

**Topic:** Inverse trigonometric derivatives**Question:** Find the derivative of the inverse trig function.

$$f(x) = \tan^{-1}(x^2 - 1)$$

**Answer choices:**

A  $f'(x) = \frac{2x}{x^4 - 2x^2 + 2}$

B  $f'(x) = \frac{1}{x^4 - 2x^2 + 2}$

C  $f'(x) = \frac{2x}{1 + 4x^2}$

D  $f'(x) = \frac{1}{1 + 4x^2}$



**Solution: A**

Apply the formula for the derivative of inverse tangent, with  $g(x) = x^2 - 1$ , in order to differentiate the function.

$$y' = \frac{g'(x)}{1 + [g(x)]^2}$$

$$y' = \frac{2x}{1 + (x^2 - 1)^2}$$

Simplify the derivative.

$$y' = \frac{2x}{1 + (x^4 - 2x^2 + 1)}$$

$$y' = \frac{2x}{x^4 - 2x^2 + 2}$$



**Topic:** Inverse trigonometric derivatives**Question:** Find the derivative of the inverse trig function.

$$y = \frac{1}{\cos^{-1} x}$$

**Answer choices:**

A  $y' = \frac{1}{1 - x^2}$

B  $y' = \frac{1}{(\cos^{-1} x)^2 \sqrt{1 - x^2}}$

C  $y' = -\sqrt{1 - x^2}$

D  $y' = \frac{\sqrt{1 - x^2}}{(\cos^{-1} x)^2}$



**Solution: B**

Rewrite the function.

$$y = \frac{1}{\arccos x}$$

$$y = (\arccos x)^{-1}$$

Use substitution with  $u = \arccos x$  and

$$u' = -\frac{1}{\sqrt{1-x^2}}$$

Then the function is

$$y = u^{-1}$$

The derivative is

$$y' = -u^{-2}u'$$

$$y' = -(\arccos x)^{-2} \left( -\frac{1}{\sqrt{1-x^2}} \right)$$

$$y' = \frac{1}{(\arccos x)^2} \left( \frac{1}{\sqrt{1-x^2}} \right)$$

$$y' = \frac{1}{(\cos^{-1} x)^2 \sqrt{1-x^2}}$$



**Topic:** Inverse trigonometric derivatives**Question:** Find the derivative of the inverse trig function.

$$y = x \sin^{-1} \sqrt{x}$$

**Answer choices:**

A  $y' = x \sin^{-1} \sqrt{x} + \frac{1}{\sqrt{1-x}}$

B  $y' = \sin^{-1} \sqrt{x} + \frac{x}{\sqrt{1-x}}$

C  $y' = x \sin^{-1} \sqrt{x} + \frac{1}{2\sqrt{x(1-x)}}$

D  $y' = \frac{\sqrt{x}}{2\sqrt{1-x}} + \sin^{-1} \sqrt{x}$



**Solution: D**

We need to apply product rule, with

$$f(x) = x$$

$$f'(x) = 1$$

and

$$g(x) = \sin^{-1} \sqrt{x}$$

$$g'(x) = \frac{1}{\sqrt{1 - (\sqrt{x})^2}} \left( \frac{1}{2} x^{-\frac{1}{2}} \right)$$

Take the derivative using product rule.

$$y' = f(x)g'(x) + f'(x)g(x)$$

$$y' = (x) \left[ \frac{1}{\sqrt{1 - (\sqrt{x})^2}} \left( \frac{1}{2} x^{-\frac{1}{2}} \right) \right] + (1)(\sin^{-1} \sqrt{x})$$

$$y' = \frac{1}{2} x \left( \frac{1}{x^{\frac{1}{2}} \sqrt{1 - x}} \right) + \sin^{-1} \sqrt{x}$$

$$y' = \frac{x}{2\sqrt{x}\sqrt{1-x}} + \sin^{-1} \sqrt{x}$$

$$y' = \frac{\sqrt{x}\sqrt{x}}{2\sqrt{x}\sqrt{1-x}} + \sin^{-1} \sqrt{x}$$



$$y' = \frac{\sqrt{x}}{2\sqrt{1-x}} + \sin^{-1} \sqrt{x}$$

