Topic: Completing the square

Question: Complete the square to solve for the variable.

$$x^2 + 4x + 2 = 0$$

Answer choices:

A $x = -2 \pm \sqrt{2}$

B $x = 2 \pm \sqrt{2}$

C $x = -2 \pm \sqrt{3}$

D $x = 2 \pm \sqrt{3}$

Solution: A

We have a quadratic polynomial of the form $x^2 + bx + c$ (with $b = 4$ and $c = 2$) on the left side of the given equation.

First, we'll subtract c (which is 2) from both sides of the equation.

$$x^2 + 4x + 2 - 2 = 0 - 2$$

$$x^2 + 4x = -2$$

Next, we'll find $(b/2)^2$. Here, $b = 4$.

$$\left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = 2^2 = 4$$

This is the number we have to add to both sides of the equation $x^2 + 4x = -2$ in order to complete the square.

$$x^2 + 4x + 4 = -2 + 4$$

Now we can factor the left-hand side as the square of a binomial.

$$(x + 2)(x + 2) = 2$$

$$(x + 2)^2 = 2$$

$$\sqrt{(x + 2)^2} = \sqrt{2}$$

$$x + 2 = \pm\sqrt{2}$$

To solve this equation for x, we subtract 2 from both sides.

$$x + 2 - 2 = -2 \pm \sqrt{2}$$

$$x = -2 \pm \sqrt{2}$$

We can't reduce $\sqrt{2}$ at all, so the roots of our equation are

$$x = -2 + \sqrt{2}$$

and

$$x = -2 - \sqrt{2}$$

Topic: Completing the square

Question: Complete the square to solve for the variable.

$$u^2 - 4u + 3 = 0$$

Answer choices:

A $u = -1, \ -3$

B $u = 1, \ -3$

C $u = 1, 3$

D $u = -1, 3$

Solution: C

Find $(b/2)^2$, where b is the coefficient of the u term. Here, $b = -4$.

$$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$$

This is the number we have to add to both sides of the equation in order to complete the square.

$$u^2 - 4u + 4 + 3 = 0 + 4$$

$$u^2 - 4u + 4 + 3 = 4$$

$$u^2 - 4u + 4 = 1$$

Factor the left-hand side as the square of a binomial.

$$(u - 2)^2 = 1$$

$$u - 2 = \pm\sqrt{1}$$

$$u = 2 \pm 1$$

$$u = 1,\ 3$$

Topic: Completing the square

Question: Complete the square to solve for the variable.

$$x^2 - 5x - 4 = 0$$

Answer choices:

A $x = \dfrac{5 \pm \sqrt{73}}{2}$

B $x = \dfrac{-5 \pm \sqrt{73}}{2}$

C $x = \dfrac{5 \pm \sqrt{41}}{2}$

D $x = \dfrac{-5 \pm \sqrt{41}}{2}$

Solution: C

To complete the square, we'll add $(b/2)^2$ to both sides of the equation, where b is equal to -5, the coefficient on the first-degree term. We'll also move the other constant term to the right side of the equation.

$$x^2 - 5x + \left(-\frac{5}{2}\right)^2 = 4 + \left(-\frac{5}{2}\right)^2$$

$$x^2 - 5x + \frac{25}{4} = \frac{16}{4} + \frac{25}{4}$$

$$x^2 - 5x + \frac{25}{4} = \frac{41}{4}$$

We'll factor the left-hand side and solve for x.

$$\left(x - \frac{5}{2}\right)^2 = \frac{41}{4}$$

$$x - \frac{5}{2} = \pm\sqrt{\frac{41}{4}}$$

$$x = \frac{5}{2} \pm \frac{\sqrt{41}}{2}$$

$$x = \frac{5 \pm \sqrt{41}}{2}$$