Topic: Coin dropped from the roof

Question: A pumpkin is dropped from the top of a building and falls 5 m to the ground. Given the position function of the pumpkin, find instantaneous velocity at $t = 3$ seconds.

$$s(t) = -6t^2 + 3t - 5$$

Answer choices:

A -40 m/s

B -39 m/s

C -33 m/s

D -50 m/s

Solution: C

Take the derivative of the position function to get the velocity function.

$$s(t) = -6t^2 + 3t - 5$$

$$s'(t) = -12t + 3$$

$$v(t) = -12t + 3$$

Substitute $t = 3$ to find instantaneous velocity at that time.

$$v(3) = -12(3) + 3$$

$$v(3) = -33$$

The instantaneous velocity at $t = 3$ is -33 m/s. Because the velocity is negative, it means that the pumpkin is falling toward the ground.

Topic: Coin dropped from the roof

Question: A baseball is dropped from the top of a bridge that's 8 m high. Find its average velocity during the first 4 seconds.

$$s(t) = -8t^2 - 4t - 8$$

Answer choices:

A -36 m/s

B -68 m/s

C -86 m/s

D -23 m/s

Solution: A

Substitute $t_1 = 0$ and $t_2 = 4$ into the formula for average velocity.

$$v_{avg} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

$$v_{avg} = \frac{s(4) - s(0)}{4 - 0}$$

$$v_{avg} = \frac{s(4) - s(0)}{4}$$

Find $s(0)$ and $s(4)$.

$$s(0) = -8(0)^2 - 4(0) - 8$$

$$s(0) = -8$$

and

$$s(4) = -8(4)^2 - 4(4) - 8$$

$$s(4) = -152$$

Substitute these values into the average velocity equation.

$$v_{avg} = \frac{-152 - (-8)}{4}$$

$$v_{avg} = \frac{-144}{4}$$

$$v_{avg} = -36 \text{ m/s}$$

Topic: Coin dropped from the roof

Question: A coin is dropped from the roof of a 400 ft building with an initial velocity of -64 ft/s. When does it hit the ground and what is the velocity at that time?

Answer choices:

A The coin hits the ground after 7.78 s at -108.48 ft/s

B The coin hits the ground after 3.39 s at -108.48 ft/s

C The coin hits the ground after 3.39 s at -172.48 ft/s

D The coin hits the ground after 7.78 s at -172.48 ft/s

Solution: C

Substitute $g = 32$ ft/s², $v_0 = -64$ ft/s, and $y_0 = 400$ into the vertical motion formula.

$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

$$y(t) = -\frac{1}{2}(32)t^2 + (-64)t + 400$$

$$y(t) = -16t^2 - 64t + 400$$

$$y(t) = -16(t^2 + 4t - 25)$$

To find velocity when the coin hits the ground, set the position function equal to 0, since height is 0 when the coin hits the ground.

$$-16(t^2 + 4t - 25) = 0$$

$$t^2 + 4t - 25 = 0$$

Use the quadratic formula to find the roots of the function.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-4 \pm \sqrt{4^2 - 4(1)(-25)}}{2(1)}$$

$$t = \frac{-4 \pm \sqrt{16 + 100}}{2}$$

$$t = \frac{-4 \pm 2\sqrt{29}}{2}$$

$$t = -2 \pm \sqrt{29}$$

$$t \approx -7.39,\ 3.39$$

A negative value for time is nonsensical, which means the coin hits the ground when $t \approx 3.39$.

To find velocity when the coin hits the ground at $t \approx 3.39$, we need to find the velocity function by taking the derivative of the position function.

$$y'(t) = -32t - 64$$

$$v(t) = -32t - 64$$

Substitute $t \approx 3.39$ to find velocity when the ball hits the ground.

$$v(3.39) = -32(3.39) - 64$$

$$v(3.39) = -108.48 - 64$$

$$v(3.39) = -172.48$$

The coin's velocity when it hits the ground is -172.48 ft/s.