Topic: Composite functions

Question: Find the composite function.

$$f(x) = \frac{1}{x^2}$$

$$g(x) = \sqrt{x - 3}$$

Answer choices:

$$\mathbf{A} \qquad g(f(x)) = \frac{1}{x - 3}$$

$$\mathsf{B} \qquad g(f(x)) = \sqrt{\frac{1}{x^2} - 3}$$

C
$$g(f(x)) = \sqrt{\frac{1}{(x-3)^2}}$$

$$D g(f(x)) = \frac{1}{\sqrt{x-3}}$$



Solution: B

To find the composite function g(f(x)), we plug f(x) into g(x), which means that we take the algebraic expression for f(x) and substitute it for x in the algebraic expression for g(x).

$$g(f(x)) = \sqrt{\frac{1}{x^2} - 3}$$



Topic: Composite functions

Question: Find g(h(x)).

$$g(x) = x^2 - x - 4$$

$$h(x) = x\sqrt{2} + 1$$

Answer choices:

A
$$\sqrt{2}(x^2 - x - 4) + 1$$

B
$$2x^2 + x\sqrt{2} - 4$$

C
$$\sqrt{2}x^2 - 2x + 3$$

D
$$2x^2 + 3x\sqrt{2} + 5$$

Solution: B

To find g(h(x)), we have to plug h(x) into g(x). Given

$$g(x) = x^2 - x - 4$$

$$h(x) = x\sqrt{2} + 1$$

we get

$$g(h(x)) = \left(x\sqrt{2} + 1\right)^2 - \left(x\sqrt{2} + 1\right) - 4$$

$$g(h(x)) = 2x^2 + 2x\sqrt{2} + 1 - x\sqrt{2} - 1 - 4$$

$$g(h(x)) = 2x^2 + x\sqrt{2} - 4$$



Topic: Composite functions

Question: Find f(g(x)) - g(f(x)).

$$f(x) = x^2 - 2x$$

$$g(x) = 3x + 1$$

Answer choices:

A
$$6x^2 - 6x + 2$$

B
$$6x^2 + 6x + 2$$

C
$$6x^2 - 6x - 2$$

D
$$6x^2 + 6x - 2$$

Solution: D

To find f(g(x)), we have to plug g(x) into f(x), and to find g(f(x)), we have to plug f(x) into g(x). Given

$$f(x) = x^2 - 2x$$

$$g(x) = 3x + 1$$

we get

$$f(g(x)) = (3x+1)^2 - 2(3x+1)$$

$$f(g(x)) = 9x^2 + 6x + 1 - 6x - 2$$

$$f(g(x)) = 9x^2 - 1$$

and

$$g(f(x)) = 3(x^2 - 2x) + 1$$

$$g(f(x)) = 3x^2 - 6x + 1$$

Therefore, the function f(g(x)) - g(f(x)), which is the difference of the composite functions f(g(x)) and g(f(x)), is

$$f(g(x)) - g(f(x)) = (9x^2 - 1) - (3x^2 - 6x + 1)$$

$$f(g(x)) - g(f(x)) = 9x^2 - 1 - 3x^2 + 6x - 1$$

$$f(g(x)) - g(f(x)) = 6x^2 + 6x - 2$$