

**Topic:** Finding a function from its inverse

**Question:** Use the given information to find  $f(x)$  if  $f^{-1}(x)$  is a linear function.

$$f^{-1}(4) = 1$$

$$f^{-1}(-2) = -2$$

**Answer choices:**

A  $f(x) = 2x + 6$

B  $f(x) = 2x - 2$

C  $f(x) = 2x - 6$

D  $f(x) = 2x + 2$



**Solution: D**

Use the points  $(4,1)$  and  $(-2, -2)$  to find the slope of the line that represents  $f^{-1}(x)$ .

$$m = \frac{-2 - 1}{-2 - 4} = \frac{-3}{-6} = \frac{1}{2}$$

Use one of the two points and  $y - y_1 = m(x - x_1)$  to find the equation of that line.

We'll use the point  $(4,1)$ .

$$y - 1 = \frac{1}{2}(x - 4)$$

$$y - 1 = \frac{1}{2}x - 2$$

$$y = \frac{1}{2}x - 1$$

Switch  $x$  with  $y$ , and then solve for  $y$  to get the equation of the line that represents  $f(x)$ .

$$x = \frac{1}{2}y - 1$$

$$x + 1 = \frac{1}{2}y$$

$$2(x + 1) = y$$

$$2x + 2 = y$$



Now replace  $y$  with  $f(x)$ .

$$f(x) = 2x + 2$$



**Topic:** Finding a function from its inverse**Question:** Use the given information to find  $f(x)$  if  $f^{-1}(x)$  is a linear function.

$$f^{-1}(2) = 5$$

$$f^{-1}(1) = -1$$

**Answer choices:**

A  $f(x) = \frac{1}{6}x + \frac{7}{6}$

B  $f(x) = \frac{1}{4}x + \frac{3}{4}$

C  $f(x) = \frac{1}{6}x - \frac{7}{6}$

D  $f(x) = \frac{1}{4}x - \frac{3}{4}$



**Solution: A**

Use the points (2,5) and (1, - 1) to find the slope of the line that represents  $f^{-1}(x)$ .

$$m = \frac{5 - (-1)}{2 - 1} = 6$$

Use one of the two points and  $y - y_1 = m(x - x_1)$  to find the equation of that line.

We'll use the point (2,5).

$$y - 5 = 6(x - 2)$$

$$y - 5 = 6x - 12$$

$$y = 6x - 7$$

Switch  $x$  with  $y$ , and then solve for  $y$  to get the equation of the line that represents  $f(x)$ .

$$x = 6y - 7$$

$$x + 7 = 6y$$

$$\frac{x + 7}{6} = y$$

$$\frac{1}{6}x + \frac{7}{6} = y$$

Now replace  $y$  with  $f(x)$ .



$$f(x) = \frac{1}{6}x + \frac{7}{6}$$



**Topic:** Finding a function from its inverse

**Question:** Use the given information to find  $f(x)$  if  $f^{-1}(x)$  is a linear function.

$$f^{-1}(3) = 4$$

$$f^{-1}(-6) = -2$$

**Answer choices:**

A  $f(x) = \frac{2}{3}x - 3$

B  $f(x) = \frac{3}{2}x - 3$

C  $f(x) = \frac{2}{3}x + 3$

D  $f(x) = \frac{3}{2}x + 3$



**Solution: B**

Use the points (3,4) and  $(-6, -2)$  to find the slope of the line that represents  $f^{-1}(x)$ .

$$m = \frac{4 - (-2)}{3 - (-6)} = \frac{6}{9} = \frac{2}{3}$$

Use one of the two points and  $y - y_1 = m(x - x_1)$  to find the equation of that line.

We'll use the point (3,4).

$$y - 4 = \frac{2}{3}(x - 3)$$

$$y - 4 = \frac{2}{3}x - 2$$

$$y = \frac{2}{3}x + 2$$

Switch  $x$  with  $y$ , and then solve for  $y$  to get the equation of the line that represents  $f(x)$ .

$$x = \frac{2}{3}y + 2$$

$$x - 2 = \frac{2}{3}y$$

$$\frac{3}{2}(x - 2) = \frac{3}{2} \left( \frac{2}{3}y \right)$$





$$\frac{3}{2}x - 3 = y$$

Now replace  $y$  with  $f(x)$ .

$$f(x) = \frac{3}{2}x - 3$$

