Calculus 1 Workbook Solutions

Other functions and trigonometry

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QUADRATIC FORMULA

1. Solve for x using the quadratic formula.

$$4x^2 - 8x - 15 = 0$$

Solution:

The quadratic formula for the expression is

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(-15)}}{2(4)}$$

$$x = \frac{8 \pm \sqrt{64 + 240}}{8}$$

$$x = \frac{8 \pm \sqrt{304}}{8}$$

$$x = \frac{8 \pm 4\sqrt{19}}{8}$$

$$x = \frac{2 \pm \sqrt{19}}{2}$$

2. Write the quadratic formula for the following quadratic equation.

$$x^2 - 5x - 24 = 0$$

Solution:

The quadratic formula for the expression is

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-24)}}{2(1)}$$

We could continue to simplify to solve for the roots.

$$x = \frac{5 \pm \sqrt{25 + 96}}{2}$$

$$x = \frac{5 \pm \sqrt{121}}{2}$$

$$x = \frac{5 \pm 11}{2}$$

$$x = -3,\ 8$$

■ 3. What went wrong in the way the quadratic formula was applied?

$$3x^2 - 5x + 10 = 0$$

$$x = \frac{-5 \pm \sqrt{(-5)^2 - 4(3)(10)}}{2(3)}$$

Solution:

The $-b$ at the beginning of the quadratic formula is written as -5, but $b=-5$. Which means it should be written as $-(-5)$.

■ 4. Solve for z using the quadratic formula.

$$z^2 = z + 3$$

Solution:

Rewrite the expression as

$$z^2 = z + 3$$

$$z^2 - z - 3 = 0$$

Then the quadratic formula gives

$$z = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)}$$

$$z = \frac{1 \pm \sqrt{13}}{2}$$

5. Fill in the blank with the correct term if the quadratic formula below was built from the quadratic equation.

$$\_\_\_\_ x^2 + 3x - 5 = 0$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(-2)(-5)}}{2(-2)}$$

Solution:

The blank should be the term -2.

6. Simplify the expression.

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(14)}}{2(1)}$$

Solution:

The expression is simplified as

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(14)}}{2(1)}$$

$$\frac{8 \pm \sqrt{64 - 56}}{2}$$

$$\frac{8 \pm \sqrt{8}}{2}$$

$$\frac{8 \pm 2\sqrt{2}}{2}$$

$$4 \pm \sqrt{2}$$

■ 7. What are two ways to solve a quadratic equation when you cannot easily factor?

Solution:

You can either use the method of completing the square or the quadratic formula.

■ 8. What went wrong if the quadratic formula below was built from the quadratic equation?

$$x^2 + 2x = 7$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(7)}}{2(1)}$$

Solution:

The expression was not written in the correct form before using the quadratic formula. It should be written as $x^2 + 2x - 7 = 0$, for which the quadratic formula would then be

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-7)}}{2(1)}$$

■ 9. Solve for t using the quadratic formula.

$$4t^2 - 1 = -8t$$

Solution:

Rewrite the expression as

$$4t^2 - 1 = -8t$$

$$4t^2 + 8t - 1 = 0$$

Then the quadratic formula is

$$t = \frac{-(8) \pm \sqrt{(8)^2 - 4(4)(-1)}}{2(4)}$$

$$t = \frac{-8 \pm \sqrt{64 + 16}}{8}$$

$$t = \frac{-8 \pm 4\sqrt{5}}{8}$$

$$t = \frac{-2 \pm \sqrt{5}}{2}$$

COMPLETING THE SQUARE

1. Solve for x by completing the square.

$$x^2 - 6x + 5 = 0$$

Solution:

Completing the square gives

$$x^2 - 6x = -5$$

$$x^2 - 6x + 9 = -5 + 9$$

$$(x - 3)^2 = 4$$

$$x - 3 = \pm 2$$

$$x = 3 \pm 2$$

$$x = 1, 5$$

2. Fill in the blank with the correct term.

$$x^2 - \underline{\hspace{2em}} + \frac{9}{4} = -2 + \frac{9}{4}$$

Solution:

The blank should be the term $3x$.

■ 3. Complete the square in the following expression, but do not solve.

$$3y^2 - 12y + 3 = 0$$

Solution:

To complete the square, we first write the expression as

$$3y^2 - 12y = -3$$

$$y^2 - 4y = -1$$

Now complete the square as

$$y^2 - 4y + 4 = -1 + 4$$

$$(y - 2)^2 = 3$$

■ 4. Solve for a by completing the square.

$$2a^2 + 8a = -4$$

Solution:

Completing the square gives

$$a^2 + 4a = -2$$

$$a^2 + 4a + 4 = -2 + 4$$

$$(a + 2)^2 = 2$$

$$a + 2 = \pm\sqrt{2}$$

$$a = -2 \pm \sqrt{2}$$

5. What is your first and second step in solving the problem by completing the square?

$$4x^2 - 16x + 28 = 0$$

Solution:

The first step is to move the 28 over to the other side. The second step is to divide everything by 4. These steps could be done in the opposite order, but they are the first two steps you must take before completing the square.

6. Explain when and why completing the square is used for factoring.

Solution:

Completing the square is used when it’s not possible to solve for the roots by factoring.

7. Solve for y by completing the square.

$$3y^2 + 9y = 3$$

Solution:

Completing the square gives

$$3y^2 + 9y = 3$$

$$y^2 + 3y = 1$$

$$y^2 + 3y + \frac{9}{4} = 1 + \frac{9}{4}$$

$$\left(y + \frac{3}{2}\right)^2 = \frac{13}{4}$$

$$y = -\frac{3}{2} \pm \frac{\sqrt{13}}{2}$$

$$y = -\frac{3 \pm \sqrt{13}}{2}$$

8. Fill in the blank with the correct term.

$$\_\_\_\_ - 4x = 6$$

$$\left(x - \frac{2}{3}\right)^2 = \frac{22}{9}$$

Solution:

The blank should be filled in with $3x^2$. We can work backwards from the second equation.

$$\left(x - \frac{2}{3}\right)^2 = \frac{22}{9}$$

$$\left(x - \frac{2}{3}\right)\left(x - \frac{2}{3}\right) = \frac{22}{9}$$

$$x^2 - \frac{4}{3}x + \frac{4}{9} = \frac{22}{9}$$

$$3x^2 - 4x + \frac{4}{3} = \frac{22}{3}$$

$$3x^2 - 4x = \frac{22}{3} - \frac{4}{3}$$

$$3x^2 - 4x = \frac{18}{3}$$

$$3x^2 - 4x = 6$$

LONG DIVISION OF POLYNOMIALS

1. Find the quotient.

$$\frac{x^2+2x-1}{x+3}$$

Solution:

Use long division to find the quotient.

$$
\begin{array}{r l}
 & x-1+\frac{2}{x+3} \\ \cline{2-2}
x+3 & \big) \, x^2+2x-1 \\
 & -(x^2+3x) \\ \cline{2-2}
 & \qquad -x-1 \\
 & \quad -(-x-3) \\ \cline{2-2}
 & \qquad\qquad 2
\end{array}
$$

2. Find the quotient.

$$\frac{2x^3-x^2-4x+5}{x-2}$$

Solution:

Use long division to find the quotient.

$$\begin{array}{r}
2x^2+3x+2+\frac{9}{x-2} \\
x-2\overline{\big)\,2x^3-x^2-4x+5} \\
-(2x^3-4x^2) \\
\hline
3x^2-4x \\
-(3x^2-6x) \\
\hline
2x+5 \\
-(2x-4) \\
\hline
9
\end{array}$$

3. Find the quotient.

$$\frac{2x^4+4x^3-x^2+5x-150}{x+4}$$

Solution:

Use long division to find the quotient.

$$
\begin{array}{r l}
 & 2x^3-4x^2+15x-55+\dfrac{70}{x+4} \\
x+4 & \overline{\big)\,2x^4+4x^3-x^2+5x-150} \\
 & -(2x^4+8x^3) \\
 & \overline{\qquad -4x^3-x^2} \\
 & \qquad -(-4x^3-16x^2) \\
 & \qquad \overline{\qquad 15x^2+5x} \\
 & \qquad\qquad -(15x^2+60x) \\
 & \qquad\qquad \overline{\qquad -55x-150} \\
 & \qquad\qquad\qquad -(-55x-220) \\
 & \qquad\qquad\qquad \overline{\qquad\qquad 70}
\end{array}
$$

4. Find the quotient.

$$\frac{3x^3 - x^2 - 7x + 5}{x - 1}$$

Solution:

Use long division to find the quotient.

$$
\begin{array}{r}
3x^2+2x-5 \\
x-1\ \overline{)\ 3x^3-x^2-7x+5} \\
-(3x^3-3x^2) \\
\hline
2x^2-7x \\
-(2x^2-2x) \\
\hline
-5x+5 \\
-(-5x+5) \\
\hline
0
\end{array}
$$

5. Find the quotient.

$$\frac{-x^2+3x+15}{x+5}$$

Solution:

Use long division to find the quotient.

$$
\begin{array}{r|l}
 & -x+8-\dfrac{25}{x+5} \\
\hline
x+5 & -x^2+3x+15 \\
 & -(-x^2-5x) \\
\hline
 & \qquad 8x+15 \\
 & \quad -(8x+40) \\
\hline
 & \qquad\quad -25
\end{array}
$$

■ 6. Find the quotient.

$$\frac{x^4+x-3}{x-2}$$

Solution:

Use long division to find the quotient. Remember to represent the missing x^3 and x^2 terms.

$$
\begin{array}{r|l}
 & x^3+2x^2+4x+9+\frac{15}{x-2} \\
\hline
x-2 & x^4+0x^3+0x^2+x-3 \\
 & -(x^4-2x^3) \\
\hline
 & \qquad 2x^3+0x^2 \\
 & \qquad -(2x^3-4x^2) \\
\hline
 & \qquad\qquad 4x^2+x \\
 & \qquad\qquad -(4x^2-8x) \\
\hline
 & \qquad\qquad\qquad 9x-3 \\
 & \qquad\qquad\qquad -(9x-18) \\
\hline
 & \qquad\qquad\qquad\qquad 15
\end{array}
$$

■ 7. Find the quotient.

$$\frac{x^3+6}{x+6}$$

Solution:

Use long division to find the quotient. Remember to represent the missing x^2 and x terms.

$$
\begin{array}{r|l}
 & x^2-6x+36-\frac{210}{x+6} \\
\cline{2-2}
x+6 & x^3+0x^2+0x+6 \\
 & -(x^3+6x^2) \\
\cline{2-2}
 & \quad -6x^2+0x \\
 & \quad -(-6x^2-36x) \\
\cline{2-2}
 & \qquad\qquad 36x+6 \\
 & \qquad\quad -(36x+216) \\
\cline{2-2}
 & \qquad\qquad\quad -210
\end{array}
$$

8. Find the quotient.

$$\frac{x^2+x}{x-3}$$

Solution:

Use long division to find the quotient. Remember to represent the missing constant term.

$$
\begin{array}{r|l}
 & x+4+\dfrac{12}{x-3} \\
\cline{2-2}
x-3 & x^2+x+0 \\
 & -(x^2-3x) \\
\cline{2-2}
 & \qquad 4x+0 \\
 & \quad -(4x-12) \\
\cline{2-2}
 & \qquad\quad 12
\end{array}
$$

9. Find the quotient.

$$\frac{x^4 - 2x^2}{x - 4}$$

Solution:

Use long division to find the quotient. Remember to represent the missing terms.

$$x^3+4x^2+14x+56+\frac{224}{x-4}$$

$$x-4 \overline{\big)\, x^4+0x^3-2x^2+0x+0}$$

$$\begin{array}{l}
-(x^4-4x^3) \\ \hline
4x^3-2x^2 \\
-(4x^3-16x^2) \\ \hline
14x^2+0x \\
-(14x^2-56x) \\ \hline
56x+0 \\
-(-56x-224) \\ \hline
224
\end{array}$$

■ 10. Find the quotient.

$$\frac{-2x^3+8x}{x+2}$$

Solution:

Use long division to find the quotient. Remember to represent the missing terms.

$$
\begin{array}{r}
-2x^2+4x \\
x+2 \overline{)\,-2x^3+0x^2+8x+0} \\
-(-2x^3-4x^2) \\
\hline
4x^2+8x \\
-(4x^2+8x) \\
\hline
0
\end{array}
$$

THE UNIT CIRCLE

■ 1. What is the coordinate point associated with $\theta = 2\pi/3$ along the unit circle?

Solution:

Looking at the unit circle shows that the coordinate point associated with $\theta = 2\pi/3$ in the second quadrant is

$$\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$$

■ 2. The terminal side of the angle θ in $[0,2\pi)$ intersects the unit circle at the given point. Find the measure of θ in degrees.

$$\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$$

Solution:

Because both the x- and y-values in the coordinate point are negative, we know the angle lies in the third quadrant. Comparing the coordinate point to points along the unit circle in the third quadrant, we see that $\theta = 240^\circ$.

■ 3. Find $\sin\theta$ if $\theta \in [0,2\pi)$ and $\cos\theta = \sin\theta$.

Solution:

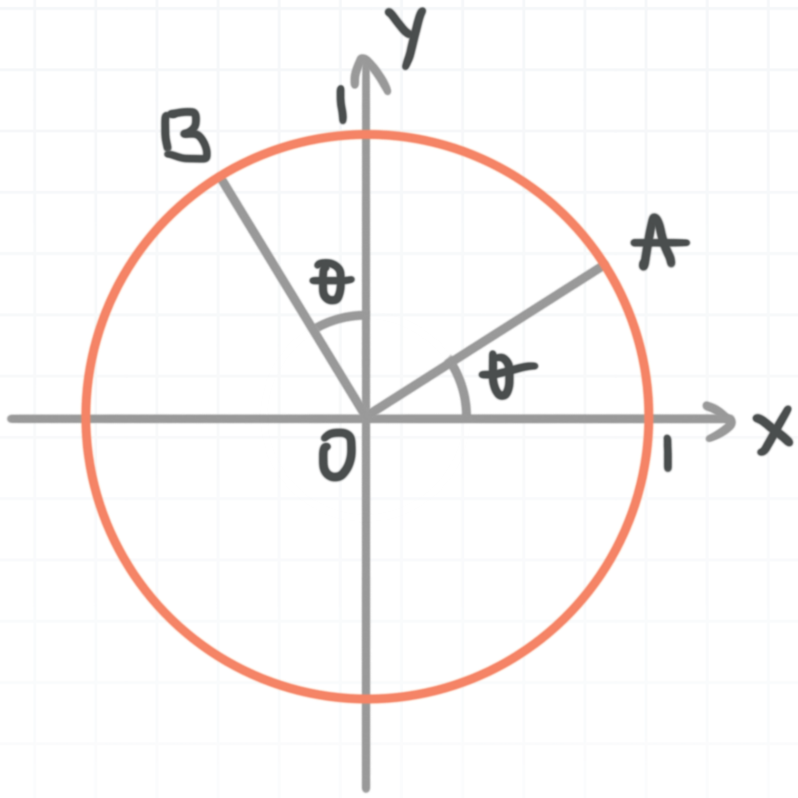
We know that $\sin\theta$ represents the y-coordinate, and $\cos\theta$ represents the x-coordinate. In the second and third quadrants, the signs of x and y are different, which means $\sin\theta$ and $\cos\theta$ cannot be equal.

If we look at the first and third quadrants of the unit circle, we can see that the x- and y-values are equal at $\theta = \pi/4$ and $\theta = 5\pi/4$.

At $\theta = \dfrac{\pi}{4}$, $\sin\theta = \dfrac{\sqrt{2}}{2}$

At $\theta = \dfrac{5\pi}{4}$, $\sin\theta = -\dfrac{\sqrt{2}}{2}$

■ 4. The points A and B lie on the unit circle in quadrants I and II respectively. The angle between OA and the positive x-axis is θ. The angle between OB and the positive y-axis is θ. Find the sine of $\angle AOB$.



Solution:

The angle between OB and the positive x-axis is $90^\circ + \theta$. So $\angle AOB$ is $(90^\circ + \theta) - \theta = 90^\circ$. Because we know $\sin 90^\circ = 1$,

$$\sin AOB = \sin 90^\circ = 1$$

■ 5. Evaluate the expression.

$$2\csc\left(\frac{49\pi}{6}\right) - 3\cos\left(\frac{13\pi}{3}\right) + \tan\left(\frac{25\pi}{4}\right)$$

Solution:

For each of the given angles, find a coterminal angle in $[0,2\pi)$.

$$\frac{49\pi}{6}=\frac{48\pi}{6}+\frac{\pi}{6}=8\pi+\frac{\pi}{6}$$

$$\frac{13\pi}{3}=\frac{12\pi}{3}+\frac{\pi}{3}=4\pi+\frac{\pi}{3}$$

$$\frac{25\pi}{4}=\frac{24\pi}{4}+\frac{\pi}{4}=6\pi+\frac{\pi}{4}$$

Which means

$$\csc\left(\frac{49\pi}{6}\right)=\csc\left(\frac{\pi}{6}\right)$$

$$\cos\left(\frac{13\pi}{3}\right)=\cos\left(\frac{\pi}{3}\right)$$

$$\tan\left(\frac{25\pi}{4}\right)=\tan\left(\frac{\pi}{4}\right)$$

Rewrite the expression with these coterminal angles.

$$2\csc\left(\frac{\pi}{6}\right)-3\cos\left(\frac{\pi}{3}\right)+\tan\left(\frac{\pi}{4}\right)$$

Apply reciprocal and quotient identities to put the expression in terms of only sine and cosine functions.

$$\frac{2}{\sin\left(\frac{\pi}{6}\right)}-3\cos\left(\frac{\pi}{3}\right)+\frac{\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)}$$

Plug in the known values from the unit circle.

$$\frac{2}{\frac{1}{2}} - 3\left(\frac{1}{2}\right) + \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$$

$$4 - \frac{3}{2} + 1$$

$$\frac{8}{2} - \frac{3}{2} + \frac{2}{2}$$

$$\frac{7}{2}$$

■ 6. Find the angle θ in the interval $[0,2\pi)$.

$$\sin\theta = \frac{1}{2} \text{ and } \cos\theta = -\frac{\sqrt{3}}{2}$$

Solution:

From the unit circle, we know the sine function takes the value $1/2$ at angles of $\pi/6$ and $5\pi/6$, so we need to evaluate cosine at both of these angles. We get

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \text{ and } \cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

Therefore, the matching angle is $\theta = 5\pi/6$.

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