

# Final Year Project Part A

May 2022

Darcy Byrne<sup>1</sup>

<sup>1</sup> Student of Mechatronics Engineering,  
The University of Newcastle, Callaghan, NSW 2308, AUSTRALIA  
Student Number: 3256634  
E-mail: [D.Byrne@uon.edu.au](mailto:D.Byrne@uon.edu.au)

---

## Abstract

Remember that your abstract may include the following information:

- Defines the intention of the report.
- Places the report in context so the reader knows why it is important to read it.
- Why is it important?
- What problem is addressed?
- Briefly states the results
- Briefly presents the implications and recommendations

Ensure that your abstract is less than 200 words.

## Acknowledgements

You may like to say thank you to someone that helped you with your project.

(Kajita et al. 2002) - A realtime pattern generator for biped walking

(Kajita et al. 2003) - ZMP

(KATAYAMA et al. 1985) - Preview Control Discrete

(Perez 2014) - Fundamentals of Mechanical Systems Vec Calc etc.

(Renton 2021) - MCHA 4100

(Grimble et al. 2009) - Robotics: Modelling Planning Control

(Raibert 1986) - Legged Robots that balance

(Kato 1973) - The WABOT-1 -; First known Quasi Static Biped ?

(Vukobratović & Stokić 1983) - Is Dynamic Control Needed in Robotic Systems ?

(Hirose 1984) - Design and Control of a Quadruped Walking Vehicle ?

## Contents

<b>1. Introduction</b>	<b>5</b>
1.1. Subsection title . . . . .	5
1.2. Subsection title . . . . .	5
1.2.1. Subsubsection 1 . . . . .	5
1.2.2. Subsubsection 2 . . . . .	5
<b>2. Background</b>	<b>6</b>
2.1. Mathematical Notation . . . . .	6
2.2. Numerical Optimisation . . . . .	6
2.3. Kinematics . . . . .	6
2.4. Kinematic Chains . . . . .	6
2.5. Walking . . . . .	6
<b>3. Models</b>	<b>7</b>
3.1. 2D Model . . . . .	7
3.2. 3D Model . . . . .	9
<b>4. Quasi Static Locomotion</b>	<b>12</b>
4.1. Quasi Static concept . . . . .	12
4.2. 2D model implementation . . . . .	12
4.3. 3D model implementation . . . . .	12
<b>5. Zero Moment Point Locomotion</b>	<b>13</b>
5.1. Zero Moment Point concept . . . . .	13
5.2. 3D Linear Inverted Pendulum . . . . .	13
5.3. Preview Control . . . . .	13
5.4. 3D Model Implementation . . . . .	13
<b>6. Results</b>	<b>14</b>
6.1. 2D Model . . . . .	14
6.2. 3D Model . . . . .	14
<b>7. Conclusion</b>	<b>16</b>
<b>A. Example of a Table</b>	<b>18</b>

## List of Figures

1.	Sketch defining coordinate systems <b>0</b> to <b>6</b> . . . . .	7
2.	2D model plotted in 3D (left) and 2D (right). . . . .	8
3.	Sketch defining coordinate system <b>0</b> , and coordinate systems <b>E<sub>L</sub></b> through to <b>E<sub>R</sub></b> . . . . .	9
4.	3D model plotted using FKM. . . . .	11
5.	2D model, plotted in 3D, Quasi Static walk sequence. . . . .	13
6.	3D model Quasi-Static walk sequence. . . . .	14
7.	3D LIPM with Preview Control. . . . .	15
8.	3D LIPM X Y components. . . . .	15
9.	3D model using ZMP walking with Preview Control. . . . .	16

## 1. Introduction

To organise your introduction section you can use the following structure:

- **Position:** Show there is a problem and that it is important to solve it.
- **Problem:** Describe the specifics of the problem you are trying to address
- **Proposal:** Discuss how you are going to address this problem. Use the literature to back-up your approach to the problem, or to highlight that what you are doing has not been done before

Here you need to sell why what you are doing is important, and what benefits will it bring if you are successful and solve the problem?

### 1.1. Subsection title

You can use subsections within any section of the report.

### 1.2. Subsection title

Recall that you need at least two subsections per section.

#### 1.2.1. Subsubsection 1

Do not use more than 2 levels of sub-sectioning.

#### 1.2.2. Subsubsection 2

Do not use more than 2 levels of sub-sectioning.

The rest of the report is organised as follows. Section [2](#) describes items related to the core content. Section [7](#) concludes the report. Appendix [A](#) shows an example of how to make a Table.

## **2. Background**

**2.1. Mathematical Notation**

**2.2. Numerical Optimisation**

**2.3. Kinematics**

**2.4. Kinematic Chains**

**2.5. Walking**

### 3. Models

Each model is a simplistic representation of a bipedal robot. As the focus is simulating different walking strategies, each model is comprised of the lower half of a humanoid, that being the feet, legs and waist. Both models were constructed via a series of homogenous transforms that describe the positions of each ankle in global coordinates.

#### 3.1. 2D Model

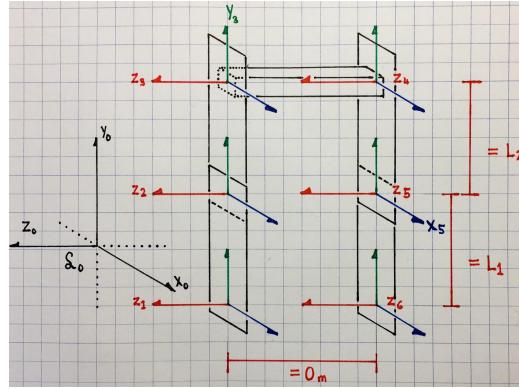


Figure 1: Sketch defining coordinate systems 0 to 6.

Beginning with a sketch, seen in Figure 1, the following series of Homogenous Transforms, equation(3.1) forms a kinematic chain that describes the position of the right ankle, the end effector, with respect to the left ankle.

$$\mathbf{T}_6^1 = \mathbf{A}_2^1(\phi_1, +L_1)\mathbf{A}_3^2(\phi_2, +L_2)\mathbf{A}_4^3(\phi_3, -H)\mathbf{A}_5^4(\phi_4, -L_2)\mathbf{A}_6^5(\phi_5, -L_1)\mathbf{A}_6^6(\phi_6) \quad (3.1)$$

Subsequently, the kinematic chain from the right ankle to the position of the left ankle, the end effector when the right ankle is fixed, is equation(3.2):

$$\mathbf{T}_1^6 = (\mathbf{T}_6^1)^{-1} \quad (3.2)$$

An additonal homogenous transform was applied to each equation to express the end effectors in global coordinates.

$$\begin{aligned} \mathbf{T}_6^{Global} &= \mathbf{A}_1^0 \mathbf{T}_6^1 \\ \mathbf{T}_1^{Global} &= \mathbf{A}_6^0 \mathbf{T}_1^6 \end{aligned} \quad (3.3)$$

...where;

$$\mathbf{A}_1^0 = \begin{bmatrix} \mathbf{I} & \dots & \dots & \text{LeftAnkle}_x \\ \vdots & \ddots & \dots & \text{LeftAnkle}_y \\ \vdots & \vdots & \ddots & \text{LeftAnkle}_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.4)$$

... and;

$$\mathbf{A}_6^0 = \begin{bmatrix} \mathbf{I} & \dots & \dots & \text{RightAnkle}_x \\ \vdots & \ddots & \dots & \text{RightAnkle}_y \\ \vdots & \vdots & \ddots & \text{RightAnkle}_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.5)$$

Equations(3.3) are the Foward kinematic Model (FKM) which were then written in Matlab, with the joint angles found in table 1 and the parameters found in table 2.

Table 1: Joint Angles

Joint Angles		
Joint	Angle (rads)	Variable Name
1	$-\frac{\pi}{6}$	$\phi_1$
2	$+\frac{2\pi}{6}$	$\phi_2$
3	$-\frac{\pi}{6}$	$\phi_3$
4	$+\frac{\pi}{6}$	$\phi_4$
5	$-\frac{2\pi}{6}$	$\phi_5$
6	$+\frac{\pi}{6}$	$\phi_6$

Table 2: Link Lengths

Link Lengths			
Link	Length (m)	Variable Name	Joints
2	0.4	$L_1$	1 to 2
3	0.4	$L_2$	2 to 3
4	0.0, however, $\approx 0.2$	$H$	3 to 4
5	0.4	$L_2$	4 to 5
6	0.4	$L_1$	5 to 6

The model was then plotted in both 2D and 3D (figure 2). Although 2D, plotting the model in 3D with an arbitrary distance between the hip joints, 3 and 4, provided more context regarding the location and orientation of the right side (blue crosses). Additonally, plotting the 2D model in 3D reduced the workload for developing the 3D model as a vast proportion of the matlab script code be reused.

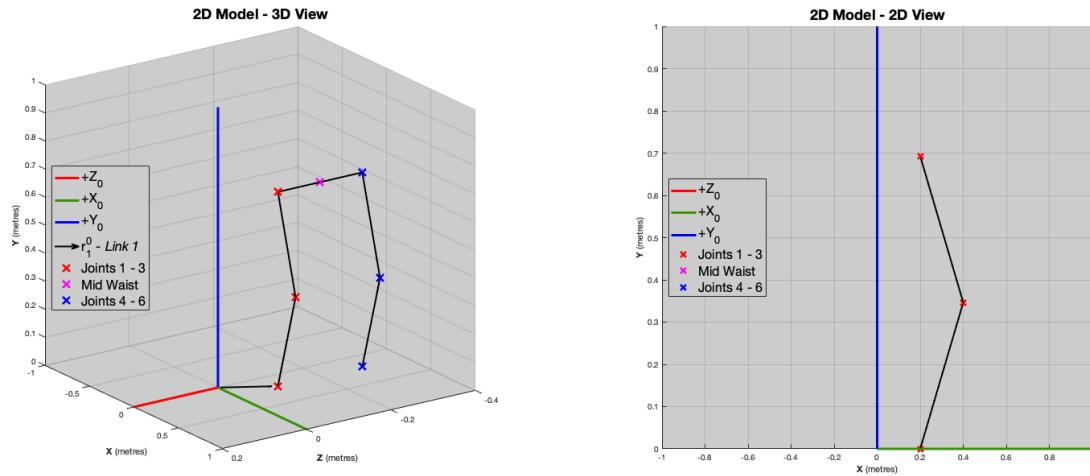


Figure 2: 2D model plotted in 3D (left) and 2D (right).

### 3.2. 3D Model

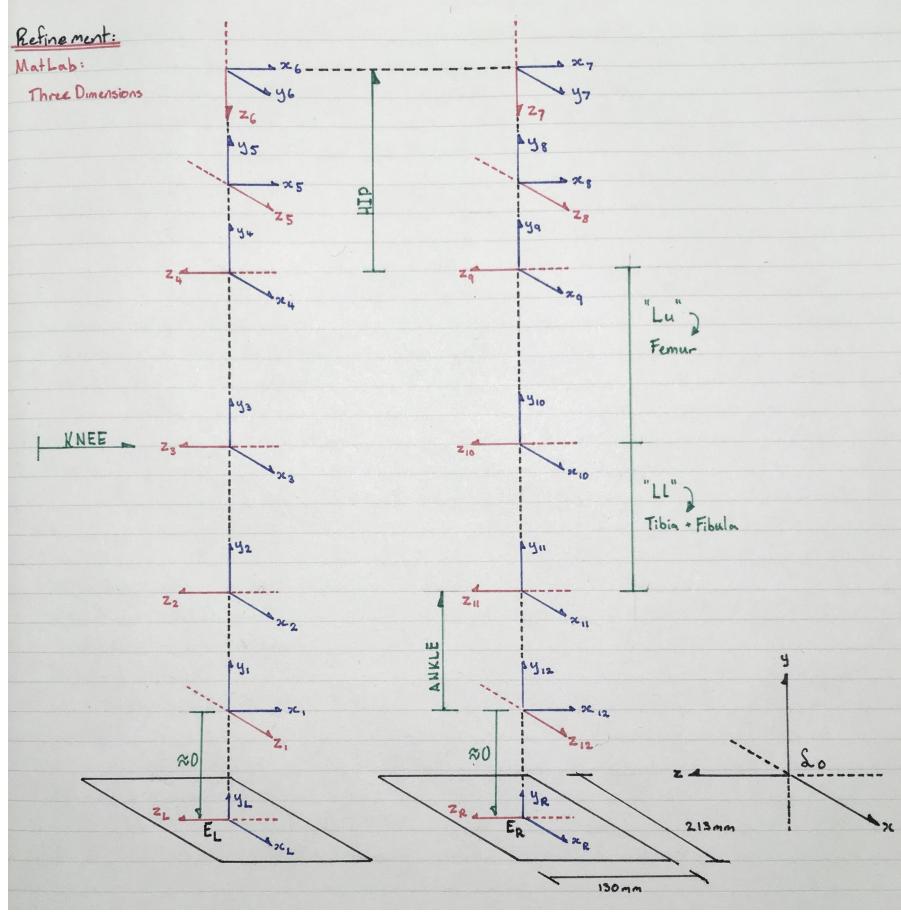


Figure 3: Sketch defining coordinate system  $\mathbf{0}$ ,  
and coordinate systems  $\mathbf{E}_L$  through to  $\mathbf{E}_R$ .

Beginning with a sketch, seen in Figure 3, the following series of Homogenous Transforms, equation(3.6) forms a kinematic chain that describes the position of the right foot, the end effector, with respect to the left foot.

$$\begin{aligned} \mathbf{T}_{E_R}^{E_L} = & \mathbf{A}_1^{E_L} \mathbf{A}_2^1(\phi_1) \mathbf{A}_3^2(\phi_2, +L_{lower}) \mathbf{A}_4^3(\phi_3, +L_{upper}) \mathbf{A}_5^4(\phi_4) \mathbf{A}_6^5(\phi_5) \mathbf{A}_7^6(\phi_6) \\ & \dots \mathbf{A}_8^7(\phi_7) \mathbf{A}_9^8(\phi_8) \mathbf{A}_{10}^9(\phi_9, -L_{upper}) \mathbf{A}_{11}^{10}(\phi_{10}, -L_{lower}) \mathbf{A}_{12}^{11}(\phi_{11}) \mathbf{A}_{E_R}^{12}(\phi_{12}) \end{aligned} \quad (3.6)$$

Subsequently, the kinematic chain from the right foot to the position of the left foot, the end effector when the right foot is fixed, is equation(3.7):

$$\mathbf{T}_{E_L}^{E_R} = (\mathbf{T}_{E_R}^{E_L})^{-1} \quad (3.7)$$

An additional homogenous transform was applied to each equation to express the end effectors in global coordinates.

$$\begin{aligned} \mathbf{T}_{E_R}^{Global} &= \mathbf{A}_{E_L}^0 \mathbf{T}_{E_R}^{E_L} \\ \mathbf{T}_{E_L}^{Global} &= \mathbf{A}_{E_R}^0 \mathbf{T}_{E_L}^{E_R} \end{aligned} \quad (3.8)$$

...where;

$$\mathbf{A}_{E_L}^0 = \begin{bmatrix} \mathbf{R}(\Theta)_{E_L}^0 & \mathbf{r}_{E_L}^0 \\ \mathbf{0} & 1 \end{bmatrix} \quad (3.9)$$

... and;

$$\mathbf{A}_{E_R}^0 = \begin{bmatrix} \mathbf{R}(\Theta)_{E_R}^0 & \mathbf{r}_{E_R}^0 \\ \mathbf{0} & 1 \end{bmatrix}. \quad (3.10)$$

Equations(3.8) are the FKM which were then written in Matlab, with the joint angles found in table 3 and the parameters found in table 4.

Table 3: Joint Angles

Joint Angles			
Joint	Angle (rads)	Variable Name	
1	0	$\phi_1$	
2	$-\frac{\pi}{6}$	$\phi_2$	
3	$+\frac{2\pi}{6}$	$\phi_3$	
4	$-\frac{\pi}{6}$	$\phi_4$	
5	0	$\phi_5$	
6	0	$\phi_6$	
7	0	$\phi_7$	
8	0	$\phi_8$	
9	$+\frac{\pi}{6}$	$\phi_9$	
10	$-\frac{2\pi}{6}$	$\phi_{10}$	
11	$+\frac{\pi}{6}$	$\phi_{11}$	
12	0	$\phi_{12}$	

Table 4: Link Lengths

Link Lengths			
Link	Length (m)	Variable Name	Joints
3	0.4	$L_{lower}$	3 to 4
4	0.4	$L_{upper}$	4 to 5
6	0.2	$H$	6 to 7
9	0.4	$L_{upper}$	9 to 10
10	0.4	$L_{lower}$	10 to 11

The model was then plotted in 3D (figure 4). Distances between joints at the ankle and hips represent physical displacements between axes found on most dual servo motors. Additionally, these distances provide context as to the movement of the sets of joints that make up the hip joint and the ankle joint.

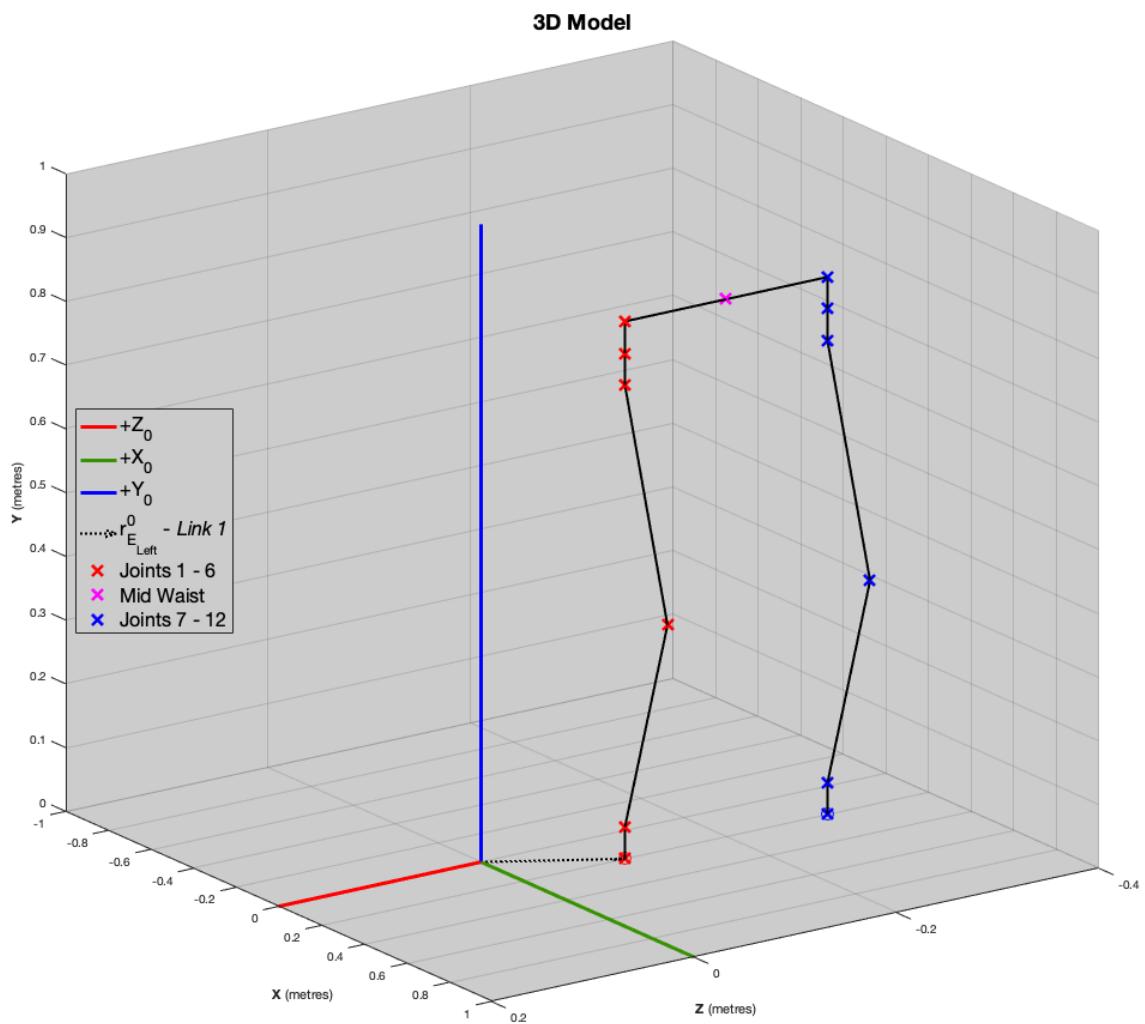


Figure 4: 3D model plotted using FKM.

## 4. Quasi Static Locomotion

### 4.1. Quasi Static concept

The core idea of Quasi-Static locomotion is that the centre of mass (CoM) remains over the support polygon (SP) for all time. This is achieved using the FKM, Inverse Kinematic Model (IKM), Trajectory Generation and Numerical Optimisation.

### 4.2. 2D model implementation

Equation(4.1) is optimised while one foot is fixed and the other moves forward to the next step.

$$\mathbf{q}^* = \arg \min_{\mathbf{q}} \mathbf{K}_q \|\mathbf{q}_0 - \mathbf{q}\| + \mathbf{K}_e \|k(\mathbf{q}) - \mathbf{x}_e\| + \mathbf{K}_{CoM} \|\mathbf{r}_{CoM}^0 - \mathbf{r}_{\text{Fixed}}^0\| \quad (4.1)$$

Equation(4.2) is during the double support phase, moving the CoM from the trailing ankle/support polygon, to the forward ankle. Throughout this phase the CoM remains over the support polygon.

$$\mathbf{q}^* = \arg \min_{\mathbf{q}} \mathbf{K}_q \|\mathbf{q}_0 - \mathbf{q}\| + \mathbf{K}_e \|k(\mathbf{q})\| + \mathbf{K}_{CoM} \|\mathbf{r}_{CoM}^0 - \mathbf{r}_{\text{Forward Ankle}}^0\| \quad (4.2)$$

### 4.3. 3D model implementation

## 5. Zero Moment Point Locomotion

### 5.1. Zero Moment Point concept

### 5.2. 3D Linear Inverted Pendulum

### 5.3. Preview Control

### 5.4. 3D Model Implementation

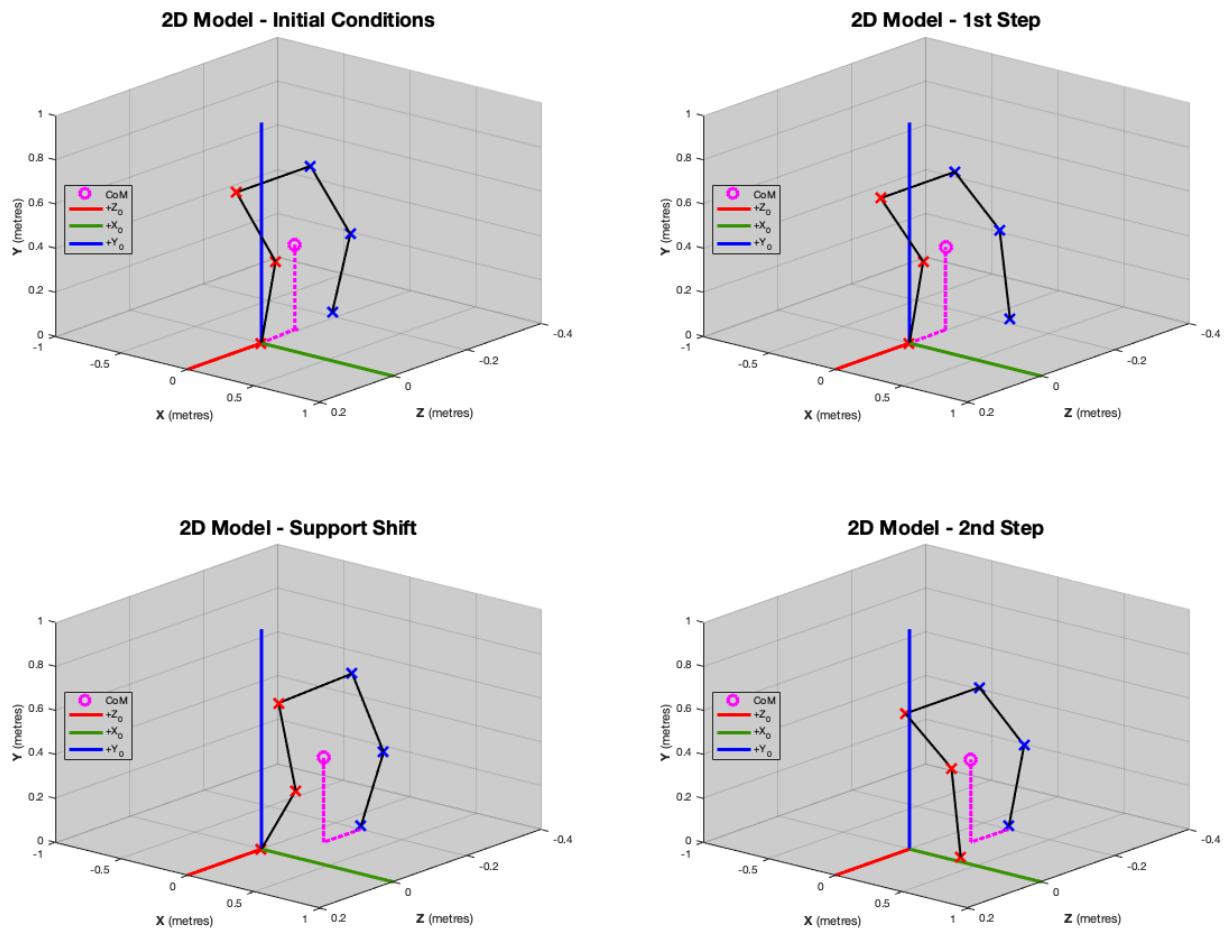


Figure 5: 2D model, plotted in 3D, Quasi Static walk sequence.

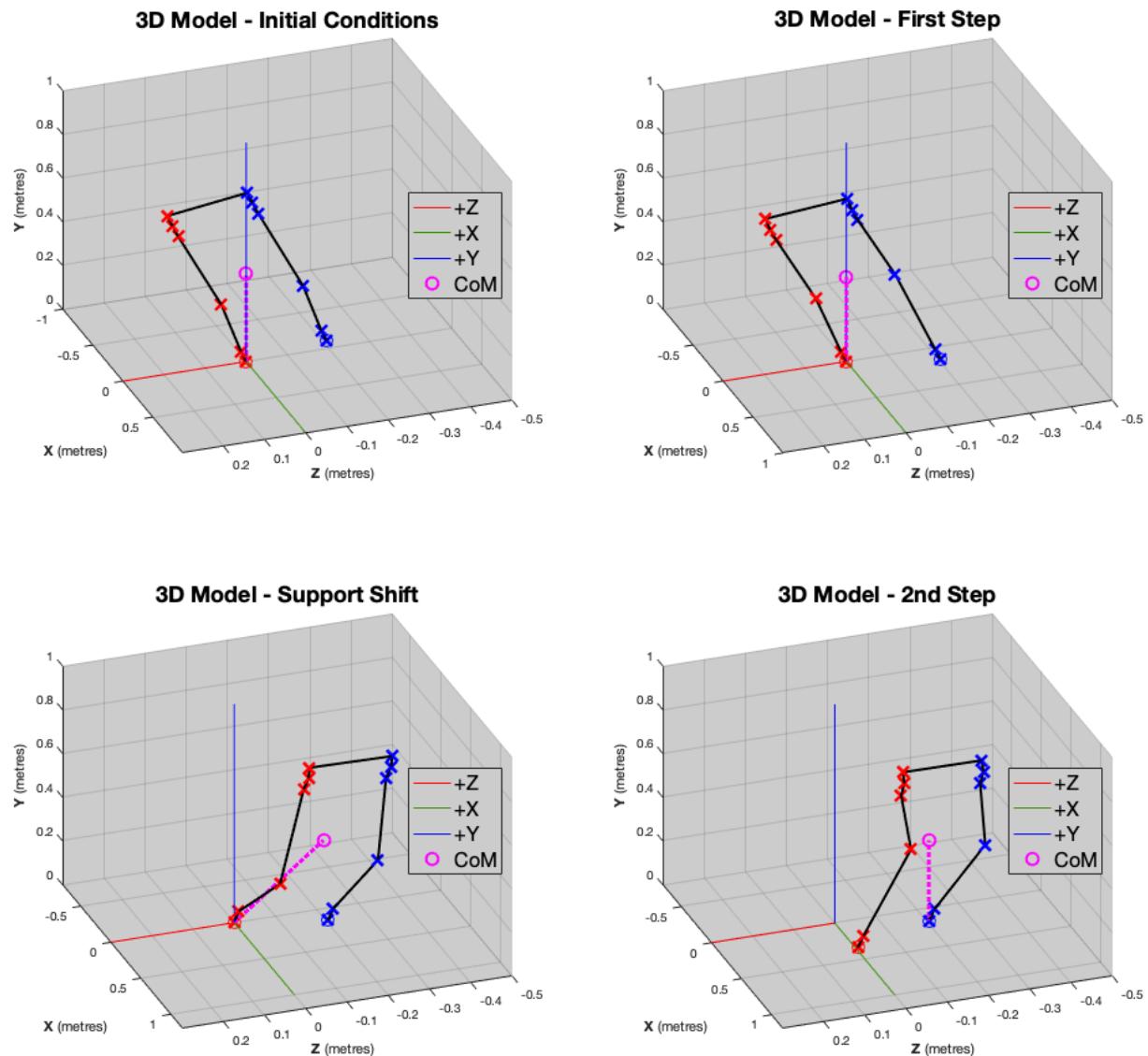


Figure 6: 3D model Quasi-Static walk sequence.

## 6. Results

### 6.1. 2D Model

### 6.2. 3D Model

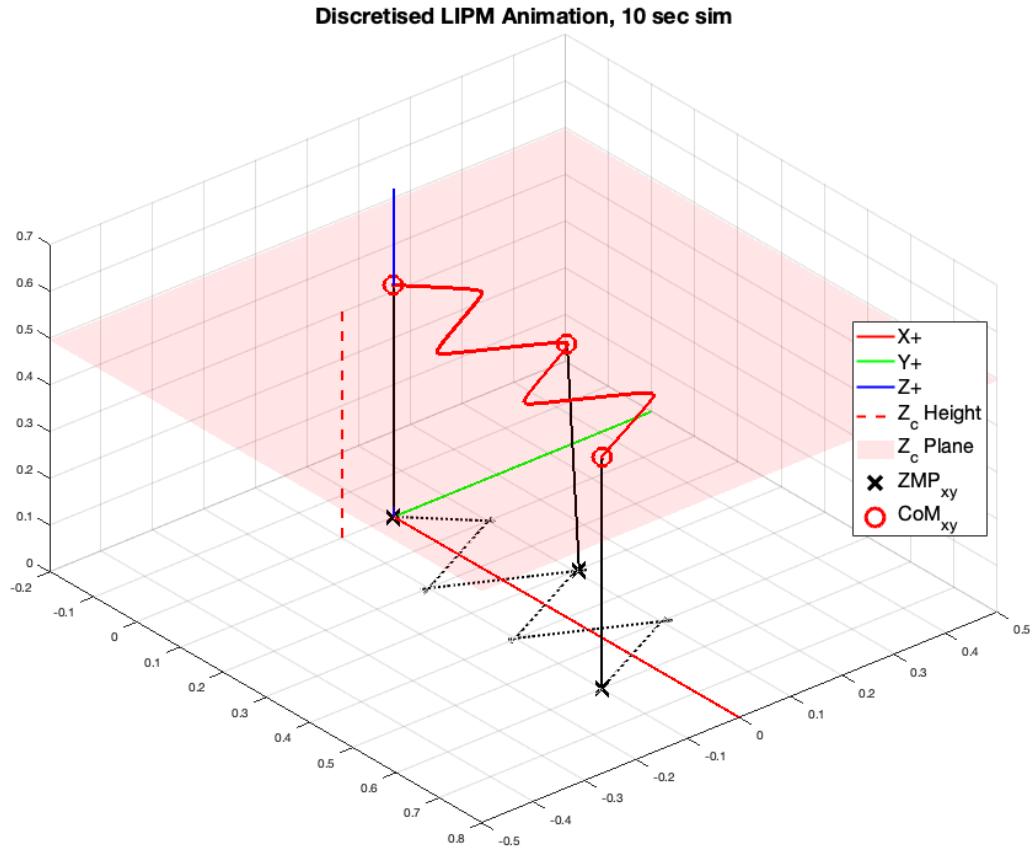


Figure 7: 3D LIPM with Preview Control.

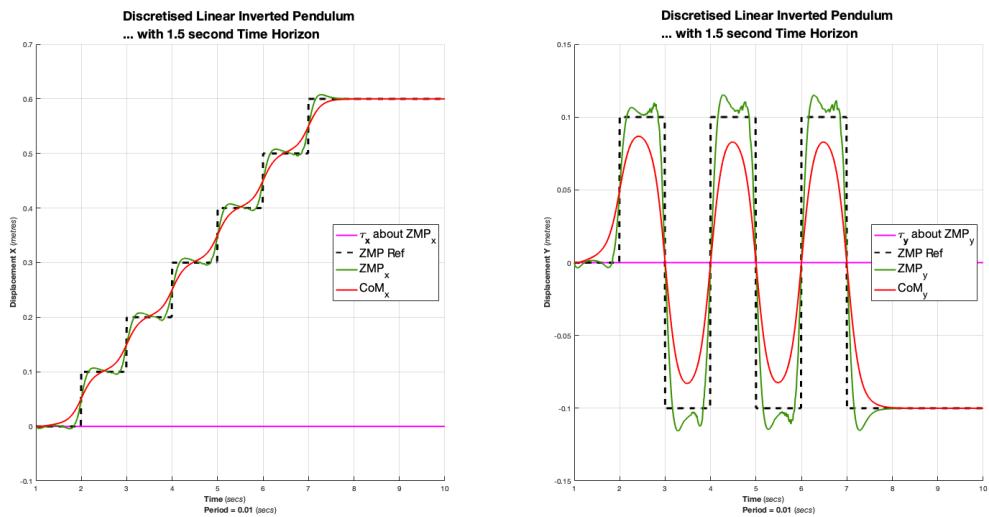


Figure 8: 3D LIPM X Y components.

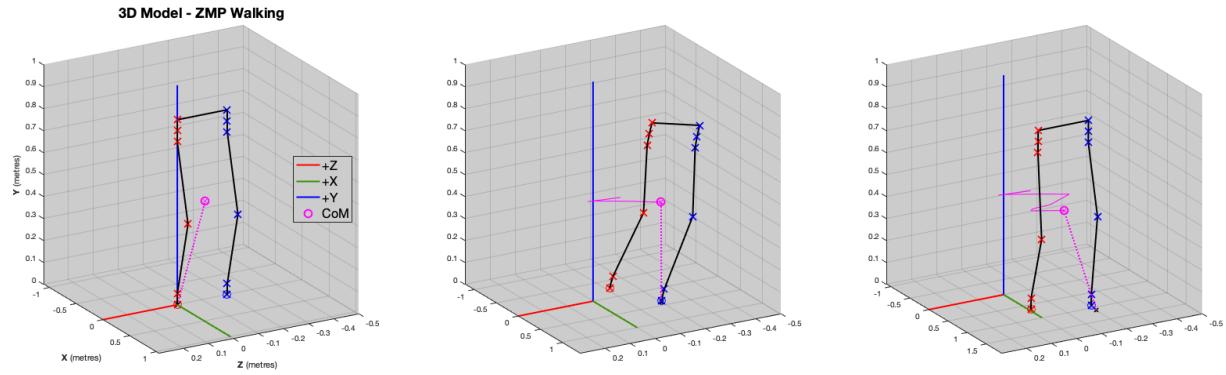


Figure 9: 3D model using ZMP walking with Preview Control.

## 7. Conclusion

This is one of the most important parts of the report. In the conclusion section, you should

- briefly summarise the results,
- reflect on the work presented,
- make recommendations,
- suggest future work or improvements.

## References

- Grimble, M. J., Johnson, M. A., Siciliano, B., Sciavicco, L., Villani, L. & Oriolo, G. (2009), *Robotics*, London Springer London.
- Hirose, S. (1984), 'A study of design and control of a quadruped walking vehicle', *The International Journal of Robotics Research* **3**, 113–133.
- Kajita, S., Kanehiro, F., Kaneko, K., Fujiwara, K., Harada, K., Yokoi, K. & Hirukawa, H. (2003), 'Biped walking pattern generation by using preview control of zero-moment point', *2003 IEEE International Conference on Robotics and Automation (Cat. No.03CH37422)* **3**.
- Kajita, S., Kanehiro, F., Kaneko, K., Fujiwara, K., Yokoi, K. & Hirukawa, H. (2002), 'A realtime pattern generator for biped walking', *Proceedings 2002 IEEE International Conference on Robotics and Automation (Cat. No.02CH37292)* **4**.
- KATAYAMA, T., OHKI, T., INOUE, T. & KATO, T. (1985), 'Design of an optimal controller for a discrete-time system subject to previewable demand', *International Journal of Control* **41**, 677–699.
- Kato, I. (1973), 'The wabot-1', *Bulletin of Science and Engineering Research Laboratory, Waseda University Special Issue on WABOT*.
- Perez, T. (2014), *Fundamentals of mechanical systems*, The University of Newcastle.
- Raibert, M. H. (1986), *Legged robots that balance*, Mit Press.
- Renton, C. (2021), 'Lecture notes: Mcha4100 - mechatronics systems'.
- Vukobratović, M. & Stokić, D. (1983), 'Is dynamic control needed in robotic systems, and, if so, to what extent?', *The International Journal of Robotics Research* **2**, 18–34.

## A. Example of a Table