Trigonometric derivatives

We've learned about the basic derivative rules, including chain rule, and now we want to shift our attention toward the derivatives of specific kinds of functions. In this section we'll be looking at the derivatives of trigonometric functions, and later on we'll look at the derivatives of exponential and logarithmic functions.

Trigonometric derivatives

There are six basic trig functions, and we should know the derivative of each one.

Trigonometric function	**Its derivative**
$y = \sin x$	$y' = \cos x$
$y = \cos x$	$y' = -\sin x$
$y = \tan x$	$y' = \sec^2 x$
$y = \cot x$	$y' = -\csc^2 x$
$y = \sec x$	$y' = \sec x \tan x$
$y = \csc x$	$y' = -\csc x \cot x$

When we differentiate a trig function, we always have to apply chain rule. For instance, in $y = \sin x$, the $\sin$ and x are **not** multiplied together. Instead, the x is the argument of the sine function. What this means for us is that the argument is always the “inside function,” so when we differentiate, we need to always multiply by the derivative of the argument.

With these six basic trig functions, the argument is x, and the derivative of x is 1, so applying chain rule and multiplying by 1 doesn’t change the value of the derivative.

But when the derivative of the argument is anything other than 1, then applying chain rule will of course have an actual effect on the value of the derivative. For instance, the derivative of $y = \sin(2x)$ will be $y' = 2\cos(2x)$, and the derivative of $y = \sec(3x^2)$ will be $y' = 6x\sec(3x^2)\tan(3x^2)$.

Let’s do a full example.

Example

Find the derivative.

$$y = 4\cos(2x)$$

The derivative of $y = \cos x$ is $y' = -\sin x$. The argument of $2x$ has a derivative of 2, so the derivative will be

$$y' = 4(-\sin(2x))(2)$$

$$y' = -8\sin(2x)$$

Let’s do another example with a little more going on.

Example

Find the derivative.

$$y = 8x^5 - 9\cot(7x^4)$$

Dealing with one term at a time, and remembering to use chain rule to handle the derivative of $-9\cot(7x^4)$, we get

$$y' = 8(5)x^{5-1} - 9(-\csc^2(7x^4))(7(4)x^{4-1})$$

$$y' = 40x^4 - 9(-\csc^2(7x^4))(28x^3)$$

$$y' = 40x^4 + 252x^3\csc^2(7x^4)$$

Let's try one more.

Example

Find the derivative.

$$y = \sec(7x^3) - 7x^5\sin x + 3\csc(5x^7)$$

Let's look at one term at a time. The derivative of $\sec(7x^3)$ is

$$(\sec(7x^3)\tan(7x^3))(7(3)x^{3-1})$$

$$(\sec(7x^3)\tan(7x^3))(21x^2)$$

$$21x^2\sec(7x^3)\tan(7x^3)$$

To find the derivative of $-7x^5\sin x$, we'll need to use product rule. If $f(x) = -7x^5$ and $f'(x) = -35x^4$, and $g(x) = \sin x$ and $g'(x) = \cos x$, then we can plug directly into the product rule formula.

$$f(x)g'(x) + f'(x)g(x)$$

$$(-7x^5)(\cos x) + (-35x^4)(\sin x)$$

$$-7x^5\cos x - 35x^4\sin x$$

The derivative of $3\csc(5x^7)$ is

$$(-3\csc(5x^7)\cot(5x^7))(5(7)x^{7-1})$$

$$(-3\csc(5x^7)\cot(5x^7))(35x^6)$$

$$-105x^6\csc(5x^7)\cot(5x^7)$$

Putting these derivatives together, we get

$$y' = 21x^2\sec(7x^3)\tan(7x^3) - 7x^5\cos x - 35x^4\sin x - 105x^6\csc(5x^7)\cot(5x^7)$$
