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Homework 1
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1. The points in the representational space $\mathbb{R}^3$, that are the homogeneous coordinates of origin in the physical space $\mathbb{R}^2$, are of the form

$$\begin{pmatrix} 0 \\ 0 \\ k \end{pmatrix}$$

where k is a real number not equal to zero.

2. All the points at the infinity in the physical plane $\mathbb{R}^2$ are not the same. We know the Homogeneous Coordinate representation of an Ideal Point is

$$\begin{pmatrix} u \\ v \\ 0 \end{pmatrix}$$

If u = 1 and v = 1, then the corresponding point in physical space $\mathbb{R}^2$ is at infinity along the line x = y. And if u = 1 and v = -1, then the corresponding point in physical space $\mathbb{R}^2$ is at infinity along the line x = -y.

3. The Homogeneous Coordinates of the degenerate conic with the two intersecting lines being l and m respectively is as follows:

$$C = lm^T + ml^T$$

We know that the rank of a matrix resultant from a vector outer product is 1 as its columns are linearly dependent. Therefore the rank of the above matrix, which is the sum of two vector outer products, cannot exceed 2.

4. The equation of a conic in the physical space $\mathbb{R}^2$ can be given as

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

Case 1:
The conic does not pass through the origin. This implies that $f \neq 0$. Therefore, we can rewrite the conic equation as follows:

$$\tfrac{a}{f}x^2 + \tfrac{b}{f}xy + \tfrac{c}{f}y^2 + \tfrac{d}{f}x + \tfrac{e}{f}y + 1 = 0$$

So there are 5 unknowns ($\frac{a}{f}, \frac{b}{f}, \frac{c}{f}, \frac{d}{f}, \frac{e}{f}$) that we must find in order to define a particular conic.

Case 2:
The conic passes through the origin. This implies that $f = 0$. Therefore we can rewrite the conic equation as follows:

$$ax^2 + bxy + cy^2 + dx + ey = 0$$

So there are 5 unknowns (a, b, c, d, e) that we must find in order to define a particular conic.

As there are 5 unknowns to be found in both the cases, we need 5 linear equations to fully solve for the unknowns. This implies that we need 5 points to define a conic.

5. **Part 1**

The following are the steps to find the intersection point of lines l_1 and l_2 :

(a) We find the Homogeneous Coordinates of the line l_1 passing through the points (0,0) and (1,2) as follows:
l_1 =
$$\begin{pmatrix}0\\0\\1\end{pmatrix} \times \begin{pmatrix}1\\2\\1\end{pmatrix} = \begin{pmatrix}-2\\1\\0\end{pmatrix}$$

(b) And the Homogeneous Coordinates of the line l_2 passing through the points (3,4) and (5,6) are as follows:
l_2 =
$$\begin{pmatrix}3\\4\\1\end{pmatrix} \times \begin{pmatrix}5\\6\\1\end{pmatrix} = \begin{pmatrix}-2\\2\\-2\end{pmatrix}$$

(c) We can find the point of intersection of the lines l_1 and l_2 in Homogeneous Coordinates as follows:
$l_1 \times$l$_2$ =
$$\begin{pmatrix}-2\\1\\0\end{pmatrix} \times \begin{pmatrix}-2\\2\\-2\end{pmatrix} = \begin{pmatrix}-2\\-4\\-2\end{pmatrix}$$

Therefore the point of intersection of the lines l_1 and l_2 in physical space is (1,2).

Part 2
The Homogeneous Coordinates of the new line l_2 passing through the points (7,-8) and (-7,8) can be given as:

l_2 =

$$\begin{pmatrix} 7 \\ -8 \\ 1 \end{pmatrix} \times \begin{pmatrix} -7 \\ 8 \\ 1 \end{pmatrix} = \begin{pmatrix} -16 \\ -14 \\ 0 \end{pmatrix}$$

As the constant value "c" of the lines l_2 and l_1 are both 0, it means that that they both pass through the origin. Therefore, their point of intersection is the (0,0). So as you can see, we need only 2 steps to find the point of intersection in this case.

6. We find the Homogeneous Coordinates of the line l_1 passing through the points (-4,0) and (-2,8) as follows:
   l_1 =

$$\begin{pmatrix} -4 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} -2 \\ 8 \\ 1 \end{pmatrix} = \begin{pmatrix} -8 \\ 2 \\ -32 \end{pmatrix}$$

   And the Homogeneous Coordinates of the line l_2 passing through the points (0,-2) and (4,14) is as follows:
   l_2 =

$$\begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 4 \\ 14 \\ 1 \end{pmatrix} = \begin{pmatrix} -16 \\ 4 \\ 8 \end{pmatrix}$$

   We can find the point of intersection of the lines l_1 and l_2 in Homogeneous Coordinates as follows:

   $l_1 \times$l$_2$ =

$$\begin{pmatrix} -8 \\ 2 \\ 32 \end{pmatrix} \times \begin{pmatrix} -16 \\ 4 \\ 8 \end{pmatrix} = \begin{pmatrix} -112 \\ -448 \\ 0 \end{pmatrix}$$

   As the last element of the above Homogeneous coordinate is zero, it clearly represents an Ideal Point. Therefore the point of intersection of the lines l_1 and l_2 in physical space is at infinity along y = 4x line.

7. The line equation of x=1 can be given as
   l_1 =

$$\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

   And the line equation of y=-1 can be given as
   l_2 =

$$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

   We can find the point of intersection of the lines l_1 and l_2 in Homogeneous Coordinates as follows:

$l_1 \times \mathrm{l}_2$ =

$$\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

Therefore the point of intersection of the lines x=1 and y=-1 in physical space is at (1,-1).

8. The equation of the ellipse centered at (2,3) with a= 1/2 and b = 1, where a and b are the semi-major and semi-minor axis respectively, is as follows:

$$\frac{(x-2)^2}{(1/2)^2} + \frac{(y-3)^2}{(1)^2} = 1$$
$$\frac{(x^2-4x+4)}{(1/4)} + \frac{(y^2-6y+9)}{1} = 1$$
$$4x^2 - 16x + 16 + y^2 - 6y + 9 = 1$$
$$4x^2 + y^2 - 16x - 6y + 24 = 0$$

Using the above equation, which represents the ellipse in the physical space $\mathbb{R}^2$, we get its Homogeneous Coordinate representation as follows:

$$C = \begin{pmatrix} 4 & 0 & -8 \\ 0 & 1 & -3 \\ -8 & -3 & 24 \end{pmatrix}$$

The Homogeneous Coordinates of the origin can be given as:

$$p = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

We can then obtain the Homogeneous Coordinates of the polar line as follows:

$$l = C \times \mathrm{p}$$

$$l = \begin{pmatrix} 4 & 0 & -8 \\ 0 & 1 & -3 \\ -8 & -3 & 24 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$l = \begin{pmatrix} -8 \\ -3 \\ 24 \end{pmatrix}$$

The x-axis passes through the origin and (1,0). So its Homogeneous Coordinates can be obtained as follows:

X =

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Similarly, the y-axis passes through the origin and (0,1). So its Homogeneous Coordinates can be obtained as follows:

Y =

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

Therefore the point of intersection of the polar line and the x-axis in Homogeneous Coordinates is as follows:

$l \times X$ =

$$\begin{pmatrix} -8 \\ -3 \\ 24 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -24 \\ 0 \\ -8 \end{pmatrix}$$

And the point of intersection of the polar line and the y-axis in Homogeneous Coordinates is as follows:

$l \times Y$ =

$$\begin{pmatrix} -8 \\ -3 \\ 24 \end{pmatrix} \times \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -24 \\ -3 \end{pmatrix}$$

Hence the x intercept of the polar line is (3,0) and the y intercept is (0,8) in the physical space $\mathbb{R}^2$.