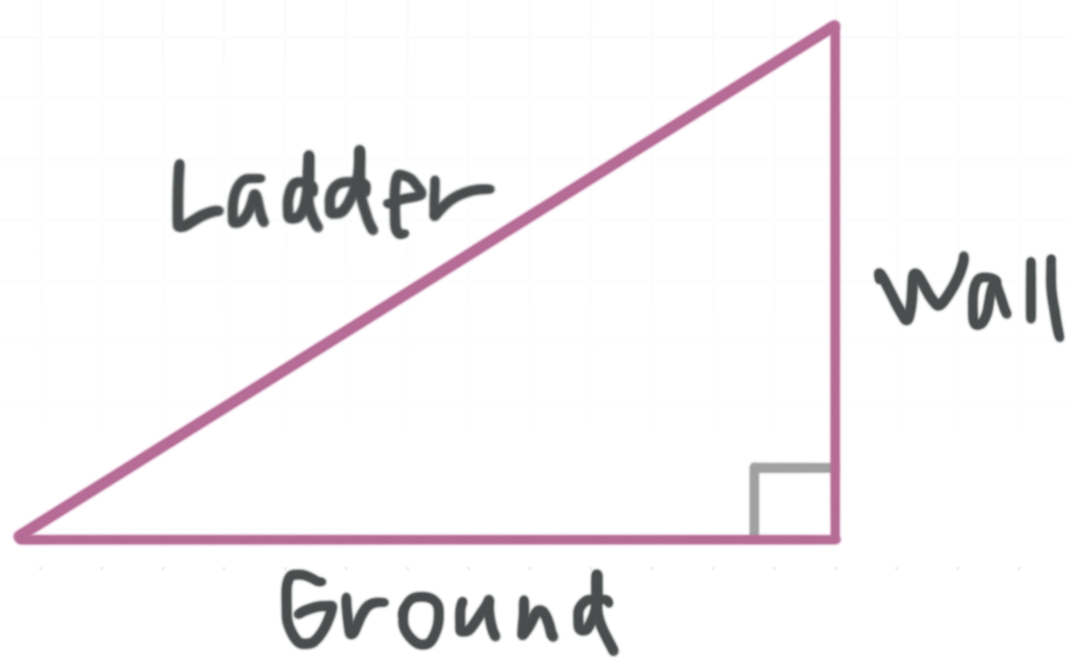
Ladder sliding down the wall

The last type of related rates problem we want to focus on is the common "ladder sliding down the wall" problem. Sometimes this is a ladder sliding down a wall, sometimes it's a shovel sliding down a garden fence, but the idea is always the same.

Like in the "observer and the airplane" lesson, we'll use the Pythagorean theorem $a^2 + b^2 = c^2$ in these problems, because the ladder, the wall, and the ground will form a right triangle.



One of the things to keep in mind here is that the length of the ladder is constant. Its length won't change as it falls. But as the ladder slides, the height of the wall between the ground and the top of the ladder will decrease. At the same time, the length of the ground between the wall and the bottom base of the ladder will increase.

We also often need to use trigonometric functions to work with the angles inside the triangle formed by the ladder, the wall, and the ground. The trig functions we want to focus on are sine, cosine, and tangent. As a reminder, the sine of an angle inside the triangle is equal to the length of the opposite side, divided by the length of the hypotenuse; the cosine of an angle inside the triangle is equal to the length of the adjacent side, divided by the length of the hypotenuse; and the tangent of an angle inside the triangle is equal to the length of the opposite side, divided by the length of the adjacent side.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

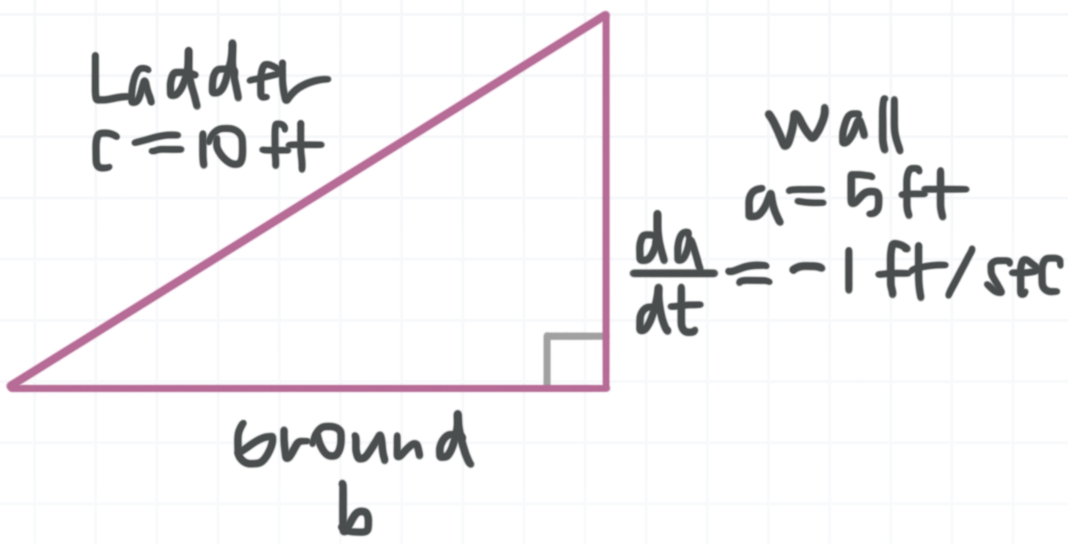
We may also need to utilize the fact that the three interior angles of a triangle always sum to 180°.

With all of this background laid out now, let's work through an example.

Example

A 10 foot ladder is sliding down a wall at a rate of 1 foot per second. How quickly is the base of the ladder sliding along the ground away from wall when the top of the ladder is 5 feet off the ground?

The first thing we want to do is sketch a diagram, and include everything we know.



Next, we start with the Pythagorean theorem equation, $a^2 + b^2 = c^2$. We don't need to preserve dc/dt in the derivative equation, because we haven't been asked to solve for the rate of change of the length of the ladder, and we know that the length of the ladder stays constant at $c = 10$ ft. So we'll substitute $c = 10$.

$$a^2 + b^2 = 10^2$$

$$a^2 + b^2 = 100$$

Because, at the moment we're interested in, we know the length of sides a and c, we can use the Pythagorean theorem to solve for b.

$$5^2 + b^2 = 10^2$$

$$25 + b^2 = 100$$

$$b^2 = 75$$

$$b = \sqrt{75}$$

$$b = 5\sqrt{3}$$

Use implicit differentiation to find the derivative,

$$2a\left(\frac{da}{dt}\right)+2b\left(\frac{db}{dt}\right)=0$$

then substitute what we know.

$$2(5)(-1)+2(5\sqrt{3})\left(\frac{db}{dt}\right)=0$$

We've been asked to find the rate of change of the length of side b, which is the db/dt value that remains in the equation. We need to solve for that value.

$$-10+10\sqrt{3}\left(\frac{db}{dt}\right)=0$$

$$10\sqrt{3}\left(\frac{db}{dt}\right)=10$$

$$\frac{db}{dt}=\frac{10}{10\sqrt{3}}$$

$$\frac{db}{dt}=\frac{1}{\sqrt{3}}$$

$$\frac{db}{dt}\approx 0.58$$

The answer tells us that the length of side b, which is the distance between the wall and the base of the ladder, is increasing at a rate of 0.58 feet per second as the base of the ladder slides away from the wall.
