Product rule, two functions

We know how to use power rule to take the derivative of a power function, and now we'll learn how to use product rule to take the derivative of a product.

If a function is, itself, the product of two functions, then we have to use product rule to find the derivative of that function. Given a function

$$y = f(x)g(x)$$

the **product rule** says that its derivative will be

$$y' = f(x)g'(x) + f'(x)g(x)$$

In other words, to use the product rule, we'll multiply the first function by the derivative of the second function, then add the derivative of the first function times the second function to that result.

Let's do an example where the function is the product of two power functions.

Example

Find the derivative of the function.

$$y = (x^2)(6x^3)$$

The two functions in this problem are x^2 and $6x^3$. It doesn't matter which one we choose for $f(x)$ and which one we choose for $g(x)$, since the product rule just has us adding $f(x)g'(x)$ and $f'(x)g(x)$. We'll get the correct answer either way, as long as we stay consistent.

Let's choose $f(x) = x^2$ and $g(x) = 6x^3$, and then list out these two functions, along with their derivatives.

$$f(x) = x^2$$

$$f'(x) = 2x$$

and

$$g(x) = 6x^3$$

$$g'(x) = 18x^2$$

Once we've got these all listed out, we can plug them directly into the product rule formula.

$$y' = f(x)g'(x) + f'(x)g(x)$$

$$y' = (x^2)(18x^2) + (2x)(6x^3)$$

$$y' = 18x^4 + 12x^4$$

$$y' = 30x^4$$

We can use the power rule double-check the answer that we got in this last example. If, instead of using product rule, we had instead started the problem by multiplying out the function, we'd get

$$h(x) = (x^2)(6x^3)$$

$$h(x) = 6x^5$$

Then, once the function is multiplied out, we can apply power rule to take the derivative.

$$h'(x) = 6(5)x^{5-1}$$

$$h'(x) = 30x^4$$

So we get the same answer both ways.