Topic: Limits of composites

Question: If $f(x) = x^3$ and $g(x) = x^2 + 3$, evaluate the limit.

$$\lim_{x \to 5} f(g(x))$$

Answer choices:

A $\lim_{x \to 5} f(g(x)) = 21{,}952$

B $\lim_{x \to 5} f(g(x)) = 81$

C $\lim_{x \to 5} f(g(x)) = 15{,}628$

D $\lim_{x \to 5} f(g(x)) = 253$

Solution: A

First find the composite $f(g(x))$, when $f(x) = x^3$ and $g(x) = x^2 + 3$.

$$f(x) = x^3$$

$$f(g(x)) = (x^2 + 3)^3$$

Then find the limit of the composite function.

$$\lim_{x \to 5} f(g(x))$$

$$\lim_{x \to 5} (x^2 + 3)^3$$

$$(5^2 + 3)^3$$

$$28^3$$

$$21{,}952$$

Topic: Limits of composites

Question: If $f(x) = \cos x$ and $g(x) = x + 4$, evaluate the limit.

$$\lim_{x \to -4} f(g(x))$$

Answer choices:

A $\lim_{x \to -4} f(g(x)) = -1$

B $\lim_{x \to -4} f(g(x)) = 0$

C $\lim_{x \to -4} f(g(x)) = 1$

D The limits does not exist (DNE)

Solution: C

First find the composite $f(g(x))$, when $f(x) = \cos x$ and $g(x) = x + 4$.

$$f(x) = \cos x$$

$$f(g(x)) = \cos(x + 4)$$

Then find the limit of the composite function.

$$\lim_{x \to -4} f(g(x))$$

$$\lim_{x \to -4} \cos(x + 4)$$

$$\cos(-4 + 4)$$

$$\cos(0)$$

$$1$$

Topic: Limits of composites

Question: If $f(x) = x^2 - 2x - 4$ and $g(x) = 5x - 5$, evaluate the limit.

$$\lim_{x \to 3} f(g(x))$$

Answer choices:

A $\lim_{x \to 3} f(g(x)) = 0$

B $\lim_{x \to 3} f(g(x)) = 76$

C $\lim_{x \to 3} f(g(x)) = 10$

D $\lim_{x \to 3} f(g(x)) = \infty$

Solution: B

First find the composite $f(g(x))$, when $f(x) = x^2 - 2x - 4$ and $g(x) = 5x - 5$.

$$f(x) = x^2 - 2x - 4$$

$$f(g(x)) = (5x - 5)^2 - 2(5x - 5) - 4$$

$$f(g(x)) = (5x - 5)(5x - 5) - 2(5x - 5) - 4$$

$$f(g(x)) = 25x^2 - 25x - 25x + 25 - 2(5x - 5) - 4$$

$$f(g(x)) = 25x^2 - 25x - 25x + 25 - 10x + 10 - 4$$

$$f(g(x)) = 25x^2 - 60x + 31$$

Then find the limit of the composite function.

$$\lim_{x \to 3} f(g(x))$$

$$\lim_{x \to 3} 25x^2 - 60x + 31$$

$$25(3)^2 - 60(3) + 31$$

$$25(9) - 180 + 31$$

$$225 - 180 + 31$$

$$76$$