Topic: Squeeze theorem

Question: Use squeeze theorem to evaluate the limit.

$$\lim_{x \to 0} x^2 \cos x$$

Answer choices:

A ∞

B -1

C 0

D 1

Solution: C

We know the value of the cosine function oscillates back and forth between -1 and 1, so we'll start with

$$-1 \leq \cos x \leq 1$$

Multiply through the inequality by x^2 to get the function at the center of the inequality to match the one we were given.

$$-x^2 \leq x^2 \cos x \leq x^2$$

Apply the limit throughout the inequality.

$$\lim_{x \to 0} -x^2 \leq \lim_{x \to 0} x^2 \cos x \leq \lim_{x \to 0} x^2$$

$$-0^2 \leq \lim_{x \to 0} x^2 \cos x \leq 0^2$$

$$0 \leq \lim_{x \to 0} x^2 \cos x \leq 0$$

We've squeezed the limit we wanted to find in the inequality, so we can say the value of the limit is

$$\lim_{x \to 0} x^2 \cos x = 0$$

Topic: Squeeze theorem

Question: Use squeeze theorem to evaluate the limit.

$$\lim_{x \to \infty} \frac{\sin(6x)}{x^2}$$

Answer choices:

A -1

B 0

C ∞

D 1

Solution: B

We know the value of the sine function oscillates back and forth between -1 and 1, so we'll start with

$$-1 \leq \sin(6x) \leq 1$$

Divide through the inequality by x^2 to get the function at the center of the inequality to match the one we were given.

$$-\frac{1}{x^2} \leq \frac{\sin(6x)}{x^2} \leq \frac{1}{x^2}$$

Apply the limit throughout the inequality.

$$\lim_{x\to\infty} -\frac{1}{x^2} \leq \lim_{x\to\infty} \frac{\sin(6x)}{x^2} \leq \lim_{x\to\infty} \frac{1}{x^2}$$

$$-\frac{1}{\infty^2} \leq \lim_{x\to\infty} \frac{\sin(6x)}{x^2} \leq \frac{1}{\infty^2}$$

$$0 \leq \lim_{x\to\infty} \frac{\sin(6x)}{x^2} \leq 0$$

We've squeezed the limit we wanted to find in the inequality, so we can say the value of the limit is

$$\lim_{x\to\infty} \frac{\sin(6x)}{x^2} = 0$$

Topic: Squeeze theorem

Question: Use squeeze theorem to evaluate the limit.

$$\lim_{x \to \infty} \frac{2x^3 + \sin(4x)}{6x^3 + 5}$$

Answer choices:

A $\frac{1}{3}$

B ∞

C 0

D 3

Solution: A

We know the value of the sine function oscillates back and forth between -1 and 1, so we'll start with

$$-1 \le \sin(4x) \le 1$$

Add $2x^3$ to each part of the inequality.

$$2x^3 - 1 \le 2x^3 + \sin(4x) \le 2x^3 + 1$$

Divide through the inequality by $6x^3 + 5$ to get the function at the center of the inequality to match the one we were given.

$$\frac{2x^3 - 1}{6x^3 + 5} \le \frac{2x^3 + \sin(4x)}{6x^3 + 5} \le \frac{2x^3 + 1}{6x^3 + 5}$$

Apply the limit throughout the inequality.

$$\lim_{x\to\infty} \frac{2x^3 - 1}{6x^3 + 5} \le \lim_{x\to\infty} \frac{2x^3 + \sin(4x)}{6x^3 + 5} \le \lim_{x\to\infty} \frac{2x^3 + 1}{6x^3 + 5}$$

$$\frac{2}{6} \le \lim_{x\to\infty} \frac{2x^3 + \sin(4x)}{6x^3 + 5} \le \frac{2}{6}$$

$$\frac{1}{3} \le \lim_{x\to\infty} \frac{2x^3 + \sin(4x)}{6x^3 + 5} \le \frac{1}{3}$$

We've squeezed the limit we wanted to find in the inequality, so we can say the value of the limit is

$$\lim_{x\to\infty} \frac{2x^3 + \sin(4x)}{6x^3 + 5} = \frac{1}{3}$$