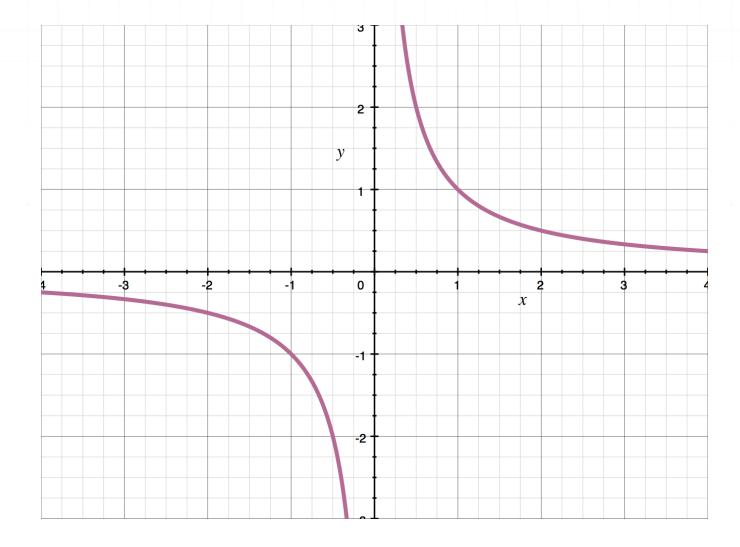
## Infinite limits and vertical asymptotes

There's a difference between "limits at infinity" and "infinite limits." When we see *limits at infinity*, it means we're talking about the limit of the function as we approach  $\infty$  or  $-\infty$ . Contrast that with *infinite limits*, which means that the value of the limit is  $\infty$  or  $-\infty$  as we approach a particular point.

## Limits at infinity, infinite limits

In the graph of f(x) = 1/x,





the function has infinite, one-sided limits at x = 0. There's a vertical asymptote there, and we can see that the function approaches  $-\infty$  from the left, and  $\infty$  from the right.

$$\lim_{x \to 0^{-}} \frac{1}{x} = -\infty$$

$$\lim_{x \to 0^+} \frac{1}{x} = \infty$$

Talking about limits at infinity for this function, we can see that the function approaches 0 as we approach either  $\infty$  or  $-\infty$ .

$$\lim_{x \to -\infty} \frac{1}{x} = 0$$

$$\lim_{r \to \infty} \frac{1}{r} = 0$$

## How to find infinite limits

Infinite limits exist around vertical asymptotes in the function. Of course, we get a vertical asymptote whenever the denominator of a rational function in lowest terms is equal to 0.

Here's an example of a rational function in lowest terms, meaning that we can't factor and cancel anything from the fraction.

$$\lim_{x \to 1} \frac{1}{(x-1)^2}$$



We can see that setting x = 1 gives 0 in the denominator, which means that we have a vertical asymptote at x = 1. Therefore, we know we'll have infinite limits on either side of x = 1.

Once we've established that this is a rational function in lowest terms and that a vertical asymptote exists, all that's left to determine is whether the one-sided limits around x = 1 approach  $\infty$  or  $-\infty$ .

In order to do that, we can substitute values very close to x=1. If the result is positive, the limit will be  $\infty$ ; if the result is negative, the limit will be  $-\infty$ .

$$f(0.99) = \frac{1}{(0.99 - 1)^2} = \frac{1}{(-0.01)^2} = \frac{1}{0.0001} = 10,000 = \infty$$

$$f(1.01) = \frac{1}{(1.01 - 1)^2} = \frac{1}{(0.01)^2} = \frac{1}{0.0001} = 10,000 = \infty$$

Because the value of the function tends toward  $\infty$  on both sides of the vertical asymptote, we can say that the general limit of the function as  $x \to 1$  is  $\infty$ .

$$\lim_{x \to 1} \frac{1}{(x-1)^2} = \infty$$

