MI Exercise Sheet 8 — 2seriously Cool Guys

Exercise H8.1

$$\tilde{C}(p,N) = 2\sum_{k=0}^{N}\binom{p-1}{k}$$

$$(x+y)^n = \sum_{k=0}^{n}\binom{n}{k} x^{n-k} y^k$$

$$\tilde{C}(N+1,N) = 2\sum_{k=0}^{N}\binom{N}{k} \quad ②$$

$$2^n = (1+1)^n = \sum_{k=0}^{n}\binom{n}{k} 1^{n-k} 1^k = \sum_{k=0}^{n}\binom{n}{k} 1^n = \sum_{k=0}^{n}\binom{n}{k} \quad ①$$

$$①,② \Rightarrow \tilde{C}(N+1,N) = 2 \times (1+1)^N = 2^{N+1} \checkmark$$

$$\tilde{C}(N+2,N) = 2\sum_{k=0}^{N+1}\binom{N+1}{k} = \tilde{C}(N+1,N) + \tilde{C}(N+1,N-1) \quad ③$$

$$\Rightarrow \tilde{C}(N+1,N-1) = 2\sum_{k=0}^{N-1}\binom{N}{k} = 2\left[\underbrace{\sum_{k=0}^{N}\binom{N}{k}}_{2^N} - \binom{N}{N}\right]$$

$$\Rightarrow \tilde{C}(N+1,N-1) = 2^{N+1} - 2 \quad ④$$

$$④,③ \Rightarrow \tilde{C}(N+1,N-1) + \tilde{C}(N+1,N) = 2^{N+1} + 2^{N+1} - 2 = 2^{N+2} - 2$$

$$\Rightarrow \tilde{C}(N+2,N) = 2^{N+2} - 2 < 2^{N+2} \checkmark$$