Center and radius of a circle

In this lesson we'll look at how to write the equation of a circle in standard form in order to find the center and radius of the circle.

The standard form for the equation of a circle is

$$(x-h)^2+(y-k)^2=r^2$$

where r is the radius and (h,k) are the coordinates of the center.

Sometimes in order to write the equation of a circle in standard form, you'll need to complete the square - on x only, on y only, or on both x and y (separately).

Example

Find the center and radius of the circle.

$$x^2+y^2+24x+10y+160=0$$

In order to find the center and radius, we need to convert the equation of the circle to standard form, $(x-h)^2+(y-k)^2=r^2$, where h and k are the coordinates of the center and r is the radius.

In order to get the given equation into standard form, we have to complete the square on both x and y.

Grouping the terms in x separately from the terms in y, and moving the constant term to the right side, we get

$$(x^2 + 24x) + (y^2 + 10y) = -160$$

To complete the square on x, we need to find the number a that satisfies the equation

$$x^2 + 24x + a^2 = (x + a)^2$$

That is, we need to find the number a for which

$$x^2 + 24x + a^2 = x^2 + 2ax + a^2$$

This means that the coefficient of the x term of the expression inside the first set of parentheses must be equal to $2a$. That coefficient is 24, so we'll set $2a$ equal to 24 and solve for a.

$$2a = 24 \quad \to \quad a = 12$$

To keep our equation balanced, we need to add and subtract a^2 (144) inside that set of parentheses and then regroup.

$$(x^2 + 24x) + (y^2 + 10y) = -160$$

$$(x^2 + 24x + 144 - 144) + (y^2 + 10y) = -160$$

$$(x^2 + 24x + 144) - 144 + (y^2 + 10y) = -160$$

To complete the square on y, we need to find the number b that satisfies the equation

$$y^2 + 10y + b^2 = (y + b)^2$$

That is, we need to find the number b for which

$$y^2 + 10y + b^2 = y^2 + 2by + b^2$$

This means that the coefficient of the y term of the expression inside the second set of parentheses must be equal to $2b$. That coefficient is 10, so we'll set $2b$ equal to 10 and solve for b.

$$2b = 10 \quad \to \quad b = 5$$

To keep our equation balanced, we need to add and subtract b^2 (25) inside that set of parentheses and then regroup.

$$(x^2 + 24x + 144) - 144 + (y^2 + 10y) = -160$$

$$(x^2 + 24x + 144) - 144 + (y^2 + 10y + 25 - 25) = -160$$

$$(x^2 + 24x + 144) - 144 + (y^2 + 10y + 25) - 25 = -160$$

Moving the -144 and -25 to the right side, we have

$$(x^2 + 24x + 144) + (y^2 + 10y + 25) = -160 + 144 + 25$$

Factoring the expressions in parentheses and simplifying the right side, we obtain.

$$(x + 12)^2 + (y + 5)^2 = 9$$

If you think of $x + 12$, $y + 5$, and 9 as $x - (-12)$, $y - (-5)$, and 3^2, respectively, you'll see that the center of the circle is at $(h, k) = (-12, -5)$ and its radius is $r = 3$. Remember that r must be positive, because it's a length, so we can rule out the possibility that $r = -\sqrt{9} = -3$.

Let's do another.

Example

Find the center and radius of the circle.

$$6x^2 + 6y^2 + 12x - 13 = 0$$

In order to find the center and radius, we need to convert the equation of the circle to standard form, $(x-h)^2 + (y-k)^2 = r^2$, where h and k are the coordinates of the center and r is the radius.

Let's begin by grouping the terms in x and moving the -13 to the right side.

$$6x^2 + 12x + 6y^2 = 13$$

In standard form, the coefficients of the x^2 term and the y^2 term must be equal to 1. Since the coefficient of each of those terms is now 6, we'll first factor out a 6 on the left side of the equation and then divide both sides by 6.

$$6(x^2 + 2x + y^2) = 13$$

$$x^2 + 2x + y^2 = \frac{13}{6}$$

Now we'll complete the square on x. There's no need to complete the square on y, because y^2 is already a perfect square.

$$(x^2 + 2x) + y^2 = \frac{13}{6}$$

To complete the square on x, we need to find the number a that satisfies the equation

$$x^2 + 2x + a^2 = (x + a)^2$$

That is, we need to find the number a for which

$$x^2 + 2x + a^2 = x^2 + 2ax + a^2$$

This means that the coefficient of the x term of the expression inside the parentheses must be equal to $2a$. that coefficient is 2, so we'll set $2a$ equal to 2 and solve for a.

$$2a = 2 \quad \to \quad a = 1$$

To keep our equation balanced, we need to add and subtract a^2 (1) inside the parentheses and then regroup.

$$(x^2 + 2x) + y^2 = \frac{13}{6}$$

$$(x^2 + 2x + 1 - 1) + y^2 = \frac{13}{6}$$

$$(x^2 + 2x + 1) - 1 + y^2 = \frac{13}{6}$$

We'll therefore add 1 to both sides, and get

$$(x^2 + 2x + 1) + y^2 = \frac{13}{6} + 1$$

Factoring the expression in parentheses and simplifying the right hand side, we get

$$(x + 1)^2 + y^2 = \frac{19}{6}$$

If you think of $x + 1$, y, and $19/6$ as $x - (-1)$, $y - 0$, and $(\sqrt{19/6})^2$, respectively, you'll see that the center of the circle is at $(h, k) = (-1,0)$ and the radius is $r = \sqrt{19/6}$.