Equation modeling

In this lesson we'll look at how to solve a word problem in which we're told to write an equation that gives one of the quantities in the problem in terms of another one.

Let's start by working through an example.

Example

An RV and a motorcycle were driven for a month. The motorcycle traveled $1{,}000$ miles more than the RV. The fuel mileage for the RV was 15 miles per gallon (mpg), and the fuel mileage for the motorcycle was 43 mpg.

Write an equation which gives the total amount of fuel, g (in gallons), that was used by the two vehicles during that month in terms of the distance m (in miles) traveled by the motorcycle. We can use a table to show the fuel mileages and distances traveled.

	RV	Motorcycle
Mileage	15 mpg	43 mpg
Distance	r miles	m miles

We'll start by writing an equation that gives the distance r (in miles) traveled by the RV in terms of m. We know that $m = r + 1{,}000$ because the motorcycle traveled $1{,}000$ miles more than the RV. So $r = m - 1{,}000$, and we'll replace "r miles" in our table with "$m - 1{,}000$ miles."

	RV	Motorcycle
Mileage	15 mpg	43 mpg
Distance	m-1,000 miles	m miles

You can calculate the amount of fuel used by each vehicle, by dividing its distance by its fuel mileage. We'll add a row to our table, to show the amounts of fuel used by the vehicles.

	RV	Motorcycle
Mileage	15 mpg	43 mpg
Distance	m-1,000 miles	m miles
Fuel used	(m-1,000)/15 gallons	m/43 gallons

Now we know that

$$g = \frac{m - 1{,}000}{15} + \frac{m}{43}$$

$$g = \frac{43}{43} \cdot \frac{m - 1{,}000}{15} + \frac{15}{15} \cdot \frac{m}{43}$$

$$g = \frac{43m - 43{,}000}{645} + \frac{15m}{645}$$

$$g = \frac{43m - 43{,}000 + 15m}{645}$$

$$g = \frac{58m - 43{,}000}{645}$$

Let's look at a few more.

Example

A rock is thrown at a speed of 16 ft/s straight downward from a high platform. The distance through which it has fallen is $D = 16t^2 + 16t$, where t is the amount of time (in seconds) that it's been falling. The average speed of any falling object between time 0 (the time at which it starts falling) and time t is given by the ratio of the distance through which it has fallen to the elapsed time (t). Therefore, the average speed of this rock between time 0 and time t is $V = D/t$. Write an equation that gives V in terms of t.

Start with $V = D/t$, and substitute $16t^2 + 16t$ for D.

$$V = \frac{16t^2 + 16t}{t}$$

$$V = 16t + 16$$

Solve for t in terms of V.

$$16t = V - 16$$

$$t = \frac{V - 16}{16}$$

One last example.

Example

Each employee at a certain level of employment with a company is paid a salary of $\$42{,}000$ per year. In addition, the owner of the company wants to divide a bonus of $\$120{,}000$ evenly among these employees over the course of a year.

If each employee receives his/her salary and bonus in equal monthly installments throughout the year, write an equation which gives the total amount a that each employee is paid per month in terms of e, the number of employees.

Each employee is paid a salary of $\$42{,}000 \div 12 = \$3{,}500$ per month.

The total amount of the bonus is $\$120{,}000.00 \div 12 = \$10{,}000.00$ per month, but that has to be divided by the number of employees. So the monthly amount of the bonus for each employee is

$$\frac{\$10{,}000}{e}$$

The total amount that each employee is paid per month is

$$a = \$3{,}500 + \frac{\$10{,}000}{e}$$