Calculus 1 Workbook Solutions

Idea of the limit

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MATH

IDEA OF THE LIMIT

1. The table below shows some values of a function $g(x)$. What does the table show for the value of $\lim_{x\to 4} g(x)$?

x	$g(x)$
3.9	1.9748
3.99	1.9975
3.999	1.9997
4.001	2.0002
4.01	2.0025
4.1	2.0248

Solution:

2

2. How would you express, mathematically, the limit of the function $f(x) = x^2 - x + 2$ as x approaches 3?

Solution:

$$\lim_{x\to 3} x^2 - x + 2$$

3. How would you write the limit of $g(x)$ as x approaches ∞, using correct mathematical notation?

$$g(x) = \frac{5x^2 - 7}{3x^2 + 8}$$

Solution:

$$\lim_{x\to\infty} \frac{5x^2 - 7}{3x^2 + 8}$$

ONE-SIDED LIMITS

■ 1. Find the limit.

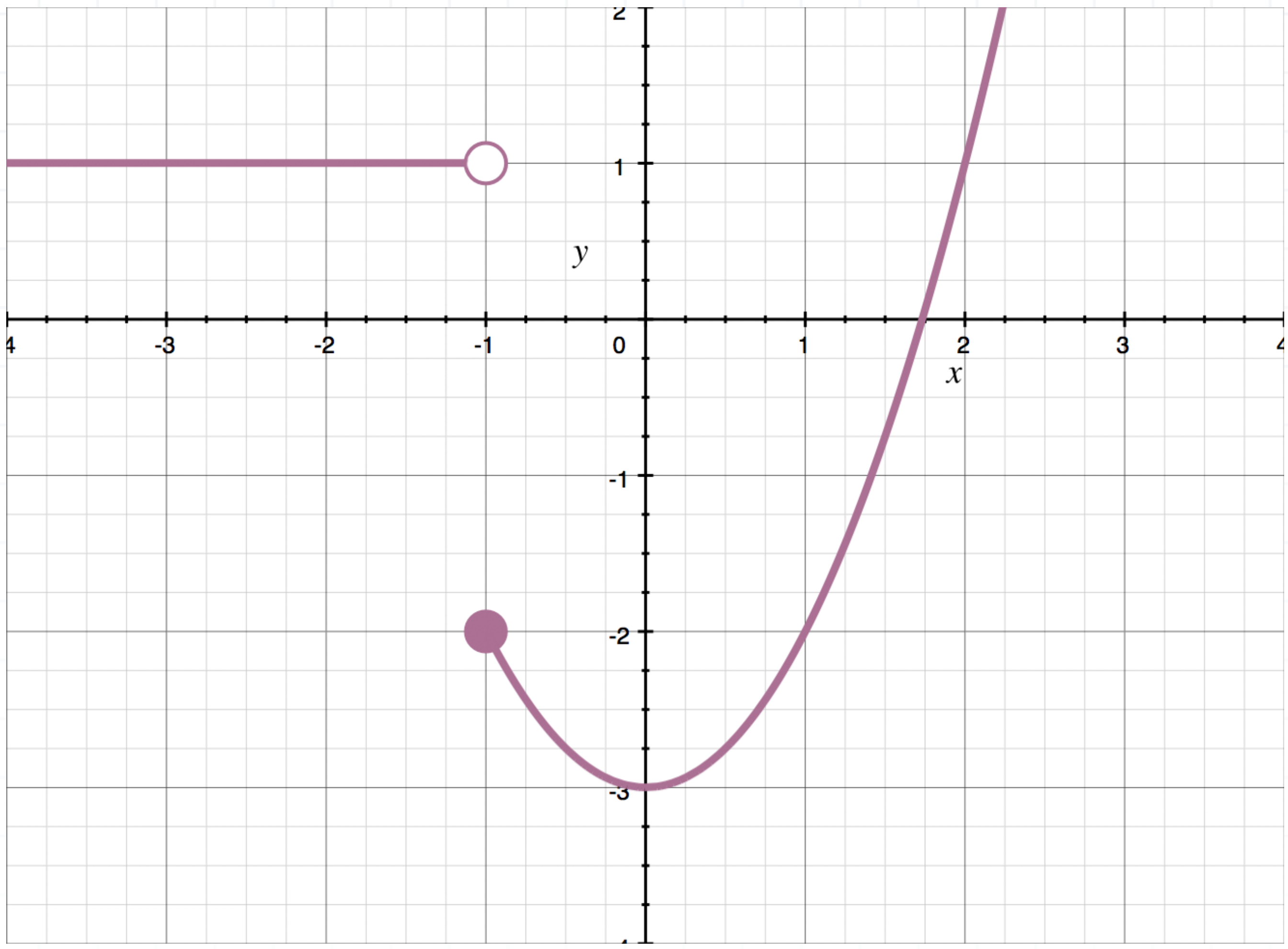
$$\lim_{x\to -7^+} x^2\sqrt{x+7}$$

Solution:

The value of the limit is 0.

x	-6.09	-6.9	-6.99	-6.999	-6.9999	-7
Value	35.38	15.056	4.886	1.5481	0.49	0

■ 2. What does the graph of $f(x)$ say about the value of $\lim_{x\to -1^+} f(x)$?



Solution:

From the graph, the limit is

$$\lim_{x \to -1^+} f(x) = -2$$

■ 3. The table shows values of $k(x)$. What is $\lim_{x \to -5^-} k(x)$?

x	-5.1	-5.01	-5.0001	-5	-4.999	-4.99	-4.9
k(x)	-392.1	-3,812	-38,012	?	37,988	3,788	368.1

Solution:

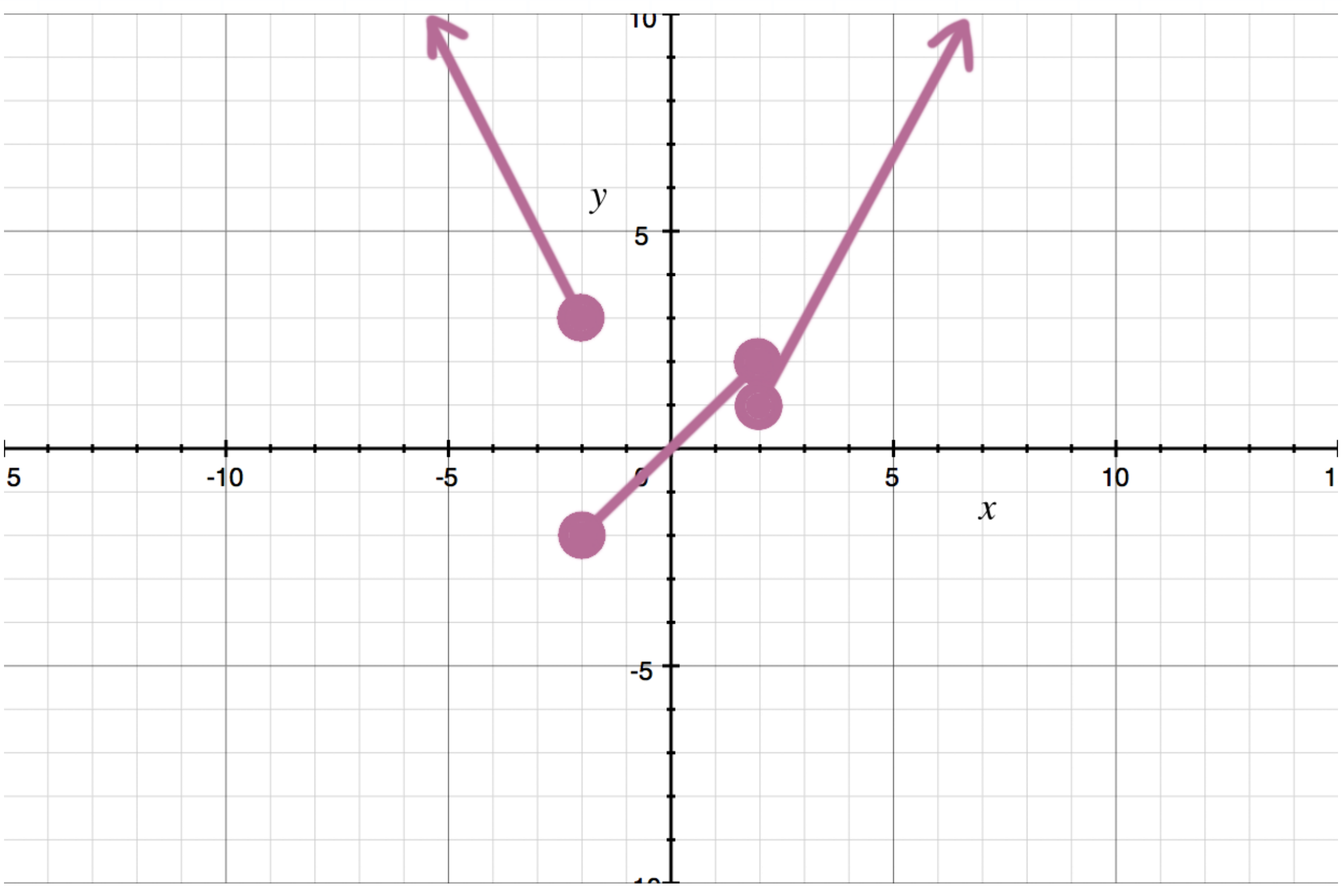
$$\lim_{x\to -5^-} k(x) = -\infty$$

■ 4. What is $\lim_{x\to -2^-} h(x)$?

$$h(x) = \begin{cases} -2x-1 & x < -2 \\ x & -2 \le x < 2 \\ 2x-3 & x \ge 2 \end{cases}$$

Solution:

The graph of $h(x)$ is



Based on the graph, the limit is 3. Or we could plug into the first piece of the function, which is the piece that approaches $x = -2$ from the left side.

$$\lim_{x \to -2^-} h(x) = \left[-2(-2) - 1\right] = 3$$

5. What is $\lim_{x \to 6^+} g(x)$?

$$g(x) = \frac{x^2 + x - 42}{x - 6}$$

Solution:

We could tell that the limit is 13 by making a table,

x	6	6.001	6.01	6.1
g(x)	?	13.001	13.01	13.1

Alternatively, we could have factored the numerator, canceled like terms, and then evaluated at the limit.

$$g(x) = \frac{x^2 + x - 42}{x - 6}$$

$$g(x) = \frac{(x + 7)(x - 6)}{x - 6}$$

$$g(x) = x + 7$$

Then the limit is

$$\lim_{x \to 6^+} x + 7$$

$$6 + 7$$

$$13$$

PROVING THAT THE LIMIT DOES NOT EXIST

1. Prove that the limit does not exist.

$$\lim_{x \to 0} \frac{-2|3x|}{3x}$$

Solution:

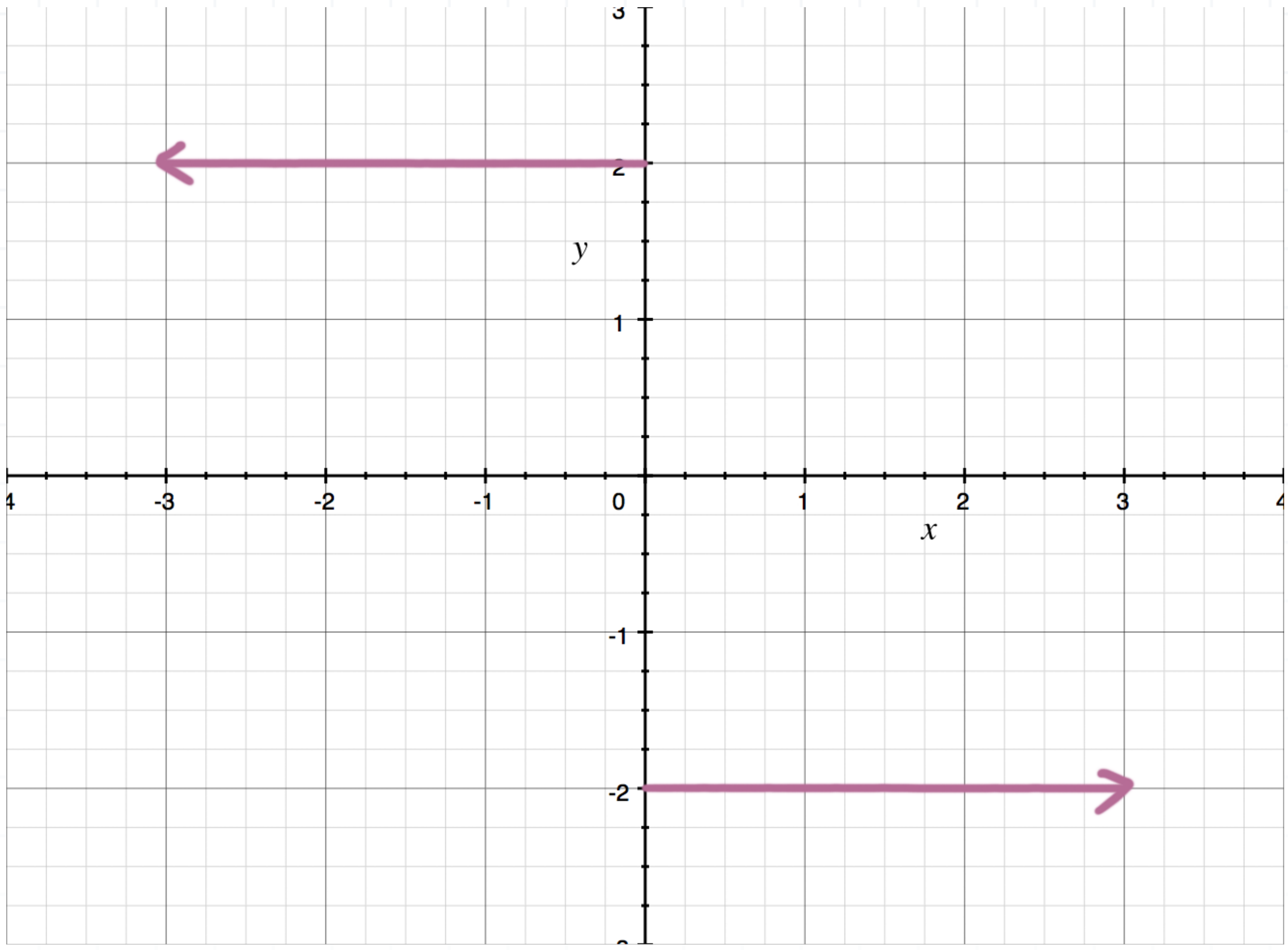
The left-hand limit is

$$\lim_{x \to 0^-} \frac{-2|3x|}{3x} = \frac{6x}{3x} = 2$$

The right-hand limit is

$$\lim_{x \to 0^+} \frac{-2|3x|}{3x} = \frac{-6x}{3x} = -2$$

Since the left- and right-hand limits aren't equal, the limit does not exist. The graph of the function would also prove that the limit doesn't exist.



2. Prove that the limit does not exist.

$$\lim_{x \to -5} \frac{x^2 + 7x + 9}{x^2 - 25}$$

Solution:

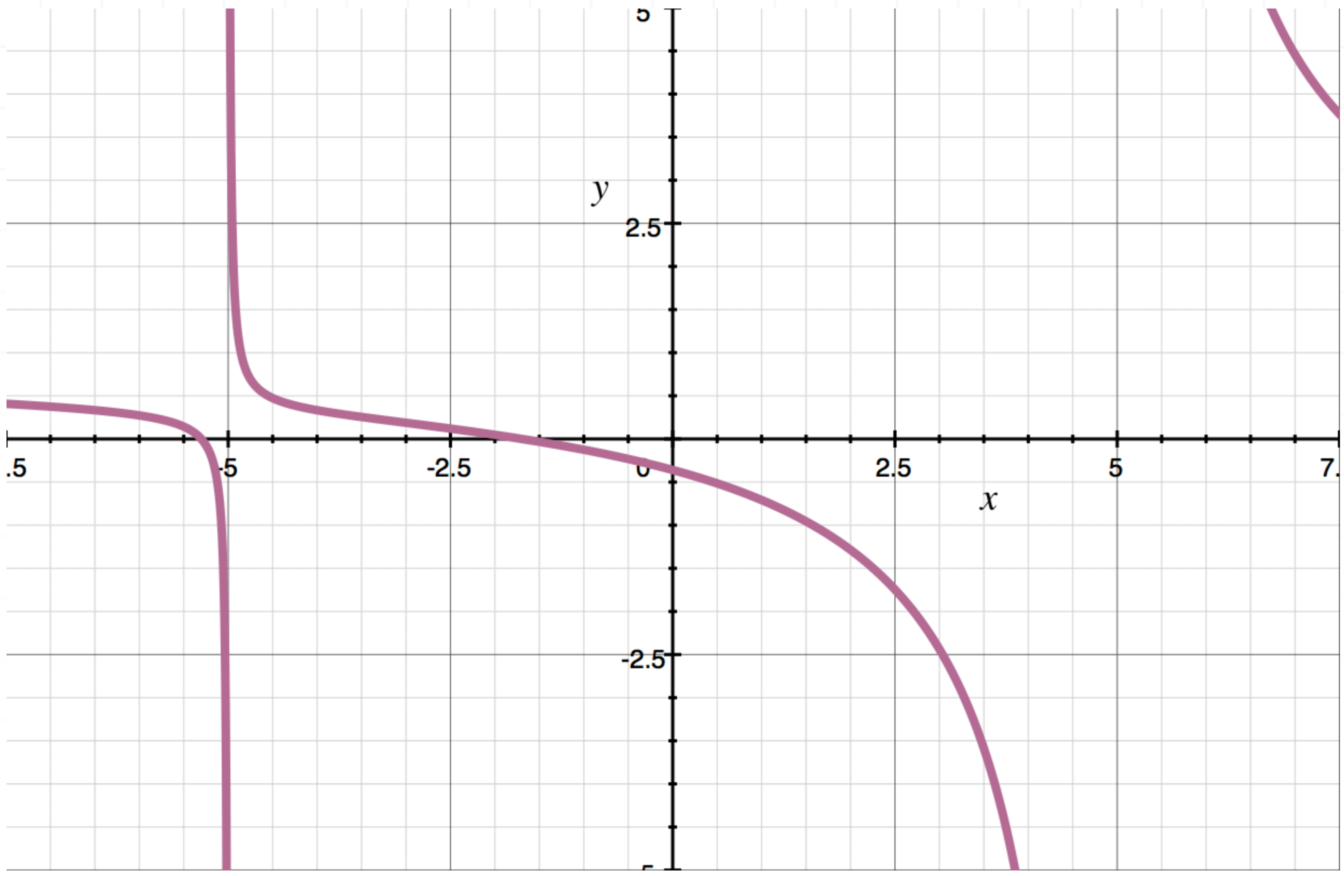
The left-hand limit is

$$\lim_{x \to -5^-} \frac{x^2 + 7x + 9}{x^2 - 25} = -\infty$$

The right-hand limit is

$$\lim_{x \to -5^+} \frac{x^2+7x+9}{x^2-25} = \infty$$

Since the left- and right-hand limits aren't equal, the limit does not exist. The graph of the function would also prove that the limit doesn't exist.



3. Prove that $\lim_{x \to 1} f(x)$ does not exist.

$$f(x) = \begin{cases} -3x+2 & x < 1 \\ 3x-2 & x \geq 1 \end{cases}$$

Solution:

The left-hand limit is

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (-3x + 2) = \left[-3(1) + 2\right] = -1$$

The right-hand limit is

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (3x - 2) = \left[3(1) - 2\right] = 1$$

Because the left- and right-hand limits aren't equal, the limit does not exist.

$$\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$$

PRECISE DEFINITION OF THE LIMIT

■ 1. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \to 4} 5x - 16 = 4$$

Solution:

If $0 < |x - 4| < \delta$, then $|(5x - 16) - 4| < \epsilon$. So,

$$|5x - 20| < \epsilon$$

$$|5(x - 4)| < \epsilon$$

$$|5| \cdot |x - 4| < \epsilon$$

$$5 \cdot |x - 4| < \epsilon$$

$$|x - 4| < \frac{\epsilon}{5}$$

Now if $|x - 4| < \epsilon/5$ and $0 < |x - 4| < \delta$, then if $\epsilon > 0$ then $\delta < \epsilon/5$. Therefore, the limit equation is true.

■ 2. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \to -7} -2x + 15 = 29$$

Solution:

If $0 < |x - (-7)| < \delta$ then $|-2x + 15 - 29| < \epsilon$. Or we could rewrite this as $0 < |x + 7| < \delta$ and $|-2x - 14| < \epsilon$. So,

$$|(-2)(x + 7)| < \epsilon$$

$$|-2| \cdot |x + 7| < \epsilon$$

$$2 \cdot |x + 7| < \epsilon$$

$$|x + 7| < \frac{\epsilon}{2}$$

Now if $|x + 7| < \epsilon/2$ and $0 < |x + 7| < \delta$, then if $\epsilon > 0$ then $\delta < \epsilon/2$. Therefore, the limit equation is true.

■ 3. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \to 16} \left(\frac{2}{5}x - \frac{17}{5} \right) = 3$$

Solution:

If $0 < |x - 16| < \delta$ then $\left| \left((2/5)x - (17/5) \right) - 3 \right| < \epsilon$. Or we could rewrite this as $0 < |x - 16| < \delta$ and

$$\left|\left(\frac{2}{5}x - \frac{17}{5}\right) - \frac{15}{5}\right| < \epsilon$$

$$\left|\frac{2}{5}x - \frac{32}{5}\right| < \epsilon$$

$$\left|\frac{2}{5}(x - 16)\right| < \epsilon$$

$$\left|\frac{2}{5}\right| |x - 16| < \epsilon$$

$$|x - 16| < \frac{5}{2}\epsilon$$

Now if $|x - 16| < (5/2)\epsilon$ and $0 < |x - 16| < \delta$, then if $\epsilon > 0$, then $\delta < (5/2)\epsilon$. Therefore, the limit equation is true.

■ 4. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \to 7} \left(\frac{x^2 - 15x + 56}{x - 7}\right) = -1$$

Solution:

We'll apply the precise definition to the given limit.

If $0 < |x-7| < \delta$, then $\left| \left(\frac{x^2-15x+56}{x-7} \right) - (-1) \right| < \epsilon$.

If $0 < |x-7| < \delta$, then $\left| \left(\frac{x^2-15x+56}{x-7} \right) - \frac{-1(x-7)}{x-7} \right| < \epsilon$.

So,

$$\left| \left(\frac{x^2-15x+56}{x-7} \right) + \frac{x-7}{x-7} \right| < \epsilon$$

$$\left| \frac{x^2-14x+49}{x-7} \right| < \epsilon$$

$$\left| \frac{(x-7)(x-7)}{x-7} \right| < \epsilon$$

$$|x-7| < \varepsilon$$

Now, if $|x-7| < \epsilon$ and $0 < |x-7| < \delta$, then if $\epsilon > 0$ and $\delta < \epsilon$. Therefore, the limit equation is true.

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