Topic: Second derivatives with implicit differentiation

Question: Use implicit differentiation to find the second derivative.

$$x^3 + y^3 = 9$$

Answer choices:

A $y'' = -\dfrac{2x(y^3 - x^3)}{y^6}$

B $y'' = -\dfrac{2x(x^3 + y^3)}{y^5}$

C $y'' = -\dfrac{2x(y^3 - x^3)}{y^5}$

D $y'' = -\dfrac{2x(x^3 + y^3)}{y^6}$

Solution: B

The first derivative is

$$3x^2 + 3y^2y' = 0$$

Solve for y'.

$$3y^2y' = -3x^2$$

$$y' = \frac{-3x^2}{3y^2}$$

$$y' = \frac{-x^2}{y^2}$$

Use quotient rule to find the second derivative.

$$y'' = \frac{(-2x)(y^2) - (-x^2)(2yy')}{(y^2)^2}$$

$$y'' = \frac{-2xy^2 + 2x^2yy'}{y^4}$$

$$y'' = \frac{2xy(-y + xy')}{y^4}$$

$$y'' = \frac{2x(-y + xy')}{y^3}$$

Substitute for the first derivative.

$$y'' = \frac{2x\left[-y + x\left(\frac{-x^2}{y^2}\right)\right]}{y^3}$$

$$y'' = \frac{2x\left(-y - \frac{x^3}{y^2}\right)}{y^3}$$

Find a common denominator within the numerator, then combine the fractions in the numerator into one fraction.

$$y'' = \frac{2x\left(\frac{-y^3 - x^3}{y^2}\right)}{y^3}$$

$$y'' = 2x\left(\frac{-y^3 - x^3}{y^5}\right)$$

$$y'' = -\frac{2x(x^3 + y^3)}{y^5}$$

Topic: Second derivatives with implicit differentiation

Question: Use implicit differentiation to find the second derivative.

$$x^2 + 2xy + y^3 = 25$$

Answer choices:

A $$y'' = \frac{8x^2 - 24x^2y + 16xy - 48xy^2 - 18y^4}{(2x + 3y^2)^3}$$

B $$y'' = -\frac{8x^2 - 24x^2y + 16xy - 48xy^2 - 18y^4}{(2x + 3y^2)^3}$$

C $$y'' = -\frac{24x^2 + 18y^4}{(2x + 3y^2)^3}$$

D $$y'' = \frac{24x^2 + 18y^4}{(2x + 3y^2)^3}$$

Solution: A

The first derivative is

$$2x + [(2)(y) + (2x)(1)(y')] + 3y^2y' = 0$$

$$2x + 2y + 2xy' + 3y^2y' = 0$$

Solve for y'.

$$2xy' + 3y^2y' = -2x - 2y$$

$$y'(2x + 3y^2) = -2x - 2y$$

$$y' = \frac{-2x - 2y}{2x + 3y^2}$$

Use quotient rule to find the second derivative.

$$y'' = \frac{(-2 - 2(1)(y'))(2x + 3y^2) - (-2x - 2y)(2 + 6yy')}{(2x + 3y^2)^2}$$

$$y'' = \frac{(-2 - 2y')(2x + 3y^2) + (2x + 2y)(2 + 6yy')}{(2x + 3y^2)^2}$$

$$y'' = \frac{-4x - 6y^2 - 4xy' - 6y^2y' + 4x + 12xyy' + 4y + 12y^2y'}{(2x + 3y^2)^2}$$

Collect like terms, then factor out y'.

$$y'' = \frac{-4xy' + 12xyy' + 6y^2y' + 4y - 6y^2}{(2x + 3y^2)^2}$$

$$y'' = \frac{y'(-4x + 12xy + 6y^2) + 4y - 6y^2}{(2x + 3y^2)^2}$$

Substitute for the first derivative.

$$y'' = \frac{\frac{-2x - 2y}{2x + 3y^2}(-4x + 12xy + 6y^2) + 4y - 6y^2}{(2x + 3y^2)^2}$$

$$y'' = \frac{\frac{(-2x - 2y)(-4x + 12xy + 6y^2)}{2x + 3y^2} + \frac{(4y - 6y^2)(2x + 3y^2)}{2x + 3y^2}}{(2x + 3y^2)^2}$$

$$y'' = \frac{\frac{(-2x - 2y)(-4x + 12xy + 6y^2) + (4y - 6y^2)(2x + 3y^2)}{2x + 3y^2}}{(2x + 3y^2)^2}$$

$$y'' = \frac{(-2x - 2y)(-4x + 12xy + 6y^2) + (4y - 6y^2)(2x + 3y^2)}{(2x + 3y^2)^3}$$

Expand the numerator, then simplify.

$$y'' = \frac{(8x^2 - 24x^2y - 12xy^2 + 8xy - 24xy^2 - 12y^3) + (8xy + 12y^3 - 12xy^2 - 18y^4)}{(2x + 3y^2)^3}$$

$$y'' = \frac{8x^2 - 24x^2y - 12xy^2 + 8xy - 24xy^2 - 12y^3 + 8xy + 12y^3 - 12xy^2 - 18y^4}{(2x + 3y^2)^3}$$

$$y'' = \frac{8x^2 - 24x^2y + 16xy - 48xy^2 - 18y^4}{(2x + 3y^2)^3}$$

Topic: Second derivatives with implicit differentiation

Question: Use implicit differentiation to find the second derivative.

$$x^2y^2 + 3xy = 100$$

Answer choices:

A $y'' = -\frac{y}{x^2}$

B $y'' = \frac{y}{x^2}$

C $y'' = -\frac{2y}{x^2}$

D $y'' = \frac{2y}{x^2}$

Solution: D

The first derivative is

$$[(2x)(y^2) + (x^2)(2y)(y')] + [(3)(y) + (3x)(1)(y')] = 0$$

$$2xy^2 + 2x^2yy' + 3y + 3xy' = 0$$

Solve for y'.

$$2x^2yy' + 3xy' = -2xy^2 - 3y$$

$$y'(2x^2y + 3x) = -2xy^2 - 3y$$

$$y' = \frac{-2xy^2 - 3y}{2x^2y + 3x}$$

Factor and simplify the first derivative.

$$y' = \frac{-y(2xy + 3)}{x(2xy + 3)}$$

$$y' = -\frac{y}{x}$$

Use the quotient rule to find the second derivative.

$$y'' = -\frac{(y')(x) - (y)(1)}{x^2}$$

$$y'' = -\frac{xy' - y}{x^2}$$

Substitute for the first derivative.

$$y'' = -\frac{x\left(-\frac{y}{x}\right) - y}{x^2}$$

$$y'' = -\frac{-y - y}{x^2}$$

$$y'' = \frac{y + y}{x^2}$$

$$y'' = \frac{2y}{x^2}$$