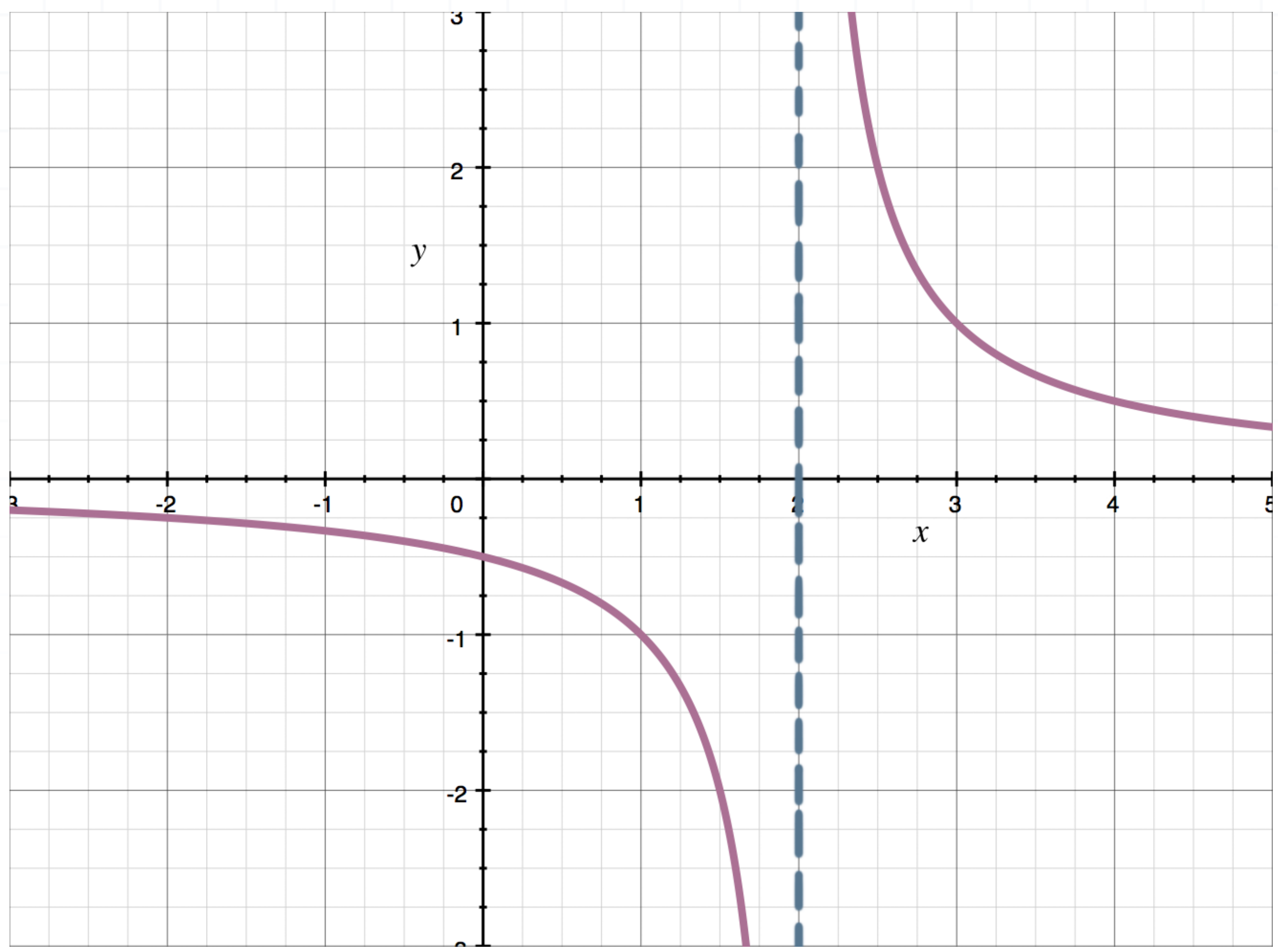
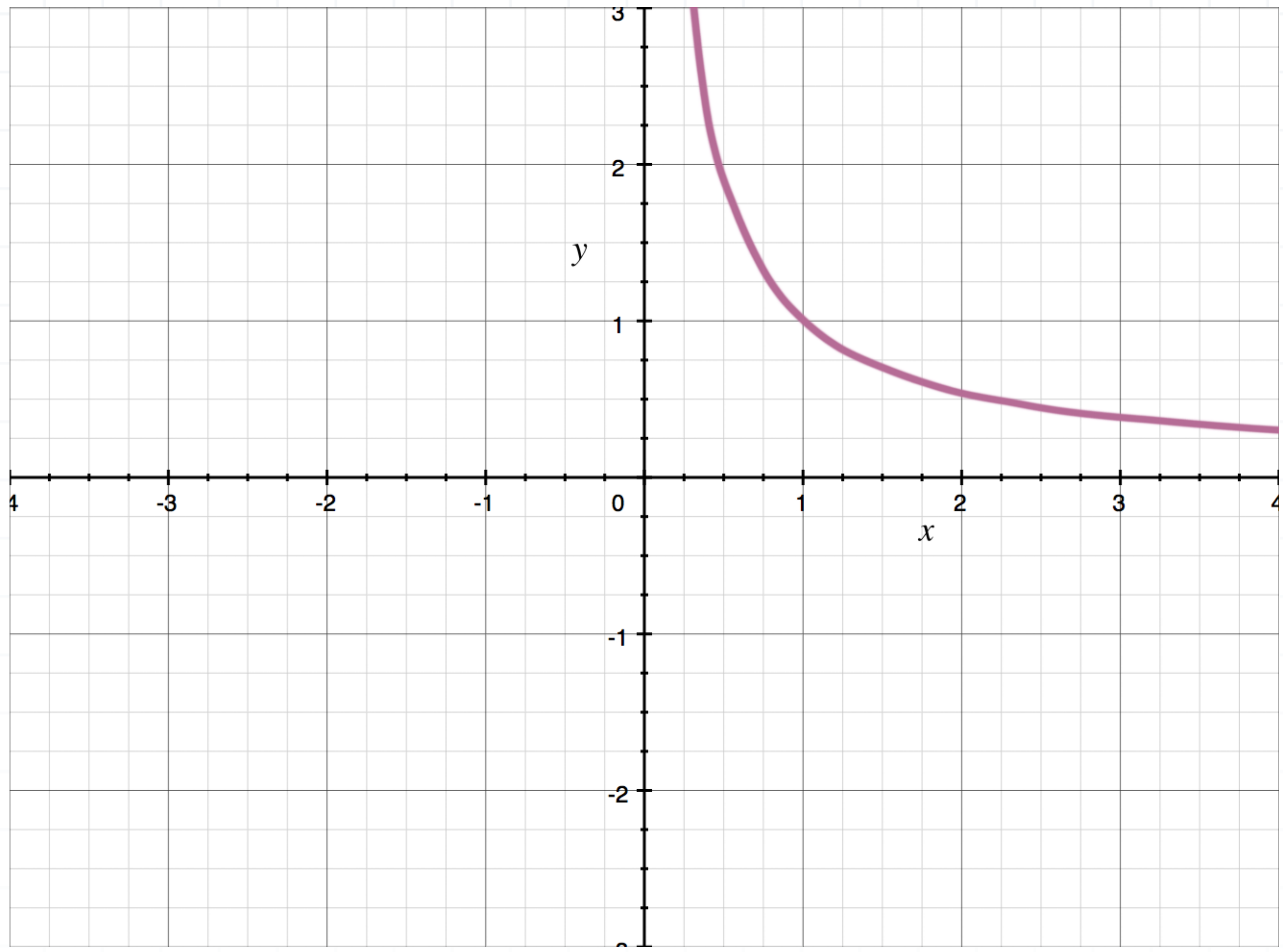
Infinite discontinuities

An **infinite discontinuity**, also called an essential discontinuity, is the kind of discontinuity that occurs at an asymptote. When the graph exists on both sides of a vertical asymptote, you can say that the graph has an infinite discontinuity at the asymptote.



The vertical asymptote in the graph below at $x = 0$ is not a discontinuity, because the graph doesn't exist on both sides of the asymptote, which means the asymptote doesn't break up any part of the graph.



Just like with point discontinuities, we get removable discontinuities when we have rational functions. A vertical asymptote will exist whenever the denominator is 0, but when the discontinuity can't be removed.

Going back to the example we used previously for point discontinuities,

$$f(x) = \frac{x-2}{x^2+x-6}$$

we factored the denominator to get

$$f(x) = \frac{x-2}{(x-2)(x+3)}$$

In this form, we can see that the denominator is 0 at both $x = 2$ and $x = -3$. Because the $x - 2$ can be canceled,

$$f(x) = \frac{1}{x+3}$$

there's a point discontinuity at $x = 2$. Since the $x + 3$ can't be canceled, and, no matter how much we simplify the fraction, $x = -3$ will always make the denominator 0, that tells us there's a vertical asymptote at $x = -3$, and therefore an infinite discontinuity there.

The general limit may or may not exist at an infinite discontinuity. If the function approaches $-\infty$ on one side and ∞ on the other side, then the general limit doesn't exist, because the left- and right-hand limits aren't equal.

The story gets a little more complicated when the left- and right-hand limits both approach $-\infty$ or both approach ∞. Technically, the function must approach a finite value for the limit to be defined, and neither $-\infty$ or ∞ are finite values, so the general limit would never exist at a vertical asymptote.

But oftentimes, for the sake of providing more information about how the function behaves, we'll say that the value of the general limit is ∞ when both the left- and right-hand limits are ∞, or that the general limit is $-\infty$ when both the left- and right-hand limits are $-\infty$.