12. Petri Nets

Oscar Nierstrasz

J. L. Peterson, *Petri Nets Theory and the Modelling of Systems*, Prentice Hall, 1983.

Roadmap

> Definition:
  - places, transitions, inputs, outputs
  - firing enabled transitions
> Modelling:
  - concurrency and synchronization
> Properties of nets:
  - liveness, boundedness
> Implementing Petri net models:
  - centralized and decentralized schemes

Roadmap

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Petri nets: a definition

A <u>Petri net</u> $C = \langle P,T,I,O \rangle$ consists of:

1. A finite set P of *places*
2. A finite set T of *transitions*
3. An *input function* $I: T \rightarrow Nat^P$ (maps to bags of places)
4. An *output function* $O: T \rightarrow Nat^P$

A <u>marking</u> of C is a mapping $m: P \rightarrow Nat$

Example:

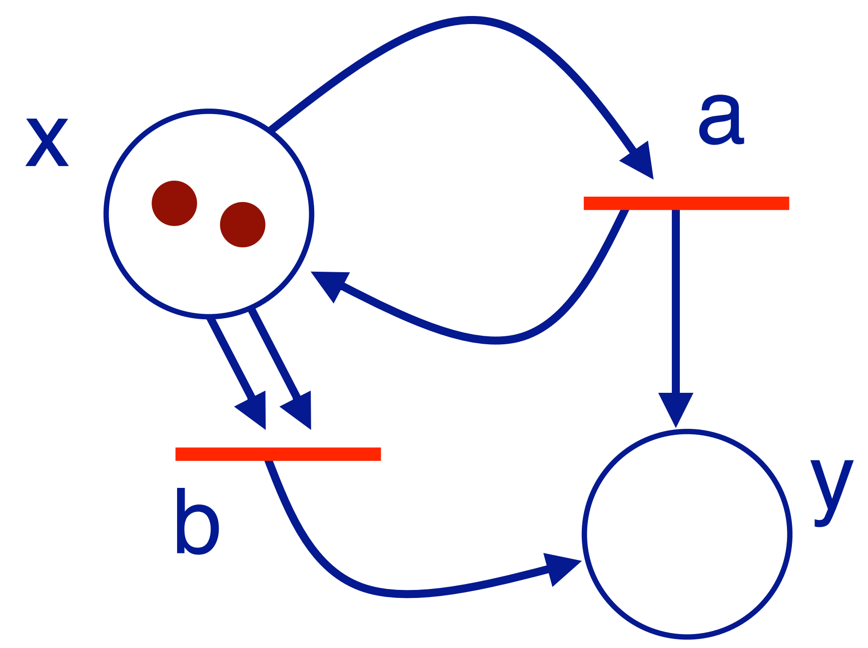
$P = \{ x, y \}$

$T = \{ a, b \}$

$I(a) = \{ x \}$, $I(b) = \{ x, x \}$

$O(a) = \{ x, y \}$, $O(b) = \{ y \}$

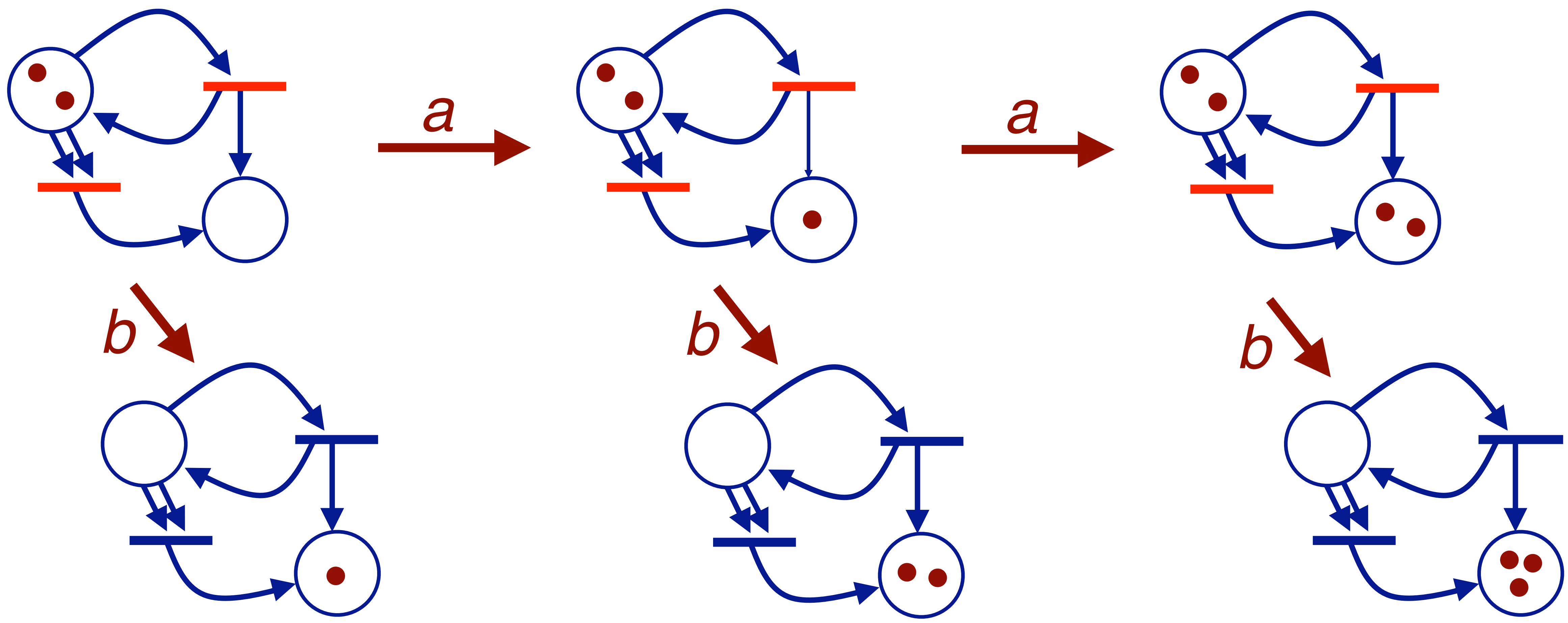
$m = \{ x, x \}$



Firing transitions

To <u>fire</u> a transition t:

1. t must be <u>enabled</u>: $m \geq I(t)$
2. *consume* inputs and *generate* output: $m' = m - I(t) + O(t)$



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Modelling with Petri nets

Petri nets are good for modelling:

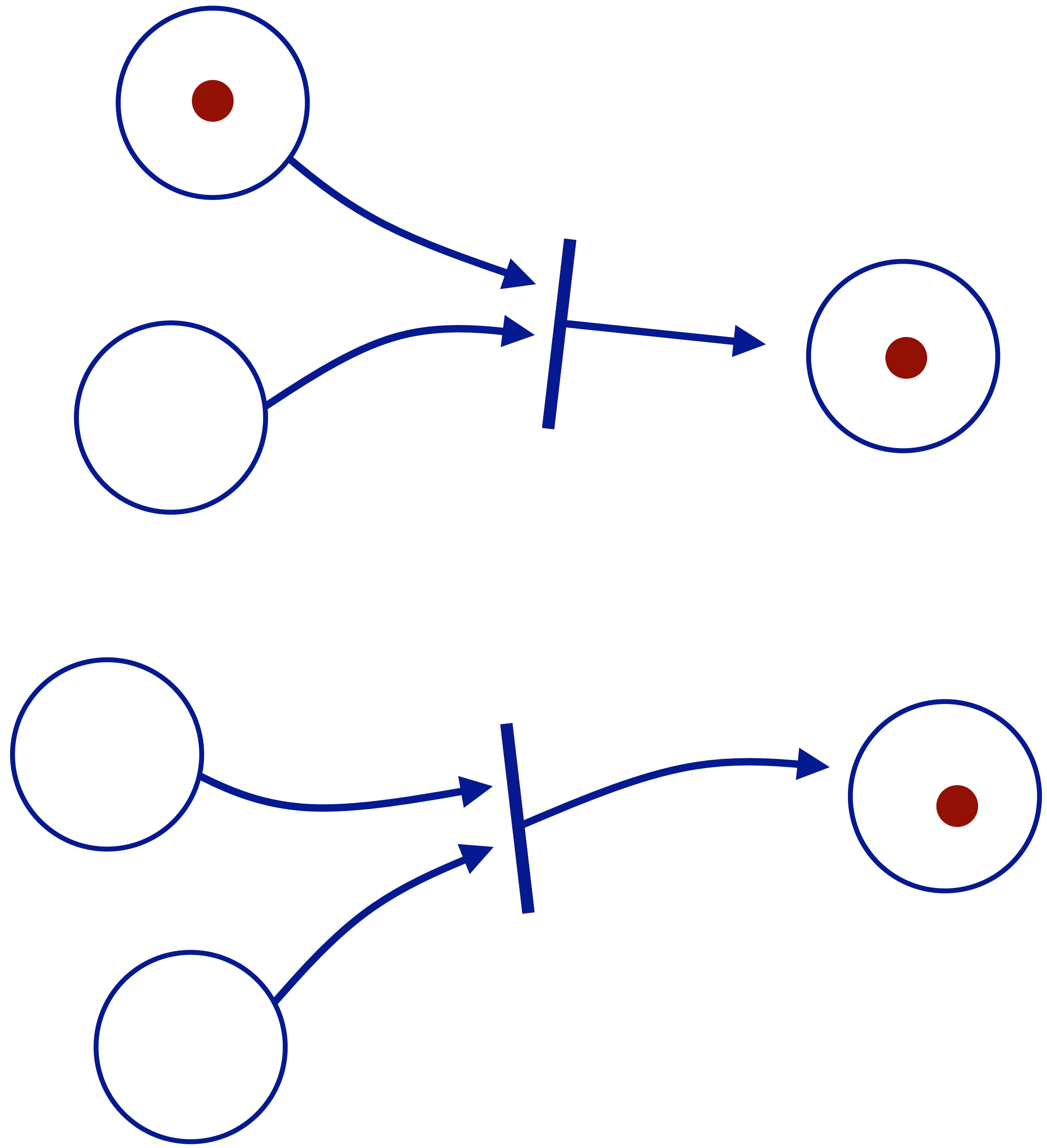
> concurrency
> synchronization

Tokens can represent:

> resource availability
> jobs to perform
> flow of control
> synchronization conditions ...

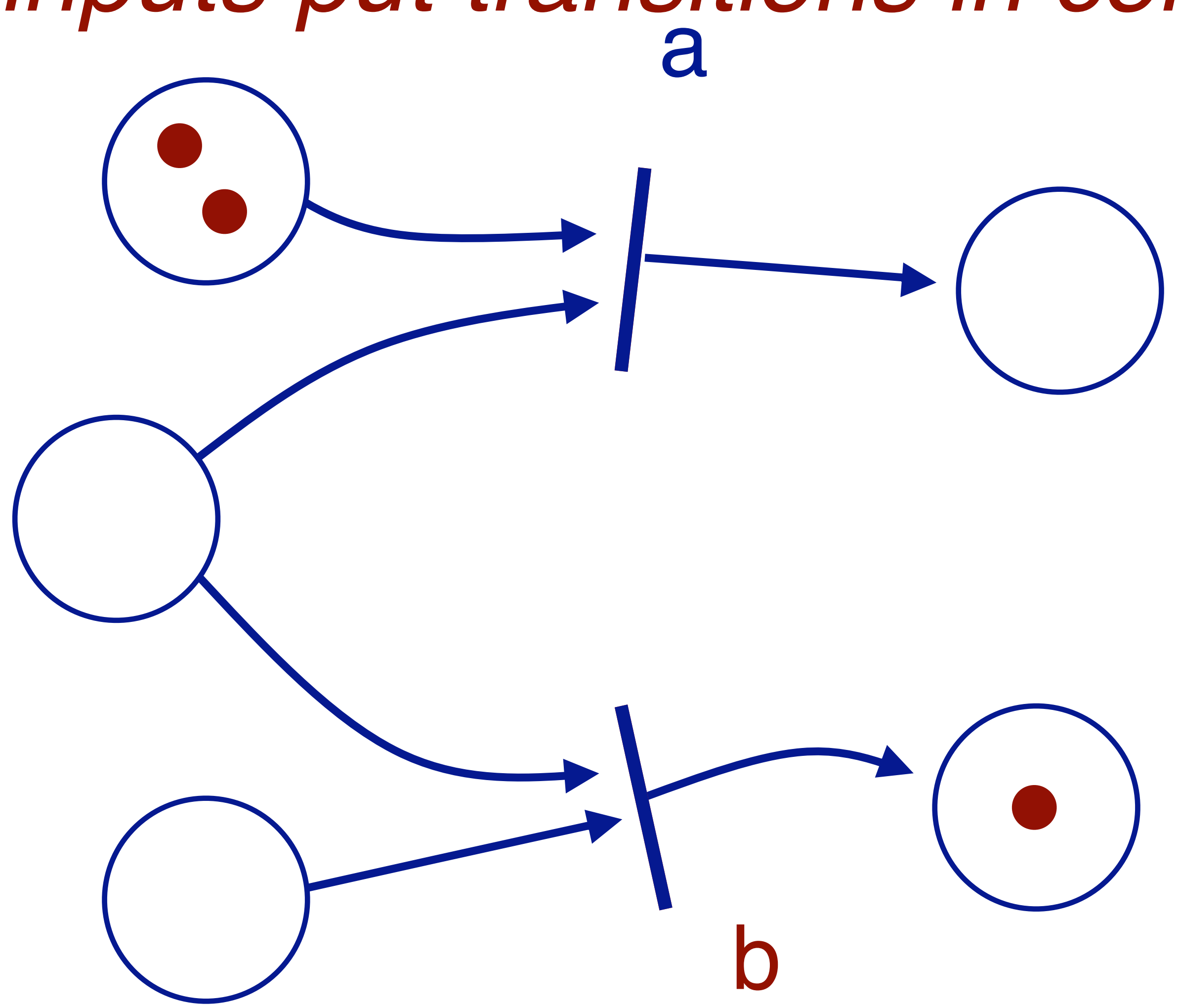
Concurrency

Independent inputs permit “concurrent” firing of transitions



Conflict

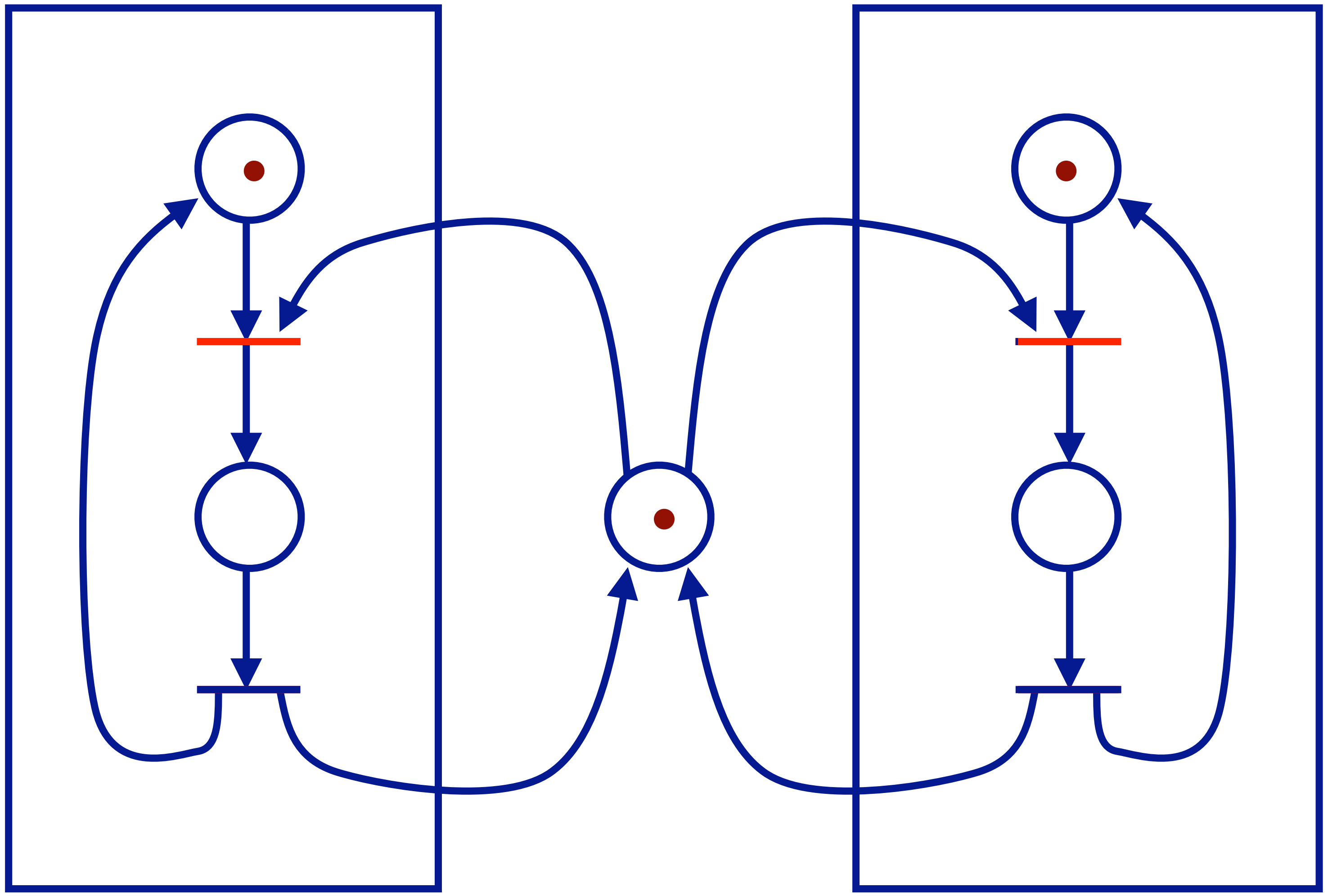
Overlapping inputs put transitions in conflict



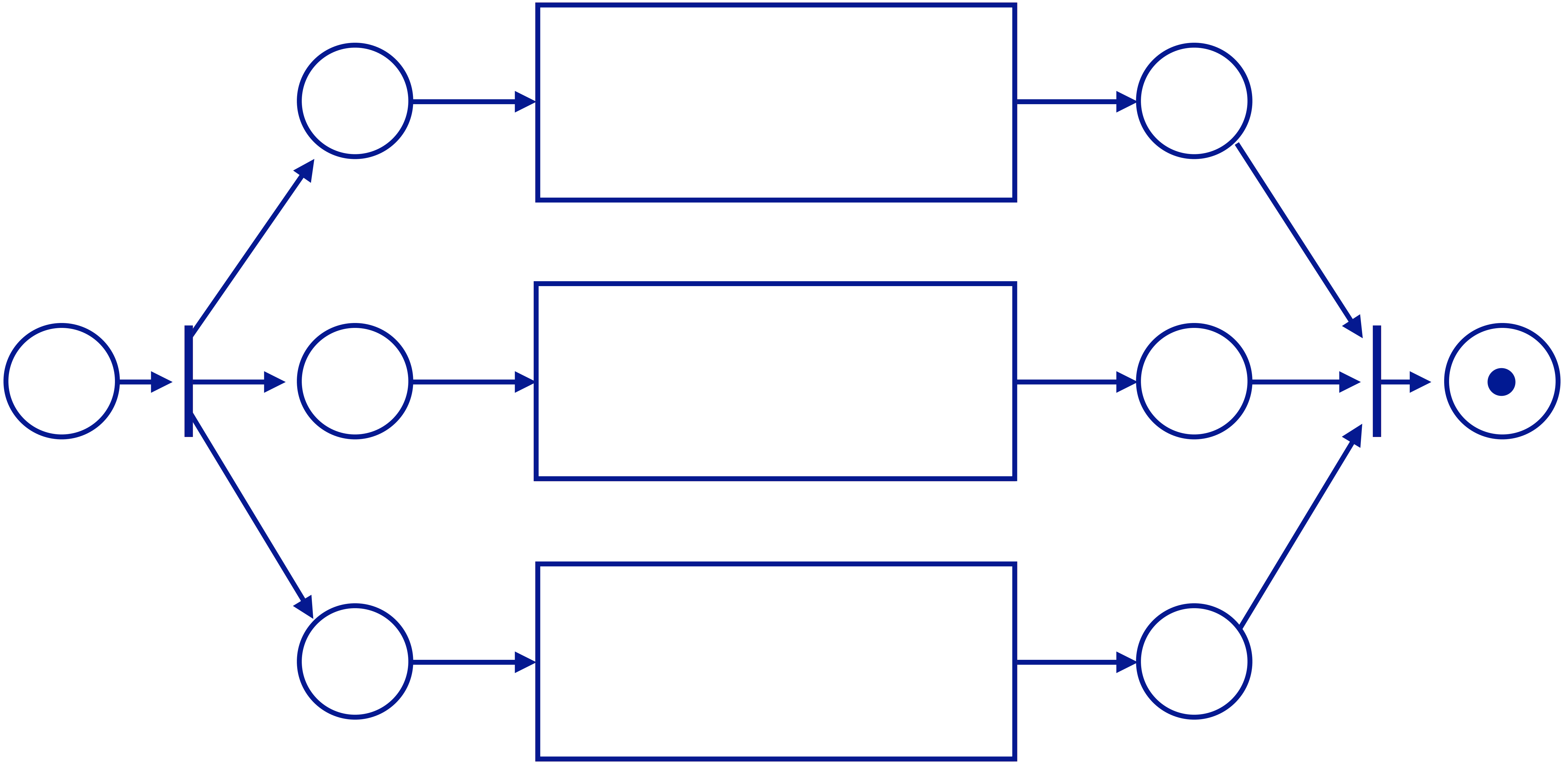
Only one of a or b may fire

Mutual Exclusion

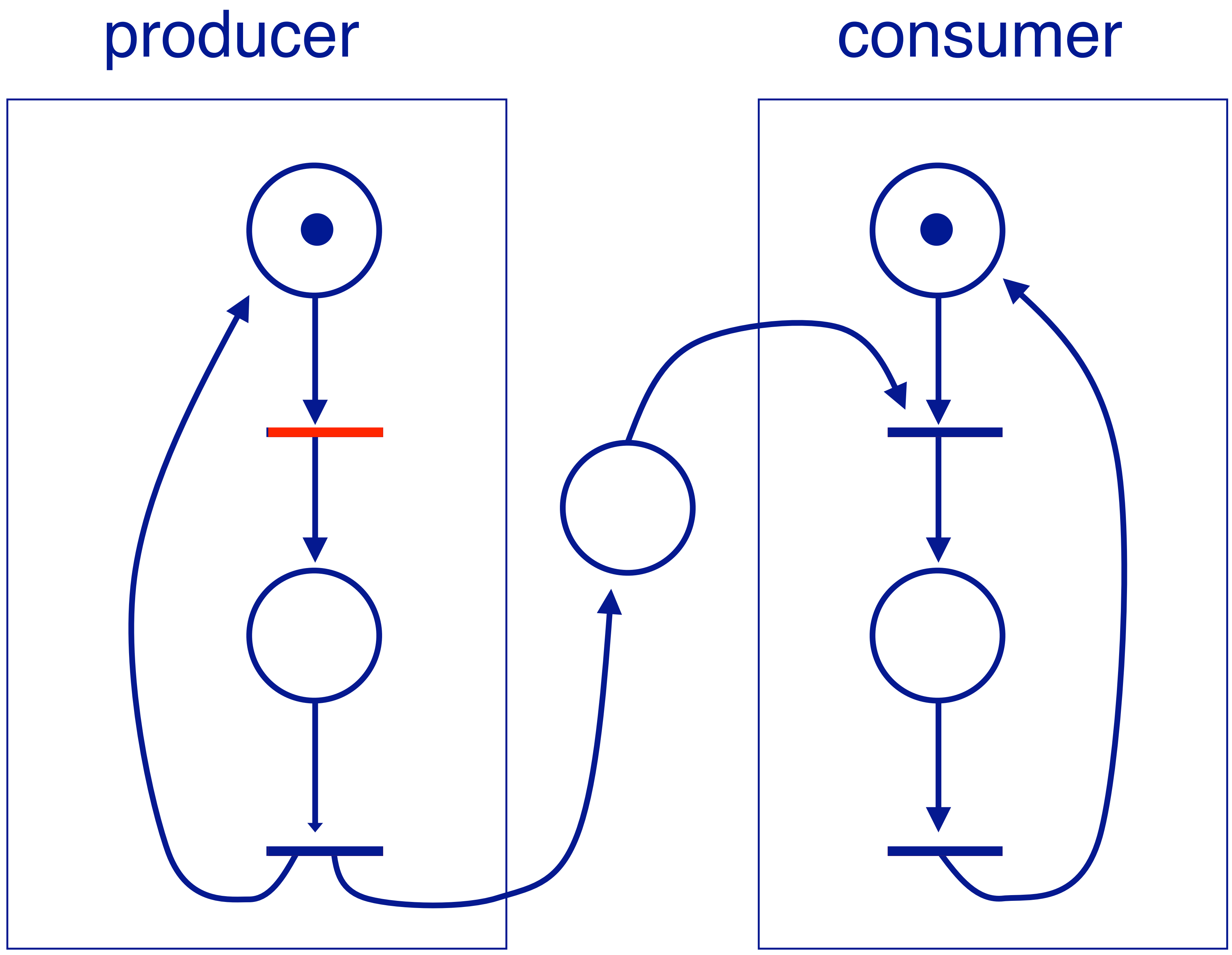
The two subnets are forced to synchronize



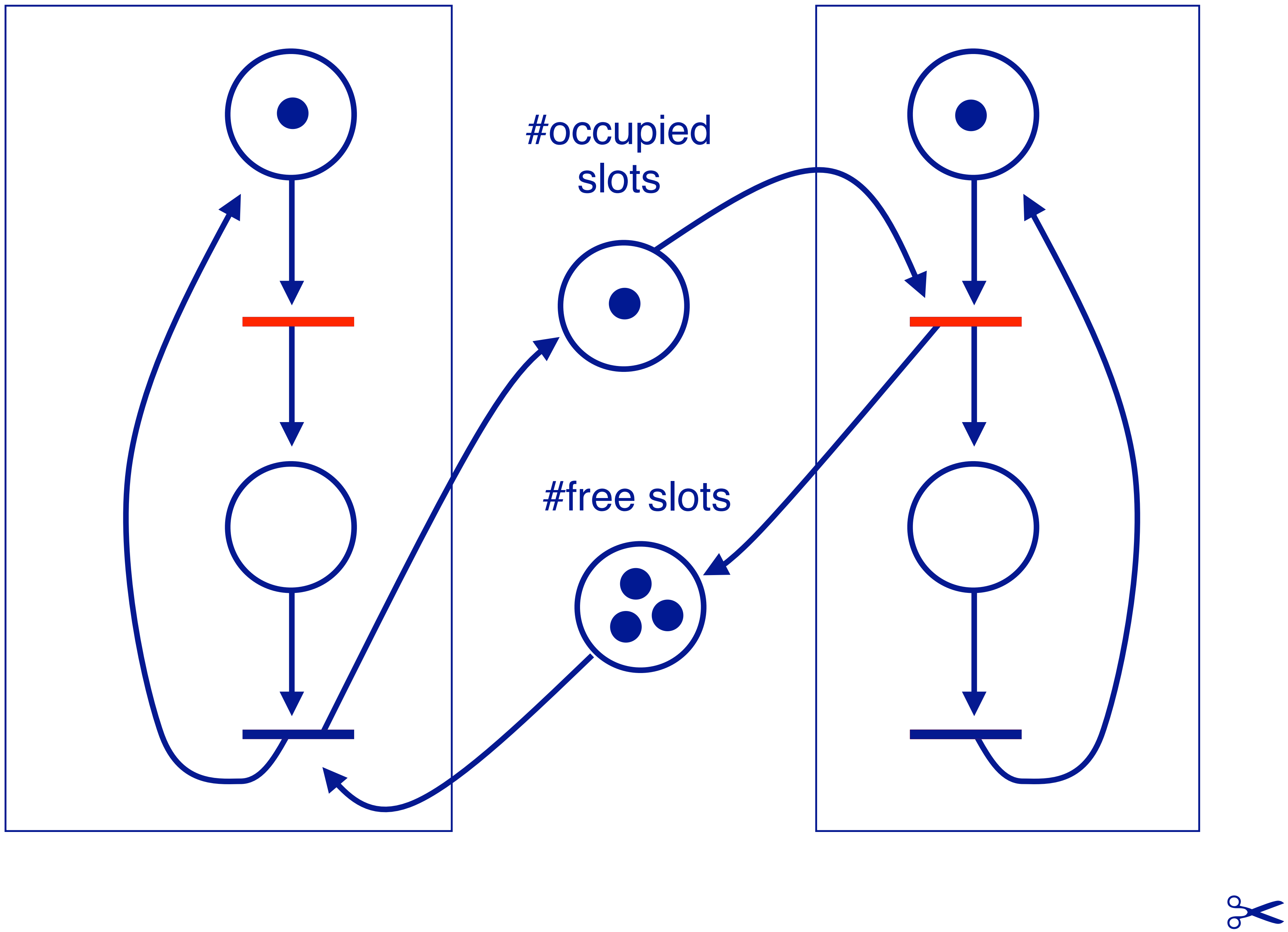
Fork and Join



Producers and Consumers



Bounded Buffers



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Reachability and Boundedness

Reachability:

> The reachability set $R(C,\mu)$ of a net C is the set of all markings μ' *reachable from initial marking m*.

Boundedness:

> A net C with initial marking μ is safe if places always hold *at most 1 token*.

> A marked net is (k-)bounded if places *never hold more than k tokens*.

> A marked net is conservative if the number of tokens is *constant*.

Liveness and Deadlock

Liveness:

> A transition is <u>deadlocked</u> if it can *never fire.*

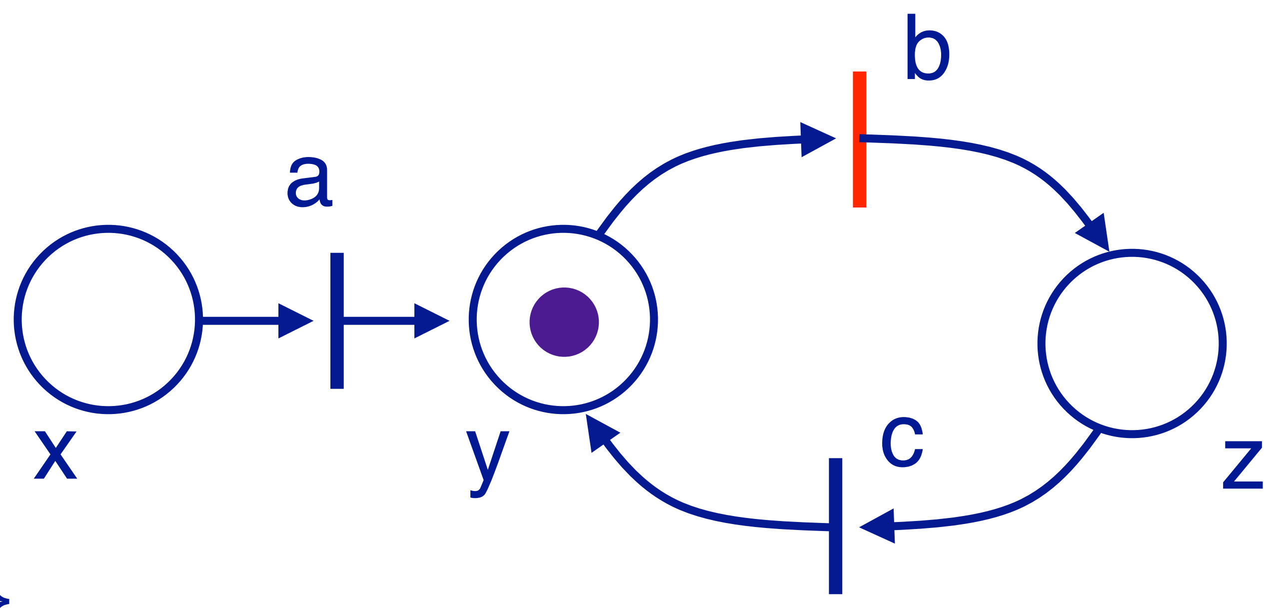
> A transition is <u>live</u> if it can *never deadlock.*

This net is both *safe and conservative*.

Transition a is *deadlocked*.

Transitions b and c are *live*.

The reachability set is {{y}, {z}}.



Are the examples we have seen bounded? Are they live?

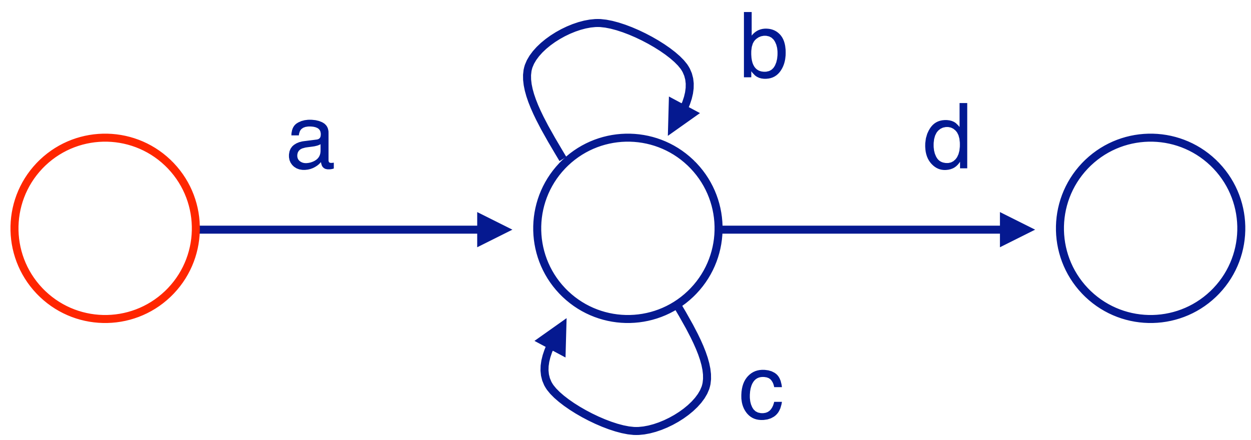
Related Models

Finite State Processes

> Equivalent to *regular expressions*

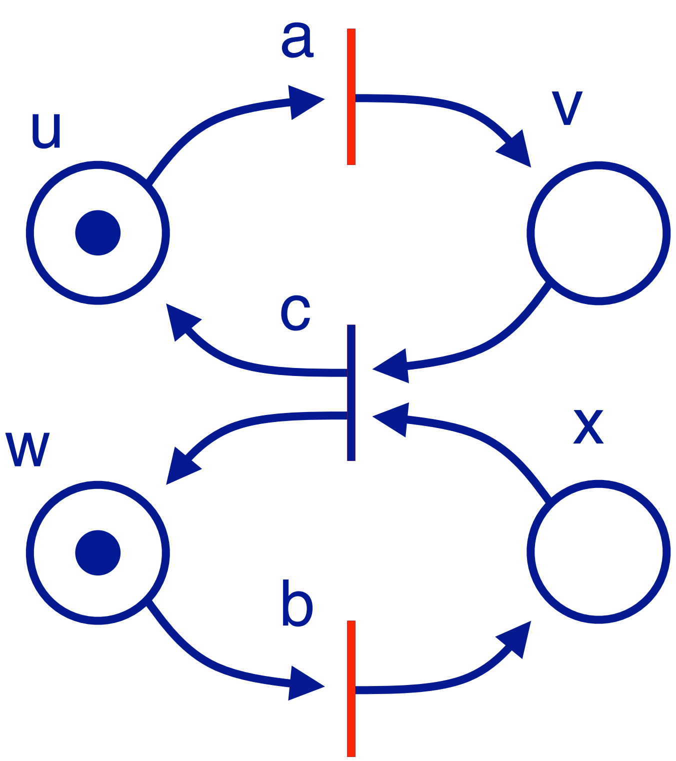
> Can be modelled by *one-token conservative nets*

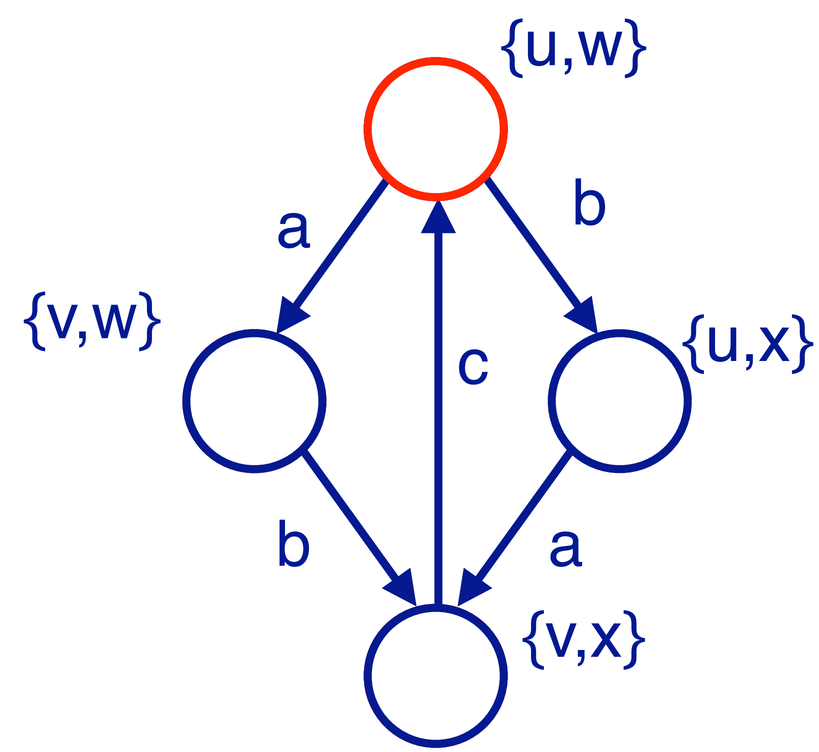
The FSA for: a(blc)*d



Finite State Nets

Some Petri nets can be modelled by FSPs





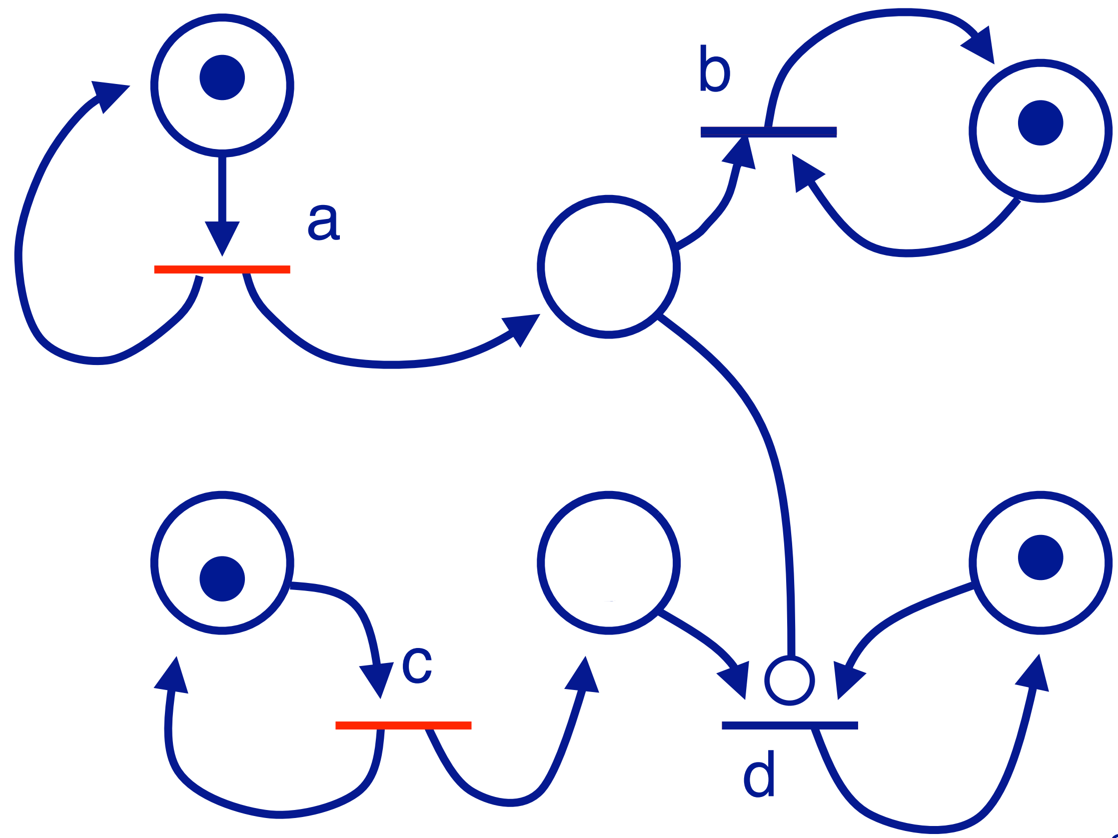
Precisely which nets can (cannot) be modelled by FSPs?

Zero-testing Nets

Petri nets are not computationally complete

> Cannot model "zero testing"
> Cannot model priorities

<u>A zero-testing net:</u> An equal number of a and b transitions may fire as a sequence during any sequence of matching c and d transitions.
(#a ≥ #b, #c ≥ #d)



Other Variants

There exist countless variants of Petri nets

Coloured Petri nets:

> Tokens are “coloured” to represent different kinds of resources

Augmented Petri nets:

> Transitions additionally depend on external conditions

Timed Petri nets:

> A duration is associated with each transition

Applications of Petri nets

Modelling information systems:

- Workflow
- Hypertext (possible transitions)
- Dynamic aspects of OODB design

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- —**centralized and decentralized schemes**

Implementing Petri nets

We can implement Petri net structures in either *centralized* or *decentralized* fashion:

Centralized:

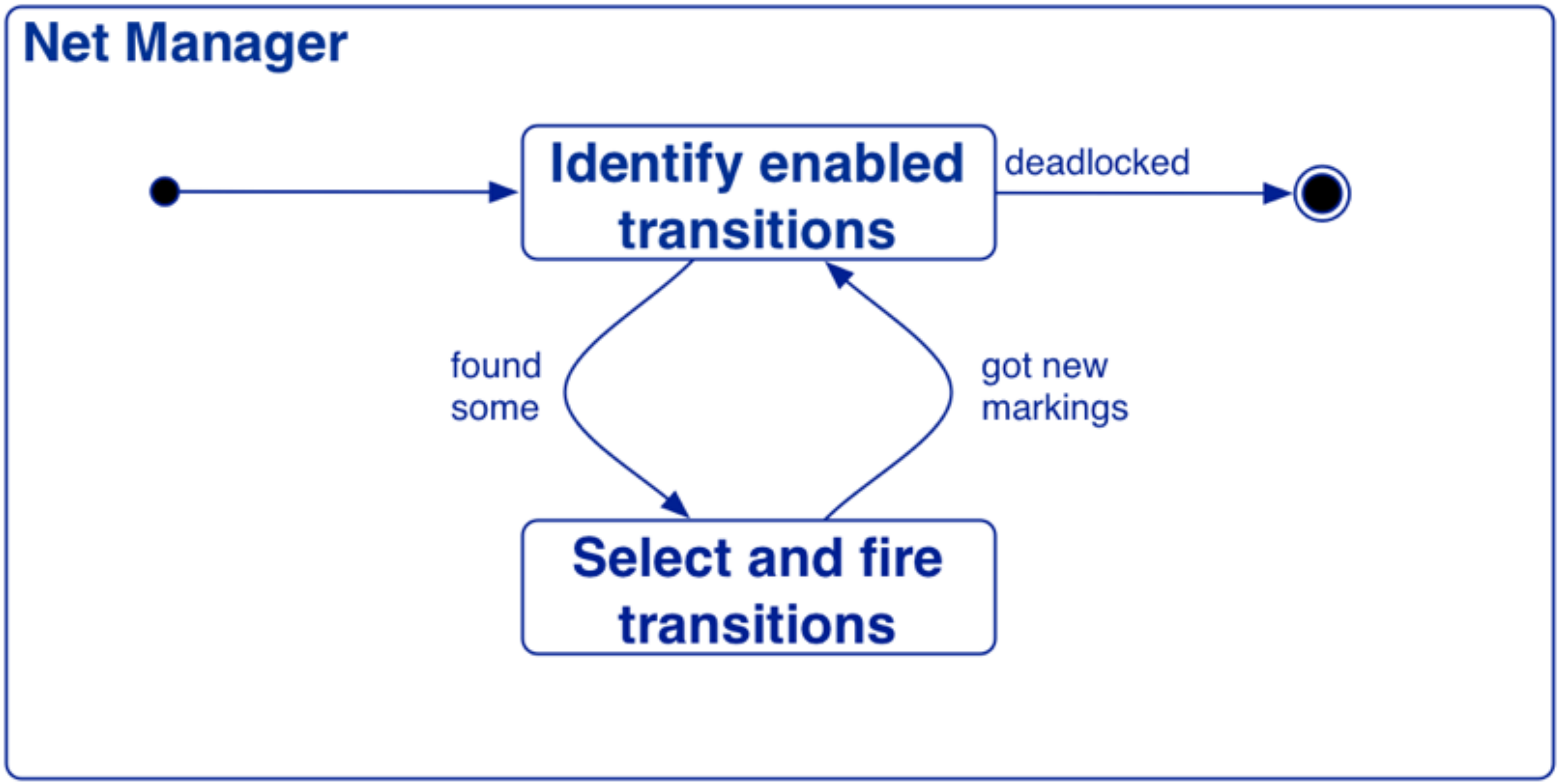
> A single “net manager” monitors the current state of the net, and fires enabled transitions.

Decentralized:

> Transitions are processes, places are shared resources, and transitions compete to obtain tokens.

Centralized schemes

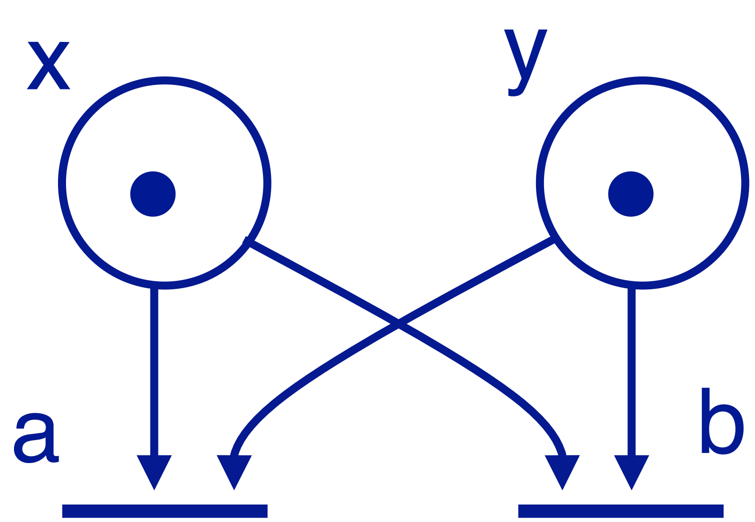
In one possible centralized scheme, the Manager selects and fires enabled transitions.

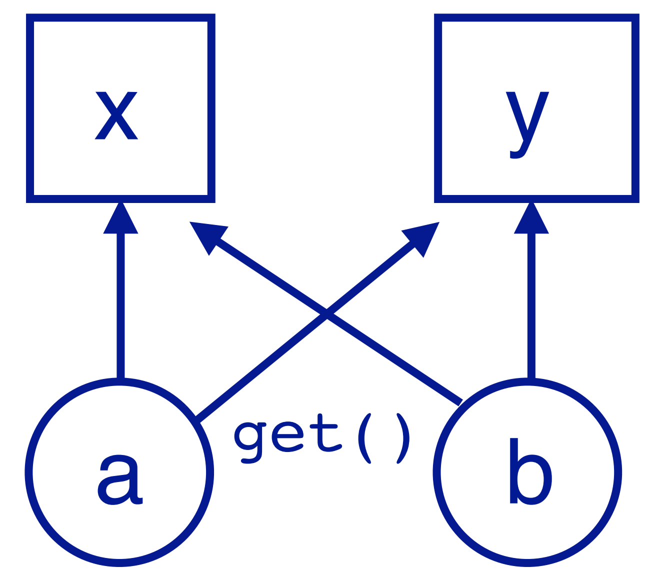


Concurrently enabled transitions can be fired in parallel.

Decentralized schemes

In decentralized schemes transitions are processes and tokens are resources held by places:

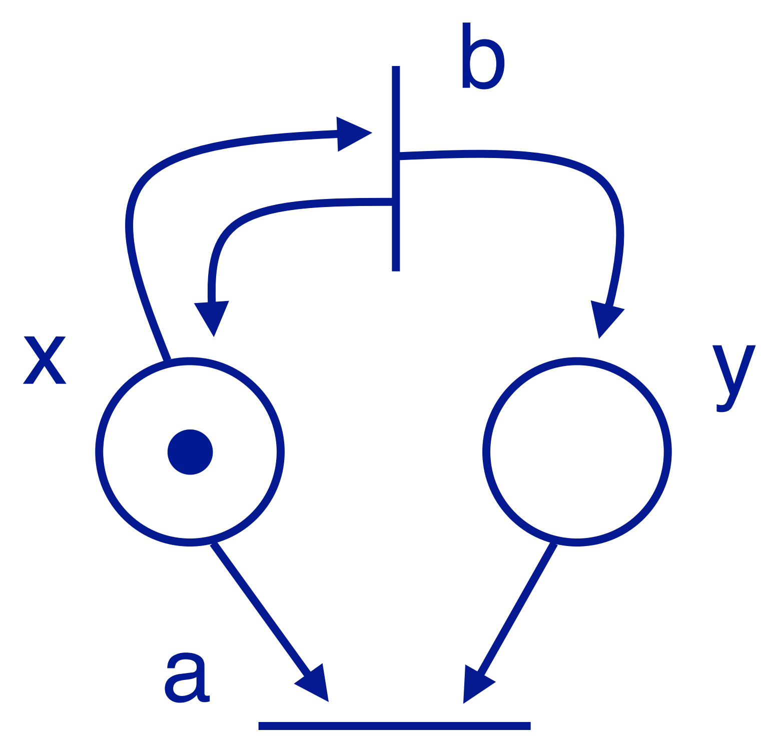




Transitions can be implemented as *thread-per-message gateways* so the same transition can be fired more than once if enough tokens are available.

Transactions

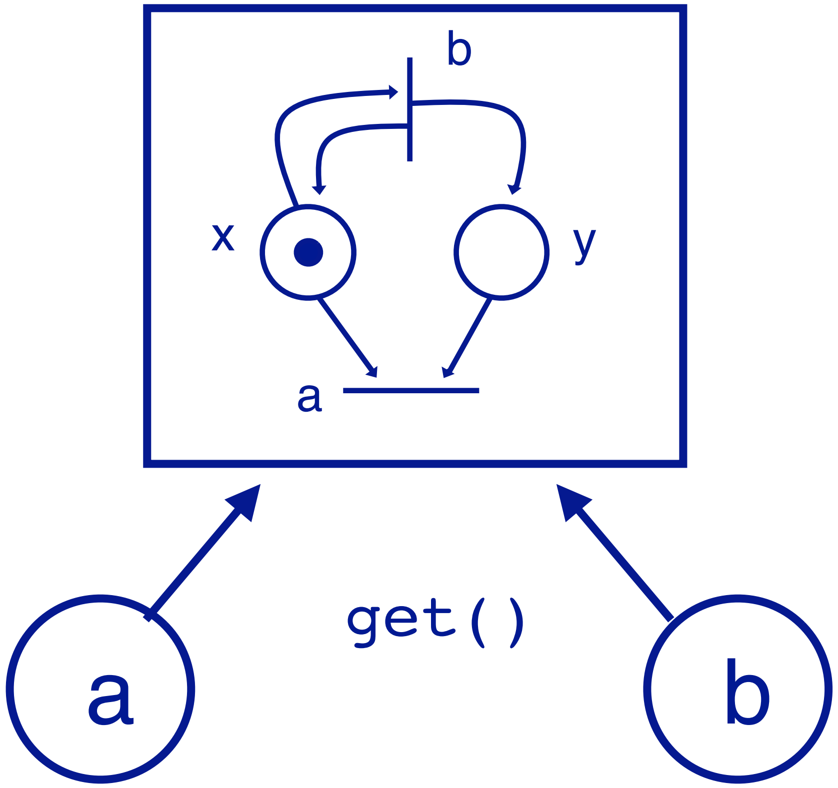
Transitions attempting to fire must grab their input tokens as an atomic transaction, or the net may deadlock even though there are enabled transitions!



If a and b are implemented by independent processes, and x and y by shared resources, this net can deadlock even though b is enabled if a (incorrectly) grabs x and waits for y.

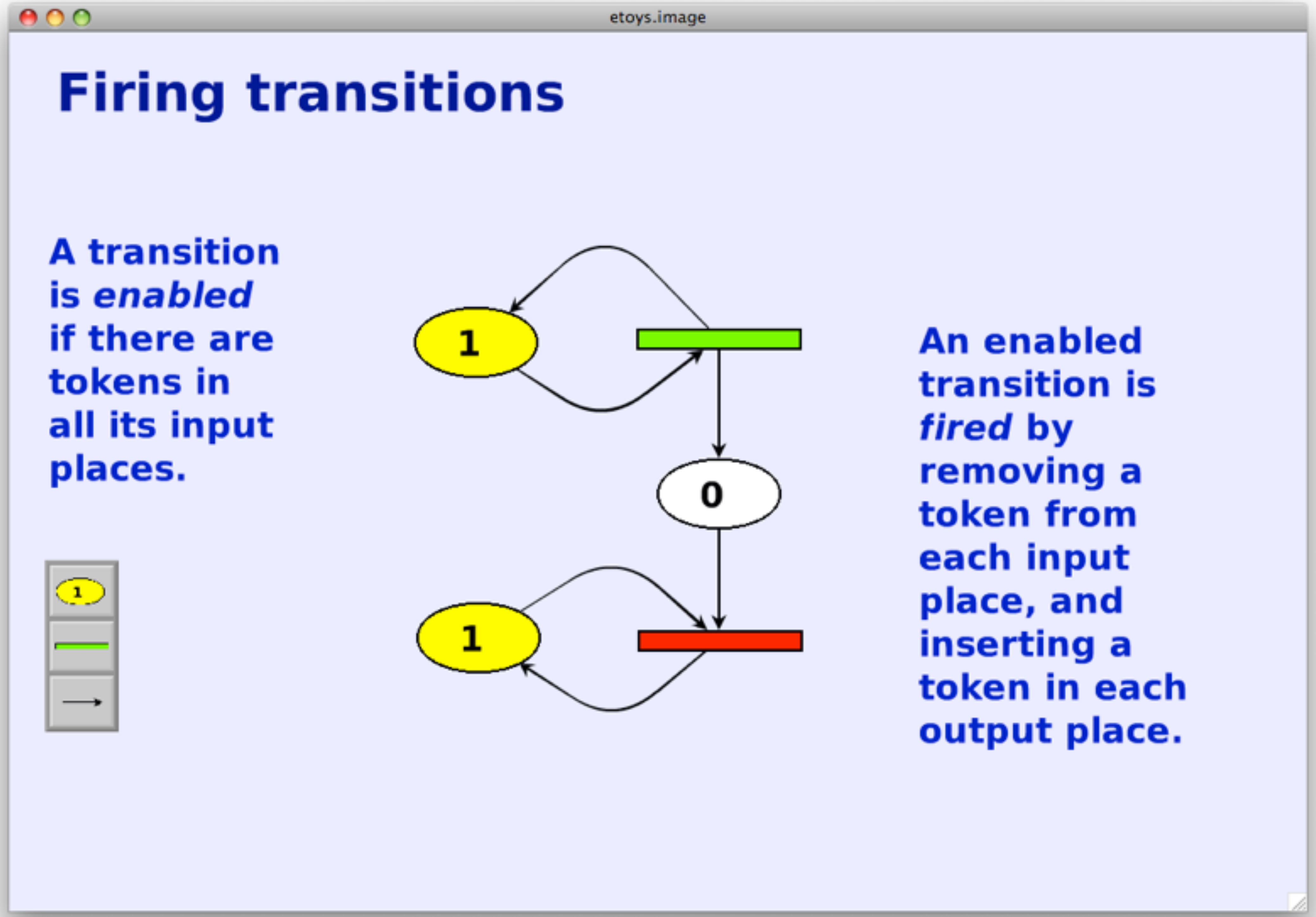
Coordinated interaction

A simple solution is to treat the state of the entire net as a single, shared resource:

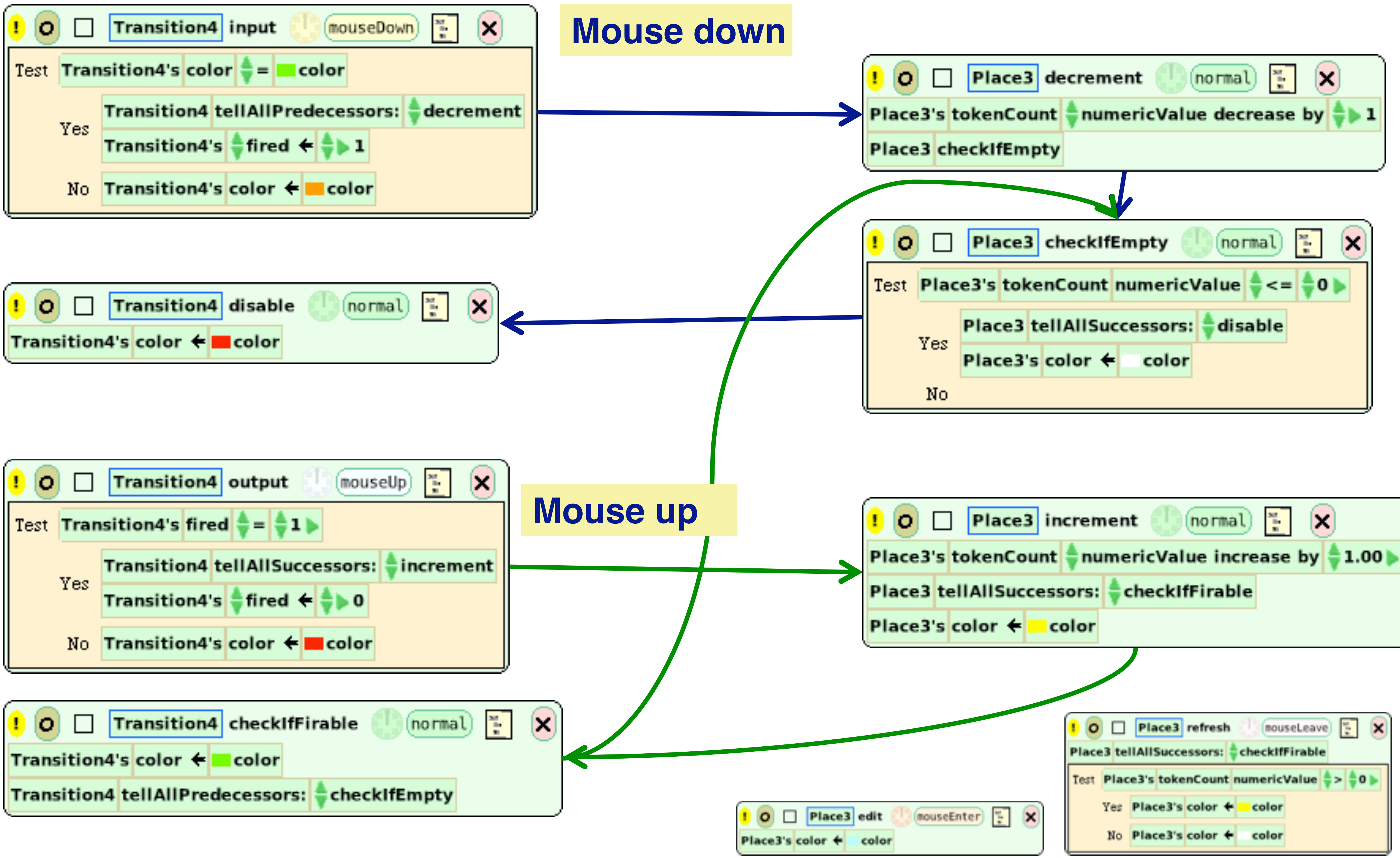


After a transition fires, it notifies waiting transitions.

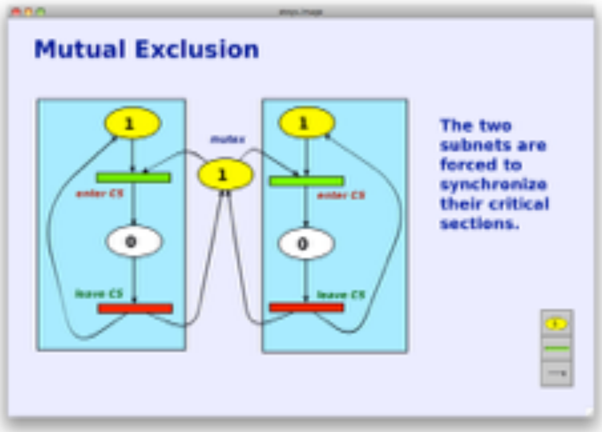
Petit Petri — a Petri Net Editor built with Etoys

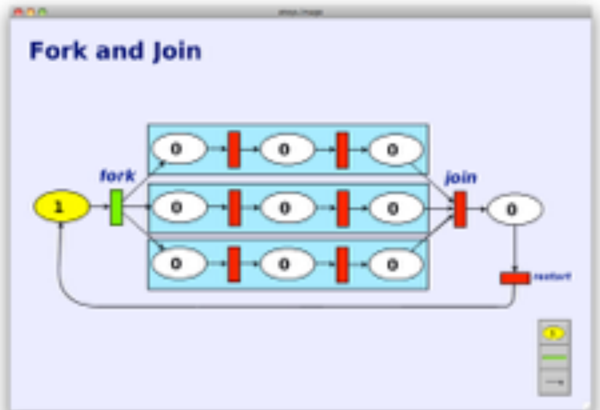


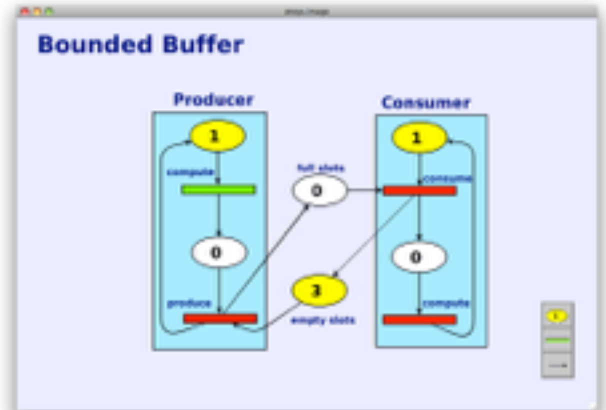
Etoys implementation

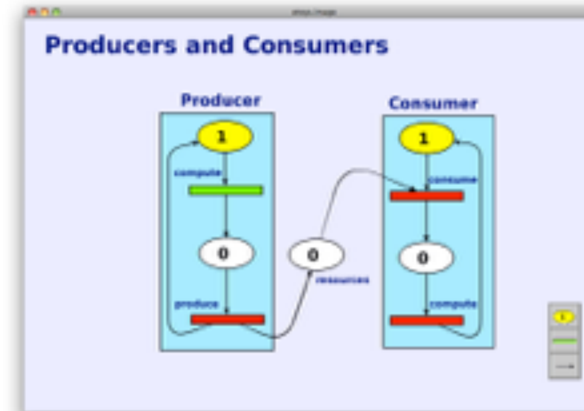


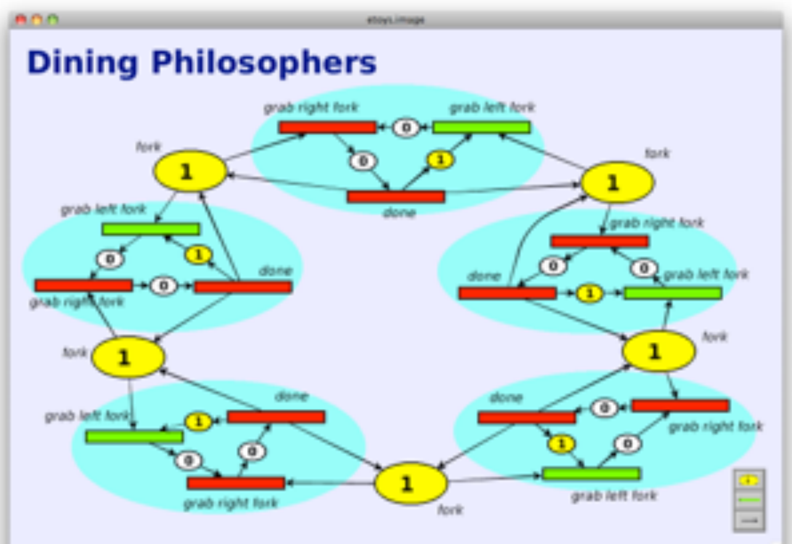
Examples

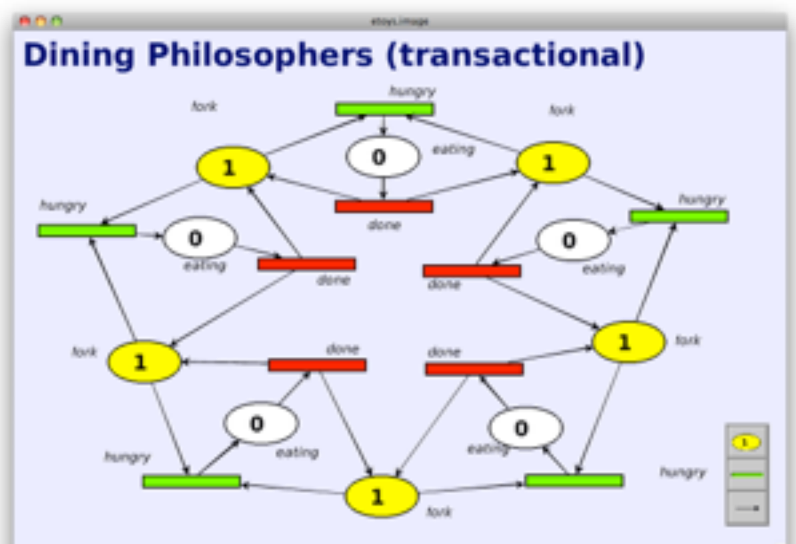

Mutual Exclusion
The two subnets are forced to synchronize their critical sections.

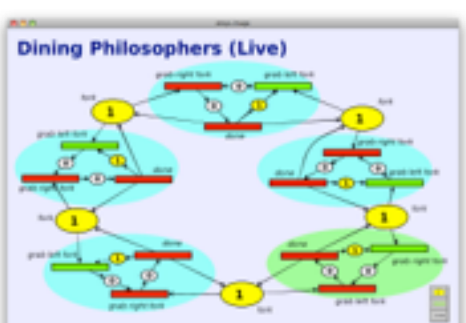

Fork and Join
fork
join

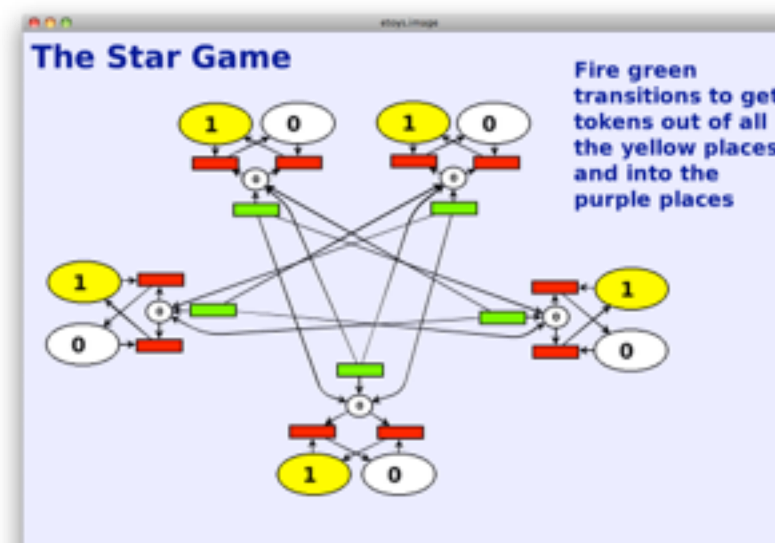

Bounded Buffer
Producer
Consumer


Producers and Consumers
Producer
Consumer


Dining Philosophers


Dining Philosophers (transactional)


Dining Philosophers (Live)


The Star Game
Fire green transitions to get tokens out of all the yellow places and into the purple places

What you should know!

- *How are Petri nets formally specified?*
- *How can nets model concurrency and synchronization?*
- *What is the “reachability set” of a net? How can you compute this set?*
- *What kinds of Petri nets can be modelled by finite state processes?*
- *How can a (bad) implementation of a Petri net deadlock even though there are enabled transitions?*
- *If you implement a Petri net model, why is it a good idea to realize transitions as “thread-per-message gateways”?*

Can you answer these questions?

> *What are some simple conditions for guaranteeing that a net is bounded?*
> *How would you model the Dining Philosophers problem as a Petri net? Is such a net bounded? Is it conservative? Live?*
> *What could you add to Petri nets to make them Turing-complete?*
> *What constraints could you put on a Petri net to make it fair?*