

**Topic:** Intermediate value theorem with an interval

**Question:** Which statement is true?

**Answer choices:**

- A      The IVT only applies to discontinuous functions.
- B      The IVT only applies when there's no interval.
- C      The IVT only applies to open intervals.
- D      The IVT only applies to closed intervals.



**Solution: D**

The Intermediate Value Theorem states that for a function on a closed interval  $[a, b]$  where the function is continuous on the interval, a point  $c$  exists on the interval where  $f(c) = k$ .

$$f(a) < k < f(b) \text{ and } a < c < b$$



**Topic:** Intermediate value theorem with an interval

**Question:** Does the Intermediate Value Theorem prove that the function  $f(x) = x^2 + 2x - 35$  has a root on the interval  $[0,10]$ ?

**Answer choices:**

- A The IVT shows that the function has a root on the open interval.
- B The IVT shows that the function has a root on the closed interval.
- C The IVT shows that there's not root on the closed interval.
- D The IVT is inconclusive.



**Solution: B**

Evaluate the function at both endpoints of the interval  $(0,10)$ .

$$f(0) = 0^2 + 2(0) - 35$$

$$f(0) = -35$$

and

$$f(10) = 10^2 + 2(10) - 35$$

$$f(10) = 85$$

Because the function is below the  $x$ -axis at the left edge of the interval, and above the  $x$ -axis at the right edge of the interval, we can say  $f(a) < 0 < f(b)$ , or more specifically,  $-35 < 0 < 85$ .

Therefore, by the intermediate value theorem, it must be true that the function has a root on the interval  $[0,10]$ .



**Topic:** Intermediate value theorem with an interval

**Question:** Is there a root for the function  $f(x) = x^2 - 4$  on the interval  $(1,6)$ ?

**Answer choices:**

- A Yes, there's a root at  $(0,4)$ .
- B Yes, there's a root at  $(0, -4)$ .
- C Yes, there's a root at  $(2,0)$ .
- D Yes, there's a root at  $(-2,0)$ .



**Solution: C**

Evaluate the function at both endpoints of the interval (1,6).

$$f(1) = 1^2 - 4$$

$$f(1) = 1 - 4$$

$$f(1) = -3$$

and

$$f(6) = 6^2 - 4$$

$$f(6) = 36 - 4$$

$$f(6) = 32$$

The IVT confirms that the function has a root on the interval, because the function's value crosses from below the  $x$ -axis to above the  $x$ -axis at some point within that interval.

To find the root, which is the point where the graph of the function crosses the  $x$ -axis, we'll set the function equal to 0.

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

Therefore, the root in the interval (1,6) is at  $x = 2$ , and that coordinate point is (2,0).

