Machine Intelligence 1 **Exercise Sheet 2**
WS 2019/20, Obermayer/Kashef due: 31.10.2018 at 23:55

Connectionist Neurons and Multi Layer Perceptrons

Please remember to upload exactly *one* ZIP file per group and name the file according to the respective group name: `yourgroupname.zip`
The ZIP file should contain a single Jupyter notebook source file as well as a single PDF file that is generated from the Jupyter notebook. Please do **not** include any folder structure, exercise PDF or data files.

Exercise T2.1: Terminology (tutorial)

(a) What does a connectionist neuron compute?

(b) Which effect do the *weights* and the *bias* have?

(c) Why is a nonlinear transfer function beneficial compared to a linear one?

(d) What is a feedforward multilayer perceptron (MLP)?

Exercise H2.1: Connectionist Neuron (homework, 6 points)

The dataset[1] `applesOranges.csv` contains 200 measurements (`x.1` and `x.2`) from two types of objects as indicated by the column `y`. In this exercise, you will use a connectionist neuron with a "binary" transfer function $f(h)$ to classify the objects, i.e., obtain the predicted class y for a data point $\underline{\mathbf{x}} \in \mathbb{R}^2$ by

$$y(\underline{\mathbf{x}}) := f(\underline{\mathbf{w}}^\top \underline{\mathbf{x}} - \theta)$$

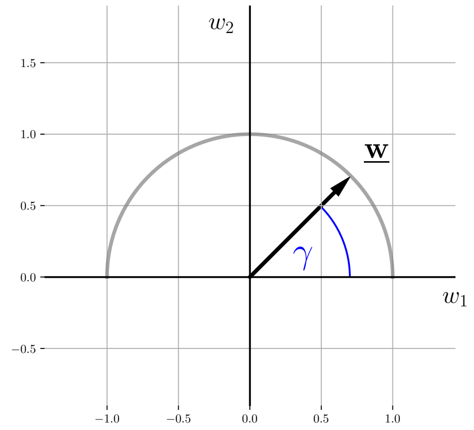
with

$$f(h) := \begin{cases} 1 & \text{for } h \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

where $h := \underline{\mathbf{w}}^\top \underline{\mathbf{x}} - \theta$ is the total input to the neuron.

(a) Plot the data in a scatter plot (x_2 vs. x_1). Mark the points with different colors to indicate the type of each object.

[1] This data file (and those required for future exercise sheets) is available on ISIS.



(b) Set the bias $\theta = 0$. Create a set of 19 weight vectors $\underline{\mathbf{w}} = (w_1, w_2)^\top$ pointing from the origin to the upper semi-circle with radius 1 (i.e. if γ denotes the angle between the weight vector and the x-axis, for each $\gamma = 0, 10, \ldots, 180$ (equally spaced) such that $||\underline{\mathbf{w}}||_2 = 1$, $w_1 \in [-1, 1], w_2 \in [0, 1]$).
For each of these weight vectors $\underline{\mathbf{w}}$,

(i) determine the classification performance ρ (% correct classifications) of the corresponding neuron and

(ii) plot a curve showing ρ as a function of γ.

(c) From the weight vectors generated above, pick the $\underline{\mathbf{w}}$ that yields the best performance. Now vary the bias $\theta \in [-3, 3]$ and pick the value of θ that gives the best performance.

(d) Plot the data points and color them according to the predicted classification when using the $\underline{\mathbf{w}}$ and θ that led to the highest performance. Plot the weight vector $\underline{\mathbf{w}}$ in the same plot. How do you interpret your results?

(e) Find the best combination of $\underline{\mathbf{w}}$ and θ by exploring all combinations of γ and θ (within a reasonable range and precision). Compute and plot the performance of all combinations in a heatmap.

(f) Can the optimization procedure used in (e) be applied to any classification problem? Discuss potential problems and give an application example in which the above method must fail.

Exercise H2.2: Multilayer Perceptrons (MLP) (homework, 4 points)

For an MLP with input $x \in \mathbb{R}$ and 1 hidden layer and 1 output node. The input-output function can be computed as

$$y(x) = \sum_{i=1}^{N_{\text{hid}}} w_{1i}^{21} f(w_{i1}^{10} x - b_i))$$

with output weights w_{1i}^{21} and parameters w_{i1}^{10} and b_i for the i-th hidden unit. The output node has no bias.

(a) Create 50 independent MLPs with $N_{\text{hid}} = 10$ hidden units by sampling for each MLP a set of random parameters $\{w_{1i}^{21}, w_{i1}^{10}, b_i\}, i = 1, ..., 10$.

- Use $f(\cdot) := \tanh(\cdot)$ as the transfer function.
- Use normally distributed $w_{1i}^{21} \sim \mathcal{N}(0, 1)$
- Use normally distributed $w_{i1}^{10} \sim \mathcal{N}(0, 2)$ and uniformly distributed $b_i \sim \mathcal{U}(-2, 2)$.

(b) Plot the input-output functions (i.e. the response $y(x)$) of these 50 MLPs for $x \in [-2, 2]$.

(c) Repeat this procedure using instead $w_{i1}^{10} \sim \mathcal{N}(0, 0.5)$. What difference can you observe?

(d) Compute the mean squared error between each of these 2x50 (50 from each of the above two initialization procedures) input-output functions and the function $g(x) = -x$. For each of the two procedures, which MLP approximates g best? Plot $y(x)$ for these two MLPs

Total 10 points.