Topic: Water level in the tank

Question: The water level in an inverted, cone-shaped funnel is decreasing at a rate of 0.5 m/s. How fast is the water volume decreasing when the top surface of the water has radius $r = 2$?

Answer choices:

A $-\frac{3}{2}\pi$ m^3/s

B $-\frac{4}{3}\pi$ m^3/s

C $-\frac{2}{3}\pi$ m^3/s

D $-\frac{1}{3}\pi$ m^3/s

Solution: C

The formula for the volume of a cone is

$$V = \frac{1}{3}\pi r^2 h$$

From the question, we know that $r = 2$, so plug that in.

$$V = \frac{1}{3}\pi (2)^2 h$$

$$V = \frac{4}{3}\pi h$$

Use implicit differentiation to take the derivative of both sides.

$$(1)\frac{dV}{dt} = \frac{4}{3}\pi (1)\frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \frac{dh}{dt}$$

From the question, we know that $dh/dt = -1/2$, so make that substitution.

$$\frac{dV}{dt} = \frac{4}{3}\pi \left(-\frac{1}{2}\right)$$

$$\frac{dV}{dt} = -\frac{2}{3}\pi$$

Topic: Water level in the tank

Question: Water is being pumped from a cylindrical tank with a radius of 3 ft at a rate of 18 cubic feet per minute. How fast is the water level falling when the water is 2 ft deep?

Answer choices:

A $-\frac{2}{\pi}$ ft/min

B -2 ft/min

C $-\pi$ ft/min

D $-\frac{\pi}{2}$ ft/min

Solution: A

The formula for the volume of a cylinder is

$$V = \pi r^2 h$$

From the question, we know that $r = 3$, so plug that in.

$$V = \pi(3)^2 h$$

$$V = 9\pi h$$

Use implicit differentiation to take the derivative of both sides.

$$(1)\frac{dV}{dt} = 9\pi(1)\frac{dh}{dt}$$

$$\frac{dV}{dt} = 9\pi\frac{dh}{dt}$$

From the question, we know that $dV/dt = -18$, so make that substitution.

$$-18 = 9\pi\frac{dh}{dt}$$

$$\frac{dh}{dt} = -\frac{18}{9\pi}$$

$$\frac{dh}{dt} = -\frac{2}{\pi}$$

Topic: Water level in the tank

Question: Water is being pumped from a cylindrical tank with a radius of 2 ft at a rate of 10 cubic feet per minute. How fast is the water level falling when the water is 5 ft deep?

Answer choices:

A $-\frac{2\pi}{5}$ ft/min

B $-\frac{5}{2\pi}$ ft/min

C $-\frac{3}{2\pi}$ ft/min

D $-\frac{2\pi}{3}$ ft/min

Solution: B

The formula for the volume of a cylinder is

$$V = \pi r^2 h$$

From the question, we know that $r = 2$, so plug that in.

$$V = \pi(2)^2 h$$

$$V = 4\pi h$$

Use implicit differentiation to take the derivative of both sides.

$$(1)\frac{dV}{dt} = 4\pi(1)\frac{dh}{dt}$$

$$\frac{dV}{dt} = 4\pi\frac{dh}{dt}$$

From the question, we know that $dV/dt = -10$, so make that substitution.

$$-10 = 4\pi\frac{dh}{dt}$$

$$\frac{dh}{dt} = -\frac{10}{4\pi}$$

$$\frac{dh}{dt} = -\frac{5}{2\pi}$$