4SeriouslySexyStudents-Assignment#4

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1.1 EX#1.2

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In [1]: import numpy as np
        import matplotlib.pyplot as plt
```

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In [24]: def p(x, m=1, b=2):
             return 1 / (2 * b) * np.exp(-(np.abs(x - m) / b))

         def f_inverse(y, m=1, b=2):
             return np.where(y >= 0.5, m - b * np.log(2 - 2 * y), m + b * np.log(2 * y))

         samples = np.asarray([f_inverse(y) for y in np.random.random(500)])

         x = np.linspace(samples.min(), samples.max(), 700)
         pdf = p(x)

         plt.hist(samples, bins=70, density=True)
         plt.plot(x, pdf)
         plt.xlabel('samples')
         plt.ylabel('pdf')
         plt.show()
```

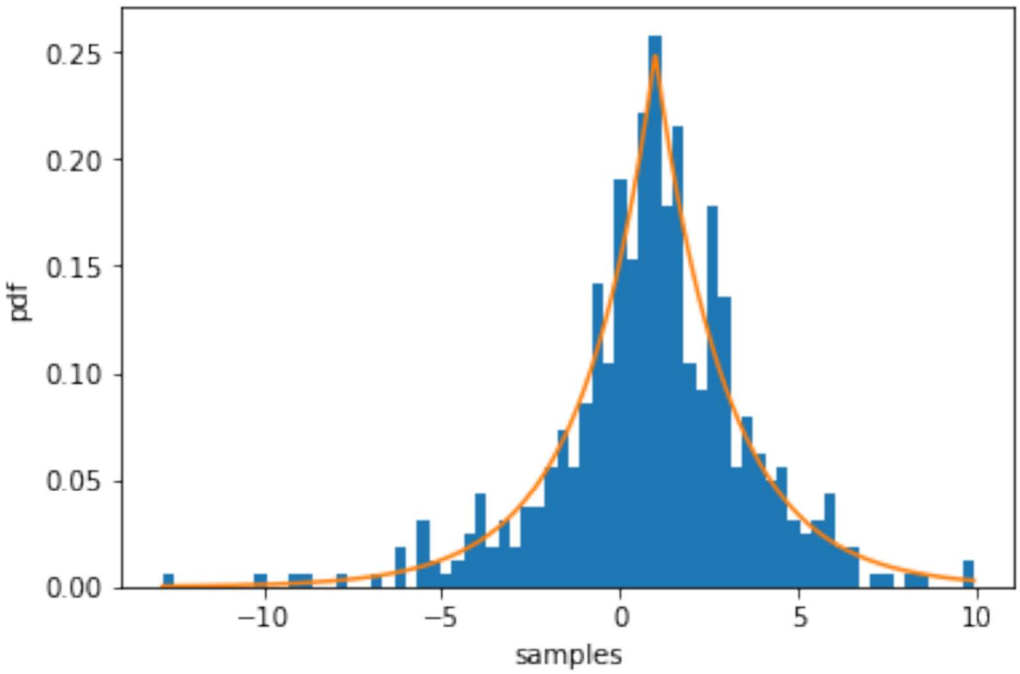
pdf
0.25
0.20
0.15
0.10
0.05
0.00
−10
−5
0
5
10
samples

Q#1

$$P_X(x) = \frac{1}{2b} * e^{\frac{-|x-\mu|}{b}}$$

$$= \begin{cases} \frac{1}{2b} \cdot e^{\frac{-x-\mu}{b}} & x \geq \mu \\ \frac{1}{2b} \cdot e^{\frac{x-\mu}{b}} & x < \mu \end{cases}$$

⇒ Compute CDF

$$F_X(x) = \begin{cases} \int_{-\infty}^{\mu} \frac{1}{2b} \cdot e^{\frac{-x-\mu}{b}} + \int_{\mu}^{x} \frac{1}{2b} \cdot e^{-\frac{x-\mu}{b}} = \frac{1}{2}(1) - \frac{1}{2}e^{\frac{-x-\mu}{b}} + \frac{1}{2} \\ \int_{-\infty}^{x} \frac{1}{2b} \cdot e^{\frac{x-\mu}{b}} = \frac{1}{2}\left[e^{\frac{x-\mu}{b}} - 0\right] \end{cases}$$

$$= \begin{cases} 1 - \frac{1}{2} e^{\frac{-x-\mu}{b}} & x \geq \mu \\ \frac{1}{2} e^{\frac{x-\mu}{b}} & x < \mu \end{cases}$$

⇒ Compute inverse CDF

(a) $y = 1 - \frac{1}{2} e^{\frac{-x-\mu}{b}} \Rightarrow x = -b \cdot \ln(2-2y) + \mu$

b) $y = \frac{1}{2} e^{\frac{x-\mu}{b}} \Rightarrow x = b \cdot \ln(2y) + \mu$

$$F_X^{-1}(x) = \begin{cases} \mu - b \cdot \ln(2-2y) & y \geq 0.5 \\ \mu + b \cdot \ln(2y) & y < 0.5 \end{cases}$$

Q #2

(a) $P_x(x) = e^{-x}$

$u(x) = e^{-x} \Rightarrow X_{(u)} = -\ln(y)$

$$\boxed{y = e^{-x} \Rightarrow x = -\ln(y)}$$

$$P_u(x)(u) = e^{\ln(u)} \cdot \left| \frac{\partial x(u)}{\partial u} \right|$$

$$= \frac{u}{u} - !$$

(b) $u_1 = \sqrt{-2\ln x_1} \cdot \cos(2\pi x_2)$, $u_2 = \sqrt{-2\ln x_1} \cdot \sin(2\pi x_2)$

$$\frac{u_2}{u_1} = \frac{\sin(2\pi x_2)}{\cos(2\pi x_2)} = \tan(2\pi x_2)$$

$$\Rightarrow x_2 = \tan^{-1}\left(\frac{u_2}{u_1}\right) * 1/2\pi$$

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$$u_1^2 + u_2^2 = (-2\ln x_1)\overbrace{\left(\cos^2(2\pi x_2) + \sin^2(2\pi x_2)\right)}^{1}$$

$$x_1 = e^{\frac{u_1^2+u_2^2}{-2}} \quad \text{--- (1)}$$

$$J = \begin{bmatrix} \frac{\partial x_1}{\partial u_1} & \frac{\partial x_1}{\partial u_2} \\ \frac{\partial x_2}{\partial u_1} & \frac{\partial x_2}{\partial u_2} \end{bmatrix} = \begin{bmatrix} -u_1 \cdot e^{\frac{u_1^2+u_2^2}{-2}} & -u_2 \cdot e^{\frac{u_1^2+u_2^2}{-2}} \\ \frac{u_2}{2\pi(u_1^2+u_2^2)} & \frac{-u_1}{2\pi(u_1^2+u_2^2)} \end{bmatrix}$$

$$\frac{\partial x_1}{\partial u_1} = -u_1 \cdot e^{\frac{u_1^2+u_2^2}{-2}}$$

$$\frac{\partial x_1}{\partial u_2} = -u_2 \cdot e^{\frac{u_1^2+u_2^2}{-2}}$$

$$\frac{\partial x_2}{\partial u_1} = \frac{u_2}{2\pi(u_1^2+u_2^2)}$$

$$\frac{\partial x_2}{\partial u_2} = \frac{-u_1}{2\pi(u_1^2+u_2^2)}$$

$$\det J = \frac{\partial x_1}{\partial u_1} * \frac{\partial x_2}{\partial u_2} - \frac{\partial x_1}{\partial u_2} \cdot \frac{\partial x_2}{\partial u_1}$$

$$= \frac{u_1^2 \cdot e^{\frac{u_1^2+u_2^2}{-2}}}{2\pi(u_1^2+u_2^2)} + \frac{u_2^2 \cdot e^{\frac{u_1^2+u_2^2}{-2}}}{2\pi(u_1^2+u_2^2)}$$

$$= 1/2\pi \cdot e^{\frac{u_1^2+u_2^2}{-2}}$$

<u>In Gaussian</u> $\stackrel{\circ}{=} \dfrac{1}{\sqrt{2\pi\delta^2}} \cdot e^{\frac{-(x-\mu)^2}{2\delta}}$

$$= \frac{1}{\sqrt{2\pi}} \cdot e^{\frac{-x^2}{2}}$$

$\delta = 1$
$\mu = 0$