Calculus 1 Workbook Solutions

Physics

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POSITION, VELOCITY, AND ACCELERATION

■ 1. Find the velocity $v(t)$, speed, and acceleration $a(t)$ at $t=2$ of the position function.

$$s(t) = -\frac{t^3}{3} + t^2 + 3t - 1$$

Solution:

Velocity is given by the first derivative of the position function.

$$s'(t) = v(t) = -t^2 + 2t + 3$$

$$v(2) = -(2)^2 + 2(2) + 3$$

$$v(2) = 3$$

Acceleration is given by the second derivative of the position function.

$$s''(t) = v'(t) = a(t) = -2t + 2$$

$$a(2) = -2(2) + 2$$

$$a(2) = -2$$

Speed is the absolute value of velocity. So speed is

$$|v(2)| = |3| = 3$$

■ 2. Find the velocity $v(t)$, speed, and acceleration $a(t)$ at $t=1$ of the position function.

$$s(t)=\frac{t^2-3}{t^3}$$

Solution:

Velocity is given by the first derivative of the position function.

$$s'(t)=v(t)=\frac{(t^3)(2t)-(t^2-3)(3t^2)}{(t^3)^2}=\frac{2t^4-3t^4+9t^2}{t^6}=\frac{-t^4+9t^2}{t^6}=\frac{-t^2+9}{t^4}$$

$$v(1)=\frac{-1^2+9}{1^4}=-1+9$$

$$v(1)=8$$

Acceleration is given by the second derivative of the position function.

$$s''(t)=v'(t)=a(t)=\frac{(-2t)(t^4)-(-t^2+9)(4t^3)}{(t^4)^2}=\frac{-2t^5+4t^5-36t^3}{t^8}=\frac{2t^2-36}{t^5}$$

$$a(1)=\frac{2(1)^2-36}{1^5}=2-36$$

$$a(1)=-34$$

Speed is the absolute value of velocity. So speed is

$$|v(1)| = |8| = 8$$

■ 3. Find the velocity $v(t)$, speed, and acceleration $a(t)$ at $t = 4$ of the position function.

$$s(t) = \frac{t^2}{2t+4}$$

Solution:

Velocity is given by the first derivative of the position function.

$$s'(t) = v(t) = \frac{(2t)(2t+4) - (t^2)(2)}{(2t+4)^2} = \frac{4t^2 + 8t - 2t^2}{4t^2 + 16t + 16} = \frac{t^2 + 4t}{2t^2 + 8t + 8} = \frac{t(t+4)}{2(t+2)(t+2)}$$

$$v(4) = \frac{4(4+4)}{2(4+2)(4+2)} = \frac{4(8)}{2(6)(6)}$$

$$v(4) = \frac{32}{72} = \frac{4}{9}$$

Acceleration is given by the second derivative of the position function.

$$s''(t) = v'(t) = a(t) = \frac{(2t+4)(2t^2+8t+8) - (t^2+4t)(4t+8)}{(2t^2+8t+8)^2}$$

$$a(t) = \frac{16t+32}{(2t^2+8t+8)^2} = \frac{16(t+2)}{4(t^2+4t+4)^2} = \frac{4(t+2)}{(t+2)^4} = \frac{4}{(t+2)^3}$$

$$a(4) = \frac{4}{(4+2)^3} = \frac{4}{216}$$

$$a(4)=\frac{1}{54}$$

Speed is the absolute value of velocity. So speed is

$$|v(4)|=\left|\frac{4}{9}\right|=\frac{4}{9}$$

BALL THROWN UP FROM THE GROUND

■ 1. A ball is thrown straight upward from the ground with an initial velocity of $v_0 = 86$ ft/sec. Assuming constant gravity, find the maximum height, in feet, that the ball attains, the time, in seconds, that it's in the air, as well as the ball's velocity, in ft/sec, when it hits the ground.

Solution:

Plugging everything we know into the formula for standard projectile motion, we get

$$y(t) = \frac{1}{2}gt^2 - v_0t - y_0$$

$$h(t) = -\frac{1}{2}(32)t^2 + 86t + 0$$

$$h(t) = -16t^2 + 86t$$

When the ball is at its maximum height, velocity is 0, so find $h'(t)$ and set it equal to 0.

$$h'(t) = -32t + 86$$

$$-32t + 86 = 0$$

$$32t = 86$$

$$t = \frac{43}{16} \approx 2.69 \text{ seconds}$$

Next, find the maximum height.

$$h(t) = -16\left(\frac{43}{16}\right)^2 + 86\left(\frac{43}{16}\right) = \frac{1{,}849}{16} \approx 115.56 \text{ feet}$$

To find the time the ball stays in the air, set the height equal to 0 and solve for t.

$$h(t) = -16t^2 + 86t$$

$$-16t^2 + 86t = 0$$

$$t(43 - 8t) = 0$$

$$t = 0, \frac{43}{8} \approx 5.38 \text{ seconds}$$

Now, find the final velocity of the ball when it hits the ground. Substitute the time the ball lands into the velocity function.

$$h'\left(\frac{43}{8}\right) = -32\left(\frac{43}{8}\right) + 86 = -86 \text{ ft/sec}$$

■ 2. A ball is thrown straight upward from the top of a building, which is 56 feet above the ground, with an initial velocity of $v_0 = 48$ ft/sec. Assuming constant gravity, find the maximum height, in feet, that the ball attains, the time, in seconds, that it's in the air, as well as the ball's velocity, in ft/sec, when it hits the ground.

Solution:

Plugging everything we know into the formula for standard projectile motion, we get

$$y(t) = \frac{1}{2}gt^2 - v_0t - y_0$$

$$h(t) = -\frac{1}{2}(32)t^2 + 48t + 56$$

$$h(t) = -16t^2 + 48t + 56$$

When the ball is at its maximum height, velocity is 0, so find $h'(t)$ and set it equal to 0.

$$h'(t) = -32t + 48$$

$$-32t + 48 = 0$$

$$32t = 48$$

$$t = \frac{3}{2} = 1.5 \text{ seconds}$$

Next, find the maximum height.

$$h(t) = -16\left(\frac{3}{2}\right)^2 + 48\left(\frac{3}{2}\right) + 56 = 92 \text{ feet}$$

To find the time the ball stays in the air, set the height equal to 0 and solve for t.

$$-16t^2 + 48t + 56 = 0$$

$$2t^2 - 6t - 7 = 0$$

$$t = \frac{3 + \sqrt{23}}{2} \approx 3.90 \text{ seconds}$$

Now, find the final velocity of the ball when it hits the ground. Substitute the time the ball lands into the velocity function.

$$h'\left(\frac{3 + \sqrt{23}}{2}\right) = -32\left(\frac{3 + \sqrt{23}}{2}\right) + 48 \approx -76.73 \text{ ft/sec}$$

■ 3. A ball is thrown straight upward from a bridge, which is 24 meters above the water, with an initial velocity of $v_0 = 20$ m/sec. Assuming constant gravity, find the maximum height, in meters, that the ball attains, the time, in seconds, that it's in the air, as well as the ball's velocity, in m/sec, when it hits the water below.

Solution:

Plugging everything we know into the formula for standard projectile motion, we get

$$y(t) = \frac{1}{2}gt^2 - v_0t - y_0$$

$$h(t) = -\frac{1}{2}(9.8)t^2 + 20t + 24$$

$$h(t) = -4.9t^2 + 20t + 24$$

When the ball is at its maximum height, velocity is 0, so find $h'(t)$ and set it equal to 0.

$$h'(t) = -9.8t + 20$$

$$-9.8t + 20 = 0$$

$$9.8t = 20$$

$$t = \frac{20}{9.8} \approx 2.041 \text{ seconds}$$

Next, find the maximum height.

$$h(t) = -4.9\left(\frac{100}{49}\right)^2 + 20\left(\frac{100}{49}\right) + 24 \approx 44.41 \text{ meters}$$

To find the time the ball stays in the air, set the height equal to 0 and solve for t.

$$h(t) = -4.9t^2 + 20t + 24$$

$$-4.9t^2 + 20t + 24 = 0$$

$$t \approx 5.05 \text{ seconds}$$

Now, find the final velocity of the ball when it hits the water. Substitute the time the ball lands into the velocity function.

$$h'(5.05) = -9.8(5.05) + 20 \approx -29.5 \text{ m/sec}$$

COIN DROPPED FROM THE ROOF

1. A rock is dropped from the top of an 800 foot tall cliff, with an initial velocity of $v_0 = 0$ ft/sec. Assuming constant gravity, when does the rock hit the ground, and what is its velocity when it hits the ground?

Solution:

Plugging everything we know into the formula for standard projectile motion, we get

$$y(t) = \frac{1}{2}gt^2 - v_0t - y_0$$

$$y(t) = \frac{1}{2}(32)t^2 - 0t - 800$$

$$y(t) = 16t^2 - 800$$

The rock hits the ground when its height is 0.

$$16t^2 - 800 = 0$$

$$16t^2 = 800$$

$$t^2 = 50$$

$t = \sqrt{50} \approx 7$ seconds

To find the velocity of the rock when it hits the ground, find $y'(t)$ and evaluate it at the time the rock hits the ground.

$$y'(t) = 32t$$

$$y'(7.071) = 32(7.071) \approx 226.27 \text{ ft/sec}$$

However, since the rock is falling downward, the correct expression for the velocity is -226.27 ft/sec.

■ 2. A rock is tossed from the top of a 300 foot tall cliff, with an initial velocity of $v_0 = 15$ ft/sec. Assuming constant gravity, when does the rock hit the ground, and what is its velocity when it hits the ground?

Solution:

Plugging everything we know into the formula for standard projectile motion, we get

$$y(t) = \frac{1}{2}gt^2 - v_0t - y_0$$

$$y(t) = \frac{1}{2}(32)t^2 - 15t - 300$$

$$y(t) = 16t^2 - 15t - 300$$

The rock hits the ground when its height is 0.

$$16t^2 - 15t - 300 = 0$$

$$t = \frac{15 \pm \sqrt{15^2 - 4(16)(-300)}}{2(16)} = \frac{15 \pm 5\sqrt{777}}{32} \approx 4.8242 \text{ seconds}$$

To find the velocity of the rock when it hits the ground, find $y'(t)$ and evaluate it at the time the rock hits the ground.

$$y'(t) = 32t - 15$$

$$y'(4.8242) = 32(4.8242) - 15 \approx 139.37 \text{ ft/sec}$$

However, since the rock is falling downward, the correct expression for the velocity is -139.37 ft/sec.

■ 3. A coin is tossed downward from the top of a 36 meter tall building, with an initial velocity of $v_0 = 6$ m/sec. Assuming constant gravity, when does the rock hit the ground, and what is its velocity when it hits the ground?

Solution:

Plugging everything we know into the formula for standard projectile motion, we get

$$y(t) = \frac{1}{2}gt^2 - v_0 t - y_0$$

$$y(t) = \frac{1}{2}(9.8)t^2 - 6t - 36$$

$$y(t) = 4.9t^2 - 6t - 36$$

The rock hits the ground when its height is 0.

$$4.9t^2 - 6t - 36 = 0$$

$$t = \frac{6 \pm \sqrt{6^2 - 4(4.9)(-36)}}{2(4.9)} = \frac{6 \pm \sqrt{741.6}}{9.8} \approx 3.391 \text{ seconds}$$

To find the velocity of the rock when it hits the ground, find $y'(t)$ and evaluate it at the time the rock hits the ground.

$$y'(t) = 9.8t - 6$$

$$y'(3.391) = 9.8(3.391) - 6 \approx 27.23 \text{ m/sec}$$

However, since the rock is falling downward, the correct expression for the velocity is -27.23 m/sec.

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