Calculus 1 Workbook Solutions

Applied optimization

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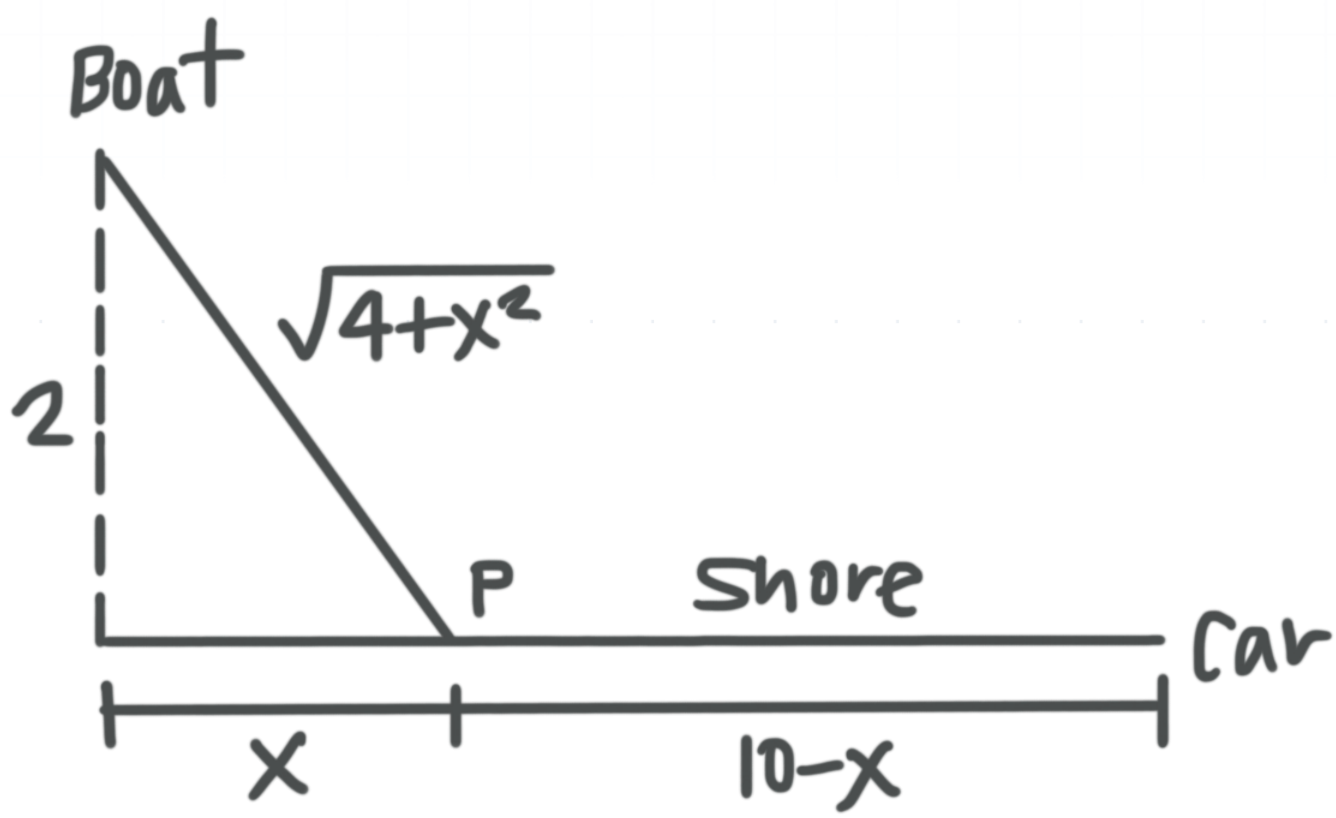
APPLIED OPTIMIZATION

■ 1. A boater finds herself 2 miles from the nearest point to a straight shoreline, which is 10 miles down the shore from where she parked her car. She plans to row to shore and then walk to her car. If she can walk 4 miles per hour but only row 3 miles per hour, toward what point on the shore should she row in order to reach her car in in the least amount of time?

Solution:

Draw a diagram.



From the diagram, the distance to point P is $\sqrt{4+x^2}$ and the distance from point P to the car is $10 - x$. So the total time to reach the car is

$$T = \frac{\sqrt{4+x^2}}{3} + \frac{10-x}{4} \text{ with } 0 \le x \le 10$$

Find the derivative of T.

$$dT = \frac{x}{3\sqrt{4+x^2}} - \frac{1}{4}$$

Set $dT = 0$ and solve for x.

$$\frac{x}{3\sqrt{4+x^2}} = \frac{1}{4}$$

$$\frac{4x}{3} = \sqrt{4+x^2}$$

$$\frac{16x^2}{9} = 4 + x^2$$

$$\frac{7}{9}x^2 = 4$$

$$x^2 = \frac{36}{7}$$

$$x = \frac{36}{\sqrt{7}} \approx 2.2678$$

If she rows to point P, where $x = 2.2678$ miles down the shoreline, it will take her

$$T = \frac{\sqrt{4+(2.2678)^2}}{3} + \frac{10-(2.2678)}{4} \approx 2.9409 \text{ hours}$$

If she rows directly to the shore, where $x = 0$, it will take her

$$T = \frac{2}{3} + \frac{10}{4} \approx 3.167 \text{ hours}$$

Find the distance directly to her car using the Pythagorean Theorem.

$$d^2 = 2^2 + 10^2 = 104$$

$$d = 2\sqrt{26} \text{ miles}$$

If she rows directly to her car, where $x = 10$,

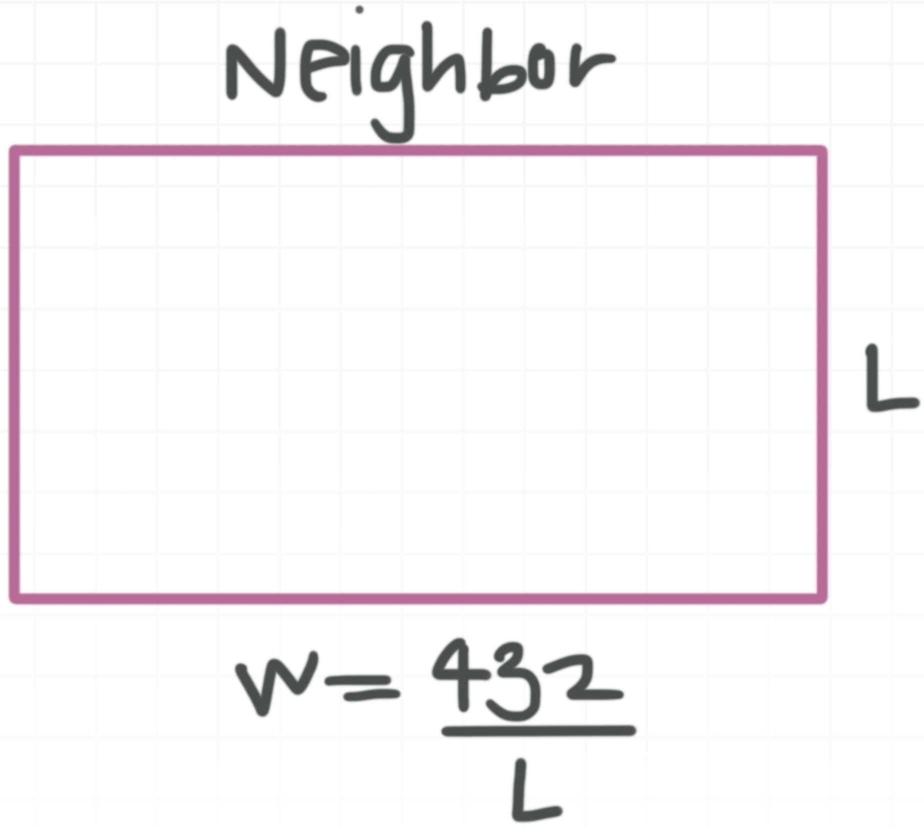
$$T = \frac{2\sqrt{26}}{3} = 3.399 \text{ hours}$$

She will minimize her time by rowing to a point that is 2.2678 miles down shore toward her car.

■ 2. Mr. Quizna wants to build in a completely fenced-in rectangular garden. The fence will be built so that one side is adjacent to his neighbor's property. The neighbor agrees to pay for half of that part of the fence because it borders his property. The garden will contain 432 square meters. What dimensions should Mr. Quizna select for his garden in order to minimize his cost?

Solution:

Draw a diagram.



The area is

$$A = L \cdot W$$

$$432 = L \cdot W$$

$$W = \frac{432}{L}$$

Let C be the total cost and y be the cost per meter. Then,

$$C = 2L \cdot y + \frac{432}{L} \cdot y + \frac{216}{L} \cdot y = 2yL + 648yL^{-1} = y(2L + 648L^{-1})$$

Take the derivative of the cost equation.

$$dC = y(2 - 648L^{-2})$$

Set the derivative equal to 0 and solve for L.

$$2 = \frac{648}{L^2}$$

$$2L^2 = 648$$

$$L^2 = 324$$

$$L = 18$$

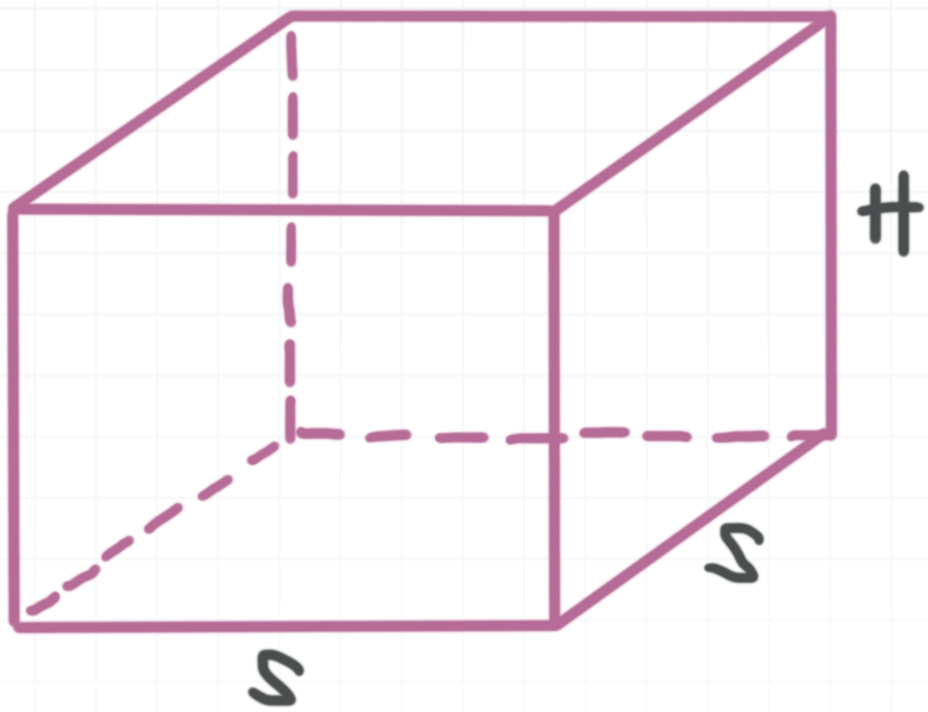
The length of the garden should be $L = 18$ meters and the width of the garden should be

$$W = \frac{432}{L} = \frac{432}{18} = 24 \text{ meters}$$

■ 3. A company is designing shipping crates and wants the volume of each crate to be 6 cubic feet, and each crate's base to be a square between 1.5 feet and 2.0 feet per side. The material for the bottom of the crate costs $\$5$ per square foot, the sides $\$3$ per square foot, and the top $\$1$ per square foot. What dimensions will minimize the cost of the shipping crates?

Solution:

Draw a diagram.



Based on the given information,

$$V = S \cdot S \cdot H$$

$$6 = S \cdot S \cdot H$$

$$H = \frac{6}{S^2}$$

The surface area of the bottom is S^2, the surface area of the top is S^2, and the surface area of the four sides is

$$4 \cdot S \cdot \frac{6}{S^2} = \frac{24}{S}$$

Create a cost function.

$$C = 5 \cdot S^2 + 1 \cdot S^2 + 3 \cdot \frac{24}{S} = 6S^2 + \frac{72}{S} = 6S^2 + 72S^{-1}$$

Differentiate the cost function.

$$dC = 12S - \frac{72}{S^2}$$

Set the derivative equal to 0 and solve for S.

$$12S = \frac{72}{S^2}$$

$$12S^3 = 72$$

$$S^3 = 6$$

$$S = \sqrt[3]{6}$$

$$S \approx 1.82$$

The dimensions that will give the minimum cost are $S = 1.82$ feet and $H = 1.81$ feet.

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