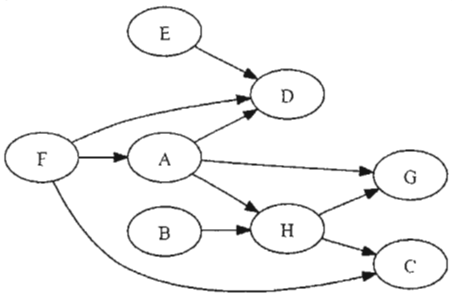
Bayesian Networks for Inference

Exercise H11.1: Directed Acylic Graphs (DAG) (homework, 4 points)

Consider the following DAG:



(a) (1 point) Give a possible topological sorting of the nodes in this DAG.

(b) (2 point) The joint distribution of the corresponding n random variables can be factorized as

$$P(X) = \prod_{i}^{n} P(X_i|\text{parents}(X_i)) \,.$$

Write down the factorization for this DAG.

(c) (1 point) Indicate which nodes belong to the Markov blanket of node A and create the moral graph of the DAG.

Exercise H11.2: Software (homework, 2 points)

Familiarize yourself with software packages for your programming language of choice (e.g., gRain[1] for R or BayesNetToolbox[2] for Matlab or BayesPy[3] for Python).

Implement the "water sprinkler" Bayesian network example in the tutorial by K. Murphy[4]. What is the probability that the sprinkler was active (S=true) after observing that the grass is wet (W=true)? What is the probability after the additional observation that it rained recently (R=true)?

Exercise H11.3: Construction of a DAG (homework, 4 points)

Consider the binary random variables B (Burglary), E (Earthquake), A (Alarm), and R (Radio broadcast) which can all take values that are either "true" (t) or "false" (f).

Assume our knowledge about their co-occurrence is given by the (conditional) probabilities: $P(B = t) = 0.01$, $P(E = t) = 10^{-6}$, $P(R = t|E = f) = 0$, $P(R = t|E = t) = 1$, and

B	E	$P(A = t\|B = b, E = e)$
f	f	0.001
f	t	0.41
t	f	0.95
t	t	0.98

(a) (1 point) Create a DAG representing the corresponding factorization of the joint distribution.

(b) (2 points) Implement the DAG with a software package of your choice (see above) and calculate $P(A = t)$, $P(A|R = t)$, $P(B = t|A = t)$ and $P(B = t|A = t, R = t)$.

(c) (1 point) Explain the phenomenon of *explaining away* using the examples obtained in (b).

Total 10 points.

[1] gRain `http://cran.r-project.org/web/views/gR.html`

[2] BayesNetToolbox `http://code.google.com/p/bnt`

[3] BayesPy `http://www.bayespy.org/intro.html` . Alternative Python packages that may be easier to use are `https://pgmpy.org/` and `https://github.com/eBay/bayesian-belief-networks` .

[4] K. Murphy. A brief introduction to graphical models and Bayesian networks. `http://people.cs.ubc.ca/~murphyk/Bayes/bnintro.html`