Equation of a line in point-slope form

You have two options for writing the equation of a line: point-slope form and slope-intercept form. Both of them require that you know at least two of the following pieces of information about the line:

1. A point

2. Another point

3. The slope, m

4. The y-intercept, b (the y-coordinate of the point at which the graph of the line crosses the y-axis)

If you know any two of these things, you can find the equation of the line.

Point-slope form

The equation of a line in point-slope form can be written as

$$y - y_1 = m(x - x_1)$$

In this form, (x_1, y_1) is a point on the line, and m is the slope. To use this form when you know two points on the line but you don't know the slope, first find m using

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Then simply plug the slope m and the coordinates of one point (x_1, y_1) into the point-slope form of the equation of a line.

Let's do an example where we know the slope and one point on the line.

Example

Write the equation of the line in point-slope form.

$$m = -\frac{1}{4}$$

$$(-6,1)$$

Since we've been given the slope of the line and a point on the line, we can use the point-slope form to find the equation of the line. We'll plug $m = -1/4$ and the coordinates of the point $(-6,1)$ into the point-slope form of the equation of a line.

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{1}{4}(x - (-6))$$

$$y - 1 = -\frac{1}{4}(x + 6)$$

Let's try an example where we know two points on the line.

Example

Find the point-slope form of the equation of the line that passes through the points $(-2, -4)$ and $(3,5)$. Use $(-2, -4)$ for (x_1, y_1).

First, we need to find the slope of the line. It's best to label the points before we plug them into the slope formula. We'll say

$$(-2, -4) = (x_1, y_1)$$

$$(3,5) = (x_2, y_2)$$

Plug these into the formula for the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5 - (-4)}{3 - (-2)}$$

$$m = \frac{9}{5}$$

Next, substitute $m = 9/5$ and the coordinates of the point $(-2, -4)$ into the equation $y - y_1 = m(x - x_1)$. If you know two or more points on the line, as we do in this problem, you can use the coordinates of any point on the line, and you'll get a correct equation for the line.

$$y - (-4) = \frac{9}{5}(x - (-2))$$

$$y + 4 = \frac{9}{5}(x + 2)$$

Let's try another example where we know two points on the line and need to find the equation of the line in point-slope form.

Example

Find the point-slope form of the equation of the line that passes through the points $(4,2)$ and $(6,3)$.

We start by finding the slope.

$$m = \frac{3 - 2}{6 - 4} = \frac{1}{2}$$

Now plug in the slope and the coordinates of one of the points into the point-slope form of the equation of a line. We'll use the point $(4,2)$.

$$y - 2 = \frac{1}{2}(x - 4)$$

Even though we could simplify this further (by distributing the $1/2$ over the two terms inside the parentheses), we'd end up with something that isn't in point-slope form, so we leave it as is.