Calculus 1 Formulas

krista king MATH

Limits & Continuity

Idea of the limit

Limit

The limit of a function is the value the function approaches at a given value of x, regardless of whether the function actually reaches that value.

One-sided limits

The one-sided limits are the left- and right-hand limits. The left-hand limit is the limit of the function as we approach from the left side (or negative side), whereas the right-hand limit is the limit of the function as we approach from the right side (or positive side).

General limit

The general limit exists at a point $x = c$ if

1. the left-hand limit exists at $x = c$,
2. the right-hand limit exists at $x = c$, and
3. those left- and right-hand limits are equal to one another.

The general limit does not exist (DNE) at $x = c$ if

1. the left-hand limit does not exist at $x = c$, and/or

2. the right-hand limit does not exist at $x = c$, and/or

3. the left- and right-hand limits both exist, but aren't equal to one another.

Precise definition of the limit

The precise definition of the limit of the function tells us that, at $x = a$, the limit is L,

$$\lim_{x \to a} f(x) = L$$

if for every number $\epsilon > 0$ there is some number $\delta > 0$ such that

$$|f(x) - L| < \epsilon \quad \text{whenever} \quad 0 < |x - a| < \delta$$

Combinations and composites

Limit of the combination

There are two ways to find the limit of the combination.

1. Find the combination of the functions, then take the limit of the combination.

2. Take the limit of each function, then find the combination of the limits.

Composite function

A "function of a function"

Continuity

Continuity

If we can draw the graph of the function without ever lifting our pencil off the paper as we sketch it out from left to right, then the function is continuous everywhere. At any point where we have to lift our pencil off the paper in order to continue sketching it, there must be a discontinuity at that point.

Point (removable) discontinuity

A point discontinuity exists wherever there's a hole in the graph at one specific point. These are also called "removable discontinuities" because we can "remove" the discontinuity redefining the function at that particular point. Point discontinuities exist when a factor in rational functions when a factor that would have made the denominator 0 is cancelled from the function. The general limit always exists at a point discontinuity.

Jump discontinuity

A jump discontinuity exists wherever there's a big break in the graph that isn't caused by an asymptote. Jump discontinuities usually occur in piecewise-defined functions. The general limit never exists at a jump discontinuity.

Infinite (essential) discontinuity

An infinite discontinuity is the kind of discontinuity that occurs at an asymptote. Infinite discontinuities exist in rational functions in factors that make the denominator equal to 0 and can't be cancelled from the denominator.

Endpoint discontinuity

Endpoint discontinuities exist at a and b when a function is defined over a particular interval $[a, b]$. The general limit never exists at an endpoint discontinuity.

Intermediate value theorem

Root

The **root** of a function, graphically, is a point where the graph of the function crosses the x-axis. Algebraically, the root of a function is the point where the function's value is equal to 0.

Intermediate value theorem

Let $f(x)$ be a function which is continuous on the closed interval $[a, b]$ and let y_0 be a real number lying between $f(a)$ and $f(b)$. If $f(a) < y_0 < f(b)$ or $f(b) < y_0 < f(a)$, then there will be at least one c on the interval (a, b) where $y_0 = f(c)$.

Solving limits

Process for solving limits

Try direct substitution first, then factoring, then conjugate method

Conjugate

The **conjugate** of an expression is an expression with the same two terms, but with the opposite sign between the terms.

Limits at infinity

The limit at infinity is the limit of the function as we approach ∞ or $-\infty$.

Infinite limits

A limit is infinite when the value of the limit is ∞ or $-\infty$ as we approach a particular point.

Degree in a rational function

The degree of the numerator or denominator is the exponent on the term with the largest exponent.

N < D: If the degree of the numerator is less than the degree of the denominator, then the horizontal asymptote is given by $y = 0$.

N > D: If the degree of the numerator is greater than the degree of the denominator, then the function doesn't have a horizontal asymptote.

N = D: If the degree of the numerator is equal to the degree of the denominator, then the horizontal asymptote is given by the ratio of the coefficients on the highest-degree terms.

Trigonometric limits

Limit problems with trigonometric functions usually revolve around three key limit values.

$$\lim_{x \to 0} \sin x = 0$$

$$\lim_{x\to 0} \cos x = 1$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

Reciprocal identities

$$\sin x = \frac{1}{\csc x}$$

$$\cos x = \frac{1}{\sec x}$$

$$\tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

Pythagorean identities

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Squeeze theorem

The **squeeze theorem** allows us to find the limit of a function at a particular point, even when the function is undefined at that point. The way that we do it is by showing that our function can be "squeezed" between two other functions at the given point, and proving that the limits of these other functions are equal.

Derivatives

Definition of the derivative

Secant line, average rate of change

A secant line is a line that runs right through the graph, crossing it at a point. The slope of the secant line is the average rate of change of the function over the points $(c, f(c))$ and $(c+\Delta x, f(c+\Delta x))$ at which the secant line intersects the function.

$$m = \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

Tangent line, instantaneous rate of change, difference quotient

A tangent line is a line that *just* barely touches the edge of a graph, intersecting it at exactly one specific point. The line doesn't cross the graph, it skims along the graph and stays along the same side of the graph. The slope of the tangent line is the instantaneous rate of change of the function at the point at which the tangent line intersects the function.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}$$

Derivative rules

Power rule

The power rule lets us take the derivative of power functions.

$$(a \cdot n)x^{n-1}$$

Derivative of a constant

The derivative of a constant is 0.

Power rule for negative powers

$$x^{-a} = \frac{1}{x^a}$$

$$\frac{1}{x^{-a}} = x^a$$

Power rule for fractional powers

$$x^{\frac{a}{b}} = \sqrt[b]{x^a}$$

Product rule

For $y = f(x)g(x)$, the derivative is

$$y' = f(x)g'(x) + f'(x)g(x)$$

For $y = f(x)g(x)h(x)$, the derivative is

$$y' = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$

Quotient rule

For $y = \frac{f(x)}{g(x)}$, the derivative is

$$y' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Reciprocal rule

For $y = \frac{a}{g(x)}$, the derivative is

$$y' = \frac{-ag'(x)}{[g(x)]^2}$$

Chain rule

The chain rule

The chain rule lets us calculate derivatives of nested functions, where one function is the “outside” function and one function is the “inside function.

For $y = g[f(x)]$, the derivative is

$$y' = g'[f(x)]f'(x)$$

Applying the chain rule requires just two simple steps:

1. Take the derivative of the “outside” function, leaving the “inside” function untouched.

2. Multiply that result by the derivative of the “inside” function.

Derivatives of trig functions

Derivatives of the six trig functions

Trigonometric function	Its derivative
$y = \sin x$	$y' = \cos x$
$y = \cos x$	$y' = -\sin x$
$y = \tan x$	$y' = \sec^2 x$
$y = \cot x$	$y' = -\csc^2 x$

$y = \sec x$	$y' = \sec x \tan x$
$y = \csc x$	$y' = -\csc x \cot x$

Derivatives of the inverse trig functions

Function	Derivative	With $g(x)$ argument
$y = \sin^{-1} x$	$y' = \dfrac{1}{\sqrt{1 - x^2}}$	$y' = \dfrac{g'(x)}{\sqrt{1 - [g(x)]^2}}$
$y = \cos^{-1} x$	$y' = -\dfrac{1}{\sqrt{1 - x^2}}$	$y' = -\dfrac{g'(x)}{\sqrt{1 - [g(x)]^2}}$
$y = \tan^{-1} x$	$y' = \dfrac{1}{1 + x^2}$	$y' = \dfrac{g'(x)}{1 + [g(x)]^2}$
$y = \sec^{-1} x$	$y' = \dfrac{1}{\lvert x \rvert \sqrt{x^2 - 1}}$	$y' = -\dfrac{g'(x)}{\lvert g(x) \rvert \sqrt{[g(x)]^2 - 1}}$
$y = \csc^{-1} x$	$y' = -\dfrac{1}{\lvert x \rvert \sqrt{x^2 - 1}}$	$y' = \dfrac{g'(x)}{\lvert g(x) \rvert \sqrt{[g(x)]^2 - 1}}$
$y = \cot^{-1} x$	$y' = -\dfrac{1}{1 + x^2}$	$y' = -\dfrac{g'(x)}{1 + [g(x)]^2}$

Derivatives of hyperbolic functions

Function	Derivative	With $g(x)$ argument

$y = \sinh x$	$y' = \cosh x$	$y' = \cosh[g(x)][g'(x)]$
$y = \cosh x$	$y' = \sinh x$	$y' = \sinh[g(x)][g'(x)]$
$y = \tanh x$	$y' = \text{sech}^2 x$	$y' = \text{sech}^2[g(x)][g'(x)]$
$y = \text{csch} x$	$y' = -\text{csch} x \coth x$	$y' = -\text{csch}[g(x)]\coth[g(x)][g'(x)]$
$y = \text{sech} x$	$y' = -\text{sech} x \tanh x$	$y' = -\text{sech}[g(x)]\tanh[g(x)][g'(x)]$
$y = \coth x$	$y' = -\text{csch}^2 x$	$y' = -\text{csch}^2[g(x)][g'(x)]$

Derivatives of inverse hyperbolic functions

Function	Derivative	With $g(x)$ argument
$y = \sinh^{-1} x$	$y' = \dfrac{1}{\sqrt{x^2+1}}$	$y' = \dfrac{g'(x)}{\sqrt{[g(x)]^2+1}}$
$y = \cosh^{-1} x$	$y' = \dfrac{1}{\sqrt{x^2-1}}, x > 1$	$y' = \dfrac{g'(x)}{\sqrt{[g(x)]^2-1}}, g(x) > 1$
$y = \tanh^{-1} x$	$y' = \dfrac{1}{1-x^2}, \ \lvert x \rvert < 1$	$y' = \dfrac{g'(x)}{1-[g(x)]^2}, \ \lvert g(x) \rvert < 1$
$y = \text{csch}^{-1} x$	$y' = -\dfrac{1}{\lvert x \rvert\sqrt{x^2+1}}, x \neq 0$	$y' = -\dfrac{g'(x)}{\lvert g(x) \rvert\sqrt{[g(x)]^2+1}}, g(x) \neq 0$
$y = \text{sech}^{-1} x$	$y' = -\dfrac{1}{x\sqrt{1-x^2}}, 0 < x < 1$	$y' = -\dfrac{g'(x)}{g(x)\sqrt{1-[g(x)]^2}}, 0 < g(x) < 1$

$y = \coth^{-1} x$	$y' = \dfrac{1}{1 - x^2}, \ \lvert x \rvert > 1$	$y' = \dfrac{g'(x)}{1 - [g(x)]^2}, \ \lvert g(x) \rvert > 1$

Derivatives of $\ln x$ and e^x

The exponential

The constant $e \approx 2.718281828459045...$

Derivatives of exponential functions

Functions	Derivative	With $g(x)$ argument
$y = e^x$	$y' = e^x$	$y' = e^{g(x)}g'(x)$
$y = a^x$	$y' = a^x(\ln a)$	$y' = a^{g(x)}(\ln a)g'(x)$

The logarithm

$\ln a$ is the natural logarithm, evaluated at a

Derivatives of logarithmic functions

Function	Derivative	With $g(x)$ argument

$$y = \log_a x \qquad y' = \frac{1}{x \ln a} \qquad y' = \frac{1}{g(x)\ln a} g'(x)$$

$$y = \ln x \qquad y' = \frac{1}{x} \qquad y' = \frac{1}{g(x)} g'(x)$$

Logarithmic differentiation

Logarithmic differentiation is a problem-solving method in which we start by applying the natural log function to both sides of the equation.

Laws of logs and natural logs

Laws of logs	Laws of natural logs
$\log_a a^x = x$	$\ln(e^x) = x$
$a^{\log_a x} = x$	$e^{\ln x} = x$
$\log_a x^r = r \log_a x$	$\ln(x^a) = a \ln x$
$\log_a(xy) = \log_a x + \log_a y$	$\ln(xy) = \ln x + \ln y$
$\log_a \frac{x}{y} = \log_a x - \log_a y$	$\ln \frac{x}{y} = \ln x - \ln y$

Tangent and normal lines

Equation of the tangent line

A tangent line is a line that touches the graph of a function at exactly one point, the point $x = a$.

$$y = f(a) + f'(a)(x - a)$$

Differentiability

A function is differentiable at a particular point if it's continuous and smooth at that point.

1. A piecewise function will be continuous at its break point if the one-sided limits of the function's value at the break point are equal

2. A piecewise function will be smooth if the one-sided limits of the slopes (value of the derivative) of each piece at the break point are equal

Normal line

The normal line to a function at a particular point is the line that's perpendicular to the tangent line to the function at that same point. So if the slope of the tangent line is m, then the slope of the normal line is the negative reciprocal of m, or $-1/m$.

We can find the equation of the normal line by following these steps:

1. Take the derivative of the original function, and evaluate it at the given point. This is the slope of the tangent line, which we'll call m.

2. Find the negative reciprocal of m, which will be $-1/m$. This is the slope of the normal line, which we'll call n. So $n = -1/m$.

3. Plug n and the given point into the point-slope formula for the equation of the line, $(y - y_1) = n(x - x_1)$.

4. Simplify the normal line equation by solving for y.

Average rate of change

$$\frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Implicit differentiation

Implicit differentiation

Implicit differentiation allows us to take the derivative of a function that contains both x and y on the same side of the equation.

To use implicit differentiation, we'll follow these steps:

1. Differentiate both sides with respect to x.

2. Whenever we encounter y, we differentiate it like we would x, but we multiply that term by the derivative of y, which we write as y' or as dy/dx.

3. Move all terms involving dy/dx to the left side and everything else to the right.

4. Factor out dy/dx on the left and divide both sides by the other left-side factor so that dy/dx is the only thing remaining on the left.

Tangent line to an implicit function

To find the equation of the tangent line to an implicitly-defined function at a particular point,

1. Find the derivative using implicit differentiation.

2. Evaluate the derivative at the given point to find the slope of the tangent line.

3. Plug the slope of the tangent line and the given point into the point-slope formula for the equation of a line, $y - y_1 = m(x - x_1)$.

4. Simplify the tangent line equation.

First and second derivatives

The first derivative is exactly the value we've been finding all along; it's what you get when you take the derivative. The second derivative is the derivative of the derivative; it's the derivative of the first derivative.

First derivative	**Second derivative**
y'	y''
$\frac{dy}{dx}$	$\frac{d^2y}{dx^2}$

Applications of Derivatives

Optimization and sketching graphs

Extrema

Extrema are the function's "extreme points," including

- Global (absolute) maximum: the highest value the function reaches. There can only be one global maximum, but the function can reach that maximum in multiple locations.

- Global (absolute) minimum: the lowest value the function reaches. There can only be one global minimum, but the function can reach that minimum in multiple locations.

- Local (relative) maximum: the highest value the function reaches in a particular area of the graph. There can be an infinite number of local maxima, all of which can have different values and occur at different points.

- Local (relative) minimum: the lowest value the function reaches in a particular area of the graph. There can be an infinite number of local minima, all of which can have different values and occur at different points.

In general, a local/relative maximum exists wherever the function changes direction from increasing to decreasing. If a local maximum also happens to be the function's highest point anywhere in its domain, then it's also the global/absolute maximum. A function can have infinitely many local/relative maxima, but it'll only one (or no) global/absolute maximum.

A local/relative minimum exists wherever the function changes direction from decreasing to increasing. If a local minimum also happens to be the function's lowest point anywhere in its domain, then it's also the global/absolute minimum. A function can have infinitely many local/relative minima, but it'll only one (or no) global/absolute minimum.

Critical points

Critical points exist where the derivative is equal to 0 (or possibly where the derivative is undefined), and they represent points at which the graph of the function will change direction, either from decreasing to increasing, or from increasing to decreasing.

Because the function changes direction at critical points, the function will always have at least a local maximum or minimum at the critical point, if not a global maximum or minimum there.

Increasing and decreasing

Where the derivative is positive, the function is increasing. A function is increasing when it moves up as we move from left to right. Where the derivative is negative, the function is decreasing. A function is decreasing when it moves down as we move from left to right.

First derivative test

- If the derivative is negative to the left of the critical point and positive to the right of it, the graph has a local minimum at that point (and it's possible this local minimum *might* be a global minimum).

- If the derivative is positive to the left of the critical point and negative to the right of it, the graph has a local maximum at that point (and it's possible this local maximum *might* be a global maximum).

Inflection points

A point at which the function changes from concave up to concave down, or from concave down to concave up.

Concave up and concave down

- Where the second derivative is positive, the function is concave up. A function is **concave up** when it's "scooping" upwards, like a bowl or a cup.

- Where the second derivative is negative, the function is concave down. A function is **concave down** when it's "scooping" downwards, like a hat or a dome.

Second derivative test

- If $f''(x) > 0$ at a critical point, there's a local minimum there.

- If $f''(x) < 0$ at a critical point, there's a local maximum there.

Intercepts

The points at which the function crosses the x- and y-axes.

Asymptotes

An asymptote is a line which a function's graph approaches, but never crosses.

- Vertical asymptotes only exist where the function is undefined.
- If the degree of the numerator is less than the degree of the denominator, then the x-axis is a horizontal asymptote. If the degree of the numerator is equal to the degree of the denominator, then the ratio of the coefficients on these highest-degree terms is the equation of the horizontal asymptote. If the degree of the numerator is greater than the degree of the denominator, there is no horizontal asymptote.
- A slant asymptote exists when the degree of the numerator is exactly one greater than the degree of the denominator.

Sketching $f(x)$ from $f'(x)$

$f(x)$	$f'(x)$	$f''(x)$
Critical point	0 **(x-intercept)**	
Increasing	**Positive (above the x-axis)**	
Decreasing	**Negative (below the x-axis)**	
Inflection point	**Critical point**	0 **(x-intercept)**
Concave up	**Increasing**	**Positive (above the x-axis)**
Concave down	**Decreasing**	**Negative (below the x-axis)**

Inflection point	**Critical point**
Concave up	**Increasing**
Concave down	**Decreasing**

Linear approximation

Linear approximation

When we use the tangent line equation as this kind of approximation tool, we call it the linear approximation (or linearization) equation, instead of the tangent line equation.

$$L(x) = f(a) + f'(a)(x - a)$$

Absolute error

The absolute error for an estimation at a particular point a is the absolute value of the difference between the function's actual value and the linear approximation at that point.

$$E_A(a) = |f(a) - L(a)|$$

Relative error

Relative error is the amount of error in the approximation, *relative* to the function's actual value.

$$E_R(a) = \frac{E_A(a)}{f(a)} = \frac{|f(a) - L(a)|}{f(a)}$$

Percentage error

The percentage error is simply the relative error expressed as a percentage.

$$E_P(a) = 100\% \cdot E_R(a) = 100\% \cdot \frac{E_A(a)}{f(a)} = 100\% \cdot \frac{|f(a) - L(a)|}{f(a)}$$

Related rates

Related rates problems

In general, in order to solve related rates problems, we want to follow these steps:

1. Build an equation containing all the relevant variables, solving for some of them using other information, if necessary.

2. Implicitly differentiate the equation with respect to time t, before plugging in any of the values we know.

3. Plug in all the values we know, leaving only the one we're trying to solving for.

4. Solve for the unknown variable.

Volumes of three-dimensional figures

Tank shape	Volume formula
Cube	$V = s^3$
Rectangular prism	$V = lwh$
Triangular prism	$V = (1/2)whl$
Pyramid	$V = (1/3)s^2h$
Cone	$V = (1/3)\pi r^2h$
Sphere	$V = (4/3)\pi r^3$
Cylinder	$V = \pi r^2h$

Right triangle

A triangle with one 90° angle

Hypotenuse

The longest side of a right triangle, opposite of the 90° angle

Pythagorean theorem

The sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse.

$$a^2 + b^2 = c^2$$

SOH-CAH-TOA

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Applied optimization

Solving optimization problems

1. Write an equation in one variable that represents the value we're tying to maximize or minimize.

2. Take the derivative, set it equal to 0 to find critical points, and use the first derivative test to determine where the function is increasing and decreasing.

3. Based on the increasing/decreasing behavior of the function, identify the function's maxima and minima.

4. Use the extrema to answer the question being asked.

Derivative theorems

Mean Value Theorem

The Mean Value Theorem tells us that, as long as the function is continuous (unbroken) and differentiable (smooth) everywhere inside the interval we've chosen, then there must be a line tangent to the curve somewhere in the interval, which is parallel to this line we've just drawn that connects the endpoints.

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

The consequence of the Mean Value Theorem is that the instantaneous rate of change at $x = c$ will be equal to the average rate of change over the interval.

Rolle's Theorem

Rolle's Theorem tells us that, as long as the function is continuous (unbroken) and differentiable (smooth) inside the interval, then there must be a tangent line that's parallel to the horizontal line that connects the endpoints.

Rolle's Theorem can prove all of the following:

- The existence of a horizontal tangent line in the interval
- A point at which the derivative is 0 in the interval
- The existence of a critical point in the interval
- A point at which the function changes direction in the interval, either from increasing to decreasing, or from decreasing to increasing

Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

L'Hospital's Rule

If substitution into the function gives an indeterminate form ($\pm\infty / \pm\infty$, $0/0$, or $(0)(\pm\infty)$), we apply L'Hospital's rule by replacing both the numerator and denominator of the fraction with their own derivatives.

Physics

Position, velocity, and acceleration

Position	$x(t)$
Velocity	$v(t) = x'(t)$
Acceleration	$a(t) = v'(t) = x''(t)$

Speed

Speed is always positive (it has no direction).

$$s(t) = |v(t)|$$

Speed is **increasing** when velocity and acceleration have the same sign: $v(t),\ a(t) > 0$ or $v(t),\ a(t) < 0$

Speed is **decreasing** when velocity and acceleration have opposite signs: $v(t) > 0$ with $a(t) < 0$, or $v(t) < 0$ with $a(t) > 0$

Velocity

Velocity can be positive or negative, depending on its direction

$$v(t) = x'(t)$$

Object is **moving forward** (to the right) when $v(t) > 0$

Object is **moving backward** (to the left) when $v(t) < 0$

Object is **at rest** (not moving) when $v(t) = 0$

Velocity is **increasing** when $a(t) > 0$

Velocity is **decreasing** when $a(t) < 0$

Acceleration

Acceleration can be positive or negative

$$a(t) = v'(t) = x''(t)$$

Instantaneous vs. average velocity

To find instantaneous velocity, we simply evaluate the velocity function $v(t)$ at $t = a$. But to find average velocity, we'll use

$$\Delta v(a, b) = \frac{x(b) - x(a)}{b - a}$$

Economics

Revenue, cost, and profit

If p is the demand function for the product, F is fixed cost, and $V(x)$ is variable cost, then the formulas for revenue, cost, and profit are

Revenue $\quad R(x) = xp$

Cost $\quad C(x) = F + V(x)$

Profit $$P(x) = R(x) - C(x)$$

Marginal revenue, cost, and profit

The marginal revenue function models the revenue generated by selling one more unit, the marginal cost function models the cost of making one more unit, and the marginal profit function models the profit made by selling one more unit.

Exponential growth and decay

Exponential decay

The basic equation for exponential decay is

$$y = Ce^{kt}$$

where C is the amount of a substance that we're starting with, k is the decay constant, and y is the amount of the substance we have remaining after time t. Since substances decay at different rates, k will vary depending on the substance.

Half life

Half life is the amount of time required for exactly half of our original substance to decay, leaving exactly half of what we started with.

Newton's Law of Cooling

Newton's Law of Cooling models the way in which a warm object in a cooler environment cools down until it matches the temperature of its environment.

The Newton's Law of Cooling formula is

$$\frac{dT}{dt} = -k(T - T_a) \text{ with } T(0) = T_0$$

where T is the temperature over time t, k is the decay constant, T_a is the temperature of the environment ("ambient temperature"), and T_0 is the initial, or starting, temperature of the hot object.

Exponential decay

If sales of a product are consistently declining at an exponential rate, we can model that decline with the formula

$$F = Pe^{rt}$$

where P is the number of items being sold, r is the rate of decline, and F is the number of items being sold after sales have continued to decline for some specified amount of time t.

Compound interest

When interest is **compounded**, it means that the interest gets added to the principal that we initially deposited, and therefore will itself also earn interest. Because of this compounding, we'll earn more and more interest each year.

Exponential growth

Exponential growth is modeled by

$$A = Pe^{rt}$$

where P is the initial investment (the principal), r is annual interest rate, and A is the amount in the account after time t. Sometimes we'll write this formula as

$$FV = PVe^{rt}$$

where $A = FV$ is the "future value" in the account, and $P = PV$ is the "present value" in the account.

W W W . K R I S T A K I N G M A T H . C O M