Topic: Horizontal and slant asymptotes

Question: Find the function's horizontal asymptote(s).

$$f(x) = \frac{4x^3 - 2x^2 + 1}{2x^3 - 3x}$$

Answer choices:

A $\quad y = 0$

B $\quad y = -3$

C $\quad y = 2$

D $\quad y = \pm 2$

Solution: C

To find the horizontal asymptote, take the limit of the function when $x \to \infty$.

$$\lim_{x \to \infty} \frac{4x^3 - 2x^2 + 1}{2x^3 - 3x}$$

$$\lim_{x \to \infty} \frac{4x^3 - 2x^2 + 1}{2x^3 - 3x} \left(\frac{\frac{1}{x^3}}{\frac{1}{x^3}} \right)$$

$$\lim_{x \to \infty} \frac{\frac{4x^3 - 2x^2 + 1}{x^3}}{\frac{2x^3 - 3x}{x^3}}$$

$$\lim_{x \to \infty} \frac{4 - \frac{2}{x} + \frac{1}{x^3}}{2 - \frac{3}{x^2}}$$

Evaluate at the limit.

$$\frac{4 - 0 + 0}{2 - 0}$$

$$\frac{4}{2}$$

$$2$$

So $y = 2$ is the horizontal asymptote.

Topic: Horizontal and slant asymptotes

Question: Find the function's horizontal asymptote(s).

$$y = \frac{x^5 - x + 6}{x^7 - x^4 + 3x^2 - 1}$$

Answer choices:

A The function has a horizontal asymptote at $y = 1$

B The function has a horizontal asymptote at $y = 5/7$

C The function has a horizontal asymptote at $y = 0$

D The function has no horizontal asymptote

Solution: C

The x^5 term is the highest-degree term in the numerator, and the x^7 term is the highest-degree term in the denominator.

Because the degree of the numerator is less than the degree of the denominator, the function has a horizontal asymptote at $y = 0$.

Topic: Horizontal and slant asymptotes

Question: Find the function's slant asymptote(s).

$$f(x) = \frac{x^2 - x + 3}{x + 1}$$

Answer choices:

A The function has a slant asymptote at $y = x + 2 + \frac{5}{x+1}$

B The function has a slant asymptote at $y = x - 2 + \frac{5}{x+1}$

C The function has a slant asymptote at $y = x - 2$

D The function has a slant asymptote at $y = x + 2$

Solution: C

We want to do polynomial long division with the function, which we set up as

$$x+1\,\overline{)\,x^2-x+3}$$

If we work through this division, we end up with

$$f(x) = x - 2 + \frac{5}{x+1}$$

The slant asymptote is what we get when we remove the remainder from this rewritten function. If we remove the remainder, we get

$$f(x) = x - 2$$

So the equation of the slant asymptote is

$$y = x - 2$$