Solving with factoring

When we can't use substitution to evaluate a limit, because plugging in the value we're approaching gives an undefined value, factoring is the next approach we should try.

We usually use factoring when we're finding the limit of a rational function. Our goal will be to factor both the numerator and denominator as completely as possible, and then cancel any common factors.

Our hope is that, once we've canceled common factors, we'll be left with a function that can then be evaluated using substitution.

Example

Evaluate the limit.

$$\lim_{x\to 4} \frac{x^2-16}{x-4}$$

Using substitution to evaluate the limit means we'll plug $x = 4$ into the function.

$$\frac{4^2-16}{4-4}$$

$$\frac{16-16}{4-4}$$

$$\frac{0}{0}$$

When we use substitution, we get an undefined result, so we should try factoring, instead. There's no factoring to be done in the denominator, but the numerator can be factored as the difference of squares.

$$\lim_{x\to4}\frac{(x-4)(x+4)}{x-4}$$

Once the numerator is factored, we can see that there's a common factor of $(x-4)$ in both the numerator and denominator. We can cancel it.

$$\lim_{x\to4}\frac{x-4}{x-4}(x+4)$$

$$\lim_{x\to4}1(x+4)$$

$$\lim_{x\to4}x+4$$

Now that the function has been factored and simplified, we'll try substitution with the simplified function. Substituting $x=4$ into the function gives

$$4+4$$

$$8$$

Therefore, the limit of the function as $x\to4$ is 8.

$$\lim_{x\to4}\frac{x^2-16}{x-4}=8$$