Calculus 1 Workbook Solutions

Derivatives of ln(x) and e^x

krista king
MATH

EXPONENTIAL DERIVATIVES

■ 1. Find $f'(x)$ if $f(x)=(x^3-x)e^{2x}$.

Solution:

Use product rule to take the derivative.

$$f'(x)=\frac{d}{dx}(x^3-x)\cdot e^{2x}+(x^3-x)\cdot\frac{d}{dx}e^{2x}$$

$$f'(x)=(3x^2-1)\cdot e^{2x}+(x^3-x)\cdot 2e^{2x}$$

Factor to simplify.

$$f'(x)=e^{2x}\left(3x^2-1+2x^3-2x\right)$$

$$f'(x)=e^{2x}\left(2x^3+3x^2-2x-1\right)$$

■ 2. Find $g'(x)$ if $g(x)=5x^2e^{2x^2}-7x+1$.

Solution:

Use product rule to take the derivative of the first term.

$$g'(x)=\frac{d}{dx}\left(5x^2\right)\cdot e^{2x^2}+5x^2\cdot\frac{d}{dx}\left(e^{2x^2}\right)-7+0$$

$$g'(x) = 10x \cdot e^{2x^2} + 5x^2 \cdot e^{2x^2}(4x) - 7$$

$$g'(x) = 10xe^{2x^2} + 20x^3e^{2x^2} - 7$$

Factor to simplify.

$$g'(x) = 10xe^{2x^2}(1 + 2x^2) - 7$$

■ 3. Find $h'(x)$ if $h(x) = \sin(4x)e^{3x^2+4}$.

Solution:

Use product rule to take the derivative.

$$h'(x) = \frac{d}{dx}\sin(4x) \cdot e^{3x^2+4} + \sin(4x) \cdot \frac{d}{dx}e^{3x^2+4}$$

$$h'(x) = \cos(4x)(4) \cdot e^{3x^2+4} + \sin(4x) \cdot e^{3x^2+4}(6x)$$

$$h'(x) = 4e^{3x^2+4}\cos(4x) + 6xe^{3x^2+4}\sin(4x)$$

Factor to simplify.

$$h'(x) = 2e^{3x^2+4}\left(3x\sin(4x) + 2\cos(4x)\right)$$

LOGARITHMIC DERIVATIVES

■ 1. Find $f'(x)$.

$$f(x) = \ln(x^2 + 6x + 9)$$

Solution:

Take the derivative, remembering to apply chain rule.

$$f'(x) = \frac{1}{x^2 + 6x + 9} \cdot (2x + 6) = \frac{2x + 6}{x^2 + 6x + 9} = \frac{2(x + 3)}{(x + 3)(x + 3)} = \frac{2}{x + 3}$$

■ 2. Find $g'(x)$.

$$g(x) = \ln \sqrt{x^3 + x}$$

Solution:

Take the derivative, remembering to apply chain rule.

$$g'(x) = \frac{1}{\sqrt{x^3 + x}} \cdot \frac{1}{2}(x^3 + x)^{-\frac{1}{2}} \cdot (3x^2 + 1)$$

$$g'(x) = \frac{(3x^2 + 1)(x^3 + x)^{-\frac{1}{2}}}{2\sqrt{x^3 + x}}$$

$$g'(x) = \frac{3x^2 + 1}{2\sqrt{x^3 + x}\sqrt{x^3 + x}}$$

$$g'(x) = \frac{3x^2 + 1}{2(x^3 + x)}$$

$$g'(x) = \frac{3x^2 + 1}{2x(x^2 + 1)}$$

■ 3. Find $h'(x)$.

$$h(x) = \ln\left(\frac{x^3}{x^2 + 3}\right)$$

Solution:

Take the derivative, remembering to apply chain rule.

$$h'(x) = \frac{1}{\frac{x^3}{x^2 + 3}} \cdot \frac{3x^2(x^2 + 3) - x^3(2x)}{(x^2 + 3)^2}$$

$$h'(x) = \frac{x^2 + 3}{x^3} \cdot \frac{3x^2(x^2 + 3) - x^3(2x)}{(x^2 + 3)^2}$$

$$h'(x) = \frac{1}{x} \cdot \frac{3(x^2 + 3) - x(2x)}{x^2 + 3}$$

$$h'(x) = \frac{3x^2 + 9 - 2x^2}{x(x^2 + 3)}$$

$$h'(x) = \frac{x^2 + 9}{x(x^2 + 3)}$$

LOGARITHMIC DIFFERENTIATION

■ 1. Use logarithmic differentiation to find dy/dx.

$$y = x^4 e^x \sqrt{x}$$

Solution:

Take the natural log of both sides.

$$\ln y = \ln \left(x^4 e^x \sqrt{x} \right)$$

Use properties of logarithms to rewrite the equation.

$$\ln y = \ln x^4 + \ln e^x + \ln \sqrt{x}$$

$$\ln y = \ln x^4 + x + \ln x^{\frac{1}{2}}$$

$$\ln y = 4 \ln x + x + \frac{1}{2} \ln x$$

Differentiate, remembering to apply chain rule, then solve for dy/dx.

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{4}{x} + 1 + \frac{1}{2x}$$

$$\frac{dy}{dx} = y \left(\frac{4}{x} + 1 + \frac{1}{2x} \right)$$

Substitute for y.

$$\frac{dy}{dx} = x^4 e^x \sqrt{x}\left(\frac{4}{x} + 1 + \frac{1}{2x}\right)$$

You could leave the answer like this, or try to simplify.

$$\frac{dy}{dx} = x^4 e^x \sqrt{x}\left(\frac{8}{2x} + \frac{2x}{2x} + \frac{1}{2x}\right)$$

$$\frac{dy}{dx} = x^4 e^x \sqrt{x}\left(\frac{2x+9}{2x}\right)$$

$$\frac{dy}{dx} = \frac{x^3 e^x \sqrt{x}(2x+9)}{2}$$

■ 2. Use logarithmic differentiation to find dy/dx.

$$y = 5x^4 e^{3x} \sqrt[4]{x}$$

Solution:

Take the natural log of both sides.

$$\ln y = \ln\left(5x^4 e^{3x} \sqrt[4]{x}\right)$$

Use properties of logarithms to rewrite the equation.

$$\ln y = \ln 5x^4 + \ln e^{3x} + \ln \sqrt[4]{x}$$

$$\ln y = 4 \ln 5x + 3x + \ln x^{\frac{1}{4}}$$

$$\ln y = 4\ln 5x + 3x + \frac{1}{4}\ln x$$

Differentiate, remembering to apply chain rule, then solve for dy/dx.

$$\frac{1}{y} \cdot \frac{dy}{dx} = 0 + (5)\frac{4}{5x} + 3 + \frac{1}{4x}$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{4}{x} + 3 + \frac{1}{4x}$$

$$\frac{dy}{dx} = y\left(\frac{4}{x} + 3 + \frac{1}{4x}\right)$$

Substitute for y.

$$\frac{dy}{dx} = 5x^4e^{3x}\sqrt[4]{x}\left(\frac{4}{x} + 3 + \frac{1}{4x}\right)$$

You could leave the answer like this, or try to simplify.

$$\frac{dy}{dx} = 5x^4e^{3x}\sqrt[4]{x}\left(\frac{16}{4x} + \frac{12x}{4x} + \frac{1}{4x}\right)$$

$$\frac{dy}{dx} = 5x^4e^{3x}\sqrt[4]{x}\left(\frac{12x+17}{4x}\right)$$

$$\frac{dy}{dx} = \frac{5x^3e^{3x}\sqrt[4]{x}(12x+17)}{4}$$

■ 3. Use logarithmic differentiation to find dy/dx.

$$y = x^3 e^{2x} \sqrt{5x}$$

Solution:

Take the natural log of both sides.

$$\ln y = \ln \left(x^3 e^{2x} \sqrt{5x} \right)$$

Use properties of logarithms to rewrite the equation.

$$\ln y = \ln x^3 + \ln e^{2x} + \ln \sqrt{5x}$$

$$\ln y = 3 \ln x + 2x + \ln(5x)^{\frac{1}{2}}$$

$$\ln y = 3 \ln x + 2x + \frac{1}{2} \ln(5x)$$

Differentiate, remembering to apply chain rule, then solve for dy/dx.

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{3}{x} + 2 + \frac{5}{2(5x)}$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{3}{x} + 2 + \frac{1}{2x}$$

$$\frac{dy}{dx} = y \left(\frac{3}{x} + 2 + \frac{1}{2x} \right)$$

Substitute for y.

$$\frac{dy}{dx} = x^3 e^{2x} \sqrt{5x} \left(\frac{3}{x} + 2 + \frac{1}{2x} \right)$$

You could leave the answer like this, or try to simplify.

$$\frac{dy}{dx} = x^3 e^{2x}\sqrt{5x}\left(\frac{6}{2x} + \frac{4x}{2x} + \frac{1}{2x}\right)$$

$$\frac{dy}{dx} = x^3 e^{2x}\sqrt{5x}\left(\frac{4x+7}{2x}\right)$$

$$\frac{dy}{dx} = \frac{x^2 e^{2x}\sqrt{5x}(4x+7)}{2}$$

■ 4. Use logarithmic differentiation to find dy/dx.

$$y = \frac{(2e)^{\cos x}}{(3e)^{\sin x}}$$

Solution:

Take the natural log of both sides.

$$\ln y = \ln\left(\frac{(2e)^{\cos x}}{(3e)^{\sin x}}\right)$$

Use properties of logarithms to rewrite the equation.

$$\ln y = \ln(2e)^{\cos x} - \ln(3e)^{\sin x}$$

$$\ln y = \cos x \ln(2e) - \sin x \ln(3e)$$

$$\ln y = (\cos x)(\ln 2 + \ln e) - (\sin x)(\ln 3 + \ln e)$$

$$\ln y = (\cos x)(\ln 2 + 1) - (\sin x)(\ln 3 + 1)$$

Differentiate, remembering to apply chain rule, then solve for dy/dx.

$$\frac{1}{y} \cdot \frac{dy}{dx} = (-\sin x)(\ln 2 + 1) + (\cos x)(0) - \left[(\cos x)(\ln 3 + 1) + (\sin x)(0)\right]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = -(\ln 2 + 1)\sin x - (\ln 3 + 1)\cos x$$

$$\frac{dy}{dx} = -y\left[(\ln 2 + 1)\sin x + (\ln 3 + 1)\cos x\right]$$

Substitute for y.

$$\frac{dy}{dx} = -\frac{(2e)^{\cos x}}{(3e)^{\sin x}}\left[(\ln 2 + 1)\sin x + (\ln 3 + 1)\cos x\right]$$

■ 5. Use logarithmic differentiation to find dy/dx.

$$y = e^x(2e)^{\sin x}(3e)^{\cos x}$$

Solution:

Take the natural log of both sides.

$$\ln y = \ln\left(e^x(2e)^{\sin x}(3e)^{\cos x}\right)$$

Use properties of logarithms to rewrite the equation.

$$\ln y = \ln e^x + \ln(2e)^{\sin x} + \ln(3e)^{\cos x}$$

$$\ln y = x + \sin x \ln(2e) + \cos x \ln(3e)$$

$$\ln y = x + \sin x(\ln 2 + \ln e) + \cos x(\ln 3 + \ln e)$$

$$\ln y = x + \sin x(\ln 2 + 1) + \cos x(\ln 3 + 1)$$

Differentiate, remembering to apply chain rule, then solve for dy/dx.

$$\frac{1}{y} \cdot \frac{dy}{dx} = 1 + (\ln 2 + 1)\cos x - (\ln 3 + 1)\sin x$$

$$\frac{dy}{dx} = y\left[1 + (\ln 2 + 1)\cos x - (\ln 3 + 1)\sin x\right]$$

Substitute for y.

$$\frac{dy}{dx} = e^x(2e)^{\sin x}(3e)^{\cos x}\left[1 + (\ln 2 + 1)\cos x - (\ln 3 + 1)\sin x\right]$$

WWW.KRISTAKINGMATH.COM