Topic: Trigonometric limits

Question: Evaluate the limit.

$$\lim_{x \to 0} \frac{\cos x \sin x}{x}$$

Answer choices:

A $\quad 0$

B $\quad -1$

C $\quad 1$

D $\quad$ Does not exist (DNE)

Solution: C

If we use direct substitution to evaluate the limit, we get the undefined value $0/0$.

$$\frac{\cos(0)\sin(0)}{0}$$

$$\frac{1(0)}{0}$$

$$\frac{0}{0}$$

But if we rewrite the limit as

$$\lim_{x \to 0} \cos x \frac{\sin x}{x}$$

then we see that we have the product of two of the three key trig limit formulas,

$\lim_{x \to 0} \cos x = 1$ and $\lim_{x \to 0} \frac{\sin x}{x} = 1$

So we can evaluate the limit using these formulas.

$$\lim_{x \to 0} \cos x \cdot \lim_{x \to 0} \frac{\sin x}{x}$$

$$1 \cdot 1$$

$$1$$

Topic: Trigonometric limits

Question: Use a reciprocal identity to move the function toward one of the key trig limits, and then evaluate the limit.

$$\lim_{x \to 0} \frac{7}{x \csc x}$$

Answer choices:

A 0

B 7

C -7

D ∞

Solution: B

Rewrite the function as using the reciprocal identity that relates $\sin x$ and $\csc x$.

$$\lim_{x\to 0} \frac{7}{x \csc x}$$

$$\lim_{x\to 0} \frac{7}{\frac{x}{\sin x}}$$

$$\lim_{x\to 0} \frac{7 \sin x}{x}$$

We know the value of the trig limit

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

Therefore,

$$7(1)$$

$$7$$

Topic: Trigonometric limits

Question: Use conjugate method, then the Pythagorean identity $\sin^2 x + \cos^2 x = 1$, to evaluate the limit.

$$\lim_{h \to 0} \frac{\cos h - 1}{h}$$

Answer choices:

A 0

B 1

C -1

D ∞

Solution: A

If we use direct substitution to evaluate the limit, we get the undefined value $0/0$.

$$\frac{\cos(0) - 1}{0}$$

$$\frac{1 - 1}{0}$$

$$\frac{0}{0}$$

But we've been asked to start with conjugate method, anyway. We'll multiply both the numerator and denominator of the function by the conjugate of $\cos h - 1$.

$$\lim_{h \to 0} \frac{\cos h - 1}{h} \left(\frac{\cos h + 1}{\cos h + 1} \right)$$

$$\lim_{h \to 0} \frac{\cos^2 h + \cos h - \cosh h - 1}{h(\cos h + 1)}$$

$$\lim_{h \to 0} \frac{\cos^2 h - 1}{h(\cos h + 1)}$$

If we factor out a negative sign, we can rewrite the limit as

$$\lim_{h \to 0} -\frac{1 - \cos^2 h}{h(\cos h + 1)}$$

We were told in the question to use the Pythagorean identity $\sin^2 x + \cos^2 x = 1$, which we can rewrite.

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

Now that the right side of this trigonometric identity matches the numerator of the function, we can make a substitution.

$$\lim_{h \to 0} -\frac{\sin^2 h}{h(\cos h + 1)}$$

Now we'll rewrite the limit

$$\lim_{h \to 0} -\sin h \frac{\sin h}{h} \left(\frac{1}{\cos h + 1} \right)$$

One of the three key trig limits is

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

which means we can simplify the limit to

$$\lim_{h \to 0} -\sin h \left(\frac{1}{\cos h + 1} \right)$$

$$\lim_{h \to 0} -\frac{\sin h}{\cos h + 1}$$

Now we can use substitution to evaluate the limit.

$$-\frac{\sin(0)}{\cos(0) + 1}$$

$$-\frac{0}{1 + 1}$$

$$-\frac{0}{2}$$

$$0$$