Calculus 1 Workbook Solutions

Modifying functions

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MATH

COMBINATIONS OF FUNCTIONS

1. Find $(f+g)(x)$.

$$f(x)=2x^2-x+5$$

$$g(x)=x^2+4x-7$$

Solution:

The combination $(f+g)(x)$ is the same as $f(x)+g(x)$, so we need to add the equations together.

$$(f+g)(x)=f(x)+g(x)$$

$$(f+g)(x)=2x^2-x+5+x^2+4x-7$$

Group like terms together and combine.

$$(f+g)(x)=2x^2+x^2-x+4x+5-7$$

$$(f+g)(x)=3x^2+3x-2$$

2. Find $(f-g)(x)$.

$$f(x)=4x^2-2$$

$$g(x)=3x^2-5x$$

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Solution:

The combination $(f-g)(x)$ is the same as $f(x)-g(x)$, so we need to find the difference.

$$(f-g)(x)=f(x)-g(x)$$

$$(f-g)(x)=4x^2-2-(3x^2-5x)$$

$$(f-g)(x)=4x^2-2-3x^2+5x$$

Group like terms together and combine.

$$(f-g)(x)=4x^2-3x^2+5x-2$$

$$(f-g)(x)=x^2+5x-2$$

■ 3. Find $(f-g)(x)$.

$$f(x)=x^2-3x+1$$

$$g(x)=2x-3$$

Solution:

The combination $(f-g)(x)$ is the same as $f(x)-g(x)$, so we need to find the difference.

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$$(f-g)(x) = f(x) - g(x)$$

$$(f-g)(x) = x^2 - 3x + 1 - (2x - 3)$$

$$(f-g)(x) = x^2 - 3x + 1 - 2x + 3$$

Group like terms together and combine.

$$(f-g)(x) = x^2 - 3x - 2x + 1 + 3$$

$$(f-g)(x) = x^2 - 5x + 4$$

4. Find $(f \cdot g)(x)$.

$$f(x) = 2x - 3$$

$$g(x) = 3x^2 + 2$$

Solution:

The combination $(f \cdot g)(x)$ is the same as $f(x) \cdot g(x)$, so we need to find the product.

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$(f \cdot g)(x) = (2x - 3)(3x^2 + 2)$$

Multiply using the FOIL method.

$$(f \cdot g)(x) = 6x^3 + 4x - 9x^2 - 6$$

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$$(f \cdot g)(x) = 6x^3 - 9x^2 + 4x - 6$$

5. Find $(f \cdot g)(x)$.

$$f(x) = x - 3$$

$$g(x) = x + 4$$

Solution:

The combination $(f \cdot g)(x)$ is the same as $f(x) \cdot g(x)$, so we need to find the product.

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$(f \cdot g)(x) = (x - 3)(x + 4)$$

Multiply using the FOIL method.

$$(f \cdot g)(x) = x^2 + 4x - 3x - 12$$

$$(f \cdot g)(x) = x^2 + x - 12$$

6. Find $(f \div g)(x)$.

$$f(x) = x^2 + 6x$$

$$g(x) = x$$

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Solution:

The combination $(f \div g)(x)$ is the same as $f(x)/g(x)$, so we need to find the quotient.

$$(f \div g)(x) = \frac{x^2 + 6x}{x}$$

$$(f \div g)(x) = \frac{x(x+6)}{x}$$

$$(f \div g)(x) = x + 6$$

7. Find $(g \div f)(x)$.

$$f(x) = x^2 + 6x$$

$$g(x) = x$$

Solution:

The combination $(g \div f)(x)$ is the same as $g(x)/f(x)$, so we need to find the quotient.

$$(g \div f)(x) = \frac{x}{x^2 + 6x}$$

$$(g \div f)(x) = \frac{x}{x(x+6)}$$

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$$(g \div f)(x) = \frac{1}{x+6}$$

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COMPOSITE FUNCTIONS

1. Find the composite function $(f \circ g)(x)$.

$$f(x) = \sqrt{2x - 1}$$

$$g(x) = 3x^2$$

Solution:

When we take the composite $(f \circ g)(x)$, we plug $g(x)$ into $f(x)$.

$$(f \circ g)(x) = f(g(x)) = \sqrt{2(3x^2) - 1}$$

$$(f \circ g)(x) = f(g(x)) = \sqrt{6x^2 - 1}$$

2. Find the composite function $(g \circ f)(x)$.

$$f(x) = \sqrt{2x - 1}$$

$$g(x) = 3x^2$$

Solution:

When we take the composite $(f \circ g)(x)$, we plug $f(x)$ into $g(x)$.

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$$(g \circ f)(x) = g(f(x)) = 3(\sqrt{2x-1})^2$$

$$(g \circ f)(x) = g(f(x)) = 3(2x-1)$$

$$(g \circ f)(x) = g(f(x)) = 6x-3$$

3. Find the composite function $f(g(x))$.

$$f(x) = x^2 - 4x + 3$$

$$g(x) = 2x + 1$$

Solution:

When we take the composite $f(g(x))$, we plug $g(x)$ into $f(x)$.

$$f(g(x)) = (2x+1)^2 - 4(2x+1) + 3$$

$$f(g(x)) = (2x+1)(2x+1) - 8x - 4 + 3$$

$$f(g(x)) = 4x^2 + 2x + 2x + 1 - 8x - 1$$

Group and combine like terms.

$$f(g(x)) = 4x^2 + 2x + 2x - 8x + 1 - 1$$

$$f(g(x)) = 4x^2 + 4x - 8x$$

$$f(g(x)) = 4x^2 - 4x$$

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4. Find the composite function $g(f(x))$.

$$f(x) = x^2 - 4x + 3$$

$$g(x) = 2x + 1$$

Solution:

When we take the composite $g(f(x))$, we plug $f(x)$ into $g(x)$.

$$g(f(x)) = 2(x^2 - 4x + 3) + 1$$

$$g(f(x)) = 2x^2 - 8x + 6 + 1$$

$$g(f(x)) = 2x^2 - 8x + 7$$

5. Find the composite function $(g \circ h)(x)$.

$$g(x) = \frac{8}{x^3}$$

$$h(x) = \sqrt[3]{x + 4}$$

Solution:

When we take the composite $(g \circ h)(x)$, we plug $h(x)$ into $g(x)$.

$$(g \circ h)(x) = g(h(x)) = \frac{8}{(\sqrt[3]{x + 4})^3}$$

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$$(g \circ h)(x) = g(h(x)) = \frac{8}{x+4}$$

6. Find the composite function $(h \circ g)(x)$.

$$g(x) = \frac{8}{x^3}$$

$$h(x) = \sqrt[3]{x+4}$$

Solution:

When we take the composite $(h \circ g)(x)$, we plug $g(x)$ into $h(x)$.

$$(h \circ g)(x) = h(g(x)) = \sqrt[3]{\frac{8}{x^3} + 4}$$

$$(h \circ g)(x) = h(g(x)) = \sqrt[3]{\frac{8}{x^3} + 4\frac{x^3}{x^3}}$$

$$(h \circ g)(x) = h(g(x)) = \sqrt[3]{\frac{8+4x^3}{x^3}}$$

Take the root of the numerator and denominator separately.

$$(h \circ g)(x) = h(g(x)) = \frac{\sqrt[3]{8+4x^3}}{\sqrt[3]{x^3}}$$

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$$(h \circ g)(x) = h(g(x)) = \frac{\sqrt[3]{8 + 4x^3}}{x}$$

■ 7. Find the composite function $g(h(x))$.

$$g(x) = \frac{1}{x}$$

$$h(x) = 3x^2 - x$$

Solution:

When we take the composite $g(h(x))$, we plug $h(x)$ into $g(x)$.

$$g(h(x)) = \frac{1}{3x^2 - x}$$

■ 8. Find the composite function $h(g(x))$.

$$g(x) = \frac{1}{x}$$

$$h(x) = 3x^2 - x$$

Solution:

When we take the composite $h(g(x))$, we plug $g(x)$ into $h(x)$.

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$$h(g(x)) = 3\left(\frac{1}{x}\right)^2 - \frac{1}{x}$$

$$h(g(x)) = \frac{3}{x^2} - \frac{1}{x}$$

Find a common denominator to combine the fractions.

$$h(g(x)) = \frac{3}{x^2} - \frac{1}{x}\left(\frac{x}{x}\right)$$

$$h(g(x)) = \frac{3}{x^2} - \frac{x}{x^2}$$

$$h(g(x)) = \frac{3-x}{x^2}$$

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COMPOSITE FUNCTIONS, DOMAIN

1. What is the domain of $f \circ g$?

$$f(x) = x^2 - 2$$

$$g(x) = \sqrt{x+3}$$

Solution:

Find the domain of $g(x)$. Remember that you can't take the square root of negative numbers, since negative roots can't be defined by real numbers, so the expression inside the root must be positive or equal to 0.

$$x + 3 \geq 0$$

$$x \geq -3$$

Now find $f \circ g$.

$$f \circ g = (\sqrt{x+3})^2 - 2$$

$$f \circ g = x + 3 - 2$$

$$f \circ g = x + 1$$

We need to consider the domain of this composite function. The composite is a simple binomial with no domain restrictions. There's only the domain restriction that we found for $g(x)$, but all restrictions of $g(x)$

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need to be included in the domain for $f(g(x))$. Therefore, the domain of $f(g(x))$ is $x \geq -3$.

2. What is the domain of $f \circ g$?

$$f(x) = \frac{1}{x}$$

$$g(x) = x + 5$$

Solution:

Find the domain of $g(x)$. In this case, $g(x)$ is a simple binomial with no domain restrictions.

Now find $f \circ g$.

$$f \circ g = \frac{1}{x+5}$$

We need to consider the domain of this composite function. Remember that we can't divide by 0, which means the denominator of a fraction can never equal 0.

$$x + 5 \neq 0$$

$$x \neq -5$$

There were no restrictions on the domain of $g(x)$, so the domain of $f(g(x))$ is just $x \neq -5$.

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3. What is the domain of $f \circ g$?

$$f(x) = \frac{2}{x-1}$$

$$g(x) = \sqrt{x-4}$$

Solution:

Find the domain of $g(x)$. Remember that you can't take the square root of negative numbers, since negative roots can't be defined by real numbers, so the expression inside the root must be positive or equal to 0.

$$x - 4 \geq 0$$

$$x \geq 4$$

Now find $f \circ g$.

$$f \circ g = \frac{2}{\sqrt{x-4} - 1}$$

We need to consider the domain of this composite function. Remember that we can't divide by 0, which means the denominator of a fraction can never equal 0.

$$\sqrt{x-4} - 1 \neq 0$$

$$\sqrt{x-4} \neq 1$$

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$$x - 4 \neq 1$$

$$x \neq 5$$

The domain of the composite has to account for domain restrictions on $g(x)$ and the composite function $f \circ g$ itself, so the domain of $f(g(x))$ is $x \geq 4$, $x \neq 5$.

■ 4. What is the domain of $f \circ g$?

$$f(x) = \frac{1}{x} + 4$$

$$g(x) = \frac{3}{2x - 7}$$

Solution:

Find the domain of $g(x)$. Remember that we can't divide by 0, which means the denominator of a fraction can never equal 0.

$$2x - 7 \neq 0$$

$$2x \neq 7$$

$$x \neq \frac{7}{2}$$

Now find $f \circ g$.

$$f \circ g = \frac{1}{\frac{3}{2x-7}} + 4$$

$$f \circ g = \frac{2x-7}{3} + 4$$

$$f \circ g = \frac{2x-7}{3} + \frac{12}{3}$$

$$f \circ g = \frac{2x-7+12}{3}$$

$$f \circ g = \frac{2x+5}{3}$$

We need to consider the domain of this composite function. But there are no domain restrictions on $f \circ g$, so the only restriction we need to consider is the one on $g(x)$. The domain of $f(g(x))$ is $x \neq 7/2$.

■ 5. What is the domain of $f \circ g$?

$$f(x) = \frac{2}{x-3}$$

$$g(x) = \frac{4}{x+2}$$

Solution:

Find the domain of $g(x)$. Remember that we can't divide by 0, which means the denominator of a fraction can never equal 0.

$$x+2\neq 0$$

$$x\neq -2$$

Now find $f\circ g$.

$$f\circ g=\frac{2}{\frac{4}{x+2}-3}$$

$$f\circ g=\frac{2}{\frac{4}{x+2}-3\left(\frac{x+2}{x+2}\right)}$$

$$f\circ g=\frac{2}{\frac{4}{x+2}-\frac{3x+6}{x+2}}$$

Combine fractions within the denominator.

$$f\circ g=\frac{2}{\frac{4-3x-6}{x+2}}$$

$$f\circ g=\frac{2}{\frac{-3x-2}{x+2}}$$

$$f\circ g=\frac{2(x+2)}{-3x-2}$$

$$f\circ g=\frac{2x+4}{-3x-2}$$

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$$f \circ g = -\frac{2x+4}{3x+2}$$

We need to consider the domain of this composite function. Remember that we can't divide by 0, which means the denominator of a fraction can never equal 0.

$$3x + 2 \neq 0$$

$$3x \neq -2$$

$$x \neq -\frac{2}{3}$$

Putting this restriction together with the one we found for $g(x)$, we can say that the domain of $f(g(x))$ is $x \neq -2, \ -2/3$.

■ 6. What is the domain of $f \circ g$?

$$f(x) = \frac{1}{x^2 - 3}$$

$$g(x) = \sqrt{x-1}$$

Solution:

Find the domain of $g(x)$. Remember that you can't take the square root of negative numbers, since negative roots can't be defined by real numbers, so the expression inside the root must be positive or equal to 0.

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$$x - 1 \geq 0$$

$$x \geq 1$$

Now find $f \circ g$.

$$f \circ g = \frac{1}{(\sqrt{x-1})^2 - 3}$$

$$f \circ g = \frac{1}{x - 1 - 3}$$

$$f \circ g = \frac{1}{x - 4}$$

We need to consider the domain of this composite function. Remember that we can't divide by 0, which means the denominator of a fraction can never equal 0.

$$x - 4 \neq 0$$

$$x \neq 4$$

Putting this restriction together with the one we found for $g(x)$, we can say that the domain of $f(g(x))$ is $x \geq 1$, $x \neq 4$.

■ 7. What is the domain of $f \circ g$?

$$f(x) = 2x^2 - x + 1$$

$$g(x) = x - 3$$

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Solution:

Find the domain of $g(x)$. In this case $g(x)$ is a simple binomial with no domain restrictions.

Now find $f \circ g$.

$$f \circ g = 2(x-3)^2 - (x-3) + 1$$

$$f \circ g = 2(x^2 - 6x + 9) - x + 3 + 1$$

$$f \circ g = 2x^2 - 12x + 18 - x + 4$$

$$f \circ g = 2x^2 - 13x + 22$$

We need to consider the domain of this composite function. But there are no domain restrictions on $f \circ g$. And, since there are also no domain restrictions for $g(x)$, the domain of $f(g(x))$ is all real numbers.

■ 8. What is the domain of $f \circ g$?

$$f(x) = x^2 + 4x - 10$$

$$g(x) = x + 6$$

Solution:

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Find the domain of $g(x)$. In this case $g(x)$ is a simple binomial with no domain restrictions.

Now find $f \circ g$.

$$f \circ g = (x+6)^2 + 4(x+6) - 10$$

$$f \circ g = x^2 + 12x + 36 + 4x + 24 - 10$$

$$f \circ g = x^2 + 16x + 50$$

There are no domain restrictions for $f \circ g$. And, since there are also no domain restrictions for $g(x)$, the domain of $f(g(x))$ is all real numbers.

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