Calculus 1 Workbook Solutions

Derivatives of trig functions

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TRIGONOMETRIC DERIVATIVES

■ 1. Find $f'(x)$ if $f(x) = 5x^7 + 8\sin(7x^7)$.

Solution:

Differentiate one term at a time, remembering to apply chain rule as you go.

$$f'(x) = 5(7)x^6 + 8\cos(7x^7)(49x^6)$$

$$f'(x) = 35x^6 + 392x^6\cos(7x^7)$$

$$f'(x) = 7x^6(5 + 56\cos(7x^7))$$

■ 2. Find $g'(x)$ if $g(x) = 3\sin(4x^3) - 4\cos(6x) + 3\sec(2x^4)$.

Solution:

Differentiate one term at a time, remembering to apply chain rule as you go.

$$g'(x) = 3\cos(4x^3)(12x^2) - 4(-\sin(6x))(6) + 3\sec(2x^4)\tan(2x^4)(8x^3)$$

$$g'(x) = 36x^2\cos(4x^3) + 24\sin(6x) + 24x^3\tan(2x^4)\sec(2x^4)$$

$$g'(x) = 12(3x^2\cos(4x^3) + 2\sin(6x) + 2x^3\tan(2x^4)\sec(2x^4))$$

■ 3. Find $h'(x)$ if $h(x) = 5\tan(4x^6) + 6\cot(6x^4)$.

Solution:

Differentiate one term at a time, remembering to apply chain rule as you go.

$$h'(x) = 5\sec^2(4x^6)(24x^5) + 6(-\csc^2(6x^4))(24x^3)$$

$$h'(x) = 120x^5\sec^2(4x^6) - 144x^3\csc^2(6x^4)$$

$$h'(x) = 24x^3(5x^2\sec^2(4x^6) - 6\csc^2(6x^4))$$

INVERSE TRIGONOMETRIC DERIVATIVES

■ 1. Find $f'(t)$.

$$f(t)=4\sin^{-1}\left(\frac{t}{4}\right)$$

Solution:

The derivative of inverse sine is given by

$$\frac{d}{dt}a\sin^{-1}(y(t))=a\cdot\frac{y'(t)}{\sqrt{1-\left[y(t)\right]^2}}$$

If $a=4$ and $y(t)=t/4$, then $y'(t)=1/4$. Then the derivative is

$$f'(t)=4\cdot\frac{\frac{1}{4}}{\sqrt{1-\left(\frac{t}{4}\right)^2}}=\frac{1}{\sqrt{\frac{16}{16}-\frac{t^2}{16}}}=\frac{1}{\sqrt{\frac{16-t^2}{16}}}=\frac{1}{\frac{\sqrt{16-t^2}}{4}}=\frac{4}{\sqrt{16-t^2}}$$

■ 2. Find $g'(t)$.

$$g(t)=-6\cos^{-1}(2t+3)$$

Solution:

The derivative of inverse cosine is given by

$$\frac{d}{dt} a\cos^{-1}(y(t)) = a \cdot -\frac{y'(t)}{\sqrt{1-\left[y(t)\right]^2}}$$

If $a=-6$ and $y(t)=2t+3$, then $y'(t)=2$. Then the derivative is

$$g'(t) = -6 \cdot -\frac{2}{\sqrt{1-(2t+3)^2}} = \frac{12}{\sqrt{1-4t^2-12t-9}} = \frac{6}{\sqrt{-(t+1)(t+2)}}$$

■ 3. Find $h'(t)$.

$$h(t) = 3\tan^{-1}(6t^2)$$

Solution:

The derivative of inverse tangent is given by

$$\frac{d}{dt} a\tan^{-1}(y(t)) = a \cdot \frac{y'(t)}{1+\left[y(t)\right]^2}$$

If $a=3$ and $y(t)=6t^2$, then $y'(t)=12t$. Then the derivative is

$$h'(t) = 3 \cdot \frac{12t}{1+(6t^2)^2} = \frac{36t}{1+36t^4}$$

HYPERBOLIC DERIVATIVES

■ 1. Find $f'(\theta)$ if $f(\theta) = 3\sinh(2\theta^2 - 5\theta + 2)$.

Solution:

The derivative of hyperbolic sine is given by

$$\frac{d}{d\theta} a\sinh(y(\theta)) = a \cdot \cosh(y(\theta)) \cdot y'(\theta)$$

If $a = 3$ and $y(\theta) = 2\theta^2 - 5\theta + 2$, then $y'(\theta) = 4\theta - 5$. Then the derivative is

$$f'(\theta) = 3\cosh(2\theta^2 - 5\theta + 2)(4\theta - 5)$$

$$f'(\theta) = 3(4\theta - 5)\cosh(2\theta^2 - 5\theta + 2)$$

■ 2. Find $g'(\theta)$ if $g(\theta) = 2\cosh(5\theta^{\frac{3}{2}} + 6\theta)$.

Solution:

The derivative of hyperbolic cosine is given by

$$\frac{d}{d\theta} a\cosh(y(\theta)) = a \cdot \sinh(y(\theta)) \cdot y'(\theta)$$

If $a = 2$ and $y(\theta) = 5\theta^{\frac{3}{2}} + 6\theta$, then $y'(\theta) = 5(3/2)\theta^{\frac{1}{2}} + 6$. Then the derivative is

$$g'(\theta) = 2\sinh(5\theta^{\frac{3}{2}} + 6\theta)\left(\frac{15}{2}\theta^{\frac{1}{2}} + 6\right)$$

$$g'(\theta) = (15\theta^{\frac{1}{2}} + 12)\sinh(5\theta^{\frac{3}{2}} + 6\theta)$$

$$g'(\theta) = 3(5\theta^{\frac{1}{2}} + 4)\sinh(5\theta^{\frac{3}{2}} + 6\theta)$$

■ 3. Find $h'(\theta)$ if $h(\theta) = 9\tanh\left(3\theta^2 - \theta^{\sqrt{3}}\right)$.

Solution:

The derivative of hyperbolic tangent is given by

$$\frac{d}{d\theta}a\tanh(y(\theta)) = a \cdot \text{sech}^2(y(\theta)) \cdot y'(\theta)$$

If $a = 9$ and $y(\theta) = 3\theta^2 - \theta^{\sqrt{3}}$, then $y'(\theta) = 6\theta - \sqrt{3} \cdot \theta^{\sqrt{3}-1}$. Then the derivative is

$$h'(\theta) = 9\left(6\theta - \sqrt{3} \cdot \theta^{\sqrt{3}-1}\right)\text{sech}^2\left(3\theta^2 - \theta^{\sqrt{3}}\right)$$

INVERSE HYPERBOLIC DERIVATIVES

■ 1. Find $f'(t)$ if $f(t) = 7\sinh^{-1}(5t^4)$.

Solution:

The derivative of inverse hyperbolic sine is given by

$$\frac{d}{dt}a\sinh^{-1}(y(t)) = a \cdot \frac{y'(t)}{\sqrt{\left[y(t)\right]^2 + 1}}$$

If $a = 7$ and $y(t) = 5t^4$, then $y'(t) = 20t^3$. Then the derivative is

$$f'(t) = 7 \cdot \frac{20t^3}{\sqrt{(5t^4)^2 + 1}} = \frac{140t^3}{\sqrt{25t^8 + 1}}$$

■ 2. Find $g'(t)$ if $g(t) = 4\cosh^{-1}(2t - 3)$.

Solution:

The derivative of inverse hyperbolic cosine is given by

$$\frac{d}{dt}a\cosh^{-1}(y(t)) = a \cdot \frac{y'(t)}{\sqrt{\left[y(t)\right]^2 - 1}}$$

If $a = 4$ and $y(t) = 2t - 3$, then $y'(t) = 2$. Then the derivative is

$$g'(t) = 4 \cdot \frac{2}{\sqrt{(2t-3)^2 - 1}}$$

$$g'(t) = \frac{8}{\sqrt{4t^2 - 12t + 9 - 1}}$$

$$g'(t) = \frac{8}{\sqrt{4t^2 - 12t + 8}}$$

$$g'(t) = \frac{8}{\sqrt{4(t-1)(t-2)}}$$

$$g'(t) = \frac{4}{\sqrt{(t-1)(t-2)}}$$

■ 3. Find $h'(t)$ if $h(t) = 9\tanh^{-1}(-7t+2)$.

Solution:

The derivative of inverse hyperbolic tangent is given by

$$\frac{d}{dt} a \tanh^{-1}(y(t)) = a \cdot \frac{y'(t)}{1 - \left[y(t)\right]^2}$$

If $a = 9$ and $y(t) = -7t + 2$, then $y'(t) = -7$. Then the derivative is

$$h'(t) = 9 \cdot \frac{-7}{1 - (-7t + 2)^2}$$

$$h'(t) = -\frac{63}{1 - (49t^2 - 28t + 4)}$$

$$h'(t) = -\frac{63}{1 - 49t^2 + 28t - 4}$$

$$h'(t) = -\frac{63}{-49t^2 + 28t - 3}$$

$$h'(t) = \frac{63}{49t^2 - 28t + 3}$$

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