Coin dropped from the roof

In the previous lesson, we were looking at a vertical motion pattern in which we threw an object up from the ground, or some other height, and the object traveled upward, eventually reached a maximum height, and then fell back down to earth, eventually stopping when it hits the ground.

In this lesson, we're looking at a different vertical motion pattern. This time, we're dropping a coin, or some other other object, from a height, letting it fall straight to the ground, eventually stopping when it hits the ground.

Let's work through an example of how to find different values from a position function that models this pattern of vertical motion.

Example

A watermelon is dropped from the top of a building 28 meters high. Find instantaneous velocity at $t=4$, average velocity between $t=2$ and $t=5$, and find the time when the watermelon hits the ground.

$$x(t)=-4t^2+28$$

Find the velocity function by differentiating the position function.

$$v(t)=x'(t)=-8t$$

To find instantaneous velocity at $t=4$, substitute $t=4$ into the velocity function.

$$v(4) = -8(4)$$

$$v(4) = -32$$

This is the instantaneous velocity at $t = 4$.

Find average velocity over $t = [2,5]$.

$$\Delta v(a, b) = \frac{x(b) - x(a)}{b - a}$$

$$\Delta v(2,5) = \frac{x(5) - x(2)}{5 - 2}$$

$$\Delta v(2,5) = \frac{-4(5)^2 + 28 - (-4(2)^2 + 28)}{3}$$

$$\Delta v(2,5) = \frac{-100 + 28 - (-16 + 28)}{3}$$

$$\Delta v(2,5) = \frac{-100 + 28 + 16 - 28}{3}$$

$$\Delta v(2,5) = \frac{-84}{3}$$

$$\Delta v(2,5) = -28$$

This is the average velocity of the watermelon between $t = 2$ and $t = 5$.

The watermelon will hit the ground when $x(t) = 0$, so we'll set the position function equal to 0 and then solve for t.

$$-4t^2 + 28 = 0$$

$$4t^2 = 28$$

$$t^2 = 7$$

$$t = \sqrt{7}$$

$$t \approx 2.65$$

The watermelon hits the ground after $t \approx 2.65$ seconds.