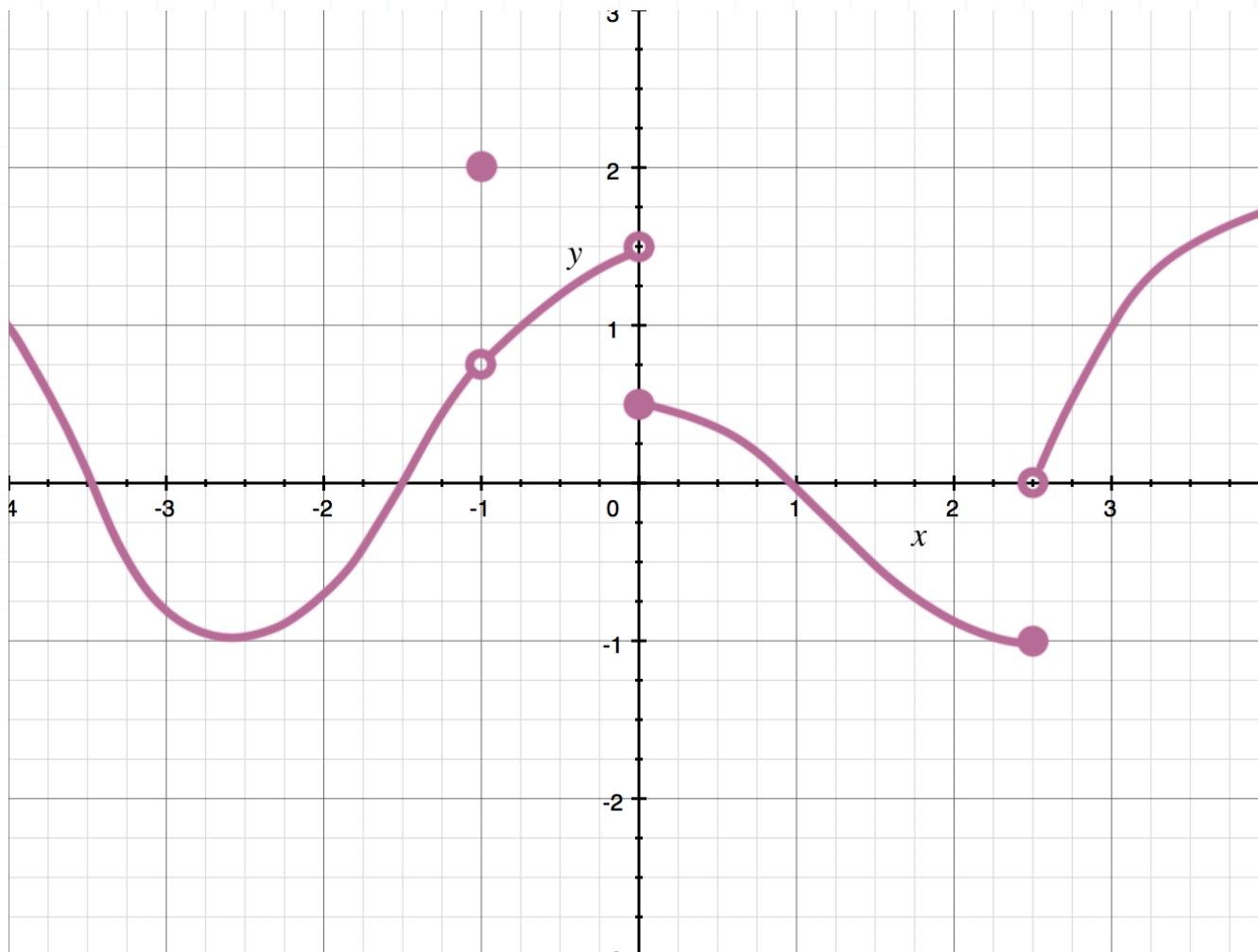# Endpoint discontinuities

Sometimes we'll define a function on a particular interval, such that the function does not extend beyond that defined interval. For instance, let's say we've been given this graph,

and told that the graph does not extend beyond the boundaries of what's shown. In other words, we're saying that the graph ends at $x = -4$ on the left, and isn't defined anywhere left of that value, and also that the graph ends at $x = 4$ on the right, and isn't defined anywhere right of that value.

In this case, we'd say that the graph has **endpoints** at $x = -4$ and $x = 4$. Regular functions don't typically have endpoints; endpoints are something that we usually artificially impose upon the function.

The limit at an endpoint

Either way, when we have defined endpoints for a function, the general limit will not exist at those endpoints. To see why, think about the endpoint $x = -4$ from the graph above. The right-hand limit at that point is 1, but because the graph doesn't extend to the left of $x = -4$, the left-hand limit does not exist there.

$$\lim_{x \to -4^-} f(x) = \text{DNE}$$

$$\lim_{x \to -4^+} f(x) = 1$$

Because the left-hand limit does not exist, that means the general limit does not exist. In the same way, think about the endpoint $x = 4$ from the same graph. The left-hand limit at that point is about $7/4$, but because the graph doesn't extend to the right of $x = 4$, the right-hand limit does not exist there.

$$\lim_{x \to 4^-} f(x) = \frac{7}{4}$$

$$\lim_{x \to 4^+} f(x) = \text{DNE}$$

Again, the takeaway here is that the general limit does not exist at an endpoint. The left-hand limit might exist, or the right-hand limit might exist, but the general limit won't exist because one of those one-sided limits won't exist.