Topic: Normal lines

Question: Find the equation of the normal line to the function at (1,2).

$$f(x) = 2x^4$$

Answer choices:

A
$$y = 8x - 6$$

$$B y = -\frac{1}{8}x - \frac{17}{8}$$

C
$$y = -\frac{1}{8}x + \frac{17}{8}$$

D
$$y = 8x - 10$$

Solution: C

Take the derivative of the function,

$$f'(x) = 8x^3$$

and then evaluate it at (1,2).

$$f'(1) = 8(1)^3$$

$$f'(1) = 8$$

This is the slope of the tangent line at (1,2). Since m=8, we'll take the negative reciprocal to find n, the slope of the normal line.

$$n = -\frac{1}{8}$$

We'll plug n = -1/8 and the point (1,2) into the point-slope formula for the equation of the line. Once we simplify, we'll have the equation of the normal line to the function at (1,2).

$$y - y_1 = n(x - x_1)$$

$$y - 2 = -\frac{1}{8}(x - 1)$$

$$y - 2 = -\frac{1}{8}x + \frac{1}{8}$$

$$y = -\frac{1}{8}x + \frac{1}{8} + \frac{16}{8}$$

$$y = -\frac{1}{8}x + \frac{17}{8}$$



Topic: Normal lines

Question: Find the equation of the normal line to the function at (3,6).

$$f(x) = x\sqrt{x+1}$$

Answer choices:

$$A \qquad y = -\frac{4}{11}x + \frac{78}{11}$$

B
$$y = \frac{11}{4}x - \frac{57}{4}$$

C
$$y = \frac{11}{4}x - \frac{9}{4}$$

$$D y = -\frac{4}{11}x - \frac{54}{11}$$

Solution: A

Take the derivative of the function,

$$f'(x) = (1)(\sqrt{x+1}) + (x)\left(\frac{1}{2}(x+1)^{-\frac{1}{2}}\right)$$

$$f'(x) = \sqrt{x+1} + \frac{x}{2\sqrt{x+1}}$$

and then evaluate it at (3,6).

$$f'(3) = \sqrt{3+1} + \frac{3}{2\sqrt{3+1}}$$

$$f'(3) = 2 + \frac{3}{2(2)}$$

$$f'(3) = \frac{8}{4} + \frac{3}{4}$$

$$f'(3) = \frac{11}{4}$$

This is the slope of the tangent line at (3,6). Since m = 11/4, we'll take the negative reciprocal to find n, the slope of the normal line.

$$n = -\frac{4}{11}$$

We'll plug n = -4/11 and the point (3,6) into the point-slope formula for the equation of the line. Once we simplify, we'll have the equation of the normal line to the function at (3,6).

$$y - y_1 = n(x - x_1)$$

$$y - 6 = -\frac{4}{11}(x - 3)$$

$$y - 6 = -\frac{4}{11}x + \frac{12}{11}$$

$$y = -\frac{4}{11}x + \frac{12}{11} + \frac{66}{11}$$

$$y = -\frac{4}{11}x + \frac{78}{11}$$



Topic: Normal lines

Question: Find the equation of the normal line to the function at (2,2).

$$f(x) = \frac{2x^2}{x+2}$$

Answer choices:

$$A \qquad y = -\frac{2}{3}x - \frac{2}{3}$$

$$B \qquad y = \frac{3}{2}x - 1$$

$$C y = \frac{3}{2}x - 5$$

D
$$y = -\frac{2}{3}x + \frac{10}{3}$$

Solution: D

Take the derivative of the function,

$$f'(x) = \frac{(4x)(x+2) - (2x^2)(1)}{(x+2)^2}$$

$$f'(x) = \frac{4x^2 + 8x - 2x^2}{(x+2)^2}$$

$$f'(x) = \frac{2x^2 + 8x}{(x+2)^2}$$

and then evaluate it at (2,2).

$$f'(2) = \frac{2(2)^2 + 8(2)}{(2+2)^2}$$

$$f'(2) = \frac{2(4) + 16}{4^2}$$

$$f'(2) = \frac{8+16}{16}$$

$$f'(2) = \frac{3}{2}$$

This is the slope of the tangent line at (2,2). Since m=3/2, we'll take the negative reciprocal to find n, the slope of the normal line.

$$n = -\frac{2}{3}$$



We'll plug n = -2/3 and the point (2,2) into the point-slope formula for the equation of the line. Once we simplify, we'll have the equation of the normal line to the function at (2,2).

$$y - y_1 = n(x - x_1)$$

$$y - 2 = -\frac{2}{3}(x - 2)$$

$$y - 2 = -\frac{2}{3}x + \frac{4}{3}$$

$$y = -\frac{2}{3}x + \frac{4}{3} + \frac{6}{3}$$

$$y = -\frac{2}{3}x + \frac{10}{3}$$