Topic: Equation of the tangent line with implicit differentiation

Question: Find the equation of the tangent line to the curve at $(3,2)$.

$$\frac{x^2}{9}+\frac{y^2}{4}=2$$

Answer choices:

A $\quad y=-\frac{2}{3}x+4$

B $\quad y=-\frac{2}{3}x-4$

C $\quad y=\frac{2}{3}x+4$

D $\quad y=\frac{2}{3}x-4$

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Solution: A

Using implicit differentiation to find the derivative of the curve, we get

$$\frac{2x}{9}+\frac{2y}{4}y'=0$$

Simplify and solve for y'.

$$\frac{2y}{4}y'=-\frac{2x}{9}$$

$$y'=-\frac{2x(4)}{9(2y)}$$

$$y'=-\frac{8x}{18y}$$

$$y'=-\frac{4x}{9y}$$

Evaluate the derivative at $(3,2)$ to find the slope of the tangent line.

$$m=-\frac{4(3)}{9(2)}$$

$$m=-\frac{12}{18}$$

$$m=-\frac{2}{3}$$

Plug the slope $m=-2/3$ and the point of tangency $(3,2)$ into the point-slope formula for the equation of a line.

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$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{2}{3}(x - 3)$$

$$y - 2 = -\frac{2}{3}x + 2$$

$$y = -\frac{2}{3}x + 4$$

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Topic: Equation of the tangent line with implicit differentiation

Question: Find the equation of the tangent line to the curve at $(0,1)$.

$$5x^2 + y^2 + 4xy = 1$$

Answer choices:

A $y = -2x - 1$

B $y = -2x + 1$

C $y = 2x - 1$

D $y = 2x + 1$

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Solution: B

Using implicit differentiation to find the derivative of the curve, we get

$$10x + 2yy' + [(4)(y) + (4x)(1)(y')] = 0$$

$$10x + 2yy' + 4y + 4xy' = 0$$

Simplify and solve for y'.

$$2yy' + 4xy' = -10x - 4y$$

$$y'(2y + 4x) = -10x - 4y$$

$$y' = -\frac{10x + 4y}{2y + 4x}$$

Evaluate the derivative at $(0,1)$ to find the slope of the tangent line.

$$m = -\frac{10(0) + 4(1)}{2(1) + 4(0)}$$

$$m = -\frac{4}{2}$$

$$m = -2$$

Plug the slope $m = -2$ and the point of tangency $(0,1)$ into the point-slope formula for the equation of a line.

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -2(x - 0)$$

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$$y - 1 = -2x$$

$$y = -2x + 1$$

Topic: Equation of the tangent line with implicit differentiation

Question: Find the equation of the tangent line to the curve at $(1,4)$.

$$8 + x^2y^2 - 5xy = 4$$

Answer choices:

A $y = 4x - 8$

B $y = 4x + 8$

C $y = -4x - 8$

D $y = -4x + 8$

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Solution: D

Using implicit differentiation to find the derivative of the curve, we get

$$0 + [(2x)(y^2) + (x^2)(2yy')] - [(5)(y) + (5x)(1)(y')] = 0$$

$$2xy^2 + 2x^2yy' - 5y - 5xy' = 0$$

Simplify and solve for y'.

$$2x^2yy' - 5xy' = 5y - 2xy^2$$

$$y'(2x^2y - 5x) = 5y - 2xy^2$$

$$y' = \frac{5y - 2xy^2}{2x^2y - 5x}$$

Evaluate the derivative at $(1,4)$ to find the slope of the tangent line.

$$m = \frac{5(4) - 2(1)(4)^2}{2(1)^2(4) - 5(1)}$$

$$m = \frac{20 - 32}{8 - 5}$$

$$m = \frac{-12}{3}$$

$$m = -4$$

Plug the slope $m = -4$ and the point of tangency $(1,4)$ into the point-slope formula for the equation of a line.

$$y - y_1 = m(x - x_1)$$

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$$y - 4 = -4(x - 1)$$

$$y - 4 = -4x + 4$$

$$y = -4x + 8$$

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