Topic: Values that make the function differentiable

Question: Which values of a and b would make the function differentiable?

$$f(x) = \begin{cases} 3x^2 & x > 1 \\ bx^2 - a & x \le 1 \end{cases}$$

Answer choices:

A	$a = 3$	and	$b = 0$
B	$a = 0$	and	$b = 0$
C	$a = 0$	and	$b = 3$
D	$a = 3$	and	$b = 3$

Solution: C

The break point of the function is at $x = 1$, because that's where the first piece of the function ends and the second piece takes over.

We'll work on continuity first by setting the one-sided limits at the break point $x = 1$ equal to one another.

$$\lim_{x\to 1^+} 3x^2 = \lim_{x\to 1^-} bx^2 - a$$

$$3(1)^2 = b(1)^2 - a$$

$$3 = b - a$$

$$b - a = 3$$

Now we'll work on smoothness by setting the one-sided limits of the derivatives of each piece at the break point $x = 1$ equal to one another.

$$\lim_{x\to 1^+} 6x = \lim_{x\to 1^-} 2bx$$

$$6(1) = 2b(1)$$

$$6 = 2b$$

$$b = 3$$

Pull together these two equations into a system of equations.

$$b = 3$$

$$b - a = 3$$

We need to solve the system, which we can do by substituting the first equation $b = 3$ into the second equation.

$$3 - a = 3$$

$$-a = 0$$

$$a = 0$$

Therefore, the values of the constants a and b that make $f(x)$ differentiable are $a = 0$ and $b = 3$.

Topic: Values that make the function differentiable

Question: Which values of a and b would make the function differentiable?

$$f(x) = \begin{cases} x^2 - 5 & x > 3 \\ 4x^2 - 2ax + b & x \le 3 \end{cases}$$

Answer choices:

A $a = 22$ and $b = 9$

B $a = 22$ and $b = 22$

C $a = 9$ and $b = 9$

D $a = 9$ and $b = 22$

Solution: D

The break point of the function is at $x = 3$, because that's where the first piece of the function ends and the second piece takes over.

We'll work on continuity first by setting the one-sided limits at the break point $x = 3$ equal to one another.

$$\lim_{x \to 3^+} x^2 - 5 = \lim_{x \to 3^-} 4x^2 - 2ax + b$$

$$3^2 - 5 = 4(3)^2 - 2a(3) + b$$

$$9 - 5 = 4(9) - 6a + b$$

$$4 = 36 - 6a + b$$

$$-6a + b = -32$$

$$6a - b = 32$$

Now we'll work on smoothness by setting the one-sided limits of the derivatives of each piece at the break point $x = 3$ equal to one another.

$$\lim_{x \to 3^+} 2x = \lim_{x \to 3^-} 8x - 2a$$

$$2(3) = 8(3) - 2a$$

$$6 = 24 - 2a$$

$$-18 = -2a$$

$$a = 9$$

Pull together these two equations into a system of equations.

$$a = 9$$

$$6a - b = 32$$

We need to solve the system, which we can do by substituting the first equation $a = 9$ into the second equation.

$$6(9) - b = 32$$

$$54 - b = 32$$

$$-b = -22$$

$$b = 22$$

Therefore, the values of the constants a and b that make $f(x)$ differentiable are $a = 9$ and $b = 22$.

Topic: Values that make the function differentiable

Question: Which values of a and b would make the function differentiable?

$$f(x) = \begin{cases} ax^2 + 10 & x \le 2 \\ x^2 - 6x + b & x > 2 \end{cases}$$

Answer choices:

A $a = \frac{1}{2}$ and $b = 16$

B $a = -\frac{1}{2}$ and $b = -16$

C $a = \frac{1}{2}$ and $b = -16$

D $a = -\frac{1}{2}$ and $b = 16$

Solution: D

The break point of the function is at $x = 2$, because that's where the first piece of the function ends and the second piece takes over.

We'll work on continuity first by setting the one-sided limits at the break point $x = 2$ equal to one another.

$$\lim_{x \to 2^-} ax^2 + 10 = \lim_{x \to 2^+} x^2 - 6x + b$$

$$a(2)^2 + 10 = 2^2 - 6(2) + b$$

$$4a + 10 = 4 - 12 + b$$

$$4a - b = -18$$

Now we'll work on smoothness by setting the one-sided limits of the derivatives of each piece at the break point $x = 2$ equal to one another.

$$\lim_{x \to 2^-} 2ax = \lim_{x \to 2^+} 2x - 6$$

$$2a(2) = 2(2) - 6$$

$$4a = 4 - 6$$

$$4a = -2$$

$$a = -\frac{1}{2}$$

Pull together these two equations into a system of equations.

$$a = -\frac{1}{2}$$

$$4a - b = -18$$

We need to solve the system, which we can do by substituting the first equation $a = -1/2$ into the second equation.

$$4\left(-\frac{1}{2}\right) - b = -18$$

$$-2 - b = -18$$

$$-b = -16$$

$$b = 16$$

Therefore, the values of the constants a and b that make $f(x)$ differentiable are $a = -1/2$ and $b = 16$.