Calculus 1 Workbook Solutions

Tangent and normal lines

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TANGENT LINES

1. Find the equation of the tangent line to the graph of the equation at $(1/2,\pi)$.

$$f(x) = 4\arctan 2x$$

Solution:

The derivative of $\arctan x$ is given by

$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

So the derivative is

$$f'(x) = \frac{4}{1+(2x)^2}\cdot 2$$

$$f'(x) = \frac{8}{1+4x^2}$$

Evaluating the derivative at $(1/2,\pi)$, we get

$$f'\left(\frac{1}{2}\right) = \frac{8}{1+4\left(\frac{1}{2}\right)^2} = \frac{8}{1+1} = \frac{8}{2} = 4$$

Now we can find the equation of the tangent line by plugging the slope $f'(1/2)=4$ and the point $(1/2,\pi)$ into the formula for the equation of the tangent line.

$$y=f(a)+f'(a)(x-a)$$

$$y=f\left(\frac{1}{2}\right)+4\left(x-\frac{1}{2}\right)$$

$$y=\pi+4\left(x-\frac{1}{2}\right)$$

$$y=\pi+4x-2$$

$$y=4x+\pi-2$$

■ 2. Find the equation of the tangent line to the graph of the equation at $(-1,-9)$.

$$g(x)=x^3-2x^2+x-5$$

Solution:

The derivative is

$$g'(x)=3x^2-4x+1$$

Evaluating the derivative at $(-1,-9)$, we get

$$g'(-1)=3(-1)^2-4(-1)+1$$

$$g'(-1) = 3(1) + 4(1) + 1$$

$$g'(-1) = 3 + 4 + 1$$

$$g'(-1) = 8$$

Now we can find the equation of the tangent line by plugging the slope $g'(-1) = 8$ and the point $(-1, -9)$ into the formula for the equation of the tangent line.

$$y = f(a) + f'(a)(x - a)$$

$$y = g(-1) + 8(x - (-1))$$

$$y = -9 + 8(x + 1)$$

$$y = -9 + 8x + 8$$

$$y = 8x - 1$$

■ 3. Find the equation of the tangent line to the graph of the equation at $(0, -4)$.

$$h(x) = -4e^{-x} + 3x$$

Solution:

The derivative is

$$h'(x) = -4(-1)e^{-x} + 3$$

$$h'(x) = 4e^{-x} + 3$$

Evaluating the derivative at $(0, -4)$, we get

$$h'(0) = 4e^{-0} + 3$$

$$h'(0) = 4(1) + 3$$

$$h'(0) = 7$$

Now we can find the equation of the tangent line by plugging the slope $h'(0) = 7$ and the point $(0, -4)$ into the formula for the equation of the tangent line.

$$y = f(a) + f'(a)(x - a)$$

$$y = h(0) + 7(x - 0)$$

$$y = -4 + 7(x - 0)$$

$$y = -4 + 7x$$

$$y = 7x - 4$$

■ 4. Find the equation of the tangent line to the graph of the equation at $(1,1)$.

$$f(x) = -6x^4 + 4x^3 - 3x^2 + 5x + 1$$

Solution:

The derivative is

$$f'(x) = -24x^3 + 12x^2 - 6x + 5$$

Evaluating the derivative at $(1,1)$, we get

$$f'(1) = -24(1)^3 + 12(1)^2 - 6(1) + 5$$

$$f'(1) = -24 + 12 - 6 + 5$$

$$f'(1) = -13$$

Now we can find the equation of the tangent line by plugging the slope $f'(1) = -13$ and the point $(1,1)$ into the formula for the equation of the tangent line.

$$y = f(a) + f'(a)(x - a)$$

$$y = f(1) - 13(x - 1)$$

$$y = 1 - 13x + 13$$

$$y = -13x + 14$$

VALUE THAT MAKES TWO TANGENT LINES PARALLEL

■ 1. Find the value of a such that the tangent lines to $f(x)=2x^3+2$ at $x=a$ and $x=a+1$ are parallel.

Solution:

We want to find the equation of the tangent lines at $x=a$ and $x=a+1$. We know that

$$f(a)=2a^3+2$$

$$f'(x)=6x^2$$

$$f'(a)=6a^2$$

So the equation of the tangent line at $x=a$ is

$$y=f(a)+f'(a)(x-a)$$

$$y=2a^3+2+(6a^2)(x-a)$$

Now we'll do the same thing at $x=a+1$. We know that

$$f(a+1)=2(a+1)^3+2$$

$$f(a+1)=2a^3+6a^2+6a+4$$

$$f'(x)=6x^2$$

$$f'(a+1) = 6a^2 + 12a + 6$$

So the equation of the tangent line at $x = a + 1$ is

$$y = f(a+1) + f'(a+1)(x - (a+1))$$

$$y = 2a^3 + 6a^2 + 6a + 4 + (6a^2 + 12a + 6)(x - (a+1))$$

For the two tangent lines to be parallel, set the slopes from the tangent lines equal to each other and solve for a.

$$6a^2 = 6a^2 + 12a + 6$$

$$0 = 12a + 6$$

$$-12a = 6$$

$$a = -\frac{1}{2}$$

The slope of $f(x)$ at $x = -1/2$ is $3/2$, and the slope of $f(x)$ at $x = 1/2$ is $3/2$.

■ 2. Find the value of a such that the tangent lines to $g(x) = x^3 + x^2 + 7$ at $x = a$ and $x = a + 1$ are parallel.

Solution:

We want to find the equation of the tangent lines at $x = a$ and $x = a + 1$. We know that

$$g(a) = a^3 + a^2 + 7$$

$$g'(x) = 3x^2 + 2x$$

$$g'(a) = 3a^2 + 2a$$

So the equation of the tangent line at $x = a$ is

$$y = g(a) + g'(a)(x - a)$$

$$y = a^3 + a^2 + 7 + (3a^2 + 2a)(x - a)$$

Now we'll do the same thing at $x = a + 1$. We know that

$$g(a + 1) = (a + 1)^3 + (a + 1)^2 + 7$$

$$g(a + 1) = a^3 + a^2 + 2a^2 + 2a + a + 1 + a^2 + a + a + 1 + 7$$

$$g(a + 1) = a^3 + 4a^2 + 5a + 9$$

$$g'(a) = 3a^2 + 2a$$

$$g'(a + 1) = 3a^2 + 8a + 5$$

So the equation of the tangent line at $x = a + 1$ is

$$y = g(a + 1) + g'(a + 1)(x - (a + 1))$$

$$y = a^3 + 4a^2 + 4a + 10 + (3a^2 + 8a + 5)(x - (a + 1))$$

For the two tangent lines to be parallel, set the slopes from the tangent lines equal to each other and solve for a.

$$3a^2 + 2a = 3a^2 + 8a + 5$$

$$-6a = 5$$

$$a = -\frac{5}{6}$$

The slope of $g(x)$ at $x = -5/6$ is $5/12$, and the slope of $g(x)$ at $x = 1/6$ is $5/12$.

■ 3. Find the value of a such that the tangent lines to $h(x) = \tan^{-1} x$ at $x = a$ and $x = a + 1$ are parallel.

Solution:

We want to find the equation of the tangent lines at $x = a$ and $x = a + 1$. We know that

$$h(a) = \tan^{-1} a$$

$$h'(x) = \frac{1}{1 + x^2}$$

$$h'(a) = \frac{1}{1 + a^2}$$

So the equation of the tangent line at $x = a$ is

$$y = h(a) + h'(a)(x - a)$$

$$y = \tan^{-1} a + \frac{1}{1 + a^2}(x - a)$$

Now we'll do the same thing at $x = a + 1$. We know that

$$h(a+1)=\tan^{-1}(a+1)$$

$$h'(a)=\frac{1}{1+a^2}$$

$$h'(a+1)=\frac{1}{1+(a+1)^2}$$

$$h'(a+1)=\frac{1}{a^2+2a+2}$$

So the equation of the tangent line at $x=a+1$ is

$$y=h(a+1)+h'(a+1)(x-(a+1))$$

$$y=\tan^{-1}(a+1)+\frac{1}{a^2+2a+2}(x-(a+1))$$

For the two tangent lines to be parallel, set the slopes from the tangent lines equal to each other and solve for a.

$$\frac{1}{1+a^2}=\frac{1}{a^2+2a+2}$$

$$1+a^2=a^2+2a+2$$

$$-1=2a$$

$$a=-\frac{1}{2}$$

The slope of $h(x)$ at $x=-1/2$ is $4/5$, and the slope of $h(x)$ at $x=1/2$ is $4/5$.

■ 4. Find the value of a such that the tangent lines to $f(x) = 4x^3 - 6x + 7$ at $x = a$ and $x = a + 1$ are parallel.

Solution:

We want to find the equation of the tangent lines at $x = a$ and $x = a + 1$. We know that

$$f(a) = 4a^3 - 6a + 7$$

$$f'(x) = 12x^2 - 6$$

$$f'(a) = 12a^2 - 6$$

So the equation of the tangent line at $x = a$ is

$$y = f(a) + f'(a)(x - a)$$

$$y = 4a^3 - 6a + 7 + (12a^2 - 6)(x - a)$$

Now we'll do the same thing at $x = a + 1$. We know that

$$f(a+1) = 4(a+1)^3 - 6(a+1) + 7$$

$$f(a+1) = 4a^3 + 12a^2 + 6a + 5$$

$$f'(x) = 12x^2 - 6$$

$$f'(a+1) = 12a^2 + 24a + 6$$

So the equation of the tangent line at $x = a + 1$ is

$$y = f(a+1) + f'(a+1)(x-(a+1))$$

$$y = 4a^3 + 12a^2 + 6a + 5 + (12a^2 + 24a + 6)(x-(a+1))$$

For the two tangent lines to be parallel, set the slopes from the tangent lines equal to each other and solve for a.

$$12a^2 - 6 = 12a^2 + 24a + 6$$

$$-12 = 24a$$

$$a = -\frac{1}{2}$$

The slope of $f(x)$ at $x = -1/2$ is -3, and the slope of $f(x)$ at $x = 1/2$ is -3.

■ 5. Find the value of a such that the tangent lines to $g(x) = (x-2)^3 + x^2 + 3$ at $x = a$ and $x = a + 1$ are parallel.

Solution:

We want to find the equation of the tangent lines at $x = a$ and $x = a + 1$. We know that

$$g(a) = (a-2)^3 + a^2 + 3$$

$$g'(x) = 3(x-2)^2 + 2x$$

$$g'(a) = 3(a-2)^2 + 2a$$

So the equation of the tangent line at $x = a$ is

$$y = g(a) + g'(a)(x - a)$$

$$y = (a - 2)^3 + a^2 + 3 + (3(a - 2)^2 + 2a)(x - a)$$

Now we'll do the same thing at $x = a + 1$. We know that

$$g(a + 1) = (a + 1 - 2)^3 + (a + 1)^2 + 3$$

$$g(a + 1) = (a - 1)^3 + (a + 1)^2 + 3$$

$$g'(a) = 3(a - 2)^2 + 2a$$

$$g'(a + 1) = 3(a + 1 - 2)^2 + 2(a + 1)$$

$$g'(a + 1) = 3(a - 1)^2 + 2(a + 1)$$

So the equation of the tangent line at $x = a + 1$ is

$$y = g(a + 1) + g'(a + 1)(x - (a + 1))$$

$$y = (a - 1)^3 + (a + 1)^2 + 3 + (3(a - 1)^2 + 2(a + 1))(x - (a + 1))$$

For the two tangent lines to be parallel, set the slopes from the tangent lines equal to each other and solve for a.

$$3(a - 2)^2 + 2a = 3(a - 1)^2 + 2(a + 1)$$

$$3a^2 - 10a + 12 = 3a^2 - 4a + 5$$

$$7 = 6a$$

$$a = \frac{7}{6}$$

The slope of $g(x)$ at $x = 7/6$ is $53/12$, and the slope of $g(x)$ at $x = 13/6$ is $53/12$.

■ 6. Find the approximate value of a, rounded to the nearest hundredth, such that the tangent lines to $h(x) = e^x - 3x^2$ at $x = a$ and $x = a + 1$ are parallel.

Solution:

We want to find the equation of the tangent lines at $x = a$ and $x = a + 1$. We know that

$$h(a) = e^a - 3a^2$$

$$h'(x) = e^x - 6x$$

$$h'(a) = e^a - 6a$$

So the equation of the tangent line at $x = a$ is

$$y = h(a) + h'(a)(x - a)$$

$$y = e^a - 3a^2 + (e^a - 6a)(x - a)$$

Now we'll do the same thing at $x = a + 1$. We know that

$$h(a + 1) = e^{a+1} - 3(a + 1)^2$$

$$h'(a) = e^a - 6a$$

$$h'(a + 1) = e^{a+1} - 6(a + 1)$$

So the equation of the tangent line at $x = a + 1$ is

$$y = h(a + 1) + h'(a + 1)(x - (a + 1))$$

$$y = e^{a+1} - 3(a + 1)^2 + e^{a+1} - 6(a + 1)(x - (a + 1))$$

For the two tangent lines to be parallel, set the slopes from the tangent lines equal to each other and solve for a.

$$e^a - 6a = e^{a+1} - 6(a + 1)$$

$$e^a - 6a = e^{a+1} - 6a - 6$$

$$e^a = e^{a+1} - 6$$

$$e^{a+1} - e^a - 6 = 0$$

$$e^a e^1 + (-e^a) - 6 = 0$$

$$(-e^a)e^1 - (-e^a) + 6 = 0$$

Substitute $x = -e^a$, then solve for x.

$$xe - x + 6 = 0$$

$$x(e - 1) + 6 = 0$$

$$x(e - 1) = -6$$

$$x = -\frac{6}{e - 1}$$

Back-substitute.

$$-e^a = -\frac{6}{e-1}$$

$$e^a = \frac{6}{e-1}$$

Take the natural log of both sides.

$$\ln(e^a) = \ln\left(\frac{6}{e-1}\right)$$

$$a = \ln\left(\frac{6}{e-1}\right)$$

$$a \approx 1.25$$

The slope of $h(x)$ at $x = 1.25$ is -4, and the slope of $h(x)$ at $x = 2.25$ is -4.

VALUES THAT MAKE THE FUNCTION DIFFERENTIABLE

■ 1. What value of a and b will make the function differentiable?

$$f(x) = \begin{cases} x^2 & x \le 3 \\ ax - b & x > 3 \end{cases}$$

Solution:

To be differentiable, the function has to be continuous. To make $f(x)$ continuous at $x = 3$,

$$x^2 = ax - b$$

$$3^2 = a(3) - b$$

$$9 = 3a - b$$

$$b = 3a - 9$$

If $f(x)$ is differentiable, then the derivatives of $f(x)$ at $x = 3$ must be equal to each other. So $2x = a$, and when $x = 3$, and $a = 6$. Therefore, $a = 6$ and

$$b = 3(6) - 9$$

$$b = 9$$

■ 2. What value of a and b will make the function differentiable?

$$g(x) = \begin{cases} ax + b & x \le -1 \\ bx^2 - 1 & x > -1 \end{cases}$$

Solution:

To be differentiable, the function has to be continuous. To make $g(x)$ continuous at $x = -1$,

$$ax + b = bx^2 - 1$$

$$a(-1) + b = b(-1)^2 - 1$$

$$-a + b = b - 1$$

$$a = 1$$

If $g(x)$ is differentiable, then the derivatives of $g(x)$ at $x = -1$ must be equal to each other. So $a = 2bx$ when $x = -1$, and $a = 1$.

$$1 = 2b(-1)$$

$$b = -\frac{1}{2}$$

Therefore, $a = 1$ and $b = -1/2$.

■ 3. What value of a and b will make the function differentiable?

$$h(x) = \begin{cases} ax^3 & x \le 2 \\ x^2 - b & x > 2 \end{cases}$$

Solution:

To be differentiable, the function has to be continuous. To make $h(x)$ continuous at $x = 2$,

$$ax^3 = x^2 - b$$

$$a(2)^3 = (2)^2 - b$$

$$8a = 4 - b$$

$$8a - 4 = -b$$

$$b = 4 - 8a$$

If $h(x)$ is differentiable, then the derivatives of $h(x)$ at $x = 2$ must be equal to each other. So $3ax^2 = 2x$ when $x = 2$, and

$$3a(2)^2 = 2(2)$$

$$12a = 4$$

$$a = \frac{1}{3}$$

To get b, we'll plug in $a = 1/3$.

$$b = 4 - 8\left(\frac{1}{3}\right) = \frac{4}{3}$$

Therefore, $a = 1/3$ and $b = 4/3$.

■ 4. What value of a and b will make the function differentiable?

$$f(x) = \begin{cases} 3 - x & x \le 1 \\ ax^2 - bx & x > 1 \end{cases}$$

Solution:

To be differentiable, the function has to be continuous. To make $f(x)$ continuous at $x = 1$,

$$3 - x = ax^2 - bx$$

$$3 - (1) = a(1)^2 - b(1)$$

$$2 = a - b$$

$$a = 2 + b$$

If $f(x)$ is differentiable, then the derivatives of $f(x)$ at $x = 1$ must be equal to each other. So

$$-1 = 2ax - b$$

$$-1 = 2a(1) - b$$

$$-1 = 2a - b$$

$$-1 - 2a = -b$$

$$b = 2a + 1$$

Now, since $a = 2 + b$ and $b = 2a + 1$,

$$a = 2 + 2a + 1$$

$$-a = 3$$

$$a = -3$$

Then

$$b = 2a + 1$$

$$b = 2(-3) + 1$$

$$b = -5$$

The answer is $a = -3$ and $b = -5$.

■ 5. What value of a and b will make the function differentiable?

$$g(x) = \begin{cases} x^3 & x \le 1 \\ a(x-2)^2 - b & x > 1 \end{cases}$$

Solution:

To be differentiable, the function has to be continuous. To make $g(x)$ continuous at $x = 1$,

$$x^3 = a(x-2)^2 - b$$

$$(1)^3 = a(1-2)^2 - b$$

$$1 = a - b$$

$$-b = 1 - a$$

$$b = a - 1$$

If $g(x)$ is differentiable, then the derivatives of $g(x)$ at $x = 1$ must be equal to each other. So

$$3x^2 = 2a(x-2)$$

$$3(1) = 2a(1-2)$$

$$3 = 2a(-1)$$

$$a = -\frac{3}{2}$$

So $a = -3/2$ and $b = -(3/2) - 1 = -(5/2)$.

■ 6. What value of a and b will make the function differentiable?

$$h(x) = \begin{cases} ax^2 + b & x \leq 3 \\ bx + 4 & x > 3 \end{cases}$$

Solution:

To be differentiable, the function has to be continuous. To make $h(x)$ continuous at $x = 3$,

$$ax^2 + b = bx + 4$$

$$a(3)^2 + b = b(3) + 4$$

$$9a + b = 3b + 4$$

$$9a = 2b + 4$$

$$a = \frac{2b + 4}{9}$$

If $h(x)$ is differentiable, then the derivatives of $h(x)$ at $x = 3$ must be equal to each other. So

$$2ax = b$$

$$2a(3) = b$$

$$b = 6a$$

Plugging $b = 6a$ into the equation for a gives

$$a = \frac{2(6a) + 4}{9}$$

$$9a = 12a + 4$$

$$-3a = 4$$

$$a = -\frac{4}{3}$$

Then, $b = 6(-4/3) = -8$. The answer is $a = -4/3$ and $b = -8$.

NORMAL LINES

1. Find the equation of the normal line to the graph of $f(x) = 5x^4 + 3e^x$ at $(0,3)$.

Solution:

Begin by finding the tangent line at $(0,3)$, starting with taking the derivative. Then evaluate the derivative at $(0,3)$.

$$f'(x) = 20x^3 + 3e^x$$

$$f'(0) = 20(0)^3 + 3e^0$$

$$f'(0) = 0 + 3(1)$$

$$f'(0) = 3$$

With $f'(0) = 3$ and $(a, f(a)) = (0,3)$, the tangent line is

$$y = f(a) + f'(a)(x - a)$$

$$y = 3(x - 0) + 3$$

$$y = 3x + 3$$

Since the normal line is the line that's perpendicular to the function at the same point, the slope of the normal line is $-1/3$, so the equation of the normal line is

$$y = 3 - \frac{1}{3}(x - 0)$$

$$y = -\frac{1}{3}x + 3$$

■ 2. Find the equation of the normal line to the graph of $g(x) = \ln e^{4x} + 2x^3$ at $(2,24)$.

Solution:

Begin by finding the tangent line at $(2,24)$, starting with taking the derivative. Then evaluate the derivative at $(2,24)$.

$$g'(x) = 4 + 6x^2$$

$$g'(2) = 4 + 6(2)^2$$

$$g'(2) = 4 + 24$$

$$g'(2) = 28$$

With $g'(2) = 28$ and $(a, g(a)) = (2,24)$, the tangent line is

$$y = g(a) + g'(a)(x - a)$$

$$y = 24 + 28(x - 2)$$

$$y = 28x - 32$$

Since the normal line is the line that's perpendicular to the function at the same point, the slope of the normal line is $-1/28$, so the equation of the normal line is

$$y = 24 - \frac{1}{28}(x - 2)$$

$$y = -\frac{1}{28}x + \frac{337}{14}$$

■ 3. Find the equation of the normal line to the graph of $h(x) = 5\cos x + 5\sin x$ at $(\pi/2,5)$.

Solution:

Begin by finding the tangent line at $(\pi/2,5)$, starting with taking the derivative. Then evaluate the derivative at $(\pi/2,5)$.

$$h'(x) = -5\sin x + 5\cos x$$

$$h'\left(\frac{\pi}{2}\right) = -5\sin\left(\frac{\pi}{2}\right) + 5\cos\left(\frac{\pi}{2}\right)$$

$$h'\left(\frac{\pi}{2}\right) = -5(1) + 5(0)$$

$$h'\left(\frac{\pi}{2}\right) = -5$$

With $h'(\pi/2) = -5$ and $(a, h(a)) = (\pi/2,5)$, the tangent line is

$$y = h(a) + h'(a)(x - a)$$

$$y = 5 - 5\left(x - \frac{\pi}{2}\right)$$

Since the normal line is the line that's perpendicular to the function at the same point, the slope of the normal line is $1/5$, so the equation of the normal line is

$$y = 5 + \frac{1}{5}\left(x - \frac{\pi}{2}\right)$$

■ 4. Find the equation of the normal line to the graph of $f(x) = 7x^3 + 2x^2 - 5x + 9$ at $(2,63)$.

Solution:

Begin by finding the tangent line at $(2,63)$, starting with taking the derivative. Then evaluate the derivative at $(2,63)$.

$$f'(x) = 21x^2 + 4x - 5$$

$$f'(2) = 21(2)^2 + 4(2) - 5$$

$$f'(2) = 84 + 8 - 5$$

$$f'(2) = 87$$

With $f'(2) = 87$ and $(a, f(a)) = (2,63)$, the tangent line is

$$y = f(a) + f'(a)(x - a)$$

$$y = 63 + 87(x - 2)$$

$$y = 87x - 111$$

Since the normal line is the line that's perpendicular to the function at the same point, the slope of the normal line is $1/87$, so the equation of the normal line is

$$y = 63 - \frac{1}{87}(x - 2)$$

$$y = -\frac{1}{87}x + \frac{5{,}483}{87}$$

■ 5. Find the equation of the normal line to the graph of $g(x) = 5\sqrt{x^2 - 14x + 49}$ at $(2,25)$.

Solution:

Begin by finding the tangent line at $(2,25)$, starting with taking the derivative. Then evaluate the derivative at $(2,25)$.

$$g'(x) = \frac{5}{2\sqrt{x^2 - 14x + 49}} \cdot (2x - 14)$$

$$g'(x) = \frac{5x - 35}{\sqrt{x^2 - 14x + 49}}$$

$$g'(x) = \frac{5(x-7)}{|x-7|}$$

$$g'(2) = \frac{5(2-7)}{|2-7|}$$

$$g'(2) = \frac{-25}{5}$$

$$g'(2) = -5$$

With $g'(2) = -5$ and $(a, g(a)) = (2,25)$, the tangent line is

$$y = g(a) + g'(a)(x-a)$$

$$y = 25 - 5(x-2)$$

$$y = -5x + 35$$

Since the normal line is the line that's perpendicular to the function at the same point, the slope of the normal line is $1/5$, so the equation of the normal line is

$$y = 25 + \frac{1}{5}(x-2)$$

$$y = \frac{1}{5}x + \frac{123}{5}$$

AVERAGE RATE OF CHANGE

■ 1. Find the average rate of change of the function over the interval $[4,9]$.

$$f(x)=\frac{5\sqrt{x}-2}{3}$$

Solution:

In this question, $f(9)$ and $f(4)$ are

$$f(9)=\frac{5\sqrt{9}-2}{3}=\frac{5(3)-2}{3}=\frac{13}{3}$$

$$f(4)=\frac{5\sqrt{4}-2}{3}=\frac{5(2)-2}{3}=\frac{8}{3}$$

Therefore, average rate of change on $[a,b]=[4,9]$ is given by

$$\frac{f(b)-f(a)}{b-a}$$

$$\frac{\frac{13}{3}-\frac{8}{3}}{9-4}=\frac{\frac{5}{3}}{\frac{5}{1}}=\frac{5}{3}\cdot\frac{1}{5}=\frac{1}{3}$$

■ 2. Find the average rate of change of the function over the interval $[16,25]$.

$$g(x) = \frac{2x - 8}{\sqrt{x} - 2}$$

Solution:

In this question, $g(25)$ and $g(16)$ are

$$g(25) = \frac{2(25) - 8}{\sqrt{25} - 2} = \frac{42}{3} = 14$$

$$g(16) = \frac{2(16) - 8}{\sqrt{16} - 2} = \frac{24}{2} = 12$$

Therefore, average rate of change on $[a, b] = [16,25]$ is given by

$$\frac{g(b) - g(a)}{b - a}$$

$$\frac{14 - 12}{25 - 16} = \frac{2}{9}$$

■ 3. Find the average rate of change of the function over the interval $[0,4]$.

$$h(x) = \frac{x^3 - 8}{x^2 - 4x - 5}$$

Solution:

In this question, $g(4)$ and $g(0)$ are

$$h(4)=\frac{4^3-8}{4^2-4(4)-5}=\frac{64-8}{16-16-5}=\frac{56}{-5}=-\frac{56}{5}$$

$$h(0)=\frac{0^3-8}{0^2-4(0)-5}=\frac{0-8}{0-0-5}=\frac{-8}{-5}=\frac{8}{5}$$

Therefore, average rate of change on $[a,b]=[0,4]$ is given by

$$\frac{h(b)-h(a)}{b-a}$$

$$\frac{\frac{-56}{5}-\frac{8}{5}}{4-0}=\frac{-\frac{64}{5}}{\frac{4}{1}}=-\frac{64}{5}\cdot\frac{1}{4}=-\frac{16}{5}$$

W W W . K R I S T A K I N G M A T H . C O M