

**Topic:** Critical points and the first derivative test**Question:** Find the critical point of the function.

$$f(x) = x^2 - 10x + 2$$

**Answer choices:**

- A  $x = \frac{1}{5}$
- B  $x = 5$
- C  $x = -5$
- D  $x = -\frac{1}{5}$



**Solution: B**

Take the derivative of the function.

$$f(x) = x^2 - 10x + 2$$

$$f'(x) = 2x - 10$$

Set the derivative equal to 0 and solve for  $x$ .

$$2x - 10 = 0$$

$$2x = 10$$

$$x = 5$$

The function has one potential critical point at  $x = 5$ .



**Topic:** Critical points and the first derivative test**Question:** Where is the function increasing and decreasing?

$$f(x) = x^2$$

**Answer choices:**

- A      Increasing on  $x < 1$  and decreasing on  $x > 1$
- B      Increasing on  $x < 0$  and decreasing on  $x > 0$
- C      Increasing on  $x > 0$  and decreasing on  $x < 0$
- D      Increasing on  $x > 1$  and decreasing on  $x < 1$



**Solution: C**

Find the derivative.

$$f(x) = x^2$$

$$f'(x) = 2x$$

Set the derivative equal to 0 and solve for  $x$ .

$$0 = 2x$$

$$x = 0$$

Investigate the critical point  $x = 0$  by testing  $x = -1$  and  $x = 1$  in the first derivative.

$$f'(-1) = 2(-1)$$

$$f'(-1) = -2$$

and

$$f'(1) = 2(1)$$

$$f'(1) = 2$$

On the left side of  $x = 0$  the derivative is negative so the function is decreasing. On the right side of  $x = 0$  the derivative is positive so the function is increasing.



**Topic:** Critical points and the first derivative test**Question:** Where is the function increasing and decreasing?

$$f(x) = x^4 - 4x^3 + 4x^2 - 7$$

**Answer choices:**

- A Decreasing on  $x < 0$  and  $1/2 < x < 3/2$ , increasing on  $0 < x < 1/2$  and  $x > 3/2$
- B Decreasing on  $0 < x < 1/2$  and  $x > 3/2$ , increasing on  $x < 0$  and  $1/2 < x < 3/2$
- C Decreasing on  $x < 0$  and  $1 < x < 2$ , increasing on  $0 < x < 1$  and  $x > 2$
- D Decreasing on  $0 < x < 1$  and  $x > 2$ , increasing on  $x < 0$  and  $1 < x < 2$



**Solution: C**

Take the first derivative of the function.

$$f(x) = x^4 - 4x^3 + 4x^2 - 7$$

$$f'(x) = 4x^3 - 12x^2 + 8x$$

$$f'(x) = 4x(x^2 - 3x + 2)$$

$$f'(x) = 4x(x - 2)(x - 1)$$

Set the derivative equal to 0 and solve for  $x$ .

$$4x(x - 2)(x - 1) = 0$$

$$x = 0, 1, 2$$

Investigate each interval by evaluating the first derivative at  $x = -1$ ,  $x = 1/2$ ,  $x = 3/2$ , and  $x = 3$ .

$$f'(-1) = 4(-1)^3 - 12(-1)^2 + 8(-1)$$

$$f'(-1) = -4 - 12 - 8$$

$$f'(-1) = -24$$

and

$$f'\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 8\left(\frac{1}{2}\right)$$

$$f'\left(\frac{1}{2}\right) = 4\left(\frac{1}{8}\right) - 12\left(\frac{1}{4}\right) + 4$$



$$f'\left(\frac{1}{2}\right) = \frac{1}{2} - 3 + 4$$

$$f'\left(\frac{1}{2}\right) = \frac{3}{2}$$

and

$$f'\left(\frac{3}{2}\right) = 4\left(\frac{3}{2}\right)^3 - 12\left(\frac{3}{2}\right)^2 + 8\left(\frac{3}{2}\right)$$

$$f'\left(\frac{3}{2}\right) = 4\left(\frac{27}{8}\right) - 12\left(\frac{9}{4}\right) + 12$$

$$f'\left(\frac{3}{2}\right) = \frac{27}{2} - 27 + 12$$

$$f'\left(\frac{3}{2}\right) = -\frac{3}{2}$$

and

$$f'(3) = 4(3)^3 - 12(3)^2 + 8(3)$$

$$f'(3) = 4(27) - 12(9) + 24$$

$$f'(3) = 108 - 108 + 24$$

$$f'(3) = 24$$

To the left of  $x = 0$  the derivative is negative so the function is decreasing. Between  $x = 0$  and  $x = 1$ , the derivative is positive so the function is increasing. Between  $x = 1$  and  $x = 2$ , the derivative is negative so the



function is decreasing. To the right of  $x = 2$  the derivative is positive so the function is increasing.

The function  $f(x) = x^4 - 4x^3 + 4x^2 - 7$  is decreasing when  $x < 0$ , increasing between  $x = 0$  and  $x = 1$ , decreasing between  $x = 1$  and  $x = 2$ , and increasing when  $x > 2$ .

