

Topic: Product rule, three or more functions**Question:** Use the product rule to find the derivative of the function.

$$f(x) = (3x^2)(x)(-2x^4)$$

Answer choices:

- A $f'(x) = 6x^7$
- B $f'(x) = 42x^6$
- C $f'(x) = -6x^7$
- D $f'(x) = -42x^6$



Solution: D

Let $r(x) = 3x^2$, $s(x) = x$, and $t(x) = -2x^4$. Find each of their derivatives.

$$r(x) = 3x^2$$

$$r'(x) = 6x$$

and

$$s(x) = x$$

$$s'(x) = 1$$

and

$$t(x) = -2x^4$$

$$t'(x) = -8x^3$$

Apply product rule.

$$f'(x) = r'(x)s(x)t(x) + r(x)s'(x)t(x) + r(x)s(x)t'(x)$$

$$f'(x) = (6x)(x)(-2x^4) + (3x^2)(1)(-2x^4) + (3x^2)(x)(-8x^3)$$

Expand the derivative, then collect like terms.

$$f'(x) = (6x^2)(-2x^4) + (3x^2)(-2x^4) + (3x^3)(-8x^3)$$

$$f'(x) = -12x^6 - 6x^6 - 24x^6$$

$$f'(x) = -42x^6$$



Topic: Product rule, three or more functions**Question:** Use the product rule to find the derivative of the function.

$$f(x) = (x + 3)(2x^2 - 5)(-x^3 + 2)$$

Answer choices:

- A $f'(x) = -12x^3$
- B $f'(x) = -12x^5 - 30x^4 + 20x^3 + 57x^2 + 24x - 10$
- C $f'(x) = -22x^4 - 36x^3 + 15x^2 + 8x$
- D $f'(x) = -2x^5 + 2x^3 - x^2 + 12x - 10$



Solution: B

Let $r(x) = x + 3$, $s(x) = 2x^2 - 5$, and $t(x) = -x^3 + 2$. Find each of their derivatives.

$$r(x) = x + 3$$

$$r'(x) = 1$$

and

$$s(x) = 2x^2 - 5$$

$$s'(x) = 4x$$

and

$$t(x) = -x^3 + 2$$

$$t'(x) = -3x^2$$

Apply product rule.

$$f'(x) = r'(x)s(x)t(x) + r(x)s'(x)t(x) + r(x)s(x)t'(x)$$

$$f'(x) = (1)(2x^2 - 5)(-x^3 + 2) + (x + 3)(4x)(-x^3 + 2) + (x + 3)(2x^2 - 5)(-3x^2)$$

Expand the derivative, then collect like terms.

$$f'(x) = -2x^5 + 4x^2 + 5x^3 - 10$$

$$+ (4x^2 + 12x)(-x^3 + 2) + (2x^3 - 5x + 6x^2 - 15)(-3x^2)$$

$$f'(x) = -2x^5 + 5x^3 + 4x^2 - 10$$



$$+(-4x^5 + 8x^2 - 12x^4 + 24x) + (-6x^5 + 15x^3 - 18x^4 + 45x^2)$$

$$f'(x) = -2x^5 + 5x^3 + 4x^2 - 10 - 4x^5 + 8x^2 - 12x^4 + 24x - 6x^5 + 15x^3 - 18x^4 + 45x^2$$

$$f'(x) = -2x^5 - 4x^5 - 6x^5 - 12x^4 - 18x^4 + 5x^3 + 15x^3 + 4x^2 + 8x^2 + 45x^2 + 24x - 10$$

$$f'(x) = -12x^5 - 30x^4 + 20x^3 + 57x^2 + 24x - 10$$



Topic: Product rule, three or more functions**Question:** Use the product rule to find the derivative of the function.

$$y = (3x^5 - 4)^3$$

Answer choices:

A $y' = 45x^4(3x^5 - 4)^2$

B $y' = (15x^4)^3$

C $y' = 3(15x^4)^2(3x^5 - 4)$

D $y' = 3(15x^4)^3$



Solution: A

Let $r(x) = 3x^5 - 4$, $s(x) = 3x^5 - 4$, and $t(x) = 3x^5 - 4$. Find each of their derivatives.

$$r(x) = 3x^5 - 4$$

$$r'(x) = 15x^4$$

and

$$s(x) = 3x^5 - 4$$

$$s'(x) = 15x^4$$

and

$$t(x) = 3x^5 - 4$$

$$t'(x) = 15x^4$$

Apply product rule.

$$f'(x) = (15x^4)(3x^5 - 4)(3x^5 - 4) + (3x^5 - 4)(15x^4)(3x^5 - 4) + (3x^5 - 4)(3x^5 - 4)(15x^4)$$

$$f'(x) = 3(15x^4)(3x^5 - 4)(3x^5 - 4)$$

$$f'(x) = 45x^4(3x^5 - 4)(3x^5 - 4)$$

$$f'(x) = 45x^4(3x^5 - 4)^2$$

