Calculus 1 Workbook Solutions

Solving limits

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SOLVING WITH SUBSTITUTION

■ 1. What is the value of the limit?

$$\lim_{x \to 3} -x^4 + x^3 + 2x^2$$

Solution:

Use substitution and plug $x = 3$ into the function.

$$\lim_{x \to 3} -x^4 + x^3 + 2x^2$$

$$-(3)^4 + (3)^3 + 2(3)^2$$

$$-36$$

■ 2. What is the value of the limit?

$$\lim_{x \to 7} \frac{x^2 - 5}{x^2 + 5}$$

Solution:

Use substitution and plug $x = 7$ into the function.

$$\lim_{x\to 7}\frac{x^2-5}{x^2+5}$$

$$\frac{7^2-5}{7^2+5}$$

$$\frac{44}{54}=\frac{22}{27}$$

■ 3. What is the value of the limit.

$$\lim_{x\to -2}\frac{x^3-5x^2+4x-6}{x^2+7x+6}$$

Solution:

Use substitution and plug $x=-2$ into the function.

$$\lim_{x\to -2}\frac{x^3-5x^2+4x-6}{x^2+7x+6}$$

$$\frac{(-2)^3-5(-2)^2+4(-2)-6}{(-2)^2+7(-2)+6}$$

$$\frac{21}{2}$$

SOLVING WITH FACTORING

1. What is the value of the limit?

$$\lim_{x\to-7}\frac{6x^3+42x^2}{2x^2+26x+84}$$

Solution:

If the limit is evaluated using substitution, the limit is undefined. However, we can factor it.

$$\lim_{x\to-7}\frac{6x^3+42x^2}{2x^2+26x+84}$$

$$\lim_{x\to-7}\frac{6x^2(x+7)}{2(x+6)(x+7)}$$

$$\lim_{x\to-7}\frac{6x^2}{2(x+6)}$$

Now we can evaluate the limit at $x=-7$ using substitution.

$$\frac{6(-7)^2}{2\left(-7+6\right)}$$

$$-147$$

■ 2. What is the value of the limit?

$$\lim_{x\to 10}\frac{3x^2-39x+90}{x^2-3x-70}$$

Solution:

If the limit is evaluated using substitution, the limit is undefined. However, we can factor it.

$$\lim_{x\to 10}\frac{3x^2-39x+90}{x^2-3x-70}$$

$$\lim_{x\to 10}\frac{3(x-10)(x-3)}{(x-10)(x+7)}$$

$$\lim_{x\to 10}\frac{3(x-3)}{x+7}$$

Now we can evaluate the limit at $x=10$ using substitution.

$$\frac{3(10-3)}{10+7}$$

$$\frac{21}{17}$$

■ 3. What is the value of the limit?

$$\lim_{x\to -8}\frac{2x^2+10x-48}{8x+64}$$

Solution:

If the limit is evaluated using substitution, the limit is undefined. However, we can factor it.

$$\lim_{x \to -8} \frac{2x^2 + 10x - 48}{8x + 64}$$

$$\lim_{x \to -8} \frac{2(x + 8)(x - 3)}{8(x + 8)}$$

$$\lim_{x \to -8} \frac{x - 3}{4}$$

Now we can evaluate the limit at $x = -8$ using substitution.

$$\frac{-8 - 3}{4}$$

$$-\frac{11}{4}$$

4. What is the value of the limit?

$$\lim_{x \to 7} \frac{x^3 - x^2 - 42x}{2x^2 - 20x + 42}$$

Solution:

If the limit is evaluated using substitution, the limit is undefined. However, we can factor it.

$$\lim_{x \to 7} \frac{x^3 - x^2 - 42x}{2x^2 - 20x + 42}$$

$$\lim_{x \to 7} \frac{x(x-7)(x+6)}{2(x-3)(x-7)}$$

$$\lim_{x \to 7} \frac{x(x+6)}{2(x-3)}$$

Now we can evaluate the limit at $x = 7$ using substitution.

$$\frac{7(7+6)}{2(7-3)}$$

$$\frac{91}{8}$$

■ 5. What is the value of the limit?

$$\lim_{x \to 8} \frac{x^2 + 2x - 80}{2x^3 - 24x^2 + 64x}$$

Solution:

If the limit is evaluated using substitution, the limit is undefined. However, we can factor it.

$$\lim_{x \to 8} \frac{x^2 + 2x - 80}{2x^3 - 24x^2 + 64x}$$

$$\lim_{x \to 8} \frac{(x + 10)(x - 8)}{2x(x - 8)(x - 4)}$$

$$\lim_{x \to 8} \frac{x + 10}{2x(x - 4)}$$

Now we can evaluate the limit at $x = 8$ using substitution.

$$\frac{8 + 10}{2(8)(8 - 4)}$$

$$\frac{18}{64} = \frac{9}{32}$$

SOLVING WITH CONJUGATE METHOD

1. What is the value of the limit?

$$\lim_{x \to 16} \frac{3(x-16)}{\sqrt{x}-4}$$

Solution:

Since the limit cannot be evaluated using substitution or factoring, use conjugate method.

$$\lim_{x \to 16} \frac{3(x-16)(\sqrt{x}+4)}{(\sqrt{x}-4)(\sqrt{x}+4)}$$

$$\lim_{x \to 16} \frac{3(x-16)(\sqrt{x}+4)}{x-16}$$

$$\lim_{x \to 16} 3(\sqrt{x}+4)$$

Then use substitution to evaluate the limit.

$$3(\sqrt{16}+4)$$

$$3(4+4)$$

$$24$$

2. What is the value of the limit?

$$\lim_{x\to 9} \frac{5(\sqrt{x}-3)}{x-9}$$

Solution:

Since the limit cannot be evaluated using substitution or factoring, use conjugate method.

$$\lim_{x\to 9} \frac{5(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)}$$

$$\lim_{x\to 9} \frac{5(x-9)}{(x-9)(\sqrt{x}+3)}$$

$$\lim_{x\to 9} \frac{5}{\sqrt{x}+3}$$

Then use substitution to evaluate the limit.

$$\frac{5}{\sqrt{9}+3}$$

$$\frac{5}{3+3}$$

$$\frac{5}{6}$$

3. What is the value of the limit?

$$\lim_{x\to 25} \frac{2(x-25)}{\sqrt{x}-5}$$

Solution:

Since the limit cannot be evaluated using substitution or factoring, use conjugate method.

$$\lim_{x\to 25} \frac{2(x-25)(\sqrt{x}+5)}{(\sqrt{x}-5)(\sqrt{x}+5)}$$

$$\lim_{x\to 25} \frac{2(x-25)(\sqrt{x}+5)}{x-25}$$

$$\lim_{x\to 25} 2(\sqrt{x}+5)$$

Then use substitution to evaluate the limit.

$$2(\sqrt{25}+5)$$

$$2(5+5)$$

$$20$$

4. What is the value of the limit?

$$\lim_{x \to 49} \frac{x - 49}{3(\sqrt{x} - 7)}$$

Solution:

Since the limit cannot be evaluated using substitution or factoring, use conjugate method.

$$\lim_{x \to 49} \frac{(x - 49)(\sqrt{x} + 7)}{3(\sqrt{x} - 7)(\sqrt{x} + 7)}$$

$$\lim_{x \to 49} \frac{(x - 49)(\sqrt{x} + 7)}{3(x - 49)}$$

$$\lim_{x \to 49} \frac{\sqrt{x} + 7}{3}$$

Then use substitution to evaluate the limit.

$$\frac{\sqrt{49} + 7}{3}$$

$$\frac{7 + 7}{3}$$

$$\frac{14}{3}$$

■ 5. What is the value of the limit?

$$\lim_{x\to 1} \frac{8(x-1)}{3(\sqrt{x}-1)}$$

Solution:

Since the limit cannot be evaluated using substitution or factoring, use conjugate method.

$$\lim_{x\to 1} \frac{8(x-1)(\sqrt{x}+1)}{3(\sqrt{x}-1)(\sqrt{x}+1)}$$

$$\lim_{x\to 1} \frac{8(x-1)(\sqrt{x}+1)}{3(x-1)}$$

$$\lim_{x\to 1} \frac{8(\sqrt{x}+1)}{3}$$

Then use substitution to evaluate the limit.

$$\frac{8(\sqrt{1}+1)}{3}$$

$$\frac{8(1+1)}{3}$$

$$\frac{16}{3}$$

INFINITE LIMITS AND VERTICAL ASYMPTOTES

1. What is the value of the limit?

$$\lim_{x\to 2} \frac{x^2 - x - 6}{-3x^2 - 3x + 18}$$

Solution:

Factor to simplify the limit.

$$\lim_{x\to 2} \frac{x^2 - x - 6}{-3x^2 - 3x + 18}$$

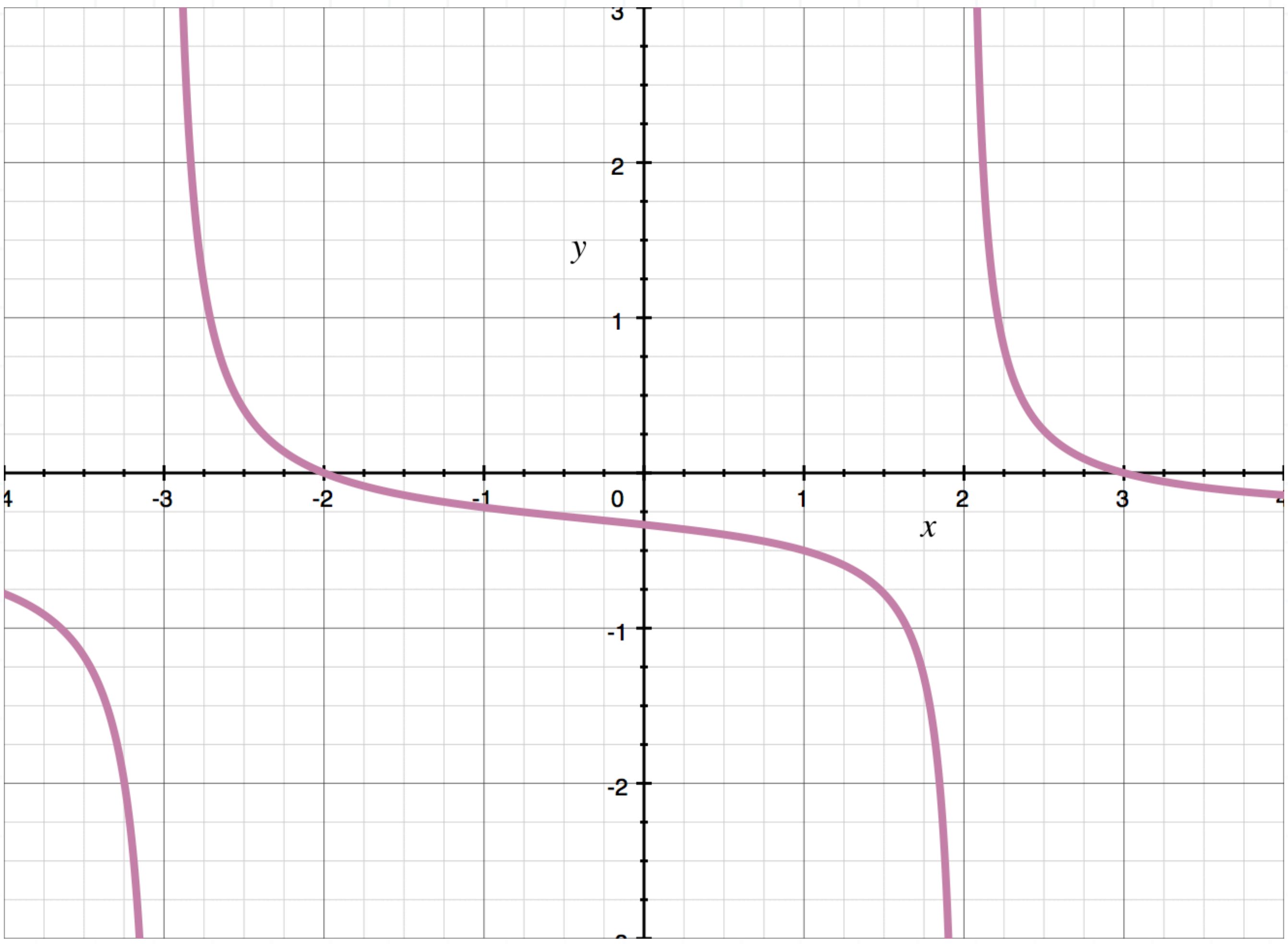
$$\lim_{x\to 2} \frac{(x-3)(x+2)}{-3(x+3)(x-2)}$$

No factors can be cancelled. The left- and right-hand limits are

$$\lim_{x\to 2^-} \frac{(x-3)(x+2)}{-3(x+3)(x-2)} = -\infty$$

$$\lim_{x\to 2^+} \frac{(x-3)(x+2)}{-3(x+3)(x-2)} = \infty$$

and they are not the same. Therefore, the limit does not exist (DNE). The graph is shown below.



2. What is the value of the limit?

$$\lim_{x \to -1} \frac{x^2 + x - 6}{4x^2 + 16x + 12}$$

Solution:

Factor to simplify the limit.

$$\lim_{x \to -1} \frac{x^2 + x - 6}{4x^2 + 16x + 12}$$

$$\lim_{x \to -1} \frac{(x+3)(x-2)}{4(x+3)(x+1)}$$

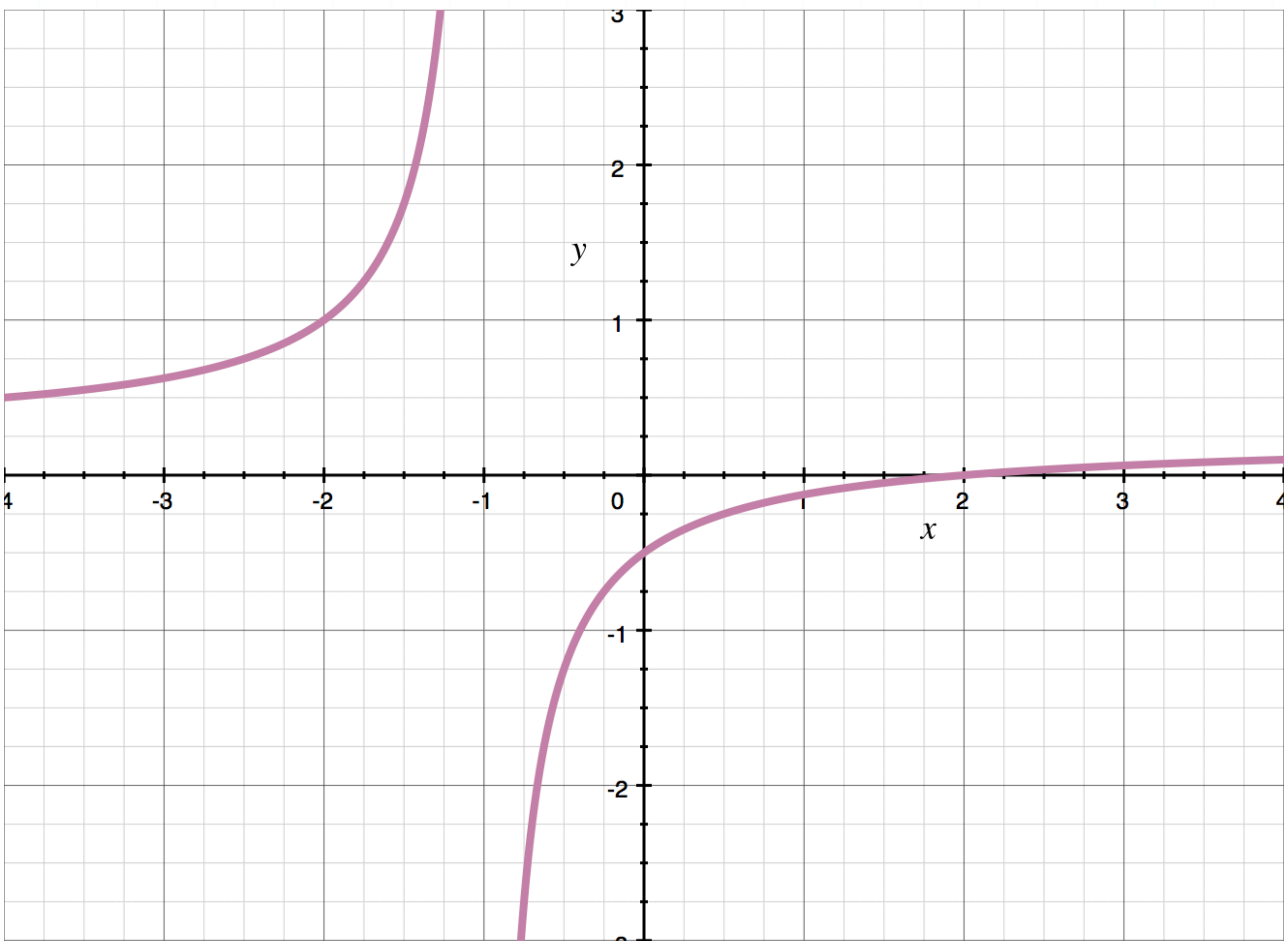
$$\lim_{x \to -1} \frac{x-2}{4(x+1)}$$

No other factors can be cancelled. The left- and right-hand limits are

$$\lim_{x \to -1^-} \frac{x-2}{4(x+1)} = \infty$$

$$\lim_{x \to -1^+} \frac{x-2}{4(x+1)} = -\infty$$

and they are not the same. Therefore, the limit does not exist. The graph is shown below.



■ 3. What is the value of the limit?

$$\lim_{x \to -4} \frac{x+5}{-4x-16}$$

Solution:

Factor to simplify the limit.

$$\lim_{x \to -4} \frac{x+5}{-4x-16}$$

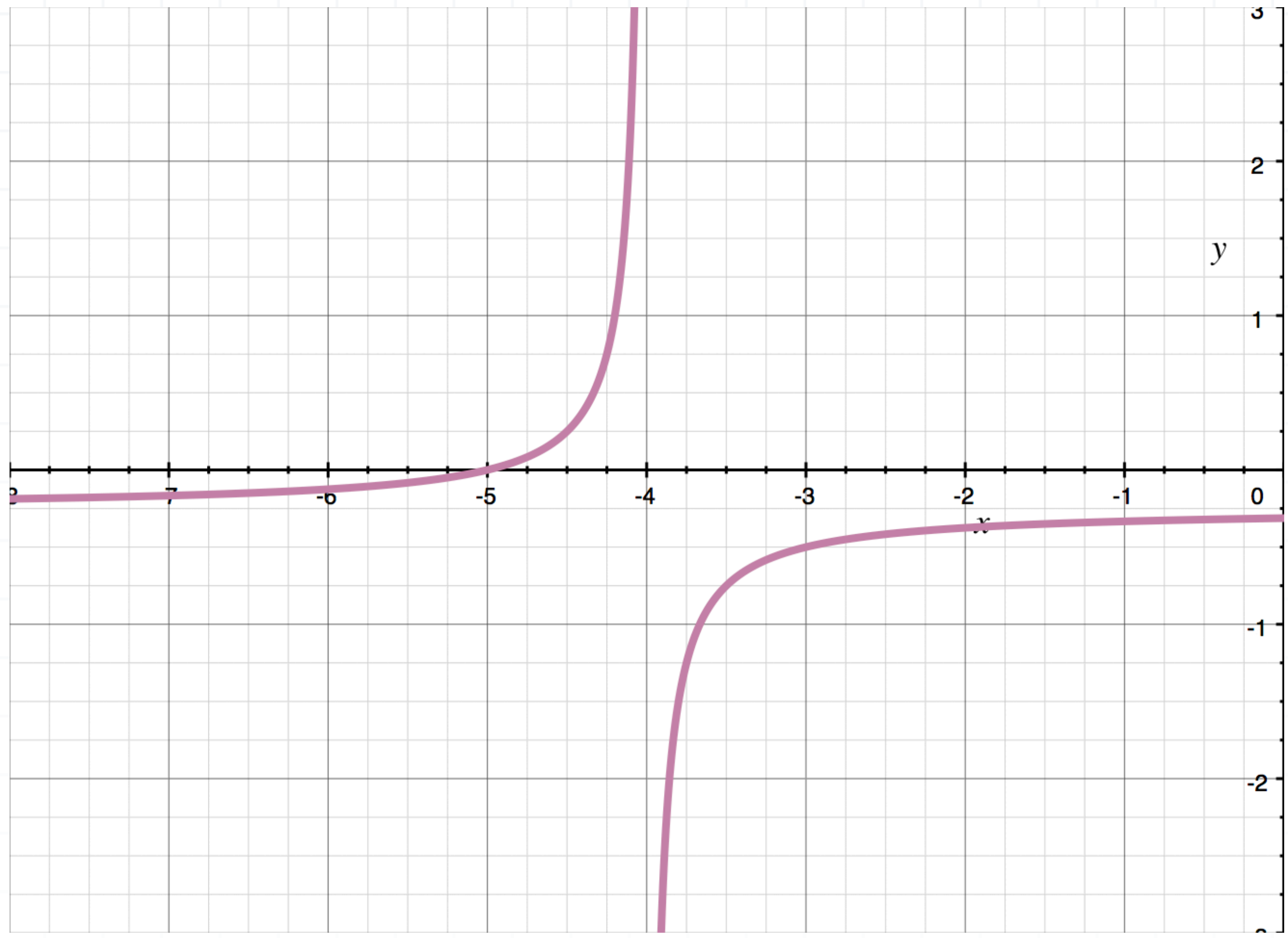
$$\lim_{x \to -4} \frac{x+5}{-4(x+4)}$$

No factors can be cancelled. The left- and right-hand limits are

$$\lim_{x \to -4^-} \frac{x+5}{-4(x+4)} = \infty$$

$$\lim_{x \to -4^+} \frac{x+5}{-4(x+4)} = -\infty$$

and they are not the same. Therefore, the limit does not exist. The graph is shown below.



4. What is the value of the limit?

$$\lim_{x \to -1} \frac{x^2 - 9}{3x^2 - 6x - 9}$$

Solution:

Factor to simplify the limit.

$$\lim_{x \to -1} \frac{x^2 - 9}{3x^2 - 6x - 9}$$

$$\lim_{x \to -1} \frac{(x + 3)(x - 3)}{3(x - 3)(x + 1)}$$

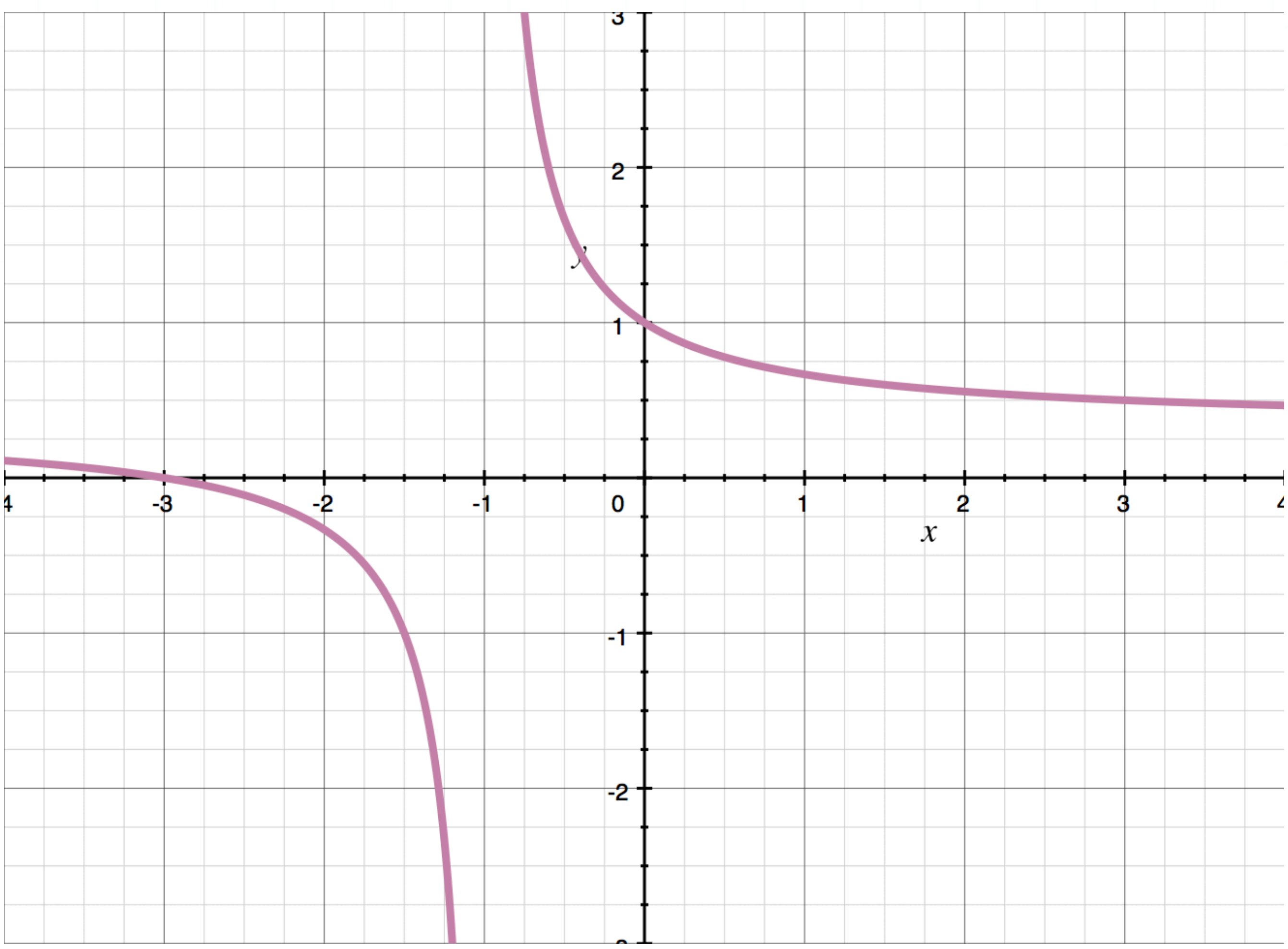
$$\lim_{x \to -1} \frac{x+3}{3(x+1)}$$

No other factors can be cancelled. The left- and right-hand limits are

$$\lim_{x \to -1^-} \frac{x+3}{3(x+1)} = -\infty$$

$$\lim_{x \to -1^+} \frac{x+3}{3(x+1)} = \infty$$

and they are not the same. Therefore, the limit does not exist. The graph is shown below.



■ 5. What is the value of the limit?

$$\lim_{x \to 3} \frac{x^2 - 4x}{x^2 - 2x - 3}$$

Solution:

Factor to simplify the limit.

$$\lim_{x \to 3} \frac{x^2 - 4x}{x^2 - 2x - 3}$$

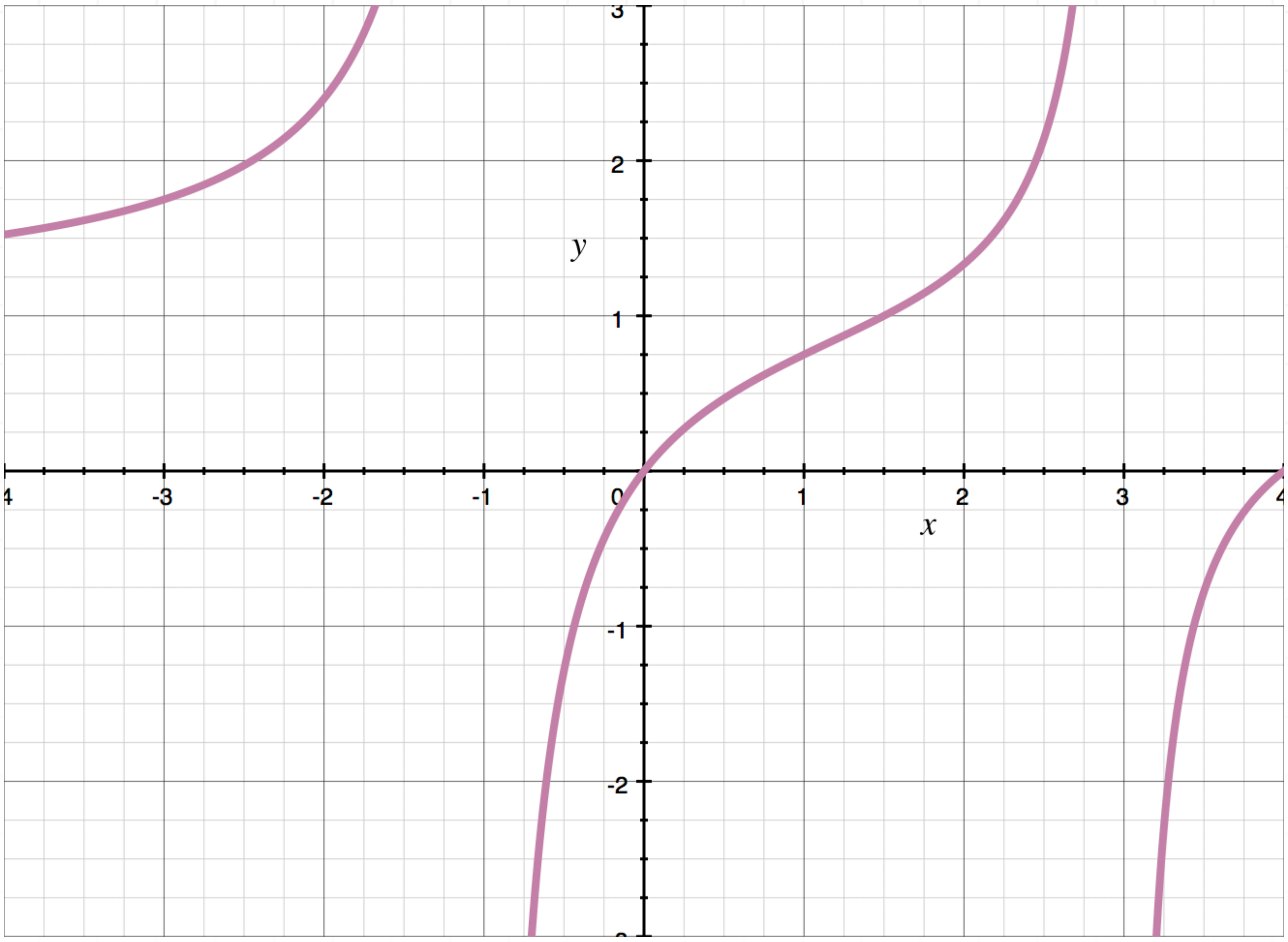
$$\lim_{x \to 3} \frac{x(x - 4)}{(x - 3)(x + 1)}$$

No factors can be cancelled. The left- and right-hand limits are

$$\lim_{x \to 3^-} \frac{x(x - 4)}{(x - 3)(x + 1)} = \infty$$

$$\lim_{x \to 3^+} \frac{x(x - 4)}{(x - 3)(x + 1)} = -\infty$$

and they are not the same. Therefore, the limit does not exist. The graph is shown below.



■ 6. What is the value of the limit?

$$\lim_{x \to -2} \frac{x^2 - 16}{-x^2 + x + 6}$$

Solution:

Factor to simplify the limit.

$$\lim_{x \to -2} \frac{x^2 - 16}{-x^2 + x + 6}$$

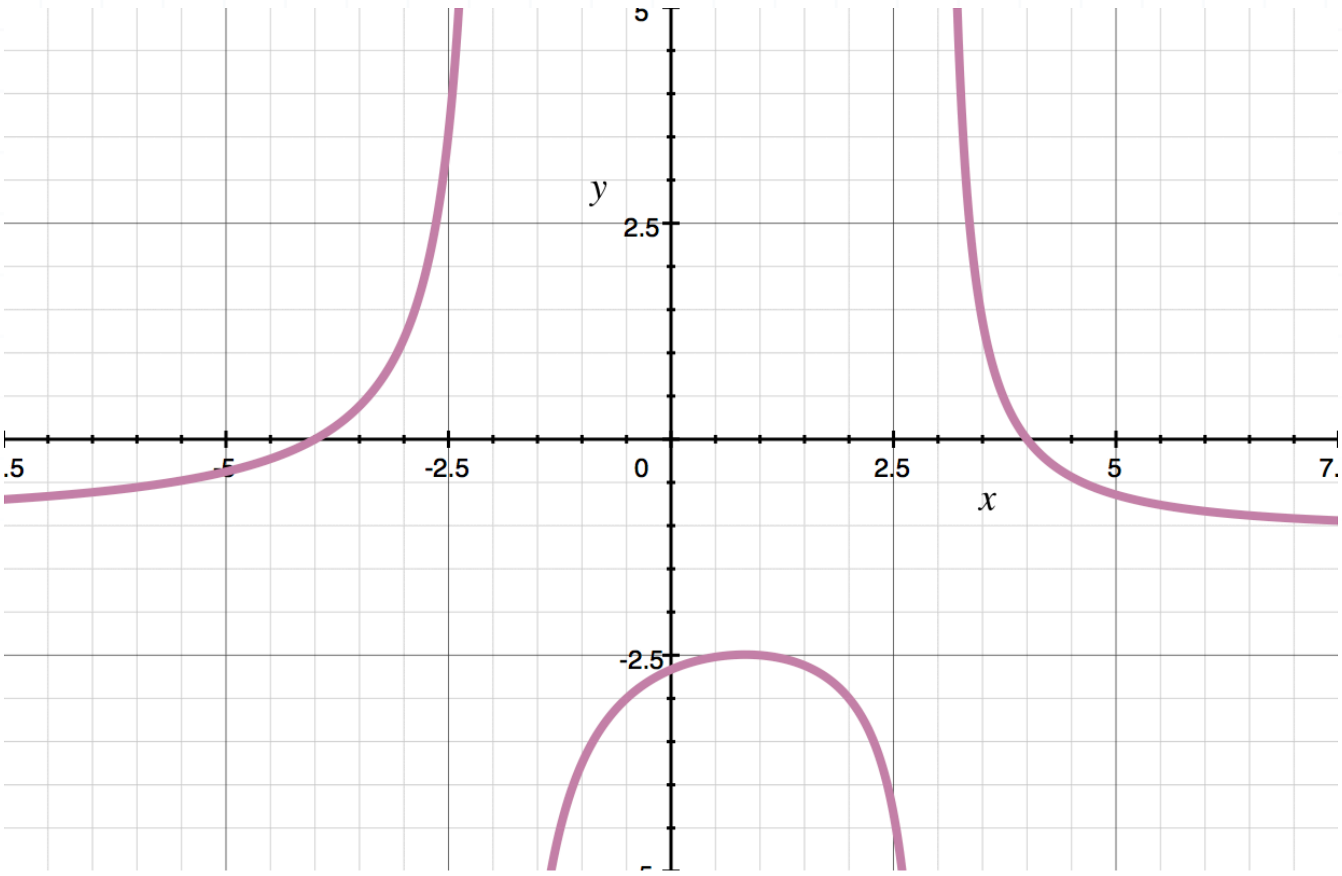
$$\lim_{x \to -2} \frac{(x+4)(x-4)}{-(x-3)(x+2)}$$

No factors can be cancelled. The left- and right-hand limits are

$$\lim_{x\to -2^-} \frac{(x+4)(x-4)}{-(x-3)(x+2)} = \infty$$

$$\lim_{x\to -2^+} \frac{(x+4)(x-4)}{-(x-3)(x+2)} = -\infty$$

and they are not the same. Therefore, the limit does not exist. The graph is shown below.



LIMITS AT INFINITY AND HORIZONTAL ASYMPTOTES

1. What is the value of the limit?

$$\lim_{x\to\infty}\frac{3x^3-5x+2}{9x^3+7x^2-x}$$

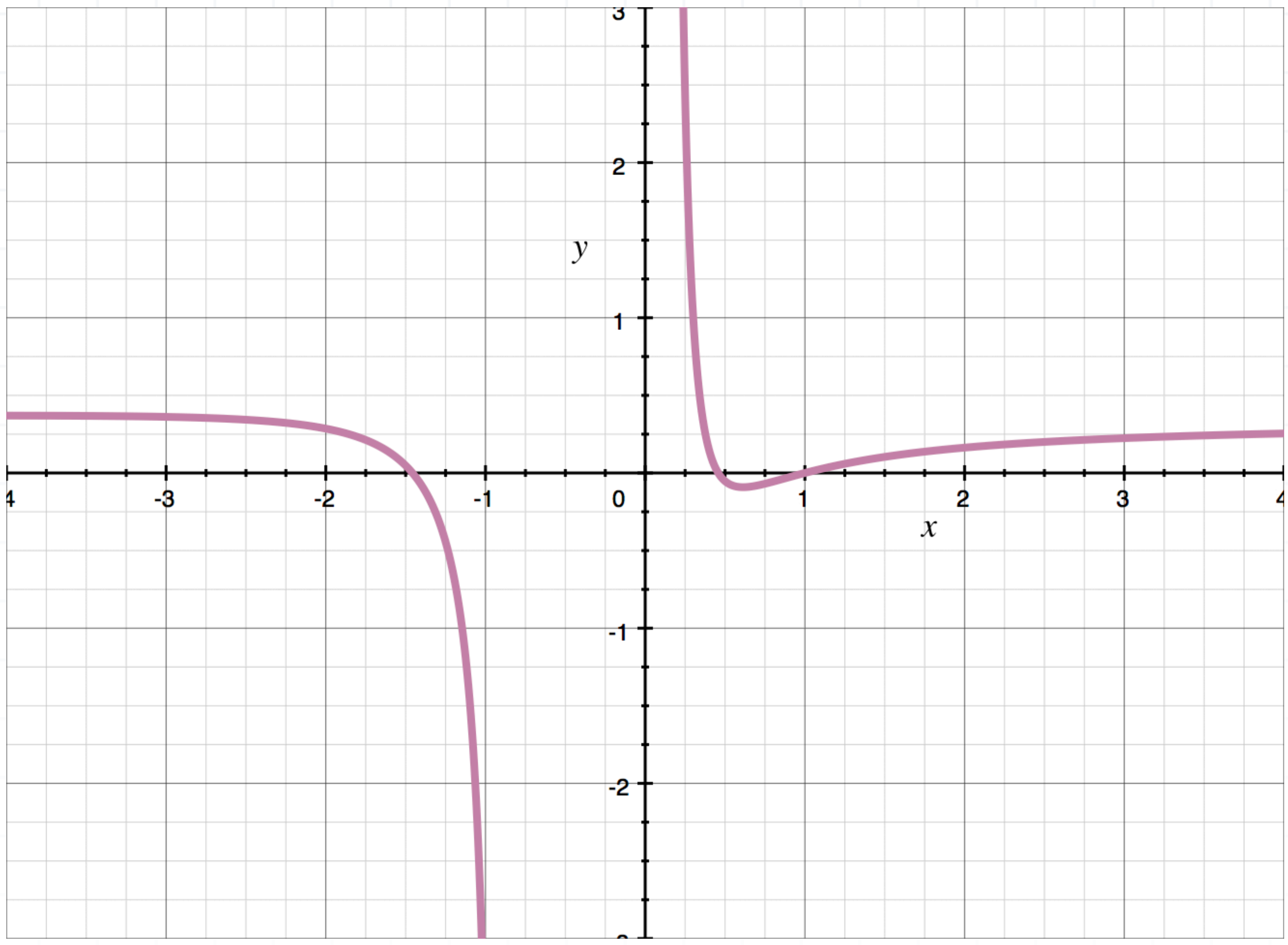
Solution:

Since this is a limit as $x \to \infty$, use the powers of the leading terms and their coefficients, since these terms dominate the end behaviors.

If the highest power in the numerator is the same as the highest power in the denominator, then the limit as $x \to \infty$ is the ratio of the leading coefficients.

$$\lim_{x\to\infty}\frac{3x^3-5x+2}{9x^3+7x^2-x}=\lim_{x\to\infty}\frac{3x^3}{9x^3}=\lim_{x\to\infty}\frac{3}{9}=\frac{3}{9}=\frac{1}{3}$$

The graph is shown below.



■ 2. What is the value of the limit?

$$\lim_{x \to -\infty} \frac{4x^2 + 3x + 5}{-2x^2 + x - 9}$$

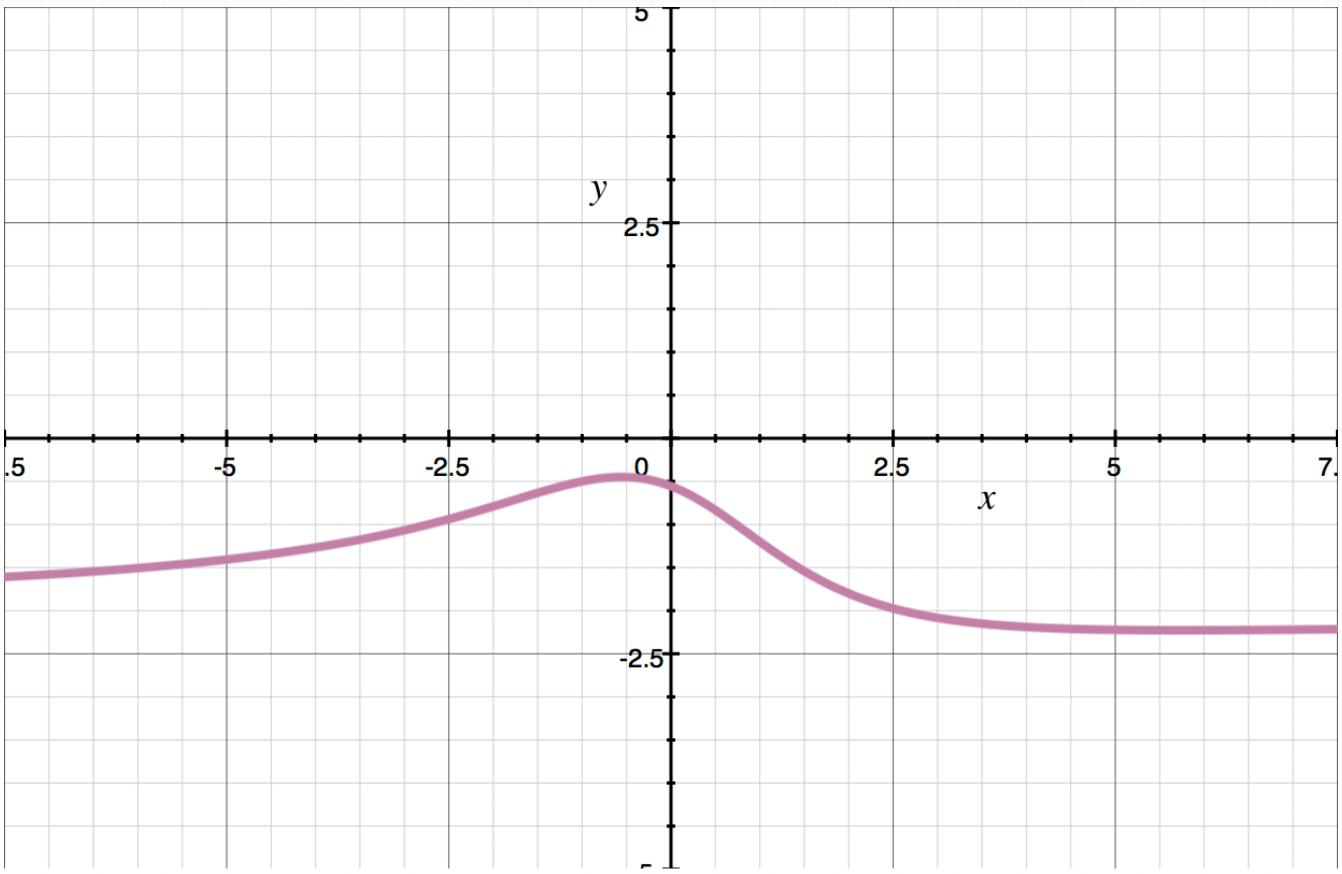
Solution:

Since this is a limit as $x \to -\infty$, use the powers of the leading terms and their coefficients, since these terms dominate the end behaviors.

If the highest power in the numerator is the same as the highest power in the denominator, then the limit as $x \to -\infty$ is the ratio of the leading coefficients.

$$\lim_{x \to -\infty} \frac{4x^2 + 3x + 5}{-2x^2 + x - 9} = \lim_{x \to -\infty} \frac{4x^2}{-2x^2} = \lim_{x \to -\infty} \frac{4}{-2} = -2$$

The graph is shown below.



3. What is the value of the limit?

$$\lim_{x \to \infty} \frac{x^3 + 6x^2 - 4x + 1}{x^3 + 9x + 8}$$

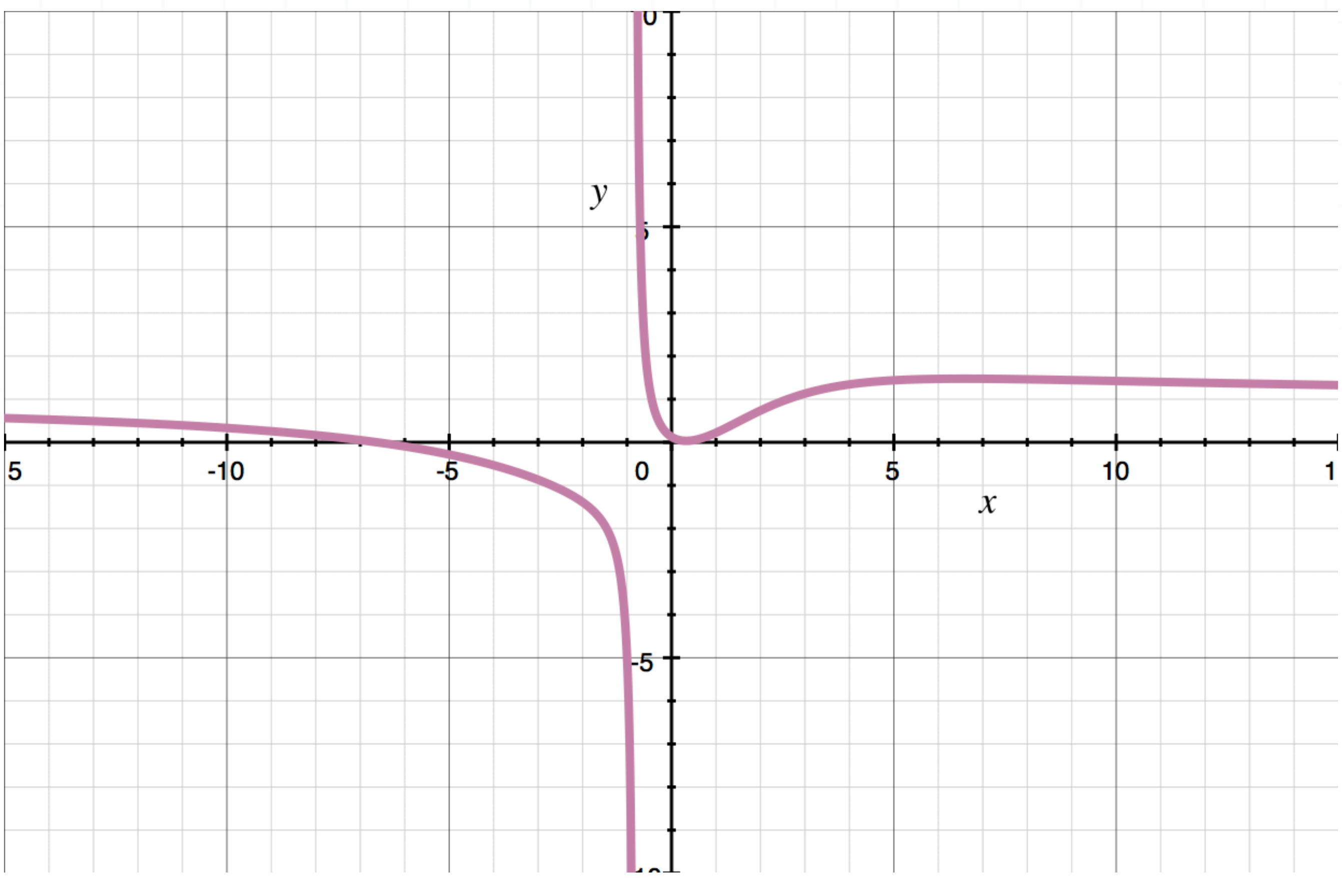
Solution:

Since this is a limit as $x \to \infty$, use the powers of the leading terms and their coefficients, since these terms dominate the end behaviors.

If the highest power in the numerator is the same as the highest power in the denominator, then the limit as $x \to \infty$ is the ratio of the leading coefficients.

$$\lim_{x\to\infty} \frac{x^3+6x^2-4x+1}{x^3+9x+8} = \lim_{x\to\infty} \frac{x^3}{x^3} = \lim_{x\to\infty} \frac{1}{1} = 1$$

The graph is shown below.



■ 4. What is the value of the limit?

$$\lim_{x\to\infty} \frac{3x^2+5x+8}{x^3-5x-9}$$

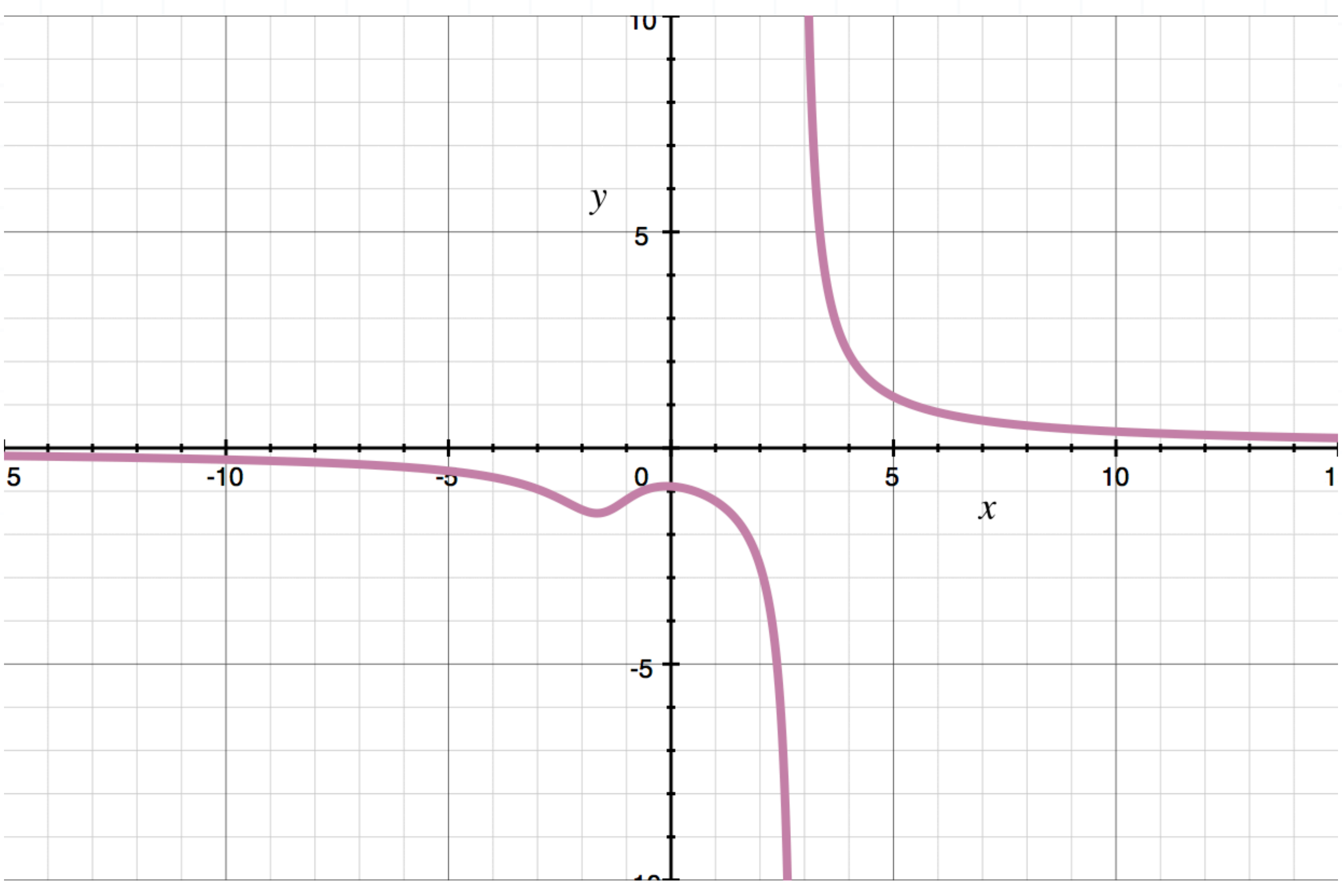
Solution:

Since this is a limit as $x \to \infty$, use the powers of the leading terms and their coefficients, since these terms dominate the end behaviors.

If the highest power in the numerator is smaller than the highest power in the denominator, then the limit as $x \to \infty$ is 0.

$$\lim_{x \to \infty} \frac{3x^2 + 5x + 8}{x^3 - 5x - 9} = \lim_{x \to \infty} \frac{3x^2}{x^3} = \lim_{x \to \infty} \frac{3}{x} = 0$$

The graph is shown below.



■ 5. What is the value of the limit?

$$\lim_{x \to -\infty} \frac{19x + 21}{x^3 + 15x + 11}$$

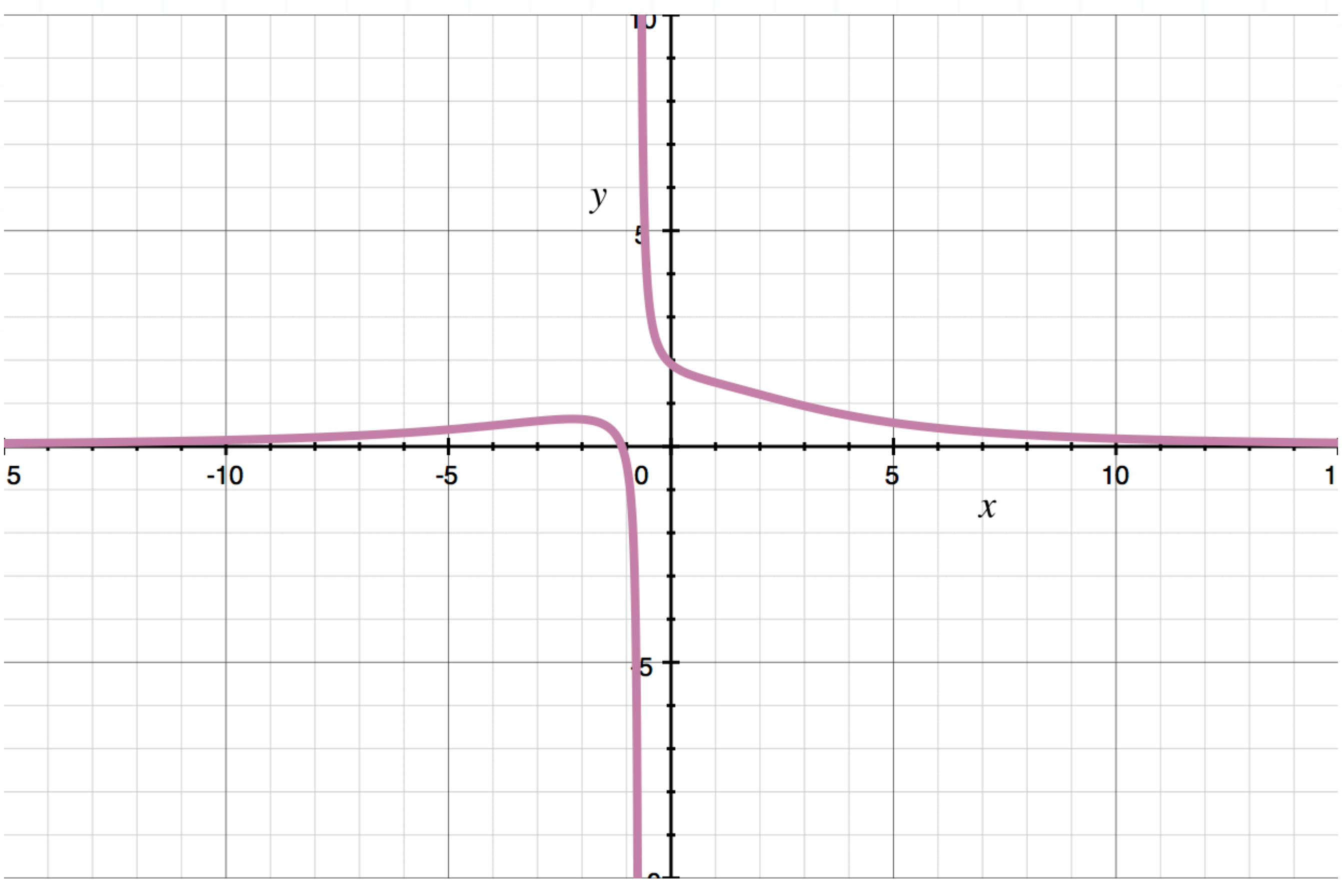
Solution:

Since this is a limit as $x \to -\infty$, use the powers of the leading terms and their coefficients, since these terms dominate the end behaviors.

If the highest power in the numerator is smaller than the highest power in the denominator, then the limit as $x \to -\infty$ is 0.

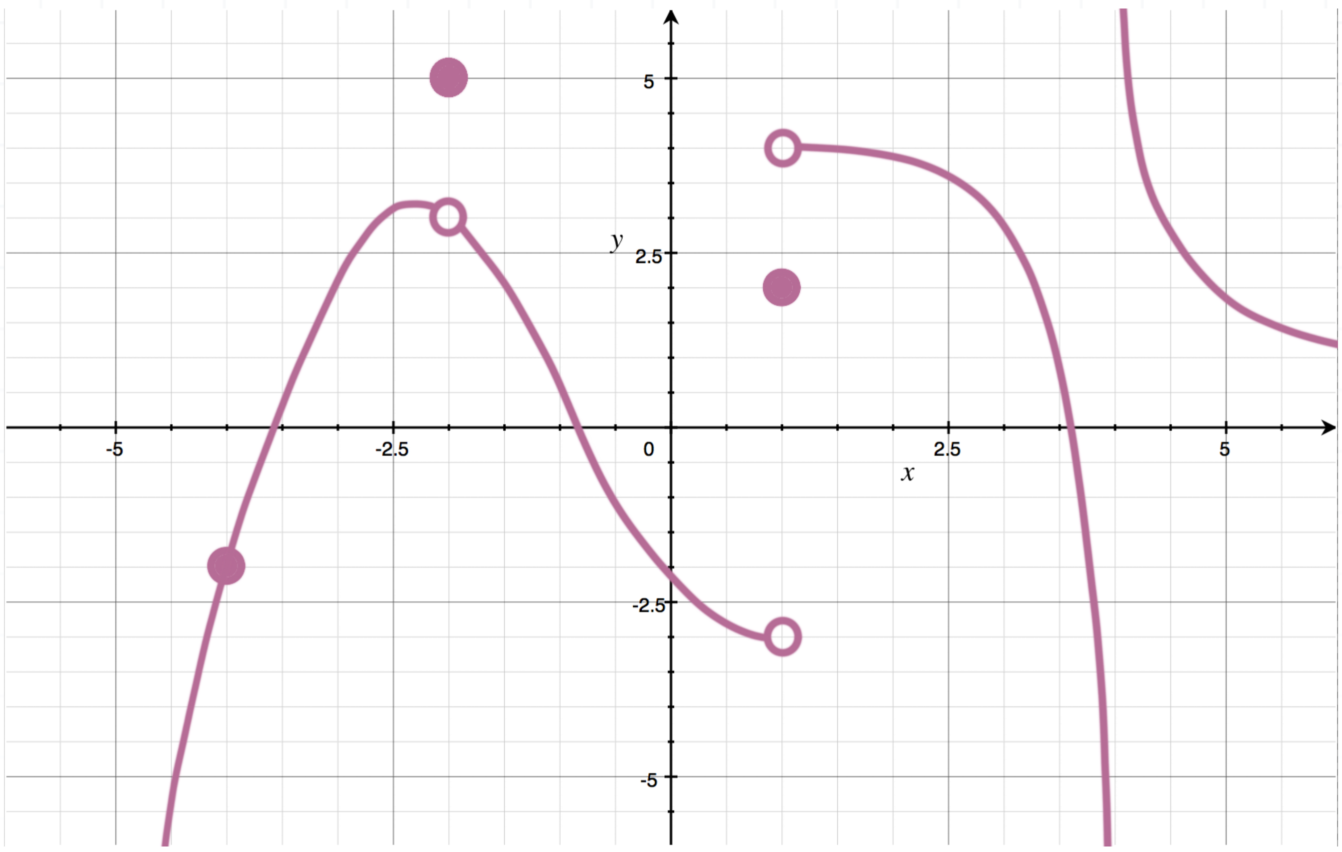
$$\lim_{x \to -\infty} \frac{19x + 21}{x^3 + 15x + 11} = \lim_{x \to -\infty} \frac{19x}{x^3} = \lim_{x \to -\infty} \frac{19}{x^2} = 0$$

The graph is shown below.



CRAZY GRAPHS

1. Use the graph to find the value of $\lim_{x \to 1} f(x)$.



Solution:

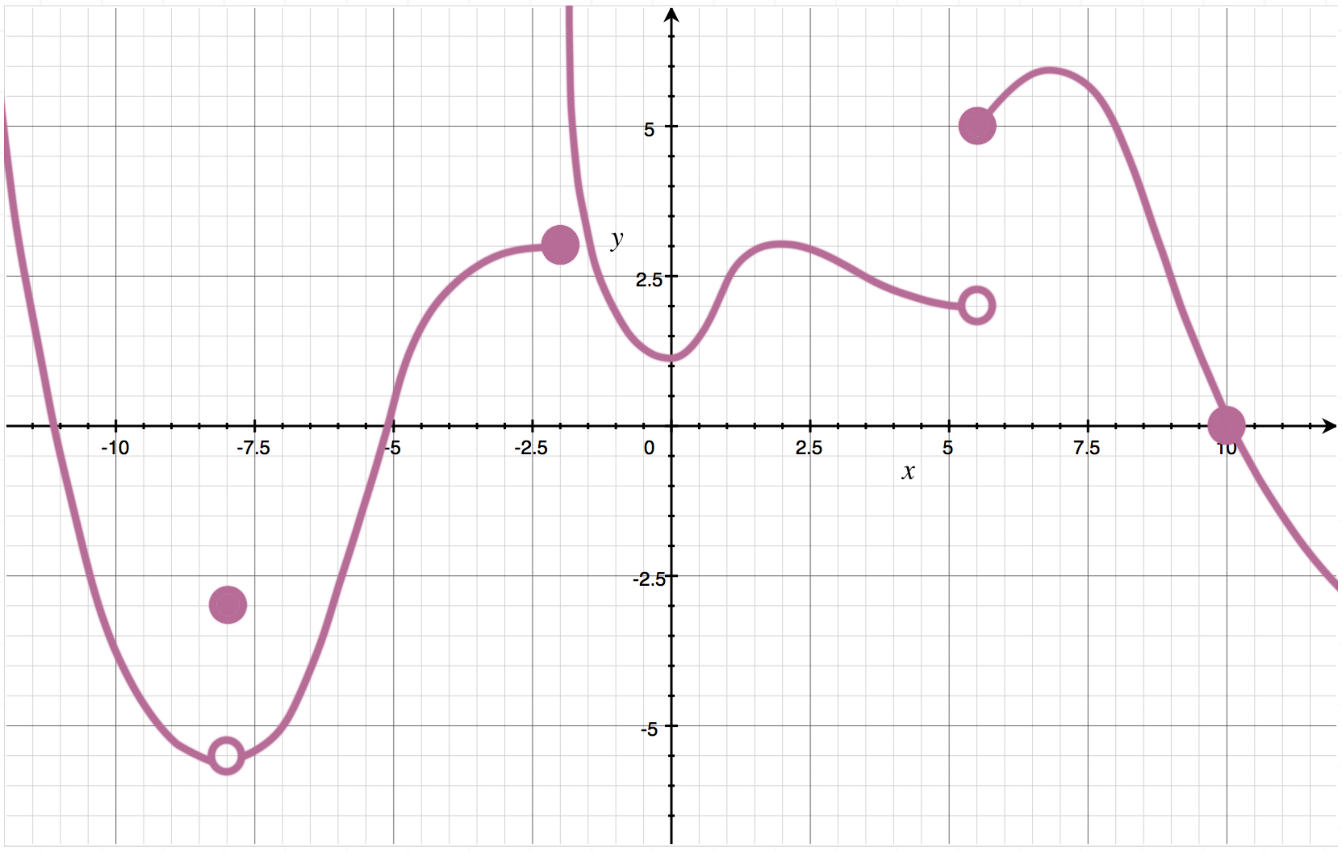
Notice in the graph that $f(x)$ has a jump discontinuity at $x = 1$.

$$\lim_{x \to 1^-} f(x) = -3$$

$$\lim_{x \to 1^+} f(x) = 4$$

Because these limits are unequal, the limit does not exist (DNE).

■ 2. Use the graph to find the value of $\lim_{x\to 5.5} g(x)$.



Solution:

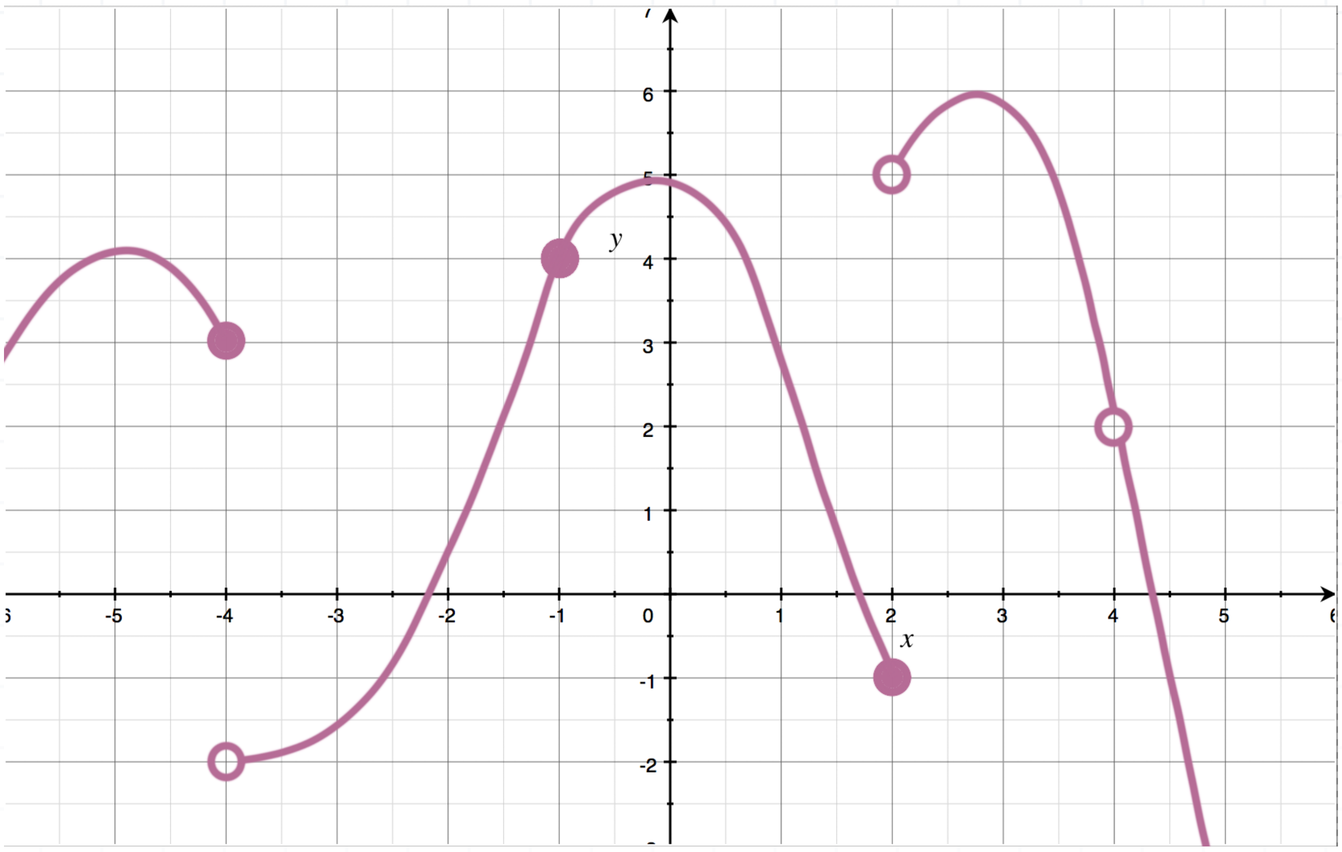
Notice in the graph that $g(x)$ has a jump discontinuity at $x = 5.5$.

$$\lim_{x\to 5.5^-} g(x) = 2$$

$$\lim_{x\to 5.5^+} g(x) = 5$$

Because these limits are unequal, the limit does not exist (DNE).

■ 3. Use the graph to find the value of $\lim_{x\to 4} h(x)$.



Solution:

Notice in the graph that $h(x)$ has a discontinuity at $x = 4$.

$$\lim_{x \to 4^-} h(x) = 2$$

$$\lim_{x \to 4^+} h(x) = 2$$

These limits are the same, which means

$$\lim_{x \to 4} h(x) = 2$$

TRIGONOMETRIC LIMITS

■ 1. Find $\lim_{x\to\pi} f(x)$ if $f(x) = 3\cos x - 2$.

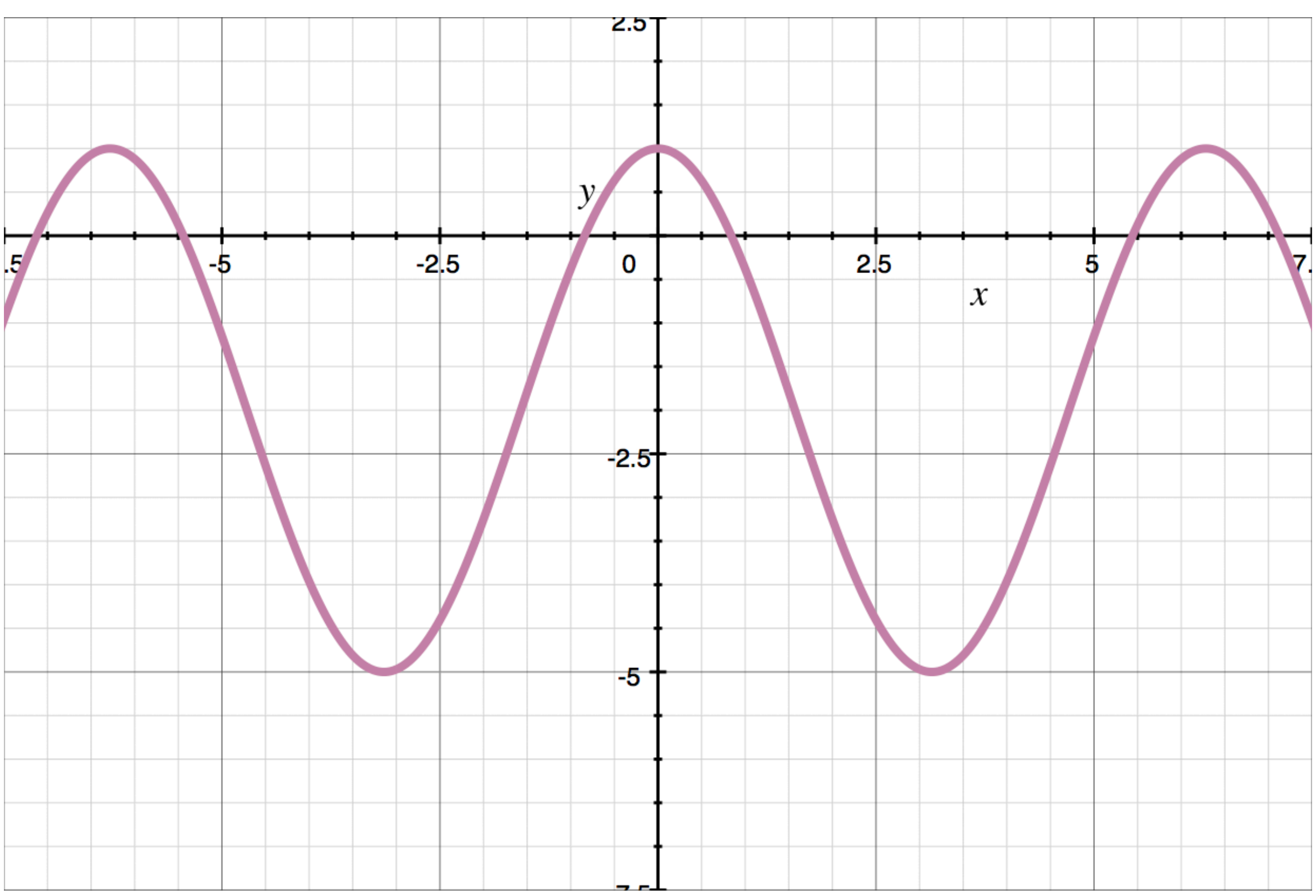
Solution:

The one-sided limits are

$$\lim_{x\to\pi^-} 3\cos x - 2 = -5$$

$$\lim_{x\to\pi^+} 3\cos x - 2 = -5$$

as shown in the graph below. Therefore, $\lim_{x\to\pi} f(x) = -5$.



■ 2. Find $\lim_{x\to\frac{3\pi}{2}} g(x)$ if $g(x) = 4\sin x + 1$.

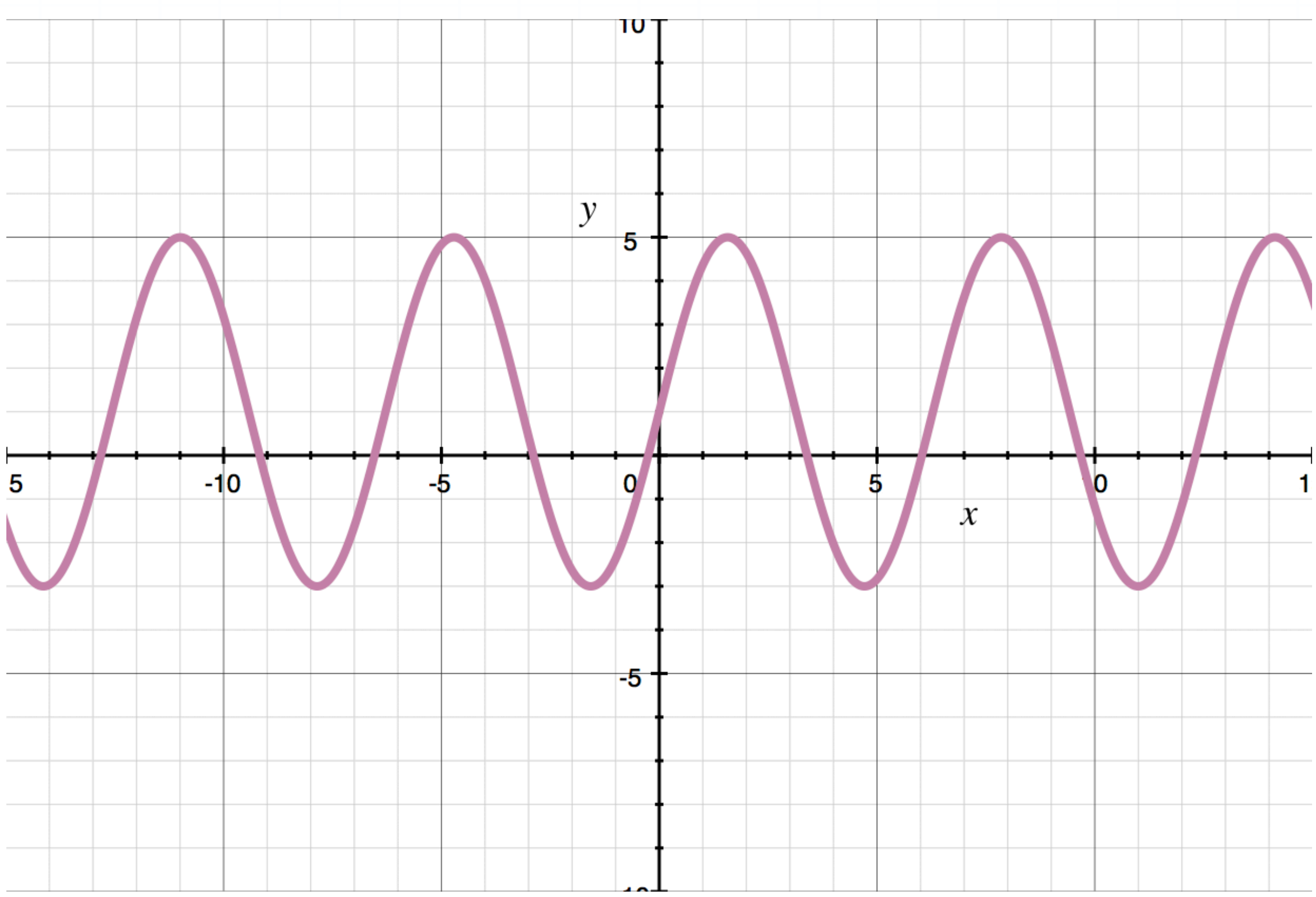
Solution:

The one-sided limits are

$$\lim_{x\to\frac{3\pi}{2}^-} 4\sin x + 1 = -3$$

$$\lim_{x\to\frac{3\pi}{2}^+} 4\sin x + 1 = -3$$

as shown in the graph below. Therefore, $\lim_{x\to\frac{3\pi}{2}} g(x) = -3$.



3. Find $\lim_{x \to -\frac{3\pi}{2}} h(x)$ if $h(x) = \tan\left(\frac{x}{6}\right)$.

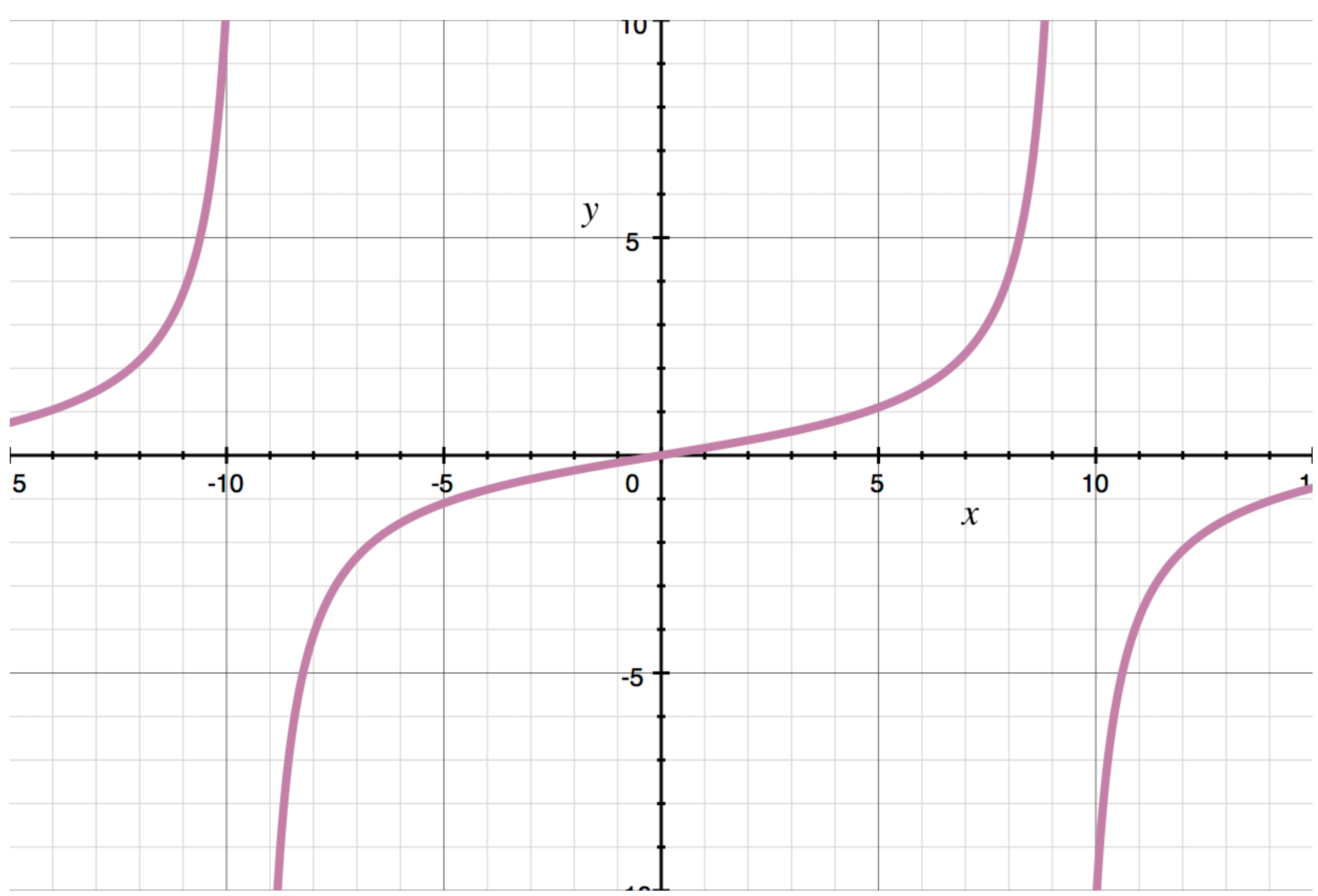
Solution:

The one-sided limits are

$$\lim_{x \to -\frac{3\pi}{2}^-} \tan\left(\frac{x}{6}\right) = -1$$

$$\lim_{x \to -\frac{3\pi}{2}^+} \tan\left(\frac{x}{6}\right) = -1$$

as shown in the graph below. Therefore, $\lim_{x \to -\frac{3\pi}{2}} h(x) = -1$.



MAKING THE FUNCTION CONTINUOUS

1. What value of c makes the function $h(x)$ continuous if c is a constant?

$$h(x) = \begin{cases} x^2 & x \le 4 \\ 3x + c & x > 4 \end{cases}$$

Solution:

Since $h(x)$ is defined as a piecewise function, $x^2 = 3x + c$ at $x = 4$.

$$4^2 = 3(4) + c$$

$$16 = 12 + c$$

$$c = 4$$

2. What value of c makes the function $f(x)$ continuous if c is a constant?

$$f(x) = \begin{cases} 5x - c & x \le 3 \\ 3x + 4 & x > 3 \end{cases}$$

Solution:

Since $f(x)$ is defined as a piecewise function, $5x - c = 3x + 4$ at $x = 3$.

$$5(3) - c = 3(3) + 4$$

$$15 - c = 9 + 4$$

$$c = 2$$

■ 3. What value of c makes the function $g(x)$ continuous if c is a constant?

$$g(x) = \begin{cases} x^2 - 4x + 8 & x \le 2 \\ cx - 2 & x > 2 \end{cases}$$

Solution:

Since $g(x)$ is defined as a piecewise function, $x^2 - 4x + 8 = cx - 2$ at $x = 2$.

$$2^2 - 4(2) + 8 = c(2) - 2$$

$$4 - 8 + 8 = 2c - 2$$

$$6 = 2c$$

$$c = 3$$

■ 4. What value of c makes the function $f(x)$ continuous if c is a constant?

$$f(x) = \begin{cases} 2x^3 - 6x^2 + 8x + 3 & x \le 1 \\ cx + 9 & x > 1 \end{cases}$$

Solution:

Since $f(x)$ is defined as a piecewise function, $2x^3 - 6x^2 + 8x + 3 = cx + 9$ at $x = 1$.

$$2(1)^3 - 6(1)^2 + 8(1) + 3 = c(1) + 9$$

$$2 - 6 + 8 + 3 = c + 9$$

$$7 = c + 9$$

$$c = -2$$

■ 5. What value of c makes the function $g(x)$ continuous if c is a constant?

$$g(x) = \begin{cases} \sqrt{x} + 18 & x \leq 16 \\ x - 2c & x > 16 \end{cases}$$

Solution:

Since $g(x)$ is defined as a piecewise function, $\sqrt{x} = 18 = 4x - 2c$ at $x = 16$.

$$\sqrt{16} + 18 = 16 - 2c$$

$$4 + 18 = 16 - 2c$$

$$22 = 16 - 2c$$

$$6 = -2c$$

$$c = -3$$

■ 6. What value of c makes the function $h(x)$ continuous if c is a constant?

$$h(x) = \begin{cases} 11x - 9 & x \le 3 \\ x^2 + 3x + c & x > 3 \end{cases}$$

Solution:

Since $h(x)$ is defined as a piecewise function, $11x - 9 = x^2 + 3x + c$ at $x = 3$.

$$11(3) - 9 = 3^2 + 3(3) + c$$

$$33 - 9 = 9 + 9 + c$$

$$24 = 18 + c$$

$$c = 6$$

W W W . K R I S T A K I N G M A T H . C O M