Topic: One-sided limits

Question: Find the left-hand limit.

$$\lim_{x \to 2^-} \frac{|x-2|}{x-2}$$

Answer choices:

A -1

B 1

C -2

D 2

Solution: A

If we try substitution to evaluate the limit, we get the undefined value $0/0$. Instead, let's try substituting a value to the left of $x = 2$ that's very close to $x = 2$, like $x = 1.9999$.

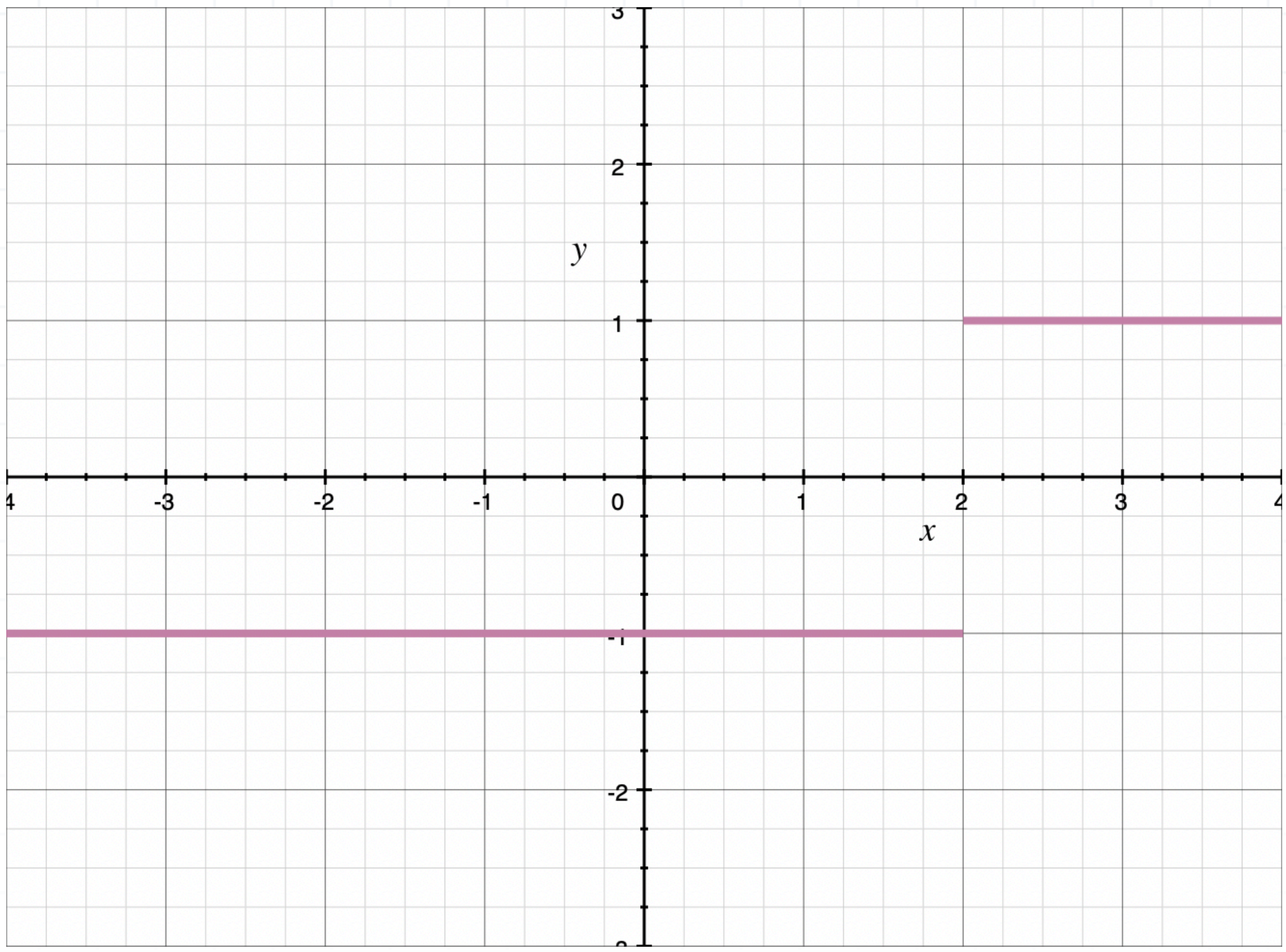
$$\frac{|1.9999 - 2|}{1.9999 - 2}$$

$$\frac{|-0.0001|}{-0.0001}$$

$$\frac{0.0001}{-0.0001}$$

$$-1$$

As we approach $x = 2$ from the left, the function is a constant -1 (the numerator is always positive and the denominator is always negative). The graph of the function confirms this value for the left-hand limit.


y
x
-4
-3
-2
-1
0
1
2
3
4
3
2
1
-1
-2
-3

Topic: One-sided limits

Question: Find the right-hand limit.

$$\lim_{x \to 2^+} \frac{|x-2|}{x-2}$$

Answer choices:

A -1

B 1

C -2

D 2

Solution: B

If we try substitution to evaluate the limit, we get the undefined value $0/0$. Instead, let's try substituting a value to the right of $x = 2$ that's very close to $x = 2$, like $x = 2.0001$.

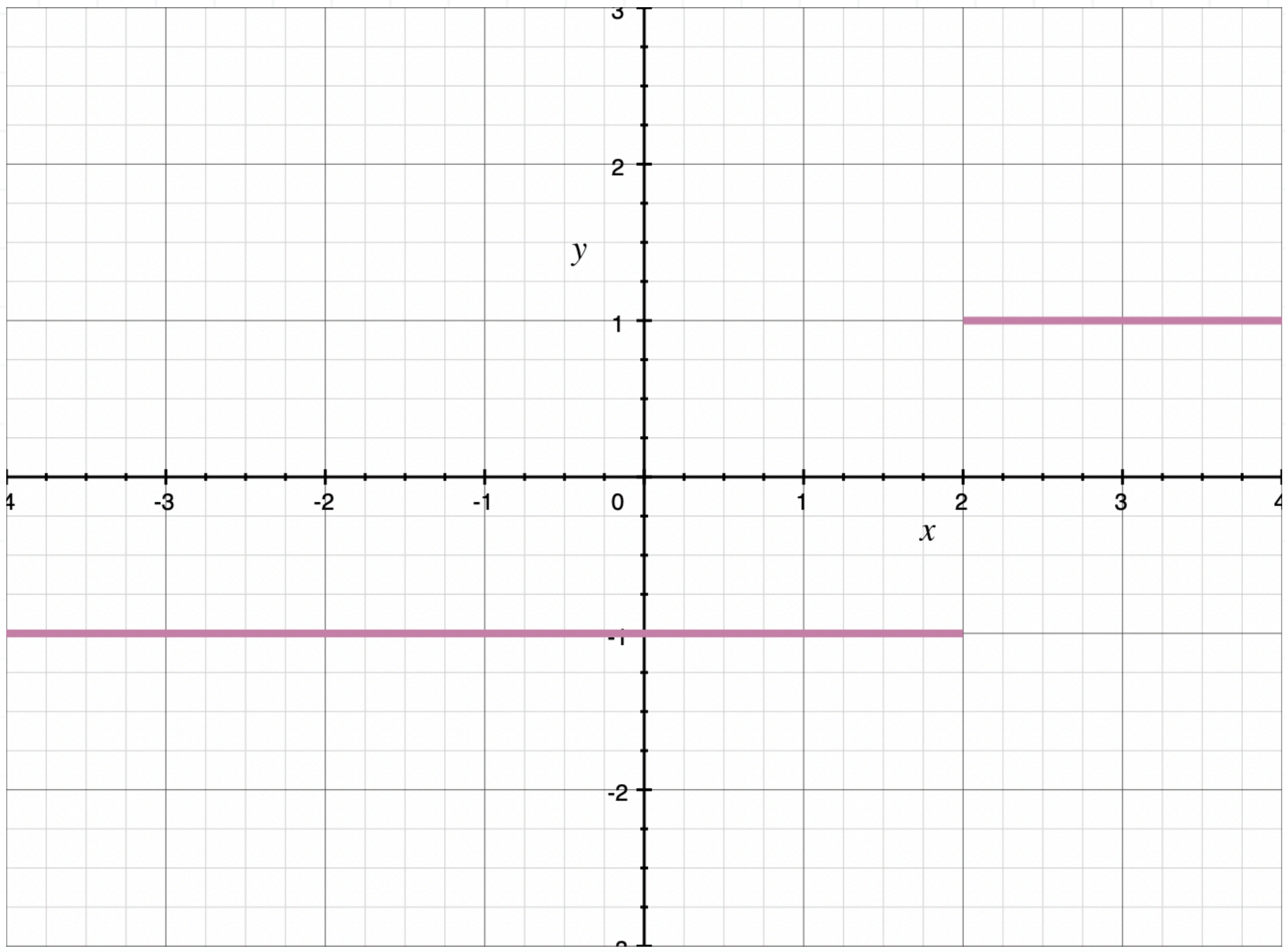
$$\frac{|2.0001 - 2|}{2.0001 - 2}$$

$$\frac{|0.0001|}{0.0001}$$

$$\frac{0.0001}{0.0001}$$

$$1$$

As we approach $x = 2$ from the right, the function is a constant 1 (the numerator is always positive and the denominator is always positive). The graph of the function confirms this value for the right-hand limit.


y
x
3
2
1
-1
-2
-4
-3
-2
-1
0
1
2
3
4

Topic: One-sided limits

Question: Find the limit.

$$\lim_{x \to 2} \frac{|x-2|}{x-2}$$

Answer choices:

A -1

B 1

C -2

D Does not exist (DNE)

Solution: D

We can see the left-hand limit of the function at $x = 2$ if we try substituting $x = 1.9999$.

$$\frac{|1.9999 - 2|}{1.9999 - 2}$$

$$\frac{|-0.0001|}{-0.0001}$$

$$\frac{0.0001}{-0.0001}$$

$$-1$$

We can see the right-hand limit of the function at $x = 2$ if we try substituting $x = 2.0001$.

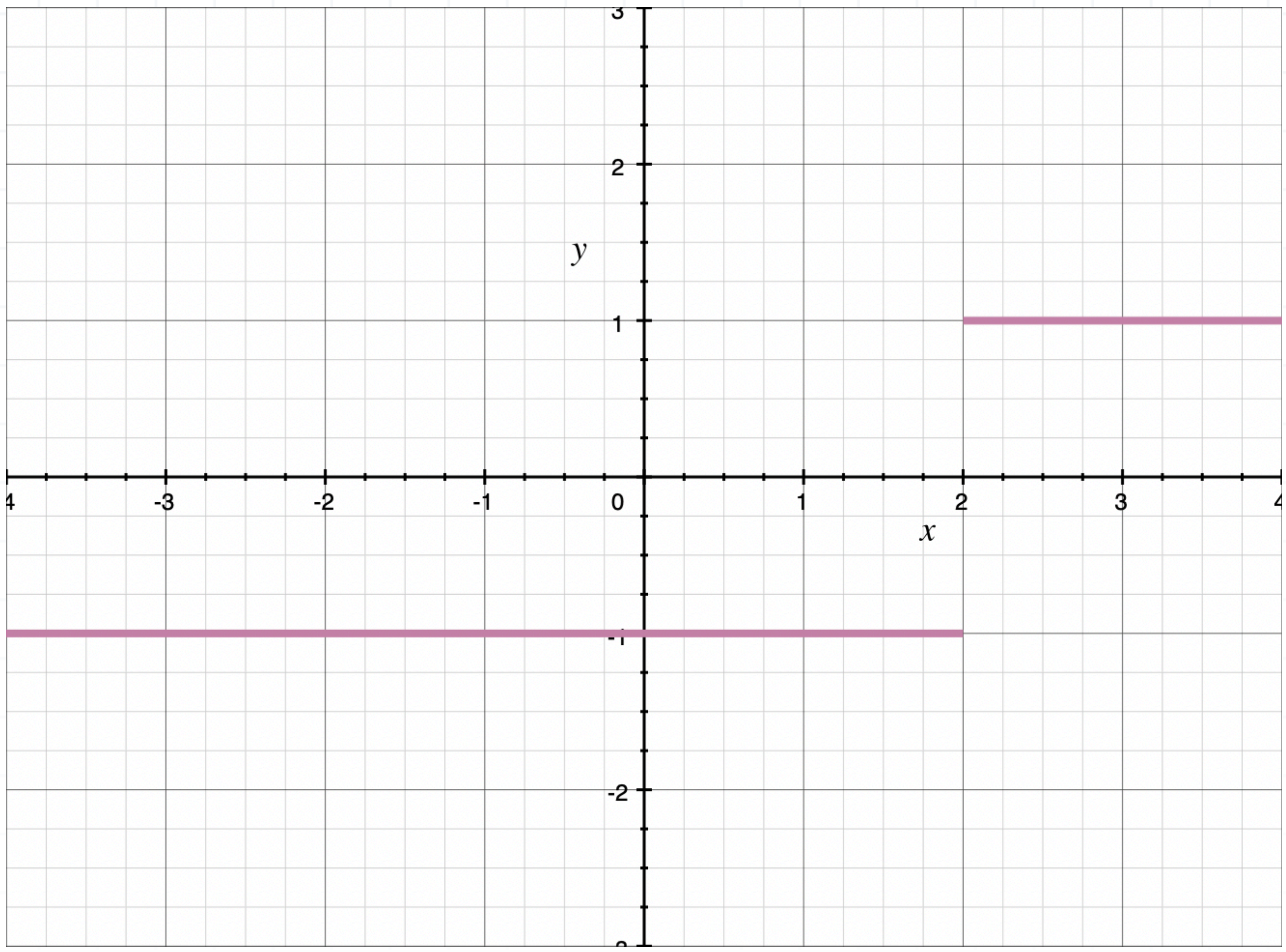
$$\frac{|2.0001 - 2|}{2.0001 - 2}$$

$$\frac{|0.0001|}{0.0001}$$

$$\frac{0.0001}{0.0001}$$

$$1$$

The graph of the function confirms these one-sided limits.



Because the one-sided limits aren't equivalent, the general limit of the function doesn't exist at $x = 2$.