Walking Gaits for the NUgus Platform

… from MCHA 4100 to idealised walking strategies

Darcy Byrne

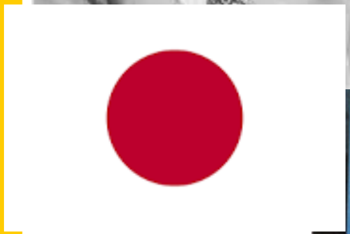
C3256634

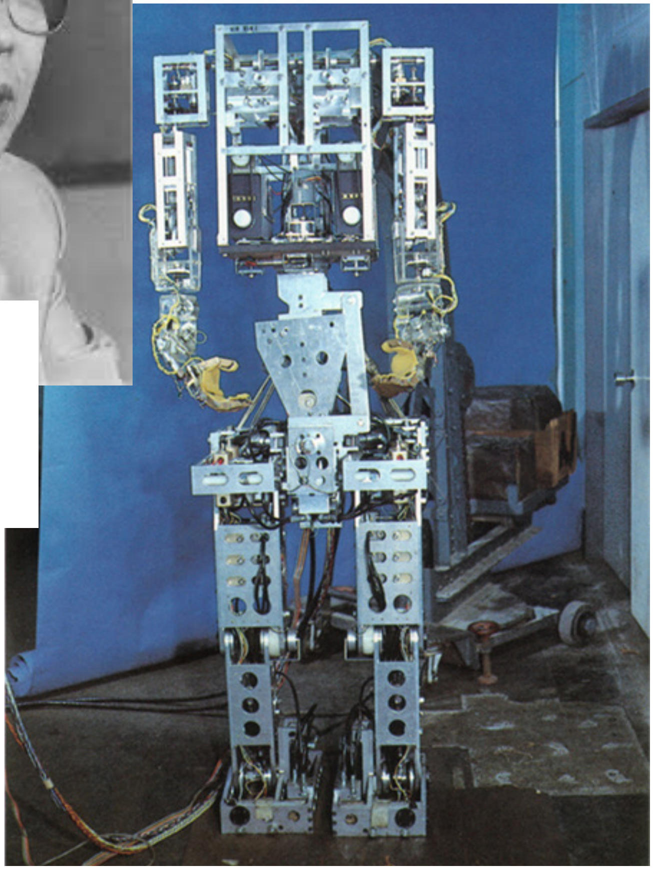
THE UNIVERSITY OF NEWCASTLE AUSTRALIA

FACULTY OF ENGINEERING AND BUILT ENVIRONMENT

www.newcastle.edu.au

History





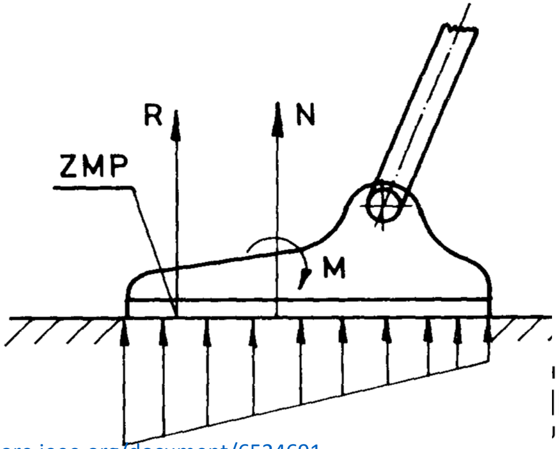
THE WABOT-1 (1973), **Ichirō Katō**

"Quasi-Static" Locomotion

Mathematical Models of General Anthropomorphic Systems (1973),

Miomir Vukobratović

"ZMP" Locomotion



https://link.springer.com/article/10.1007/BF00735435?noAccess=true

https://www.humanoid.waseda.ac.jp/booklet/kato_2.html https://ieeexplore.ieee.org/document/6524691

Mathematical Background

- Vectors

  Direction & Magnitude, with respect to a Coordinate System

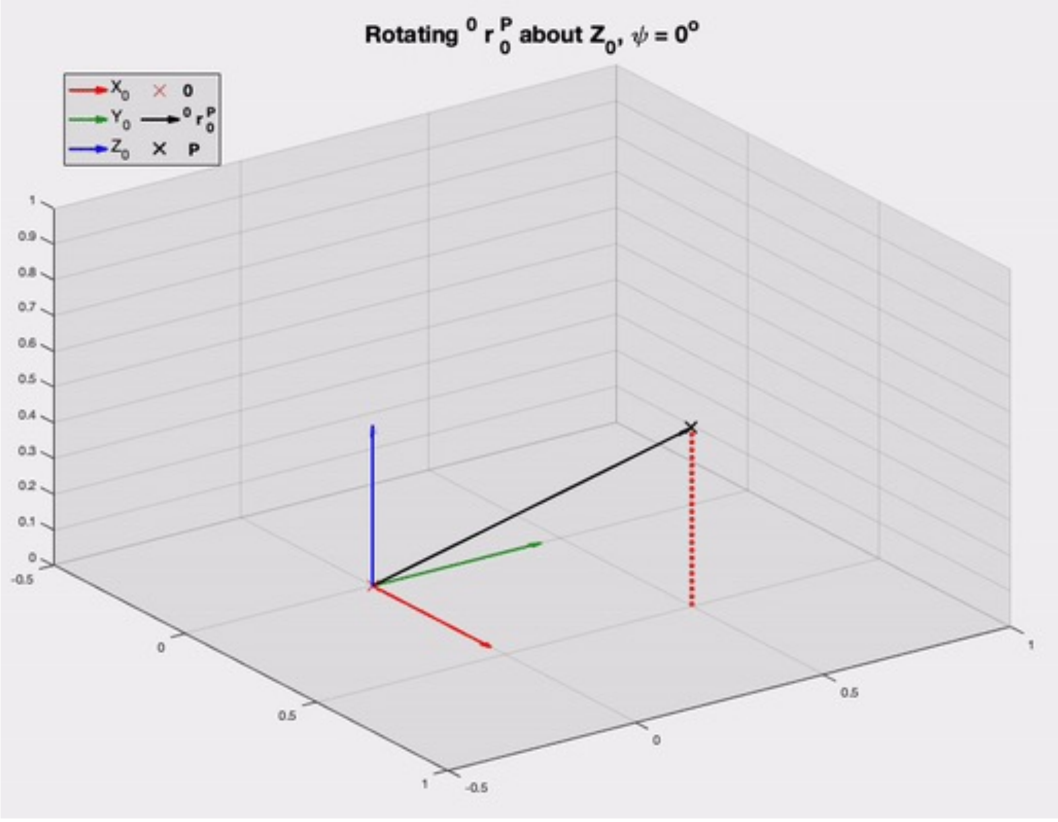
- Rotations (Passive & Active)

  Coordinate transformers

$$\boldsymbol{r}_P^0 = [0.5 \quad 0.5 \quad 0.5]^T$$

$$\boldsymbol{R}_{\boldsymbol{Z_0}}(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{R}_{\boldsymbol{Z_0}}^0 \boldsymbol{r}_P^0 \implies$$



Mathematical Background – Cont'd…

- Canonical Homogenous Transforms

$$\boldsymbol{A}_{\boldsymbol{b}}^{\boldsymbol{a}}(q) = \begin{bmatrix} \boldsymbol{R}_{\boldsymbol{b}}^{\boldsymbol{a}}(q) & \boldsymbol{r}_{\boldsymbol{b}}^{\boldsymbol{a}}(q) \\ \mathbf{0} & 1 \end{bmatrix} = \boldsymbol{A}_{\boldsymbol{a}}^{\boldsymbol{b}}(q)^{-\mathbf{1}}$$

- Kinematic Chains

$$\boldsymbol{T}_{\boldsymbol{n}}^{\mathbf{0}}(\boldsymbol{q}) = \boldsymbol{A}_{\mathbf{1}}^{\mathbf{0}}(q_1)\boldsymbol{A}_{\mathbf{2}}^{\mathbf{1}}(q_2)\ldots\boldsymbol{A}_{\boldsymbol{n}}^{\boldsymbol{n}-\mathbf{1}}(q_n) = \begin{bmatrix} \boldsymbol{R}_{\boldsymbol{n}}^{\mathbf{0}}(\boldsymbol{q}) & \boldsymbol{r}_{\boldsymbol{n}}^{\mathbf{0}}(\boldsymbol{q}) \\ \mathbf{0} & 1 \end{bmatrix}$$

… where $\boldsymbol{q}$ is an array of 'Joint Variables'

$$\boldsymbol{T}_{\boldsymbol{n}}^{\mathbf{0}}(\boldsymbol{q}) \Longrightarrow \textbf{\textit{Forward Kinematic Model}} \Longrightarrow \boldsymbol{k}(\boldsymbol{q})$$

Mathematical Background – Cont'd…

- Forward Kinematic Model (***FKM***)
  - Returns the End Effector Position, $\boldsymbol{r}_{\boldsymbol{x}_e}^{0}$, with respect to the **ZERO coordinate system**
  - Returns End Effector Orientation, $\boldsymbol{\Theta}_{\boldsymbol{x}_e}^{0}$, with respect to the **ZERO coordinate system**
    - However, this first involves the parameterisation of the Rotation Matrix $\boldsymbol{R}_{\boldsymbol{x}_e}^{0}(\boldsymbol{q})$ into Euler Angles or Unit Quaternions
    - **' Google it, mate. '**

- Comrade Adam Bandt, 2022

$$\boldsymbol{x}_e = \begin{bmatrix} \boldsymbol{r}_{\boldsymbol{x}_e}^{0} \\ \boldsymbol{\Theta}_{\boldsymbol{x}_e}^{0} \end{bmatrix} = \boldsymbol{k}(\boldsymbol{q}) = [\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{\varphi}, \boldsymbol{\theta}, \boldsymbol{\psi}]^{\boldsymbol{T}}$$

Mathematical Background – Cont'd…

- Inverse Kinematics (***IKM***)
  - Returns the array of Joint Variables necessary to achieve a desired End Effector Pose, $\boldsymbol{x_e}$
  - Usually solved with Numerical Optimisation

- Parallel Kinematics

Mathematical Background – Cont'd…

- Numerical Optimisation

  General Solution to the **IKM** problem:

$$\boldsymbol{q}^{\star} = \arg\min_{\boldsymbol{q}} \ \boldsymbol{K_q} \|\boldsymbol{q_0} - \boldsymbol{q}\| + \boldsymbol{K_e} \|\boldsymbol{k}(\boldsymbol{q}) - \boldsymbol{x_e^{\star}}\|$$

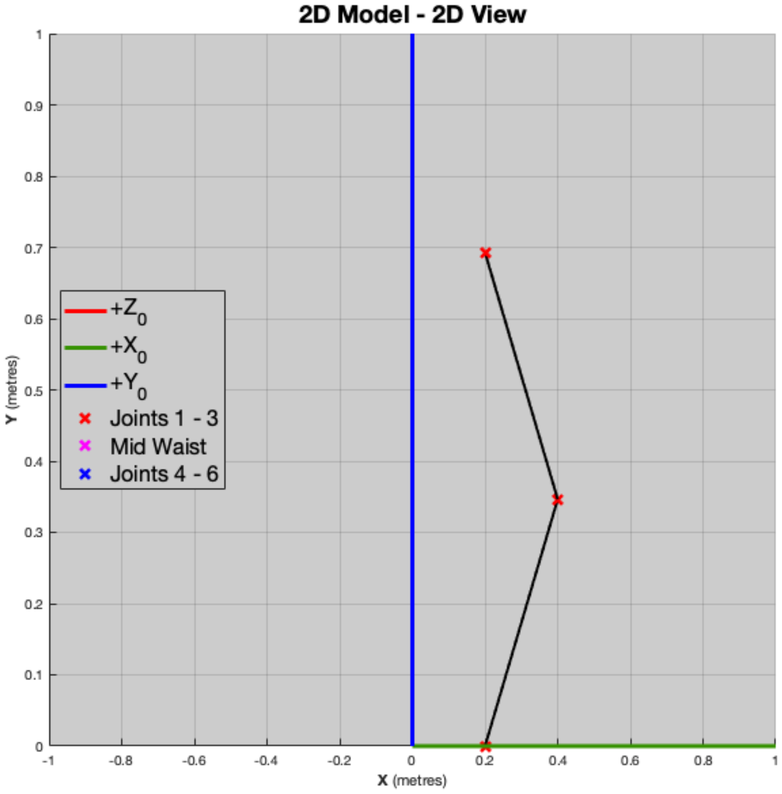
... which is to say;

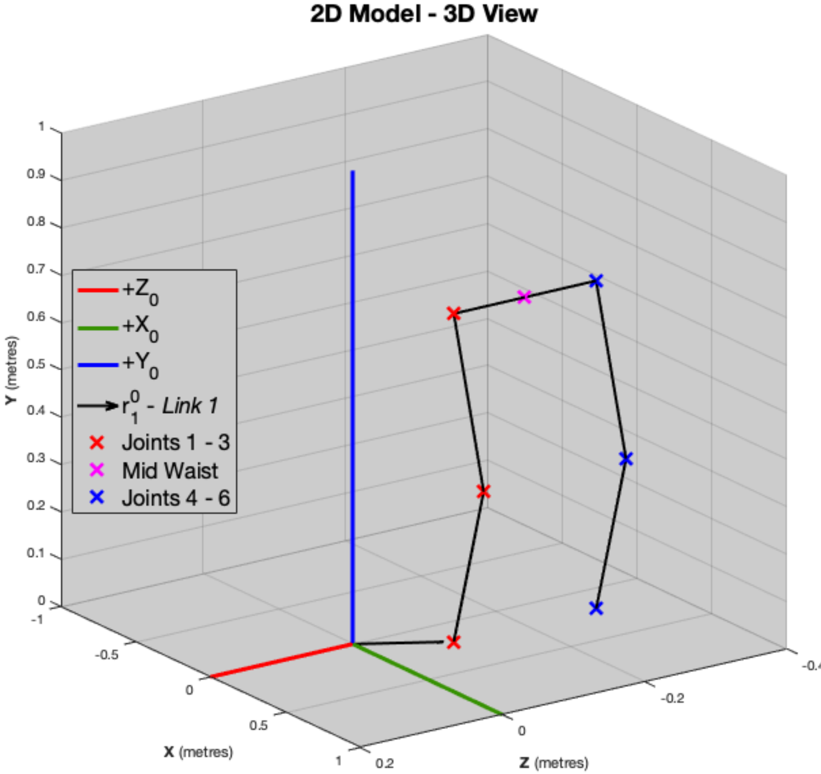
the optimal array of **State Variables**, $\boldsymbol{q}^{\star}$,

that satisfy the desired **End Effector Pose**, $\boldsymbol{x_e^{\star}}$,

can be found by minimising the above function.

2D Model





$$T_6^1 = A_2^1 A_3^2 A_4^3 A_5^4 A_6^5$$

$$T_1^6 = (T_6^1)^{-1}$$

$$T_1^{Global} = A_1^0 T_6^1$$
$$T_6^{Global} = A_6^0 T_1^6$$

$$q = [\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6]^T$$

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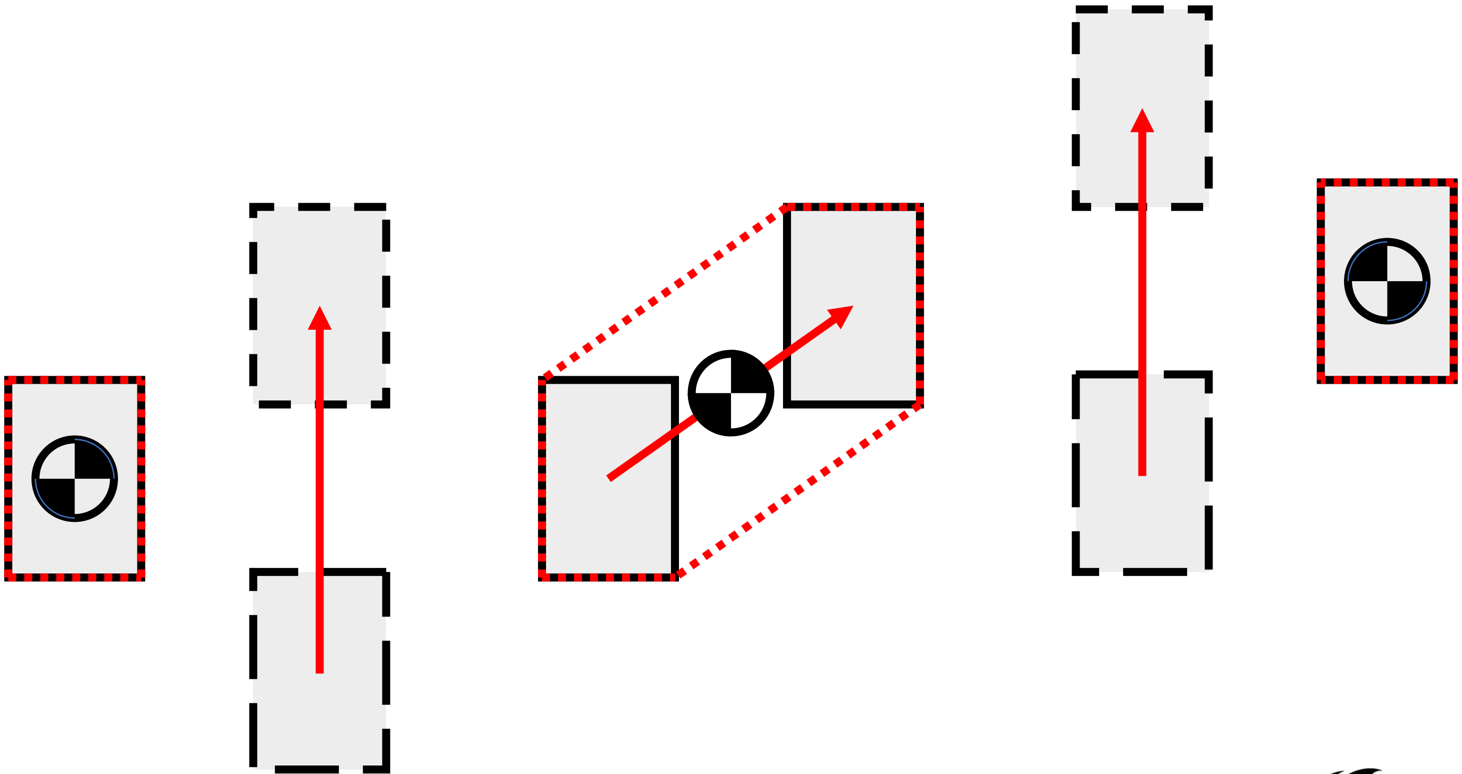
Quasi Static

$$r_{CoM}^{0} = \frac{1}{\sum Mass} \sum_{i=1}^{n} m_i r_i^0$$



“Support Polygon”

Quasi Static – Cont'd…

Single Support *IKM*:

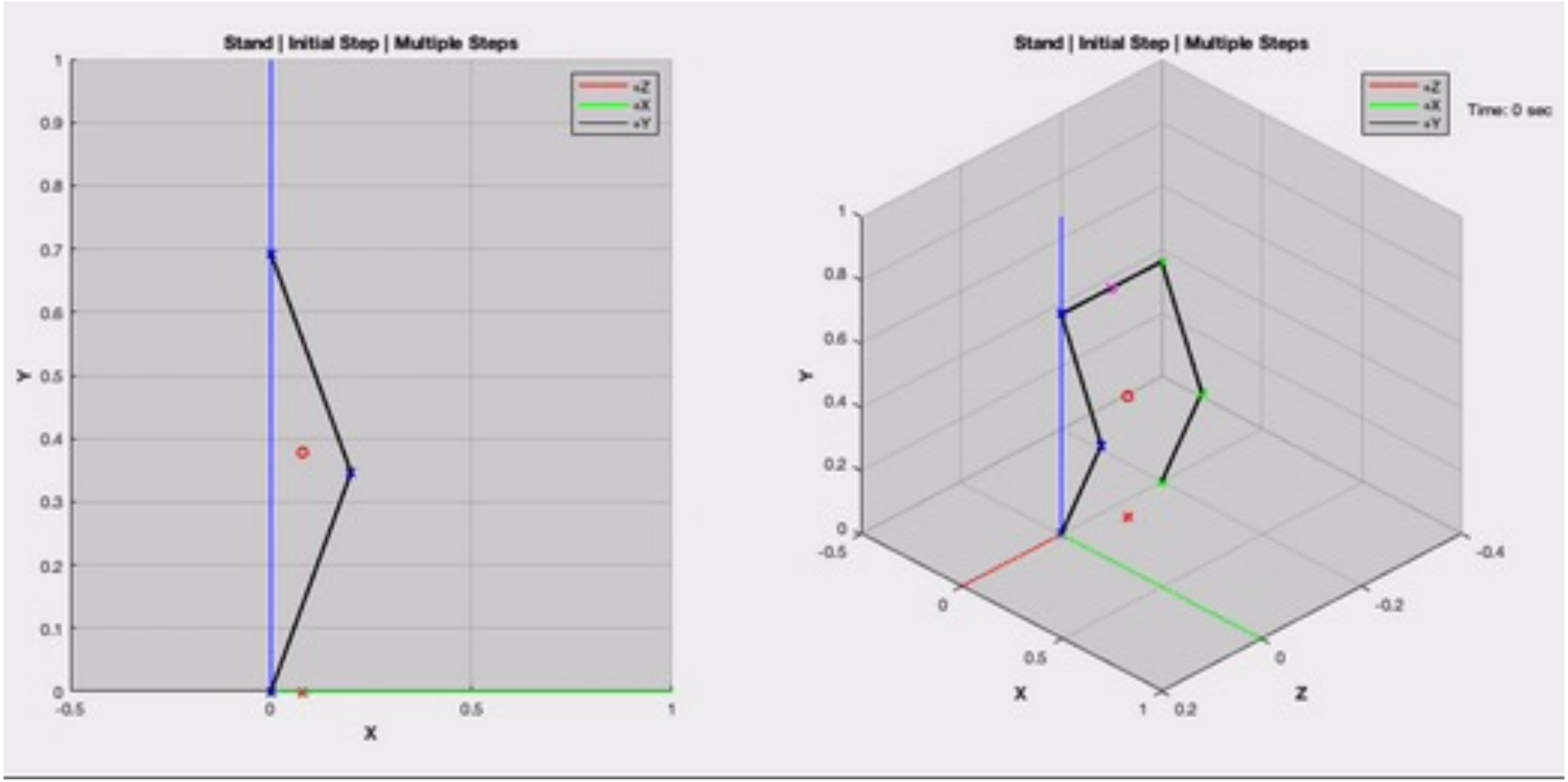
$$\mathbf{q}^* = \arg\min_{\mathbf{q}} \; \mathbf{K}_q\|\mathbf{q_0} - \mathbf{q}\| \; + \; \mathbf{K}_e\|k(\mathbf{q}) - \mathbf{x}_e\| \; + \; \mathbf{K}_{CoM}\|\mathbf{r}^0_{CoM} - \mathbf{r}^0_{\text{Fixed}}\|$$

Double Support *IKM*:

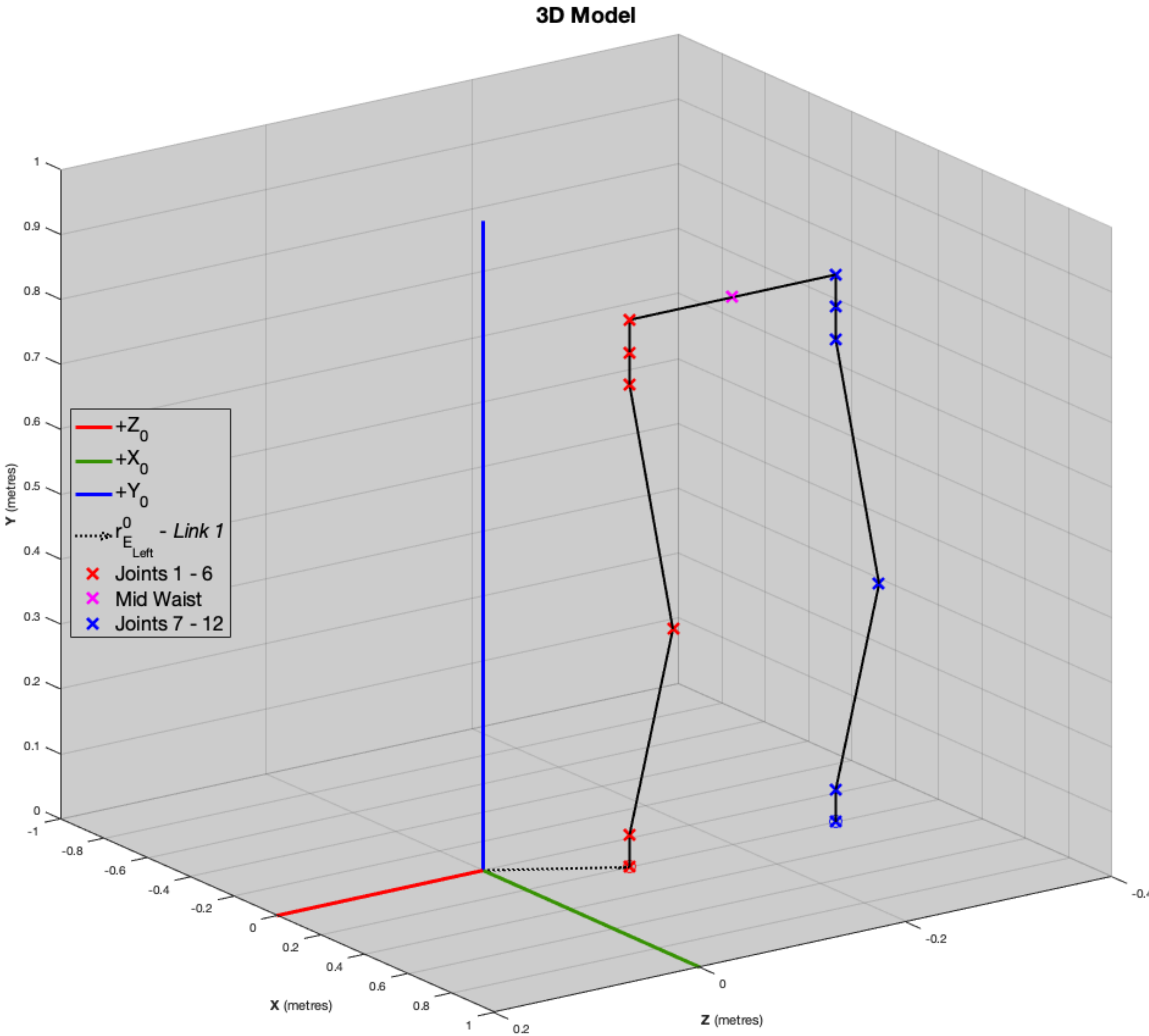
$$\mathbf{q}^* = \arg\min_{\mathbf{q}} \; \mathbf{K}_q\|\mathbf{q_0} - \mathbf{q}\| \; + \; \mathbf{K}_e\|k(\mathbf{q})\| \; + \; \mathbf{K}_{CoM}\|\mathbf{r}^0_{CoM} - \mathbf{r}^0_{\text{Forward foot}}\|$$

1. Single Support
   1. Generate Step Trajectory
   2. Recursively Solve **IKM** for each point along trajectory
2. Double Support
   1. Generate ***CoM*** Trajectory
   2. Recursively Solve **IKM** for each point along trajectory

2D Model Walking



3D Model



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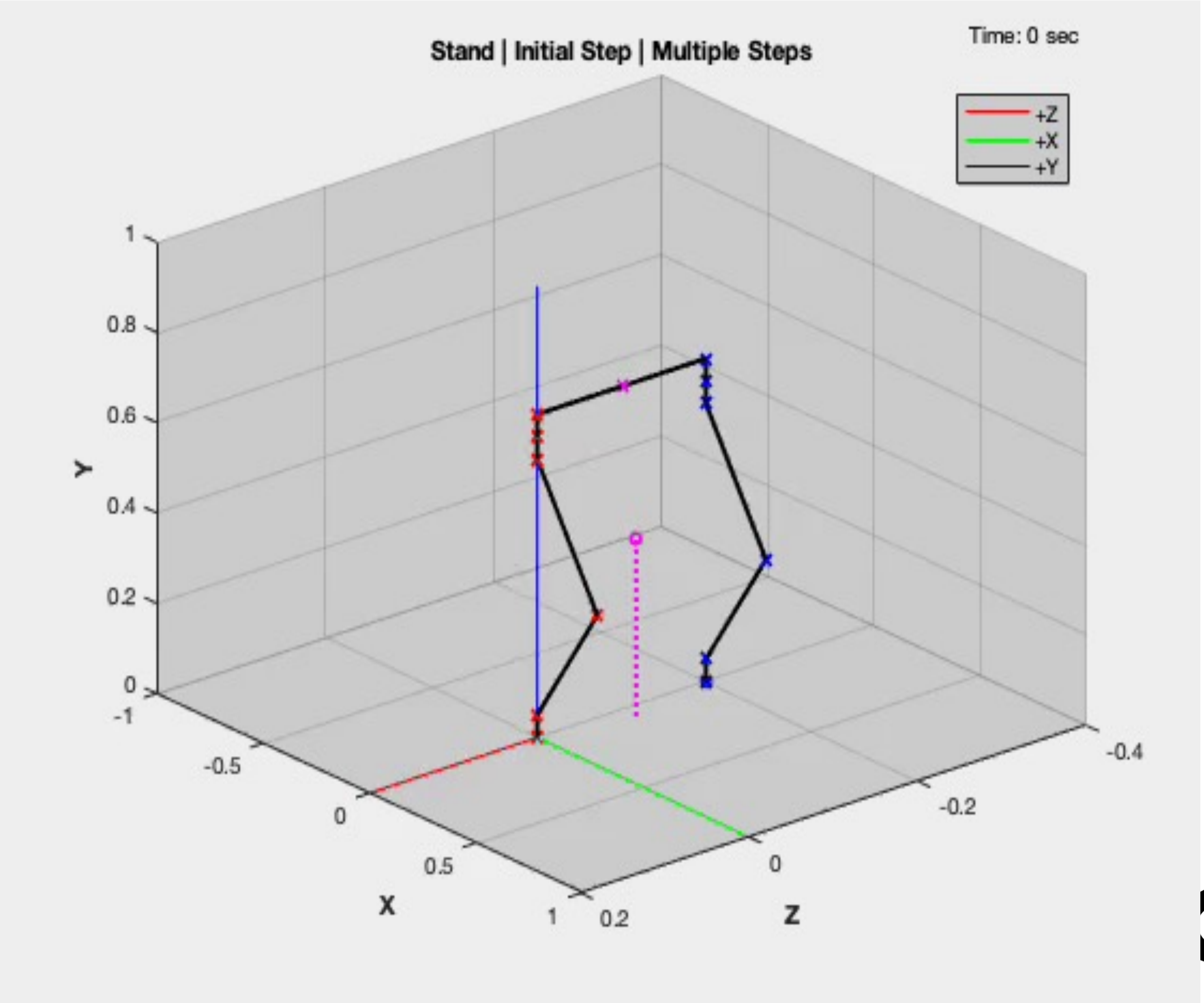
3D Model Walking – Quasi Static



Zero Moment Point

Supposed, while standing on one leg, that **YOU** are an **inverted pendulum** …

Your entire degree has led to this moment …

You begin to fall …

But at the last moment you place your foot down, shifting all your **mass** to it …

You're now walking using **ZMP**

Zero Moment Point – Cont'd

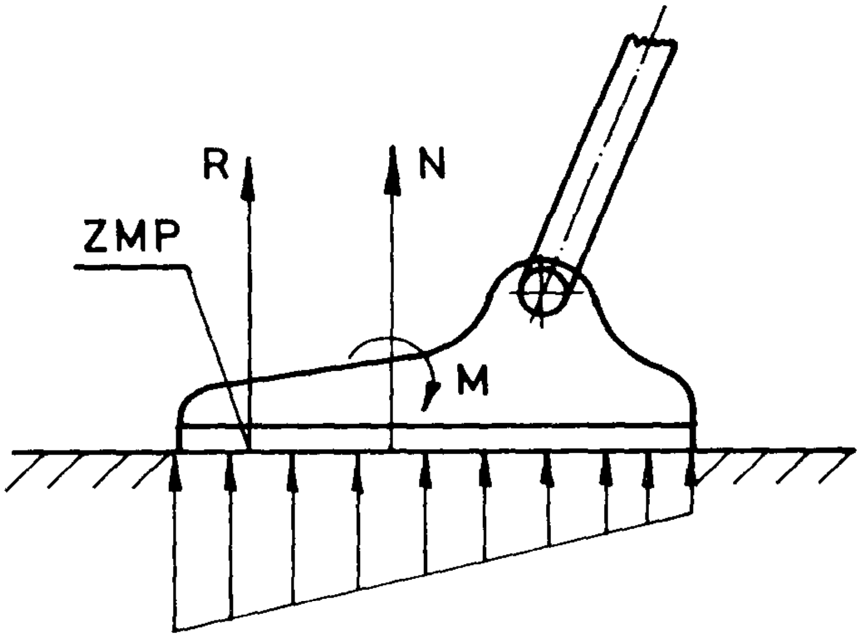


FIG. 3. Load distribution by foot longitudinal section.

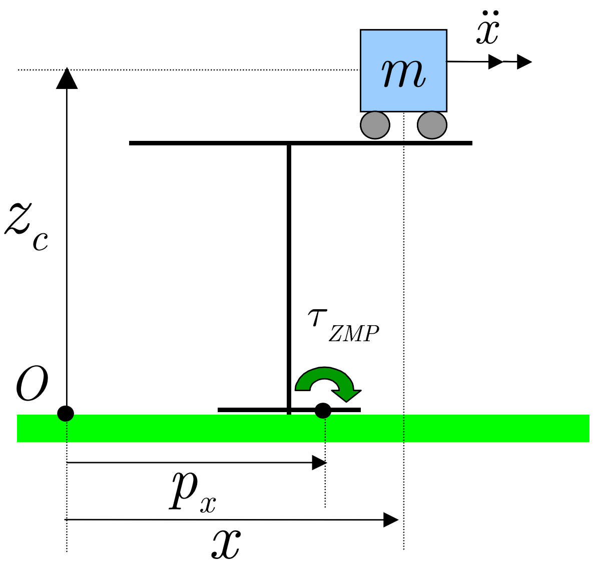


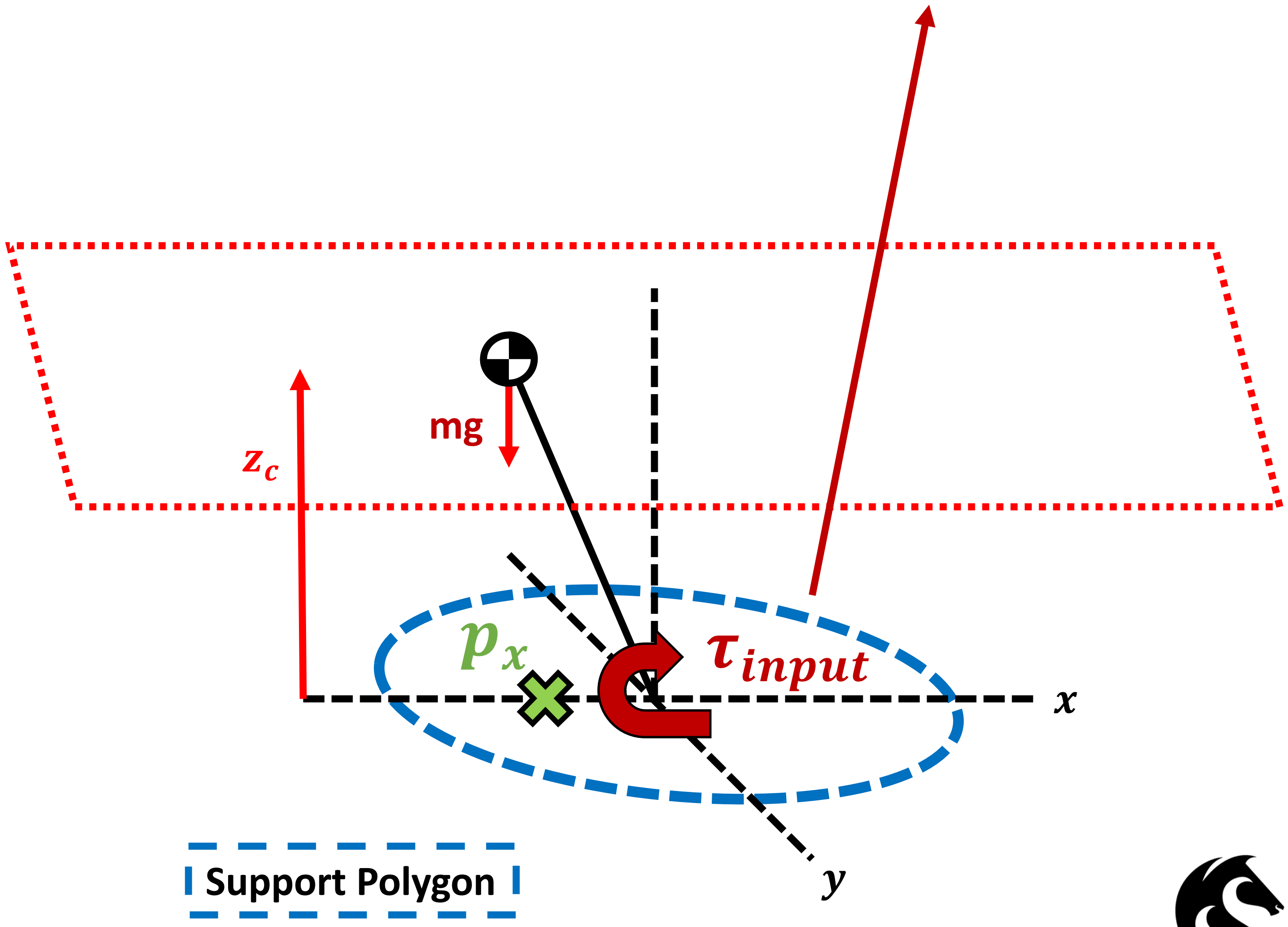
Figure 3: A cart-table model

Mathematical Model of General Anthropomorphic Systems
- Vukobratović, et al. 1973

Biped Walking Pattern Generation by using Preview Control of Zero-Moment Point
- Kajita, et al. 2003

Zero Moment Point – Cont'd

$$\tau_{ZMP} = mg(x - p_x) - m\ddot{x}z_c = 0$$



Zero Moment Point – Cont'd

$$\frac{\delta}{\delta t}\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_{(x,y)}$$

$$\mathbf{p}_{(x,y)} = \begin{bmatrix} 1 & 0 & \frac{-z_c}{g} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix}$$

$$\mathbf{x}_{(k+1)} = \mathbf{A_d}\mathbf{x}_k + \mathbf{B_d}\mathbf{u}_k$$

$$\mathbf{p}_k = \mathbf{C_d}\mathbf{x}_k$$

... where:

$$\mathbf{A_d} = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{B_d} = \begin{bmatrix} \frac{T^3}{6} \\ \frac{T^2}{2} \\ T \end{bmatrix} \quad \mathbf{C_d} = \begin{bmatrix} 1 & 0 & \frac{-z_c}{g} \end{bmatrix}$$

Zero Moment Point – Cont'd

$$J_u = \sum_{i=k}^{\infty} \mathbf{Q}_e \|\mathbf{y}(k) - \mathbf{y_d}(k)\| \; + \; \mathbf{Q}_x \|\mathbf{x}(k) - \mathbf{x}(k-1)\| \; + \; \mathbf{R}\|\mathbf{u}(k) - \mathbf{u}(k-1)\|$$

$$\mathbf{u}^*(k) = \; -\mathbf{G_e} \sum_{i=0}^{k} [\mathbf{y}(k) - \mathbf{y_d}(k)] \; - \mathbf{G_x}\mathbf{x}(k) \; - \sum_{l=1}^{N_l} \mathbf{G_d}(l)\mathbf{y_d}(k+l)$$

$$\tilde{\mathbf{B}} = \begin{bmatrix} \mathbf{C_d}\mathbf{B_d} \\ \mathbf{B_d} \end{bmatrix}, \; \tilde{\mathbf{F}} = \begin{bmatrix} \mathbf{C_d}\mathbf{A_d} \\ \mathbf{A_d} \end{bmatrix}, \; \tilde{\mathbf{Q}} = \begin{bmatrix} \mathbf{Q_e} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q_x} \end{bmatrix}, \; \tilde{\mathbf{I}} = \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix}, \; \tilde{\mathbf{A}} = [\tilde{\mathbf{I}} \quad \tilde{\mathbf{F}}]$$

... where the gains, $\mathbf{G_e}$, $\mathbf{G_x}$ and $\mathbf{G_d}(l)$ are given by;

$$\mathbf{G_e} = [\mathbf{R} + \tilde{\mathbf{B}}^\intercal \tilde{\mathbf{K}}_\mathbf{d} \tilde{\mathbf{B}}]^{-1} \tilde{\mathbf{B}}^\intercal \tilde{\mathbf{K}}_\mathbf{d} \tilde{\mathbf{I}}$$

$$\mathbf{G_x} = [\mathbf{R} + \tilde{\mathbf{B}}^\intercal \tilde{\mathbf{K}}_\mathbf{d} \tilde{\mathbf{B}}]^{-1} \tilde{\mathbf{B}}^\intercal \tilde{\mathbf{K}}_\mathbf{d} \tilde{\mathbf{F}}$$

$$\mathbf{G_d}(l) = -[\mathbf{R} + \tilde{\mathbf{B}}^\intercal \tilde{\mathbf{K}}_\mathbf{d} \tilde{\mathbf{B}}]^{-1} \tilde{\mathbf{B}}^\intercal (\tilde{\mathbf{A}}_\mathbf{c}^\intercal)^{(l-1)} \tilde{\mathbf{K}}_\mathbf{d} \tilde{\mathbf{I}}$$

... where;

$$\tilde{\mathbf{A}}_\mathbf{c} = \tilde{\mathbf{A}} - \tilde{\mathbf{B}}[\mathbf{R} + \tilde{\mathbf{B}}^\intercal \tilde{\mathbf{K}}_\mathbf{d} \tilde{\mathbf{B}}]^{-1} \tilde{\mathbf{B}}^\intercal \tilde{\mathbf{K}}_\mathbf{d} \tilde{\mathbf{A}}$$

... and $\tilde{\mathbf{K}}_\mathbf{d}$ is a solution to the Discrete Time Algebraic Riccati Equation.

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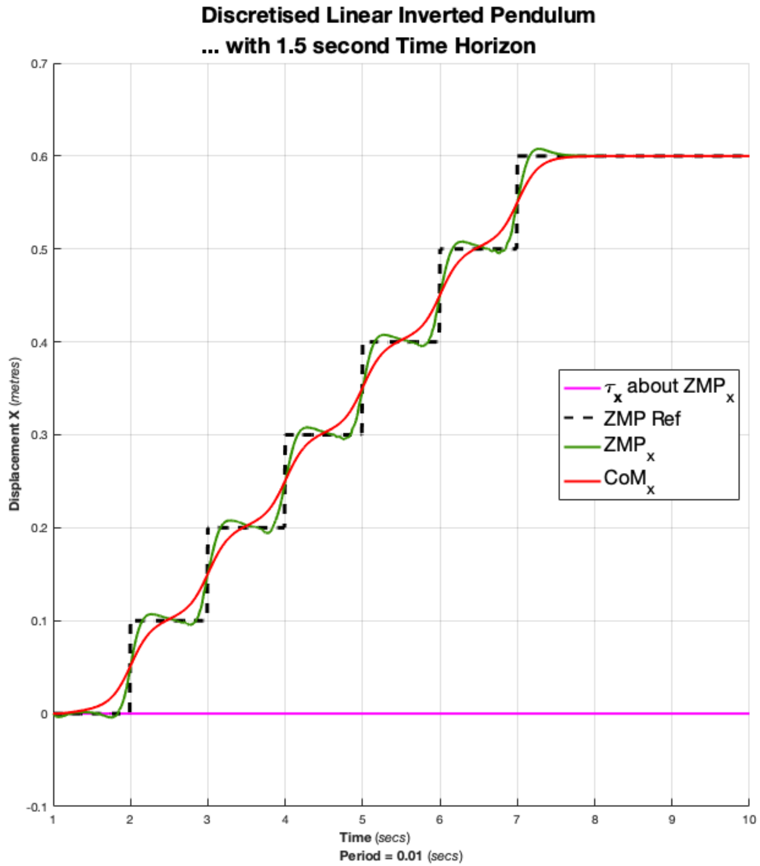
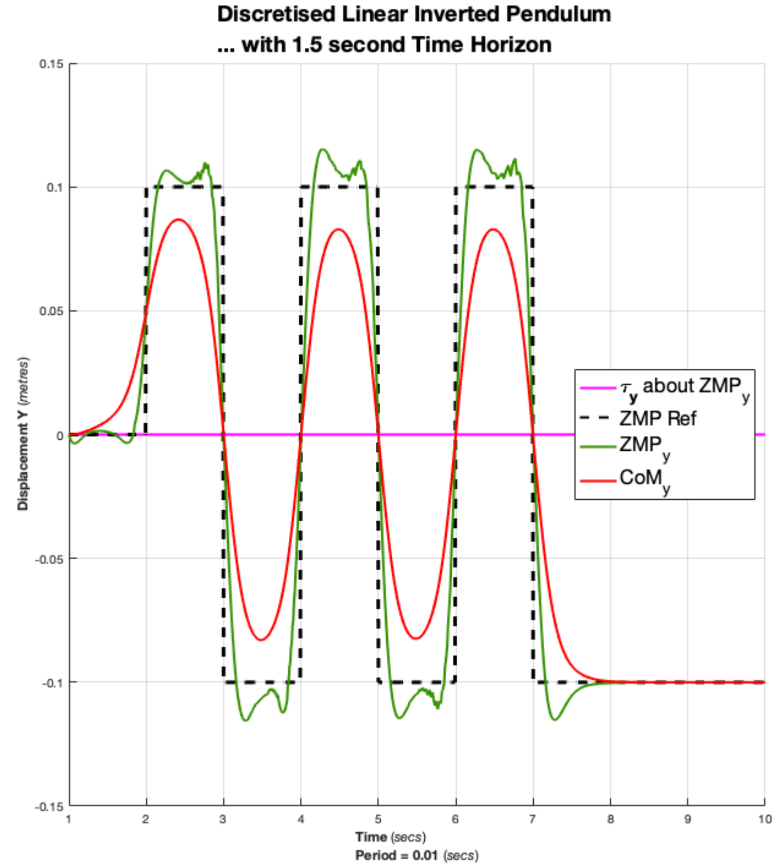
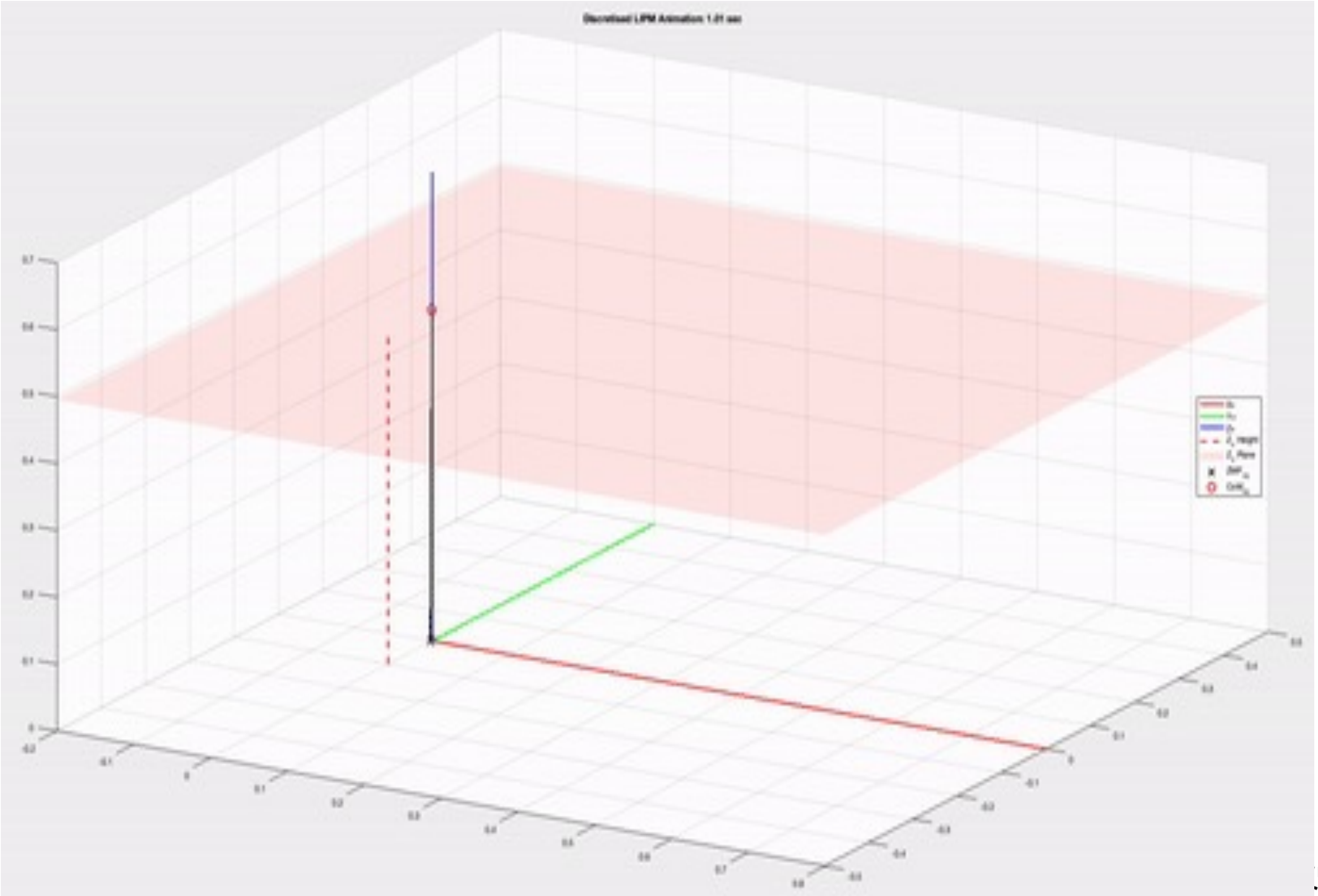
Zero Moment Point – Cont'd

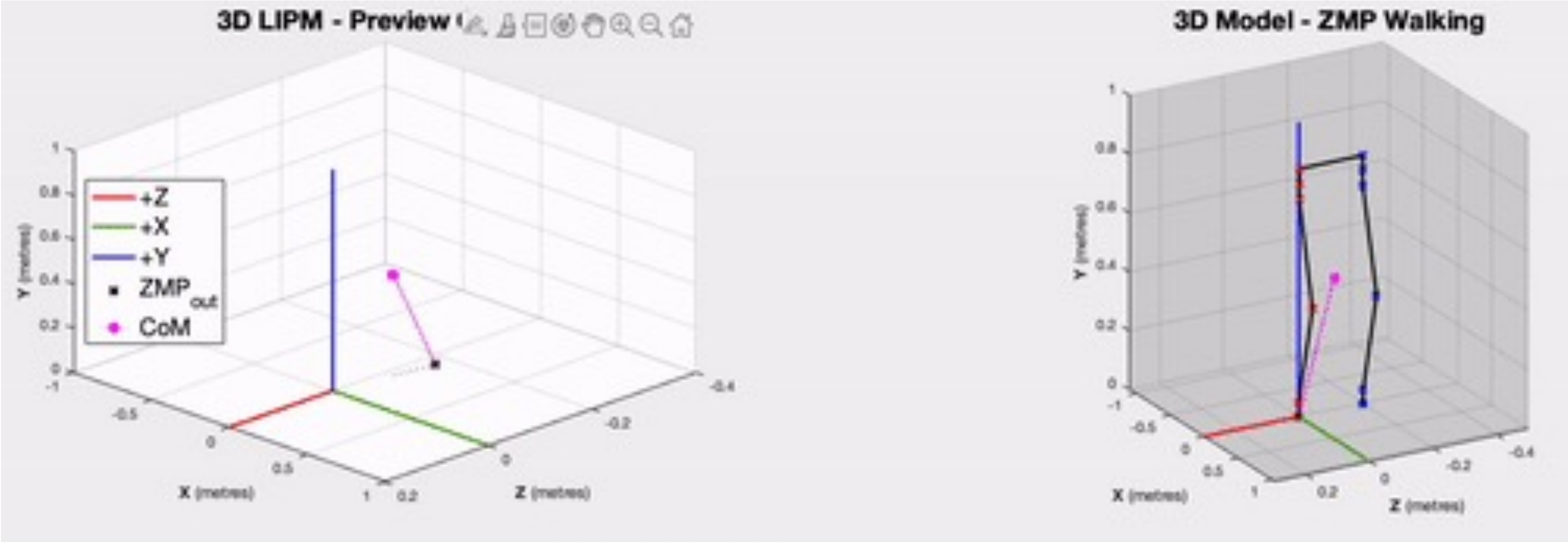




Zero Moment Point – Cont'd



3D Model Walking – ZMP



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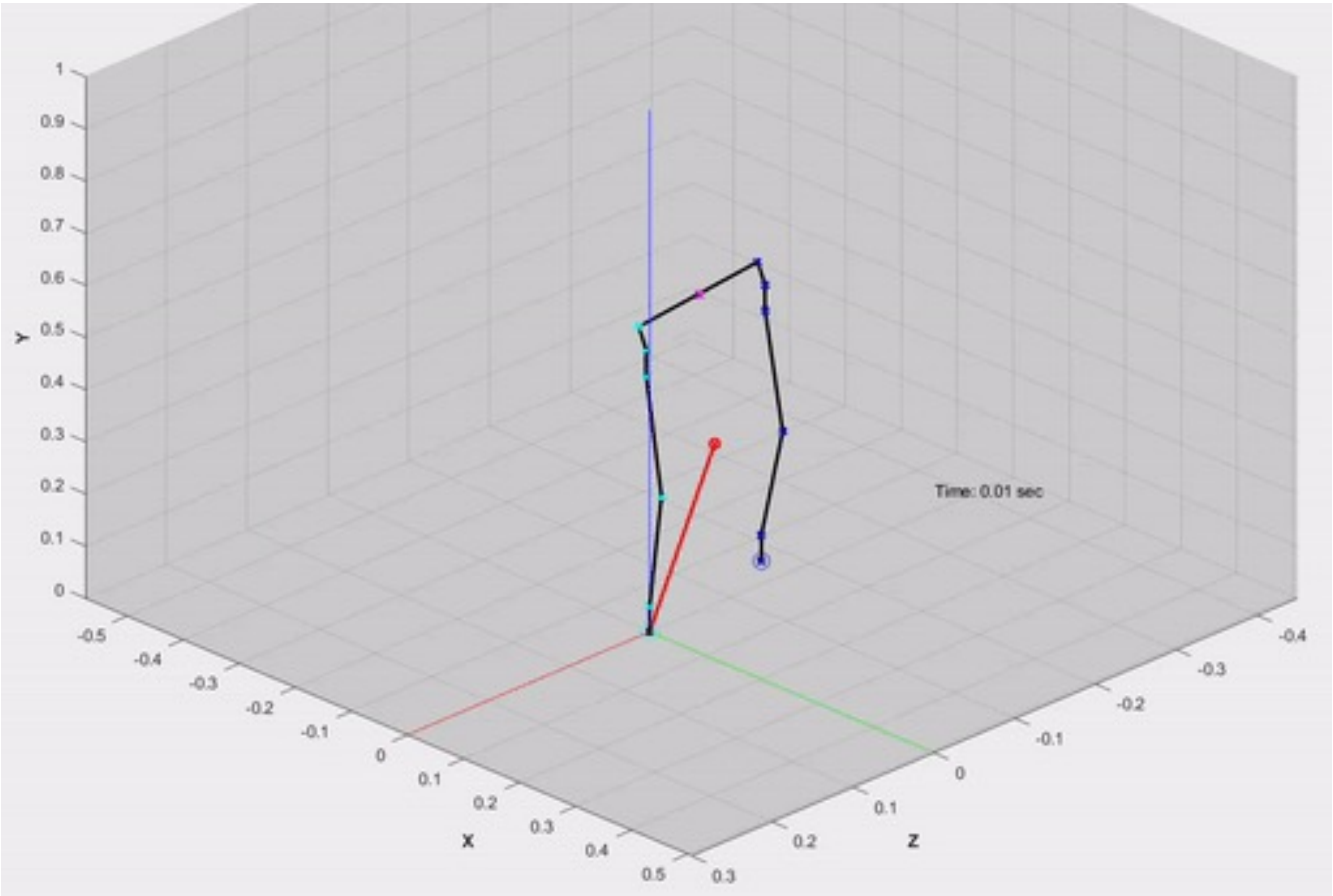
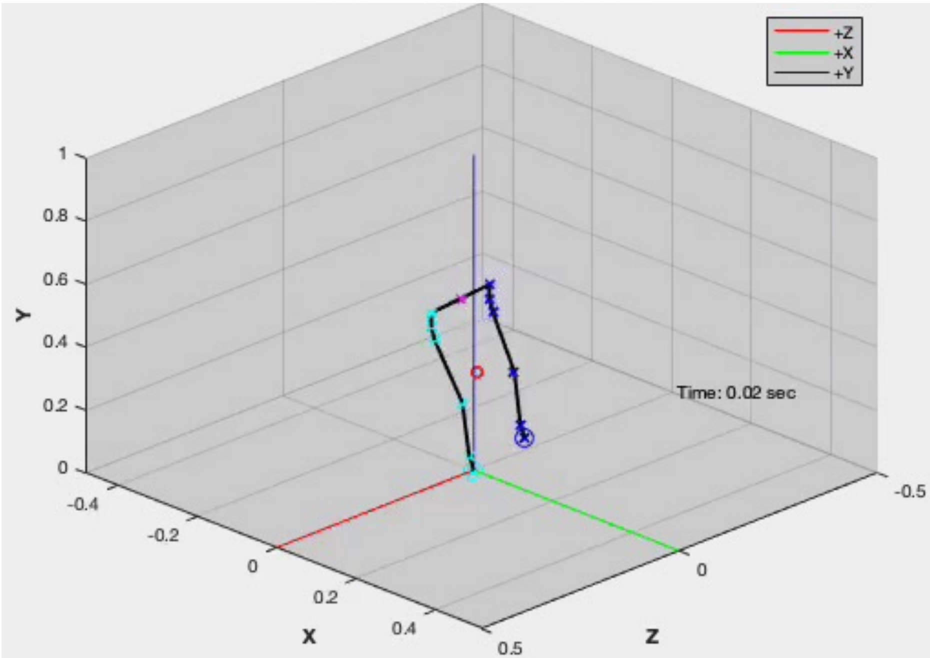
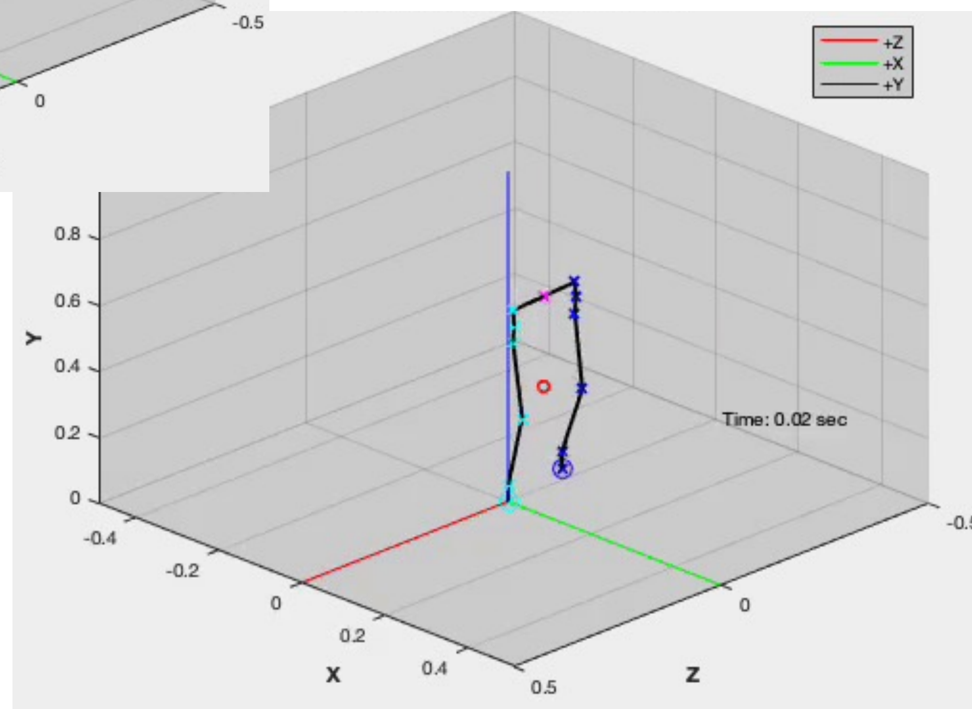
3D Model Walking – ZMP

- Zero Moment Point



FAILS


+Z
+X
+Y
Time: 0.02 sec
Y
1
0.8
0.6
0.4
0.2
0
X
-0.4
-0.2
0
0.2
0.4
0.5
Z
0
-0.5


+Z
+X
+Y
Time: 0.02 sec
Y
0.8
0.6
0.4
0.2
0
X
-0.4
-0.2
0
0.2
0.4
0.5
Z
0
-0.5

Next Steps…

- Simulation:
  1. Shift from Global Coordinate System to Body Fixed
  2. Optimal Trajectory for each foot step
  3. Torso and Arms
  4. Following a predefined path
  5. Disturbance Rejection
  6. Spring Loaded Invert Pendulum

- Reality:
  1. MatLab Plugin: `Realtime` Joint Variables to WeBots
  2. Quasi Static Locomotion implemented on NUgus
  3. Zero Moment Point Locomotion implemented on NUgus