



THE UNIVERSITY OF
NEWCASTLE
AUSTRALIA

FACULTY OF
ENGINEERING AND
BUILT ENVIRONMENT



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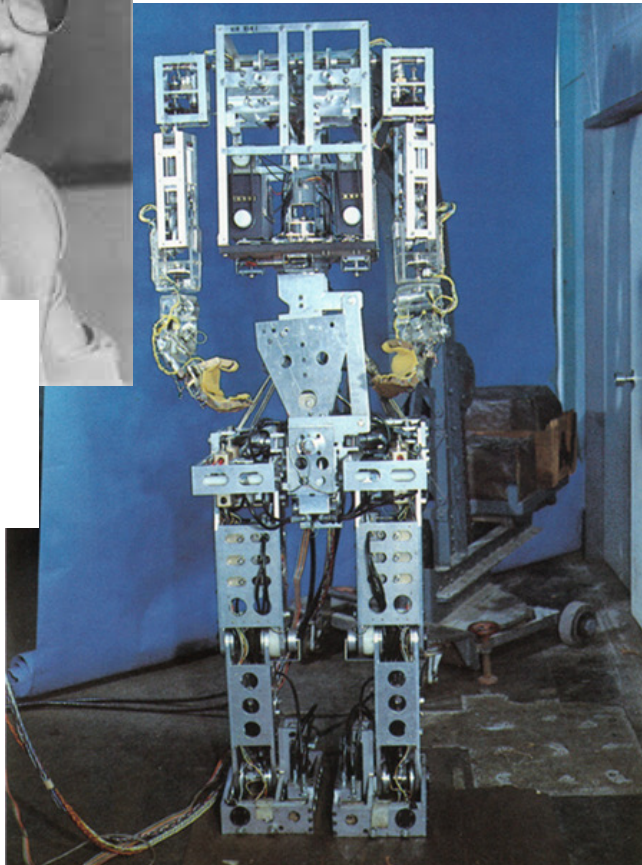
Walking Gaits for the NUgus Platform

... from MCHA 4100 to idealised walking strategies

Darcy Byrne

C3256634

History



THE WABOT-1 (1973), Ichirō Katō
“Quasi-Static” Locomotion

<https://link.springer.com/article/10.1007/BF00735435?noAccess=true>

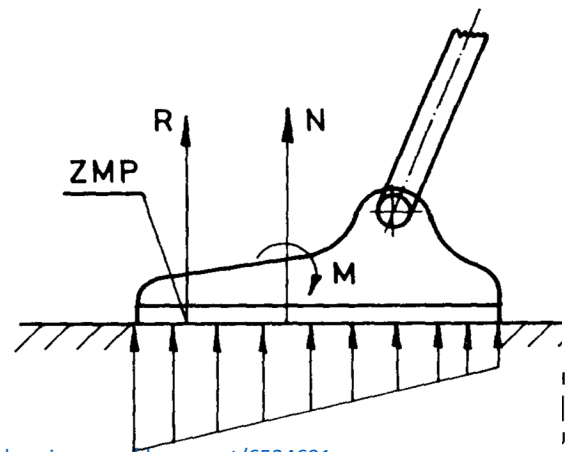
https://www.humanoid.waseda.ac.jp/booklet/kato_2.html



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Mathematical Models of General Anthropomorphic Systems (1973),
Miomir Vukobratović
“ZMP” Locomotion



Mathematical Background

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- Vectors

Direction & Magnitude, with respect to a Coordinate System

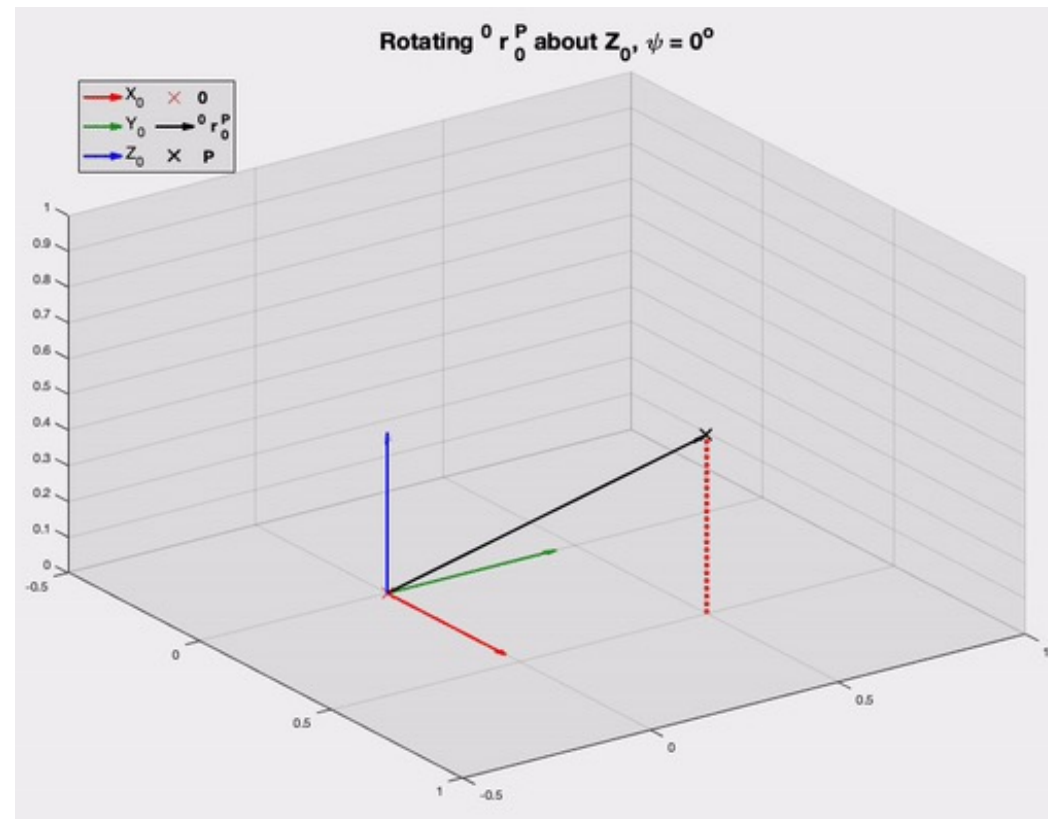
- Rotations (Passive & Active)

Coordinate transformers

$$\mathbf{r}_P^0 = [0.5 \quad 0.5 \quad 0.5]^T$$

$$\mathbf{R}_{Z_0}(\psi) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_{Z_0}^0 \mathbf{r}_P^0 \Rightarrow$$



Mathematical Background – Cont'd...

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- Canonical Homogenous Transforms

$$A_{\mathbf{b}}^{\mathbf{a}}(q) = \begin{bmatrix} R_{\mathbf{b}}^{\mathbf{a}}(q) & \mathbf{r}_{\mathbf{b}}^{\mathbf{a}}(q) \\ \mathbf{0} & 1 \end{bmatrix} = A_{\mathbf{a}}^{\mathbf{b}}(q)^{-1}$$

- Kinematic Chains

$$T_n^0(\mathbf{q}) = A_1^0(q_1)A_2^1(q_2) \dots A_n^{n-1}(q_n) = \begin{bmatrix} R_n^0(\mathbf{q}) & \mathbf{r}_n^0(\mathbf{q}) \\ \mathbf{0} & 1 \end{bmatrix}$$

... where \mathbf{q} is an array of 'Joint Variables'

$$T_n^0(\mathbf{q}) \Rightarrow \text{Forward Kinematic Model} \Rightarrow \mathbf{k}(\mathbf{q})$$

Mathematical Background – Cont'd...

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- Forward Kinematic Model (**FKM**)
 - Returns the End Effector Position, $\mathbf{r}_{x_e}^0$, with respect to the **ZERO coordinate system**
 - Returns End Effector Orientation, $\mathbf{\Theta}_{x_e}^0$, with respect to the **ZERO coordinate system**
 - However, this first involves the parameterisation of the Rotation Matrix $\mathbf{R}_{x_e}^0(\mathbf{q})$ into Euler Angles or Unit Quaternions
 - ‘Google it, mate.’

- Comrade Adam Bandt, 2022

$$\mathbf{x}_e = \begin{bmatrix} \mathbf{r}_{x_e}^0 \\ \mathbf{\Theta}_{x_e}^0 \end{bmatrix} = \mathbf{k}(\mathbf{q}) = [x, y, z, \varphi, \theta, \psi]^T$$

Mathematical Background – Cont'd...

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- Inverse Kinematics (*IKM*)
 - Returns the array of Joint Variables necessary to achieve a desired End Effector Pose, \mathbf{x}_e
 - Usually solved with Numerical Optimisation
- Parallel Kinematics

Mathematical Background – Cont'd...

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- Numerical Optimisation

General Solution to the **IKM** problem:

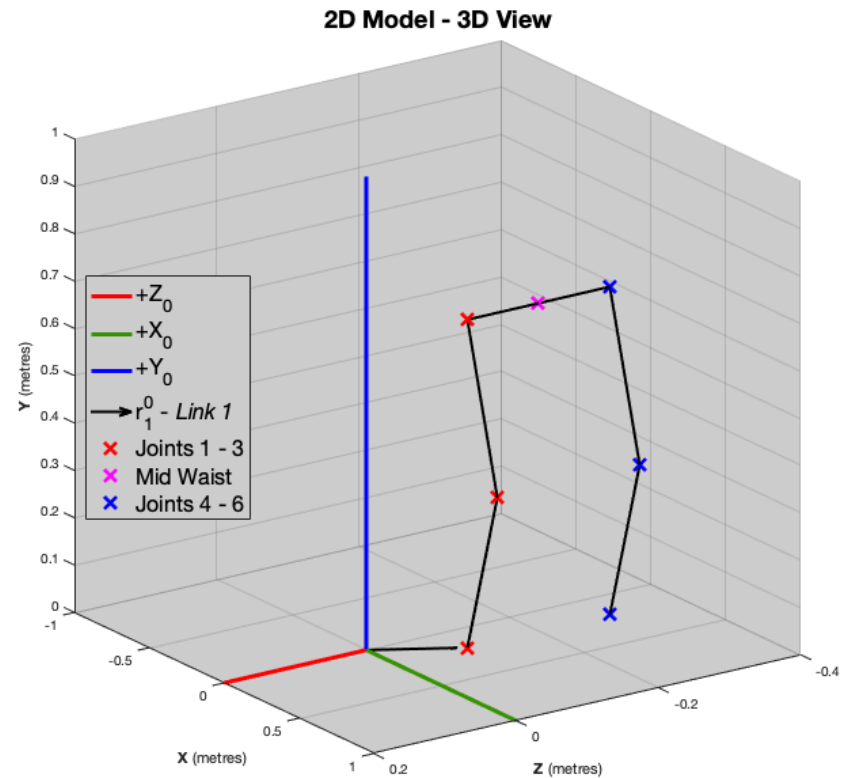
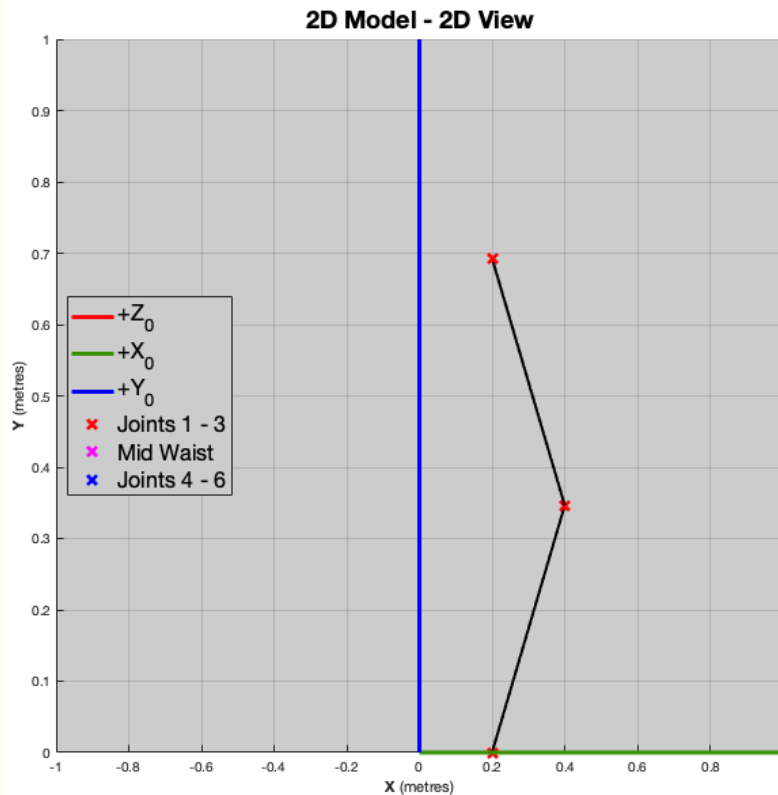
$$\mathbf{q}^* = \arg \min_{\mathbf{q}} K_q \|\mathbf{q}_0 - \mathbf{q}\| + K_e \|k(\mathbf{q}) - \mathbf{x}_e^*\|$$

... which is to say;

the optimal array of **State Variables**, \mathbf{q}^* ,
that satisfy the desired **End Effector Pose**, \mathbf{x}_e^* ,
can be found by minimising the above function.

2D Model

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$$T_6^1 = A_2^1 A_3^2 A_4^3 A_5^4 A_6^5$$

$$T_1^6 = (T_6^1)^{-1}$$

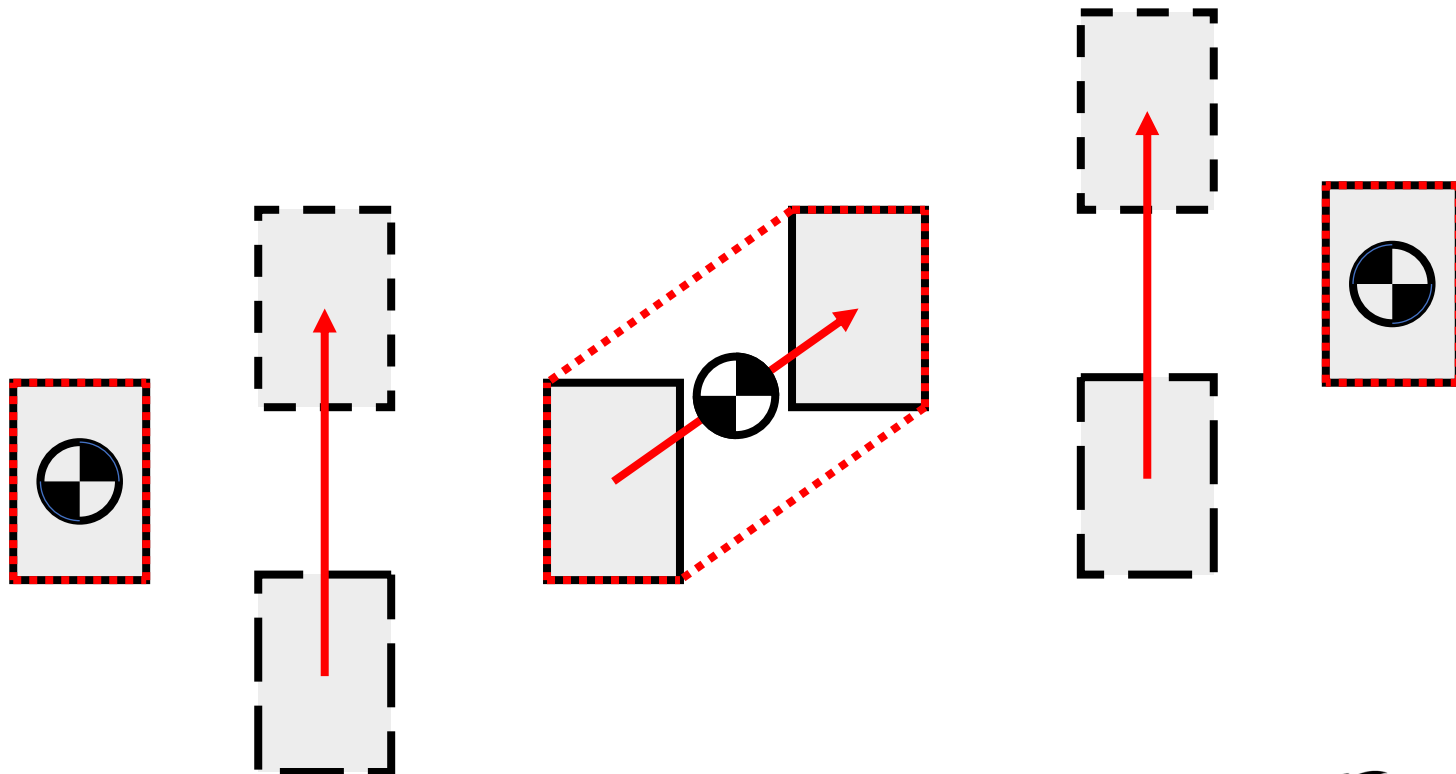
$$T_1^{Global} = A_1^0 T_6^1$$

$$T_6^{Global} = A_6^0 T_1^6$$

$$q = [\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6]^T$$

Quasi Static

$$r_{CoM}^0 = \frac{1}{\sum Mass} \sum_{i=1}^n m_i r_i^0$$



“Support Polygon”

Quasi Static – Cont'd...

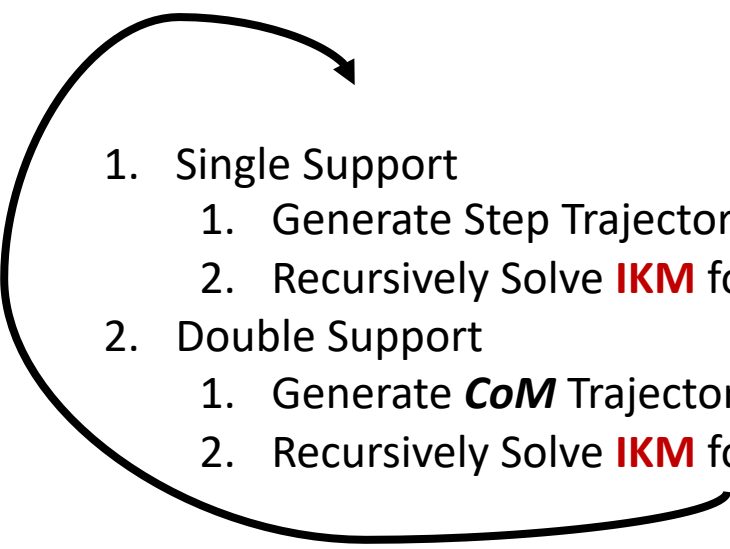
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Single Support *IKM*:

$$\mathbf{q}^* = \arg \min_{\mathbf{q}} \mathbf{K}_q \|\mathbf{q}_0 - \mathbf{q}\| + \mathbf{K}_e \|k(\mathbf{q}) - \mathbf{x}_e\| + \mathbf{K}_{CoM} \|\mathbf{r}_{CoM}^0 - \mathbf{r}_{Fixed}^0\|$$

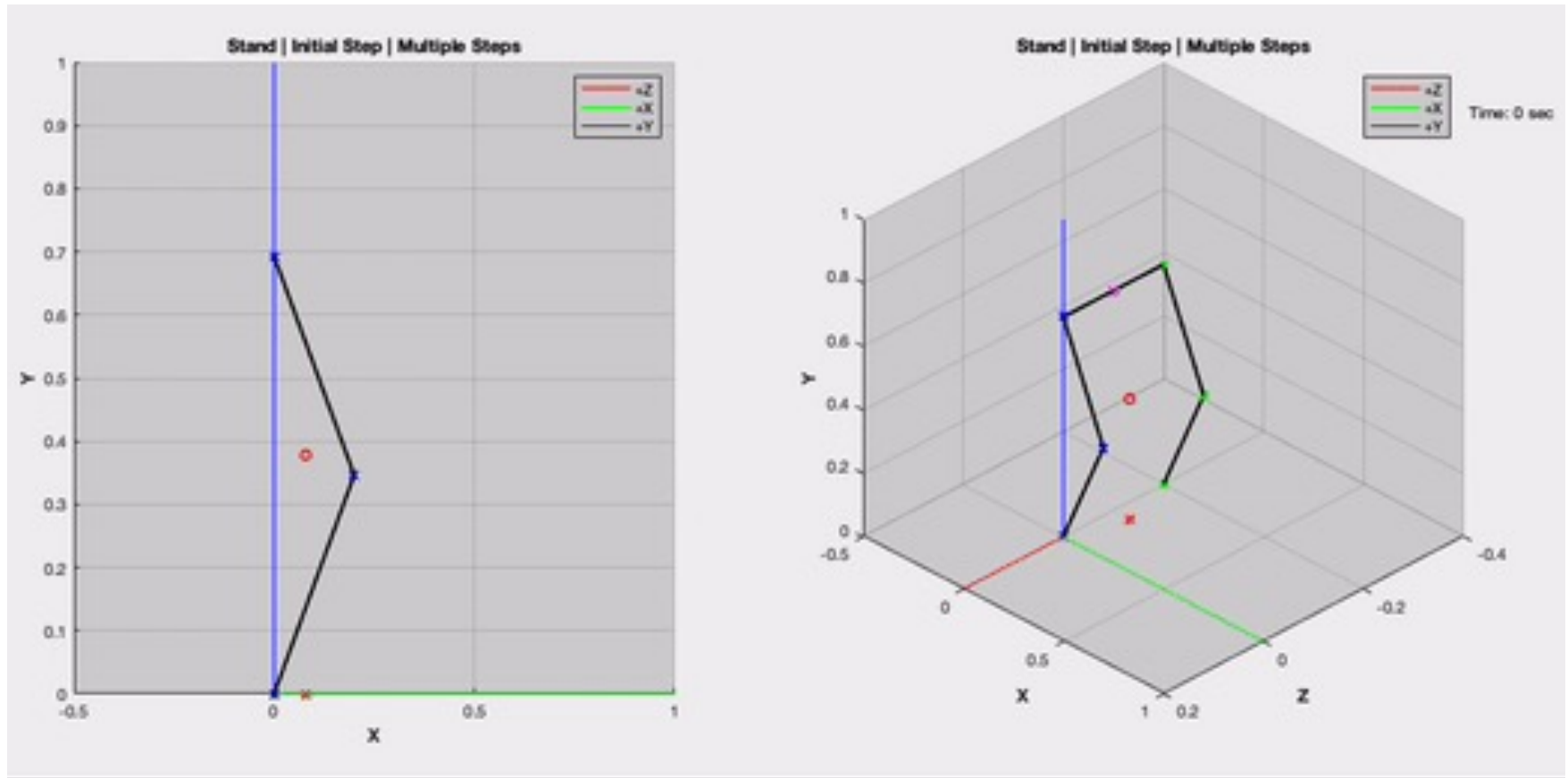
Double Support *IKM*:

$$\mathbf{q}^* = \arg \min_{\mathbf{q}} \mathbf{K}_q \|\mathbf{q}_0 - \mathbf{q}\| + \mathbf{K}_e \|k(\mathbf{q})\| + \mathbf{K}_{CoM} \|\mathbf{r}_{CoM}^0 - \mathbf{r}_{Forward\ foot}^0\|$$

- 
1. Single Support
 1. Generate Step Trajectory
 2. Recursively Solve *IKM* for each point along trajectory
 2. Double Support
 1. Generate *CoM* Trajectory
 2. Recursively Solve *IKM* for each point along trajectory

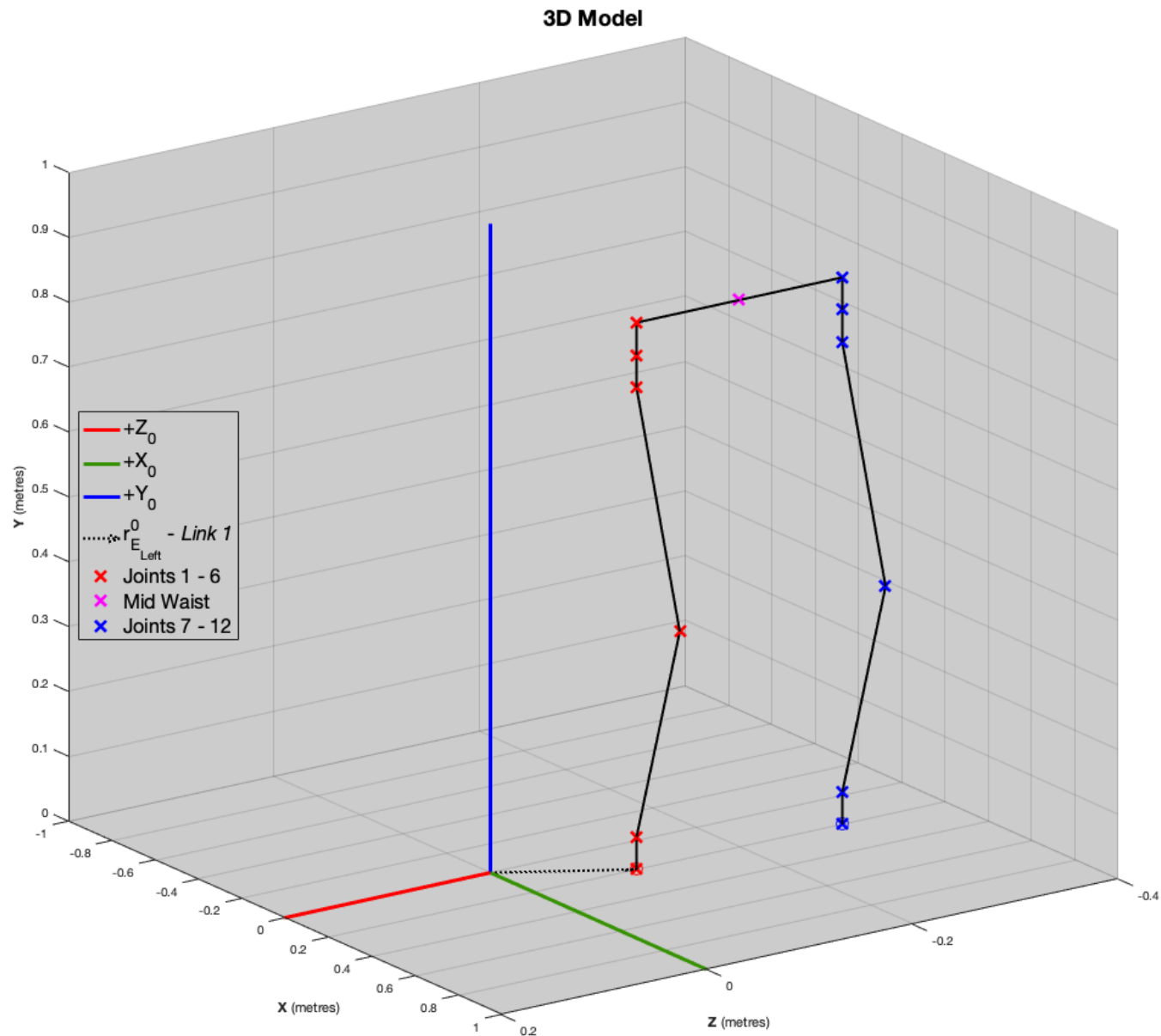
2D Model Walking

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3D Model

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Zero Moment Point

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Supposed, while standing on one leg, that **YOU** are an **inverted pendulum** ...

Your entire degree has led to this moment ...

You begin to fall ...

But at the last moment you place your foot down, shifting all your **mass** to it ...

You're now walking using **ZMP**

Zero Moment Point – Cont'd

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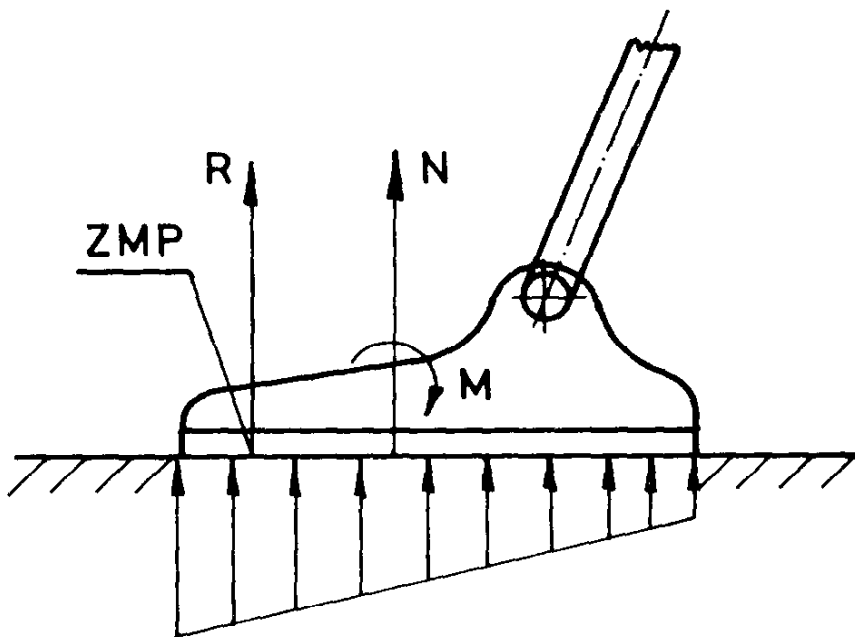


FIG. 3. Load distribution by foot longitudinal section.

Mathematical Model of General Anthropomorphic Systems

- Vukobratović, et al. 1973

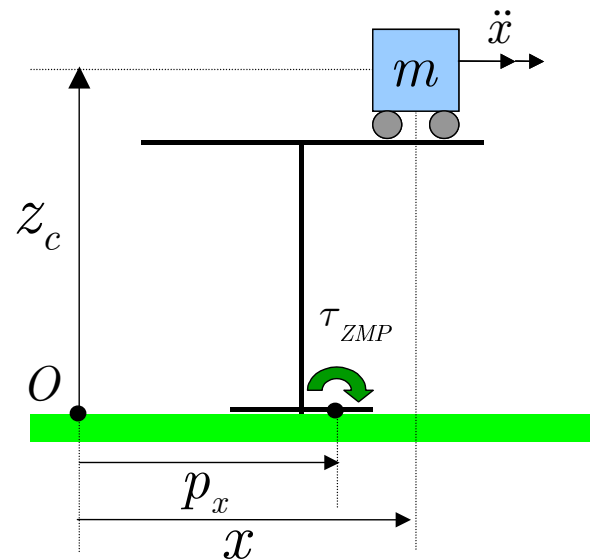


Figure 3: A cart-table model

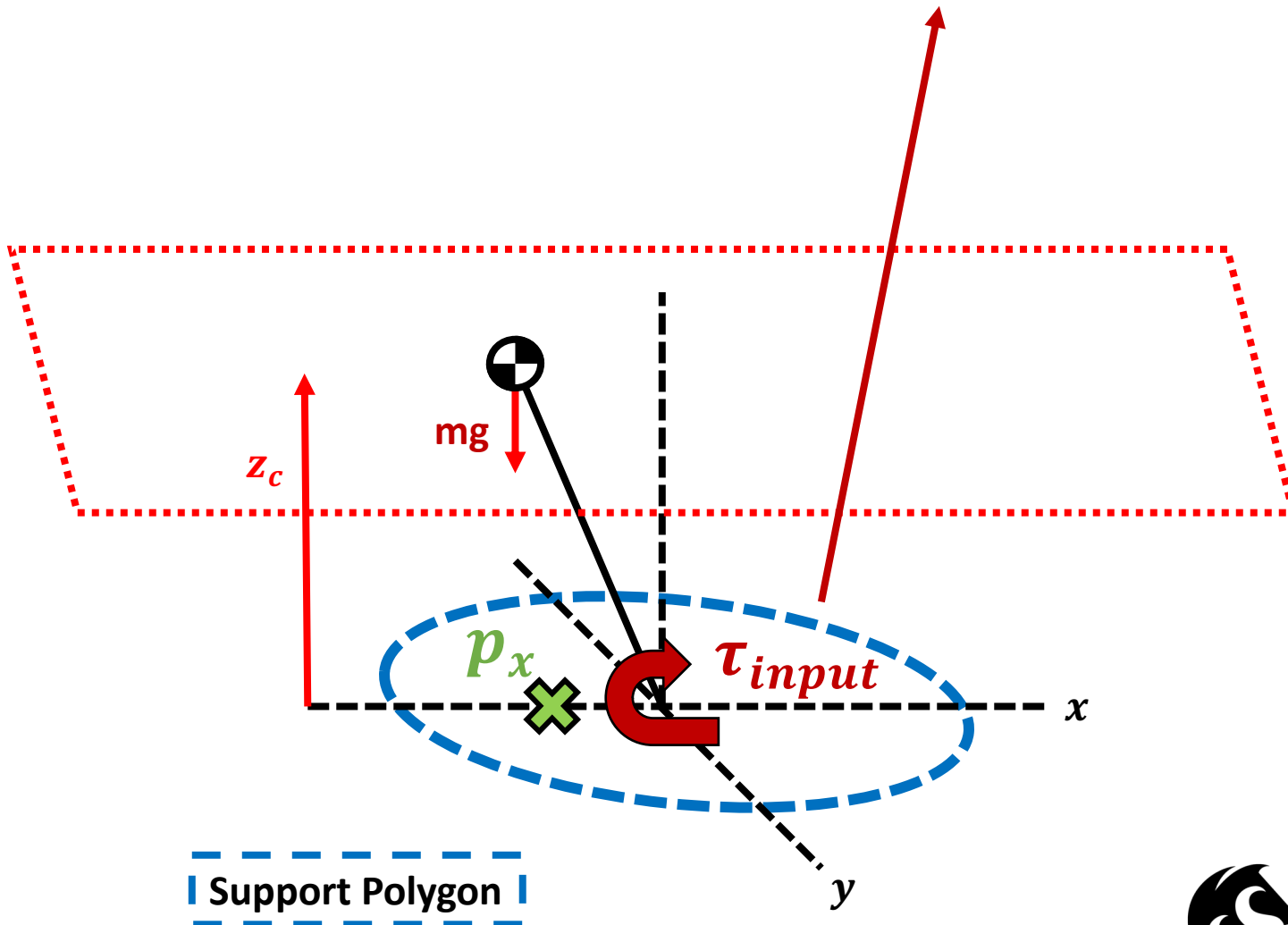
Biped Walking Pattern Generation by using Preview Control of Zero-Moment Point

- Kajita, et al. 2003

Zero Moment Point – Cont'd

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$$\tau_{ZMP} = mg(x - p_x) - m\ddot{x}z_c = 0$$



Zero Moment Point – Cont'd

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$$\frac{\delta}{\delta t} \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_{(x,y)}$$

$$\mathbf{p}_{(x,y)} = \begin{bmatrix} 1 & 0 & \frac{-z_c}{g} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix}$$

$$\mathbf{x}_{(k+1)} = \mathbf{A}_d \mathbf{x}_k + \mathbf{B}_d \mathbf{u}_k$$

$$\mathbf{p}_k = \mathbf{C}_d \mathbf{x}_k$$

... where:

$$\mathbf{A}_d = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{B}_d = \begin{bmatrix} \frac{T^3}{6} \\ \frac{T^2}{2} \\ T \end{bmatrix} \quad \mathbf{C}_d = \begin{bmatrix} 1 & 0 & \frac{-z_c}{g} \end{bmatrix}$$

Zero Moment Point – Cont'd

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$$J_u = \sum_{i=k}^{\infty} \mathbf{Q}_e \|\mathbf{y}(k) - \mathbf{y}_d(k)\| + \mathbf{Q}_x \|\mathbf{x}(k) - \mathbf{x}(k-1)\| + \mathbf{R} \|\mathbf{u}(k) - \mathbf{u}(k-1)\|$$

$$\mathbf{u}^*(k) = -\mathbf{G}_e \sum_{i=0}^k [\mathbf{y}(k) - \mathbf{y}_d(k)] - \mathbf{G}_x \mathbf{x}(k) - \sum_{l=1}^{N_l} \mathbf{G}_d(l) \mathbf{y}_d(k+l)$$

$$\tilde{\mathbf{B}} = \begin{bmatrix} \mathbf{C}_d \mathbf{B}_d \\ \mathbf{B}_d \end{bmatrix}, \quad \tilde{\mathbf{F}} = \begin{bmatrix} \mathbf{C}_d \mathbf{A}_d \\ \mathbf{A}_d \end{bmatrix}, \quad \tilde{\mathbf{Q}} = \begin{bmatrix} \mathbf{Q}_e & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_x \end{bmatrix}, \quad \tilde{\mathbf{I}} = \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix}, \quad \tilde{\mathbf{A}} = [\tilde{\mathbf{I}} \quad \tilde{\mathbf{F}}]$$

... where the gains, \mathbf{G}_e , \mathbf{G}_x and $\mathbf{G}_d(l)$ are given by;

$$\begin{aligned} \mathbf{G}_e &= [\mathbf{R} + \tilde{\mathbf{B}}^\top \tilde{\mathbf{K}}_d \tilde{\mathbf{B}}]^{-1} \tilde{\mathbf{B}}^\top \tilde{\mathbf{K}}_d \tilde{\mathbf{I}} \\ \mathbf{G}_x &= [\mathbf{R} + \tilde{\mathbf{B}}^\top \tilde{\mathbf{K}}_d \tilde{\mathbf{B}}]^{-1} \tilde{\mathbf{B}}^\top \tilde{\mathbf{K}}_d \tilde{\mathbf{F}} \\ \mathbf{G}_d(l) &= -[\mathbf{R} + \tilde{\mathbf{B}}^\top \tilde{\mathbf{K}}_d \tilde{\mathbf{B}}]^{-1} \tilde{\mathbf{B}}^\top (\tilde{\mathbf{A}}_c^\top)^{(l-1)} \tilde{\mathbf{K}}_d \tilde{\mathbf{I}} \end{aligned}$$

... where;

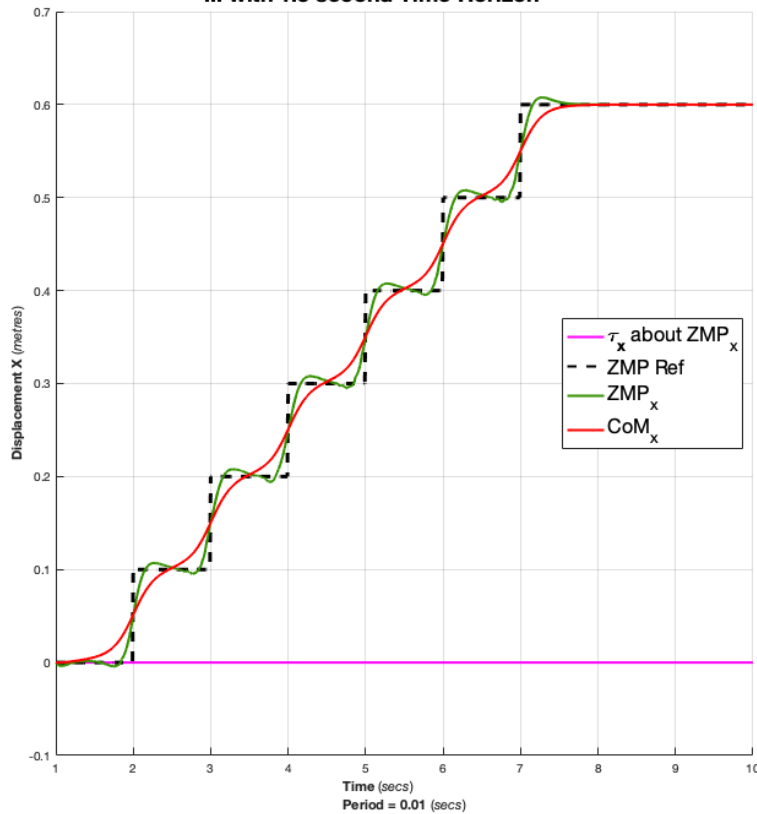
$$\tilde{\mathbf{A}}_c = \tilde{\mathbf{A}} - \tilde{\mathbf{B}} [\mathbf{R} + \tilde{\mathbf{B}}^\top \tilde{\mathbf{K}}_d \tilde{\mathbf{B}}]^{-1} \tilde{\mathbf{B}}^\top \tilde{\mathbf{K}}_d \tilde{\mathbf{A}}$$

... and $\tilde{\mathbf{K}}_d$ is a solution to the Discrete Time Algebraic Riccati Equation.

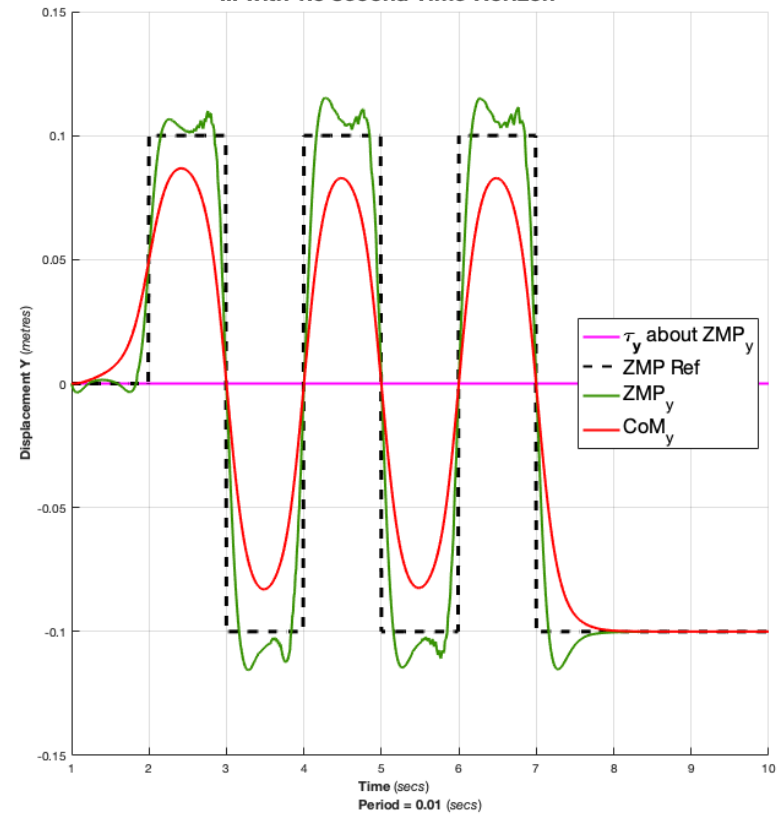
Zero Moment Point – Cont'd

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Discretised Linear Inverted Pendulum
... with 1.5 second Time Horizon

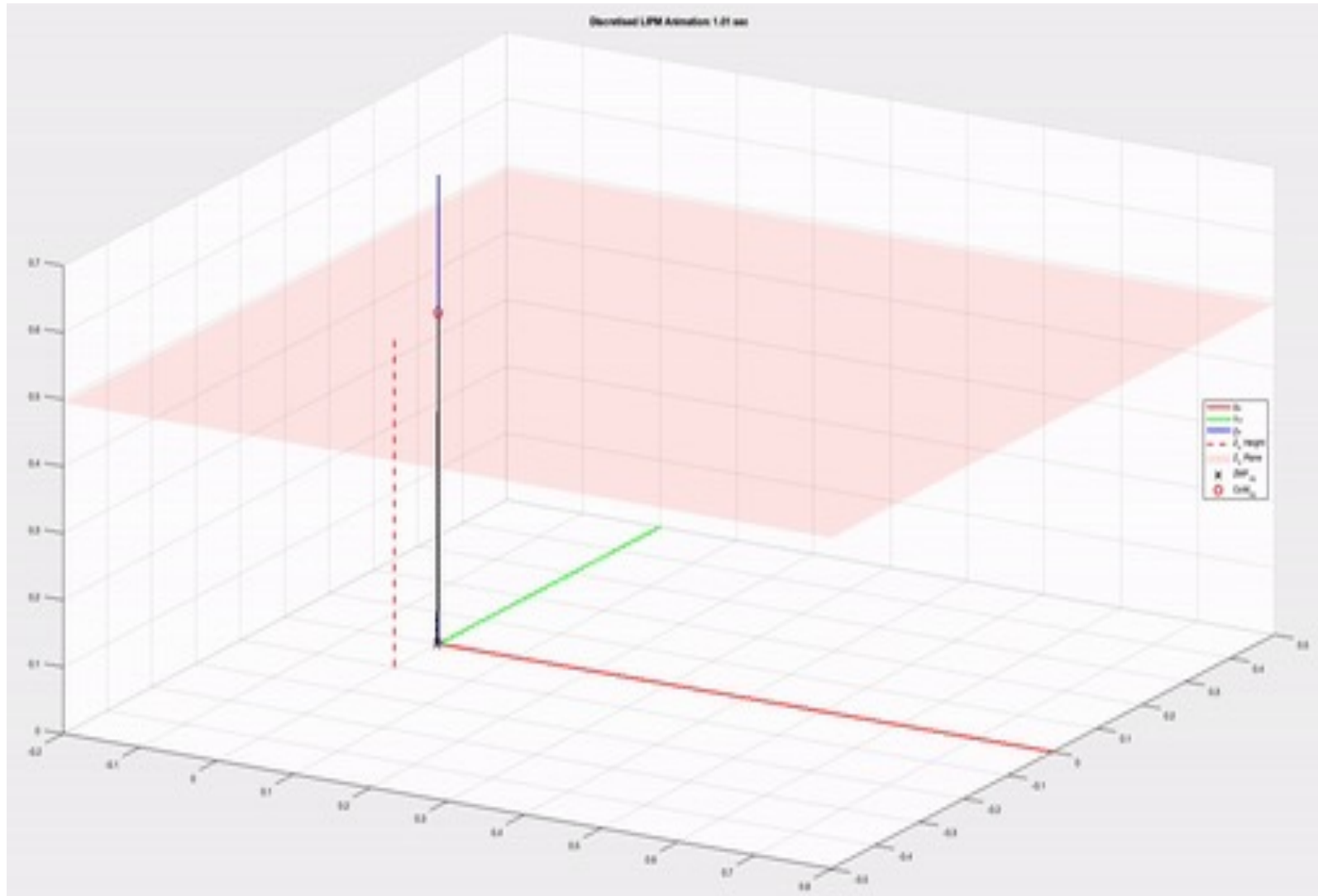


Discretised Linear Inverted Pendulum
... with 1.5 second Time Horizon



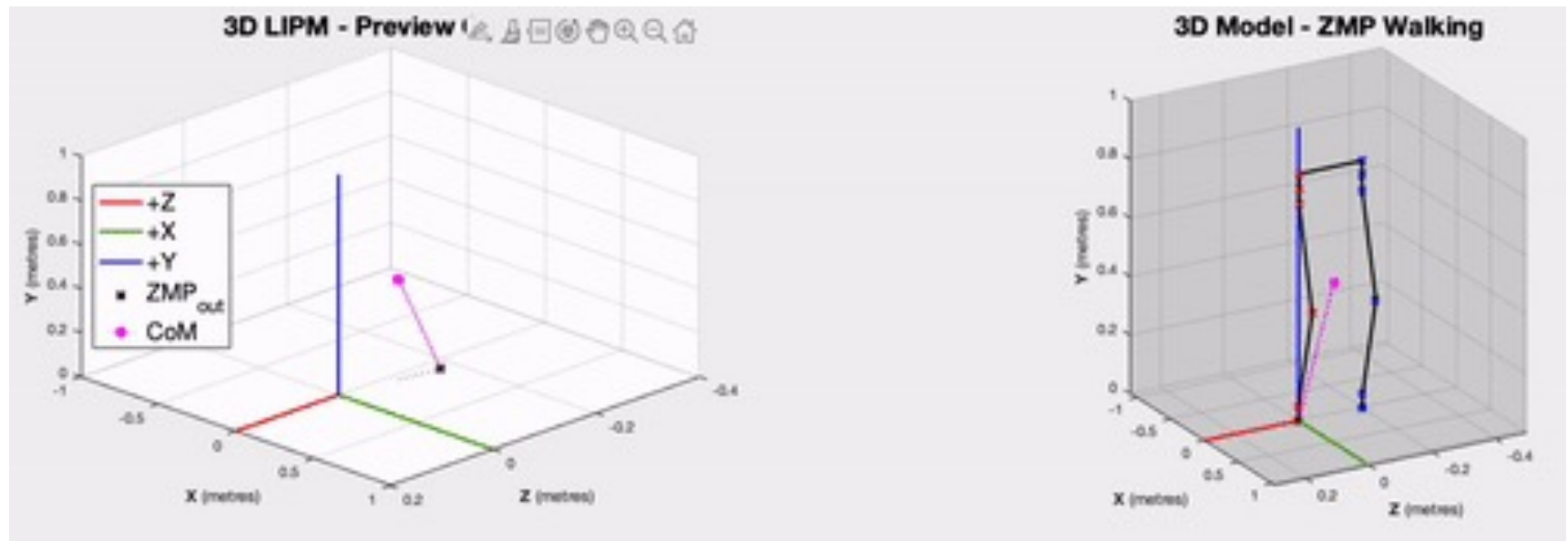
Zero Moment Point – Cont'd

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3D Model Walking – ZMP

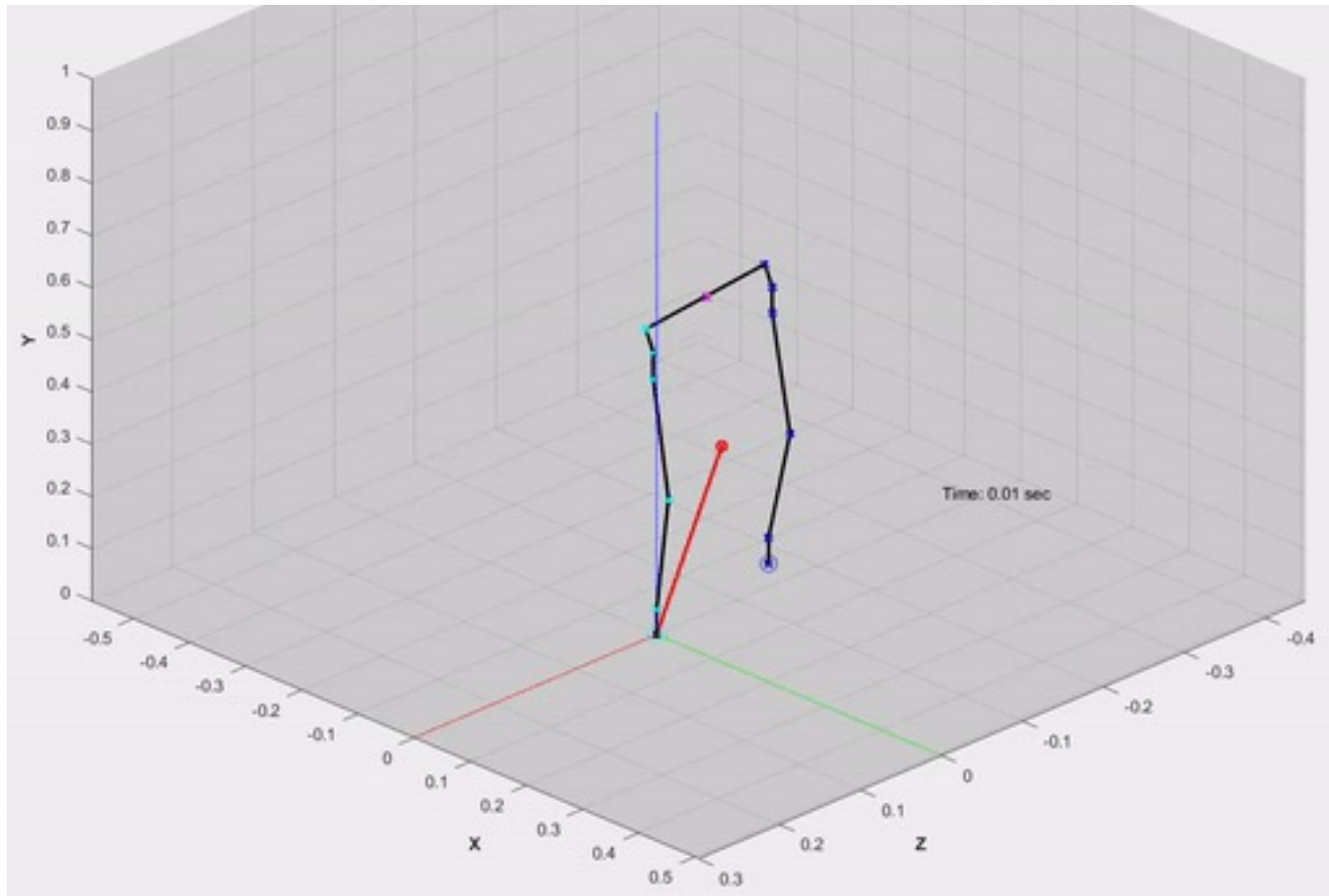
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3D Model Walking – ZMP

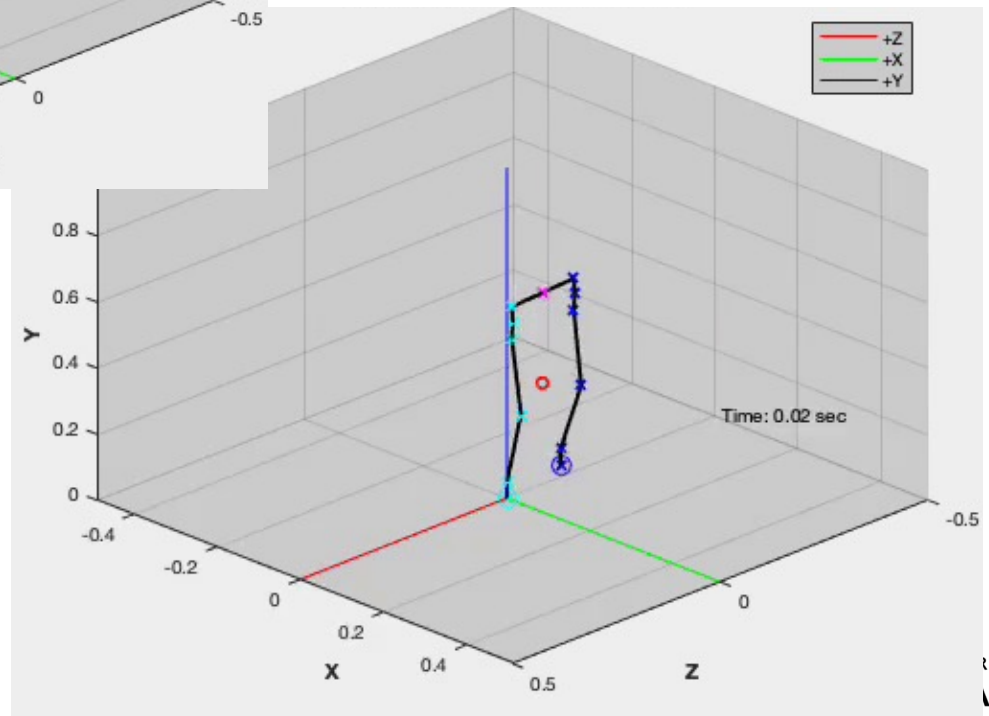
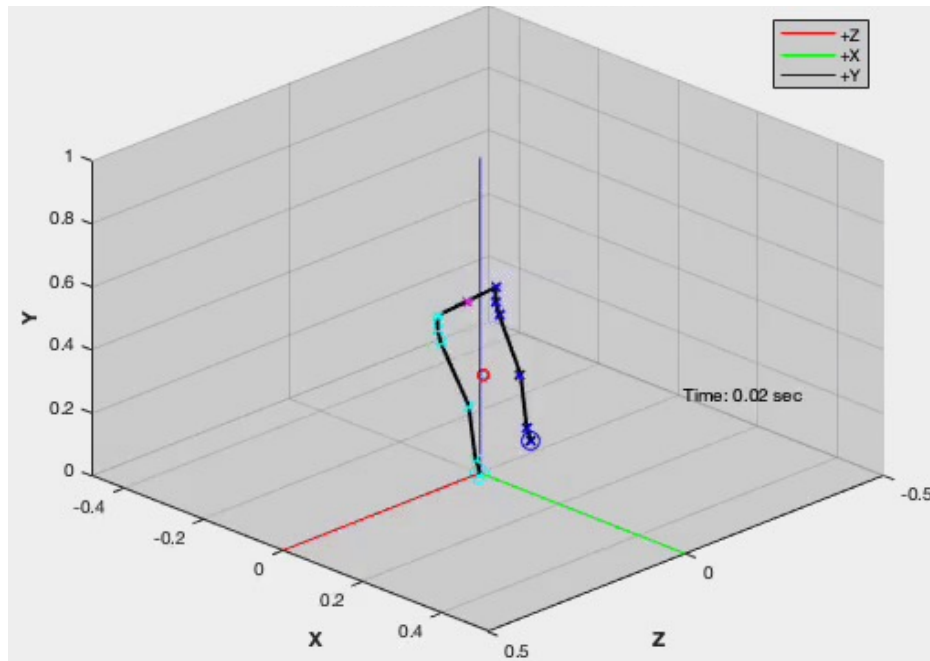
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- Zero Moment Point



FAILS

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Next Steps...

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- Simulation:
 1. Shift from Global Coordinate System to Body Fixed
 2. Optimal Trajectory for each foot step
 3. Torso and Arms
 4. Following a predefined path
 5. Disturbance Rejection
 6. Spring Loaded Invert Pendulum
- Reality:
 1. MatLab Plugin: `Realtime` Joint Variables to WeBots
 2. Quasi Static Locomotion implemented on NUgus
 3. Zero Moment Point Locomotion implemented on NUgus