Chain rule with power rule

Up to now, we've essentially only been calculating derivatives of power functions. Even when we looked at product rule and quotient rule, we used functions that were the products of power functions, or the quotients of power functions.

In other words, we're comfortable differentiating things like $3x^2$, $-2x$, or $6x^{-3}$. But what do we do when we want to differentiate a function like $3(2x+1)^2$? This is like a power function, but having $2x+1$ inside the parentheses makes this derivative a little more complicated.

The chain rule

That's where the chain rule comes in. The **chain rule** lets us calculate derivatives of nested functions, where one function is the "outside" function and one function is the "inside function. If we want to differentiate a nested function like

$$y = g[f(x)]$$

then $g[f(x)]$ is the outside function and $f(x)$ is the inside function. The derivative is

$$y' = g'[f(x)]f'(x)$$

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Notice here that we took the derivative first of the outside function, $g[f(x)]$, leaving the inside function, $f(x)$, completely untouched, and then we multiplied that result by the derivative of the inside function.

So applying the chain rule requires just two simple steps:

1. Take the derivative of the "outside" function, leaving the "inside" function untouched.

2. Multiply that result by the derivative of the "inside" function.

Sometimes, depending on the complexity of the inside function, it can be helpful to use substitution to make it easier to think about $g[f(x)]$. If we decide to use substitution, we just replace the inside function with u, and the function simplifies from $g[f(x)]$ to $y = g[u]$. Then the derivative of this simplified version is

$$y' = g'[u]u'$$

If we're going to use substitution, we need to make sure we back-substitute at the end of the problem, in order to get the final answer back in terms of the original variable.

Example

Use chain rule to find the derivative.

$$y = (4x^8 - 6)^6$$

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The outside function is the power function $(4x^8 - 6)^6$, and the inside function is $4x^8 - 6$.

Let's use the substitution method, and say that $u = 4x^8 - 6$ and $u' = 32x^7$. Then we can rewrite the original equation $y = (4x^8 - 6)^6$ as

$$y = (u)^6$$

We'll apply power rule and chain rule to find the derivative, and we'll get

$$y' = 6(u)^5(u')$$

Then we'll back-substitute for u and u'.

$$y' = 6(4x^8 - 6)^5(32x^7)$$

$$y' = 192x^7(4x^8 - 6)^5$$

We just worked an example where we used chain rule in conjunction with power rule.

We'll also need to know how to use the chain rule in combination with product rule and quotient rule, and with trigonometric functions, all of which we'll tackle in the next few lessons.

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