Topic: Newton's Method

Question: Use Newton's Method to approximate to three decimal places the root of the function on the interval $[1/2,2]$.

$$f(x) = x^2 - 1$$

Answer choices:

A $x = 1.500$

B $x = 1.000$

C $x = 2.000$

D $x = 0.500$

Solution: B

Take the derivative of the function.

$$f(x_n) = x_n^2 - 1$$

$$f'(x_n) = 2x_n$$

Then the Newton's Method formula will be

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^2 - 1}{2x_n}$$

Let's start with the left endpoint of the interval, $x_n = 1/2$, and work our problem with the number of decimals we were asked for.

$$x_0 = 0.500$$

$$x_1 = 0.500 - \frac{0.500^2 - 1}{2(0.500)} = 1.250$$

$$x_2 = 1.250 - \frac{1.250^2 - 1}{2(1.250)} = 1.025$$

$$x_3 = 1.025 - \frac{1.025^2 - 1}{2(1.025)} = 1.000$$

$$x_4 = 1.000 - \frac{1.000^2 - 1}{2(1.000)} = 1.000$$

Since these last two approximations were identical to three decimal places, we can stop and conclude that an approximation of the root of the function in the given interval is $x = 1.000$.

Topic: Newton's Method

Question: Use Newton's Method to approximate to three decimal places the root of the function on the interval $[1,2]$.

$$f(x) = 2x^2 - 3$$

Answer choices:

A $x = 1.525$

B $x = 1.255$

C $x = 1.522$

D $x = 1.225$

Solution: D

Take the derivative of the function.

$$f(x_n) = 2x_n^2 - 3$$

$$f'(x_n) = 4x_n$$

Then the Newton's Method formula will be

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{2x_n^2 - 3}{4x_n}$$

Let's start with the left endpoint of the interval, $x_n = 1$, and work our problem with the number of decimals we were asked for.

$$x_0 = 1.000$$

$$x_1 = 1.000 - \frac{2(1.000)^2 - 3}{4(1.000)} = 1.250$$

$$x_2 = 1.250 - \frac{2(1.250)^2 - 3}{4(1.250)} = 1.225$$

$$x_3 = 1.225 - \frac{2(1.225)^2 - 3}{4(1.225)} = 1.225$$

Since these last two approximations were identical to three decimal places, we can stop and conclude that an approximation of the root of the function in the given interval is $x = 1.225$.

Topic: Newton's Method

Question: Use Newton's Method to approximate to three decimal places the root of the function on the interval $[3,4]$.

$$f(x) = x^2 - 3x - 1$$

Answer choices:

A $x = 3.303$

B $x = 3.322$

C $x = 3.032$

D $x = 3.332$

Solution: A

Take the derivative of the function.

$$f(x_n) = x_n^2 - 3x_n - 1$$

$$f'(x_n) = 2x_n - 3$$

Then the Newton's Method formula will be

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^2 - 3x_n - 1}{2x_n - 3}$$

Let's start with the left endpoint of the interval, $x_n = 3$, and work our problem with the number of decimals we were asked for.

$$x_0 = 3.000$$

$$x_1 = 3.000 - \frac{(3.000)^2 - 3(3.000) - 1}{2(3.000) - 3} = 3.333$$

$$x_2 = 3.333 - \frac{(3.333)^2 - 3(3.333) - 1}{2(3.333) - 3} = 3.303$$

$$x_3 = 3.303 - \frac{(3.303)^2 - 3(3.303) - 1}{2(3.303) - 3} = 3.303$$

Since these last two approximations were identical to three decimal places, we can stop and conclude that an approximation of the root of the function in the given interval is $x = 3.303$.