**Topic**: Endpoint discontinuities

Question: Which of the following statements is true?

## **Answer choices:**

- A The endpoint of an interval is discontinuous because one of the one-sided limits will be 0.
- B The endpoint of an interval is discontinuous because one of the one-sided limits will be  $\infty$ .
- C The endpoint of an interval is discontinuous because one of the one-sided limits will not exist.
- D The endpoint of an interval is discontinuous because both of the one-sided limits will not exist.



# Solution: C

The endpoint of an interval is discontinuous because one of the one-sided limits does not exist.

Because the function stops at an endpoint, either the left-hand limit will exist while the right-hand limit does not, or the right-hand limit will exist while the left-hand limit does not.



**Topic**: Endpoint discontinuities

**Question**: If the function  $f(x) = x^2$  is only defined on [1,4], and does not extend beyond that interval, what are the discontinuities of the function?

### **Answer choices:**

- A Endpoint discontinuities at x = 0, 4.
- B A jump discontinuity at x = 0.
- C Endpoint discontinuities at x = 1, 4 and a jump discontinuity at x = 0.
- D Endpoint discontinuities at x = 1, 4.



#### Solution: D

The endpoints of an interval are discontinuous for a function because one of the one-sided limits will not exist at each endpoint.

The function  $f(x) = x^2$  is a continuous function, but the interval [1,4] means that there will be endpoint discontinuities at x = 1 and x = 4.

At x=1, only the right-hand limit exists. The left-hand limit would be outside the function's domain. By the definition of continuity (that the left-hand limit exists, the right-hand limit exists, and the left- and right-hand limits are equal), that means the function isn't continuous at x=1, so there's an endpoint discontinuity there.

At x = 4, only the left-hand limit exists. The right-hand limit is outside the function's domain. By the definition of continuity, that means the function isn't continuous at x = 4, so there's an endpoint discontinuity there.



**Topic**: Endpoint discontinuities

Question: What are the discontinuities of the function on the interval [2,5]?

$$f(x) = \sqrt{x}$$

# **Answer choices:**

- A Endpoint discontinuities at x = 2 and x = 5 and when  $x \ge 0$ .
- B Endpoint discontinuities at x = 2 and x = 5.
- C Endpoint discontinuities at x = 2 and x = 5 and when  $x \le 0$ .
- D Endpoint discontinuities at x = 0 and x = 5.



## Solution: B

The function  $f(x) = \sqrt{x}$  is a continuous function when  $x \ge 0$  but the interval [2,5] means that there will be endpoint discontinuities at the points x = 2 and x = 5.

An endpoint discontinuity exists at x=2 because the left-hand limit doesn't exist there, and an endpoint discontinuity exists at x=5 because the right-hand limit doesn't exist there.

