Calculus 1 Workbook

Idea of the limit

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MATH

IDEA OF THE LIMIT

■ 1. The table below shows some values of a function $g(x)$. What does the table show for the value of $\lim_{x \to 4} g(x)$?

x	$g(x)$
3.9	1.9748
3.99	1.9975
3.999	1.9997
4.001	2.0002
4.01	2.0025
4.1	2.0248

■ 2. How would you express, mathematically, the limit of the function $f(x) = x^2 - x + 2$ as x approaches 3?

■ 3. How would you write the limit of $g(x)$ as x approaches ∞, using correct mathematical notation?
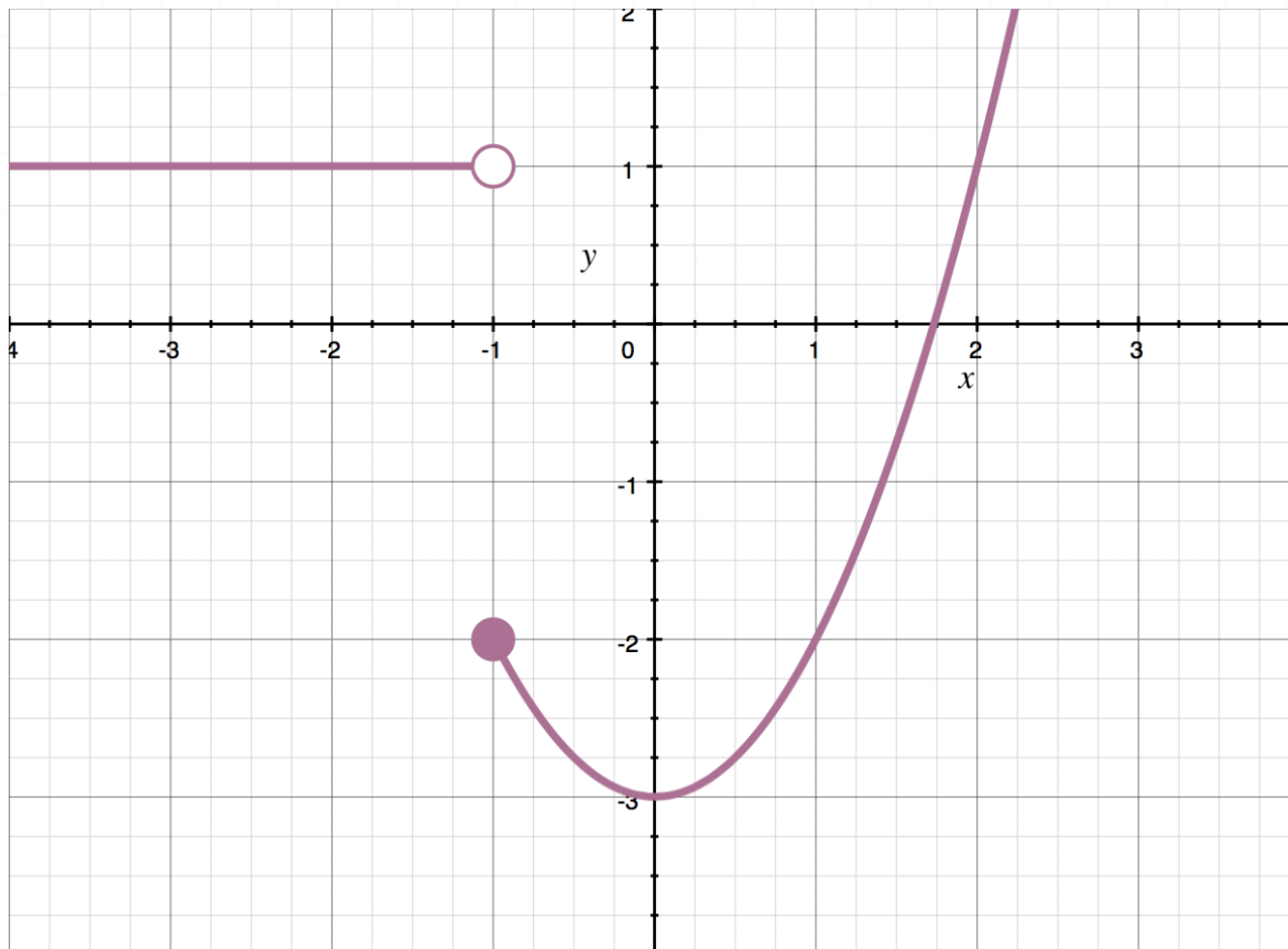
$$g(x) = \frac{5x^2 - 7}{3x^2 + 8}$$

ONE-SIDED LIMITS

■ 1. Find the limit.

$$\lim_{x \to -7^+} x^2 \sqrt{x + 7}$$

■ 2. What does the graph of $f(x)$ say about the value of $\lim_{x \to -1^+} f(x)$?

■ 3. The table shows values of $k(x)$. What is $\lim_{x \to -5^-} k(x)$?

x	-5.1	-5.01	-5.0001	-5	-4.999	-4.99	-4.9
k(x)	-392.1	-3,812	-38,012	?	37,988	3,788	368.1

■ 4. What is $\lim\limits_{x \to -2^-} h(x)$?

$$h(x) = \begin{cases} -2x - 1 & x < -2 \\ x & -2 \leq x < 2 \\ 2x - 3 & x \geq 2 \end{cases}$$

■ 5. What is $\lim\limits_{x \to 6^+} g(x)$?

$$g(x) = \frac{x^2 + x - 42}{x - 6}$$

PROVING THAT THE LIMIT DOES NOT EXIST

■ 1. Prove that the limit does not exist.

$$\lim_{x \to 0} \frac{-2\,|3x|}{3x}$$

■ 2. Prove that the limit does not exist.

$$\lim_{x \to -5} \frac{x^2 + 7x + 9}{x^2 - 25}$$

■ 3. Prove that $\lim_{x \to 1} f(x)$ does not exist.

$$f(x) = \begin{cases} -3x + 2 & x < 1 \\ 3x - 2 & x \geq 1 \end{cases}$$

PRECISE DEFINITION OF THE LIMIT

■ 1. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \to 4} 5x - 16 = 4$$

■ 2. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \to -7} -2x + 15 = 29$$

■ 3. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \to 16} \left(\frac{2}{5}x - \frac{17}{5} \right) = 3$$

■ 4. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \to 7} \left(\frac{x^2 - 15x + 56}{x - 7} \right) = -1$$