



Calculus 1 Workbook

Optimization and sketching graphs

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MATH

CRITICAL POINTS AND THE FIRST DERIVATIVE TEST

- 1. Identify the critical point(s) of the function on the interval $[-3, 2]$.

$$f(x) = x^{\frac{2}{3}}(x + 2)$$

- 2. Identify the critical point(s) of the function on the interval $[-2, 2]$.

$$g(x) = x\sqrt{4 - x^2}$$

- 3. Determine the intervals where the function is increasing and decreasing.

$$f(x) = \frac{5}{4}x^4 - 10x^2$$

- 4. Determine the intervals where the function is increasing and decreasing.

$$g(x) = -x^3 + 2x^2 + 2$$

- 5. Use the first derivative test to find the extrema of

$$f(x) = 4x^3 + 21x^2 + 36x - 5.$$



■ 6. Use the first derivative test to find the extrema of

$$g(x) = 2x^3 - 14x^2 + 22x + 3.$$



INFLECTION POINTS AND THE SECOND DERIVATIVE TEST

- 1. For $f(x) = x^3 - 3x^2 + 5$, find inflection points and identify where the function is concave up and concave down.

- 2. For $g(x) = -x^3 + 2x^2 + 3$, find inflection points and identify where the function is concave up and concave down.

- 3. For $h(x) = x^4 + x^3 - 3x^2 + 2$, find inflection points and identify where the function is concave up and concave down.

- 4. Use the second derivative test to identify the extrema of $f(x) = x^3 - 12x - 2$ as maximum values or minimum values.

- 5. Use the second derivative test to identify the extrema of $g(x) = -4x^3 + 12x^2 + 5$ as maximum values or minimum values.

- 6. Use the second derivative test to identify the extrema of $h(x) = 2x^4 - 4x^2 + 1$ as maximum values or minimum values.



INTERCEPTS AND VERTICAL ASYMPTOTES

- 1. Find any vertical asymptote(s) of the function.

$$f(x) = \frac{-x^2 + 16x - 63}{x^2 - 2x - 35}$$

- 2. Find any vertical asymptote(s) of the function.

$$g(x) = \frac{x^2 - 3x - 10}{x^2 + x - 2}$$

- 3. Find any vertical asymptote(s) of the function.

$$h(x) = \frac{40 - 27x - 12x^2 - x^3}{9x^2 + 63x - 72}$$



HORIZONTAL AND SLANT ASYMPTOTES

- 1. Find the horizontal asymptote(s) of the function.

$$f(x) = \frac{8x^4 - x^2 + 1}{4x^4 - 1}$$

- 2. Find the horizontal asymptote(s) of the function.

$$g(x) = \frac{2x^2 - 5x + 12}{3x^2 - 11x - 4}$$

- 3. Find the horizontal asymptote(s) of the function.

$$h(x) = \frac{x^3 - x^2 + 6x - 1}{7x^4 - 1}$$

- 4. Find the slant asymptote of the function.

$$f(x) = \frac{3x^4 - x^3 + x^2 - 4}{x^3 - x^2 + 1}$$

- 5. Find the slant asymptote of the function.



$$g(x) = \frac{8x^2 + 14x - 7}{4x - 1}$$

■ 6. Find the slant asymptote of the function.

$$h(x) = \frac{x^3 - 8}{x^2 - 5x + 6}$$



SKETCHING GRAPHS

- 1. Sketch the graph of the function.

$$f(x) = x^3 - 4x^2 + 8$$

- 2. Sketch the graph of the function.

$$g(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2 + 1$$

- 3. Sketch the graph of the function.

$$h(x) = \frac{x^2 + x - 6}{4x^2 + 16x + 12}$$



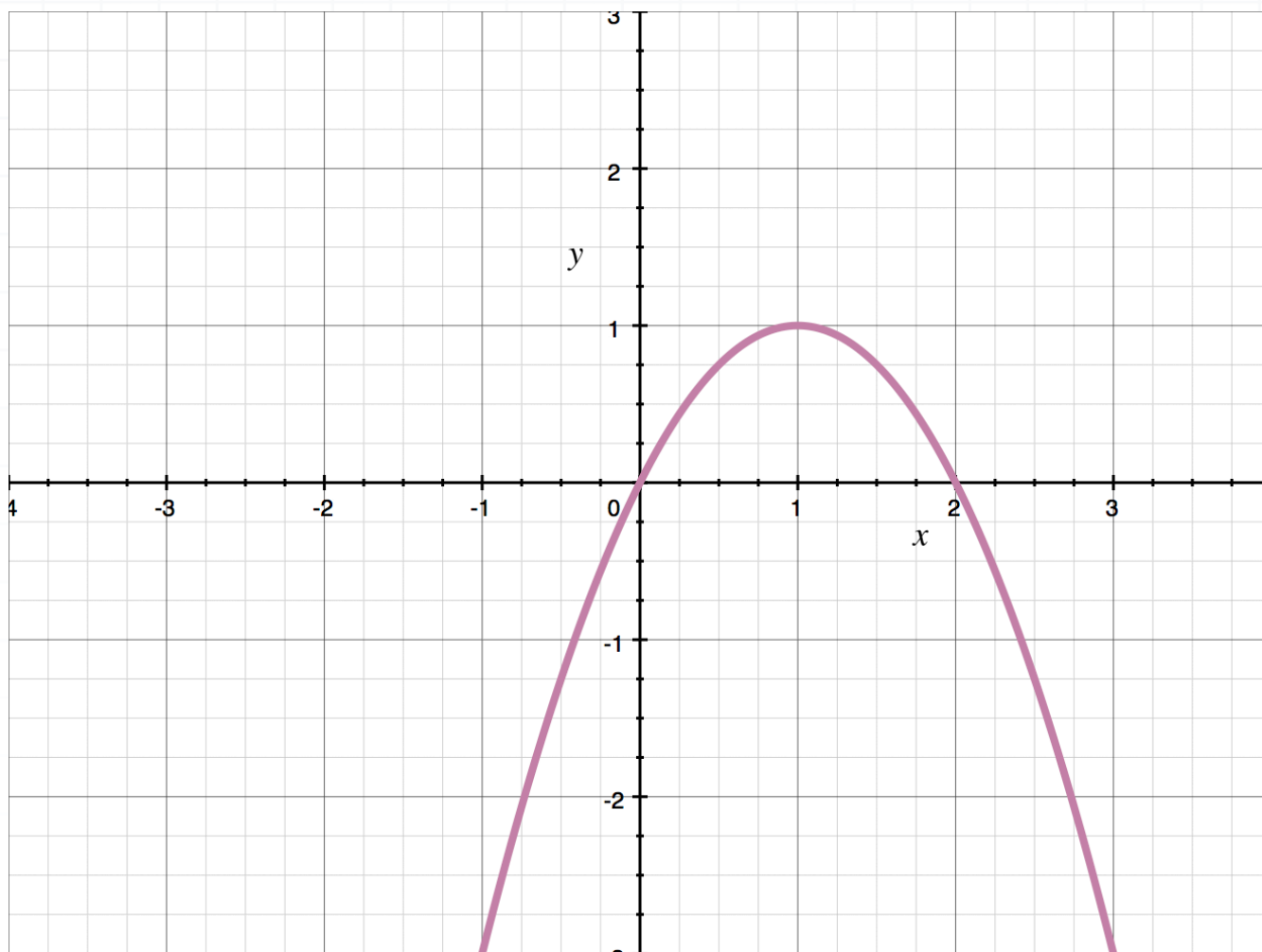
EXTREMA ON A CLOSED INTERVAL

- 1. Find the extrema of $f(x) = x^3 - 3x^2 + 5$ over the closed interval $[-3,4]$.
- 2. Find the extrema of $g(x) = \sqrt[3]{2x^2 + 3}$ over the closed interval $[-1,5]$.
- 3. Find the extrema of $h(x) = -4x^3 + 6x^2 - 3x - 2$ over the closed interval $[-4,6]$.



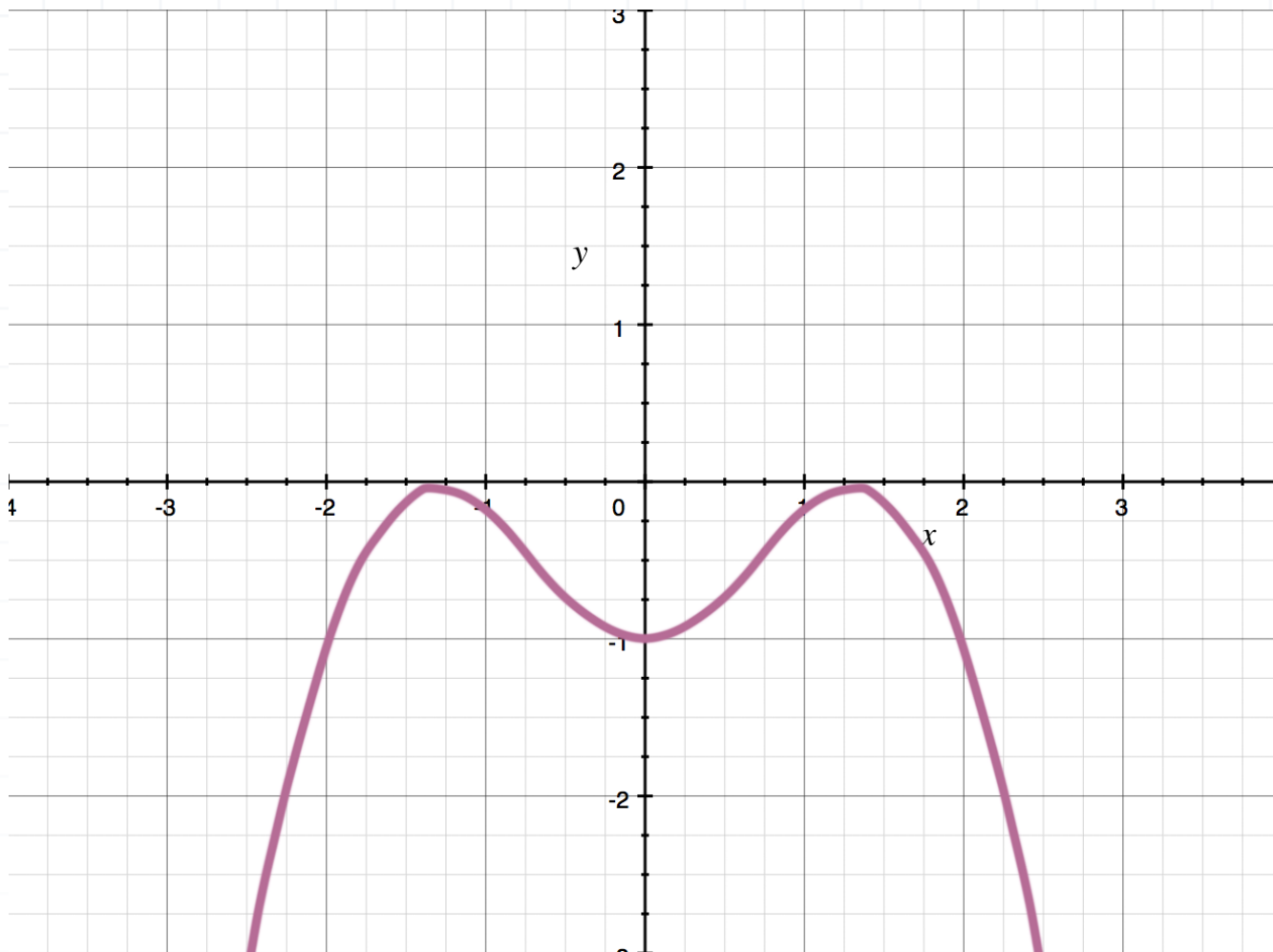
SKETCHING $F(X)$ FROM $F'(X)$

- 1. Sketch a possible graph of $f(x)$ given the graph below of $f'(x)$.



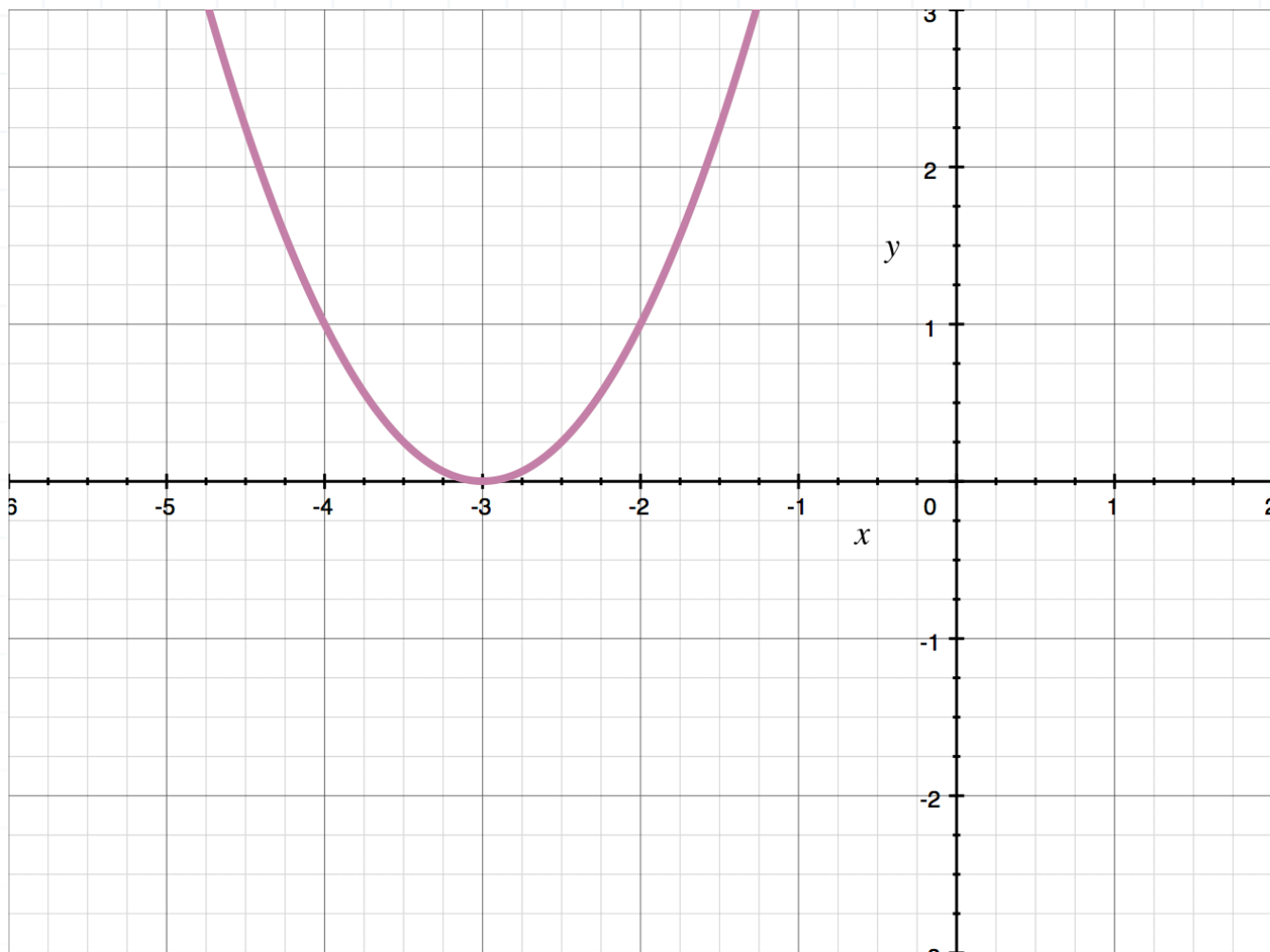
- 2. Sketch a possible graph of $g'(x)$ given the graph below of $g(x)$.



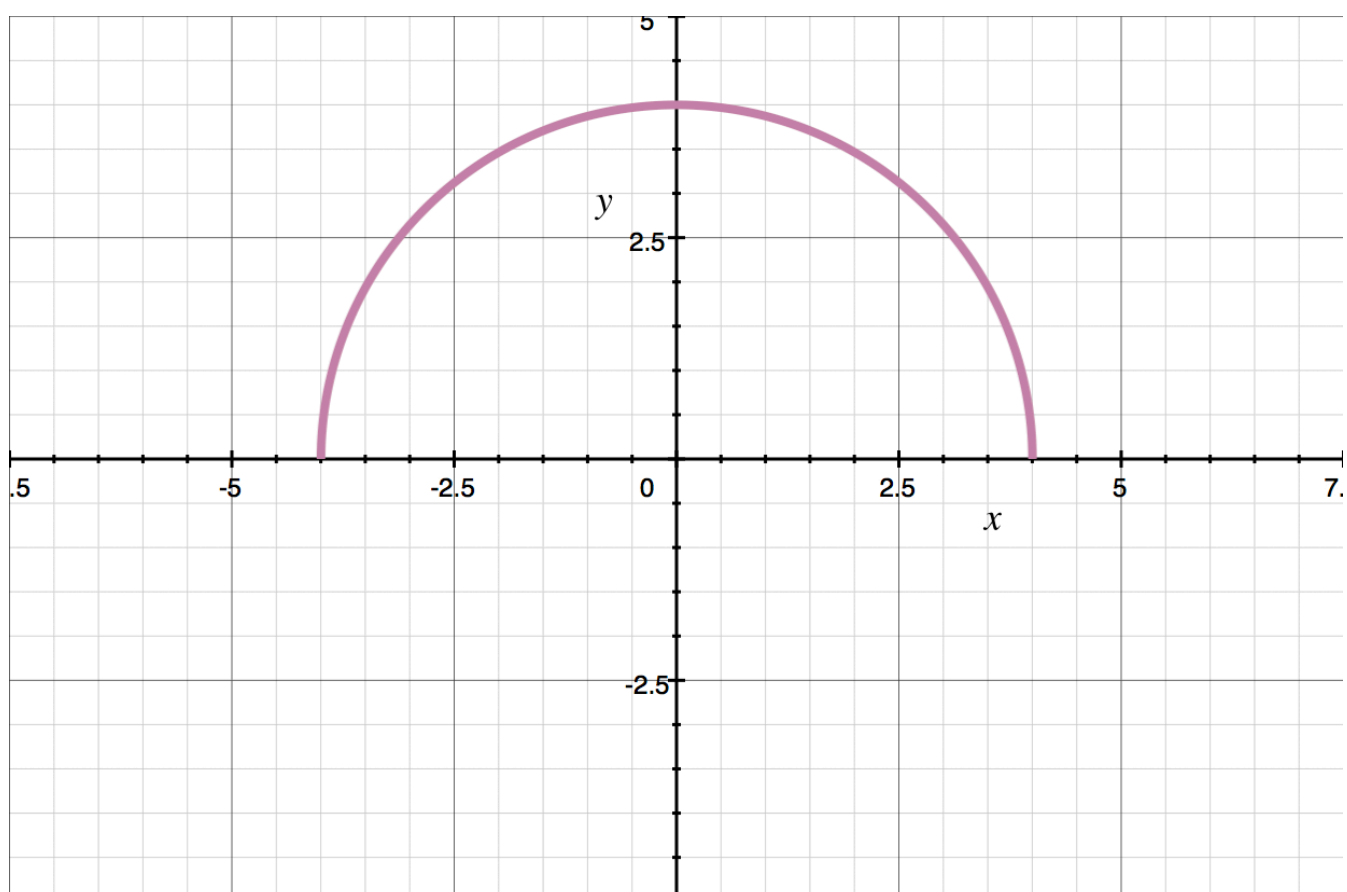


■ 3. Sketch a possible graph of $h(x)$ given the graph below of $h'(x)$.

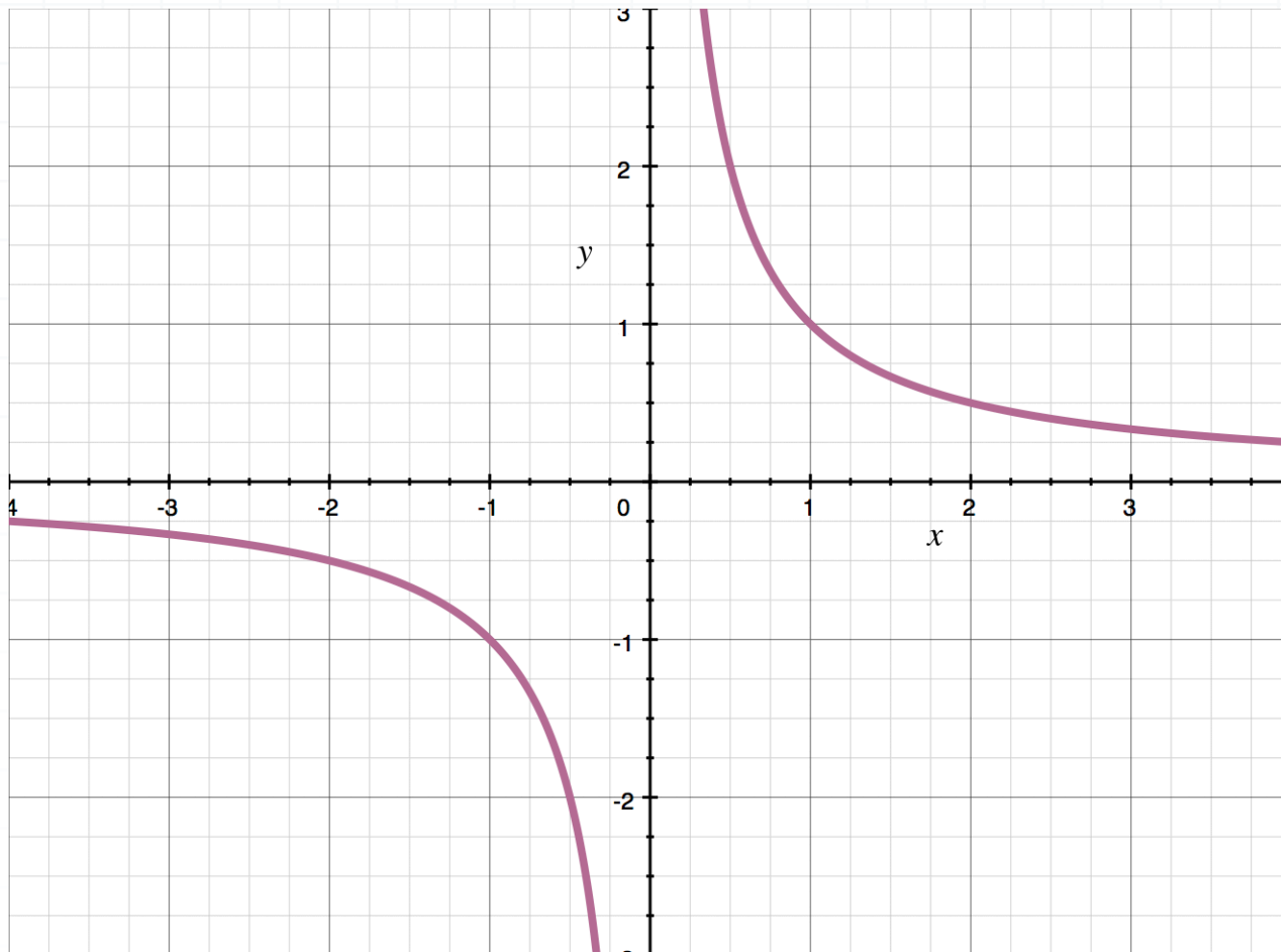




■ 4. Sketch a possible graph of $f'(x)$ given the graph below of $f(x)$.



- 5. Sketch a possible graph of $f(x)$ given the graph below of $f'(x)$.



- 6. Sketch a possible graph of $g'(x)$ given the graph below of $g(x)$.



