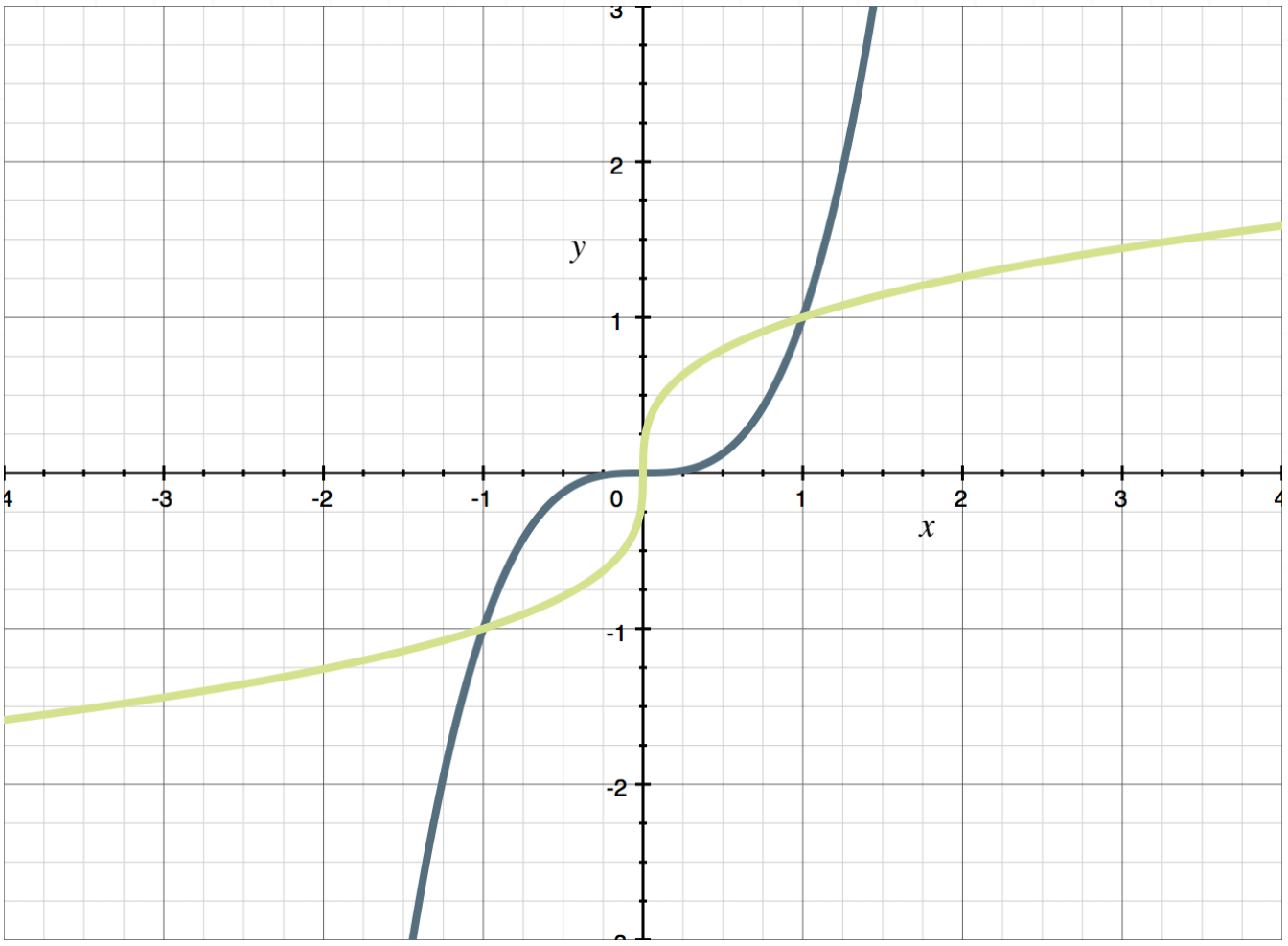
Topic: Inverse functions

Question: The blue curve is the graph of a function, and the green curve is the graph of its inverse. Which pair of functions do the graphs represent?



Answer choices:

A $f(x) = x^3$ $g(x) = -\sqrt[3]{x}$

B $f(x) = x^3$ $g(x) = \sqrt[3]{x}$

C $f(x) = -x^3$ $g(x) = -\sqrt[3]{x}$

D $f(x) = -x^3$ $g(x) = \sqrt[3]{x}$

Solution: B

The point $(1,1)$ is common to the two graphs, so let's input 1 into both functions in each pair and see if it returns 1 for both of them.

A	$f(1) = 1^3 = 1$	$g(1) = -\sqrt[3]{1} = -1$
B	$f(1) = 1^3 = 1$	$g(1) = \sqrt[3]{1} = 1$
C	$f(1) = -(1^3) = -1$	$g(1) = -\sqrt[3]{1} = -1$
D	$f(1) = -(1^3) = -1$	$g(1) = \sqrt[3]{1} = 1$

Look at answer choice B. Evaluating $f(1)$ returns a value of 1. Likewise, evaluating $g(1)$ also returns a value of 1. This tells us that $(1,1)$ is a point of the graphs of the functions $f(x)$ and $g(x)$ that are defined in answer choice B.

Topic: Inverse functions

Question: Which of these functions is the inverse of the given function?

$$f(x) = \frac{1}{x} - 2$$

Answer choices:

A $f^{-1}(x) = 2 - \frac{1}{x}$

B $f^{-1}(x) = \frac{x+1}{2}$

C $f^{-1}(x) = \frac{1}{x+2}$

D $f^{-1}(x) = \frac{x}{2} + 1$

Solution: C

To find the inverse of

$$f(x) = \frac{1}{x} - 2$$

first replace $f(x)$ with y.

$$y = \frac{1}{x} - 2$$

Next, solve for x.

$$y + 2 = \frac{1}{x}$$

$$x(y + 2) = 1$$

$$x = \frac{1}{y + 2}$$

Now switch x with y.

$$y = \frac{1}{x + 2}$$

Finally, write the inverse function by replacing y with $f^{-1}(x)$.

$$f^{-1}(x) = \frac{1}{x + 2}$$

Topic: Inverse functions

Question: Which of these is the inverse of the given function?

$$f(x) = 3x^3 - 4$$

Answer choices:

A $f^{-1}(x) = \sqrt[3]{\frac{3x}{4}}$

B $f^{-1}(x) = \frac{\sqrt[3]{x}}{3} + 4$

C $f^{-1}(x) = \sqrt[3]{3x + 4}$

D $f^{-1}(x) = \sqrt[3]{\frac{x + 4}{3}}$

Solution: D

To find the inverse of $f(x) = 3x^3 - 4$, first replace $f(x)$ with y.

$$y = 3x^3 - 4$$

$$y + 4 = 3x^3$$

$$\frac{y+4}{3} = x^3$$

$$\sqrt[3]{\frac{y+4}{3}} = x$$

Now switch x with y.

$$\sqrt[3]{\frac{x+4}{3}} = y$$

Finally, write the inverse function by replacing y with $f^{-1}(x)$ (and then turning the equation around so that $f^{-1}(x)$ is on the left side).

$$f^{-1}(x) = \sqrt[3]{\frac{x+4}{3}}$$