle *ABC* into two identical right triangles. Length *BD* is the same as length *CD*. So each of them is half of 160 meters, or 80 meters. Each right triangle has an hypotenuse of 100 meters and one leg of 80 meters. This is a 3:4:5 right triangle, with each member of the ratio multiplied by 20. So *AD* must have length **60**, and the minimum distance is 60 meters.

16. E

The average is $\frac{\text{The sum of terms}}{\text{The number of terms}}$. Here you have $y - z$ and the other number, which you can call x. The average of x and $y - z$ is $3y$, so $3y = \frac{x + y - z}{2}$. Multiplying both sides by 2 gives $6y = x + y - z$. Subtracting $y - z$ from both sides gives $5y + z = x$. So the other number, x, is $5y + z$, answer choice **(E)**.

17. B, D

Test each point. Substitute a value for x and compare the result to the given value for y in the ordered pair.

Let $x = -3$.

$$y = \frac{x^2}{x+1} = \frac{(-3)^2}{-3+1} = \frac{9}{-2} \neq -5$$

Let $x = -2$.

$$y = \frac{x^2}{x+1} = \frac{(-2)^2}{-2+1} = \frac{4}{-1} = -4$$

Let $x = -1$.

$$y = \frac{x^2}{x+1} = \frac{(-1)^2}{-1+1} = \frac{1}{0} \neq -3$$

Let $x = 1$.

$$y = \frac{x^2}{x+1} = \frac{1^2}{1+1} = \frac{1}{2}$$

Let $x = 3$.

$$y = \frac{x^2}{x+1} = \frac{3^2}{3+1} = \frac{9}{4} \neq 2\frac{1}{2}$$

So, the correct answers are **(B)** and **(D)**.

18. D

You're looking for the lowest ratio of males to females. In the double bar graph, the males outnumber females in each double bar, so you want the specialty in which the numbers of males and females are closest. Skimming the bar graphs, you can see that in pediatrics, the female graph and the male graph are closer than any of the others. **(D)** *pediatrics* is the correct answer.

19. B

How many male general surgery physicians were under 35 years old? The pie chart breaks down general surgery physicians by age, so work with that. And because you're looking for a number of general surgery physicians, you know that you're going to have to find the total number of general surgery physicians and then break it down according to the percentages on the pie chart.

The number of female general surgery physicians in the under-35 category represented 3.5% of all the general surgery physicians. What this does is break that slice of the pie for under-35 into two smaller slices, one for men under 35 and one for women under 35. Now the whole slice for under-35-year-olds is 30% of the total, and the question states that the number of females under 35 is 3.5% of the total. So the difference between 30% and 3.5% (26.5%) must be the men in the under-35 category.

From the top graph, estimate the total number of general surgery physicians as 37,000 (35,000 male plus 2,000 female). Multiply 37,000 by 26.5%: $0.265 \times 37{,}000 = 9{,}805$, which is very close to **(B)**, the correct answer.

20. E

The bar graph doesn't give the total number of general practice physicians, but if you add the number of males to the number of females, you get the total number of GP physicians. To find the percent who are male, take the number of males and put it over the total number. There are about 2,000 women and about 23,000 men, making the total about 25,000. Well, if there are around 25,000 GP physicians altogether and 2,000 of them are female, that's around 8%. About 92% are male, which is closest to 90%, **(E)**.

APPENDIX D

Math Reference

The math on the GRE covers a lot of ground—from number properties and arithmetic to basic algebra and symbol problems to geometry and statistics. Don't let yourself be intimidated.

We've highlighted the 100 most important concepts that you need to know and divided them into three levels. The GRE Quantitative sections test your understanding of a relatively limited number of mathematical concepts, all of which you will be able to master.

Level 1 consists of foundational math topics. Though these topics may seem basic, review this list so that you are aware that these skills may play a part in the questions you will answer on the GRE. Look over the Level 1 list to make sure you're comfortable with the basics.

Level 2 is where most people start their review of math. Level 2 skills and formulas come into play quite frequently on the GRE. If the skills needed to handle Level 1 or 2 topics are keeping you from feeling up to the tasks expected on the GRE Quantitative section, you might consider taking the Kaplan GRE Math Refresher course.

Level 3 represents the most challenging math concepts you'll find on the GRE. Don't spend a lot of time on Level 3 if you still have gaps in Level 2, but once you've mastered Level 2, tackling Level 3 can put you over the top.

LEVEL 1

1. How to add, subtract, multiply, and divide WHOLE NUMBERS

You can check addition with subtraction.

$$17 + 5 = 22 \qquad 22 - 5 = 17$$

You can check multiplication with division.

$$5 \times 28 = 140 \qquad 140 \div 5 = 28$$

2. How to add, subtract, multiply, and divide FRACTIONS

Find a common denominator before adding or subtracting fractions.

$$\frac{4}{5} + \frac{3}{10} = \frac{8}{10} + \frac{3}{10} = \frac{11}{10} \text{ or } 1\frac{1}{10}$$
$$2 - \frac{3}{8} = \frac{16}{8} - \frac{3}{8} = \frac{13}{8} \text{ or } 1\frac{5}{8}$$

To multiply fractions, multiply the numerators first and then multiply the denominators. Simplify if necessary.

$$\frac{3}{4} \times \frac{1}{6} = \frac{3}{24} = \frac{1}{8}$$

You can also reduce before multiplying numerators and denominators. This keeps the products small.

$$\frac{5}{8} \times \frac{2}{15} = \frac{{}^{1}\cancel{5}}{{}_{4}\cancel{8}} \times \frac{\cancel{2}^{1}}{\cancel{15}_{3}} = \frac{1}{12}$$

To divide by a fraction, multiply by its reciprocal. To write the reciprocal of a fraction, flip the numerator and the denominator.

$$5 \div \frac{1}{3} = \frac{5}{1} \times \frac{3}{1} = 15 \qquad \frac{1}{3} \div \frac{4}{5} = \frac{1}{3} \times \frac{5}{4} = \frac{5}{12}$$

3. How to add, subtract, multiply, and divide DECIMALS

To add or subtract, align the decimal points and then add or subtract normally. Place the decimal point in the answer directly below existing decimal points.

$$\begin{array}{r} 3.25 \\ +\,4.4 \\ \hline 7.65 \end{array} \qquad \begin{array}{r} 7.65 \\ -\,4.4 \\ \hline 3.25 \end{array}$$

To multiply with decimals, multiply the digits normally and count off decimal places (equal to the total number of places in the factors) from the right.

$$\begin{aligned} 2.5 \times 2.5 &= 6.25 \\ 0.06 \times 2{,}000 &= 120.00 = 120 \end{aligned}$$

To divide by a decimal, move the decimal point in the divisor to the right to form a whole number; move the decimal point in the dividend the same number of places. Divide as though there were no decimals, then place the decimal point in the quotient.

$$\begin{aligned} &6.25 \div 2.5 \\ &= 62.5 \div 25 = 2.5 \end{aligned}$$

4. How to convert FRACTIONS TO DECIMALS and DECIMALS TO FRACTIONS

To convert a fraction to a decimal, divide the numerator by the denominator.

$$\frac{4}{5} = 0.8 \quad \frac{4}{50} = 0.08 \quad \frac{4}{500} = 0.008$$

To convert a decimal to a fraction, write the digits in the numerator and use the decimal name in the denominator.

$$0.003 = \frac{3}{1{,}000} \quad 0.03 = \frac{3}{100} \quad 0.3 = \frac{3}{10}$$

5. How to add, subtract, multiply, and divide POSITIVE AND NEGATIVE NUMBERS

When addends (the numbers being added) have the same sign, add their absolute values; the sum has the same sign as the addends. But when addends have different signs, subtract the absolute values; the sum has the sign of the greater absolute value.

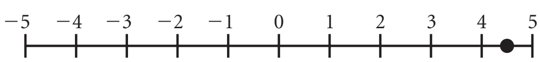
$3 + 9 = 12$, but $-3 + (-9) = -12$
$3 + (-9) = -6$, but $-3 + 9 = 6$

In multiplication and division, when the signs are the same, the product/quotient is positive. When the signs are different, the product/quotient is negative.

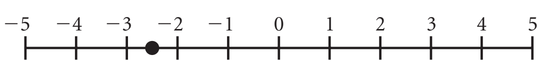
$6 \times 7 = 42$ and $-6 \times (-7) = 42$
$-6 \times 7 = -42$ and $6 \times (-7) = -42$
$96 \div 8 = 12$ and $-96 \div (-8) = 12$
$-96 \div 8 = -12$ and $96 \div (-8) = -12$

6. How to plot points on the NUMBER LINE

To plot the point 4.5 on the number line, start at 0, go right to 4.5, halfway between 4 and 5.



To plot the point −2.5 on the number line, start at 0, go left to −2.5, halfway between −2 and −3.



7. How to plug a number into an ALGEBRAIC EXPRESSION

To evaluate an algebraic expression, choose numbers for the variables or use the numbers assigned to the variables.

Evaluate $4np + 1$ when $n = -4$ and $p = 3$.

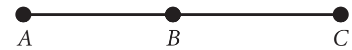
$4np + 1 = 4(-4)(3) + 1 = -48 + 1 = -47$

8. How to SOLVE a simple LINEAR EQUATION

Use algebra to isolate the variable. Do the same steps to both sides of the equation.

$$\begin{aligned} 28 &= -3x - 5 \\ 28 + 5 &= -3x - 5 + 5 \quad \text{Add 5.} \\ 33 &= -3x \\ \frac{33}{-3} &= \frac{-3x}{-3} \quad \text{Divide by } -3. \\ -11 &= x \end{aligned}$$

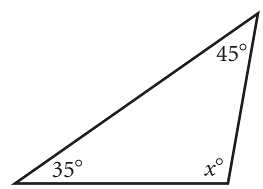
9. How to add and subtract LINE SEGMENTS



If $AB = 6$ and $BC = 8$, then $AC = 6 + 8 = 14$.
If $AC = 14$ and $BC = 8$, then $AB = 14 - 8 = 6$.

10. How to find the THIRD ANGLE of a TRIANGLE, given the other two angles

Use the fact that the sum of the measures of the interior angles of a triangle always equals 180°.



$$\begin{aligned} 35 + 45 + x &= 180 \\ 80 + x &= 180 \\ x &= 100 \end{aligned}$$

LEVEL 2

11. How to use PEMDAS

When you're given a complex arithmetic expression, it's important to know the order of operations. Just remember PEMDAS (as in "Please Excuse My Dear Aunt Sally"). What PEMDAS means is this: Clean up **Parentheses** first (nested sets of parentheses are worked from the innermost set to the outermost set); then deal with **Exponents** (or **Radicals**); then do the **Multiplication** and **Division** together, going from left to right; and finally do the **Addition** and **Subtraction** together, again going from left to right.

Example:

$$9 - 2 \times (5 - 3)^2 + 6 \div 3 =$$

Begin with the parentheses:

$$9 - 2 \times (2)^2 + 6 \div 3 =$$

Then do the exponent:

$$9 - 2 \times 4 + 6 \div 3 =$$

Now do multiplication and division from left to right:

$$9 - 8 + 2 =$$

Finally, do addition and subtraction from left to right:

$$1 + 2 = 3$$

12. How to use the PERCENT FORMULA

Identify the part, the percent, and the whole.

$$\textit{Part} = \textit{Percent} \times \textit{Whole}$$

Find the part.

Example:

What is 12 percent of 25?

Setup:

$$\textit{Part} = \frac{12}{100} \times 25 = \frac{300}{100} = 3$$

Find the percent.

Example:

45 is what percent of 9?

Setup:

$$\begin{aligned} 45 &= \frac{\text{Percent}}{100} \times 9 \\ 4{,}500 &= \text{Percent} \times 9 \\ 500 &= \text{Percent} \end{aligned}$$

Find the whole.

Example:

15 is $\frac{3}{5}$ percent of what number?

Setup:

$$\begin{aligned} 15 &= \frac{3}{5}\left(\frac{1}{100}\right) \times \textit{Whole} \\ 15 &= \frac{3}{500} \times \textit{Whole} \\ \textit{Whole} &= 15\left(\frac{500}{3}\right) = \frac{7{,}500}{3} = 2{,}500 \end{aligned}$$

13. How to use the PERCENT INCREASE/DECREASE FORMULAS

Identify the original whole and the amount of increase/decrease.

$$\textit{Percent increase} = \frac{\textit{Amount of increase}}{\textit{Original whole}} \times 100\%$$

$$\textit{Percent decrease} = \frac{\textit{Amount of decrease}}{\textit{Original whole}} \times 100\%$$

Example:

The price goes up from \$80 to \$100. What is the percent increase?

Setup:

$$\begin{aligned} \textit{Percent increase} &= \frac{20}{80} \times 100\% \\ &= 0.25 \times 100\% = 25\% \end{aligned}$$

14. How to predict whether a sum, difference, or product will be ODD or EVEN

Don't bother memorizing the rules. Just take simple numbers like 2 for even numbers and 3 for odd numbers and see what happens.

Example:

If m is even and n is odd, is the product mn odd or even?

Setup:

Say $m = 2$ and $n = 3$.
$2 \times 3 = 6$, which is even, so mn is even.

15. How to recognize MULTIPLES OF 2, 3, 4, 5, 6, 9, 10, and 12

- 2: Last digit is even.
- 3: Sum of digits is a multiple of 3.
- 4: Last two digits are a multiple of 4.
- 5: Last digit is 5 or 0.
- 6: Sum of digits is a multiple of 3, and last digit is even.
- 9: Sum of digits is a multiple of 9.
- 10: Last digit is 0.
- 12: Sum of digits is a multiple of 3, and last two digits are a multiple of 4.

16. How to find a COMMON FACTOR of two numbers

Break both numbers down to their prime factors to see which they have in common. Then multiply the shared prime factors to find all common factors.

Example:

What factors greater than 1 do 135 and 225 have in common?

Setup:

First find the prime factors of 135 and 225; $135 = 3 \times 3 \times 3 \times 5$, and $225 = 3 \times 3 \times 5 \times 5$. The numbers share $3 \times 3 \times 5$ in common. Thus, aside from 3 and 5, the remaining common factors can be found by multiplying 3, 3, and 5 in every possible combination: $3 \times 3 = 9$, $3 \times 5 = 15$, and $3 \times 3 \times 5 = 45$. Therefore, the common factors of 135 and 225 are 3, 5, 9, 15, and 45.

17. How to find a COMMON MULTIPLE of two numbers

The product of two numbers is the easiest common multiple to find, but it is not always the least common multiple (LCM).

Example:

What is the least common multiple of 28 and 42?

Setup:

$$\begin{aligned} 28 &= 2 \times 2 \times 7 \\ 42 &= 2 \times 3 \times 7 \end{aligned}$$

The LCM can be found by finding the prime factorization of each number, then seeing the greatest number of times each factor is used. Multiply each prime factor the greatest number of times it appears.

In 28, 2 is used twice. In 42, 2 is used once. In 28, 7 is used once. In 42, 7 is used once, and 3 is used once.

So you multiply each factor the greatest number of times it appears in a prime factorization:

$$\text{LCM} = 2 \times 2 \times 3 \times 7 = 84$$

18. How to find the AVERAGE or ARITHMETIC MEAN

$$Average = \frac{Sum\ of\ terms}{Number\ of\ terms}$$

Example:

What is the average of 3, 4, and 8?

Setup:

$$Average = \frac{3 + 4 + 8}{3} = \frac{15}{3} = 5$$

19. How to use the AVERAGE to find the SUM

$$Sum = (Average) \times (Number\ of\ terms)$$

Example:

17.5 is the average (arithmetic mean) of 24 numbers.

What is the sum of the 24 numbers?

Setup:

$$Sum = 17.5 \times 24 = 420$$

20. How to find the AVERAGE of CONSECUTIVE NUMBERS

The average of evenly spaced numbers is simply the average of the smallest number and the largest number. The average of all the integers from 13 to 77, for example, is the same as the average of 13 and 77:

$$\frac{13 + 77}{2} = \frac{90}{2} = 45$$

21. How to COUNT CONSECUTIVE NUMBERS

The number of integers from A to B inclusive is $B - A + 1$.

Example:

How many integers are there from 73 through 419, inclusive?

Setup:

$$419 - 73 + 1 = 347$$

22. How to find the SUM OF CONSECUTIVE NUMBERS

$$Sum = (Average) \times (Number\ of\ terms)$$

Example:

What is the sum of the integers from 10 through 50, inclusive?

Setup:

Average: $\frac{10 + 50}{2} = 30$

Number of terms: $50 - 10 + 1 = 41$

Sum: $30 \times 41 = 1{,}230$

23. How to find the MEDIAN

Put the numbers in numerical order and take the middle number.

Example:

What is the median of 88, 86, 57, 94, and 73?

Setup:

First, put the numbers in numerical order, then take the middle number:

$$57, 73, 86, 88, 94$$

The median is 86.

In a set with an even number of numbers, take the average of the two in the middle.

Example:

What is the median of 88, 86, 57, 73, 94, and 100?

Setup:

First, put the numbers in numerical order.

$$57, 73, 86, 88, 94, 100$$

Because 86 and 88 are the two numbers in the middle:

$$\frac{86 + 88}{2} = \frac{174}{2} = 87$$

The median is 87.

24. How to find the MODE

Take the number that appears most often. For example, if your test scores were 88, 57, 68, 85, 98, 93, 93, 84, and 81, the mode of the scores would be 93 because it appears more often than any other score. (If there's a tie for most often, then there's more than one mode. If each number in a set is used equally often, there is no mode.)

25. How to find the RANGE

Take the positive difference between the greatest and least values. Using the example under "How to find the MODE" above, if your test scores were 88, 57, 68, 85, 98, 93, 93, 84, and 81, the range of the scores would be 41, the greatest value minus the least value ($98 - 57 = 41$).

26. How to use actual numbers to determine a RATIO

To find a ratio, put the number associated with *of* on the top and the number associated with *to* on the bottom.

$$Ratio = \frac{of}{to}$$

The ratio of 20 oranges to 12 apples is $\frac{20}{12}$, or $\frac{5}{3}$.

Ratios should always be reduced to lowest terms. Ratios can also be expressed in linear form, such as 5:3.

27. How to use a ratio to determine an ACTUAL NUMBER

Set up a proportion using the given ratio.

Example:

The ratio of boys to girls is 3 to 4. If there are 135 boys, how many girls are there?

Setup:

$$\begin{aligned} \frac{3}{4} &= \frac{135}{g} \\ 3 \times g &= 4 \times 135 \\ 3g &= 540 \\ g &= 180 \end{aligned}$$

28. How to use actual numbers to determine a RATE

Identify the quantities and the units to be compared. Keep the units straight.

Example:

Anders typed 9,450 words in $3\frac{1}{2}$ hours. What was his rate in words per minute?

Setup:

First convert $3\frac{1}{2}$ hours to 210 minutes. Then set up the rate with words on top and minutes on bottom (because "per" means "divided by"):

$$\frac{9{,}450 \text{ words}}{210 \text{ minutes}} = 45 \text{ words per minute}$$

29. How to deal with TABLES, GRAPHS, AND CHARTS

Read the question and all labels carefully. Ignore extraneous information and zero in on what the question asks for. Take advantage of the spread in the answer choices by approximating the answer whenever possible and choosing the answer choice closest to your approximation.

30. How to count the NUMBER OF POSSIBILITIES

You can use multiplication to find the number of possibilities when items can be arranged in various ways.

Example:

How many three-digit numbers can be formed with the digits 1, 3, and 5 each used only once?

Setup:

Look at each digit individually. The first digit (or, the hundreds digit) has three possible numbers to plug in: 1, 3, or 5. The second digit (or, the tens digit) has two possible numbers, since one has already been plugged in. The last digit (or, the ones digit) has only one remaining possible number. Multiply the possibilities together: $3 \times 2 \times 1 = 6$.

31. How to calculate a simple PROBABILITY

$$\textit{Probability} = \frac{\textit{Number of desired outcomes}}{\textit{Number of total possible outcomes}}$$

Example:

What is the probability of throwing a 5 on a fair six-sided die?

Setup:

There is one desired outcome—throwing a 5. There are 6 possible outcomes—one for each side of the die.

$$\textit{Probability} = \frac{1}{6}$$

32. How to work with new SYMBOLS

If you see a symbol you've never seen before, don't be alarmed. It's just a made-up symbol whose operation is uniquely defined by the problem. Everything you need to know is in the question stem. Just follow the instructions.

33. How to SIMPLIFY BINOMIALS

A binomial is a sum or difference of two terms. To simplify two binomials that are multiplied together, use the **FOIL** method. Multiply the **F**irst terms, then the **O**uter terms, followed by the **I**nner terms and the **L**ast terms. Lastly, combine like terms.

Example:

$$\begin{aligned}(3x + 5)(x - 1) &= \\ 3x^2 - 3x + 5x - 5 &= \\ 3x^2 + 2x - 5 &\end{aligned}$$

34. How to FACTOR certain POLYNOMIALS

A polynomial is an expression consisting of the sum of two or more terms, where at least one of the terms is a variable.

Learn to spot these classic polynomial equations.

$$
\begin{aligned}
ab + ac &= a(b + c) \\
a^2 + 2ab + b^2 &= (a + b)^2 \\
a^2 - 2ab + b^2 &= (a - b)^2 \\
a^2 - b^2 &= (a - b)(a + b)
\end{aligned}
$$

35. How to solve for one variable IN TERMS OF ANOTHER

To find x "in terms of" y, isolate x on one side, leaving y as the only variable on the other.

36. How to solve an INEQUALITY

Treat it much like an equation—adding, subtracting, multiplying, and dividing both sides by the same thing. Just remember to reverse the inequality sign if you multiply or divide by a negative quantity.

Example:
Rewrite $7 - 3x > 2$ in its simplest form.

Setup:

$$7 - 3x > 2$$

First, subtract 7 from both sides:

$$7 - 3x - 7 > 2 - 7$$

$$-3x > -5$$

Now divide both sides by -3, remembering to reverse the inequality sign:

$$x < \frac{5}{3}$$

37. How to handle ABSOLUTE VALUES

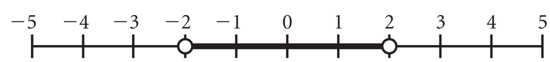
The *absolute value* of a number n, denoted by $|n|$, is defined as n if $n \geq 0$ and $-n$ if $n < 0$. The absolute value of a number is the distance from zero to the number on the number line. The absolute value of a number or expression is always positive.

$$|-5| = 5$$

If $|x| = 3$, then x could be 3 or -3.

Example:
If $|x - 3| < 2$, what is the range of possible values for x?

Setup:
Represent the possible range for $x - 3$ on a number line.



$|x - 3| < 2$, so $(x - 3) < 2$ and $(x - 3) > -2$
$x - 3 < 2$ and $x - 3 > -2$
$x < 2 + 3$ and $x > -2 + 3$
$x < 5$ and $x > 1$
So $1 < x < 5$.

38. How to TRANSLATE ENGLISH INTO ALGEBRA

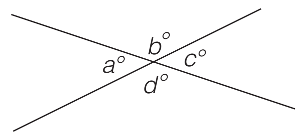
Look for the key words and systematically turn phrases into algebraic expressions and sentences into equations.

Here's a table of key words that you may have to translate into mathematical terms:

Operation	Key Words
Addition	sum, plus, and, added to, more than, increased by, combined with, exceeds, total, greater than
Subtraction	difference between, minus, subtracted from, decreased by, diminished by, less than, reduced by
Multiplication	of, product, times, multiplied by, twice, double, triple, half
Division	quotient, divided by, per, out of, ratio ___ of ___ to
Equals	equals, is, was, will be, the result is, adds up to, costs, is the same as

39. How to find an ANGLE formed by INTERSECTING LINES

Vertical angles are equal. Angles along a line add up to 180°.

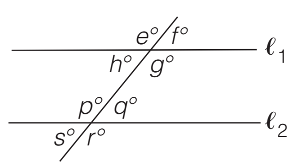


$$\begin{aligned} a^\circ &= c^\circ \\ b^\circ &= d^\circ \\ a^\circ + b^\circ &= 180^\circ \\ a^\circ + b^\circ + c^\circ + d^\circ &= 360^\circ \end{aligned}$$

40. How to find an angle formed by a TRANSVERSAL across PARALLEL LINES

When a transversal crosses parallel lines, all the acute angles formed are equal, and and all the obtuse angles formed are equal. Any acute angle plus any obtuse angle equals 180°.

Example:



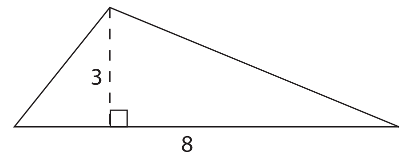
$$e^\circ = g^\circ = p^\circ = r^\circ$$
$$f^\circ = h^\circ = q^\circ = s^\circ$$
$$e^\circ + q^\circ = g^\circ + s^\circ = 180^\circ$$

41. How to find the AREA of a TRIANGLE

$$Area = \frac{1}{2}(Base)(Height)$$

Base and height must be perpendicular to each other. Height is measured by drawing a perpendicular line segment from the base—which can be any side of the triangle—to the angle opposite the base.

Example:

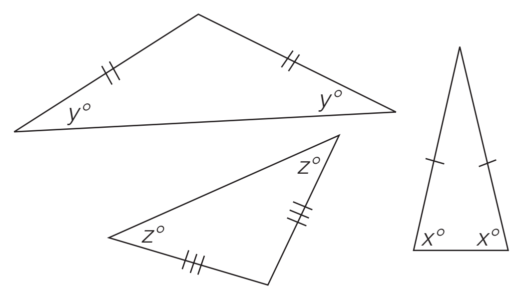


Setup:

$$Area = \frac{1}{2}(8)(3) = 12$$

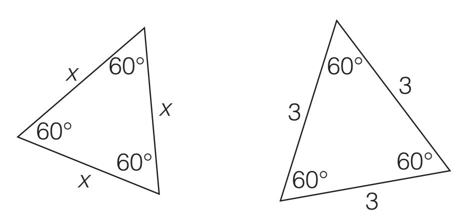
42. How to work with ISOSCELES TRIANGLES

Isosceles triangles have at least two equal sides and two equal angles. If a GRE question tells you that a triangle is isosceles, you can bet that you'll need to use that information to find the length of a side or a measure of an angle.



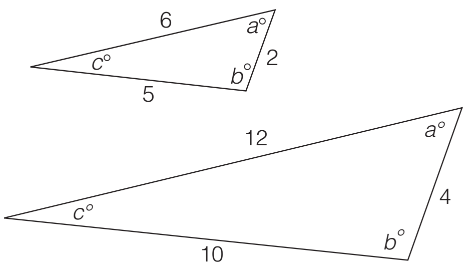
43. How to work with EQUILATERAL TRIANGLES

Equilateral triangles have three equal sides and three 60° angles. If a GRE question tells you that a triangle is equilateral, you can bet that you'll need to use that information to find the length of a side or the measure of an angle.



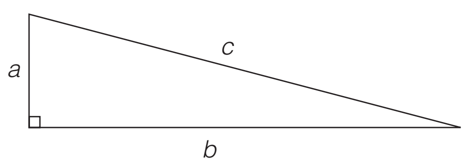
44. How to work with SIMILAR TRIANGLES

In similar triangles, corresponding angles are equal, and corresponding sides are proportional. If a GRE question tells you that triangles are similar, use the properties of similar triangles to find the length of a side or the measure of an angle.



45. How to find the HYPOTENUSE or a LEG of a RIGHT TRIANGLE

For all right triangles, the Pythagorean theorem is $a^2 + b^2 = c^2$, where a and b are the legs and c is the hypotenuse.

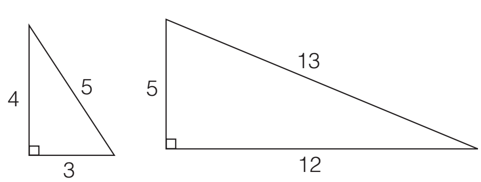


46. How to spot SPECIAL RIGHT TRIANGLES

Special right triangles are ones that are seen on the GRE with frequency. Recognizing them can streamline your problem solving.

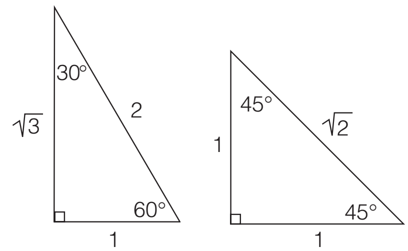
3:4:5
5:12:13

These numbers (3, 4, 5 and 5, 12, 13) represent the ratio of the side lengths of these triangles.



$30° - 60° - 90°$
$45° - 45° - 90°$

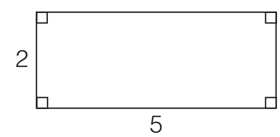
In a $30 - 60 - 90$ triangle, the side lengths are multiples of 1, $\sqrt{3}$, and 2, respectively. In a $45 - 45 - 90$ triangle, the side lengths are multiples of 1, 1, and $\sqrt{2}$, respectfully.



47. How to find the PERIMETER of a RECTANGLE

$$\textit{Perimeter} = 2(\textit{Length} + \textit{Width})$$

Example:



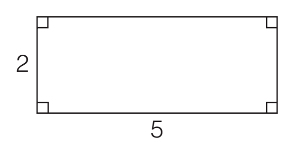
Setup:

$$\textit{Perimeter} = 2(2 + 5) = 14$$

48. How to find the AREA of a RECTANGLE

$$\textit{Area} = (\textit{Length})(\textit{Width})$$

Example:



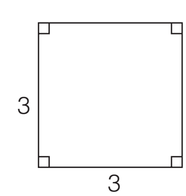
Setup:

$$\textit{Area} = 2 \times 5 = 10$$

49. How to find the AREA of a SQUARE

$$\textit{Area} = (\textit{Side})^2$$

Example:



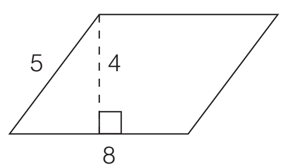
Setup:

$$Area = 3^2 = 9$$

50. How to find the AREA of a PARALLELOGRAM

$$Area = (Base)(Height)$$

Example:



Setup:

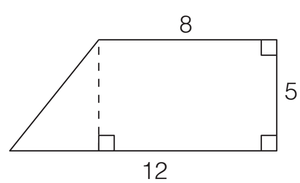
$$Area = 8 \times 4 = 32$$

51. How to find the AREA of a TRAPEZOID

A trapezoid is a quadrilateral having only two parallel sides. You can always drop a perpendicular line or two to break the figure into a rectangle and a triangle or two triangles. Use the area formulas for those familiar shapes. Alternatively, you could apply the general formula for the area of a trapezoid:

$$Area = (Average\ of\ parallel\ sides) \times (Height)$$

Example:



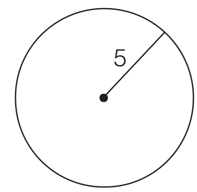
Setup:

$$\begin{aligned} Area\ of\ rectangle &= 8 \times 5 = 40 \\ Area\ of\ triangle &= \frac{1}{2}(4 \times 5) = 10 \\ Area\ of\ trapezoid &= 40 + 10 = 50 \\ Area\ of\ trapezoid &= \left(\frac{8+12}{2}\right) \times 5 = 50 \end{aligned}$$

52. How to find the CIRCUMFERENCE of a CIRCLE

$$Circumference = 2\pi \mathrm{r},\ where\ \mathrm{r}\ is\ the\ radius$$
$$Circumference = \pi \mathrm{d},\ where\ \mathrm{d}\ is\ the\ diameter$$

Example:



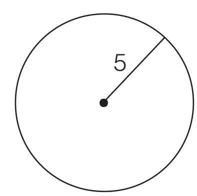
Setup:

$$Circumference = 2\pi(5) = 10\pi$$

53. How to find the AREA of a CIRCLE

$$Area = \pi \mathrm{r}^2\ where\ \mathrm{r}\ is\ the\ radius$$

Example:



Setup:

$$Area = \pi \times 5^2 = 25\pi$$

54. How to find the DISTANCE BETWEEN POINTS on the coordinate plane

If two points have the same x coordinates or the same y coordinates—that is, they make a line segment that is parallel to an axis—all you have to do is subtract the numbers that are different. Just remember that distance is always positive.

Example:

What is the distance from (2, 3) to (–7, 3)?

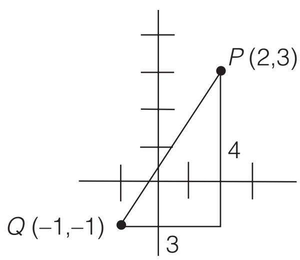
Setup:

The y's are the same, so just subtract the x's: $2 - (-7) = 9$.

If the points have different x coordinates and different y coordinates, make a right triangle and use the Pythagorean theorem or apply the special right triangle attributes if applicable.

Example:
What is the distance from (2,3) to (–1,–1)?

Setup:



It's a 3:4:5 triangle!
$PQ = 5$

55. How to find the SLOPE of a LINE

$$Slope = \frac{Rise}{Run} = \frac{Change\ in\ y}{Change\ in\ x}$$

Example:
What is the slope of the line that contains the points (1,2) and (4,–5)?

Setup:

$$Slope = \frac{-5-2}{4-1} = \frac{-7}{3} = -\frac{7}{3}$$

LEVEL 3

56 How to determine COMBINED PERCENT INCREASE/DECREASE when no original value is specified

Start with 100 as a starting value.

Example:
A price rises by 10 percent one year and by 20 percent the next. What's the combined percent increase?

Setup:
Say the original price is \$100.

Year one:
$100 + (10% of 100) = 100 + 10 = 110

Year two:
110 + (20% of 110) = 110 + 22 = 132

From 100 to 132 is a 32 percent increase.

57. How to find the ORIGINAL WHOLE before percent increase/decrease

Think of a 15 percent increase over x as $1.15x$ and set up an equation.

Example:
After decreasing by 5 percent, the population is now 57,000. What was the original population?

Setup:
$0.95 \times (\textit{Original population}) = 57{,}000$
Divide both sides by 0.95.
$\textit{Original population} = 57{,}000 \div 0.95 = 60{,}000$

58. How to solve a SIMPLE INTEREST problem

With simple interest, the interest is computed on the principal only and is given by

$Interest = Principle \times \mathrm{rt}$

In this formula, r is defined as the interest rate per payment period, and t is defined as the number of payment periods.

Example:
If \$12,000 is invested at 6 percent simple annual interest, how much interest is earned after 9 months?

Setup:
Since the interest rate is annual and we are calculating how much interest accrues after 9 months, we will express the payment period as $\frac{9}{12}$.

$$(12{,}000) \times (0.06) \times \frac{9}{12} = \$540$$

59. How to solve a COMPOUND INTEREST problem

If interest is compounded, the interest is computed on the principal as well as on any interest earned. To compute compound interest:

$$(\textit{Final balance}) = (\textit{Principal}) \times \left(1 + \frac{\textit{interest rate}}{c}\right)^{(\textit{time})(c)}$$

where c = the number of times the interest is compounded annually.

Example:

If \$10,000 is invested at 8 percent annual interest, compounded semiannually, what is the balance after 1 year?

Setup:

Final balance

$$\begin{aligned} &= (10{,}000) \times \left(1 + \frac{0.08}{2}\right)^{(1)(2)} \\ &= (10{,}000) \times (1.04)^2 \\ &= \$10{,}816 \end{aligned}$$

Semiannual interest is interest that is distributed twice a year. When an interest rate is given as an annual rate, divide by 2 to find the semiannual interest rate.

60. How to solve a REMAINDERS problem

Pick a number that fits the given conditions and see what happens.

Example:

When n is divided by 7, the remainder is 5. What is the remainder when $2n$ is divided by 7?

Setup:

Find a number that leaves a remainder of 5 when divided by 7. You can find such a number by taking any multiple of 7 and adding 5 to it. A good choice would be 12. If $n = 12$, then $2n = 24$, which when divided by 7 leaves a remainder of 3.

61. How to solve a DIGITS problem

Use a little logic—and some trial and error.

Example:

If A, B, C, and D represent distinct digits in the addition problem below, what is the value of D?

$$\begin{array}{r} AB \\ +\,BA \\ \hline CDC \end{array}$$

Setup:

Two 2-digit numbers will add up to at most something in the 100s, so $C = 1$. B plus A in the units column gives a 1, and since A and B in the tens column don't add up to C, it can't simply be that $B + A = 1$. It must be that $B + A = 11$, and a 1 gets carried. In fact, A and B can be any pair of digits that add up to 11 (3 and 8, 4 and 7, etc.), but it doesn't matter what they are: they always give you the same value for D, which is 2:

$$\begin{array}{r} 47 \\ +\ \ 74 \\ \hline 121 \end{array} \qquad \begin{array}{r} 83 \\ +\ \ 38 \\ \hline 121 \end{array}$$

62. How to find a WEIGHTED AVERAGE

Give each term the appropriate "weight."

Example:

The girls' average score is 30. The boys' average score is 24. If there are twice as many boys as girls, what is the overall average?

Setup:

$$\textit{Weighted avg.} = \frac{(1 \times 30) + (2 \times 24)}{3} = \frac{78}{3} = 26$$

HINT: Don't just average the averages.

63. How to find the NEW AVERAGE when a number is added or deleted

Use the sum of the terms of the old average to help you find the new average.

Example:

Michael's average score after four tests is 80. If he scores 100 on the fifth test, what's his new average?

Setup:

Find the original sum from the original average:

$$\textit{Original sum} = 4 \times 80 = 320$$

Add the fifth score to make the new sum:

$$\textit{New sum} = 320 + 100 = 420$$

Find the new average from the new sum:

$$\textit{New average} = \frac{420}{5} = 84$$

64. How to use the ORIGINAL AVERAGE and NEW AVERAGE to figure out WHAT WAS ADDED OR DELETED

Use the sums.

$\textit{Number added} = (\textit{New sum}) - (\textit{Original sum})$
$\textit{Number deleted} = (\textit{Original sum}) - (\textit{New sum})$

Example:

The average of five numbers is 2. After one number is deleted, the new average is –3. What number was deleted?

Setup:

Find the original sum from the original average:

$$\textit{Original sum} = 5 \times 2 = 10$$

Find the new sum from the new average:

$$\textit{New sum} = 4 \times (-3) = -12$$

The difference between the original sum and the new sum is the answer.

$\textit{Number deleted} = 10 - (-12) = 22$

65. How to find an AVERAGE RATE

Convert to totals.

$$\textit{Average}\ \text{A}\ \textit{per}\ \text{B} = \frac{\textit{Total}\ \text{A}}{\textit{Total}\ \text{B}}$$

Example:

If the first 500 pages have an average of 150 words per page, and the remaining 100 pages have an average of 450 words per page, what is the average number of words per page for the entire 600 pages?

Setup:

$$\begin{aligned} \textit{Total pages} &= 500 + 100 = 600 \\ \textit{Total words} &= (500 \times 150) + (100 \times 450) \\ &= 75{,}000 + 45{,}000 \\ &= 120{,}000 \end{aligned}$$

$$\textit{Average words per page} = \frac{120{,}000}{600} = 200$$

To find an average speed, you also convert to totals.

$$\textit{Average speed} = \frac{\textit{Total distance}}{\textit{Time}}$$

Example:

Rosa drove 120 miles one way at an average speed of 40 miles per hour and returned by the same 120-mile route at an average speed of 60 miles per hour. What was Rosa's average speed for the entire 240-mile round trip?

Setup:

To drive 120 miles at 40 mph takes 3 hours. To return at 60 mph takes 2 hours. The total time, then, is 5 hours.

$$\textit{Average speed} = \frac{240 \text{ miles}}{5 \text{ hours}} = 48 \text{ mph}$$

66. How to solve a COMBINED WORK PROBLEM

In a combined work problem, you are given the rate at which people or machines perform work individually and you are asked to compute the rate at which they work together (or vice versa). The work formula states: *The inverse of the time it would take everyone working together equals the sum of the inverses of the times it would take each working individually*. In other words:

$$\frac{1}{r} + \frac{1}{s} = \frac{1}{t}$$

where r and s are, for example, the number of hours it would take Rebecca and Sam, respectively, to complete a job working by themselves, and t is the number of hours it would take the two of them working together. Remember that all these variables must stand for units of TIME and must all refer to the amount of time it takes to do the same task.

Example:

If it takes Joe 4 hours to paint a room and Pete twice as long to paint the same room, how long would it take the two of them, working together, to paint the same room, if each of them works at his respective individual rate?

Setup:

Joe takes 4 hours, so Pete takes 8 hours; thus:

$$\begin{aligned} \frac{1}{4}+\frac{1}{8} &= \frac{1}{t} \\ \frac{2}{8}+\frac{1}{8} &= \frac{1}{t} \\ \frac{3}{8} &= \frac{1}{t} \\ t &= \frac{1}{\left(\frac{3}{8}\right)} = \frac{8}{3} \end{aligned}$$

So it would take them $\frac{8}{3}$ hours, or 2 hours and 40 minutes, to paint the room together.

67. How to determine a COMBINED RATIO

Multiply one or both ratios by whatever you need in order to get the terms they have in common to match.

Example:

The ratio of a to b is 7:3. The ratio of b to c is 2:5. What is the ratio of a to c?

Setup:

Multiply each member of $a{:}b$ by 2 and multiply each member of $b{:}c$ by 3, and you get $a{:}b = 14{:}6$ and $b{:}c = 6{:}15$. Now that the values of b match, you can write $a{:}b{:}c = 14{:}6{:}15$ and then say $a{:}c = 14{:}15$.

68. How to solve a DILUTION or MIXTURE problem

In dilution or mixture problems, you have to determine the characteristics of a resulting mixture when different substances are combined. Or, alternatively, you have to determine how to combine different substances to produce a desired mixture. There are two approaches to such problems—the straightforward setup and the balancing method.

Example:

If 5 pounds of raisins that cost \$1 per pound are mixed with 2 pounds of almonds that cost \$2.40 per pound, what is the cost per pound of the resulting mixture?

Setup:

The straightforward setup:

$(\$1)(5) + (\$2.40)(2) = \$9.80 =$ total cost for 7 pounds of the mixture

The cost per pound is $\frac{\$9.80}{7} = \$1.40$.

Example:

How many liters of a solution that is 10 percent alcohol by volume must be added to 2 liters of a solution that is 50 percent alcohol by volume to create a solution that is 15 percent alcohol by volume?

Setup:

The balancing method: Make the weaker and stronger (or cheaper and more expensive, etc.) substances balance. That is, (percent difference between the weaker solution and the desired solution) $\times$ (amount of weaker solution) $=$ (percent difference between the stronger solution and the desired solution) $\times$ (amount of stronger solution).

Make n the amount, in liters, of the weaker solution.

$$\begin{aligned} n(15 - 10) &= 2(50 - 15) \\ 5n &= 2(35) \\ n &= \frac{70}{5} = 14 \end{aligned}$$

So 14 liters of the 10 percent solution must be added to the original, stronger solution.

69. How to solve an OVERLAPPING SETS problem involving BOTH/NEITHER

Some GRE word problems involve two groups with overlapping members and possibly elements that belong to neither group. It's easy to identify this type of question because the words *both* and/or *neither* appear in the question. These problems are quite workable if you just memorize the following formula:

$$\textit{Group 1} + \textit{Group 2} + \textit{Neither} - \textit{Both} = \textit{Total}$$

Example:

Of the 120 students at a certain language school, 65 are studying French, 51 are studying Spanish, and 53 are studying neither language. How many are studying both French and Spanish?

Setup:

$$\begin{aligned} 65 + 51 + 53 - \textit{Both} &= 120 \\ 169 - \textit{Both} &= 120 \\ \textit{Both} &= 49 \end{aligned}$$

70 How to solve an OVERLAPPING SETS problem involving EITHER/OR CATEGORIES

Other GRE word problems involve groups with distinct "either/or" categories (male/female, blue-collar/white-collar, etc.). The key to solving this type of problem is to organize the information in a grid.

Example:

At a certain professional conference with 130 attendees, 94 of the attendees are doctors, and the rest are dentists. If 48 of the attendees are women and $\frac{1}{4}$ of the dentists in attendance are women, how many of the attendees are male doctors?

Setup:

To complete the grid, use the information in the problem, making each row and column add up to the corresponding total:

	Doctors	Dentists	Total
Male	55	27	82
Female	39	9	48
Total	94	36	130

After you've filled in the information from the question, use simple arithmetic to fill in the remaining boxes until you get the number you are looking for—in this case, that 55 of the attendees are male doctors.

71. How to work with FACTORIALS

You may see a problem involving factorial notation, which is indicated by the ! symbol. If n is an integer greater than 1, then n factorial, denoted by $n!$, is defined as the product of all the integers from 1 to n. For example:

$$\begin{aligned} 2! &= 2 \times 1 = 2 \\ 3! &= 3 \times 2 \times 1 = 6 \\ 4! &= 4 \times 3 \times 2 \times 1 = 24, \text{etc} \end{aligned}$$

By definition, $0! = 1$.

Also note: $6! = 6 \times 5! = 6 \times 5 \times 4!$, etc. Most GRE factorial problems test your ability to factor and/or cancel.

Example:

$$\frac{8!}{6! \times 2!} = \frac{8 \times 7 \times 6!}{6! \times 2 \times 1} = 28$$

72: How to solve a PERMUTATION problem

Factorials are useful for solving questions about permutations (i.e., the number of ways to arrange elements sequentially). For instance, to figure out how many ways there are to arrange 7 items along a shelf, you would multiply the number of possibilities for the first position times the number of possibilities remaining for the second position, and so on—in other words: $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$, or 7!.

If you're asked to find the number of ways to arrange a smaller group that's being drawn from a larger group, you can either apply logic, or you can use the permutation formula:

$$_nP_k = \frac{n!}{(n-k)!}$$

where n = (the number in the larger group) and
k = (the number you're arranging).

Example:

Five runners run in a race. The runners who come in first, second, and third place will win gold, silver, and bronze medals, respectively. How many possible outcomes for gold, silver, and bronze medal winners are there?

Setup:

Any of the 5 runners could come in first place, leaving 4 runners who could come in second place, leaving 3 runners who could come in third place, for a total of $5 \times 4 \times 3 = 60$ possible outcomes for gold, silver, and bronze medal winners. Or, using the formula:

$$_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5 \times 4 \times 3 \times \cancel{2} \times \cancel{1}}{\cancel{2} \times \cancel{1}}$$
$$= 5 \times 4 \times 3 = 60$$

73: How to solve a COMBINATION problem

If the order or arrangement of the smaller group that's being drawn from the larger group does NOT matter, you are looking for the numbers of combinations, and a different formula is called for:

$$_nC_k = \frac{n!}{k!(n-k)!}$$

where n = (the number in the larger group) and
k = (the number you're choosing).

Example:

How many different ways are there to choose 3 delegates from 8 possible candidates?

Setup:

$$_nC_k = \frac{8!}{3!(8-3)!} = \frac{8!}{3! \times 5!}$$
$$= \frac{8 \times 7 \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}$$
$$= 8 \times 7 = 56$$

So there are 56 different possible combinations.

74. How to solve PROBABILITY problems where probabilities must be multiplied

Suppose that a random process is performed. Then there is a set of possible outcomes that can occur. An event is a set of possible outcomes. We are concerned with the probability of events.

When all the outcomes are all equally likely, the basic probability formula is this:

$$Probability = \frac{\textit{Number of desired outcomes}}{\textit{Number of total possible outcomes}}$$

Many more difficult probability questions involve finding the probability that several events occur. Let's consider first the case of the probability that two events occur. Call these two events A and B. The probability that both events occur is the probability that event A occurs multiplied by the probability that event B occurs given that event A

occurred. The probability that B occurs given that A occurs is called the conditional probability that B occurs given that A occurs. Except when events A and B do not depend on one another, the probability that B occurs given that A occurs is not the same as the probability that B occurs.

The probability that three events A, B, and C occur is the probability that A occurs multiplied by the conditional probability that B occurs given that A occurred multiplied by the conditional probability that C occurs given that both A and B have occurred.

This can be generalized to any number of events.

Example:
If 2 students are chosen at random to run an errand from a class with 5 girls and 5 boys, what is the probability that both students chosen will be girls?

Setup:
The probability that the first student chosen will be a girl is $\frac{5}{10} = \frac{1}{2}$, and since there would be 4 girls and 5 boys left out of 9 students, the probability that the second student chosen will be a girl (given that the first student chosen is a girl) is $\frac{4}{9}$. Thus the probability that both students chosen will be girls is $\frac{1}{2} \times \frac{4}{9} = \frac{2}{9}$. There was conditional probability here because the probability of choosing the second girl was affected by another girl being chosen first. Now let's consider another example where a random process is repeated.

Example:
If a fair coin is tossed 4 times, what's the probability that at least 3 of the 4 tosses will be heads?

Setup:
There are 2 possible outcomes for each toss, so after 4 tosses, there are $2 \times 2 \times 2 \times 2 = 16$ possible outcomes.

We can list the different possible sequences where at least 3 of the 4 tosses are heads. These sequences are

HHHT
HHTH
HTHH
THHH
HHHH

Thus, the probability that at least 3 of the 4 tosses will come up heads is:

$$\frac{\textit{Number of desired outcomes}}{\textit{Number of total possible outcomes}} = \frac{5}{16}$$

We could have also solved this question using the combinations formula. The probability of a head is $\frac{1}{2}$, and the probability of a tail is $\frac{1}{2}$. The probability of any particular sequence of heads and tails resulting from 4 tosses is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$, which is $\frac{1}{16}$.
Suppose that the result each of the four tosses is recorded in each of the four spaces.

_____ _____ _____ _____

Thus, we would record an H for head or a T for tails in each of the 4 spaces.

The number of ways of having exactly 3 heads among the 4 tosses is the number of ways of choosing 3 of the 4 spaces above to record an H for heads.

The number of ways of choosing 3 of the 4 spaces is

$$_4C_3 = \frac{4!}{3!\,(4-3)!} = \frac{4!}{3!(1)!} = \frac{4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 1} = 4$$

The number of ways of having exactly 4 heads among the 4 tosses is 1.

If we use the combinations formula, using the definition that $0! = 1$, then

$$
\begin{aligned}
{}_4C_4 &= \frac{4!}{4!(4-4)!} = \frac{4!}{4!(0)!} \\
&= \frac{4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1 \times 1} = 1
\end{aligned}
$$

Thus, ${}_4C_3 = 4$ and ${}_4C_4 = 1$. So the number of different sequences containing at least 3 heads is $4 + 1 = 5$.

The probability of having at least 3 heads is $\frac{5}{16}$.

75. How to deal with STANDARD DEVIATION

Like the terms *mean*, *mode*, *median*, and *range*, *standard deviation* is a term used to describe sets of numbers. Standard deviation is a measure of how spread out a set of numbers is (how much the numbers deviate from the mean). The greater the spread, the higher the standard deviation. You'll rarely have to calculate the standard deviation on Test Day (although this skill may be necessary for some high-difficulty questions). Here's how standard deviation is calculated:

- Find the average (arithmetic mean) of the set.
- Find the differences between the mean and each value in the set.
- Square each of the differences.
- Find the average of the squared differences.
- Take the positive square root of the average.

In addition to the occasional question that asks you to calculate standard deviation, you may also be asked to compare standard deviations between sets of data or otherwise demonstrate that you understand what standard deviation means. You can often handle these questions using estimation.

Example:

High temperatures, in degrees Fahrenheit, in two cities over five days:

September	1	2	3	4	5
City A	54	61	70	49	56
City B	62	56	60	67	65

For the five-day period listed, which city had the greater standard deviation in high temperatures?

Setup:

Even without trying to calculate them out, one can see that City A has the greater spread in temperatures and, therefore, the greater standard deviation in high temperatures. If you were to go ahead and calculate the standard deviations following the steps described above, you would find that the standard deviation in high temperatures for

City $A = \sqrt{\frac{254}{5}} \approx 7.1$ while the standard deviation for City

$B = \sqrt{\frac{74}{5}} \approx 3.8$.

76. How to MULTIPLY/DIVIDE VALUES WITH EXPONENTS

Add/subtract the exponents.

Example:

$$
\begin{aligned}
x^a \times x^b &= x^{a+b} \\
2^3 \times 2^4 &= 2^7
\end{aligned}
$$

Example:

$$
\begin{aligned}
\frac{x^a}{x^b} &= x^{a-b} \\
\frac{2^8}{2^2} &= 2^{8-2} = 2^6
\end{aligned}
$$

77. How to handle a value with an EXPONENT RAISED TO AN EXPONENT

Multiply the exponents.

Example:

$$
\begin{aligned}
(x^a)^b &= x^{ab} \\
(3^4)^5 &= 3^{20}
\end{aligned}
$$

78. How to handle EXPONENTS with a base of ZERO and BASES with an EXPONENT of ZERO

Zero raised to any nonzero exponent equals zero.

Example:

$$0^4 = 0^{12} = 0^1 = 0$$

Any nonzero number raised to the exponent 0 equals 1.

Example:

$$3^0 = 15^0 = (0.34)^0 = (-345)^0 = \pi^0 = 1$$

The lone exception is 0 raised to the 0 power, which is *undefined*.

79. How to handle NEGATIVE POWERS

A number raised to the exponent $-x$ is the reciprocal of that number raised to the exponent x.

Example:

$$n^{-1} = \frac{1}{n},\ n^{-2} = \frac{1}{n^2},\ \text{and so on.}$$

$$5^{-3} = \frac{1}{5^3} = \frac{1}{5 \times 5 \times 5} = \frac{1}{125}$$

80. How to handle FRACTIONAL POWERS

Fractional exponents relate to roots. For instance, $x^{\frac{1}{2}} = \sqrt{x}$.

Likewise, $x^{\frac{1}{3}} = \sqrt[3]{x}$, $x^{\frac{2}{3}} = \sqrt[3]{x^2}$, and so on.

Example:

$$\sqrt{x^{-2}} = (x^{-2})^{\frac{1}{2}} = x^{(-2)\frac{1}{2}} = x^{-1} = \frac{1}{x}$$

$$4^{\frac{1}{2}} = \sqrt{4} = 2$$

81. How to handle CUBE ROOTS

The cube root of x is just the number that when used as a factor 3 times (i.e., cubed) gives you x. Both positive and negative numbers have one and only one cube root, denoted by the symbol $\sqrt[3]{\ }$, and the cube root of a number is always the same sign as the number itself.

Example:

$$(-5) \times (-5) \times (-5) = -125, \text{ so } \sqrt[3]{-125} = -5$$

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}, \text{ so } \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$$

82. How to ADD, SUBTRACT, MULTIPLY, and DIVIDE ROOTS

You can add/subtract roots only when the parts inside the $\sqrt{\ }$ are identical.

Example:

$$\sqrt{2} + 3\sqrt{2} = 4\sqrt{2}$$

$$\sqrt{2} - 3\sqrt{2} = -2\sqrt{2}$$

$$\sqrt{2} + \sqrt{3} \quad \text{cannot be combined.}$$

To multiply/divide roots, deal with what's inside the $\sqrt{\ }$ and outside the $\sqrt{\ }$ separately.

Example:

$$\left(2\sqrt{3}\right)\left(7\sqrt{5}\right) = (2 \times 7)\left(\sqrt{3 \times 5}\right) = 14\sqrt{15}$$

$$\frac{10\sqrt{21}}{5\sqrt{3}} = \frac{10}{5}\sqrt{\frac{21}{3}} = 2\sqrt{7}$$

83. How to SIMPLIFY A RADICAL

Look for factors of the number under the radical sign that are perfect squares; then find the square root of those perfect squares. Keep simplifying until the term with the square root sign is as simplified as possible, that is, when there are no other perfect square factors (4, 9, 16, 25, 36, . . .) inside the $\sqrt{\ }$. Write the perfect squares as separate factors and "unsquare" them.

Example:

$$\sqrt{48} = \sqrt{16}\ \sqrt{3} = 4\sqrt{3}$$

$$\sqrt{180} = \sqrt{36}\ \sqrt{5} = 6\sqrt{5}$$

84. How to solve certain QUADRATIC EQUATIONS

Manipulate the equation (if necessary) so that it is equal to 0, factor the left side (reverse FOIL by finding two numbers whose product is the constant and whose sum is the coefficient of the term without the exponent), and break the quadratic into two simple expressions. Then find the value(s) for the variable that make either expression $= 0$.

Example:

$$\begin{aligned} x^2 + 6 &= 5x \\ x^2 - 5x + 6 &= 0 \\ (x - 2)(x - 3) &= 0 \\ x - 2 &= 0 \text{ or } x - 3 = 0 \\ x &= 2 \text{ or } 3 \end{aligned}$$

Example:

$$\begin{aligned} x^2 &= 9 \\ x &= 3 \text{ or } -3 \end{aligned}$$

85. How to solve MULTIPLE EQUATIONS

When you see two equations with two variables on the GRE, they're probably easy to combine in such a way that you get something closer to what you're looking for.

Example:

If $5x - 2y = -9$ and $3y - 4x = 6$, what is the value of $x + y$?

Setup:

The question doesn't ask for x and y separately, so don't solve for them separately if you don't have to. Look what happens if you just rearrange a little and "add'' the equations:

$$\begin{array}{rcr} 5x - 2y &=& -9 \\ +[-4x + 3y &=& 6] \\ \hline x + y &=& -3 \end{array}$$

86. How to solve a SEQUENCE problem

The notation used in sequence problems scares many test takers, but these problems aren't as bad as they look. In a sequence problem, the nth term in the sequence is generated by performing an operation, which will be defined for you, on either n or on the previous term in the sequence. The term itself is expressed as a_n. For instance, if you are referring to the fourth term in a sequence, it is called a_4 in sequence notation. Familiarize yourself with sequence notation and you should have no problem.

Example:

What is the positive difference between the fifth and fourth terms in the sequence 0, 4, 18, . . . whose nth term is $n^2(n - 1)$?

Setup:

Use the definition given to come up with the values for your terms:

$$\begin{aligned} a_5 &= 5^2(5 - 1) = 25(4) = 100 \\ a_4 &= 4^2(4 - 1) = 16(3) = 48 \end{aligned}$$

So the positive difference between the fifth and fourth terms is $100 - 48 = 52$.

87. How to solve a FUNCTION problem

You may see function notation on the GRE. An algebraic expression of only one variable may be defined as a function, usually symbolized by f or g, of that variable.

Example:

What is the minimum value of x in the function $f(x) = x^2 - 1$?

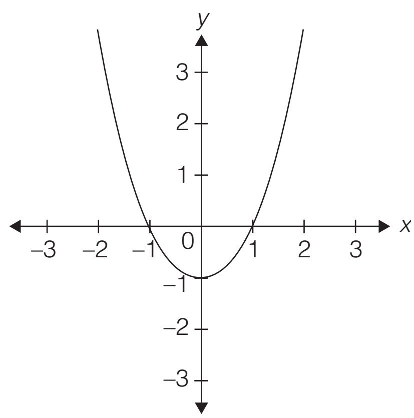
Setup:

In the function $f(x) = x^2 - 1$, if x is 1, then $f(1) = 1^2 - 1 = 0$. In other words, by inputting 1 into the function, the output $f(x) = 0$. Every number inputted has one and only one output (although the reverse is not necessarily true). You're asked to find the minimum value, so how would you minimize the expression $f(x) = x^2 - 1$?

Since x^2 cannot be negative, in this case $f(x)$ is minimized by making $x = 0$: $f(0) = 0^2 - 1 = -1$, so the minimum value of the function is -1.

88. How to handle GRAPHS of FUNCTIONS

You may see a problem that involves a function graphed onto the *xy*-coordinate plane, often called a "rectangular coordinate system" on the GRE. When graphing a function, the output, $f(x)$, becomes the *y*-coordinate. For example, in the previous example, $f(x) = x^2 - 1$, you've already determined 2 points, (1,0) and (0,−1). If you were to keep plugging in numbers to determine more points and then plotted those points on the *xy*-coordinate plane, you would come up with something like this:



This curved line is called a *parabola*. In the event that you should see a parabola on the GRE (it could be upside down or narrower or wider than the one shown), you will most likely be asked to choose which equation the parabola is describing. These questions can be surprisingly easy to answer. Pick out obvious points on the graph, such as (1,0) and (0,−1) above, plug these values into the answer choices, and eliminate answer choices that don't jibe with those values until only one answer choice is left.

89. How to handle LINEAR EQUATIONS

You may also encounter linear equations on the GRE. A linear equation is often expressed in the form

$y = mx + b$, where

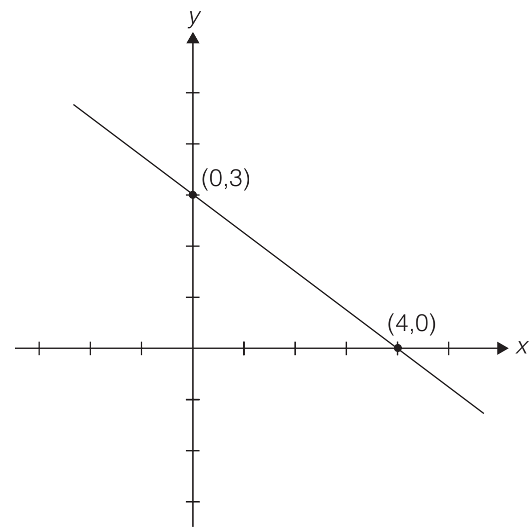
$m =$ the slope of the line $= \dfrac{rise}{run}$

$b =$ the *y*-intercept (the point where the line crosses the *y*-axis)

Example:

The graph of the linear equation

$$y = -\frac{3}{4}x + 3 \text{ is this:}$$



Note:

The equation could also be written in the form $3x + 4y = 12$, but this form does not readily describe the slope and *y*-intercept of the line.

To get a better handle on an equation written in this form, you can solve for *y* to write it in its more familiar form. Or, if you're asked to choose which equation the line is describing, you can pick obvious points, such as (0,3) and (4,0) in this example, and use these values to eliminate answer choices until only one answer is left.

90. How to find the *x*- and *y*-INTERCEPTS of a line

The *x*-intercept of a line is the value of *x* where the line crosses the *x*-axis. In other words, it's the value of *x* when $y = 0$. Likewise, the *y*-intercept is the value of *y* where the line crosses the *y*-axis (i.e., the value of *y* when $x = 0$). The *y*-intercept is also the value *b* when the equation is in the form $y = mx + b$. For instance, in the line shown in the previous example, the *x*-intercept is 4 and the *y*-intercept is 3.

91. How to find the MAXIMUM and MINIMUM lengths for a SIDE of a TRIANGLE

If you know the lengths of two sides of a triangle, you know that the third side is somewhere between the positive difference and the sum of the other two sides.

Example:

The length of one side of a triangle is 7. The length of another side is 3. What is the range of possible lengths for the third side?

Setup:

The third side is greater than the positive difference ($7 - 3 = 4$) and less than the sum ($7 + 3 = 10$) of the other two sides.

92. How to find the sum of all the ANGLES of a POLYGON and one angle measure of a REGULAR POLYGON

Sum of the interior angles in a polygon with n sides:

$$(n - 2) \times 180$$

The term *regular* means all angles in the polygon are of equal measure.

Degree measure of one angle in a regular polygon with n sides:

$$\frac{(n - 2) \times 180}{n}$$

Example:

What is the measure of one angle of a regular pentagon?

Setup:

Since a pentagon is a five-sided figure, plug $n = 5$ into the formula:

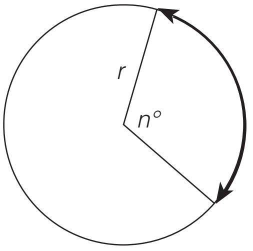
Degree measure of one angle:

$$\frac{(5 - 2) \times 180}{5} = \frac{540}{5} = 108$$

93. How to find the LENGTH of an ARC

Think of an arc as a fraction of the circle's circumference. Use the measure of an interior angle of a circle, which has 360 degrees around the central point, to determine the length of an arc.

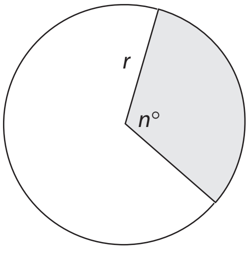
$$\textit{Length of arc} = \frac{n}{360} \times 2\pi r$$



94. How to find the AREA of a SECTOR

Think of a sector as a fraction of the circle's area. Again, set up the interior angle measure as a fraction of 360, which is the degree measure of a circle around the central point.

$$\textit{Area of sector} = \frac{n}{360} \times \pi r^2$$

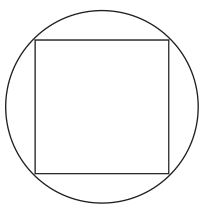


95. How to find the dimensions or area of an INSCRIBED or CIRCUMSCRIBED FIGURE

Look for the connection. Is the diameter the same as a side or a diagonal?

Example:

If the area of the square is 36, what is the circumference of the circle?



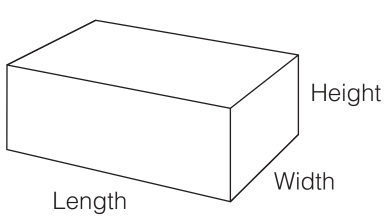
Setup:

To get the circumference, you need the diameter or radius. The circle's diameter is also the square's diagonal. The diagonal of the square is $6\sqrt{2}$. This is because the diagonal of the square transforms it into two separate 45° – 45° – 90° triangles (see #46). So, the diameter of the circle is $6\sqrt{2}$.

$$\textit{Circumference} = \pi(\textit{Diameter}) = 6\pi\sqrt{2}.$$

96. How to find the VOLUME of a RECTANGULAR SOLID

$$\textit{Volume} = \textit{Length} \times \textit{Width} \times \textit{Height}$$



97. How to find the SURFACE AREA of a RECTANGULAR SOLID

To find the surface area of a rectangular solid, you have to find the area of each face and add the areas together. Here's the formula:

Let l = length, w = width, h = height:

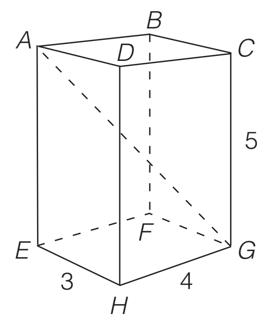
$$\textit{Surface area} = 2(lw) + 2(wh) + 2(lh)$$

98. How to find the DIAGONAL of a RECTANGULAR SOLID

Use the Pythagorean theorem twice, unless you spot "special" triangles.

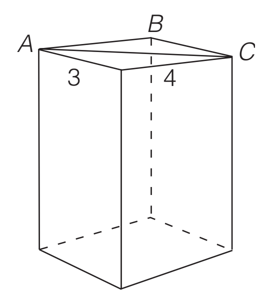
Example:

What is the length of AG?

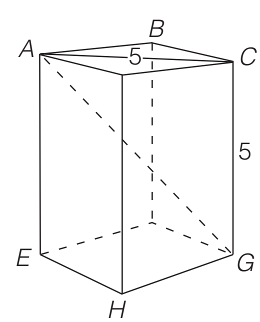


Setup:

Draw diagonal AC.



ABC is a 3:4:5 triangle, so $AC = 5$. Now look at triangle ACG:

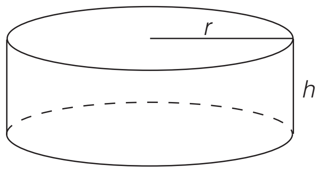


ACG is another special triangle, so you don't need to use the Pythagorean theorem. ACG is a 45° – 45° – 90° triangle, so $AG = 5\sqrt{2}$.

99. How to find the VOLUME of a CYLINDER

$$\textit{Volume} = \textit{Area of the base} \times \textit{Height} = \pi r^2 h$$

Example:



Let $r = 6$ and $h = 3$.

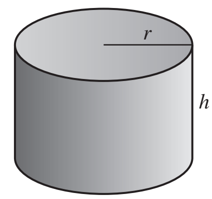
Setup:

$$\textit{Volume} = \pi r^2 h = \pi(6^2)(3) = 108\pi$$

100. How to find the SURFACE AREA of a CYLINDER

$$\textit{Surface area} = 2\pi r^2 + 2\pi rh$$

Example:



Let $r = 3$ and $h = 4$.

Setup:

$$\begin{aligned}\textit{Surface area} &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(3)^2 + 2\pi(3)(4) \\ &= 18\pi + 24\pi = 42\pi\end{aligned}$$