Calculus 1 Workbook

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MATH

VERTICAL LINE TEST

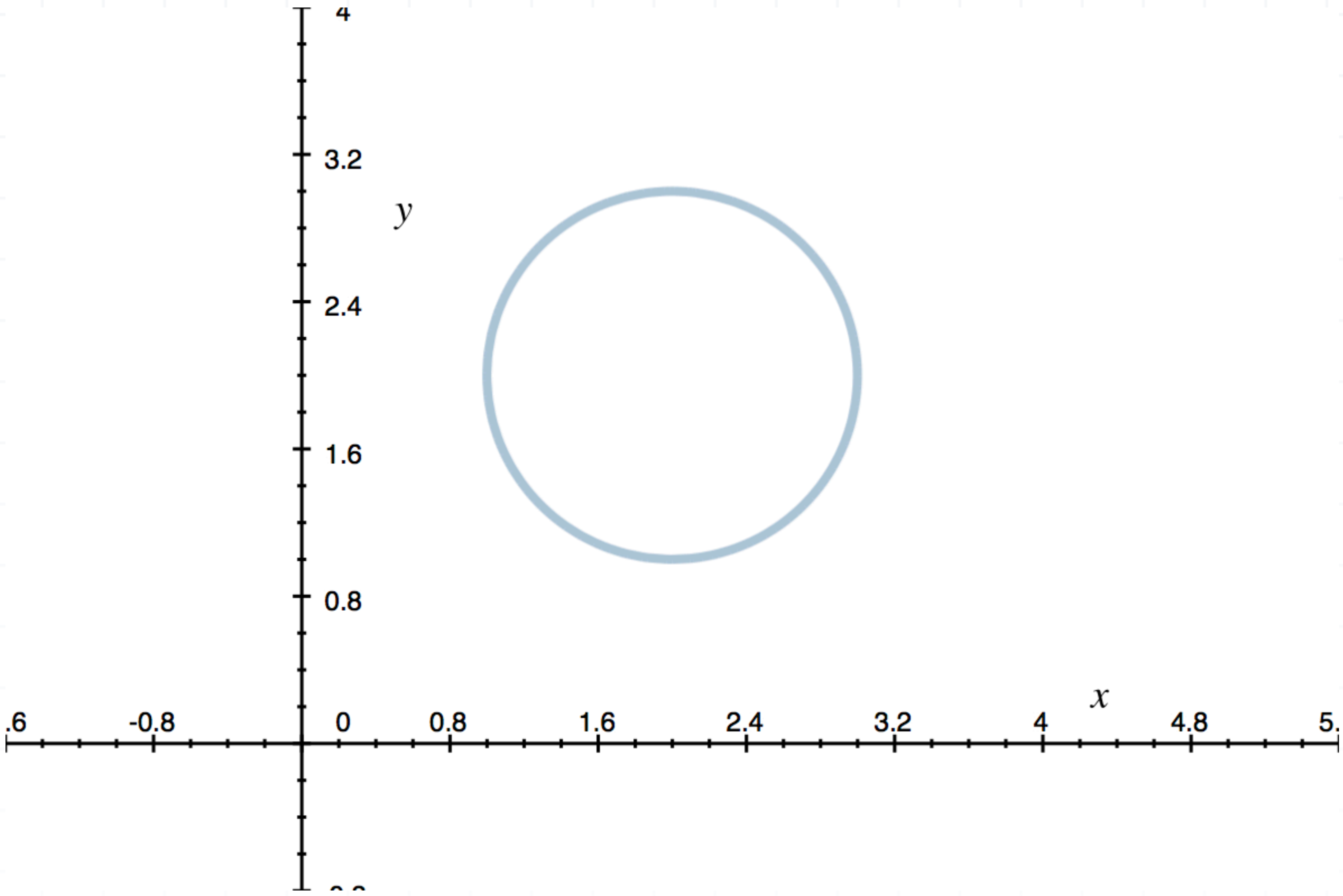
1\. Determine algebraically whether or not the equation represents a function.

$$(x-1)^2+y=3$$

2\. Fill in the blanks in the following statement using “equations,” and “functions.”

All ___________ are ___________.

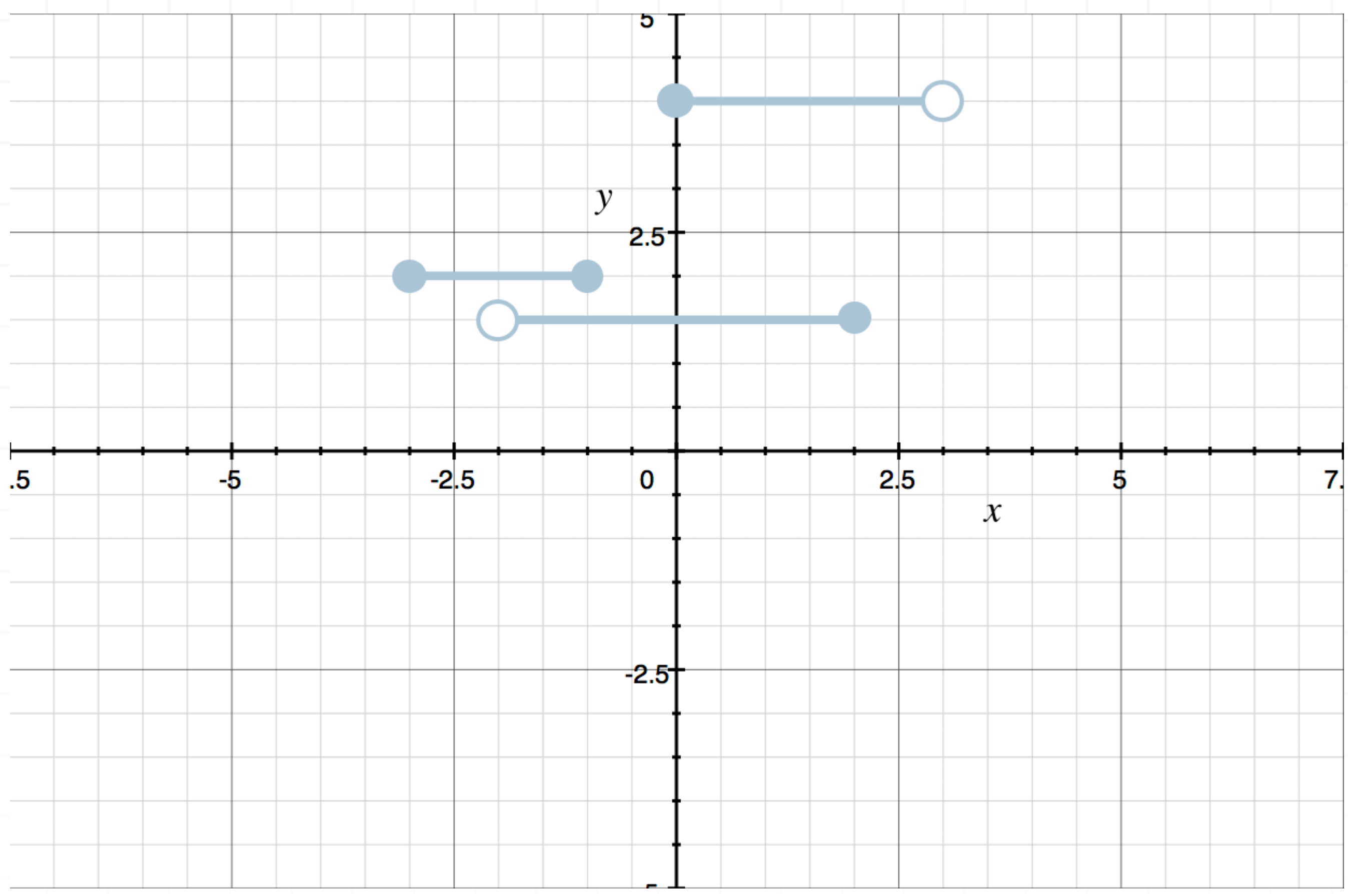
3\. Use the Vertical Line Test to determine whether or not the graph is the graph of a function.



4. Determine algebraically whether or not the equation represents a function.

$$y^2 = x + 1$$

5. Use the Vertical Line Test to determine whether or not the graph represents a function.



6. Explain why the Vertical Line Test determines whether or not a graph represents a function.

7. Fill in the blanks in the following statement using: equations, functions.

Not all ____________ are ____________.

8. Determine algebraically whether or not the equation represents a function.

$$x^3 + y = 5$$

DOMAIN AND RANGE

1. Find the domain of $f(x)$.

$$f(x) = \frac{3}{x(x+1)} + x^2$$

2. Find the domain and range of the given set.

$$(-1,-3), \quad (0,5), \quad (-3,6), \quad (0,-3)$$

3. Find the domain and range of $g(x)$.

$$g(x) = \frac{\sqrt{x-2}}{3}$$

4. Find the domain and range of the function.

$$f(x) = \frac{2}{x} + 1$$

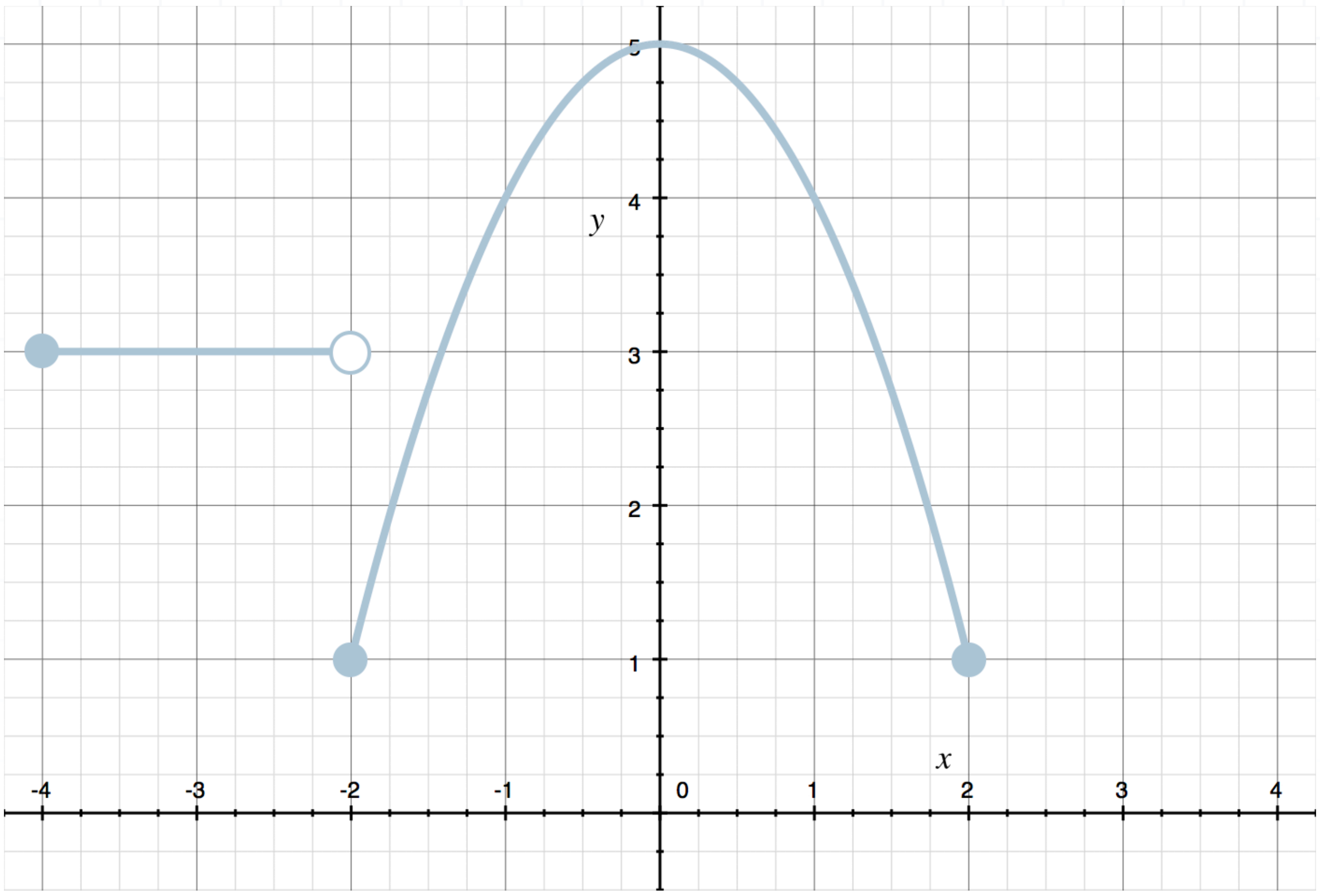
5. Give an example of a function that has a domain of $[1,\infty)$.

6. Find the domain and range of $f(x)$.

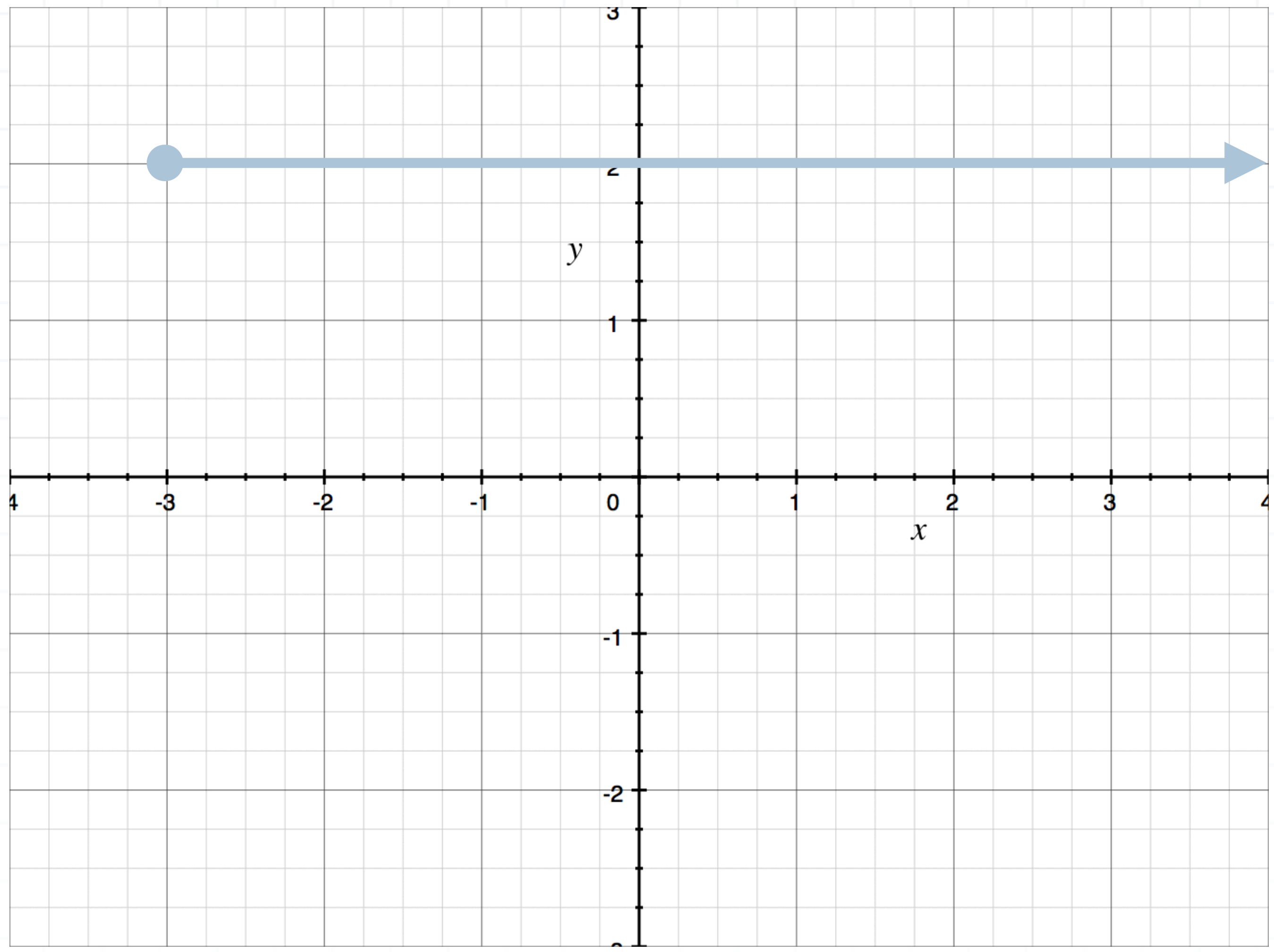
$$f(x) = \ln(x + 3) + 5$$

DOMAIN AND RANGE FROM A GRAPH

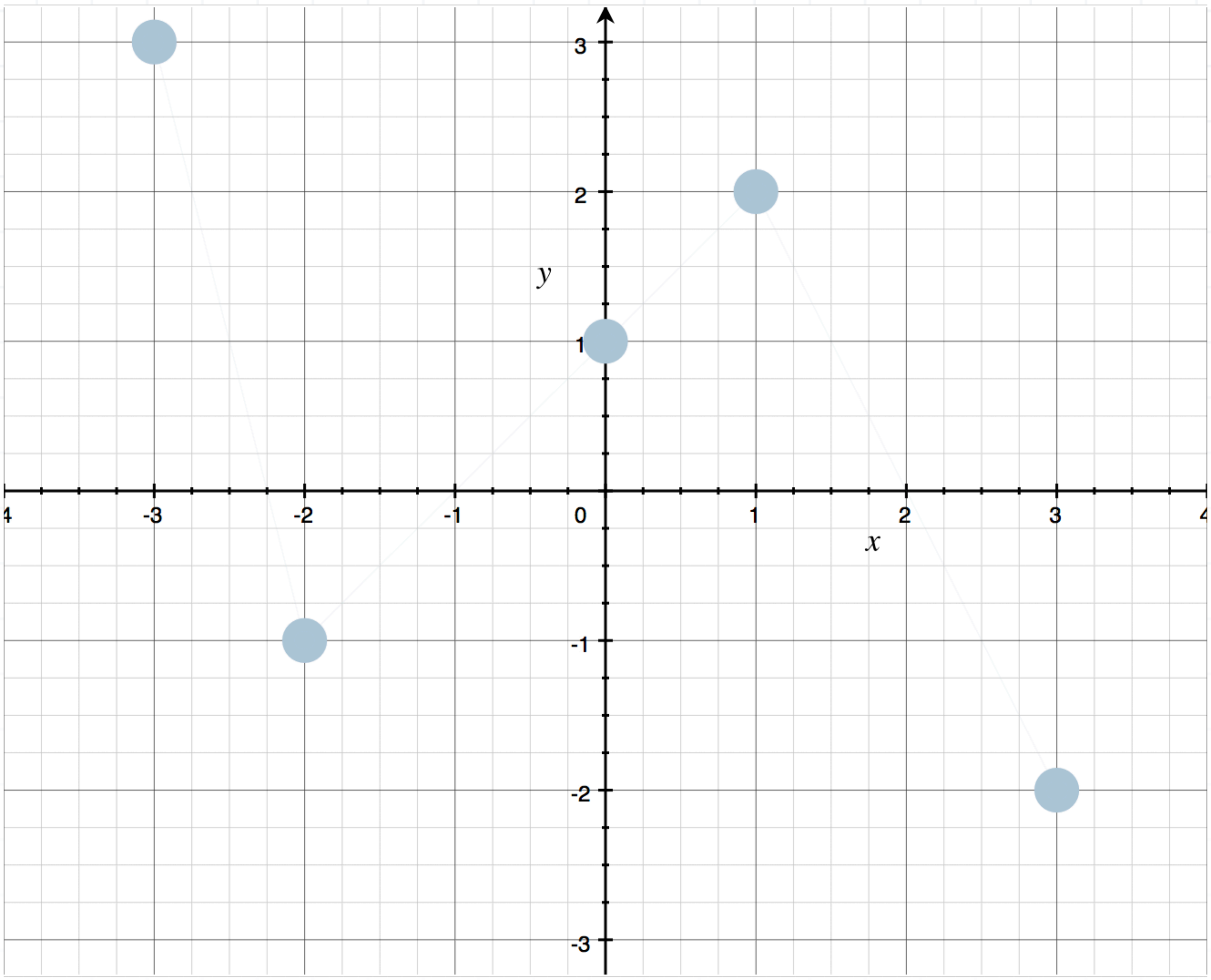
■ 1. What is the domain and range of the function? Assume the graph does not extend beyond the graph shown.



■ 2. What is the domain and range of the function?



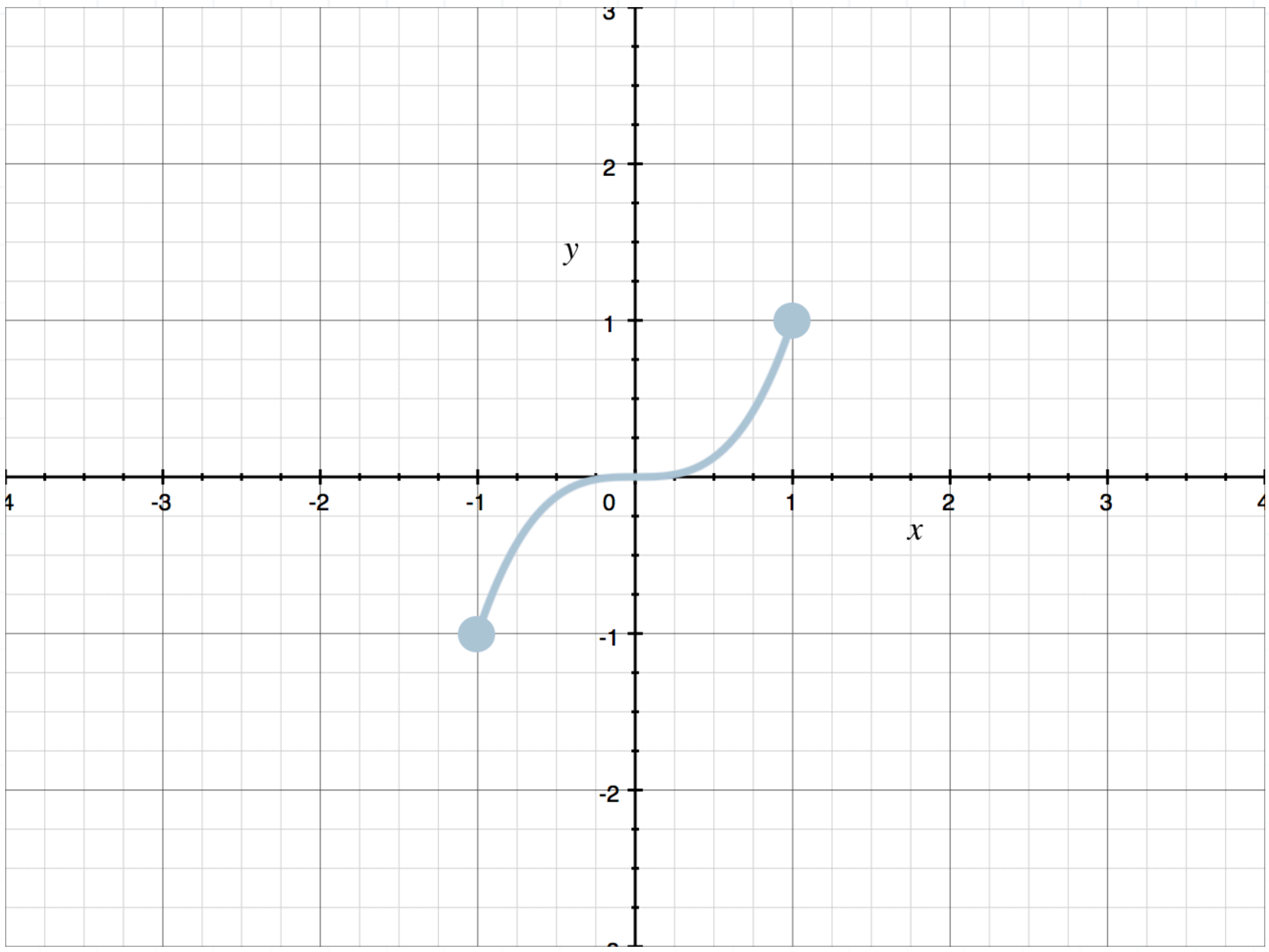
3. Determine the domain and range of the function.



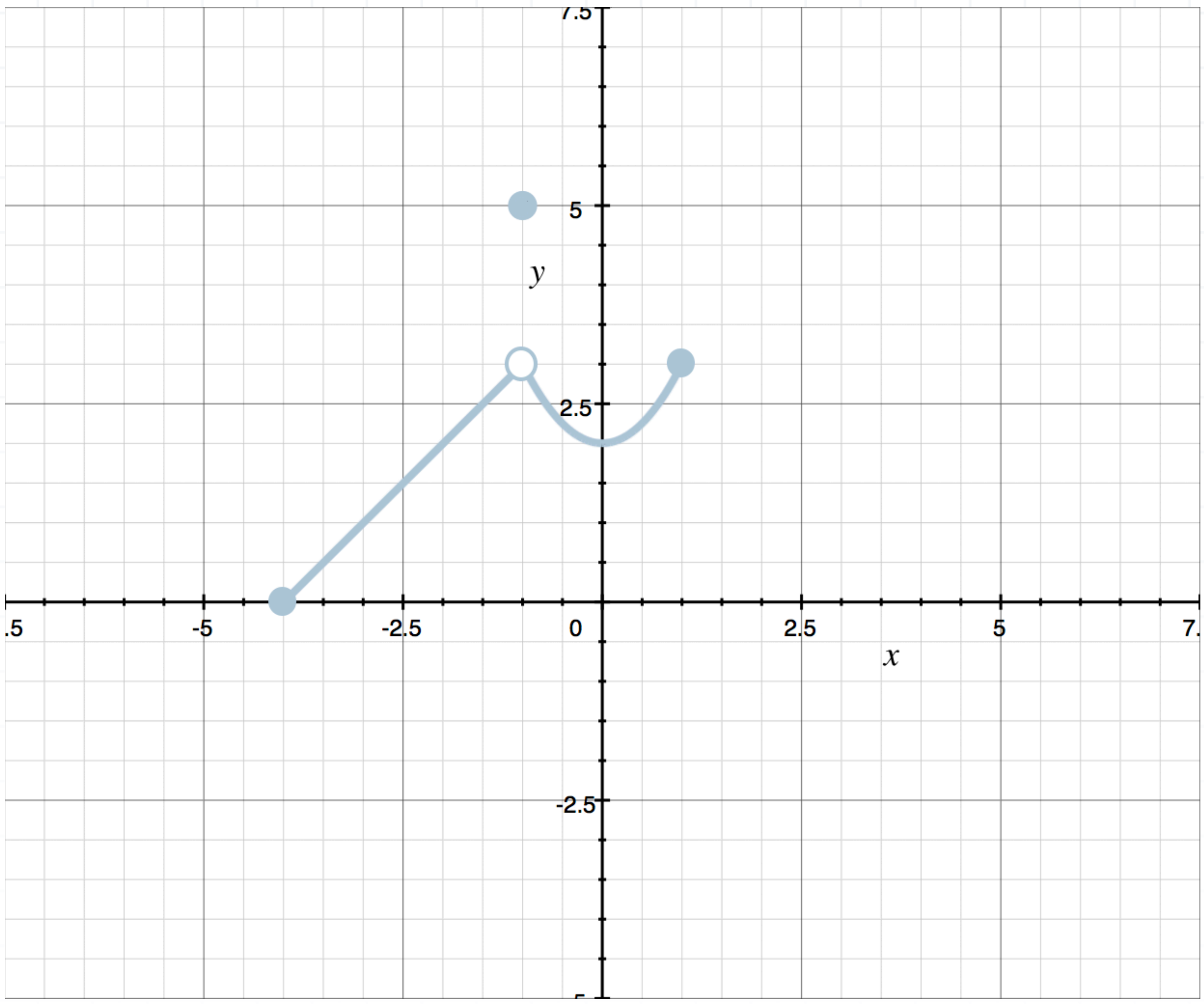
■ 4. Fill in the blanks in the following description of the domain of a graph.

"The domain is all the values of the graph from ___________ to ___________."

■ 5. What is the domain and range of the function? Assume the graph does not extend beyond the graph shown.



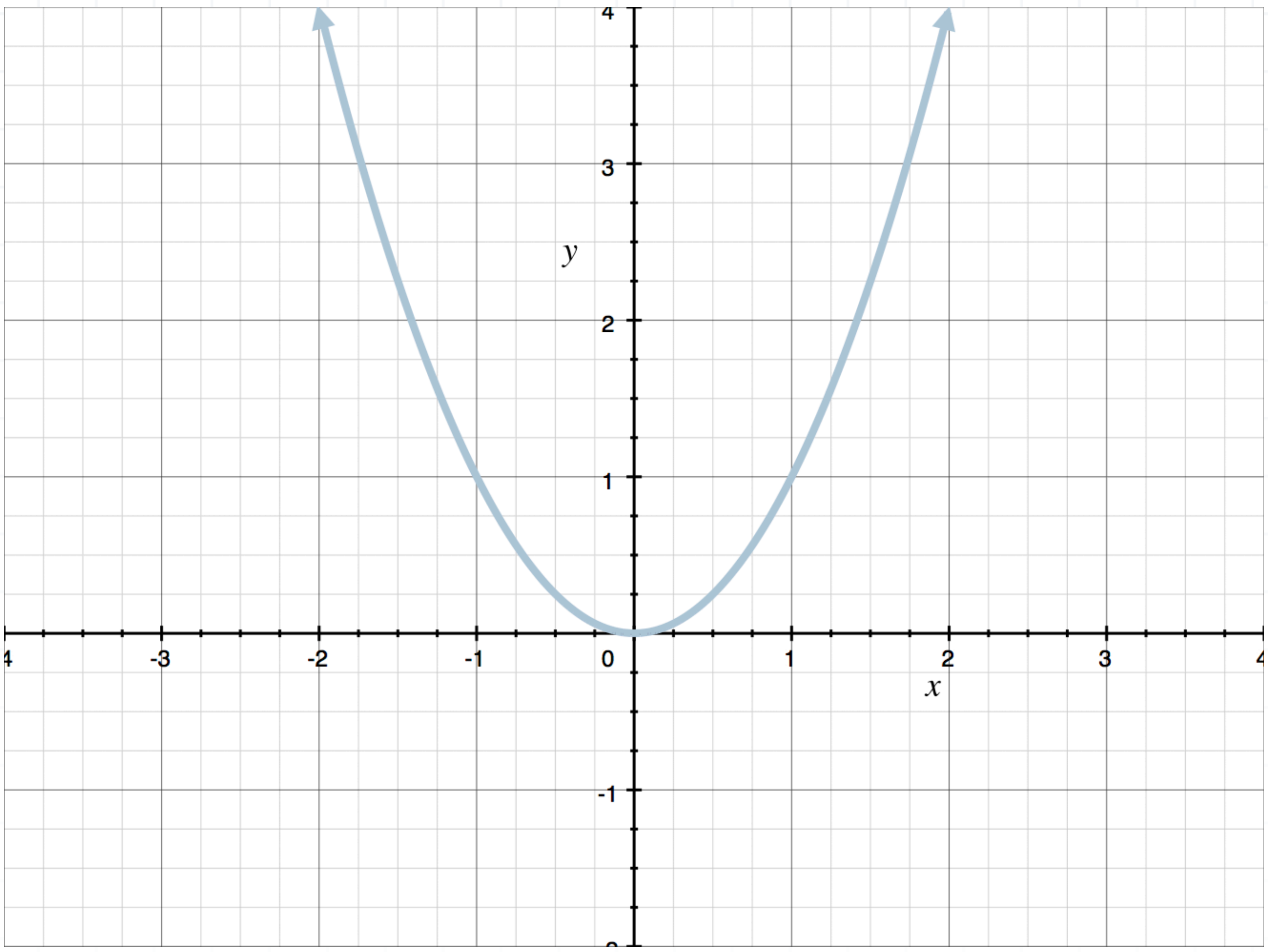
6. What is the domain and range of the function? Assume the graph does not extend beyond the graph shown.



■ 7. Fill in the blanks in the following description of the range of a graph.

"The range is all the values of the graph from __________ to __________."

■ 8. What is the domain and range of the function?



4
3
y
2
1
4
-3
-2
-1
0
1
2
3
4
x
-1

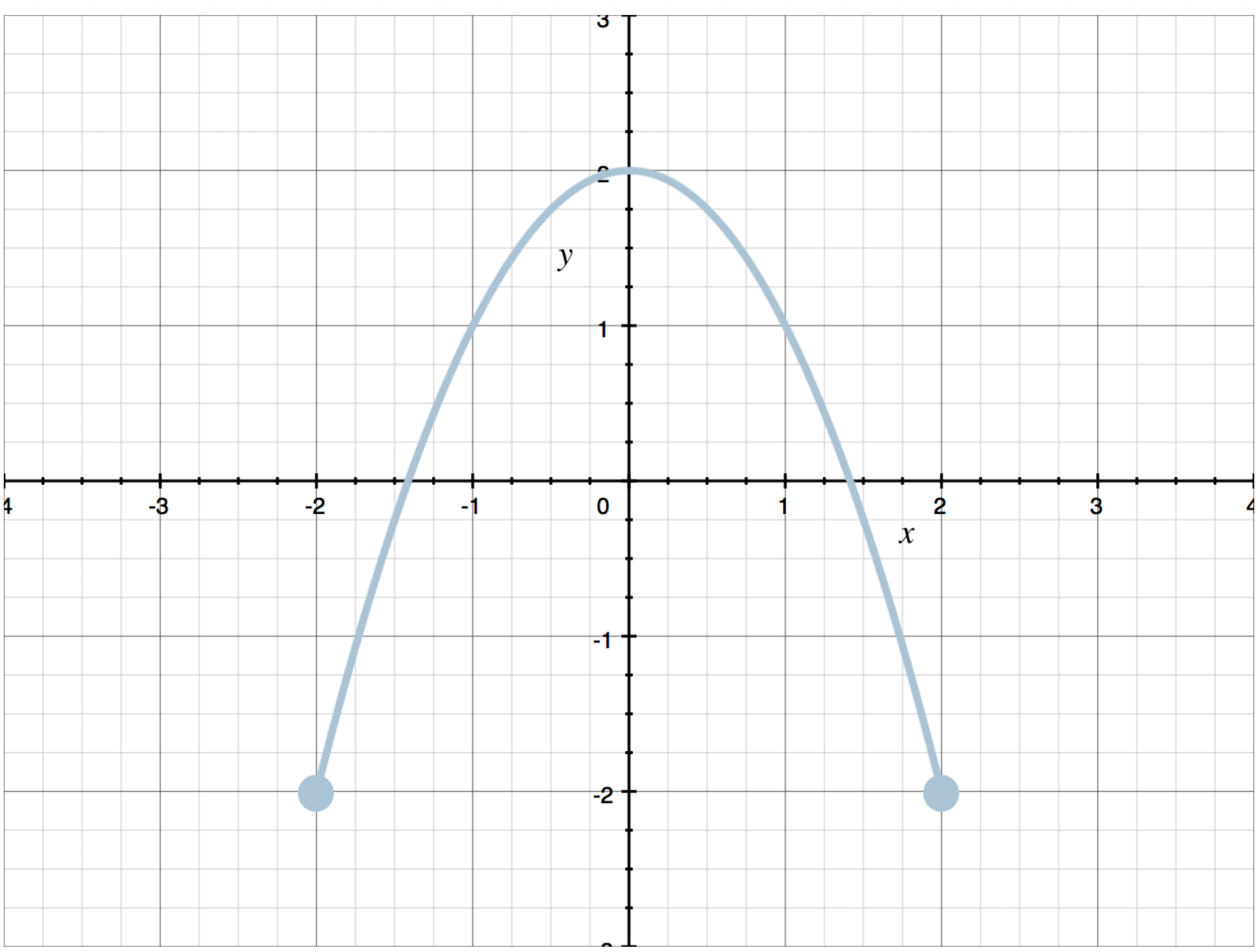
EVEN, ODD, OR NEITHER

1. Is the function even, odd, or neither?

$$f(x) = -x^5 + 2x^2 - 1$$

2. Describe the symmetry of an even function, and give an example of an even function.

3. Determine if the graph is the graph of a function that is even, odd, or neither.



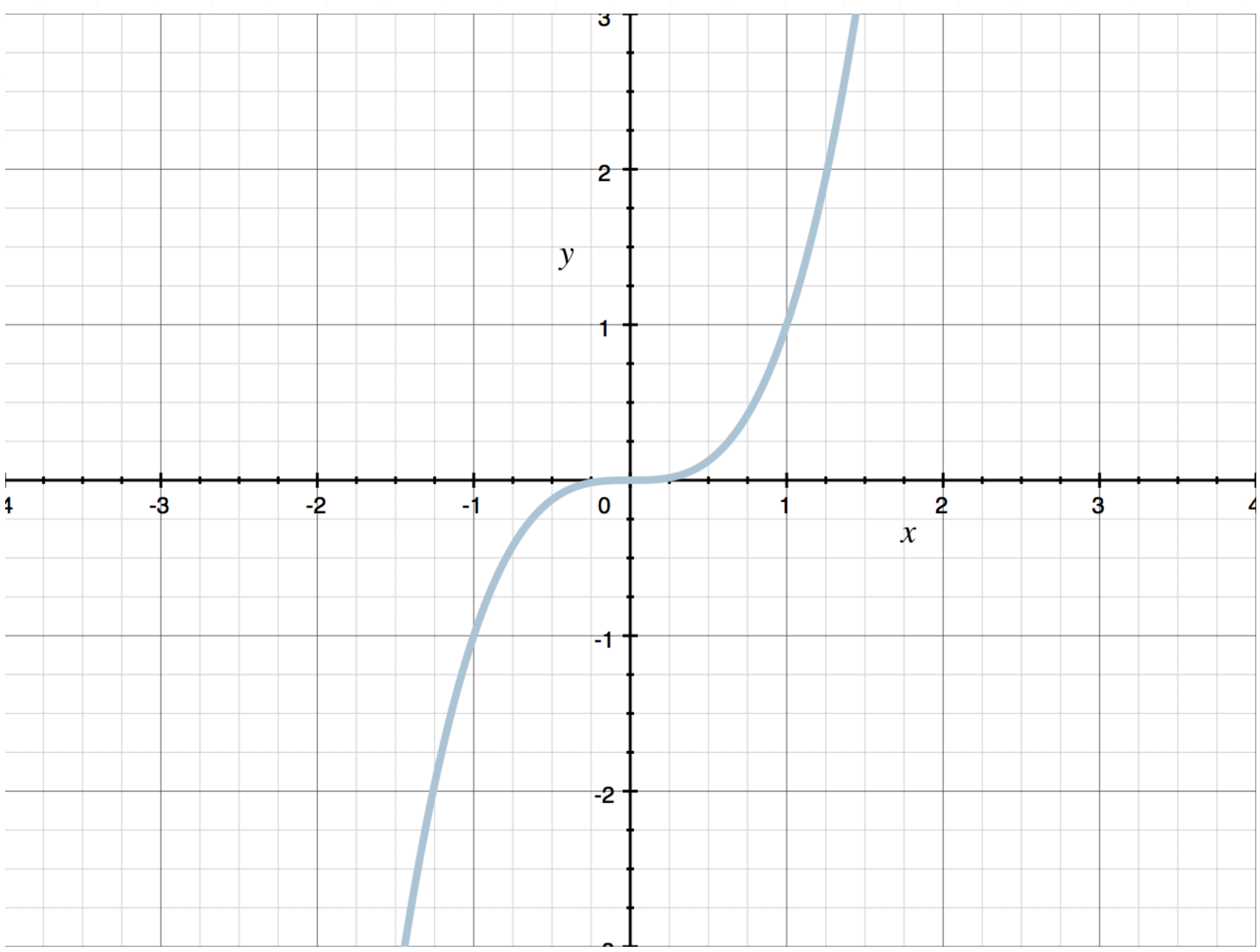
4. Is the function even, odd, or neither?

$$g(x) = -3x^2 + 5x^6$$

5. Show that the function is neither even nor odd.

$$f(x) = x^2 - 5x + 7$$

6. Determine if the graph is the graph of a function that is even, odd, or neither.



7. Is the function even, odd, or neither?

$$h(x) = x^3 - 3x$$

8. Describe the symmetry of an odd function, and give an example of an odd function.

EQUATION MODELING

1. A car and a truck were driven for a week. The car traveled 75 miles more than the truck. Each vehicle had different fuel mileage. Write an equation using t (where t is the number of miles the truck traveled) to calculate the number of gallons g, used during the week.

	Car	Truck
Mileage	28 mpg	14 mpg
Distance	c miles	t miles

2. A motorcycle and a car were driven for a month. The motorcycle traveled 120 miles more than the car. Each vehicle had different fuel mileage. Write an equation using m (where m is the number of miles the motorcycle traveled) to calculate the number of gallons g, used during the month.

	Motorcycle	Car
Mileage	33 mpg	22 mpg
Distance	m miles	c miles

3. A baseball is thrown at a speed of 21 ft/s straight down from a high platform. The distance it travels can be calculated using $D = 16t^2 + 21t$, where t is the amount of time in seconds that it's been falling. The average

speed of any object can be calculated using $V = D/t$. Write an equation giving the time of the fall in terms of V.

4. A rock is thrown at a speed of 8 ft/s straight down from a high platform. The distance it travels can be calculated using $D = 16t^2 + 8t$, where t is the amount of time in seconds that it's been falling. The average speed of any object can be calculated using $V = D/t$. Write an equation giving the time of the fall in terms of V.

5. Managers at a company are each paid $\$45{,}000$ in base salary. The company's owner wants to divide $\$162{,}000$ in annual bonus money evenly among the managers. Write an expression, in terms of the number of mangers m, that gives the amount a each manager earns per month.

6. Managers at a company are each paid $\$37{,}800$ in base salary. The company's owner wants to divide $\$102{,}000$ in annual bonus money evenly among the managers. Write an expression, in terms of the number of mangers m, that gives the amount a each manager earns per month.

7. The Jones and Anderson family go on vacation together with each family driving in their own car. The Anderson family travels 50 miles further than the Jones family. Each family averages 65 mph on the trip. Write an

equation using D_a (where D_a is the total miles the Anderson family drove) to calculate the total time T both families spent driving to their destination.

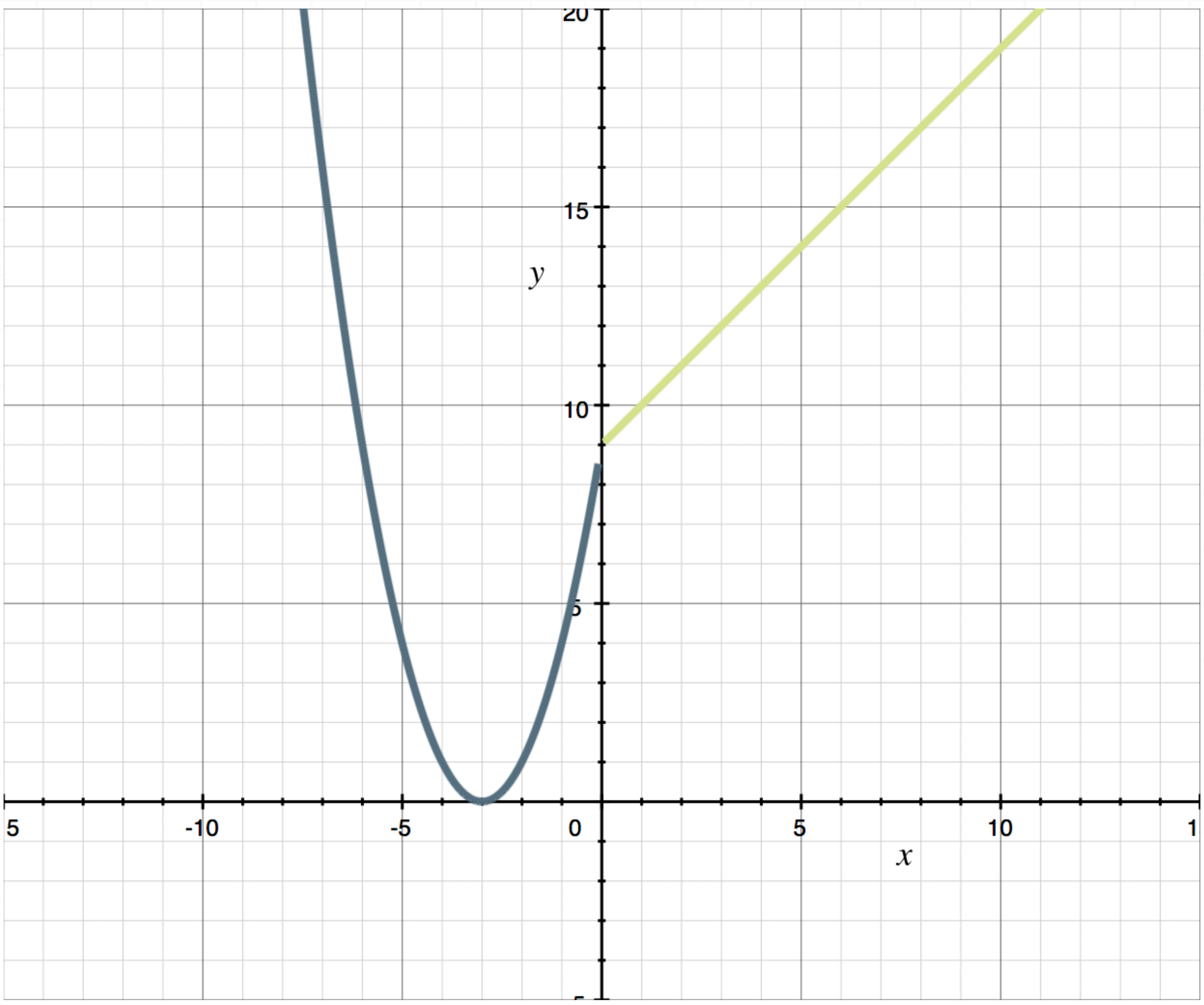
	Jones	Anderson
Distance	D_j miles	D_a miles
Rate	65 mph	65 mph
Time	T_j hours	T_a hours

■ 8. The Frank and Harrington family go on vacation together with each family driving in their own car. The Frank family travels 120 miles less than the Harrington family. Each family averages 50 mph on the trip. Write an equation using D_f (where D_f is the total miles the Frank family drove) to calculate the total time T both families spent driving to their destination.

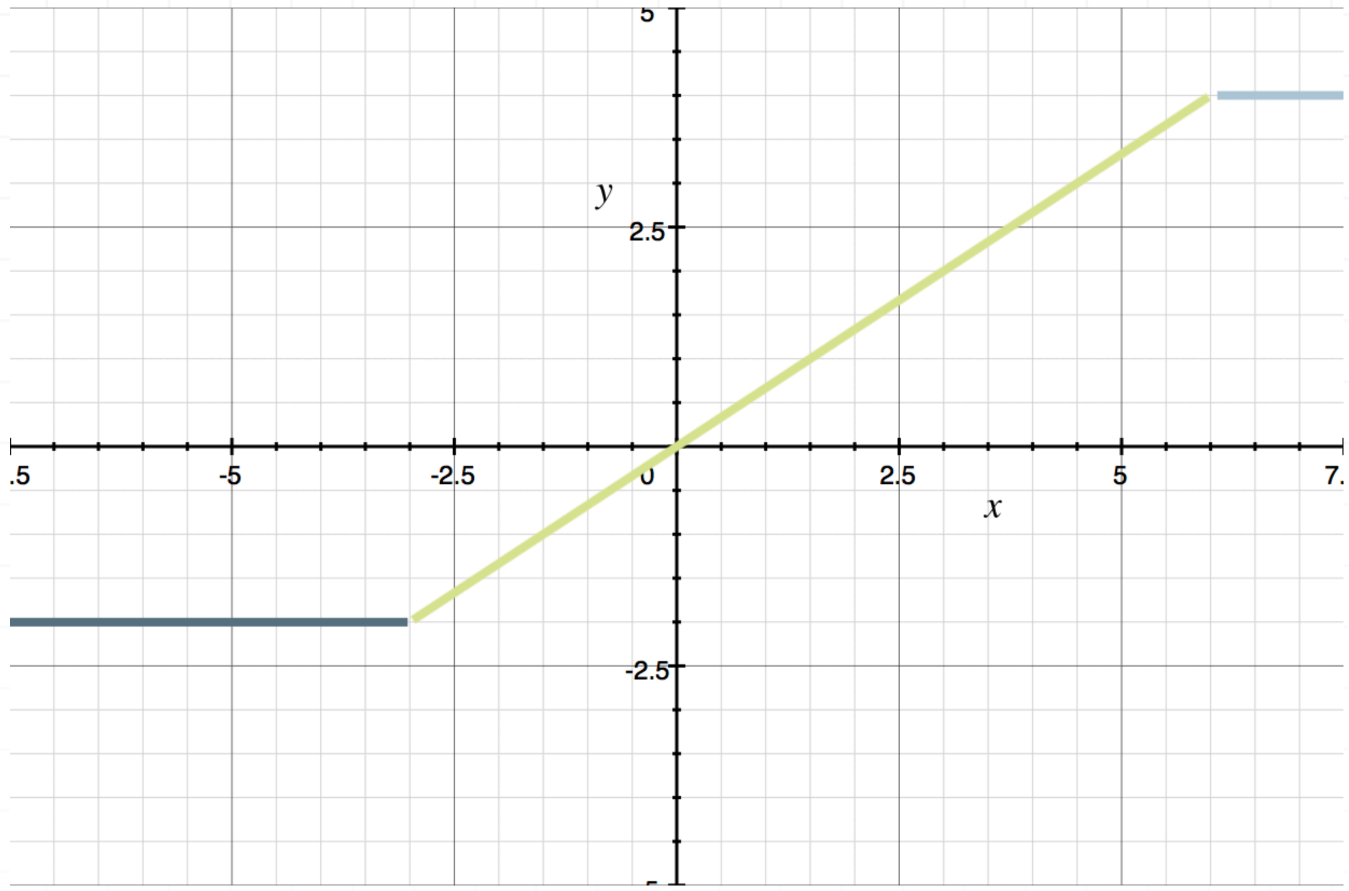
	Frank	Harrington
Distance	D_f miles	D_h miles
Rate	50 mph	50 mph
Time	T_f hours	T_h hours

MODELING A PIECEWISE-DEFINED FUNCTION

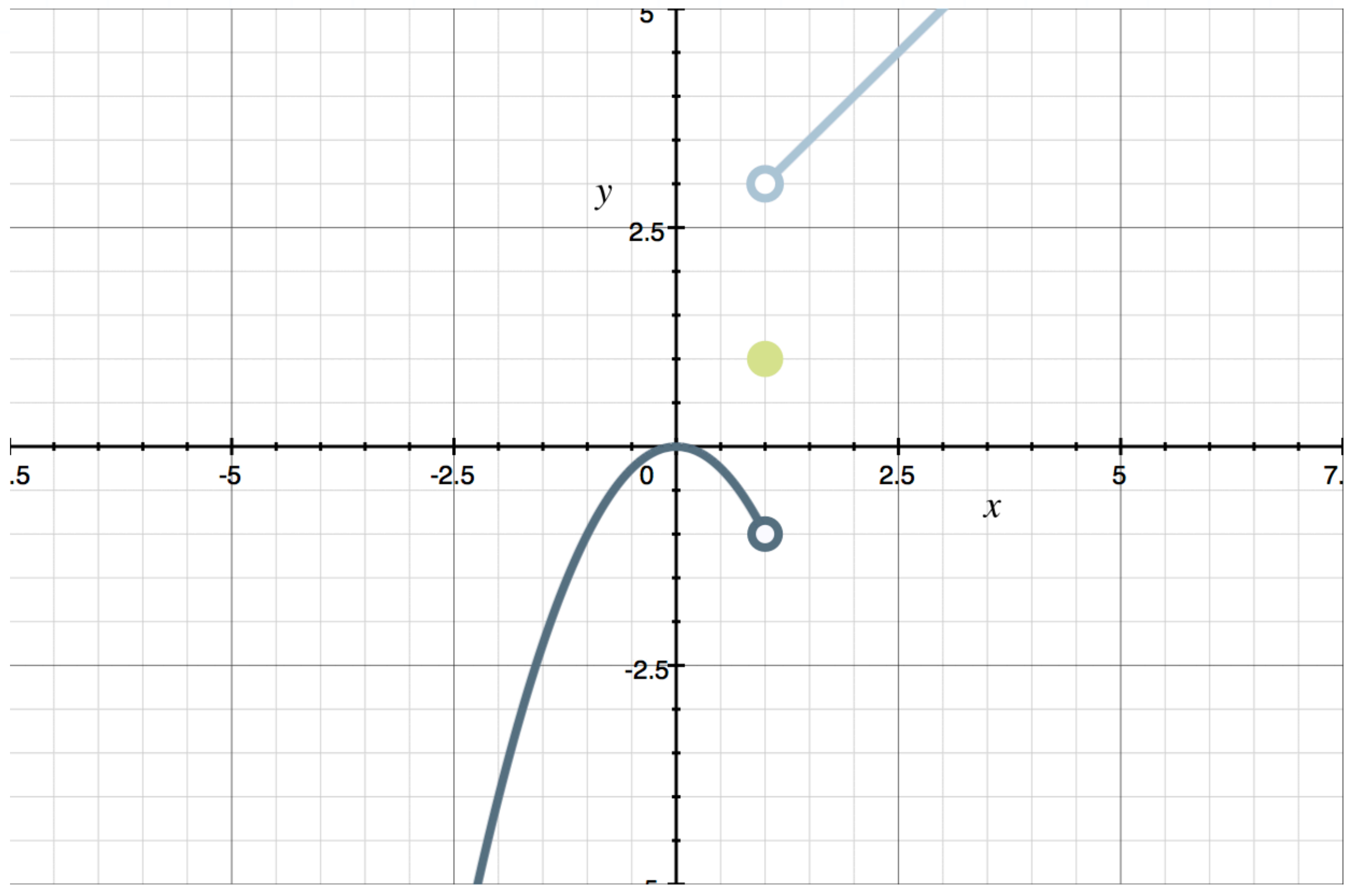
■ 1. Find the equation of the piecewise function.



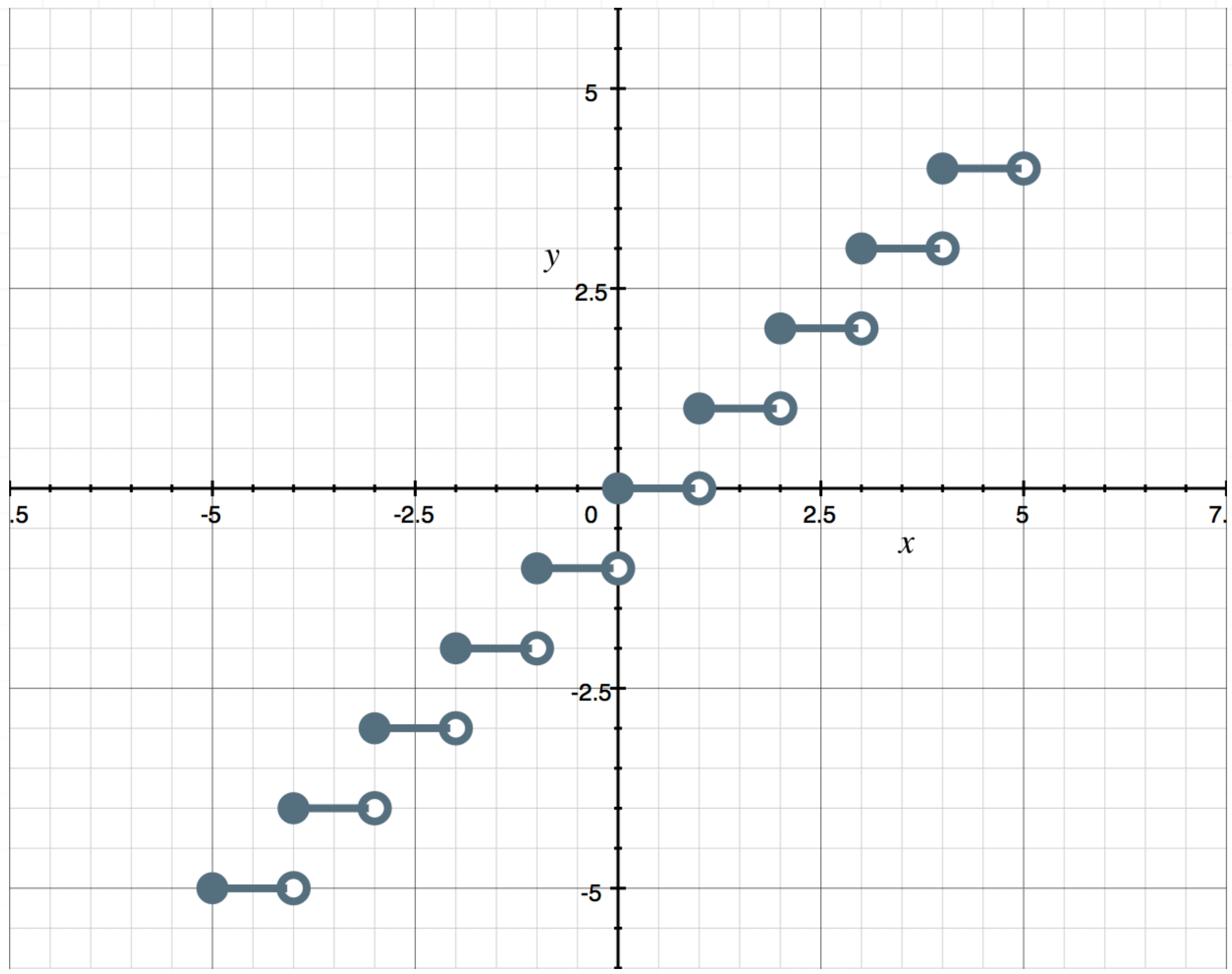
■ 2. Find the equation of the piecewise function.



3. Find the equation of the piecewise function.



4. Find the equation of the piecewise function.



5. Graph the piecewise function.

$$f(x) = \begin{cases} x^2 & x < -1 \\ \frac{1}{2}x - 3 & x \geq -1 \end{cases}$$

6. Graph the piecewise function.

$$f(x) = \begin{cases} x^2 - 2 & x < 0 \\ 4 & x = 0 \\ -x^2 + 8 & x > 0 \end{cases}$$

7. Graph the piecewise function.

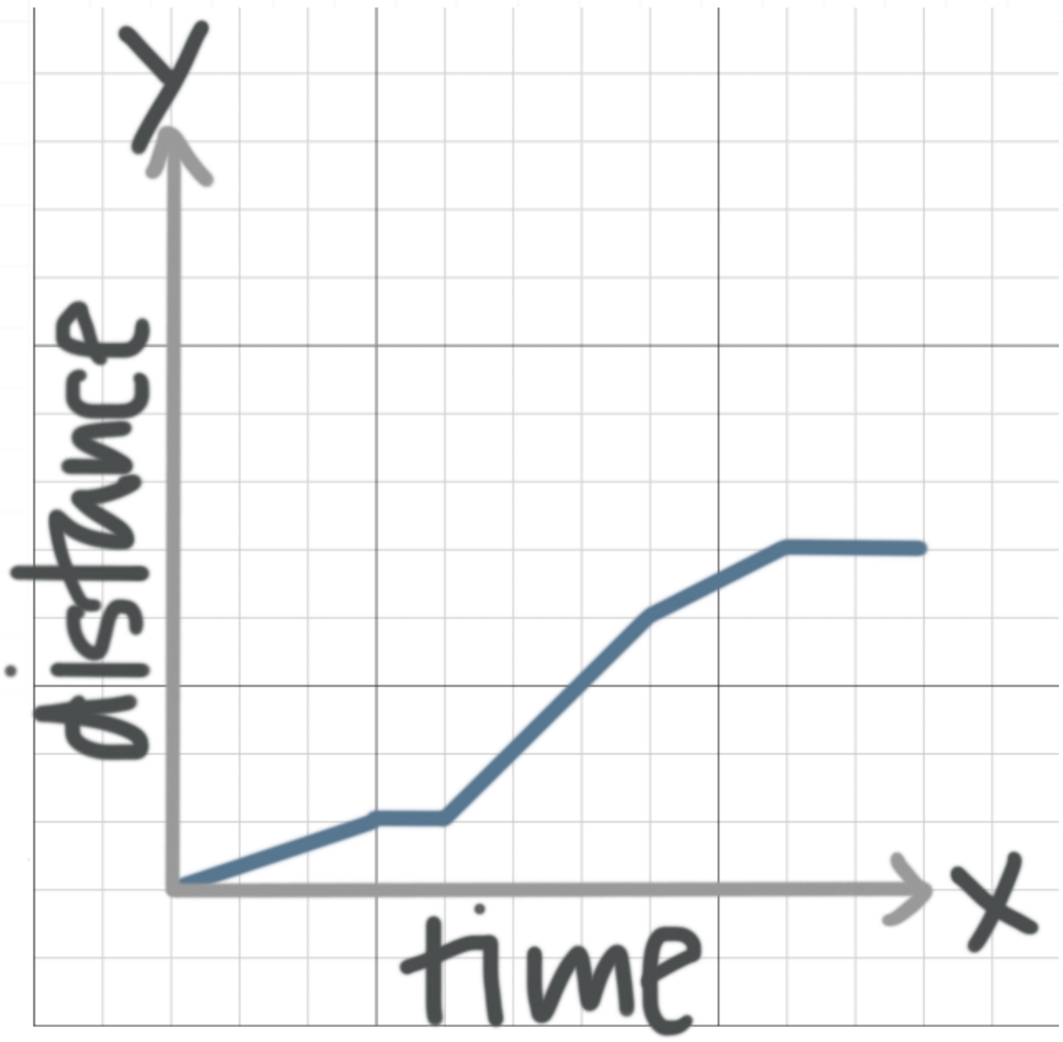
$$f(x) = \begin{cases} -x+1 & x < -4 \\ 5 & -4 < x \le 3 \\ -2x+11 & x > 3 \end{cases}$$

8. Graph the piecewise function.

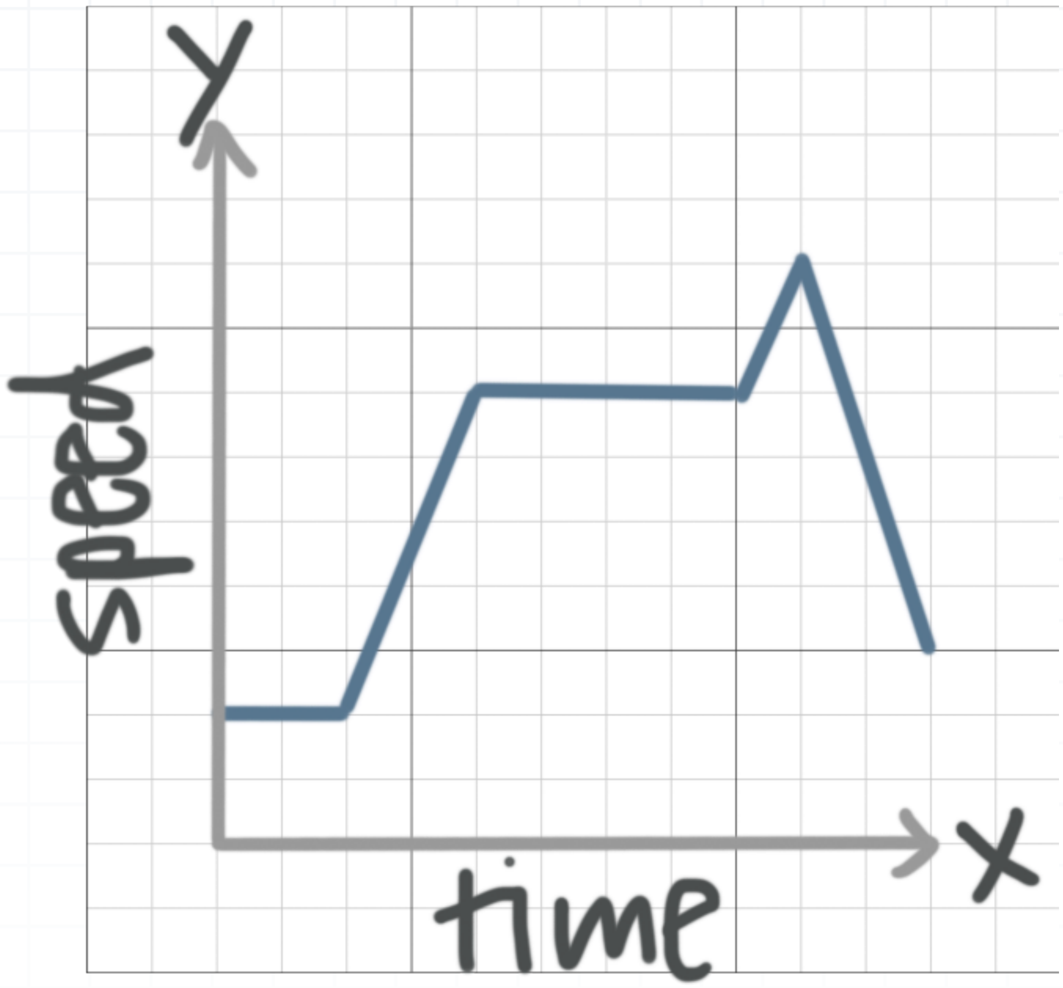
$$f(x) = [x]$$

SKETCHING GRAPHS FROM STORY PROBLEMS

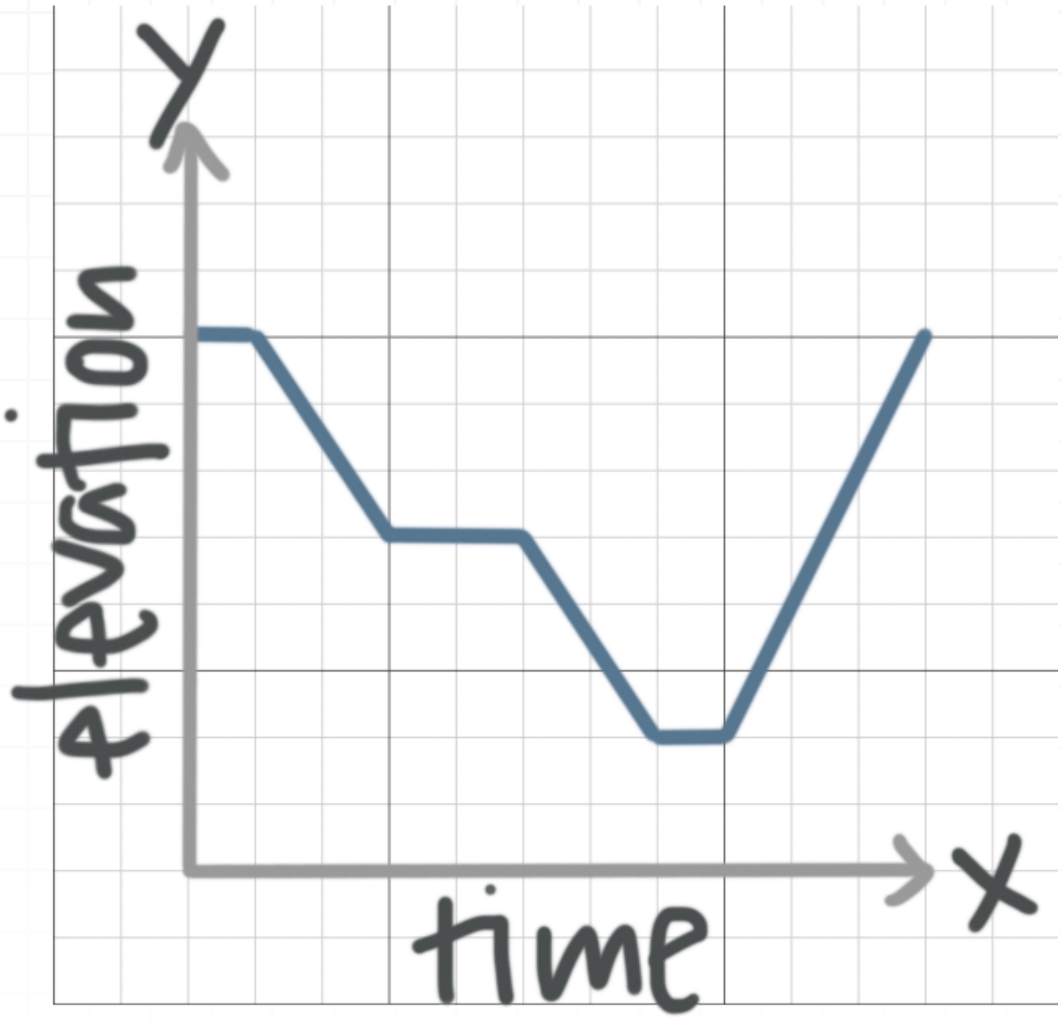
1. Alex left in his car to visit his grandparents' house. The graph shows his distance from his house over time. Write a possible story to go along with the graph.



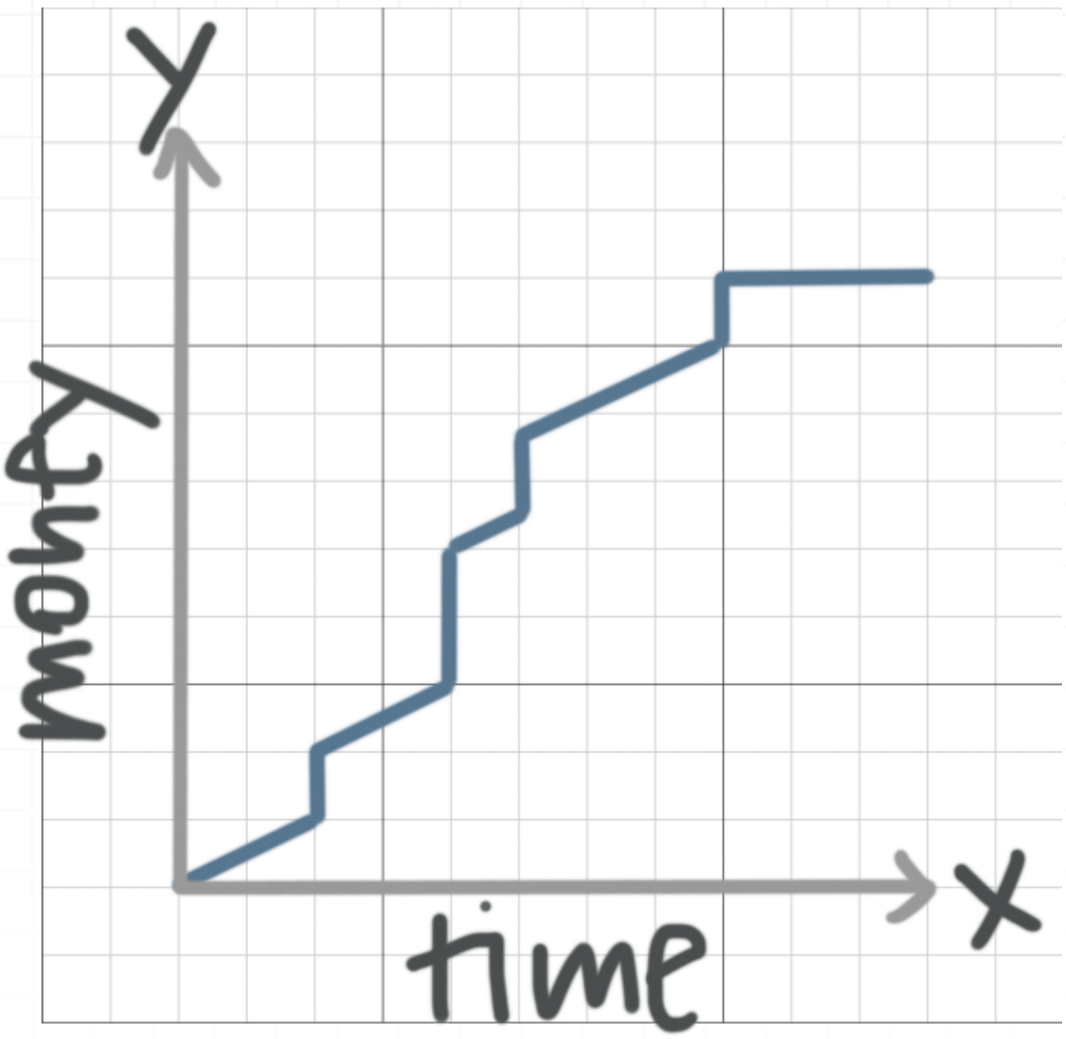
2. A horse is practicing for a race. The graph shows the horse's speed over time. Write a possible story to go with the graph.



3. A scuba diver takes a dive to explore the ocean. The graph below shows the diver's elevation over time. Write a possible story to go with the graph.



4. Janet delivers packages and get paid an hourly rate in addition to $1 for every package she delivers. The graph shows Janet's pay over the course of the day. Write a possible story to go with the graph.



5. A plane takes off and then cruises at $30{,}000$ feet for several hours before rising in elevation to $35{,}000$ feet to avoid turbulence for the last few hours. The plane then reaches its destination and lands. Sketch a graph representing the situation.

6. The temperature throughout a summer day starts at 65° F at 6:00 a.m.. Over the next few hours the temperature rises steadily until it reaches 85° F at 1:00 p.m.. At 1:15 p.m., a rainstorm begins and cools the temperature down to 75° F. The temperature then steadily decreases until it reaches 70° F at 9:00 p.m.. Sketch a graph representing the situation.

7. Brett goes for a hike up a mountain. He starts hiking up steadily for several hours with two stops for water. Then Brett stops for an hour to eat lunch and rest. He then continues up the mountain, summits, and spends a little time at the top of the mountain before climbing down. Sketch a graph representing Brett's elevation over time.

8. Heather went for a bike ride. She started at 12 mph to warm up, but quickly increased her speed to 20 mph and maintained that speed for most of the ride. Near the end of her bike ride, Heather decreased her speed to 15 mph until she reached her destination. Sketch a graph representing Heather's speed over time.

EQUATION OF A LINE IN POINT-SLOPE FORM

■ 1. Find the equation of the line that passes through $(3,0)$ with slope -2.

■ 2. Name two (of four possible) pieces of information about a line that are required to write an equation of the line in point-slope form.

■ 3. Find the equation of the line that passes through the points $(-2,3)$ and $(2,-4)$.

■ 4. Find the equation of the line that passes through $(-2,-5)$ with a slope 6.

■ 5. Identify the point (x_1, y_1) and slope m in the equation of the line.

$$y+3=\frac{1}{4}(x-6)$$

■ 6. Write the following equation in point-slope form.

$$y=-\frac{1}{2}x+4$$

7. Find the equation of the line that passes through the points $(5,-4)$ and $(6,0)$.

EQUATION OF A LINE IN SLOPE-INTERCEPT FORM

■ 1. Find the equation of a line through the point $(0,5)$ with slope -2. Write the solution in slope-intercept form.

■ 2. Identify the y-intercept and slope m defining the line.

$$y = -\frac{1}{4}(x + 12)$$

■ 3. Convert the following point-slope equation into a slope-intercept equation.

$$y - 3 = \frac{1}{3}(x - 6)$$

■ 4. Find the equation of a line that passes through the points $(1,-1)$ and $(0,3)$. Write the solution in slope-intercept form.

■ 5. Determine the y-intercept of a line with slope -3 that passes through the point $(1,1)$. Write your solution as a coordinate point.

6. Name two (of four possible) pieces of information about a line that are required to write an equation of the line in point-slope form.

7. Find the equation of a line that passes through the points $(-3,-2)$ and $(2,-4)$. Write the solution in slope-intercept form.

GRAPHING PARABOLAS

1. Write the equation in vertex form.

$$y = x^2 + 8x + 5$$

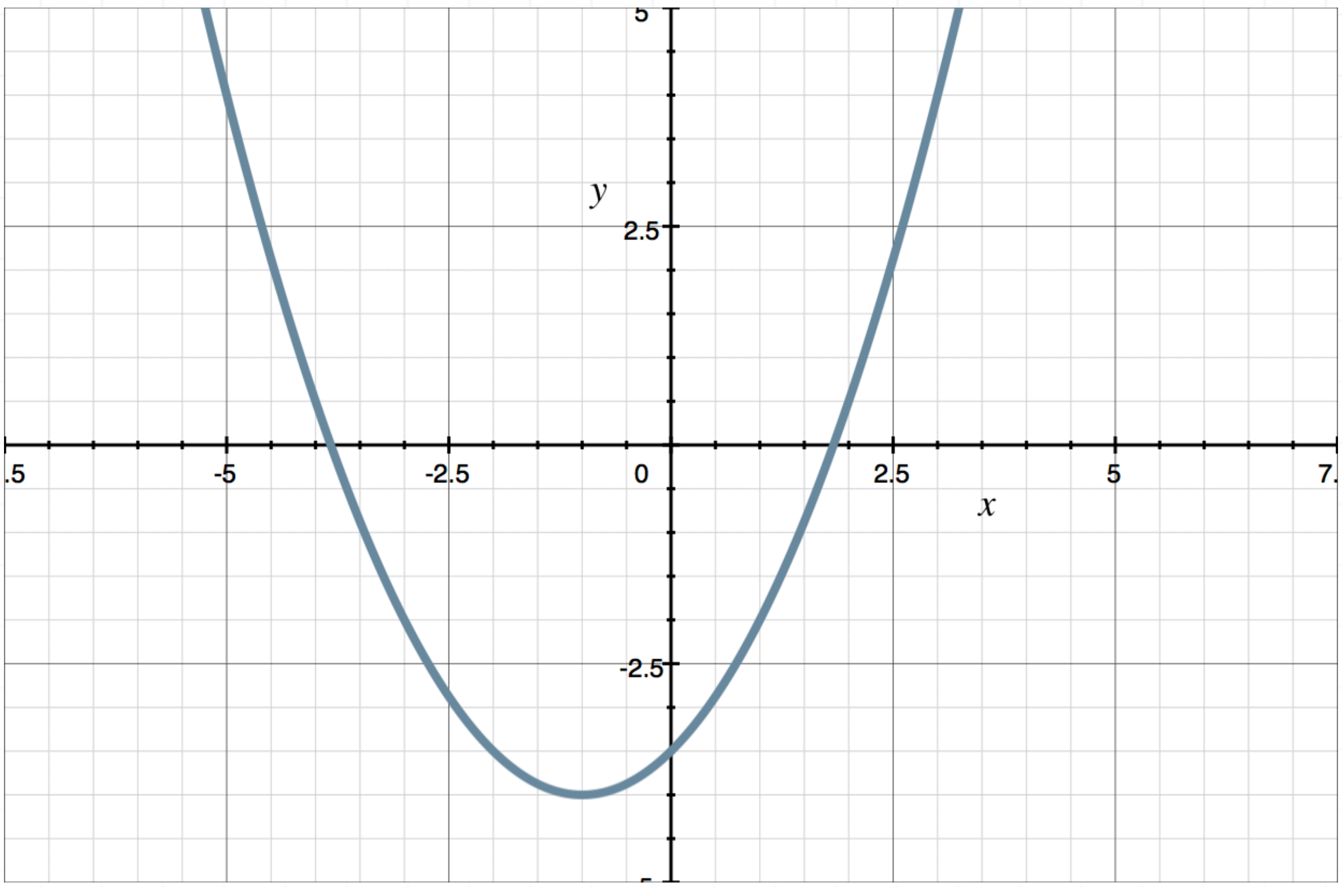
2. Write the equation in vertex form.

$$y = -2x^2 + 24x - 68$$

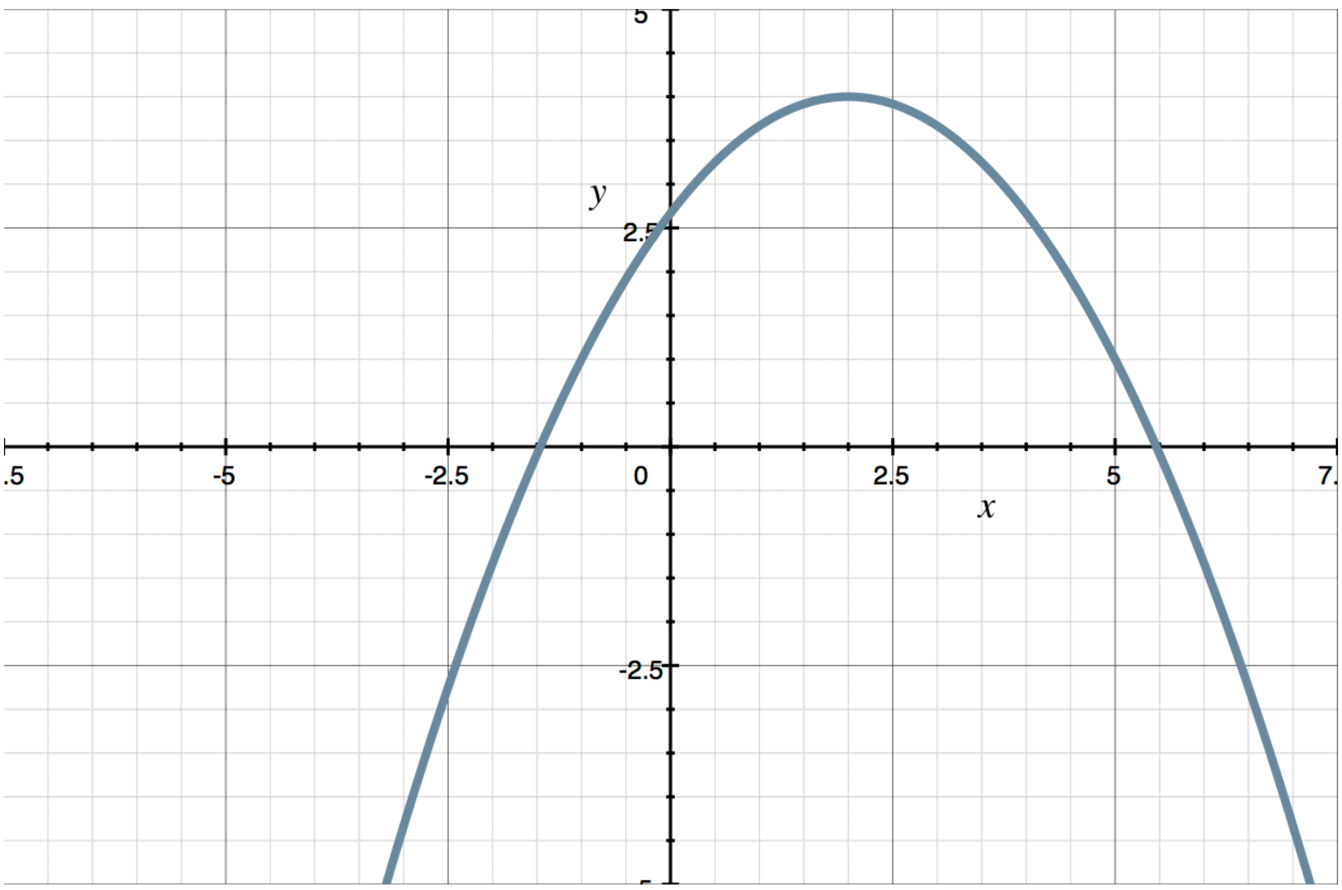
3. Find the vertex and axis of symmetry of $y = x^2 + 5x + 6$.

4. Find the vertex and axis of symmetry of $y = 3(x + 2)^2 + 6$.

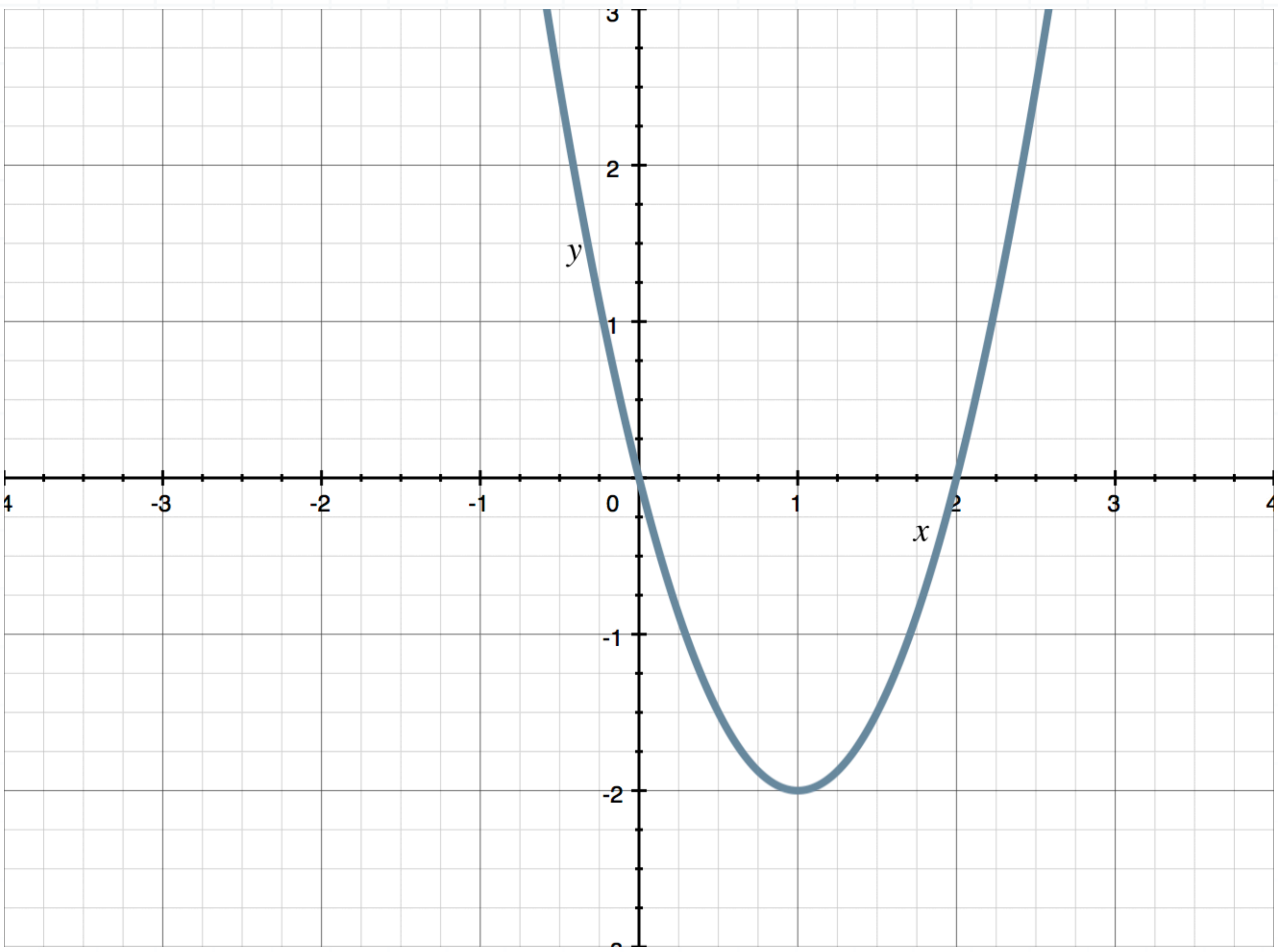
5. Identify the vertex and axis of symmetry from the graph of the parabola.



6. Using the graph below, find the equation of the parabola in standard form.



■ 7. Using the graph, find the equation of the parabola in standard form.



■ 8. Complete the square to graph the parabola $y = x^2 + 6x + 5$.

■ 9. Complete the square to graph $y = -x^2 - 4x - 6$.

FINDING CENTER AND RADIUS OF A CIRCLE

■ 1. Find the center and radius of the circle.

$$x^2 + y^2 - 2x - 3 = 0$$

■ 2. Find the center and radius of the circle.

$$x^2 + y^2 + 14x + 22y + 145 = 0$$

■ 3. Find the center and radius of the circle.

$$16x^2 + 16y^2 - 8x - 24y - 150 = 0$$

■ 4. Find the center and radius of the circle.

$$4x^2 + 4y^2 + 32x - 4y + 41 = 0$$

■ 5. Find the center and radius of the circle.

$$9x^2 + 9y^2 - 30x - 6y - 118 = 0$$

■ 6. Find the center and radius of the circle.

$$x^2 + y^2 + 4x - 2y = 0$$

■ 7. Find the center and radius of the circle.

$$x^2 + y^2 - 12x + 10y - 3 = 0$$

■ 8. Find the center and radius of the circle.

$$x^2 + y^2 - \frac{1}{4} = 0$$

■ 9. Find the center and radius of the circle.

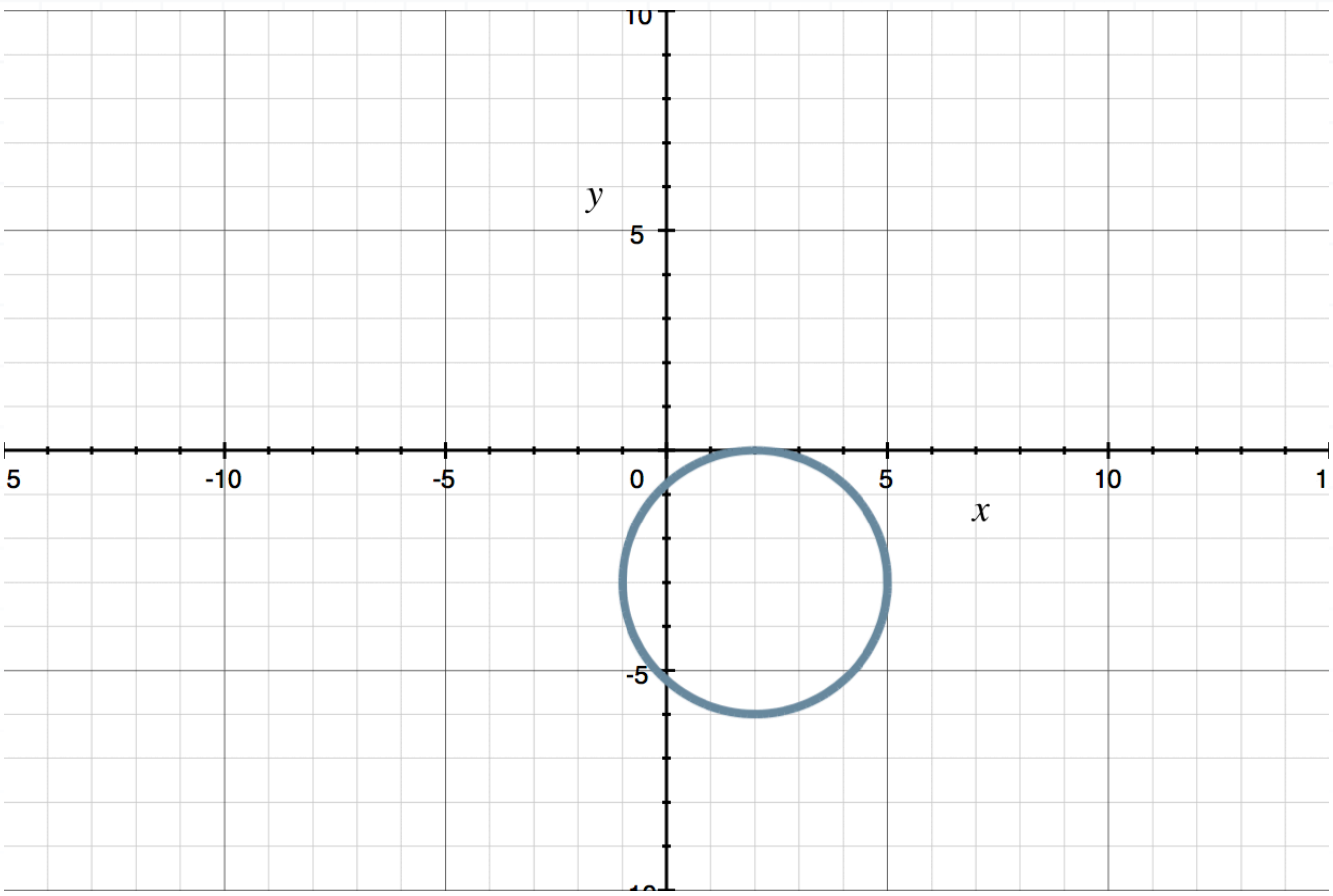
$$16x^2 + 16y^2 + 96x - 160y + 543 = 0$$

■ 10. Find the center and radius of the circle.

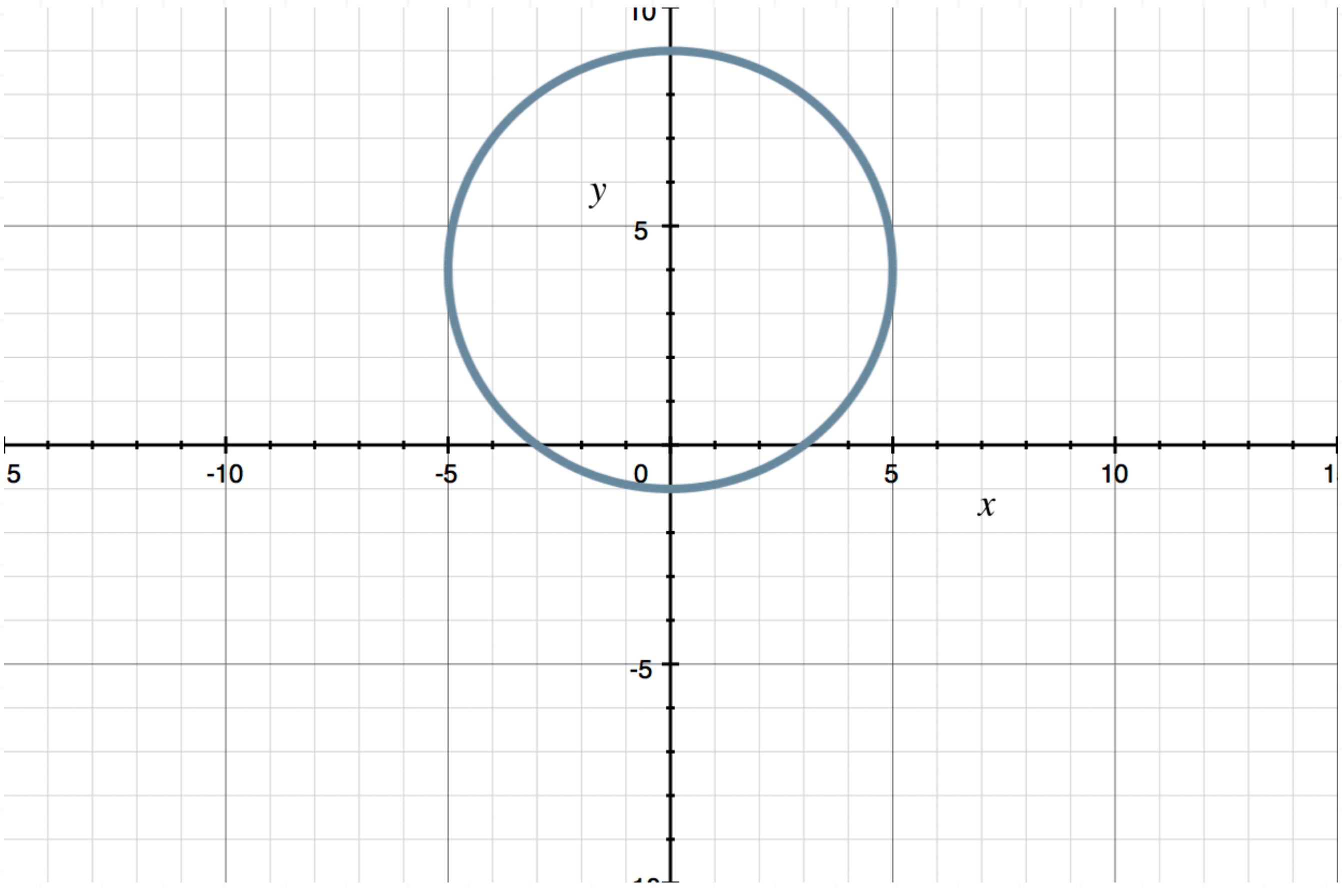
$$9x^2 + 9y^2 - 72x + 12y - 77 = 0$$

GRAPHING CIRCLES

1. Find the equation of the circle.



2. Find the equation of the circle.



■ 3. Graph the circle $(x-1)^2 + (y-2)^2 = 4$.

■ 4. Graph the circle $(x+3)^2 + (y-4)^2 = 25$.

■ 5. Graph the circle $x^2 + (y-3)^2 = 16$.

■ 6. Graph the circle $x^2 + y^2 + 2x + 2y - 14 = 0$.

■ 7. Graph the circle $x^2 + y^2 - 8x - 4y + 11 = 0$.

8. Graph the circle $x^2 + y^2 + 6x - 8y - 11 = 0$.

COMBINATIONS OF FUNCTIONS

■ 1. Find $(f+g)(x)$.

$$f(x)=2x^2-x+5$$

$$g(x)=x^2+4x-7$$

■ 2. Find $(f-g)(x)$.

$$f(x)=4x^2-2$$

$$g(x)=3x^2-5x$$

■ 3. Find $(f-g)(x)$.

$$f(x)=x^2-3x+1$$

$$g(x)=2x-3$$

■ 4. Find $(f\cdot g)(x)$.

$$f(x)=2x-3$$

$$g(x)=3x^2+2$$

5. Find $(f \cdot g)(x)$.

$$f(x) = x - 3$$

$$g(x) = x + 4$$

6. Find $(f \div g)(x)$.

$$f(x) = x^2 + 6x$$

$$g(x) = x$$

7. Find $(g \div f)(x)$.

$$f(x) = x^2 + 6x$$

$$g(x) = x$$

COMPOSITE FUNCTIONS

1. Find the composite function $(f \circ g)(x)$.

$$f(x) = \sqrt{2x - 1}$$

$$g(x) = 3x^2$$

2. Find the composite function $(g \circ f)(x)$.

$$f(x) = \sqrt{2x - 1}$$

$$g(x) = 3x^2$$

3. Find the composite function $f(g(x))$.

$$f(x) = x^2 - 4x + 3$$

$$g(x) = 2x + 1$$

4. Find the composite function $g(f(x))$.

$$f(x) = x^2 - 4x + 3$$

$$g(x) = 2x + 1$$

5. Find the composite function $(g \circ h)(x)$.

$$g(x) = \frac{8}{x^3}$$

$$h(x) = \sqrt[3]{x+4}$$

6. Find the composite function $(h \circ g)(x)$.

$$g(x) = \frac{8}{x^3}$$

$$h(x) = \sqrt[3]{x+4}$$

7. Find the composite function $g(h(x))$.

$$g(x) = \frac{1}{x}$$

$$h(x) = 3x^2 - x$$

8. Find the composite function $h(g(x))$.

$$g(x) = \frac{1}{x}$$

$$h(x) = 3x^2 - x$$

COMPOSITE FUNCTIONS, DOMAIN

1. What is the domain of $f \circ g$?

$$f(x) = x^2 - 2$$

$$g(x) = \sqrt{x+3}$$

2. What is the domain of $f \circ g$?

$$f(x) = \frac{1}{x}$$

$$g(x) = x + 5$$

3. What is the domain of $f \circ g$?

$$f(x) = \frac{2}{x-1}$$

$$g(x) = \sqrt{x-4}$$

4. What is the domain of $f \circ g$?

$$f(x) = \frac{1}{x} + 4$$

$$g(x) = \frac{3}{2x - 7}$$

5. What is the domain of $f \circ g$?

$$f(x) = \frac{2}{x - 3}$$

$$g(x) = \frac{4}{x + 2}$$

6. What is the domain of $f \circ g$?

$$f(x) = \frac{1}{x^2 - 3}$$

$$g(x) = \sqrt{x - 1}$$

7. What is the domain of $f \circ g$?

$$f(x) = 2x^2 - x + 1$$

$$g(x) = x - 3$$

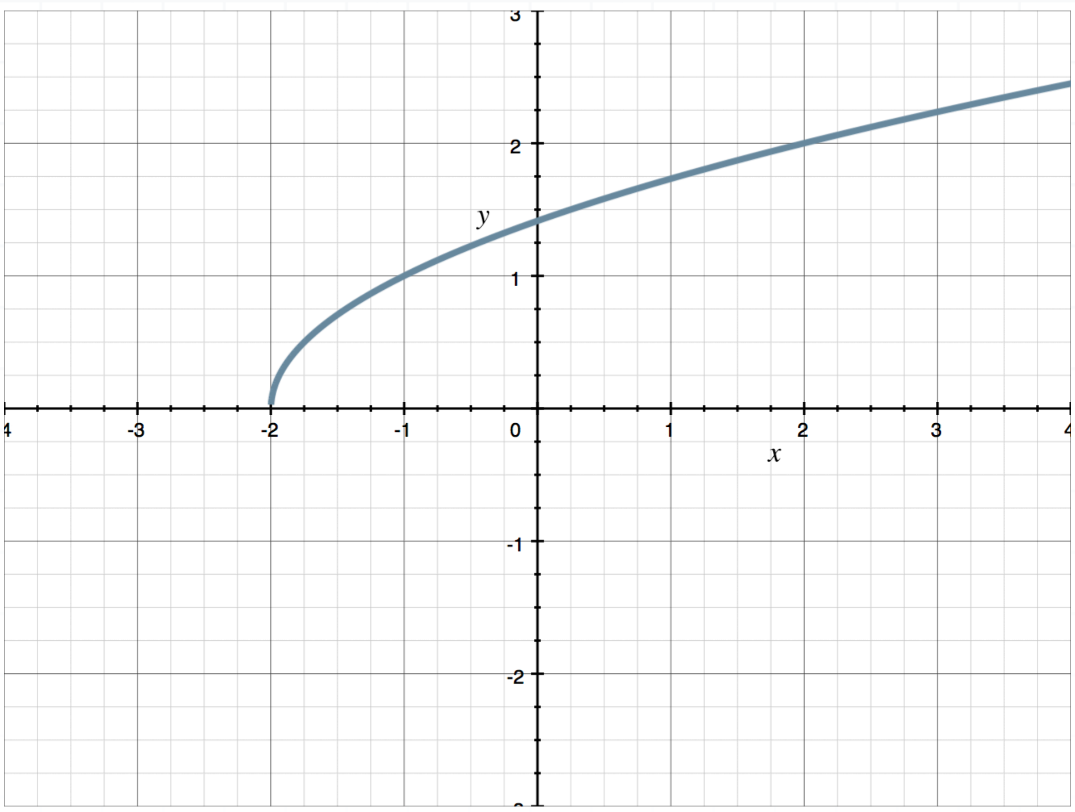
8. What is the domain of $f \circ g$?

$$f(x) = x^2 + 4x - 10$$

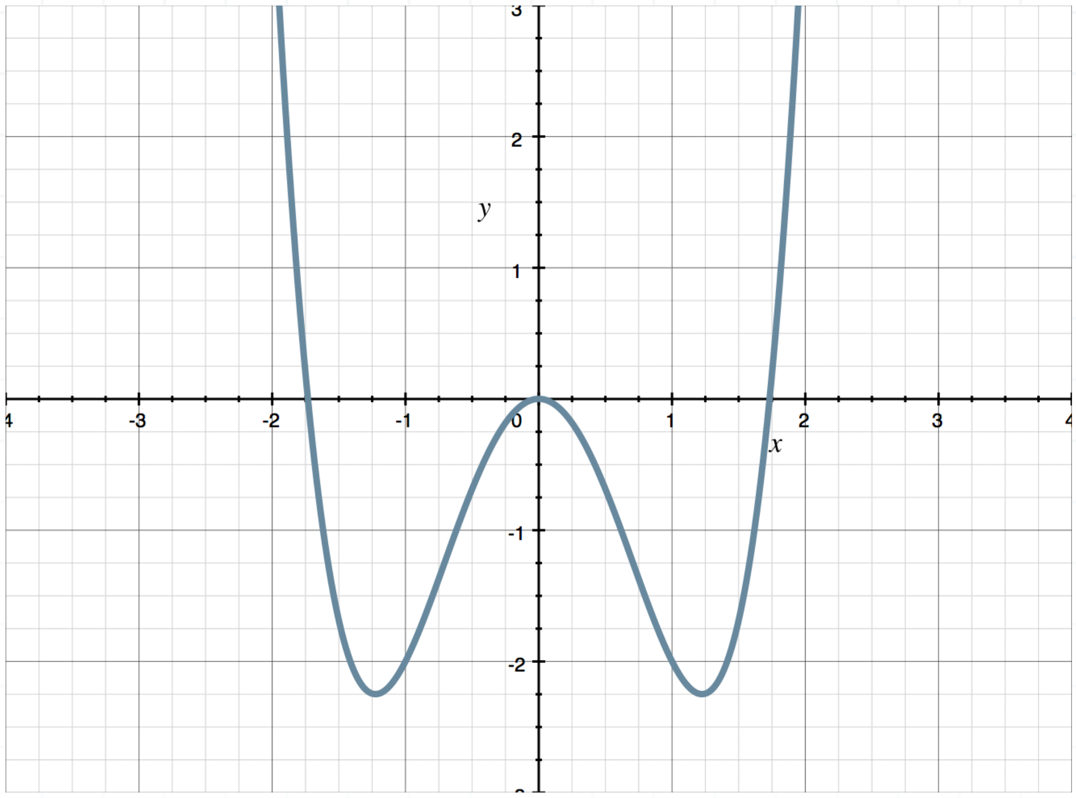
$$g(x) = x + 6$$

HORIZONTAL LINE TEST

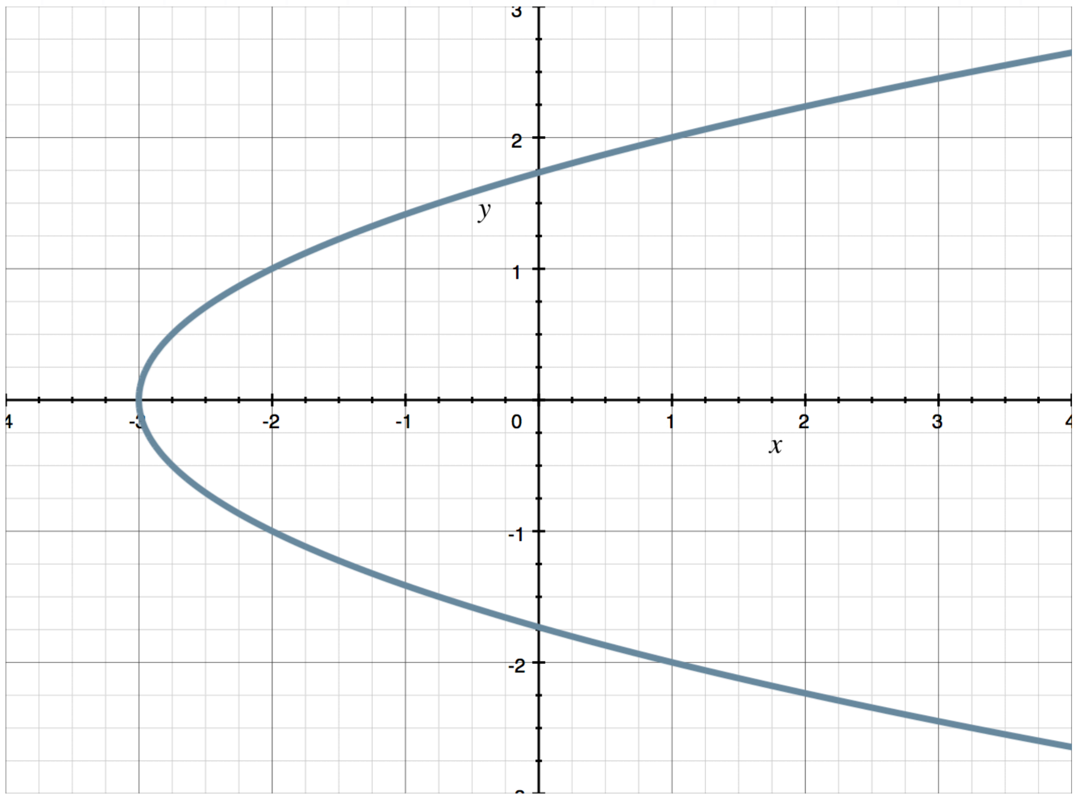
■ 1. Does the graph represent a one-to-one function?



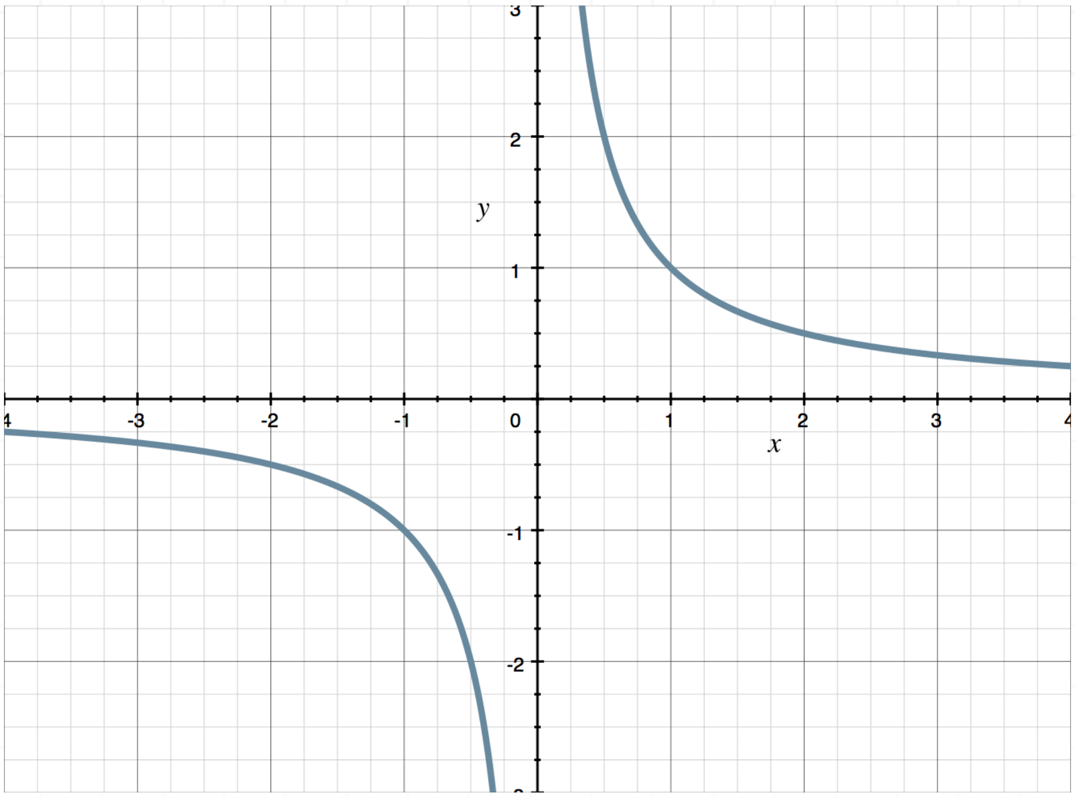
■ 2. Does the graph represent a one-to-one function?



3. Does the graph represent a one-to-one function?



4. Does the graph represent a one-to-one function?



5. Show that the function is one-to-one by showing that $f(a) = f(b)$ leads to $a = b$.

$$f(x) = 3x - 4$$

6. Show that the function is one-to-one by showing that $f(a) = f(b)$ leads to $a = b$.

$$f(x) = \frac{x+1}{x-5}$$

7. Show that the function is not one-to-one by showing that $f(a) = f(b)$ does not lead to $a = b$.

$$f(x) = x^2 - 6$$

8. Show that the function is not one-to-one by showing that $f(a)=f(b)$ does not lead to $a=b$.

$$f(x)=(x+3)(x-2)$$

INVERSE FUNCTIONS

1. What is the inverse of the function?

$$f(x) = \frac{1}{2}x - 3$$

2. What is the inverse of the function?

$$f(x) = -4x + 5$$

3. What is the inverse of the function?

$$f(x) = \frac{x}{x+2}$$

4. What is the inverse of the function?

$$f(x) = \frac{2x}{x-5}$$

5. What is the inverse of the function?

$$f(x) = \frac{1}{x} + 3$$

6. What is the inverse of the function?

$$f(x) = -\frac{3}{x-2} - 4$$

7. What is the inverse of the function?

$$f(x) = \frac{x-2}{x+3}$$

8. What is the inverse of the function?

$$f(x) = \frac{5+x}{4-x}$$

FINDING THE EQUATION OF A LINE FROM POINTS ON ITS INVERSE

1. Find $f(x)$ if $f^{-1}(x)$ is a linear function.

$$f^{-1}(1) = -2$$

$$f^{-1}(-3) = -1$$

2. Find $f(x)$ if $f^{-1}(x)$ is a linear function.

$$f^{-1}(0) = 3$$

$$f^{-1}(-2) = 1$$

3. Find $f(x)$ if $f^{-1}(x)$ is a linear function.

$$f^{-1}(2) = 5$$

$$f^{-1}(4) = 9$$

4. Find $f(x)$ if $f^{-1}(x)$ is a linear function.

$$f^{-1}(-3) = 2$$

$$f^{-1}(1) = 4$$

5. Find $f(x)$ if $f^{-1}(x)$ is a linear function.

$$f^{-1}(-4) = 7$$

$$f^{-1}(-1) = 14$$

6. Find $f(x)$ if $f^{-1}(x)$ is a linear function.

$$f^{-1}(5) = -4$$

$$f^{-1}(10) = -12$$

7. Find $f(x)$ if $f^{-1}(x)$ is a linear function.

$$f^{-1}(-3) = -4$$

$$f^{-1}(3) = 12$$

8. Find $f(x)$ if $f^{-1}(x)$ is a linear function.

$$f^{-1}(1) = 3$$

$$f^{-1}(2) = 6$$

LAWS OF LOGARITHMS

1. Write the expression as a single logarithm. Solve if possible.

$$\log_2 2 + \log_2 4$$

2. Write the expression as a single logarithm. Solve if possible.

$$\log_3 216 - \log_3 24$$

3. Write the expression as a single logarithm. Solve if possible.

$$\log_4 10 - 3\log_4 2$$

4. Write the expression as a single logarithm. Solve if possible.

$$2\log_7 4 + 3\log_7 5$$

5. Solve the equation.

$$\log_a 2 + \log_a 4 = \log_a(x + 2)$$

6. Solve the equation.

$$\log_4(x+5) - \log_4(x-2) = \log_4 3$$

7. Solve the equation.

$$2\log_b x = \log_b 49$$

8. Solve the equation.

$$\log_{12} x = \frac{3}{2}\log_{12} 16$$

QUADRATIC FORMULA

1. Solve for x using the quadratic formula.

$$4x^2 - 8x - 15 = 0$$

2. Write the quadratic formula for the following quadratic equation.

$$x^2 - 5x - 24 = 0$$

3. What went wrong in the way the quadratic formula was applied?

$$3x^2 - 5x + 10 = 0$$

$$x = \frac{-5 \pm \sqrt{(-5)^2 - 4(3)(10)}}{2(3)}$$

4. Solve for z using the quadratic formula.

$$z^2 = z + 3$$

5. Fill in the blank with the correct term if the quadratic formula below was built from the quadratic equation.

$$\underline{\hspace{2em}}x^2 + 3x - 5 = 0$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(-2)(-5)}}{2(-2)}$$

6. Simplify the expression.

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(14)}}{2(1)}$$

7. What are two ways to solve a quadratic equation when you cannot easily factor?

8. What went wrong if the quadratic formula below was built from the quadratic equation?

$$x^2 + 2x = 7$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(7)}}{2(1)}$$

9. Solve for t using the quadratic formula.

$$4t^2 - 1 = -8t$$

COMPLETING THE SQUARE

1. Solve for x by completing the square.

$$x^2 - 6x + 5 = 0$$

2. Fill in the blank with the correct term.

$$x^2 - \underline{\qquad} + \frac{9}{4} = -2 + \frac{9}{4}$$

3. Complete the square in the following expression, but do not solve.

$$3y^2 - 12y + 3 = 0$$

4. Solve for a by completing the square.

$$2a^2 + 8a = -4$$

5. What is your first and second step in solving the problem by completing the square?

$$4x^2 - 16x + 28 = 0$$

■ 6. Explain when and why completing the square is used for factoring.

■ 7. Solve for y by completing the square.

$$3y^2 + 9y = 3$$

■ 8. Fill in the blank with the correct term.

$$\underline{\qquad} - 4x = 6$$

$$\left(x - \frac{2}{3}\right)^2 = \frac{22}{9}$$

LONG DIVISION OF POLYNOMIALS

■ 1. Find the quotient.

$$\frac{x^2 + 2x - 1}{x + 3}$$

■ 2. Find the quotient.

$$\frac{2x^3 - x^2 - 4x + 5}{x - 2}$$

■ 3. Find the quotient.

$$\frac{2x^4 + 4x^3 - x^2 + 5x - 150}{x + 4}$$

■ 4. Find the quotient.

$$\frac{3x^3 - x^2 - 7x + 5}{x - 1}$$

■ 5. Find the quotient.

$$\frac{-x^2+3x+15}{x+5}$$

■ 6. Find the quotient.

$$\frac{x^4+x-3}{x-2}$$

■ 7. Find the quotient.

$$\frac{x^3+6}{x+6}$$

■ 8. Find the quotient.

$$\frac{x^2+x}{x-3}$$

■ 9. Find the quotient.

$$\frac{x^4-2x^2}{x-4}$$

■ 10. Find the quotient.

$$\frac{-2x^3 + 8x}{x + 2}$$

THE UNIT CIRCLE

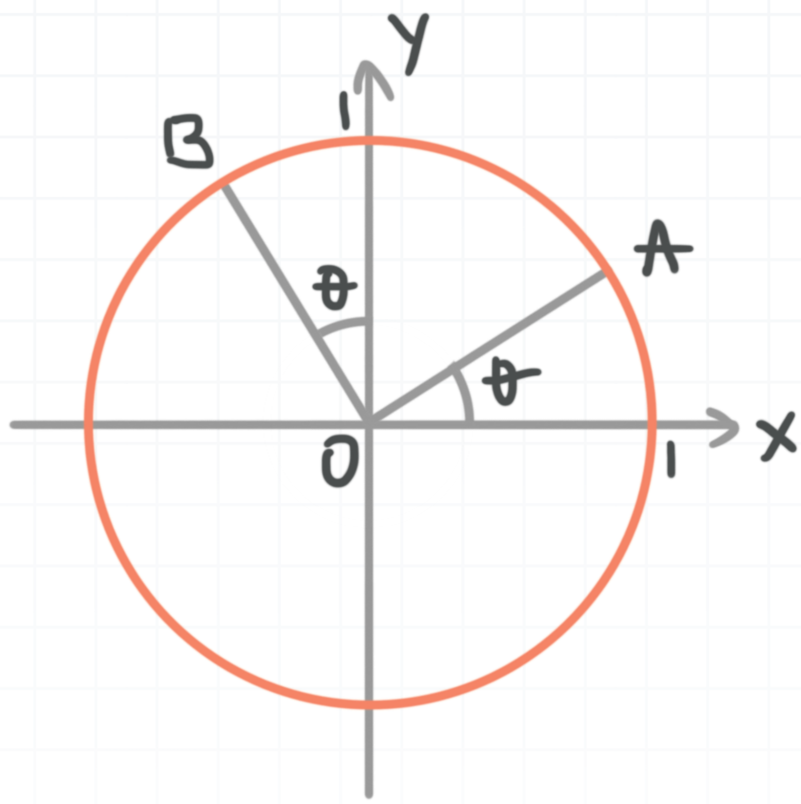
■ 1. What is the coordinate point associated with $\theta = 2\pi/3$ along the unit circle?

■ 2. The terminal side of the angle θ in $[0,2\pi)$ intersects the unit circle at the given point. Find the measure of θ in degrees.

$$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

■ 3. Find $\sin\theta$ if $\theta \in [0,2\pi)$ and $\cos\theta = \sin\theta$.

■ 4. The points A and B lie on the unit circle in quadrants I and II respectively. The angle between OA and the positive x-axis is θ. The angle between OB and the positive y-axis is θ. Find the sine of $\angle AOB$.



5. Evaluate the expression.

$$2\csc\left(\frac{49\pi}{6}\right) - 3\cos\left(\frac{13\pi}{3}\right) + \tan\left(\frac{25\pi}{4}\right)$$

6. Find the angle θ in the interval $[0,2\pi)$.

$$\sin\theta = \frac{1}{2} \text{ and } \cos\theta = -\frac{\sqrt{3}}{2}$$

IDEA OF THE LIMIT

1. The table below shows some values of a function $g(x)$. What does the table show for the value of $\lim_{x\to 4} g(x)$?

x	$g(x)$
3.9	1.9748
3.99	1.9975
3.999	1.9997
4.001	2.0002
4.01	2.0025
4.1	2.0248

2. How would you express, mathematically, the limit of the function $f(x) = x^2 - x + 2$ as x approaches 3?

3. How would you write the limit of $g(x)$ as x approaches ∞, using correct mathematical notation?

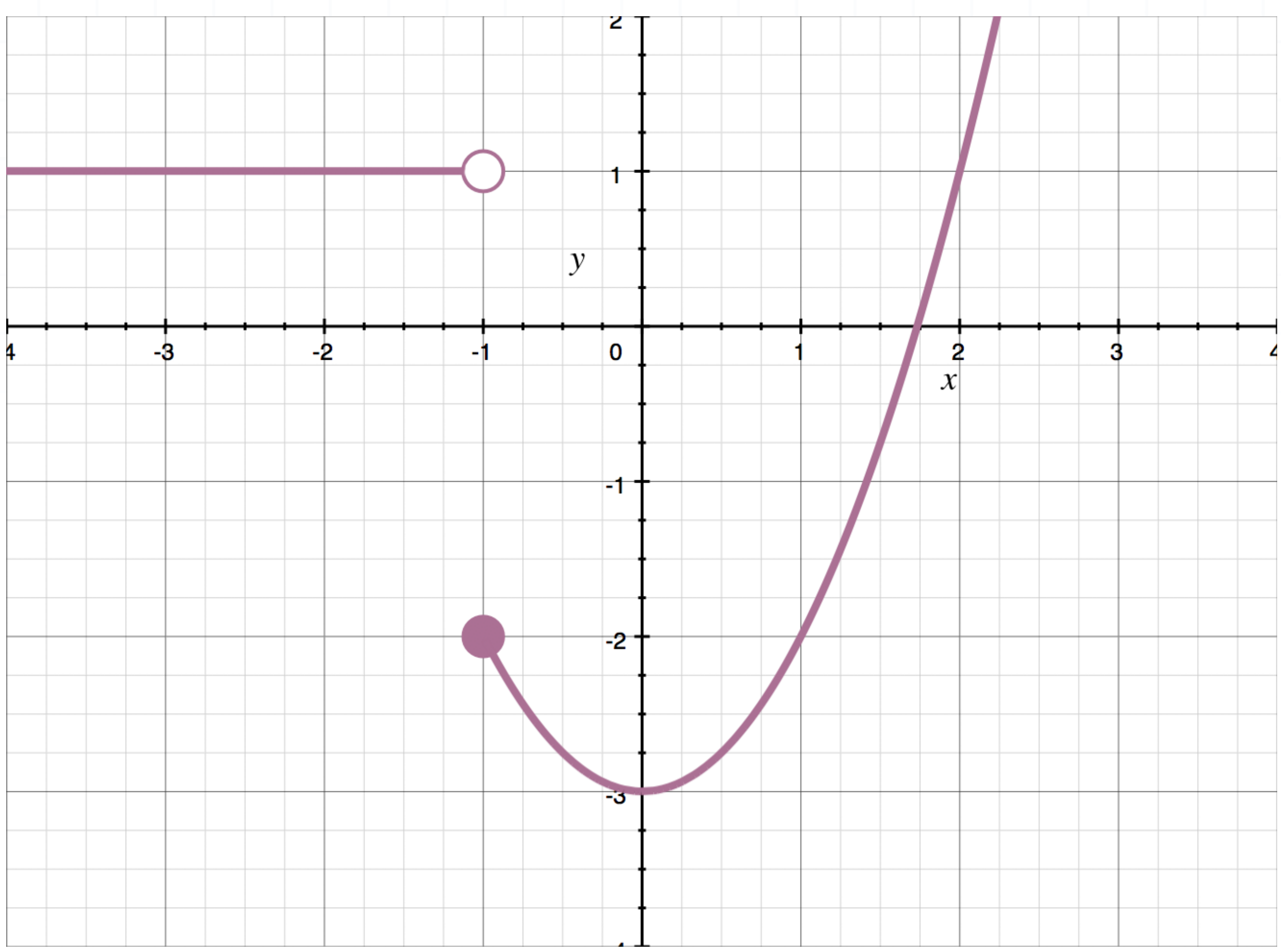
$$g(x) = \frac{5x^2 - 7}{3x^2 + 8}$$

ONE-SIDED LIMITS

■ 1. Find the limit.

$$\lim_{x \to -7^+} x^2\sqrt{x+7}$$

■ 2. What does the graph of $f(x)$ say about the value of $\lim_{x \to -1^+} f(x)$?



■ 3. The table shows values of $k(x)$. What is $\lim_{x \to -5^-} k(x)$?

x	-5.1	-5.01	-5.0001	-5	-4.999	-4.99	-4.9
k(x)	-392.1	-3,812	-38,012	?	37,988	3,788	368.1

■ 4. What is $\lim_{x\to -2^-} h(x)$?

$$h(x) = \begin{cases} -2x - 1 & x < -2 \\ x & -2 \le x < 2 \\ 2x - 3 & x \ge 2 \end{cases}$$

■ 5. What is $\lim_{x\to 6^+} g(x)$?

$$g(x) = \frac{x^2 + x - 42}{x - 6}$$

PROVING THAT THE LIMIT DOES NOT EXIST

1. Prove that the limit does not exist.

$$\lim_{x\to 0} \frac{-2|3x|}{3x}$$

2. Prove that the limit does not exist.

$$\lim_{x\to -5} \frac{x^2+7x+9}{x^2-25}$$

3. Prove that $\lim_{x\to 1} f(x)$ does not exist.

$$f(x) = \begin{cases} -3x+2 & x<1 \\ 3x-2 & x\geq 1 \end{cases}$$

PRECISE DEFINITION OF THE LIMIT

■ 1. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \to 4} 5x - 16 = 4$$

■ 2. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \to -7} -2x + 15 = 29$$

■ 3. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \to 16} \left(\frac{2}{5}x - \frac{17}{5} \right) = 3$$

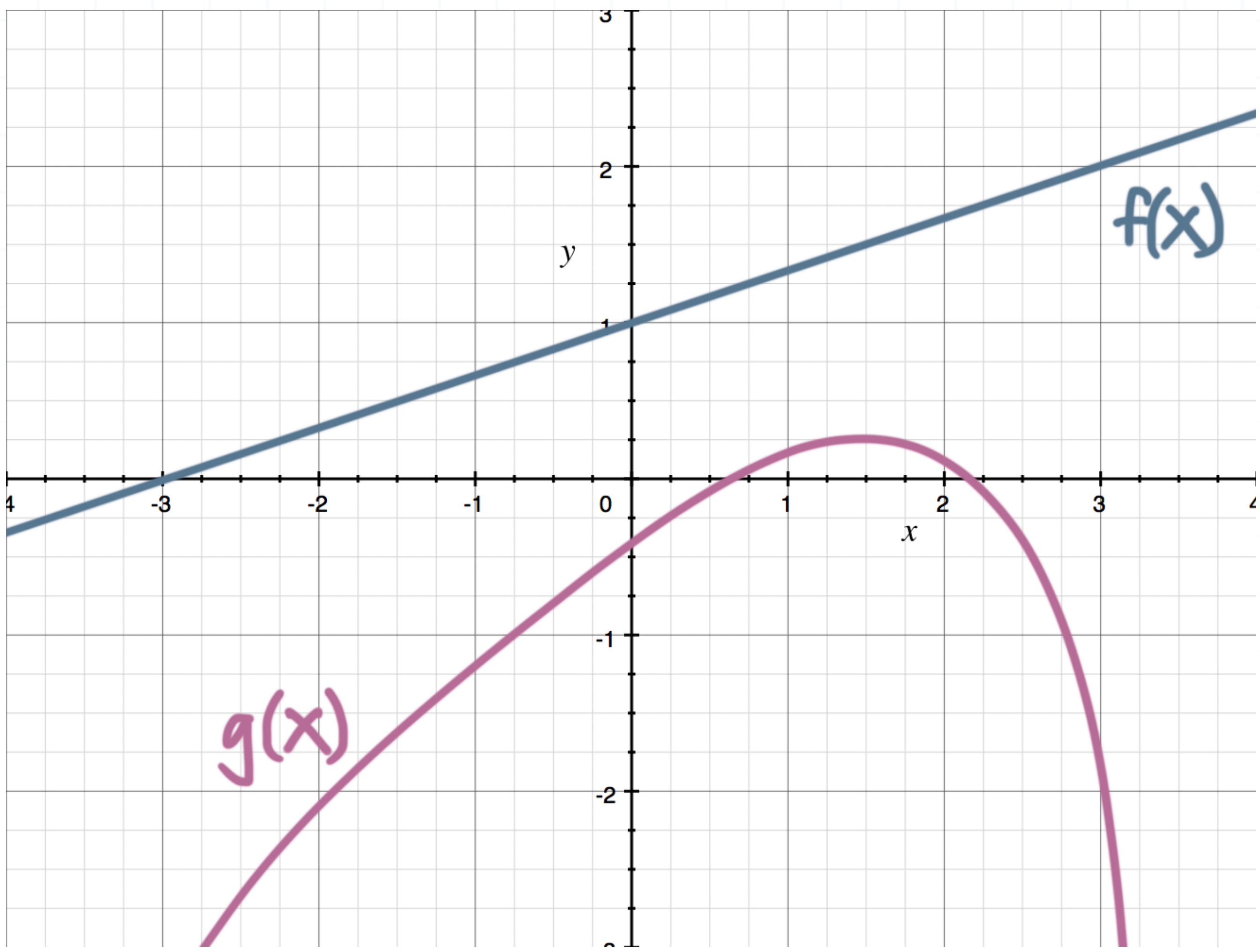
■ 4. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \to 7} \left(\frac{x^2 - 15x + 56}{x - 7} \right) = -1$$

LIMITS OF COMBINATIONS

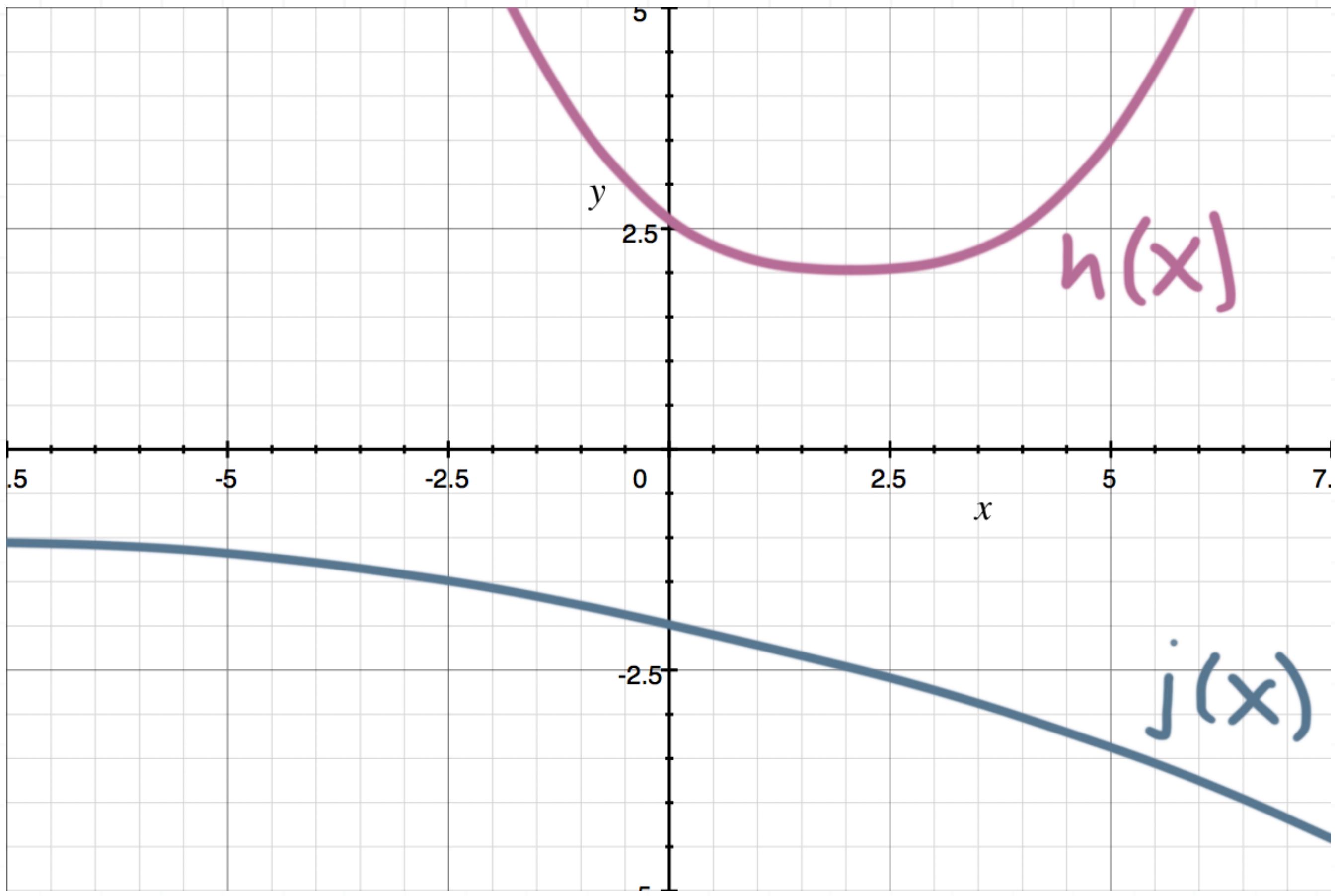
■ 1. Use limit laws and the graph below to evaluate the limit.

$$\lim_{x \to 3} \left[4f(x) - 3g(x)\right]$$



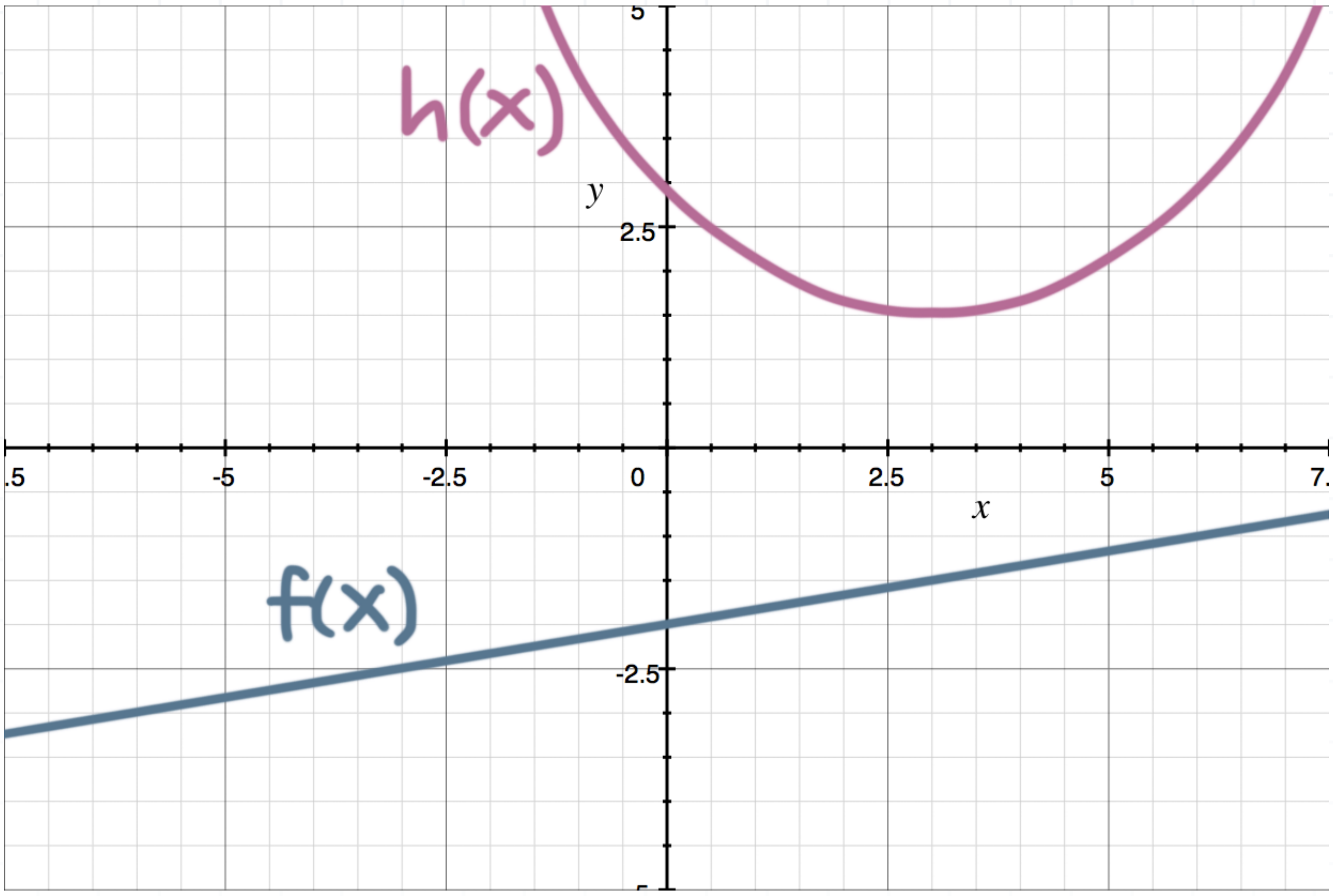
■ 2. Use limit laws and the graph below to evaluate the limit.

$$\lim_{x \to 4} \frac{h(x)}{j(x)}$$



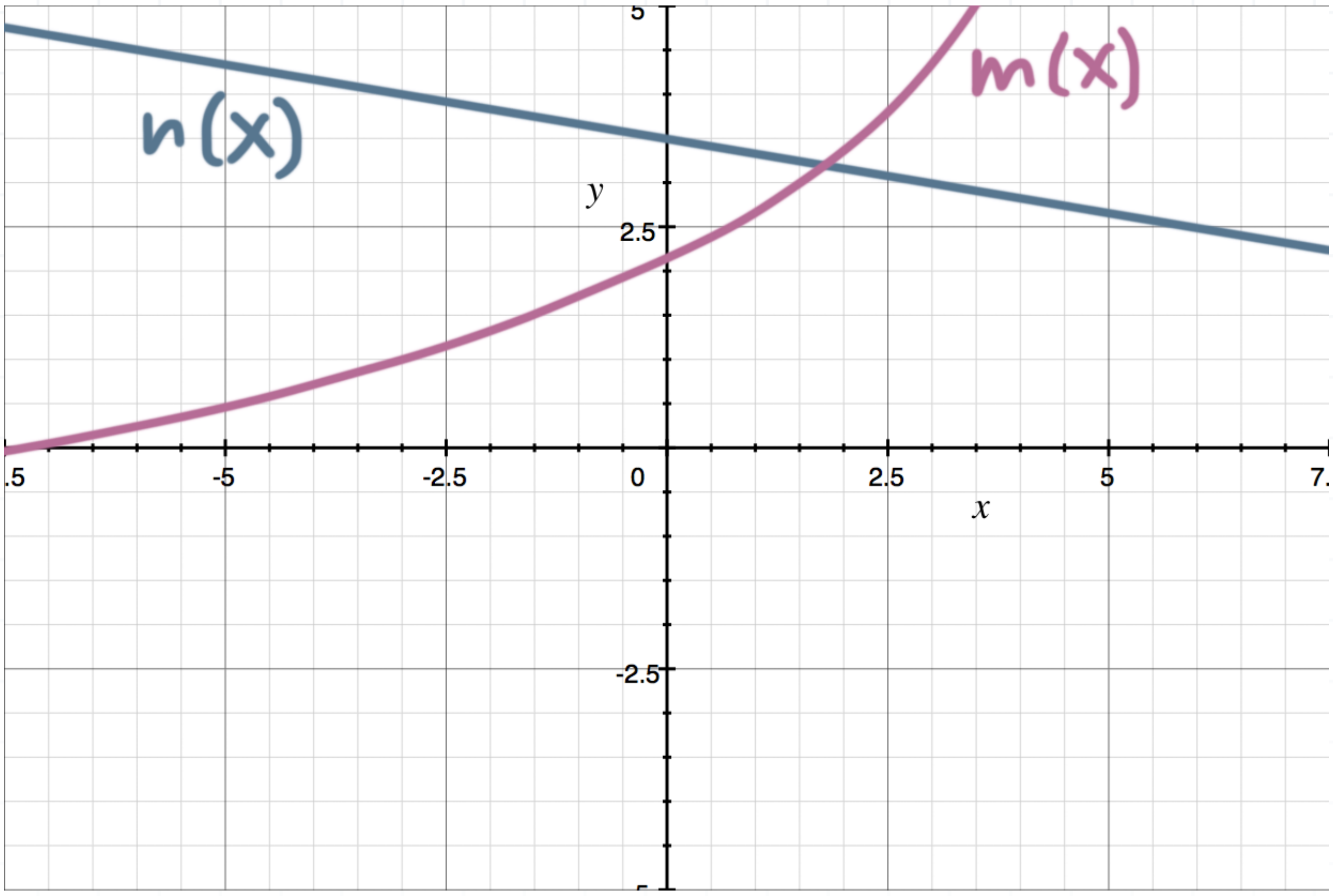
3. Use limit laws and the graph below to evaluate the limit.

$$\lim_{x \to 0} \left[2f(x) \cdot 3h(x)\right]$$



4. Use limit laws and the graph below to evaluate the limit.

$$\lim_{x\to -3}\left[\frac{5m(x)}{n(x)}-\frac{4m(x)}{n(x)}\right]$$


n(x)
m(x)
y
x
2.5
-2.5
-5
-2.5
0
2.5
5

LIMITS OF COMPOSITES

1. What is $\lim_{x\to 3} f(g(x))$ if $f(x) = 4x$ and $g(x) = 6x - 9$?

2. What is $\lim_{x\to -4} f(g(x))$ if $f(x) = 2x^2$ and $g(x) = 2x - 1$?

3. What is $\lim_{x\to \frac{\pi}{2}} f(g(x))$ if $f(x) = \sin x$ and $g(x) = x/2$?

POINT DISCONTINUITIES

■ 1. Redefine the function as a continuous piecewise function.

$$f(x) = \frac{x^2 - 6x - 27}{x + 3}$$

■ 2. Redefine the function as a continuous piecewise function.

$$g(x) = \frac{\sin x}{x}$$

■ 3. What are the removable discontinuities of the function?

$$h(x) = \frac{x^4 - 5x^2 + 4}{x^2 - 1}$$

■ 4. Identify the non-removable discontinuities of the function.

$$k(x) = \frac{x^3 + 3x^2 - 25x - 75}{x^2 + x - 12}$$

■ 5. What is the set of removable discontinuities of the function?

$$j(\theta) = \frac{\cos^2\theta \cdot \sin^2\theta}{\tan^2\theta}$$

■ 6. Redefine the function as a continuous piecewise function.

$$g(x) = \frac{x^3 - 8}{x - 2}$$

■ 7. Identify the non-removable discontinuity in the function.

$$k(x) = \begin{cases} x & x < 0 \\ 2 & x = 0 \\ x + 3 & x > 0 \end{cases}$$

■ 8. What is the removable discontinuity in the function?

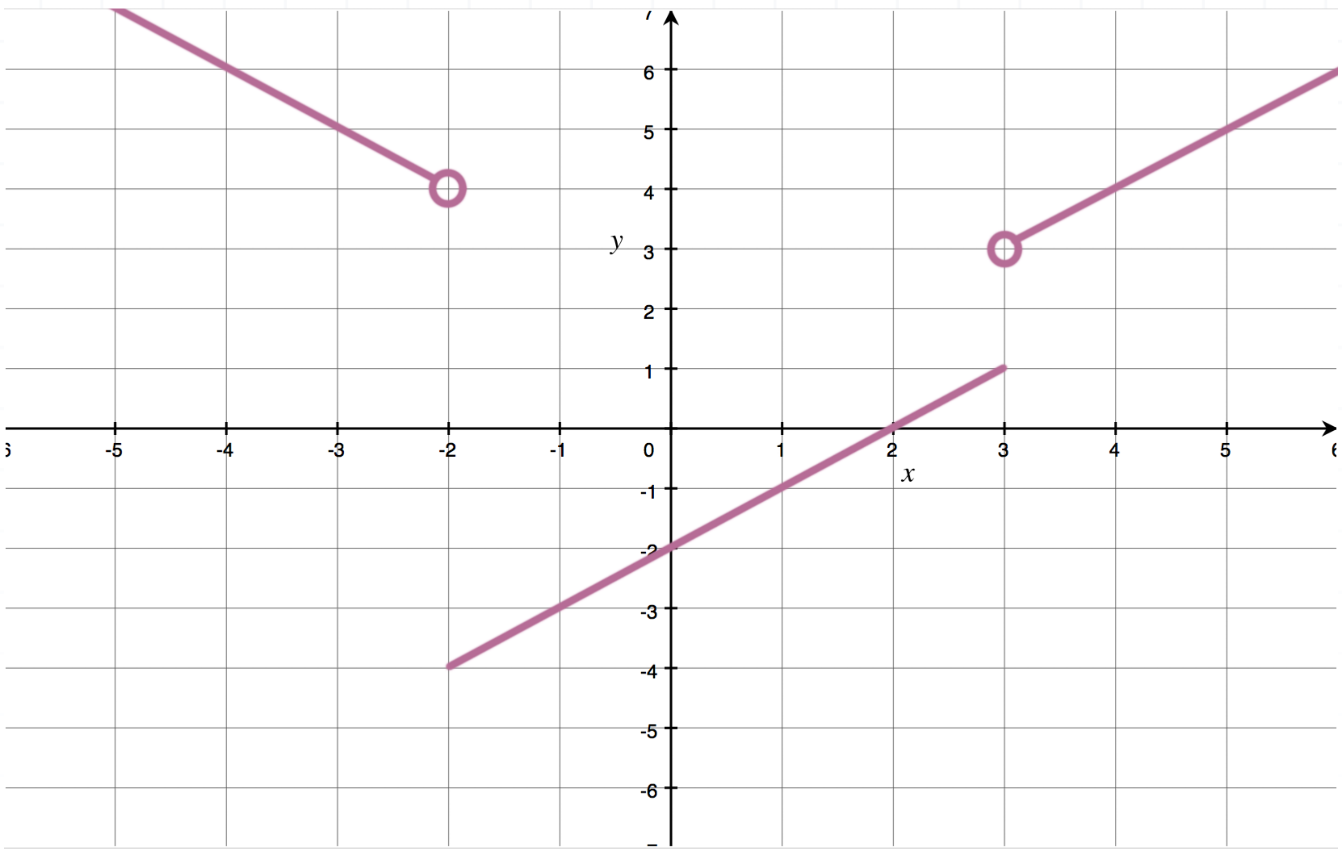
$$f(x) = \frac{x^3 + 27}{x + 3}$$

■ 9. Identify the removable discontinuities in the function.

$$k(x) = \frac{x^4 - 2x^3 - 16x^2 + 2x + 15}{x^2 - 2x - 15}$$

JUMP DISCONTINUITIES

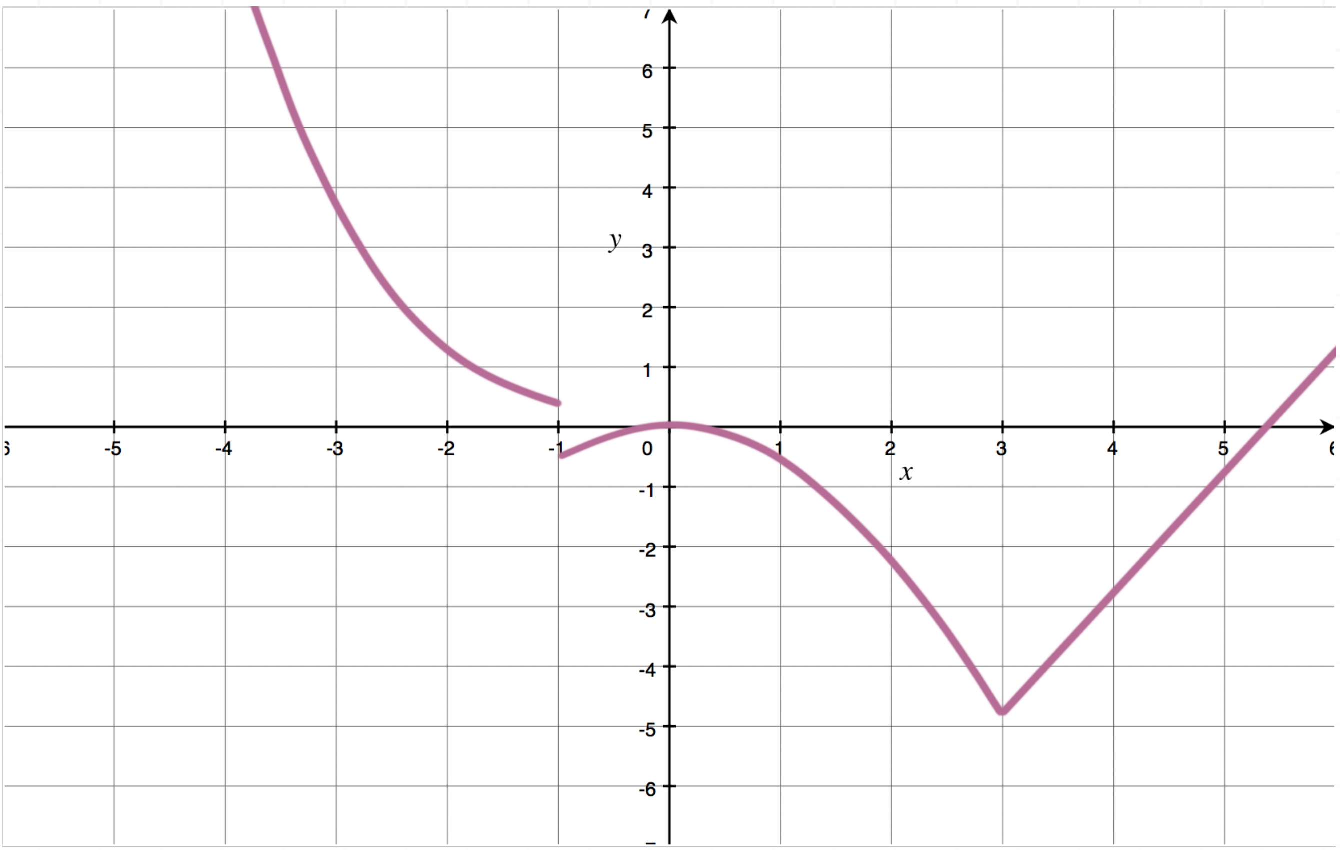
■ 1. What are the x-values where the graph of $f(x)$, shown below, has jump discontinuities?



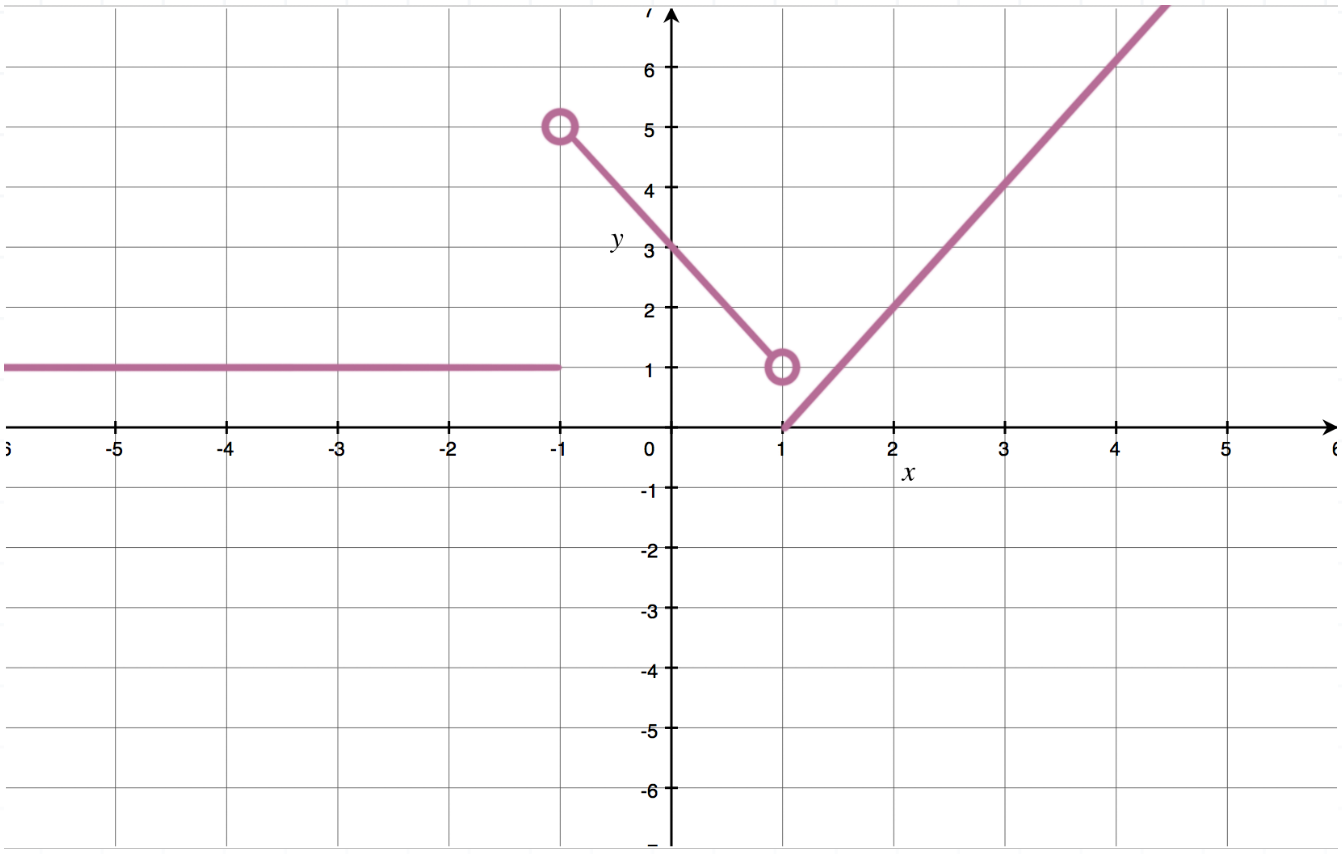
■ 2. Where are the jump discontinuities in the graph of the function?

$$h(x) = \begin{cases} -\frac{1}{3}x^2 + 2 & x < 0 \\ 3 & 0 \le x \le 1 \\ \frac{1}{3}x^2 + 4 & x > 1 \end{cases}$$

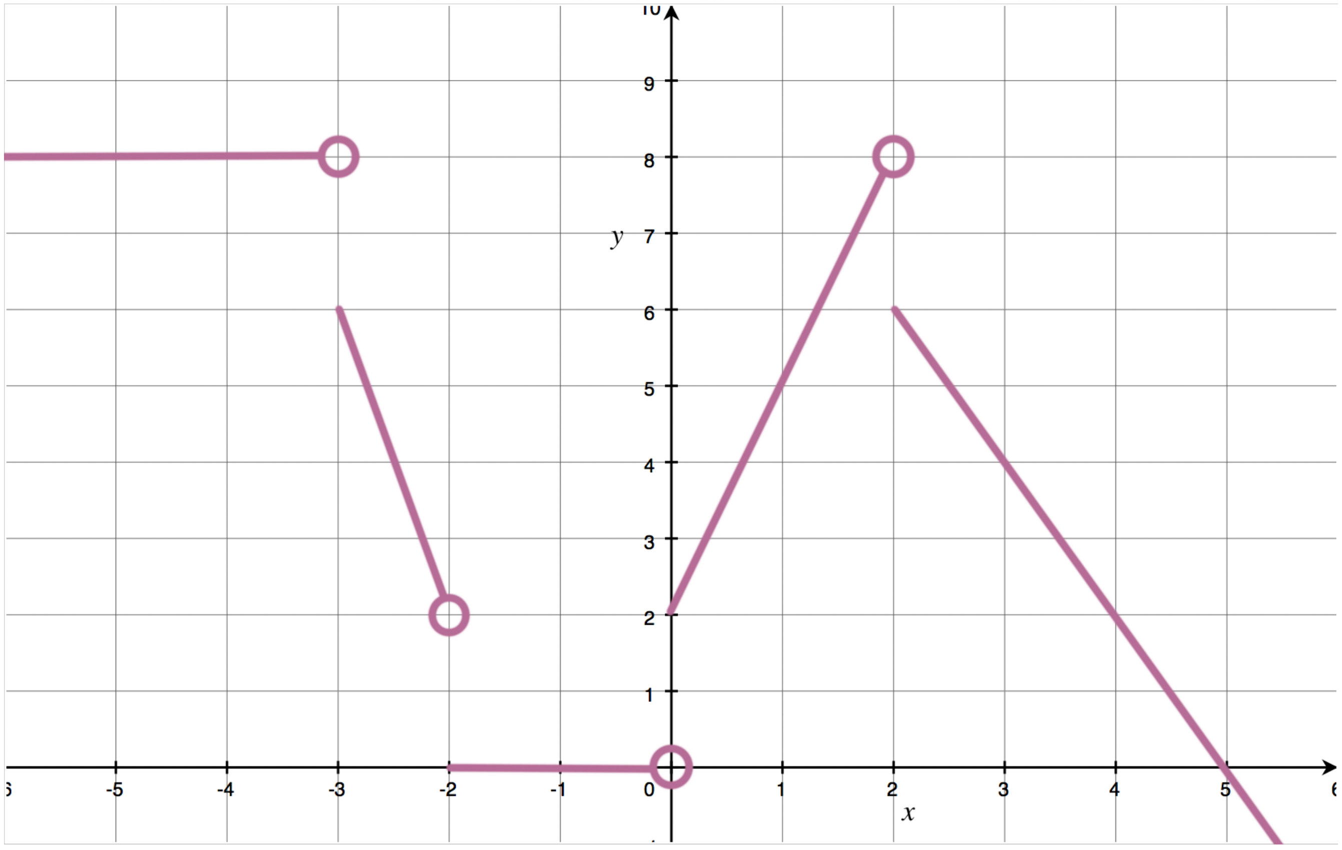
■ 3. What are the x-values where the graph of $g(x)$ has jump discontinuities?



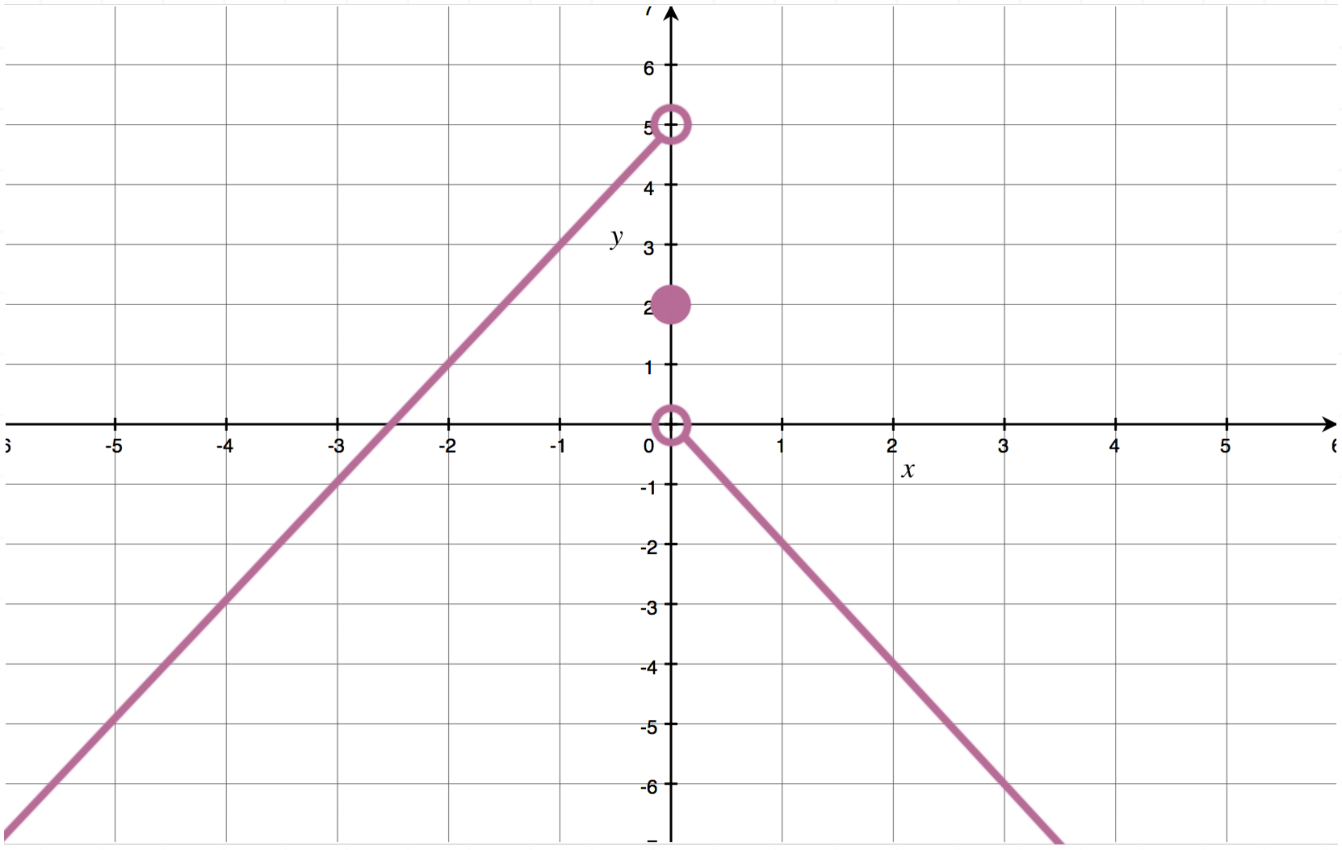
■ 4. What are the x-values where the graph of $f(x)$, shown below, has jump discontinuities?



■ 5. Where are the jump discontinuities in the graph of the function shown below?



■ 6. What are the x-values where the graph of $h(x)$, shown below, has jump discontinuities?



INFINITE DISCONTINUITIES

■ 1. At what x-values does the function have infinite discontinuities?

$$f(x) = \frac{x^2 + x - 12}{x^2 + x - 2}$$

■ 2. Where are the infinite discontinuities of the function?

$$h(x) = \frac{x^4 + 3x^3 - 8x - 24}{x^2 + 3x - 4}$$

■ 3. At what x-values does the function have infinite discontinuities?

$$g(x) = \frac{x^2 - 5x + 6}{x^2 - 1}$$

■ 4. Where are the infinite discontinuities of the function?

$$h(x) = \frac{x^2 - 6x + 9}{x^2 - 4}$$

■ 5. At what x-values does the function have infinite discontinuities?

$$h(x) = \frac{x^2 - 15x + 21}{x^2 - x - 12}$$

■ 6. Where are the infinite discontinuities of the function?

$$g(x) = \frac{x^3 + 4x^2 - 20x - 48}{x^2 + 2x - 8}$$

ENDPOINT DISCONTINUITIES

1. What is the value of the limit on the interval $[0,3]$?

$$\lim_{x\to 3} -\sqrt{x+5}$$

2. What is the value of the limit on the interval $[\pi,2\pi]$?

$$\lim_{x\to \pi} \sin x$$

3. What is the value of the limit on the interval $(-\infty,2]$.

$$\lim_{x\to 2} x^3 - x^2 + 4$$

4. What is the value of the limit on the interval $[4,\infty)$?

$$\lim_{x\to 4} -\frac{x+7}{x^2-6x+15}$$

5. What is the value of the limit on the interval $[-9/2,5/2]$?

$$\lim_{x\to \frac{5}{2}} \frac{x+3}{x^2+x+1}$$

■ 6. What is the value of the limit on the interval $(-2,2]$?

$$\lim_{x \to -2} \sqrt{2x+4}$$

■ 7. What is the value of the limit on the interval $[-\pi, \pi]$?

$$\lim_{x \to \pi} -\frac{5\cos x}{2}$$

INTERMEDIATE VALUE THEOREM WITH AN INTERVAL

■ 1. The value $c = -1$ satisfies the conditions of the Intermediate Value Theorem for the function on the interval $[-3,5]$ because $f(c)$ equals what value?

$$f(x) = \frac{1}{4}(2x+5)(x-3)^2$$

■ 2. The value $c = 2$ does not satisfy the conditions of the Intermediate Value Theorem for $g(x) = 2x^2 - 11x + 4$ on the interval $[-2,8]$ because $g(c)$ equals what value?

■ 3. What value of c is guaranteed by the Intermediate Value Theorem on the interval $[-3,3]$ if $h(x) = 3(x+1)^3$ and $h(c) = 24$?

■ 4. What value of c is guaranteed by the Intermediate Value Theorem on the interval $[-5,6]$ if $f(c) = -6$ and

$$f(x) = \begin{cases} 3x - 10 & \text{if } x \le 0 \\ x^2 + 3x - 10 & \text{if } 0 < x < 2 \\ 3x - 6 & \text{if } x \ge 2 \end{cases}$$

■ 5. The value $c=5$ satisfies the conditions of the Intermediate Value Theorem for the function on the interval $[3,9]$ because $g(c)$ equals what value?

$$g(x)=\frac{x^2-9}{x+3}$$

■ 6. What value of c is guaranteed by the Intermediate Value Theorem on the interval $[3,6]$ if c is a root of $h(x)$.

$$h(x)=\frac{x^3-4x^2-11x+30}{x^2-4}$$

INTERMEDIATE VALUE THEOREM WITHOUT AN INTERVAL

■ 1. Use the Intermediate Value Theorem to prove that the equation $2e^x = 3\cos x$ has at least one positive solution. In what interval is that solution?

■ 2. Use the Intermediate Value Theorem to prove that the equation $3\sin x + 7 = x^2 - 2x - 2$ has at least one positive solution. In what interval is that solution?

■ 3. Use the Intermediate Value Theorem to prove that the equation $x^6 - 9x^4 + 7 = x^5 - 8x^3 - 9$ has at least one positive solution. In what interval is that solution?

■ 4. Use the Intermediate Value Theorem to prove that the equation $4e^{x-3} = 2(x^3 - 5x + 9)$ has at least one negative solution. In what interval is that solution?

■ 5. Use the Intermediate Value Theorem to show that the equation has at least one positive solution. In what interval is that solution?

$$6e^{-x} = -\left(\frac{1}{5}x^2 - 4x + 9\right)$$

■ 6. Use the Intermediate Value Theorem to show that the equation $2\sin(4x - 1) = \cos(2x - 3)$ has at least one negative solution. In what interval is that solution?

SOLVING WITH SUBSTITUTION

■ 1. What is the value of the limit?

$$\lim_{x \to 3} -x^4 + x^3 + 2x^2$$

■ 2. What is the value of the limit?

$$\lim_{x \to 7} \frac{x^2 - 5}{x^2 + 5}$$

■ 3. What is the value of the limit.

$$\lim_{x \to -2} \frac{x^3 - 5x^2 + 4x - 6}{x^2 + 7x + 6}$$

SOLVING WITH FACTORING

■ 1. What is the value of the limit?

$$\lim_{x\to -7} \frac{6x^3 + 42x^2}{2x^2 + 26x + 84}$$

■ 2. What is the value of the limit?

$$\lim_{x\to 10} \frac{3x^2 - 39x + 90}{x^2 - 3x - 70}$$

■ 3. What is the value of the limit?

$$\lim_{x\to -8} \frac{2x^2 + 10x - 48}{8x + 64}$$

■ 4. What is the value of the limit?

$$\lim_{x\to 7} \frac{x^3 - x^2 - 42x}{2x^2 - 20x + 42}$$

■ 5. What is the value of the limit?

$$\lim_{x \to 8} \frac{x^2 + 2x - 80}{2x^3 - 24x^2 + 64x}$$

SOLVING WITH CONJUGATE METHOD

■ 1. What is the value of the limit?

$$\lim_{x \to 16} \frac{3(x - 16)}{\sqrt{x} - 4}$$

■ 2. What is the value of the limit?

$$\lim_{x \to 9} \frac{5(\sqrt{x} - 3)}{x - 9}$$

■ 3. What is the value of the limit?

$$\lim_{x \to 25} \frac{2(x - 25)}{\sqrt{x} - 5}$$

■ 4. What is the value of the limit?

$$\lim_{x \to 49} \frac{x - 49}{3(\sqrt{x} - 7)}$$

■ 5. What is the value of the limit?

$$\lim_{x \to 1} \frac{8(x-1)}{3(\sqrt{x}-1)}$$

INFINITE LIMITS AND VERTICAL ASYMPTOTES

■ 1. What is the value of the limit?

$$\lim_{x \to 2} \frac{x^2 - x - 6}{-3x^2 - 3x + 18}$$

■ 2. What is the value of the limit?

$$\lim_{x \to -1} \frac{x^2 + x - 6}{4x^2 + 16x + 12}$$

■ 3. What is the value of the limit?

$$\lim_{x \to -4} \frac{x + 5}{-4x - 16}$$

■ 4. What is the value of the limit?

$$\lim_{x \to -1} \frac{x^2 - 9}{3x^2 - 6x - 9}$$

■ 5. What is the value of the limit?

$$\lim_{x \to 3} \frac{x^2 - 4x}{x^2 - 2x - 3}$$

6. What is the value of the limit?

$$\lim_{x \to -2} \frac{x^2 - 16}{-x^2 + x + 6}$$

LIMITS AT INFINITY AND HORIZONTAL ASYMPTOTES

■ 1. What is the value of the limit?

$$\lim_{x\to\infty} \frac{3x^3-5x+2}{9x^3+7x^2-x}$$

■ 2. What is the value of the limit?

$$\lim_{x\to-\infty} \frac{4x^2+3x+5}{-2x^2+x-9}$$

■ 3. What is the value of the limit?

$$\lim_{x\to\infty} \frac{x^3+6x^2-4x+1}{x^3+9x+8}$$

■ 4. What is the value of the limit?

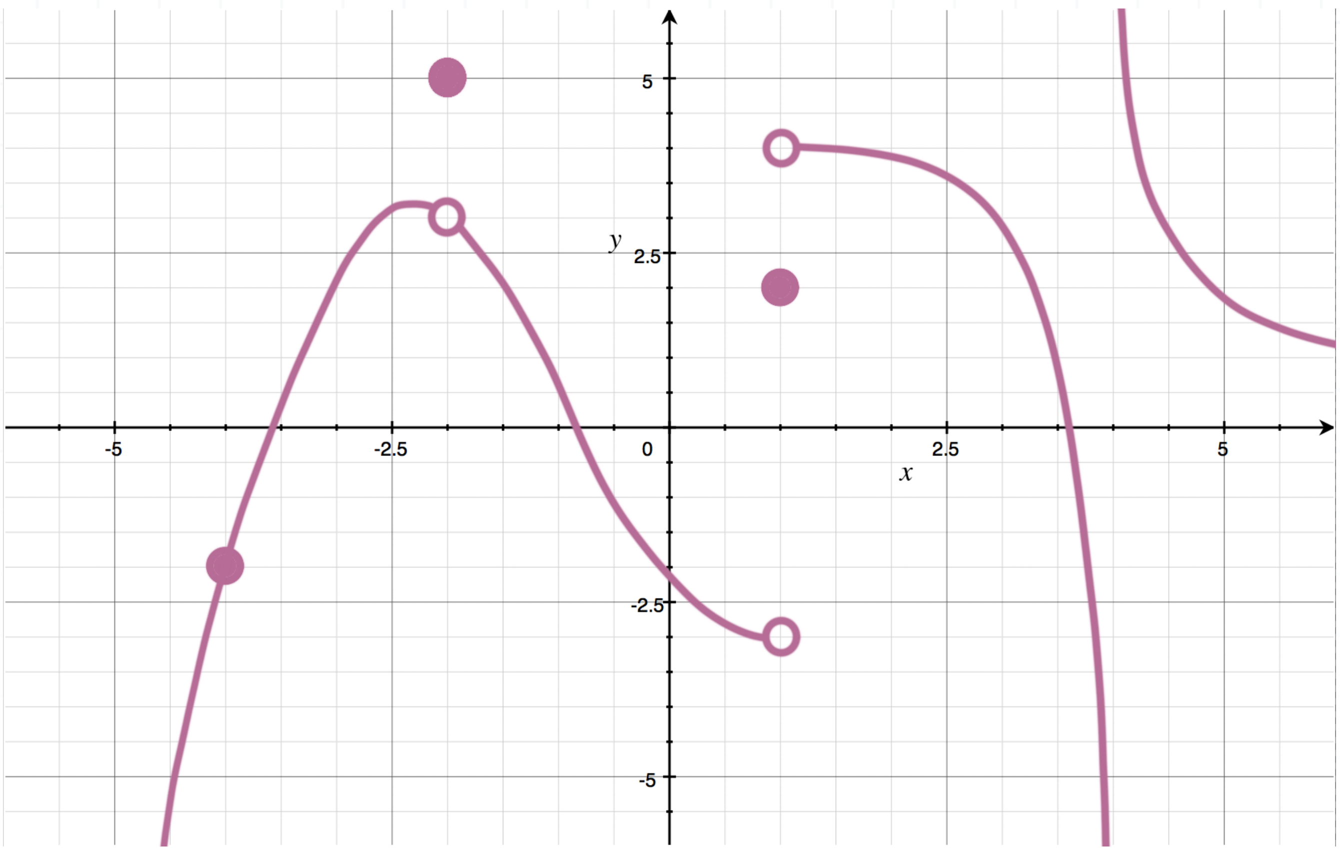
$$\lim_{x\to\infty} \frac{3x^2+5x+8}{x^3-5x-9}$$

■ 5. What is the value of the limit?

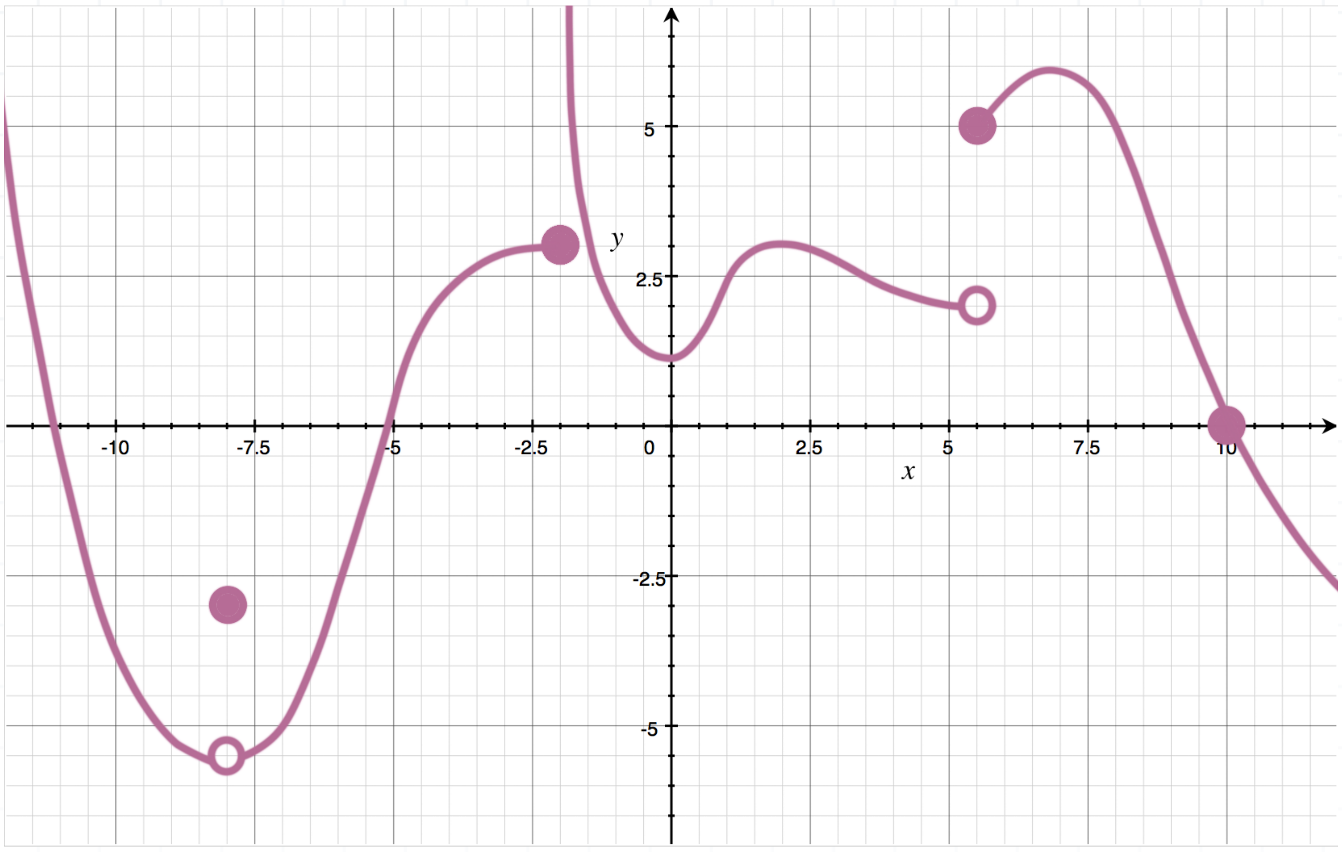
$$\lim_{x \to -\infty} \frac{19x + 21}{x^3 + 15x + 11}$$

CRAZY GRAPHS

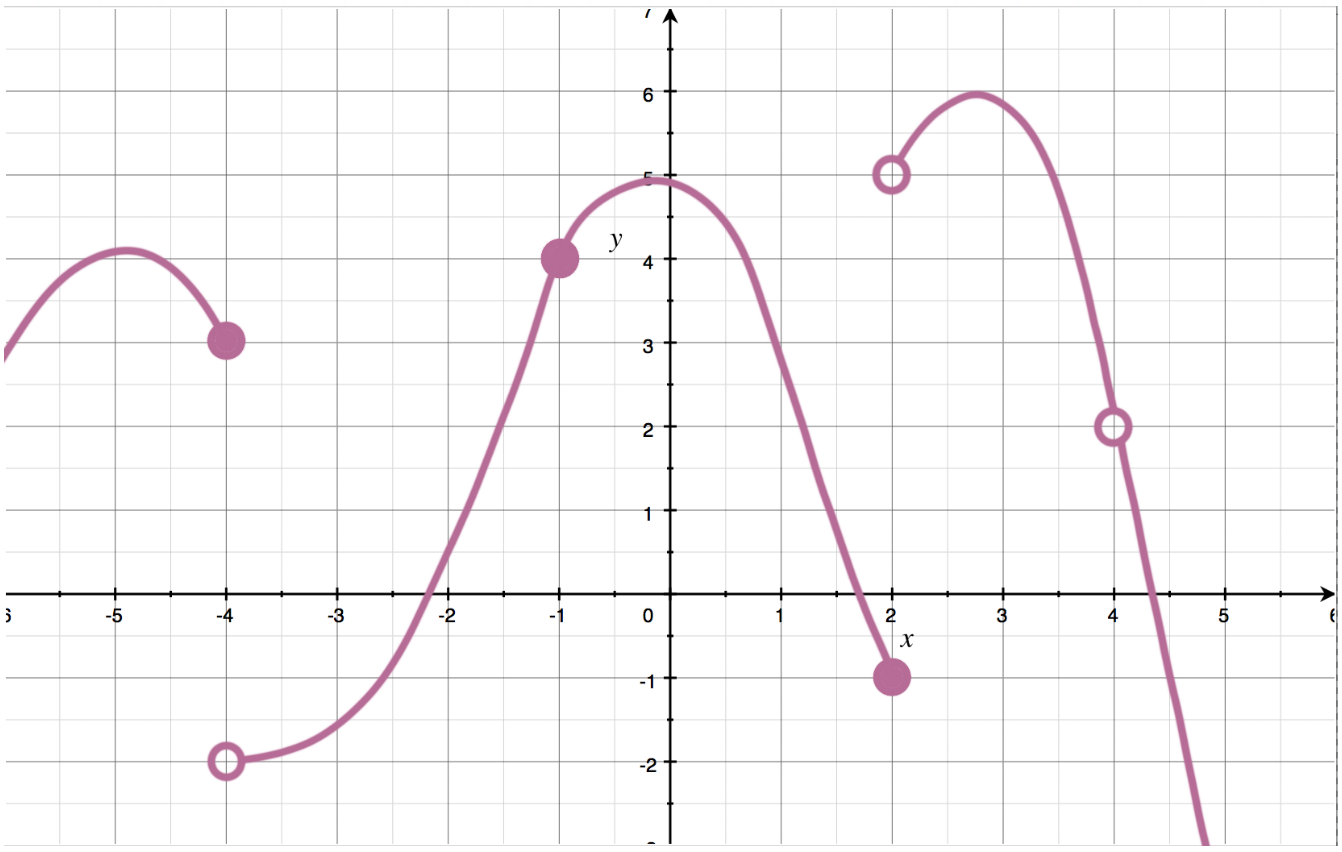
■ 1. Use the graph to find the value of $\lim_{x \to 1} f(x)$.



■ 2. Use the graph to find the value of $\lim_{x \to 5.5} g(x)$.



■ 3. Use the graph to find the value of $\lim_{x \to 4} h(x)$.



TRIGONOMETRIC LIMITS

■ 1. Find $\lim_{x\to\pi} f(x)$ if $f(x) = 3\cos x - 2$.

■ 2. Find $\lim_{x\to\frac{3\pi}{2}} g(x)$ if $g(x) = 4\sin x + 1$.

■ 3. Find $\lim_{x\to-\frac{3\pi}{2}} h(x)$ if $h(x) = \tan\left(\frac{x}{6}\right)$.

MAKING THE FUNCTION CONTINUOUS

■ 1. What value of c makes the function $h(x)$ continuous if c is a constant?

$$h(x) = \begin{cases} x^2 & x \leq 4 \\ 3x + c & x > 4 \end{cases}$$

■ 2. What value of c makes the function $f(x)$ continuous if c is a constant?

$$f(x) = \begin{cases} 5x - c & x \leq 3 \\ 3x + 4 & x > 3 \end{cases}$$

■ 3. What value of c makes the function $g(x)$ continuous if c is a constant?

$$g(x) = \begin{cases} x^2 - 4x + 8 & x \leq 2 \\ cx - 2 & x > 2 \end{cases}$$

■ 4. What value of c makes the function $f(x)$ continuous if c is a constant?

$$f(x) = \begin{cases} 2x^3 - 6x^2 + 8x + 3 & x \leq 1 \\ cx + 9 & x > 1 \end{cases}$$

■ 5. What value of c makes the function $g(x)$ continuous if c is a constant?

$$g(x) = \begin{cases} \sqrt{x} + 18 & x \leq 16 \\ x - 2c & x > 16 \end{cases}$$

6. What value of c makes the function $h(x)$ continuous if c is a constant?

$$h(x) = \begin{cases} 11x - 9 & x \leq 3 \\ x^2 + 3x + c & x > 3 \end{cases}$$

SQUEEZE THEOREM

■ 1. Use the Squeeze Theorem to evaluate the limit.

$$\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) - 2$$

■ 2. Use the Squeeze Theorem to evaluate the limit.

$$\lim_{x\to\infty} \frac{3\sin x}{4x}$$

■ 3. Use the Squeeze Theorem to evaluate the limit.

$$\lim_{x\to 0} x^2 \cos\left(\frac{1}{x^2}\right) + 1$$

■ 4. Use the Squeeze Theorem to evaluate the limit.

$$\lim_{x\to\infty} \frac{e^{-x}}{x}$$

■ 5. Use the Squeeze Theorem to evaluate the limit.

$$\lim_{x \to 0} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}}$$

■ 6. Find $\lim_{x \to 4} f(x)$ if $x^2 + 1 \leq f(x) \leq 4x + 1$.

■ 7. Find $\lim_{x \to 3} g(x)$ if $x^2 - 7 \leq g(x) \leq \sqrt{13 - x^2}$.

■ 8. Find $\lim_{x \to 5} h(x)$ if $x^2 - 6x + 9 \leq h(x) \leq x - 1$.

DEFINITION OF THE DERIVATIVE

■ 1. Use the definition of the derivative to find the derivative of $f(x) = 2x^2 + 2x - 12$ at $(4,28)$.

■ 2. Use the definition of the derivative to find the derivative of $g(x) = 3x^3 - 4x + 7$ at $(-2, -9)$.

■ 3. Use the definition of the derivative to find the derivative of $h(x) = 9x^2 - 7x - 4$ at $(2,18)$.

■ 4. Use the definition of the derivative to find the derivative of $h(x) = 8x^2 - 19x + 15$ at $(2,9)$.

POWER RULE

■ 1. Find the derivative of $f(x) = 7x^3 - 17x^2 + 51x - 25$ using the power rule.

■ 2. Find the derivative of $g(x) = 2x^4 + 8x^3 + 6x^2 - 32x + 16$ using the power rule.

■ 3. Find the derivative of $h(x) = 22x^3 - 19x^2 + 13x - 17$ using the power rule.

POWER RULE FOR NEGATIVE POWERS

1. Find the derivative of the function using the power rule.

$$f(x) = \frac{7}{x^2} - \frac{5}{x^4} + \frac{2}{x}$$

2. Find the derivative of the function using the power rule.

$$g(x) = \frac{1}{9x^4} + \frac{2}{3x^5} - \frac{1}{x}$$

3. Find the derivative of the function using the power rule.

$$h(x) = -\frac{7}{6x^6} - \frac{1}{4x^4} + \frac{9}{2x^2}$$

POWER RULE FOR FRACTIONAL POWERS

■ 1. Find the derivative of the function using the power rule.

$$f(x) = 4x^{\frac{3}{2}} - 6x^{\frac{5}{3}}$$

■ 2. Find the derivative of the function using the power rule.

$$g(x) = 6x^{\sqrt{3}} - 4x^{\sqrt{5}}$$

■ 3. Find the derivative of the function using the power rule.

$$h(x) = \frac{1}{3}x^{\frac{6}{5}} + \frac{1}{4}x^{\frac{8}{3}} - \frac{1}{5}x^{\frac{5}{2}}$$

PRODUCT RULE, TWO FUNCTIONS

■ 1. Use the product rule to find the derivative of the function.

$$h(x) = (3x + 5)\ln(5x)$$

■ 2. Use the product rule to find the derivative of the function.

$$h(x) = 8x^3e^{7x}$$

■ 3. Use the product rule to find the derivative of the function.

$$h(x) = (5x^2 - x)(e^{4x} - 6)$$

PRODUCT RULE, THREE OR MORE FUNCTIONS

■ 1. Use the product rule to find the derivative of the function.

$$y = 5x^4e^{3x}\cos(6x)$$

■ 2. Use the product rule to find the derivative of the function.

$$y = (-6x^2)(-2e^{5x})\tan(5x)$$

■ 3. Use the product rule to find the derivative of the function.

$$y = (\sin(7x))(7e^{4x})(2x^6 + 1)$$

■ 4. Use the product rule to find the derivative of the function.

$$y = (\cos(3x))(\sin(2x))(\tan(5x))(e^{2x})$$

QUOTIENT RULE

■ 1. Use the quotient rule to find the derivative of the function.

$$h(x) = \frac{2x + 6}{7x + 5}$$

■ 2. Use the quotient rule to find the derivative of the function.

$$h(x) = \frac{5x - 3}{4x - 9}$$

■ 3. Use the quotient rule to find the derivative of the function.

$$h(x) = \frac{-8x}{5x + 2}$$

■ 4. Use the quotient rule to find the derivative of the function.

$$h(x) = \frac{3x^2 + 12x}{e^x}$$

CHAIN RULE WITH POWER RULE

■ 1. Find $h'(x)$ if $h(x) = (3x^2 - 7)^4$.

■ 2. Find $h'(x)$ if $h(x) = 2(5x^2 + 2x)^3$.

■ 3. Find $h'(x)$ if $h(x) = (2x^2 - 6x + 5)^7$.

■ 4. Find $h'(x)$ if $h(x) = 2(x^3 + 4x^2 - 2x)^5$.

CHAIN RULE WITH PRODUCT RULE

■ 1. Find $y'(x)$ if $y(x) = (3x - 2)(5x^3)^5$.

■ 2. Find $h'(x)$ if $h(x) = (x^2 - 5x)^2(2x^3 - 3x^2)^5$.

■ 3. Find $h'(x)$ if $h(x) = (x + 4)^5(3x - 2)^3$.

CHAIN RULE WITH QUOTIENT RULE

■ 1. Find $h'(x)$.

$$h(x) = \frac{(2x+1)^3}{(3x-2)^2}$$

■ 2. Find $h'(x)$.

$$h(x) = \frac{(4x+5)^5}{(x+3)^2}$$

■ 3. Find $h'(x)$.

$$h(x) = \frac{(7x-4)^3}{(5x+3)^2}$$

■ 4. Find $h'(x)$.

$$h(x) = \frac{(6x-1)^4}{(8x+1)^2}$$

TRIGONOMETRIC DERIVATIVES

■ 1. Find $f'(x)$ if $f(x) = 5x^7 + 8\sin(7x^7)$.

■ 2. Find $g'(x)$ if $g(x) = 3\sin(4x^3) - 4\cos(6x) + 3\sec(2x^4)$.

■ 3. Find $h'(x)$ if $h(x) = 5\tan(4x^6) + 6\cot(6x^4)$.

INVERSE TRIGONOMETRIC DERIVATIVES

■ 1. Find $f'(t)$.

$$f(t)=4\sin^{-1}\left(\frac{t}{4}\right)$$

■ 2. Find $g'(t)$.

$$g(t)=-6\cos^{-1}(2t+3)$$

■ 3. Find $h'(t)$.

$$h(t)=3\tan^{-1}(6t^2)$$

HYPERBOLIC DERIVATIVES

■ 1. Find $f'(\theta)$ if $f(\theta) = 3\sinh(2\theta^2 - 5\theta + 2)$.

■ 2. Find $g'(\theta)$ if $g(\theta) = 2\cosh(5\theta^{\frac{3}{2}} + 6\theta)$.

■ 3. Find $h'(\theta)$ if $h(\theta) = 9\tanh\left(3\theta^2 - \theta^{\sqrt{3}}\right)$.

INVERSE HYPERBOLIC DERIVATIVES

■ 1. Find $f'(t)$ if $f(t) = 7\sinh^{-1}(5t^4)$.

■ 2. Find $g'(t)$ if $g(t) = 4\cosh^{-1}(2t - 3)$.

■ 3. Find $h'(t)$ if $h(t) = 9\tanh^{-1}(-7t + 2)$.

EXPONENTIAL DERIVATIVES

■ 1. Find $f'(x)$ if $f(x) = (x^3 - x)e^{2x}$.

■ 2. Find $g'(x)$ if $g(x) = 5x^2e^{2x^2} - 7x + 1$.

■ 3. Find $h'(x)$ if $h(x) = \sin(4x)e^{3x^2+4}$.

LOGARITHMIC DERIVATIVES

■ 1. Find $f'(x)$.

$$f(x) = \ln(x^2 + 6x + 9)$$

■ 2. Find $g'(x)$.

$$g(x) = \ln\sqrt{x^3 + x}$$

■ 3. Find $h'(x)$.

$$h(x) = \ln\left(\frac{x^3}{x^2 + 3}\right)$$

LOGARITHMIC DIFFERENTIATION

■ 1. Use logarithmic differentiation to find dy/dx.

$$y = x^4 e^x \sqrt{x}$$

■ 2. Use logarithmic differentiation to find dy/dx.

$$y = 5x^4 e^{3x} \sqrt[4]{x}$$

■ 3. Use logarithmic differentiation to find dy/dx.

$$y = x^3 e^{2x} \sqrt{5x}$$

■ 4. Use logarithmic differentiation to find dy/dx.

$$y = \frac{(2e)^{\cos x}}{(3e)^{\sin x}}$$

■ 5. Use logarithmic differentiation to find dy/dx.

$$y = e^x (2e)^{\sin x} (3e)^{\cos x}$$

TANGENT LINES

■ 1. Find the equation of the tangent line to the graph of the equation at $(1/2,\pi)$.

$$f(x) = 4 \arctan 2x$$

■ 2. Find the equation of the tangent line to the graph of the equation at $(-1,-9)$.

$$g(x) = x^3 - 2x^2 + x - 5$$

■ 3. Find the equation of the tangent line to the graph of the equation at $(0,-4)$.

$$h(x) = -4e^{-x} + 3x$$

■ 4. Find the equation of the tangent line to the graph of the equation at $(1,1)$.

$$f(x) = -6x^4 + 4x^3 - 3x^2 + 5x + 1$$

VALUE THAT MAKES TWO TANGENT LINES PARALLEL

■ 1. Find the value of a such that the tangent lines to $f(x) = 2x^3 + 2$ at $x = a$ and $x = a + 1$ are parallel.

■ 2. Find the value of a such that the tangent lines to $g(x) = x^3 + x^2 + 7$ at $x = a$ and $x = a + 1$ are parallel.

■ 3. Find the value of a such that the tangent lines to $h(x) = \tan^{-1} x$ at $x = a$ and $x = a + 1$ are parallel.

■ 4. Find the value of a such that the tangent lines to $f(x) = 4x^3 - 6x + 7$ at $x = a$ and $x = a + 1$ are parallel.

■ 5. Find the value of a such that the tangent lines to $g(x) = (x - 2)^3 + x^2 + 3$ at $x = a$ and $x = a + 1$ are parallel.

■ 6. Find the approximate value of a, rounded to the nearest hundredth, such that the tangent lines to $h(x) = e^x - 3x^2$ at $x = a$ and $x = a + 1$ are parallel.

VALUES THAT MAKE THE FUNCTION DIFFERENTIABLE

■ 1. What value of a and b will make the function differentiable?

$$f(x) = \begin{cases} x^2 & x \le 3 \\ ax - b & x > 3 \end{cases}$$

■ 2. What value of a and b will make the function differentiable?

$$g(x) = \begin{cases} ax + b & x \le -1 \\ bx^2 - 1 & x > -1 \end{cases}$$

■ 3. What value of a and b will make the function differentiable?

$$h(x) = \begin{cases} ax^3 & x \le 2 \\ x^2 - b & x > 2 \end{cases}$$

■ 4. What value of a and b will make the function differentiable?

$$f(x) = \begin{cases} 3 - x & x \le 1 \\ ax^2 - bx & x > 1 \end{cases}$$

■ 5. What value of a and b will make the function differentiable?

$$g(x) = \begin{cases} x^3 & x \le 1 \\ a(x-2)^2 - b & x > 1 \end{cases}$$

■ 6. What value of a and b will make the function differentiable?

$$h(x) = \begin{cases} ax^2 + b & x \le 3 \\ bx + 4 & x > 3 \end{cases}$$

NORMAL LINES

■ 1. Find the equation of the normal line to the graph of $f(x) = 5x^4 + 3e^x$ at $(0,3)$.

■ 2. Find the equation of the normal line to the graph of $g(x) = \ln e^{4x} + 2x^3$ at $(2,24)$.

■ 3. Find the equation of the normal line to the graph of $h(x) = 5\cos x + 5\sin x$ at $(\pi/2,5)$.

■ 4. Find the equation of the normal line to the graph of $f(x) = 7x^3 + 2x^2 - 5x + 9$ at $(2,63)$.

■ 5. Find the equation of the normal line to the graph of $g(x) = 5\sqrt{x^2 - 14x + 49}$ at $(2,25)$.

AVERAGE RATE OF CHANGE

■ 1. Find the average rate of change of the function over the interval $[4,9]$.

$$f(x) = \frac{5\sqrt{x} - 2}{3}$$

■ 2. Find the average rate of change of the function over the interval $[16,25]$.

$$g(x) = \frac{2x - 8}{\sqrt{x} - 2}$$

■ 3. Find the average rate of change of the function over the interval $[0,4]$.

$$h(x) = \frac{x^3 - 8}{x^2 - 4x - 5}$$

IMPLICIT DIFFERENTIATION

1. Use implicit differentiation to find dy/dx at $(3,4)$ for the equation.

$$4x^3 - 3xy^2 + y^3 = 28$$

2. Use implicit differentiation to find dy/dx for the equation.

$$5x^3 + xy^2 = 4x^3y^3$$

3. Use implicit differentiation to find dy/dx for the equation.

$$3x^2 = (3xy - 1)^2$$

EQUATION OF THE TANGENT LINE WITH IMPLICIT DIFFERENTIATION

■ 1. Use implicit differentiation to find the equation of the tangent line to $5y^2 = 2x^3 - 5y + 6$ at $(3,3)$.

■ 2. Use implicit differentiation to find the equation of the tangent line to $5x^3 = -3xy + 4$ at $(2, -6)$.

■ 3. Use implicit differentiation to find the equation of the tangent line to $4y^2 + 8 = 3x^2$ at $(6, -5)$.

SECOND DERIVATIVES WITH IMPLICIT DIFFERENTIATION

■ 1. Use implicit differentiation to find d^2y/dx^2.

$$2x^3 = 2y^2 + 4$$

■ 2. Use implicit differentiation to find d^2y/dx^2.

$$4x^2 = 2y^3 + 4y - 2$$

■ 3. Use implicit differentiation to find d^2y/dx^2 at $(0,3)$.

$$3x^2 + 3y^2 = 27$$

CRITICAL POINTS AND THE FIRST DERIVATIVE TEST

■ 1. Identify the critical point(s) of the function on the interval $[-3,2]$.

$$f(x) = x^{\frac{2}{3}}(x+2)$$

■ 2. Identify the critical point(s) of the function on the interval $[-2,2]$.

$$g(x) = x\sqrt{4-x^2}$$

■ 3. Determine the intervals where the function is increasing and decreasing.

$$f(x) = \frac{5}{4}x^4 - 10x^2$$

■ 4. Determine the intervals where the function is increasing and decreasing.

$$g(x) = -x^3 + 2x^2 + 2$$

■ 5. Use the first derivative test to find the extrema of $f(x) = 4x^3 + 21x^2 + 36x - 5$.

■ 6. Use the first derivative test to find the extrema of $g(x) = 2x^3 - 14x^2 + 22x + 3$.

INFLECTION POINTS AND THE SECOND DERIVATIVE TEST

■ 1. For $f(x) = x^3 - 3x^2 + 5$, find inflection points and identify where the function is concave up and concave down.

■ 2. For $g(x) = -x^3 + 2x^2 + 3$, find inflection points and identify where the function is concave up and concave down.

■ 3. For $h(x) = x^4 + x^3 - 3x^2 + 2$, find inflection points and identify where the function is concave up and concave down.

■ 4. Use the second derivative test to identify the extrema of $f(x) = x^3 - 12x - 2$ as maximum values or minimum values.

■ 5. Use the second derivative test to identify the extrema of $g(x) = -4x^3 + 12x^2 + 5$ as maximum values or minimum values.

■ 6. Use the second derivative test to identify the extrema of $h(x) = 2x^4 - 4x^2 + 1$ as maximum values or minimum values.

INTERCEPTS AND VERTICAL ASYMPTOTES

■ 1. Find any vertical asymptote(s) of the function.

$$f(x) = \frac{-x^2 + 16x - 63}{x^2 - 2x - 35}$$

■ 2. Find any vertical asymptote(s) of the function.

$$g(x) = \frac{x^2 - 3x - 10}{x^2 + x - 2}$$

■ 3. Find any vertical asymptote(s) of the function.

$$h(x) = \frac{40 - 27x - 12x^2 - x^3}{9x^2 + 63x - 72}$$

HORIZONTAL AND SLANT ASYMPTOTES

■ 1. Find the horizontal asymptote(s) of the function.

$$f(x) = \frac{8x^4 - x^2 + 1}{4x^4 - 1}$$

■ 2. Find the horizontal asymptote(s) of the function.

$$g(x) = \frac{2x^2 - 5x + 12}{3x^2 - 11x - 4}$$

■ 3. Find the horizontal asymptote(s) of the function.

$$h(x) = \frac{x^3 - x^2 + 6x - 1}{7x^4 - 1}$$

■ 4. Find the slant asymptote of the function.

$$f(x) = \frac{3x^4 - x^3 + x^2 - 4}{x^3 - x^2 + 1}$$

■ 5. Find the slant asymptote of the function.

$$g(x) = \frac{8x^2 + 14x - 7}{4x - 1}$$

6. Find the slant asymptote of the function.

$$h(x) = \frac{x^3 - 8}{x^2 - 5x + 6}$$

SKETCHING GRAPHS

■ 1. Sketch the graph of the function.

$$f(x) = x^3 - 4x^2 + 8$$

■ 2. Sketch the graph of the function.

$$g(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2 + 1$$

■ 3. Sketch the graph of the function.

$$h(x) = \frac{x^2 + x - 6}{4x^2 + 16x + 12}$$

EXTREMA ON A CLOSED INTERVAL

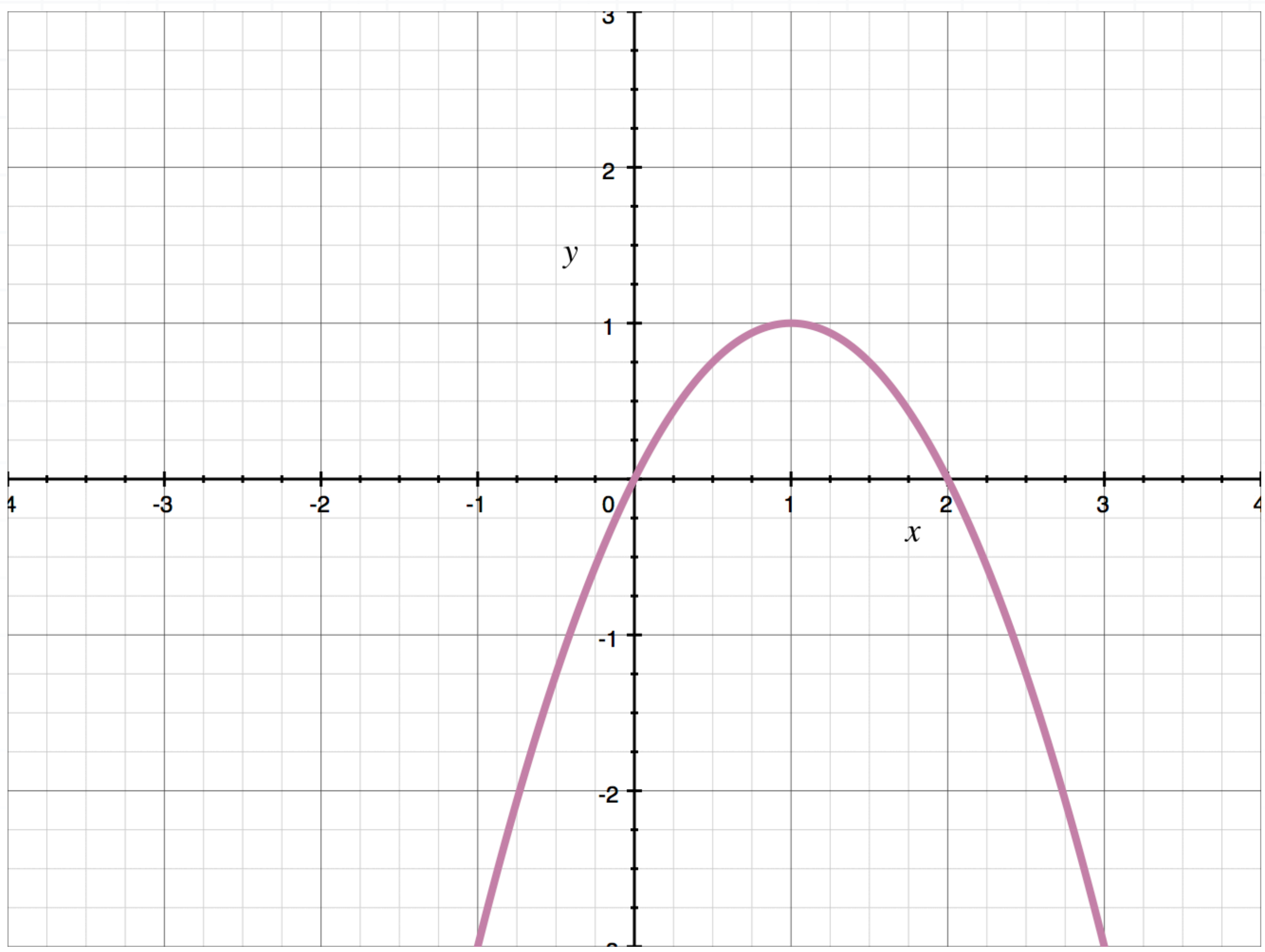
■ 1. Find the extrema of $f(x) = x^3 - 3x^2 + 5$ over the closed interval $[-3,4]$.

■ 2. Find the extrema of $g(x) = \sqrt[3]{2x^2 + 3}$ over the closed interval $[-1,5]$.

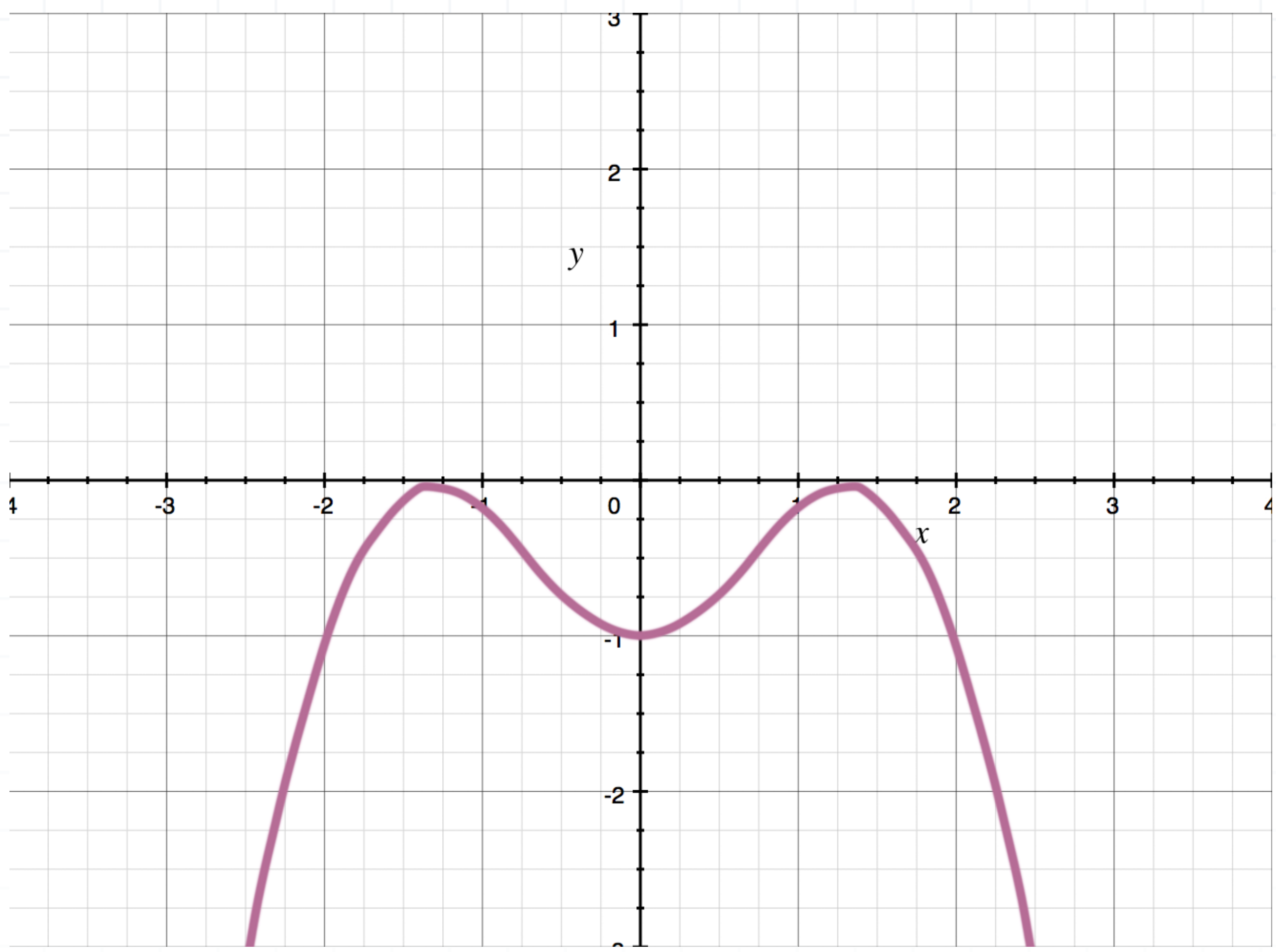
■ 3. Find the extrema of $h(x) = -4x^3 + 6x^2 - 3x - 2$ over the closed interval $[-4,6]$.

SKETCHING F(X) FROM F'(X)

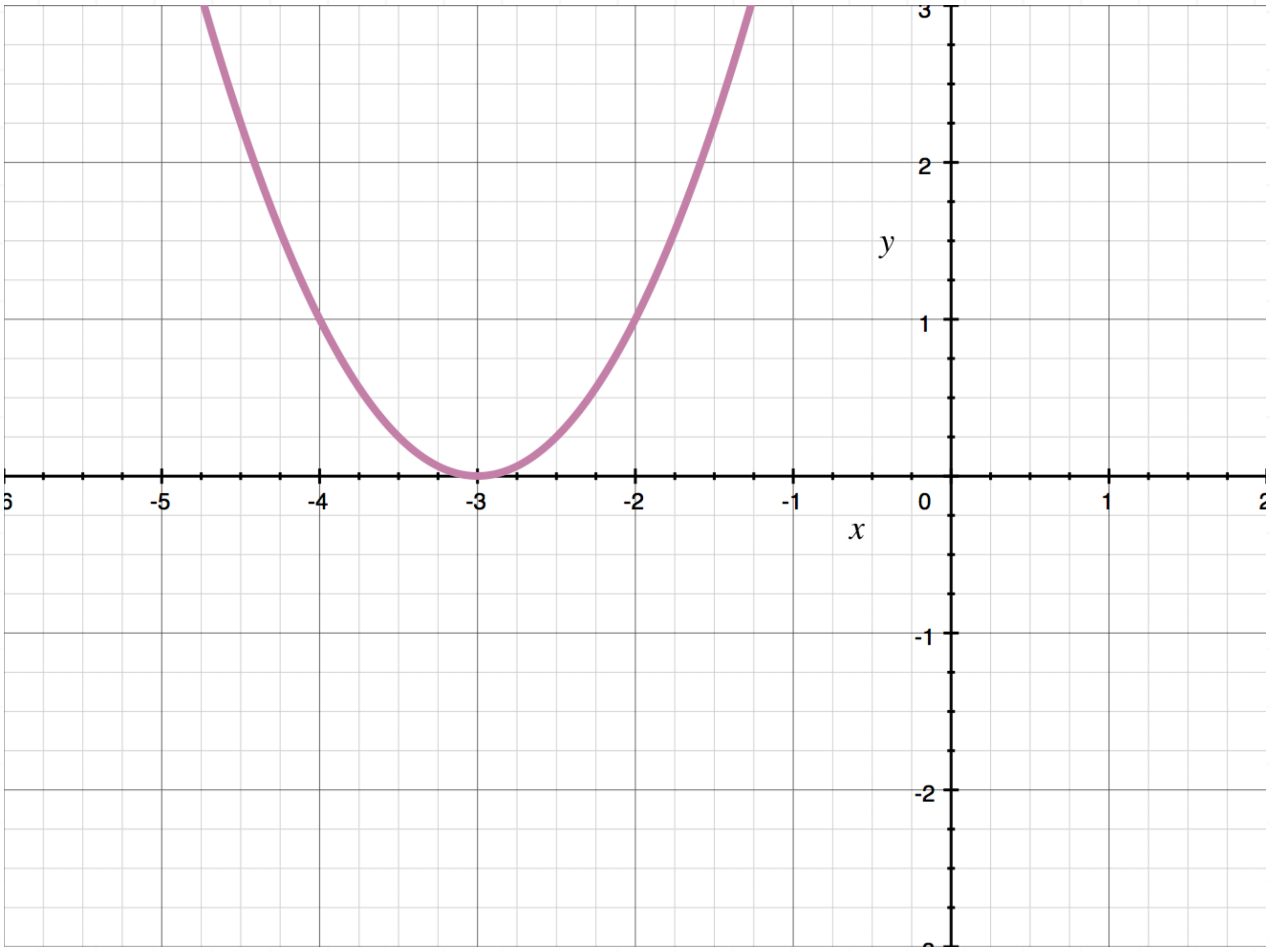
■ 1. Sketch a possible graph of $f(x)$ given the graph below of $f'(x)$.



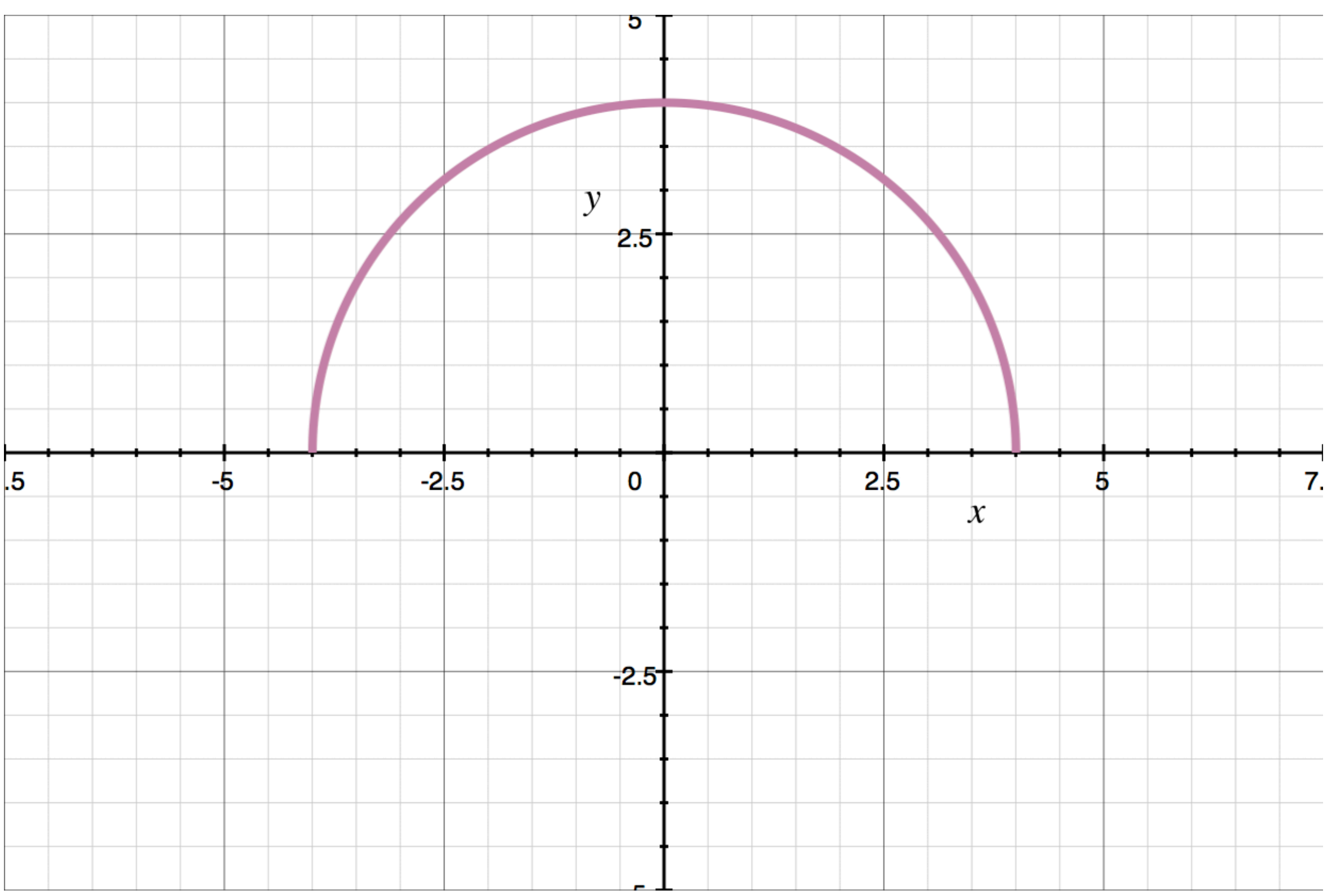
■ 2. Sketch a possible graph of $g'(x)$ given the graph below of $g(x)$.



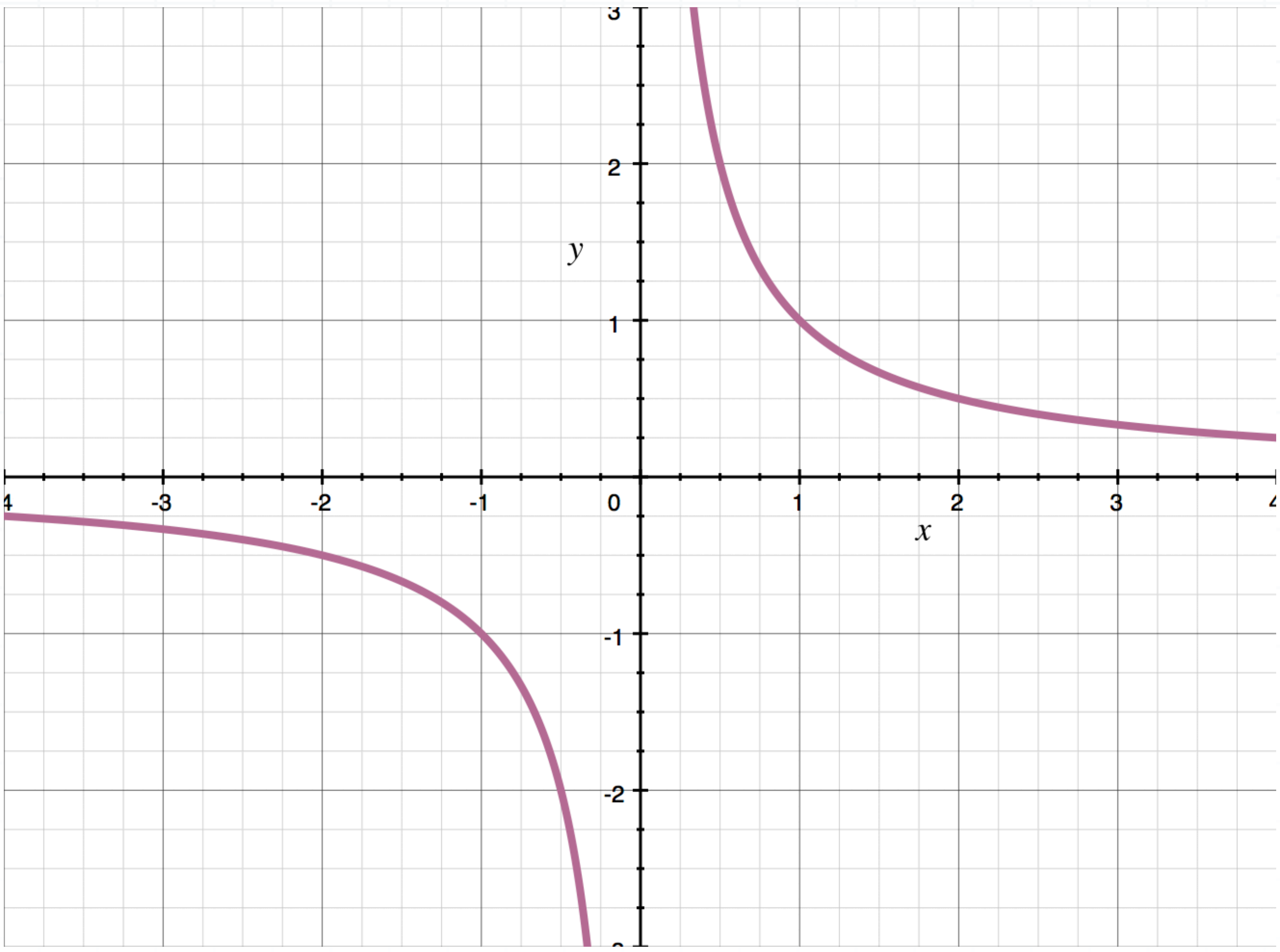
■ 3. Sketch a possible graph of $h(x)$ given the graph below of $h'(x)$.



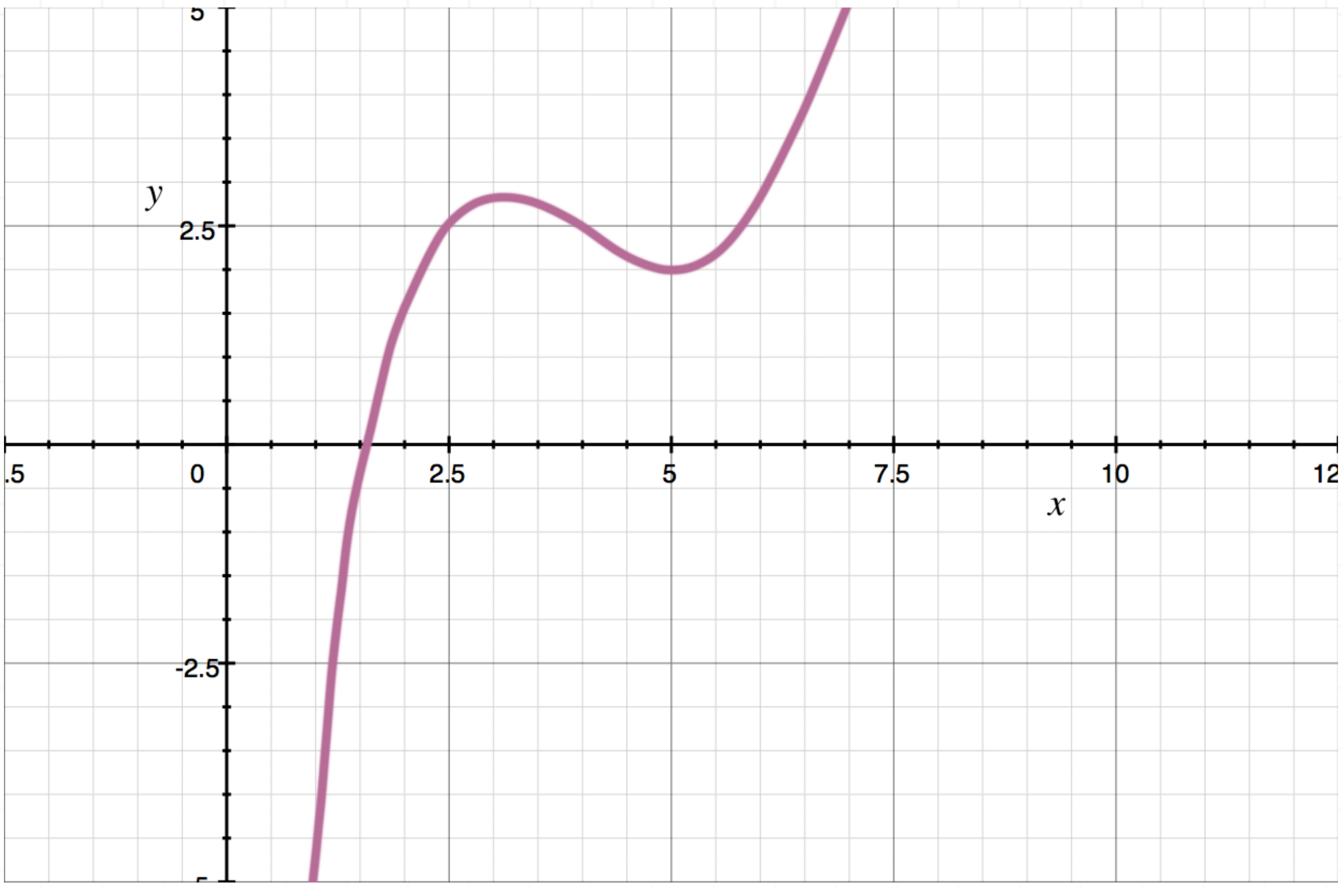
■ 4. Sketch a possible graph of $f'(x)$ given the graph below of $f(x)$.



■ 5. Sketch a possible graph of $f(x)$ given the graph below of $f'(x)$.



■ 6. Sketch a possible graph of $g'(x)$ given the graph below of $g(x)$.


y
2.5
-2.5
0
2.5
5
7.5
10
12
x

LINEAR APPROXIMATION

■ 1. Find the linearization of $f(x) = x^3 - 4x^2 + 2x - 3$ at $x = 3$ and use the linearization to approximate $f(3.02)$.

■ 2. Find the linearization of $g(x) = \sqrt{8x - 15}$ at $x = 8$ and use the linearization to approximate $f(8.05)$.

■ 3. Find the linearization of $h(x) = 2e^{x-4} + 6$ at $x = 5$ and use the linearization to approximate $h(5.1)$.

ESTIMATING A ROOT

■ 1. Use linear approximation to estimate $\sqrt[5]{34}$.

■ 2. Use linear approximation to estimate $\sqrt[8]{260}$.

■ 3. Use linear approximation to estimate $\sqrt[4]{85}$.

■ 4. Use linear approximation to estimate $\sqrt[4]{615}$.

■ 5. Use linear approximation to estimate $\sqrt{95}$.

■ 6. Use linear approximation to estimate $\sqrt[3]{700}$.

ABSOLUTE, RELATIVE, AND PERCENTAGE ERROR

■ 1. What is the absolute change of $f(x)$ from $x = \pi$ to $x = 2\pi$?

$$f(x) = 3x^2 - \cos\left(\frac{x}{2}\right)$$

■ 2. What is the relative change of $g(x)$ from $x = 2$ to $x = 3$?

$$g(x) = 2x^4 - 3x^2 - 5$$

■ 3. What is the relative change of $h(x)$ from $x = 0$ to $x = \pi$?

$$h(x) = \tan x + 4x + 2$$

RELATED RATES

■ 1. A truck is 40 miles north of an intersection, traveling toward the intersection at 35 mph. At the same time, another car is 30 miles west of the intersection, traveling away from the intersection at 45 mph. Is the distance between the vehicles increasing or decreasing at that moment? At what rate?

■ 2. Water is flowing out of a cone-shaped tank at a rate of 6 cubic inches per second. If the cone has a height of 5 inches and a base radius of 4 inches, how fast is the water level falling when the water is 3 inches deep?

■ 3. A ladder 25 feet long leans against a vertical wall of a building. If the bottom of the ladder is pulled away horizontally from the building at 3 feet per second, how fast is the angle formed by the ladder and the horizontal ground decreasing when the bottom of the ladder is 7 feet from the base of the wall?

APPLIED OPTIMIZATION

1. A boater finds herself 2 miles from the nearest point to a straight shoreline, which is 10 miles down the shore from where she parked her car. She plans to row to shore and then walk to her car. If she can walk 4 miles per hour but only row 3 miles per hour, toward what point on the shore should she row in order to reach her car in in the least amount of time?

2. Mr. Quizna wants to build in a completely fenced-in rectangular garden. The fence will be built so that one side is adjacent to his neighbor's property. The neighbor agrees to pay for half of that part of the fence because it borders his property. The garden will contain 432 square meters. What dimensions should Mr. Quizna select for his garden in order to minimize his cost?

3. A company is designing shipping crates and wants the volume of each crate to be 6 cubic feet, and each crate's base to be a square between 1.5 feet and 2.0 feet per side. The material for the bottom of the crate costs $\$5$ per square foot, the sides $\$3$ per square foot, and the top $\$1$ per square foot. What dimensions will minimize the cost of the shipping crates?

MEAN VALUE THEOREM

■ 1. Find the value(s) of c that satisfy the Mean Value Theorem for the function in the interval $[1,5]$.

$$f(x) = x^3 - 9x^2 + 24x - 18$$

■ 2. Find the value(s) of c that satisfy the Mean Value Theorem for the function in the interval $[1,4]$.

$$g(x) = \frac{x^2 - 9}{3x}$$

■ 3. Find the value(s) of c that satisfy the Mean Value Theorem for the function in the interval $[0,5]$.

$$h(x) = -\sqrt{25 - 5x}$$

ROLLE'S THEOREM

■ 1. Use Rolle's Theorem to show that the function has a horizontal tangent line in the interval $[-1,2]$. Find the value(s) of c in the interval that satisfy Rolle's Theorem.

$$f(x) = x^3 - 2x^2 - x - 3$$

■ 2. Use Rolle's Theorem to show that the function has a horizontal tangent line in the interval $[-3,5]$. Find the value(s) of c in the interval that satisfy Rolle's Theorem.

$$g(x) = \frac{x^2 - 2x - 15}{6 - x}$$

■ 3. Use Rolle's Theorem to show that the function has a horizontal tangent line in the interval $[-\pi/2,\pi/2]$. Find the value(s) of c in the interval that satisfy Rolle's Theorem.

$$h(x) = \sin(2x)$$

NEWTON'S METHOD

■ 1. Use four iterations of Newton's method to approximate the root of $g(x) = x^3 - 12$ in the interval $[1,3]$. Give the answer to the nearest three decimal places.

■ 2. Use four iterations of Newton's method to approximate the root of $f(x) = x^4 - 15$ in the interval $[-2, -1]$. Give the answer to the nearest four decimal places.

■ 3. Use four iterations of Newton's method to approximate the root of $h(x) = 3e^{x-3} - 4 + \sin x$ in the interval $[2,4]$. Give the answer to the nearest four decimal places.

L'HOSPITAL'S RULE

1. Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x\to 0}\frac{2\sqrt{x+4}-4-\frac{1}{2}x}{x^2}$$

2. Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x\to \frac{\pi}{2}}\frac{\sec x}{3+\tan x}$$

3. Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x\to \infty}\frac{\ln x}{4\sqrt{x}}$$

POSITION, VELOCITY, AND ACCELERATION

■ 1. Find the velocity $v(t)$, speed, and acceleration $a(t)$ at $t = 2$ of the position function.

$$s(t) = -\frac{t^3}{3} + t^2 + 3t - 1$$

■ 2. Find the velocity $v(t)$, speed, and acceleration $a(t)$ at $t = 1$ of the position function.

$$s(t) = \frac{t^2 - 3}{t^3}$$

■ 3. Find the velocity $v(t)$, speed, and acceleration $a(t)$ at $t = 4$ of the position function.

$$s(t) = \frac{t^2}{2t + 4}$$

BALL THROWN UP FROM THE GROUND

■ 1. A ball is thrown straight upward from the ground with an initial velocity of $v_0 = 86$ ft/sec. Assuming constant gravity, find the maximum height, in feet, that the ball attains, the time, in seconds, that it's in the air, as well as the ball's velocity, in ft/sec, when it hits the ground.

■ 2. A ball is thrown straight upward from the top of a building, which is 56 feet above the ground, with an initial velocity of $v_0 = 48$ ft/sec. Assuming constant gravity, find the maximum height, in feet, that the ball attains, the time, in seconds, that it's in the air, as well as the ball's velocity, in ft/sec, when it hits the ground.

■ 3. A ball is thrown straight upward from a bridge, which is 24 meters above the water, with an initial velocity of $v_0 = 20$ m/sec. Assuming constant gravity, find the maximum height, in meters, that the ball attains, the time, in seconds, that it's in the air, as well as the ball's velocity, in m/sec, when it hits the water below.

COIN DROPPED FROM THE ROOF

■ 1. A rock is dropped from the top of an 800 foot tall cliff, with an initial velocity of $v_0 = 0$ ft/sec. Assuming constant gravity, when does the rock hit the ground, and what is its velocity when it hits the ground?

■ 2. A rock is tossed from the top of a 300 foot tall cliff, with an initial velocity of $v_0 = 15$ ft/sec. Assuming constant gravity, when does the rock hit the ground, and what is its velocity when it hits the ground?

■ 3. A coin is tossed downward from the top of a 36 meter tall building, with an initial velocity of $v_0 = 6$ m/sec. Assuming constant gravity, when does the rock hit the ground, and what is its velocity when it hits the ground?

MARGINAL COST, REVENUE, AND PROFIT

■ 1. A company manufactures and sells basketballs for $\$9.50$ each. The company has a fixed cost of $\$395$ per week and a variable cost of $\$2.75$ per basketball. The company can make up to 300 basketballs per week. Find the marginal cost, marginal revenue, and marginal profit, if the company makes 150 basketballs.

■ 2. A company manufactures and sells high end folding tables for $\$250$ each. The company has a fixed cost of $\$3,000$ per week and variable costs of $85x + 150\sqrt{x}$, where x is the number of tables manufactured. The company can make up to 200 tables per week. Find the marginal cost, marginal revenue, and marginal profit, if the company makes 64 tables.

■ 3. A company manufactures and sells electric food mixers for $\$150$ each. The company has a fixed cost of $\$7,800$ per week and variable costs of $24x + 0.04x^2$, where x is the number of mixers manufactured. The company can make up to 200 mixers per week. Find the marginal cost, marginal revenue, and marginal profit, if the company makes 75 mixers.

HALF LIFE

■ 1. Find the half-life of Tritium if its decay constant is 0.0562.

■ 2. Find the half-life of Cobalt-60 if its decay constant is 0.1315.

■ 3. Find the half-life of Berkelium-97 if its decay constant is 0.000503.

NEWTON'S LAW OF COOLING

■ 1. A cup of coffee is 195° F when it's brewed. Room temperature is 74° F. If the coffee is 180° F after 5 minutes, to the nearest degree, how hot is the coffee after 25 minutes?

■ 2. A boiled egg that's 99° C is placed in a pan of water that's 24° C. If the egg is 62° C after 5 minutes, how much longer, to the nearest minute, will it take the egg to reach 32° C.

■ 3. Suppose a cup of soup cooled from 200° F to 161° F in 10 minutes in a room whose temperature is 68° F. How much longer will it take for the soup to cool to 105° F?

SALES DECLINE

■ 1. Suppose a pizza company stops a special sale for their three-topping pizza. They will resume the sale if sales drop to 70% of the current sales level. If sales decline to 90% during the first week, when should the company expect to start the special sale again?

■ 2. Suppose a donut store experiments with raising the price of a dozen donuts to see if sales are affected. They'll resume the sale if sales drop to 80% of the current sales level. If sales decline to 90% after two weeks, when should the store change back to the original price?

■ 3. Suppose a flower shop decides to stop ordering roses in the winter time to see if sales are affected. They will resume the sale if sales drop to 90% of the current sales level. If sales decline to 96% after three weeks, when should the shop begin ordering roses again?

COMPOUNDING INTEREST

■ 1. Suppose you borrow $\$15{,}000$ with a single payment loan, payable in 2 years, with interest growing exponentially at 1.82% per month, compounded continuously. How much will it cost to pay off the loan after 2 years?

■ 2. Your parents deposit $\$5{,}000$ into a college savings account, with interest growing exponentially at 0.875% per quarter, compounded continuously. How much will be in the account after 18 years?

■ 3. Suppose you win $\$50{,}000$ in a contest and you decide to save it for your retirement. You deposit it into an annuity account that pays 2.4% semi-annually, compounded continuously. How much will the account contain after 25 years, when you plan to retire?

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