Calculus 1 Workbook Solutions

Definition of the derivative

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DEFINITION OF THE DERIVATIVE

1. Use the definition of the derivative to find the derivative of $f(x)=2x^2+2x-12$ at $(4,28)$.

Solution:

The definition of the derivative is

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

So at $(4,28)$, the derivative will be

$$f'(4)=\lim_{h\to 0}\frac{f(4+h)-f(4)}{h}$$

$$f'(4)=\lim_{h\to 0}\frac{(2(4+h)^2+2(4+h)-12)-(2(4)^2+2(4)-12)}{h}$$

$$f'(4)=\lim_{h\to 0}\frac{(2(16+8h+h^2)+8+2h-12)-(32+8-12)}{h}$$

$$f'(4)=\lim_{h\to 0}\frac{32+16h+2h^2+8+2h-12-32-8+12}{h}$$

$$f'(4)=\lim_{h\to 0}\frac{18h+2h^2}{h}$$

$$f'(4)=\lim_{h\to 0}18+2h$$

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Evaluate the limit to find the derivative at $(4,28)$.

$$f'(4) = 18 + 2(0)$$

$$f'(4) = 18$$

■ 2. Use the definition of the derivative to find the derivative of $g(x) = 3x^3 - 4x + 7$ at $(-2, -9)$.

Solution:

The definition of the derivative is

$$g'(x) = \lim_{x \to a} \frac{g(x) - g(a)}{x - a}$$

So at $(-2, -9)$, the derivative will be

$$g'(-2) = \lim_{x \to -2} \frac{g(x) - g(-2)}{x - (-2)}$$

$$g'(-2) = \lim_{x \to -2} \frac{(3x^3 - 4x + 7) - (3(-2)^3 - 4(-2) + 7)}{x + 2}$$

$$g'(-2) = \lim_{x \to -2} \frac{(3x^3 - 4x + 7) - (-24 + 8 + 7)}{x + 2}$$

$$g'(-2) = \lim_{x \to -2} \frac{3x^3 - 4x + 16}{x + 2}$$

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$$g'(-2) = \lim_{x \to -2} \frac{(x+2)(3x^2 - 6x + 8)}{x+2}$$

$$g'(-2) = \lim_{x \to -2} 3x^2 - 6x + 8$$

Evaluate the limit to find the derivative at $(-2, -9)$.

$$g'(-2) = 3(-2)^2 - 6(-2) + 8$$

$$g'(-2) = 32$$

■ 3. Use the definition of the derivative to find the derivative of $h(x) = 9x^2 - 7x - 4$ at $(2,18)$.

Solution:

The definition of the derivative is

$$h'(x) = \lim_{h \to 0} \frac{h(x+h) - h(x)}{h}$$

So at $(2,18)$, the derivative will be

$$h'(2) = \lim_{h \to 0} \frac{h(2+h) - h(2)}{h}$$

$$h'(2) = \lim_{h \to 0} \frac{(9(2+h)^2 - 7(2+h) - 4) - (9(2)^2 - 7(2) - 4)}{h}$$

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$$h'(2) = \lim_{h \to 0} \frac{(9(4 + 4h + h^2) - 14 - 7h - 4) - (36 - 14 - 4)}{h}$$

$$h'(2) = \lim_{h \to 0} \frac{36 + 36h + 9h^2 - 14 - 7h - 4 - 36 + 14 + 4}{h}$$

$$h'(2) = \lim_{h \to 0} \frac{29h + 9h^2}{h}$$

$$h'(2) = \lim_{h \to 0} 29 + 9h$$

Evaluate the limit to find the derivative at $(2,18)$.

$$h'(2) = 29 + 9(0)$$

$$h'(2) = 29$$

■ 4. Use the definition of the derivative to find the derivative of $h(x) = 8x^2 - 19x + 15$ at $(2,9)$.

Solution:

The definition of the derivative is

$$h'(x) = \lim_{x \to a} \frac{h(x) - h(a)}{x - a}$$

So at $(2,9)$, the derivative will be

$$h'(2) = \lim_{x \to 2} \frac{h(x) - h(2)}{x - 2}$$

$$h'(2) = \lim_{x \to 2} \frac{(8x^2 - 19x + 15) - (8(2)^2 - 19(2) + 15)}{x - 2}$$

$$h'(2) = \lim_{x \to 2} \frac{8x^2 - 19x + 15 - 32 + 38 - 15}{x - 2}$$

$$h'(2) = \lim_{x \to 2} \frac{8x^2 - 19x + 6}{x + 2}$$

$$h'(2) = \lim_{x \to 2} \frac{(8x - 3)(x - 2)}{x - 2}$$

$$h'(2) = \lim_{x \to 2} 8x - 3$$

Evaluate the limit to find the derivative at $(2,9)$.

$$h'(2) = 8(2) - 3$$

$$h'(2) = 13$$

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