Topic: Extrema on a closed interval

Question: Find the absolute extrema of the function on the interval $[-2,1]$.

$$f(x) = x^3 - 2x$$

Answer choices:

A	Minimum at $(1,1)$	Maximum at $(-2,10)$
B	Minimum at $(-2,-4)$	Maximum at $(-0.82,1.09)$
C	Minima at $(\pm 0.82,0)$	Maximum at $(-2,10)$
D	Minimum at $(-0.82,-2.18)$	Maximum at $(0.82,2.18)$

Solution: B

Find the first derivative,

$$f'(x) = 3x^2 - 2$$

then set it equal to 0 and solve for x.

$$3x^2 - 2 = 0$$

$$3x^2 = 2$$

$$x^2 = \frac{2}{3}$$

$$x = \pm\sqrt{\frac{2}{3}}$$

Absolute extrema could occur at these critical points and/or at the endpoints of the interval. So we'll find the value of $f(x)$ at each of these points.

At $x = -2$,

$$f(-2) = (-2)^3 - 2(-2)$$

$$f(-2) = -8 + 4$$

$$f(-2) = -4$$

At $x = -\sqrt{2/3} \approx -0.82$,

$$f\left(-\sqrt{\frac{2}{3}}\right)=\left(-\sqrt{\frac{2}{3}}\right)^3-2\left(-\sqrt{\frac{2}{3}}\right)$$

$$f\left(-\sqrt{\frac{2}{3}}\right)=-\frac{2\sqrt{2}}{3\sqrt{3}}+\frac{2\sqrt{2}}{\sqrt{3}}$$

$$f\left(-\sqrt{\frac{2}{3}}\right)=-\frac{2\sqrt{2}}{3\sqrt{3}}+\frac{6\sqrt{2}}{3\sqrt{3}}$$

$$f\left(-\sqrt{\frac{2}{3}}\right)=\frac{4\sqrt{2}}{3\sqrt{3}}$$

$$f\left(-\sqrt{\frac{2}{3}}\right)\approx 1.09$$

At $x=\sqrt{2/3}\approx 0.82$,

$$f\left(\sqrt{\frac{2}{3}}\right)=\left(\sqrt{\frac{2}{3}}\right)^3-2\left(\sqrt{\frac{2}{3}}\right)$$

$$f\left(\sqrt{\frac{2}{3}}\right)=\frac{2\sqrt{2}}{3\sqrt{3}}-\frac{2\sqrt{2}}{\sqrt{3}}$$

$$f\left(\sqrt{\frac{2}{3}}\right)=\frac{2\sqrt{2}}{3\sqrt{3}}-\frac{6\sqrt{2}}{3\sqrt{3}}$$

$$f\left(\sqrt{\frac{2}{3}}\right)=-\frac{4\sqrt{2}}{3\sqrt{3}}$$

$$f\left(\sqrt{\frac{2}{3}}\right) \approx -1.09$$

At $x = 1$,

$$f(1) = 1^3 - 2(1)$$

$$f(1) = 1 - 2$$

$$f(1) = -1$$

If we rank these points from least to greatest in terms of the function's value, we get

$$(-2, -4)$$

$$(0.82, -1.09)$$

$$(1, -1)$$

$$(-0.82, 1.09)$$

So on the interval $[-2,1]$, the function has an absolute minimum at $(-2, -4)$ and an absolute maximum at $(-0.82, 1.09)$.

Topic: Extrema on a closed interval

Question: Find the absolute extrema of the function on the interval $[0,3]$.

$$f(x) = x^2 - 4x$$

Answer choices:

A Global minimum at $(3, -3)$; Global maximum at $(2, -4)$

B Global maximum at $(2, -4)$; Global maximum at $(3, -3)$

C Global minimum at $(0,0)$; Global maximum at $(2, -4)$

D Global minimum at $(2, -4)$; Global maximum at $(0,0)$

Solution: D

Find the first derivative,

$$f'(x) = 2x - 4$$

$$f'(x) = 2(x - 2)$$

then set it equal to 0 and solve for x.

$$2(x - 2) = 0$$

$$x - 2 = 0$$

$$x = 2$$

Absolute extrema could occur at this critical point and/or at the endpoints of the interval. So we'll find the value of $f(x)$ at each of these points.

At $x = 0$,

$$f(0) = 0^2 - 4(0)$$

$$f(0) = 0$$

At $x = 2$,

$$f(2) = 2^2 - 4(2)$$

$$f(2) = 4 - 8$$

$$f(2) = -4$$

At $x = 3$,

$$f(3) = 3^2 - 4(3)$$

$$f(3) = 9 - 12$$

$$f(3) = -3$$

If we rank these points from least to greatest in terms of the function's value, we get

$$(2, -4)$$

$$(3, -3)$$

$$(0,0)$$

So on the interval $[0,3]$, the function has an absolute minimum at $(2, -4)$ and an absolute maximum at $(0,0)$.

Topic: Extrema on a closed interval

Question: Find the absolute extrema of the function on the interval $[0,2]$.

$$f(x) = x^3 - 3x$$

Answer choices:

A Global minimum at $(1, -2)$; Global maximum at $(2,2)$

B Global minimum at $(2,2)$; Global maximum at $(1, -2)$

C Global minimum at $(-1,2)$; Global maximum at $(2,2)$

D Global minimum at $(1, -2)$; Global maxima at $(-1,2)$ and $(2,2)$

Solution: A

Find the first derivative,

$$f'(x) = 3x^2 - 3$$

$$f'(x) = 3(x^2 - 1)$$

$$f'(x) = 3(x + 1)(x - 1)$$

then set it equal to 0 and solve for x.

$$3(x + 1)(x - 1) = 0$$

$$x = -1, \ 1$$

The critical point $x = -1$ is outside the interval $[0,2]$, so we'll ignore it. Then we can say that absolute extrema could occur at just $x = 1$ and/or at the endpoints of the interval. So we'll find the value of $f(x)$ at each of these points.

At $x = 0$,

$$f(0) = 0^3 - 3(0)$$

$$f(0) = 0 - 0$$

$$f(0) = 0$$

At $x = 1$,

$$f(1) = 1^3 - 3(1)$$

$$f(1) = 1 - 3$$

$$f(1) = -2$$

At $x = 2$,

$$f(2) = 2^3 - 3(2)$$

$$f(2) = 8 - 6$$

$$f(2) = 2$$

If we rank these points from least to greatest in terms of the function's value, we get

$$(1, -2)$$

$$(0,0)$$

$$(2,2)$$

So on the interval $[0,2]$, the function has an absolute minimum at $(1, -2)$ and an absolute maximum at $(2,2)$.