Topic: Chain rule with product rule

Question: Apply product rule and chain rule to find the derivative.

$$y = (4x - 7)^2(2x + 3)$$

Answer choices:

A $y' = (4x - 7)(12x + 5)$

B $y' = 2(4x - 7)(2x + 3)$

C $y' = 2(4x - 7)(12x + 5)$

D $y' = 2(4x - 7)^3(12x + 5)$

Solution: C

Set $f(x) = (4x - 7)^2$ and $g(x) = 2x + 3$. Then

$$f(x) = (4x - 7)^2$$

$$f'(x) = 8(4x - 7)$$

and

$$g(x) = 2x + 3$$

$$g'(x) = 2$$

Now we can apply product rule.

$$y' = f(x)g'(x) + f'(x)g(x)$$

$$y' = ((4x - 7)^2)(2) + (8(4x - 7))(2x + 3)$$

The two terms $2(4x - 7)^2$ and $8(4x - 7)(2x + 3)$ share a common factor of $2(4x - 7)$, so factor that out.

$$y' = 2(4x - 7)\big[(4x - 7) + 4(2x + 3)\big]$$

$$y' = 2(4x - 7)(4x - 7 + 8x + 12)$$

$$y' = 2(4x - 7)(12x + 5)$$

Topic: Chain rule with product rule

Question: Apply product rule and chain rule to find the derivative.

$$y = 2x^2(-5x^2)^3$$

Answer choices:

A $\quad y' = -200x^7$

B $\quad y' = -200x^8$

C $\quad y' = -2{,}000x^7$

D $\quad y' = -2{,}000x^8$

Solution: C

Set $f(x) = 2x^2$ and $g(x) = (-5x^2)^3$. Then

$$f(x) = 2x^2$$

$$f'(x) = 4x$$

and

$$g(x) = (-5x^2)^3$$

$$g'(x) = -30x(-5x^2)^2$$

Now we can apply product rule.

$$y' = f(x)g'(x) + f'(x)g(x)$$

$$y' = (2x^2)(-30x(-5x^2)^2) + (4x)((-5x^2)^3)$$

$$y' = -60x^3(-5x^2)^2 + 4x(-5x^2)^3$$

The two terms $-60x^3(-5x^2)^2$ and $4x(-5x^2)^3$ share a common factor of $4x(-5x^2)^2$, so factor that out.

$$y' = 4x(-5x^2)^2\left[-15x^2 + (-5x^2)\right]$$

$$y' = 4x(-5x^2)^2(-15x^2 - 5x^2)$$

$$y' = 4x(-5x^2)^2(-20x^2)$$

$$y' = 4x(25x^4)(-20x^2)$$

$$y' = -2{,}000x^7$$

Topic: Chain rule with product rule

Question: Apply product rule and chain rule to find the derivative.

$$y = (9x)(2x^3)(-3x^2)$$

Answer choices:

A $\quad y' = -92x^4$

B $\quad y' = -92x^5$

C $\quad y' = -324x^4$

D $\quad y' = -324x^5$

Solution: D

Set $f(x) = 9x$, $g(x) = 2x^3$, and $h(x) = -3x^2$. Then

$$f(x) = 9x$$

$$f'(x) = 9$$

and

$$g(x) = 2x^3$$

$$g'(x) = 6x^2$$

and

$$h(x) = -3x^2$$

$$h'(x) = -6x$$

Now we can apply product rule.

$$y' = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$

$$y' = (9)(2x^3)(-3x^2) + (9x)(6x^2)(-3x^2) + (9x)(2x^3)(-6x)$$

Simplify the derivative.

$$y' = 18x^3(-3x^2) + 54x^3(-3x^2) + 18x^4(-6x)$$

$$y' = -54x^5 - 162x^5 - 108x^5$$

$$y' = -324x^5$$