Intermediate value theorem without an interval

When we're asked to use the intermediate value theorem to prove that the function has a root, but we're not given the interval in which to find the root, then we're forced to come up with our own interval.

Because the intermediate value theorem can only be used on closed intervals $[a, b]$, we'll need to find a closed interval to investigate.

There are different ways we can go about finding the interval, the first of which is to blindly guess. We can try picking two random values to be the ends of the interval, and we might get lucky and find that a negative value at one end and a positive value at the other.

If that doesn't work, we might be able to consider some other aspect of the function. For instance, if we're dealing with the trigonometric function $\sin x$ or $\cos x$, we know that those functions oscillate back and forth between -1 and 1. So we could choose a value where the trig function is -1, and another value where it's 1, and use those as the endpoints of the interval, since the rest of the graph will simply be an identical section to that one.

Example

Use the intermediate value theorem to prove that the function has at least one real root.

$$f(x) = (x - 1)^3$$

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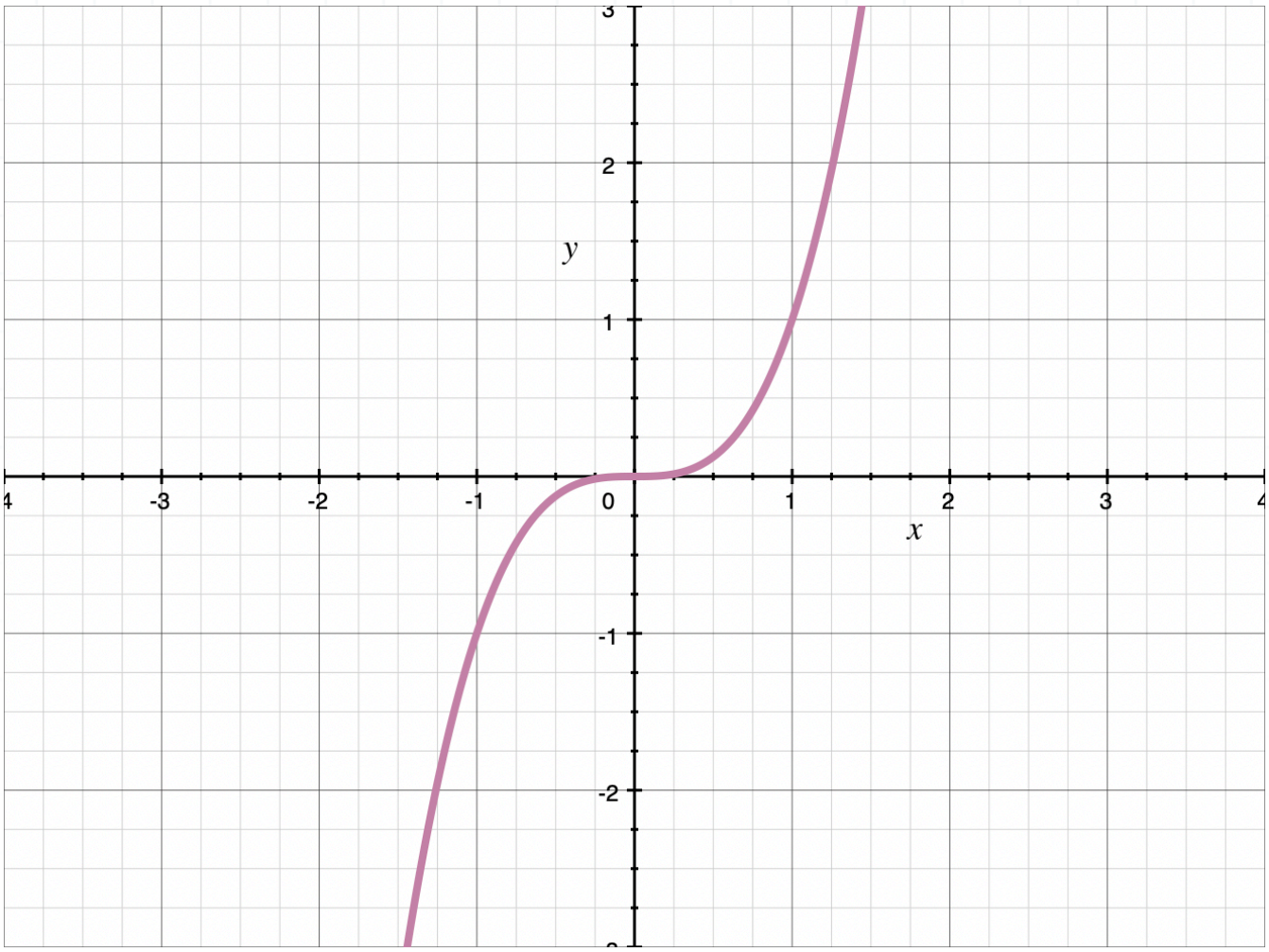
This is a fairly simple function, so we can actually sketch it, and then pick an interval around the root.

If we think first about the function $f(x) = x^3$, we can sketch it.



When we transform $f(x) = x^3$ into $f(x) = (x - 1)^3$, we're simply replacing x with $x - 1$, which means the graph shifts 1 unit to the right, and the graph of $f(x) = (x - 1)^3$ is

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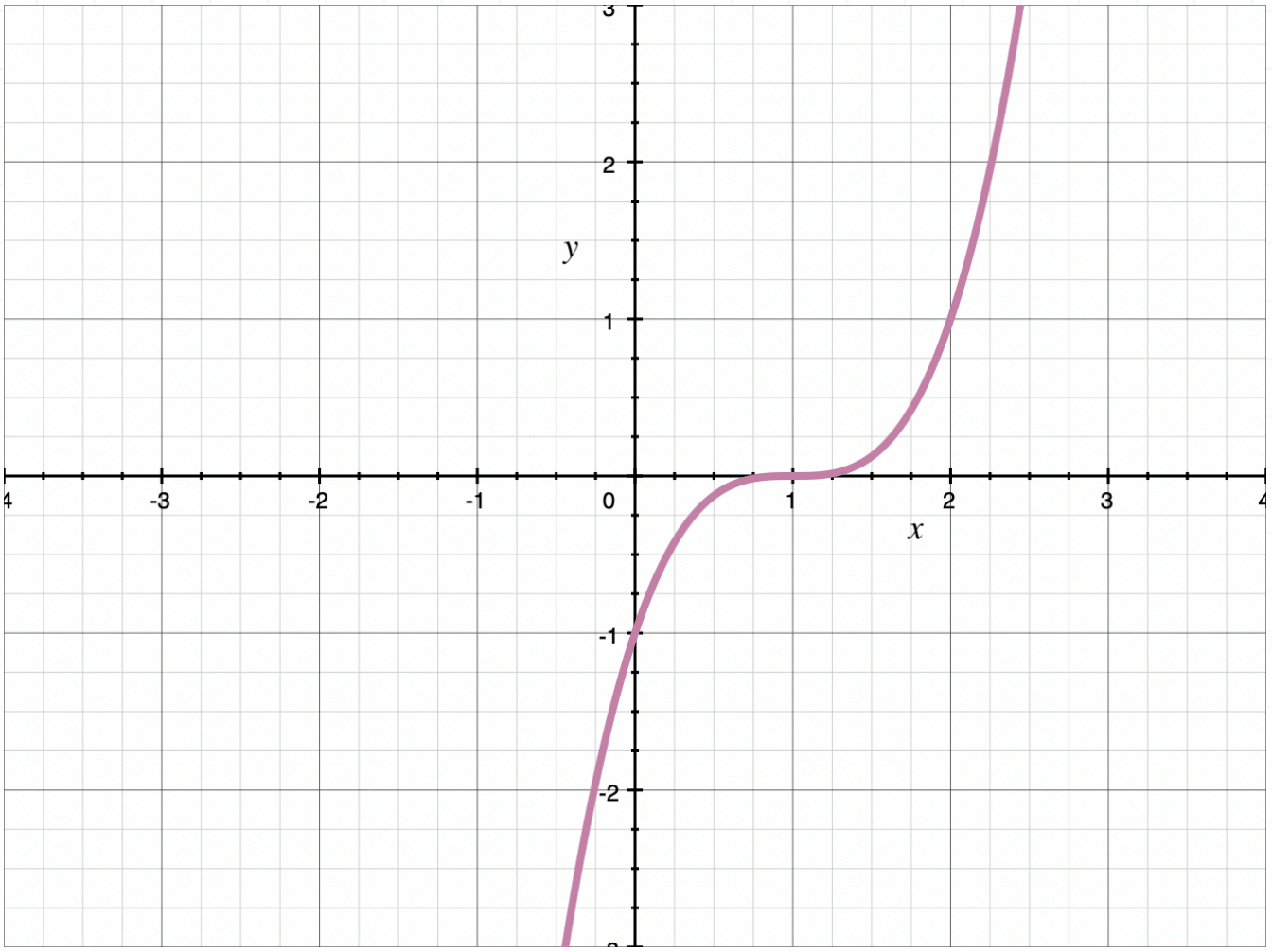

From the graph, we expect the root of the function to be at $x = 1$, so let's choose the interval $[0,2]$, and test it. For the endpoints, we get

$$f(0) = (0 - 1)^3$$

$$f(0) = (-1)^3$$

$$f(0) = -1$$

and

$$f(2) = (2 - 1)^3$$

$$f(2) = (1)^3$$

$$f(2) = 1$$

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Because we get a negative value at the left edge of the interval, and a positive value at the right edge of the interval, we've proven with the intermediate value theorem that the function $f(x) = (x - 1)^3$ has a root in the interval $[0,2]$.

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