Topic: Intermediate value theorem without an interval

Question: If we're trying to use the Intermediate Value Theorem to prove the existence of a root for the function, but no interval is given, what should we do?

Answer choices:

A We should give up, because there's no way to use the IVT if no interval is given.

B We should give up, because by definition, the function has no root if no interval is given.

C We should try to find our own interval, and the only way to do this is to guess random intervals.

D We should try to find our own interval, and to do this, we can try to consider what we might already know about the function's values, we can look at the graph of the function, or otherwise try to be strategic about how to narrow down a useful interval.

Solution: D

When no interval is given to us in which we should look for the root, it still may be possible to use the Intermediate Value Theorem to prove the existence of a root.

But we have to find an interval first. In order to do that, we can employ different techniques, and get creative in order to narrow down what interval we might be able to use.

For instance, graphing the function might show us the approximate location of the root, and we can pick values for the interval that are on either side of the root.

Or, if we know something about the shape of the graph of the function, and we can use that information to narrow down an interval, we can take that approach as well.

Topic: Intermediate value theorem without an interval

Question: There are no real roots for the function $f(x) = \sin x$.

Answer choices:

A True

B False

Solution: B

No interval is given, but the sine function oscillates back and forth between $[-1,1]$.

If it oscillates back and forth from -1 below the x-axis, to 1 above the x-axis, and because $f(x) = \sin x$ is a continuous function, it must cross the x-axis at some point, which means it has at least one real root somewhere in its domain.

Topic: Intermediate value theorem without an interval

Question: Find an interval for the function $f(x) = \cos x$ on which the function has a real root.

Answer choices:

A $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

B $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

C $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

D $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

Solution: D

We could answer this question by evaluating the function $f(x) = \cos x$ at the endpoints of each interval given in the answer choices.

If we did that, only answer choices C and D give a negative value at one edge of the interval and a positive value at the other edge of the interval. From there, only answer choice D is a closed interval, so answer choice D can be the only correct answer.

Alternatively, we could have looked at the graph of $f(x) = \cos x$ and investigated its value at the endpoints of each of the intervals. We'd see that the graph was above the x-axis everywhere from $-\pi/4$ to $\pi/4$, thereby eliminating answer choices A and B, allowing us to conclude that D must be the correct choice, since C is not a closed interval.