Topic: Average rate of change

Question: Find the average rate of change of the function on $[0,2]$.

$$f(x) = x^2$$

Answer choices:

A $\dfrac{\Delta f}{\Delta x} = 0$

B $\dfrac{\Delta f}{\Delta x} = \dfrac{1}{2}$

C $\dfrac{\Delta f}{\Delta x} = 1$

D $\dfrac{\Delta f}{\Delta x} = 2$

Solution: D

From the interval, we know $x_1 = 0$ and $x_2 = 2$. We'll find $f(x_1)$ and $f(x_2)$ by plugging these values into the function. We get

$$f(0) = 0^2$$

$$f(0) = 0$$

and

$$f(2) = 2^2$$

$$f(2) = 4$$

Now we can plug the values we've found into the formula for average rate fo change.

$$\frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\frac{\Delta f}{\Delta x} = \frac{f(2) - f(0)}{2 - 0}$$

$$\frac{\Delta f}{\Delta x} = \frac{4 - 0}{2}$$

$$\frac{\Delta f}{\Delta x} = \frac{4}{2}$$

$$\frac{\Delta f}{\Delta x} = 2$$

Topic: Average rate of change

Question: Find the average rate of change of the function on $[1,4]$.

$$f(x) = \frac{2x}{3x^2 - 1}$$

Answer choices:

A $\frac{\Delta f}{\Delta x} = -\frac{47}{13}$

B $\frac{\Delta f}{\Delta x} = -\frac{13}{47}$

C $\frac{\Delta f}{\Delta x} = \frac{47}{13}$

D $\frac{\Delta f}{\Delta x} = \frac{13}{47}$

Solution: B

From the interval, we know $x_1 = 1$ and $x_2 = 4$. We'll find $f(x_1)$ and $f(x_2)$ by plugging these values into the function. We get

$$f(1) = \frac{2(1)}{3(1)^2 - 1}$$

$$f(1) = \frac{2}{2}$$

$$f(1) = 1$$

and

$$f(4) = \frac{2(4)}{3(4)^2 - 1}$$

$$f(4) = \frac{8}{48 - 1}$$

$$f(4) = \frac{8}{47}$$

Now we can plug the values we've found into the formula for average rate fo change.

$$\frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\frac{\Delta f}{\Delta x} = \frac{f(4) - f(1)}{4 - 1}$$

$$\frac{\Delta f}{\Delta x} = \frac{\frac{8}{47} - 1}{3}$$

$$\frac{\Delta f}{\Delta x} = \frac{\frac{8}{47} - \frac{47}{47}}{3}$$

$$\frac{\Delta f}{\Delta x} = \frac{-\frac{39}{47}}{3}$$

$$\frac{\Delta f}{\Delta x} = -\frac{39}{47} \cdot \frac{1}{3}$$

$$\frac{\Delta f}{\Delta x} = -\frac{13}{47}$$

Topic: Average rate of change

Question: Find the average rate of change of the function on $[2,3]$.

$$f(x) = 6e^x - 4\sqrt{x^3}$$

Answer choices:

A $\dfrac{\Delta f}{\Delta x} = 6e^2(e-1) - 4(3\sqrt{3} - 2\sqrt{2})$

B $\dfrac{\Delta f}{\Delta x} = e^2(6e+1) + 3\sqrt{3} + 8\sqrt{2}$

C $\dfrac{\Delta f}{\Delta x} = 6e^2(e+1) + 4(3\sqrt{3} + 2\sqrt{2})$

D $\dfrac{\Delta f}{\Delta x} = e^2(6e-1) - 3\sqrt{3} + 8\sqrt{2}$

Solution: A

From the interval, we know $x_1 = 2$ and $x_2 = 3$. We'll find $f(x_1)$ and $f(x_2)$ by plugging these values into the function. We get

$$f(3) = 6e^3 - 4\sqrt{3^3}$$

$$f(3) = 6e^3 - 4\sqrt{27}$$

$$f(3) = 6e^3 - 12\sqrt{3}$$

and

$$f(2) = 6e^2 - 4\sqrt{2^3}$$

$$f(2) = 6e^2 - 4\sqrt{8}$$

$$f(2) = 6e^2 - 8\sqrt{2}$$

Now we can plug the values we've found into the formula for average rate fo change.

$$\frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\frac{\Delta f}{\Delta x} = \frac{f(3) - f(2)}{3 - 2}$$

$$\frac{\Delta f}{\Delta x} = \frac{6e^3 - 12\sqrt{3} - (6e^2 - 8\sqrt{2})}{3 - 2}$$

$$\frac{\Delta f}{\Delta x} = \frac{6e^3 - 12\sqrt{3} - 6e^2 + 8\sqrt{2}}{1}$$

$$\frac{\Delta f}{\Delta x} = 6e^3 - 12\sqrt{3} - 6e^2 + 8\sqrt{2}$$

$$\frac{\Delta f}{\Delta x} = 6e^2(e-1) - 4(3\sqrt{3} - 2\sqrt{2})$$