Topic: Observer and the airplane

Question: An airplane is flying horizontally at 720 miles/hr, 3 miles above the ground, when it passes an observer on the ground. How fast is the distance between the person and the plane increasing 10 seconds later?

Answer choices:

A About 400 miles/hr

B About 500 miles/hr

C About 600 miles/hr

D About 700 miles/hr

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Solution: A

The horizontal path of the plane, the vertical distance between the observer and that path, and the diagonal connecting the observer to the plane's location, together form a right triangle.

So we'll use the Pythagorean theorem, which relates the three side lengths of a right triangle.

$$a^2 + b^2 = c^2$$

Use implicit differentiation to take the derivative of both sides.

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

We'll call a the vertical distance, b the horizontal distance, and c the diagonal distance. We know from the question that $a = 3$. And because a stays constant, $da/dt = 0$.

$$2(3)(0) + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

Convert $t = 10$ seconds to hours,

$$x \text{ hours} = 10 \text{ seconds} \times \frac{1 \text{ minute}}{60 \text{ seconds}} \times \frac{1 \text{ hour}}{60 \text{ minutes}}$$

$$x \text{ hours} = 0.00278 \text{ hours}$$

then use it to find the horizontal distance b.

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$$b \text{ miles} = 0.00278 \text{ hours} \times \frac{720 \text{ miles}}{\text{hour}}$$

$$b \text{ miles} \approx 2 \text{ miles}$$

With $a = 3$ and $b \approx 2$, we can find c.

$$a^2 + b^2 = c^2$$

$$3^2 + 2^2 = c^2$$

$$9 + 4 = c^2$$

$$13 = c^2$$

$$c \approx 3.61$$

Substitute $b = 2$ and $c = 3.61$, along with $db/dt = 720$, into the derivative. This will give us the rate at which the distance between the observer and the airplane is increasing.

$$2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2(2)(720) = 2(3.61)\ \frac{dc}{dt}$$

$$\frac{dc}{dt} \approx 399$$

Topic: Observer and the airplane

Question: An airplane is flying horizontally at 540 miles/hr, 4 miles above the ground, when it passes an observer on the ground. How fast is the distance between the person and the plane increasing 20 seconds later?

Answer choices:

A About 424 miles/hr

B About 324 miles/hr

C About 224 miles/hr

D About 124 miles/hr

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Solution: B

The horizontal path of the plane, the vertical distance between the observer and that path, and the diagonal connecting the observer to the plane's location, together form a right triangle.

So we'll use the Pythagorean theorem, which relates the three side lengths of a right triangle.

$$a^2 + b^2 = c^2$$

Use implicit differentiation to take the derivative of both sides.

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

We'll call a the vertical distance, b the horizontal distance, and c the diagonal distance. We know from the question that $a = 4$. And because a stays constant, $da/dt = 0$.

$$2(4)(0) + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

Convert $t = 20$ seconds to hours,

$$x \text{ hours} = 20 \text{ seconds} \times \frac{1 \text{ minute}}{60 \text{ seconds}} \times \frac{1 \text{ hour}}{60 \text{ minutes}}$$

$$x \text{ hours} = 0.0056 \text{ hours}$$

then use it to find the horizontal distance b.

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$$b \text{ miles} = 0.0056 \text{ hours} \times \frac{540 \text{ miles}}{\text{hour}}$$

$$b \text{ miles} \approx 3 \text{ miles}$$

With $a = 4$ and $b \approx 3$, we can find c.

$$a^2 + b^2 = c^2$$

$$4^2 + 3^2 = c^2$$

$$16 + 9 = c^2$$

$$25 = c^2$$

$$c = 5$$

Substitute $b = 3$ and $c = 5$, along with $db/dt = 540$, into the derivative. This will give us the rate at which the distance between the observer and the airplane is increasing.

$$2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2(3)(540) = 2(5)\ \frac{dc}{dt}$$

$$\frac{dc}{dt} = 324$$

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Topic: Observer and the airplane

Question: An airplane is flying horizontally at 360 miles/hr, 5 miles above the ground, when it passes an observer on the ground. How fast is the distance between the person and the plane increasing 40 seconds later?

Answer choices:

A 125 miles/hr

B 225 miles/hr

C 325 miles/hr

D 425 miles/hr

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Solution: B

The horizontal path of the plane, the vertical distance between the observer and that path, and the diagonal connecting the observer to the plane's location, together form a right triangle.

So we'll use the Pythagorean theorem, which relates the three side lengths of a right triangle.

$$a^2 + b^2 = c^2$$

Use implicit differentiation to take the derivative of both sides.

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

We'll call a the vertical distance, b the horizontal distance, and c the diagonal distance. We know from the question that $a = 5$. And because a stays constant, $da/dt = 0$.

$$2(5)(0) + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

Convert $t = 40$ seconds to hours,

$$x \text{ hours} = 40 \text{ seconds} \times \frac{1 \text{ minute}}{60 \text{ seconds}} \times \frac{1 \text{ hour}}{60 \text{ minutes}}$$

$$x \text{ hours} = 0.0111 \text{ hours}$$

then use it to find the horizontal distance b.

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$$b \text{ miles} = 0.0111 \text{ hours} \times \frac{360 \text{ miles}}{\text{hour}}$$

$$b \text{ miles} \approx 4 \text{ miles}$$

With $a = 5$ and $b \approx 4$, we can find c.

$$a^2 + b^2 = c^2$$

$$5^2 + 4^2 = c^2$$

$$25 + 16 = c^2$$

$$41 = c^2$$

$$c \approx 6.4$$

Substitute $b = 4$ and $c = 6.4$, along with $db/dt = 360$, into the derivative. This will give us the rate at which the distance between the observer and the airplane is increasing.

$$2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2(4)(360) = 2(6.4)\ \frac{dc}{dt}$$

$$\frac{dc}{dt} = 225$$

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