Calculus 1 Workbook Solutions

Derivative rules

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POWER RULE

■ 1. Find the derivative of $f(x) = 7x^3 - 17x^2 + 51x - 25$ using the power rule.

Solution:

The power rule for a polynomial function $f(x)$ is

$$\frac{d}{dx}\left(ax^n\right) = anx^{n-1}$$

Differentiating

$$f(x) = 7x^3 - 17x^2 + 51x - 25$$

term by term gives

$$f'(x) = 7(3)x^{3-1} - 17(2)x^{2-1} + 51(1)x^{1-1} - 25(0)x^{0-1}$$

$$f'(x) = 21x^2 - 34x^1 + 51x^0 - 0x^{-1}$$

$$f'(x) = 21x^2 - 34x + 51(1) - 0$$

$$f'(x) = 21x^2 - 34x + 51$$

■ 2. Find the derivative of $g(x) = 2x^4 + 8x^3 + 6x^2 - 32x + 16$ using the power rule.

Solution:

The power rule for a polynomial function $f(x)$ is

$$\frac{d}{dx}\left(ax^n\right) = anx^{n-1}$$

Differentiating

$$g(x) = 2x^4 + 8x^3 + 6x^2 - 32x + 16$$

term by term gives

$$g'(x) = 2(4)x^{4-1} + 8(3)x^{3-1} + 6(2)x^{2-1} - 32(1)x^{1-1} + 16(0)x^{0-1}$$

$$g'(x) = 8x^3 + 24x^2 + 12x^1 - 32x^0 + 0x^{-1}$$

$$g'(x) = 8x^3 + 24x^2 + 12x - 32(1) + 0$$

$$g'(x) = 8x^3 + 24x^2 + 12x - 32$$

■ 3. Find the derivative of $h(x) = 22x^3 - 19x^2 + 13x - 17$ using the power rule.

Solution:

The power rule for a polynomial function $f(x)$ is

$$\frac{d}{dx}\left(ax^n\right) = anx^{n-1}$$

Differentiating

$$h(x) = 22x^3 - 19x^2 + 13x - 17$$

term by term gives

$$h'(x) = 22(3)x^{3-1} - 19(2)x^{2-1} + 13(1)x^{1-1} - 17(0)x^{0-1}$$

$$h'(x) = 66x^2 - 38x^1 + 13x^0 - 0x^{-1}$$

$$h'(x) = 66x^2 - 38x + 13(1) - 0$$

$$h'(x) = 66x^2 - 38x + 13$$

POWER RULE FOR NEGATIVE POWERS

1. Find the derivative of the function using the power rule.

$$f(x) = \frac{7}{x^2} - \frac{5}{x^4} + \frac{2}{x}$$

Solution:

The power rule for a polynomial function is

$$\frac{d}{dx}\left(ax^n\right) = anx^{n-1}$$

Rearrange $f(x)$ to use the power rule.

$$f(x) = 7x^{-2} - 5x^{-4} + 2x^{-1}$$

Differentiating term by term gives

$$f'(x) = 7(-2)x^{-2-1} - 5(-4)x^{-4-1} + 2(-1)x^{-1-1}$$

$$f'(x) = -14x^{-3} + 20x^{-5} - 2x^{-2}$$

Move the variables back to the denominator to make positive exponents.

$$f'(x) = -\frac{14}{x^3} + \frac{20}{x^5} - \frac{2}{x^2}$$

■ 2. Find the derivative of the function using the power rule.

$$g(x) = \frac{1}{9x^4} + \frac{2}{3x^5} - \frac{1}{x}$$

Solution:

The power rule for a polynomial function is

$$\frac{d}{dx}\left(ax^n\right) = anx^{n-1}$$

Rearrange $g(x)$ to use the power rule.

$$g(x) = \frac{1}{9}x^{-4} + \frac{2}{3}x^{-5} - x^{-1}$$

Differentiating term by term gives

$$g'(x) = \frac{1}{9}(-4)x^{-4-1} + \frac{2}{3}(-5)x^{-5-1} - (-1)x^{-1-1}$$

$$g'(x) = -\frac{4}{9}x^{-5} - \frac{10}{3}x^{-6} + x^{-2}$$

Move the variables back to the denominator to make positive exponents.

$$g'(x) = -\frac{4}{9x^5} - \frac{10}{3x^6} + \frac{1}{x^2}$$

■ 3. Find the derivative of the function using the power rule.

$$h(x) = -\frac{7}{6x^6} - \frac{1}{4x^4} + \frac{9}{2x^2}$$

Solution:

The power rule for a polynomial function is

$$\frac{d}{dx}\left(ax^n\right) = anx^{n-1}$$

Rearrange $h(x)$ to use the power rule.

$$h(x) = -\frac{7}{6}x^{-6} - \frac{1}{4}x^{-4} + \frac{9}{2}x^{-2}$$

Differentiating term by term gives

$$h'(x) = -\frac{7}{6}(-6)x^{-6-1} - \frac{1}{4}(-4)x^{-4-1} + \frac{9}{2}(-2)x^{-2-1}$$

$$h'(x) = 7x^{-7} + x^{-5} - 9x^{-3}$$

Move the variables back to the denominator to make positive exponents.

$$h'(x) = \frac{7}{x^7} + \frac{1}{x^5} - \frac{9}{x^3}$$

POWER RULE FOR FRACTIONAL POWERS

1. Find the derivative of the function using the power rule.

$$f(x) = 4x^{\frac{3}{2}} - 6x^{\frac{5}{3}}$$

Solution:

The power rule for a polynomial function is

$$\frac{d}{dx}\left(ax^n\right) = anx^{n-1}$$

Differentiating

$$f(x) = 4x^{\frac{3}{2}} - 6x^{\frac{5}{3}}$$

term by term gives

$$f'(x) = 4\left(\frac{3}{2}\right)x^{\frac{3}{2}-1} - 6\left(\frac{5}{3}\right)x^{\frac{5}{3}-1}$$

$$f'(x) = 6x^{\frac{3}{2}-\frac{2}{2}} - 10x^{\frac{5}{3}-\frac{3}{3}}$$

$$f'(x) = 6x^{\frac{1}{2}} - 10x^{\frac{2}{3}}$$

2. Find the derivative of the function using the power rule.

$$g(x) = 6x^{\sqrt{3}} - 4x^{\sqrt{5}}$$

Solution:

The power rule for a polynomial function is

$$\frac{d}{dx}\left(ax^n\right) = anx^{n-1}$$

Differentiating

$$g(x) = 6x^{\sqrt{3}} - 4x^{\sqrt{5}}$$

term by term gives

$$g'(x) = 6\sqrt{3}x^{\sqrt{3}-1} - 4\sqrt{5}x^{\sqrt{5}-1}$$

■ 3. Find the derivative of the function using the power rule.

$$h(x) = \frac{1}{3}x^{\frac{6}{5}} + \frac{1}{4}x^{\frac{8}{3}} - \frac{1}{5}x^{\frac{5}{2}}$$

Solution:

The power rule for a polynomial function is

$$\frac{d}{dx}\left(ax^n\right) = anx^{n-1}$$

Differentiating

$$h(x) = \frac{1}{3}x^{\frac{6}{5}} + \frac{1}{4}x^{\frac{8}{3}} - \frac{1}{5}x^{\frac{5}{2}}$$

term by term gives

$$h'(x) = \frac{1}{3}\left(\frac{6}{5}\right)x^{\frac{6}{5}-1} + \frac{1}{4}\left(\frac{8}{3}\right)x^{\frac{8}{3}-1} - \frac{1}{5}\left(\frac{5}{2}\right)x^{\frac{5}{2}-1}$$

$$h'(x) = \frac{2}{5}x^{\frac{6}{5}-\frac{5}{5}} + \frac{2}{3}x^{\frac{8}{3}-\frac{3}{3}} - \frac{1}{2}x^{\frac{5}{2}-\frac{2}{2}}$$

$$h'(x) = \frac{2}{5}x^{\frac{1}{5}} + \frac{2}{3}x^{\frac{5}{3}} - \frac{1}{2}x^{\frac{3}{2}}$$

PRODUCT RULE, TWO FUNCTIONS

■ 1. Use the product rule to find the derivative of the function.

$$h(x) = (3x + 5)\ln(5x)$$

Solution:

The derivative of a function $h(x) = f(x) \cdot g(x)$ using the product rule is

$$h'(x) = f(x) \cdot g'(x) + f'(x) \cdot g(x)$$

Let

$$f(x) = 3x + 5$$

$$f'(x) = 3$$

$$g(x) = \ln(5x)$$

$$g'(x) = \frac{1}{5x}(5) = \frac{1}{x}$$

Then by product rule, the derivative is

$$h'(x) = (3x + 5) \cdot \frac{1}{x} + 3 \cdot \ln(5x)$$

$$h'(x) = \frac{3x + 5}{x} + 3\ln(5x)$$

■ 2. Use the product rule to find the derivative of the function.

$$h(x) = 8x^3e^{7x}$$

Solution:

The derivative of a function $h(x) = f(x) \cdot g(x)$ using the product rule is

$$h'(x) = f(x) \cdot g'(x) + f'(x) \cdot g(x)$$

Let

$$f(x) = 8x^3$$

$$f'(x) = 24x^2$$

$$g(x) = e^{7x}$$

$$g'(x) = 7e^{7x}$$

Then by product rule, the derivative is

$$h'(x) = 8x^3 \cdot 7e^{7x} + 24x^2 \cdot e^{7x}$$

$$h'(x) = 56x^3e^{7x} + 24x^2e^{7x}$$

■ 3. Use the product rule to find the derivative of the function.

$$h(x) = (5x^2 - x)(e^{4x} - 6)$$

Solution:

The derivative of a function $h(x) = f(x) \cdot g(x)$ using the product rule is

$$h'(x) = f(x) \cdot g'(x) + f'(x) \cdot g(x)$$

Let

$$f(x) = 5x^2 - x$$

$$f'(x) = 10x - 1$$

$$g(x) = e^{4x} - 6$$

$$g'(x) = 4e^{4x}$$

Then by product rule, the derivative is

$$h'(x) = (5x^2 - x) \cdot 4e^{4x} + (10x - 1) \cdot (e^{4x} - 6)$$

$$h'(x) = 20x^2e^{4x} - 4xe^{4x} + 10xe^{4x} - 60x - e^{4x} + 6$$

$$h'(x) = 20x^2e^{4x} + 6xe^{4x} - 60x - e^{4x} + 6$$

PRODUCT RULE, THREE OR MORE FUNCTIONS

■ 1. Use the product rule to find the derivative of the function.

$$y = 5x^4e^{3x}\cos(6x)$$

Solution:

The derivative of a function $y = f(x) \cdot g(x) \cdot h(x)$ using the product rule is

$$y' = f'(x) \cdot g(x) \cdot h(x) + f(x) \cdot g'(x) \cdot h(x) + f(x) \cdot g(x) \cdot h'(x)$$

Let

$$f(x) = 5x^4$$

$$f'(x) = 20x^3$$

$$g(x) = \cos(6x)$$

$$g'(x) = -6\sin(6x)$$

$$h(x) = e^{3x}$$

$$h'(x) = 3e^{3x}$$

Then by product rule, the derivative is

$$y' = (20x^3)(\cos(6x))(e^{3x}) + (5x^4)(-6\sin(6x)(e^{3x}) + (5x^4)(\cos(6x))(3e^{3x})$$

$$y' = 20x^3e^{3x}\cos(6x) - 30x^4e^{3x}\sin(6x) + 15x^4e^{3x}\cos(6x)$$

■ 2. Use the product rule to find the derivative of the function.

$$y = (-6x^2)(-2e^{5x})\tan(5x)$$

Solution:

The derivative of a function $y = f(x) \cdot g(x) \cdot h(x)$ using the product rule is

$$y' = f'(x) \cdot g(x) \cdot h(x) + f(x) \cdot g'(x) \cdot h(x) + f(x) \cdot g(x) \cdot h'(x)$$

Let

$$f(x) = -6x^2$$

$$f'(x) = -12x$$

$$g(x) = -2e^{5x}$$

$$g'(x) = -10e^{5x}$$

$$h(x) = \tan(5x)$$

$$h'(x) = 5\sec^2(5x)$$

Then by product rule, the derivative is

$$y' = (-12x)(-2e^{5x})(\tan(5x)) + (-6x^2)(-10e^{5x})(\tan(5x)) + (-6x^2)(-2e^{5x})(5\sec^2(5x))$$

$$y' = 24xe^{5x}\tan(5x) + 60x^2e^{5x}\tan(5x) + 60x^2e^{5x}\sec^2(5x)$$

3. Use the product rule to find the derivative of the function.

$$y = (\sin(7x))(7e^{4x})(2x^6 + 1)$$

Solution:

The derivative of a function $y = f(x) \cdot g(x) \cdot h(x)$ using the product rule is

$$y' = f'(x) \cdot g(x) \cdot h(x) + f(x) \cdot g'(x) \cdot h(x) + f(x) \cdot g(x) \cdot h'(x)$$

Let

$$f(x) = \sin(7x)$$

$$f'(x) = 7\cos(7x)$$

$$g(x) = 7e^{4x}$$

$$g'(x) = 28e^{4x}$$

$$h(x) = 2x^6 + 1$$

$$h'(x) = 12x^5$$

Then by product rule, the derivative is

$$y' = (7\cos(7x))(7e^{4x})(2x^6 + 1) + (\sin(7x))(28e^{4x})(2x^6 + 1) + (\sin(7x))(7e^{4x})(12x^5)$$

$$y' = 49e^{4x}(2x^6 + 1)\cos(7x) + 28e^{4x}(2x^6 + 1)\sin(7x) + 84x^5e^{4x}\sin(7x)$$

4. Use the product rule to find the derivative of the function.

$$y = (\cos(3x))(\sin(2x))(\tan(5x))(e^{2x})$$

Solution:

The derivative of a function $y = f(x) \cdot g(x) \cdot h(x) \cdot k(x)$ using the product rule is

$$y' = f'(x) \cdot g(x) \cdot h(x) \cdot k(x) + f(x) \cdot g'(x) \cdot h(x) \cdot k(x)$$

$$+f(x) \cdot g(x) \cdot h'(x) \cdot k(x) + f(x) \cdot g(x) \cdot h(x) \cdot k'(x)$$'

Let

$$f(x) = \cos(3x)$$

$$f'(x) = -3\sin(3x)$$

$$g(x) = \sin(2x)$$

$$g'(x) = 2\cos(2x)$$

$$h(x) = \tan(5x)$$

$$h'(x) = 5\sec^2(5x)$$

$$k(x) = e^{2x}$$

$$k'(x) = 2e^{2x}$$

Then by product rule, the derivative is

$$y' = -3\sin(3x)\sin(2x)\tan(5x)e^{2x} + \cos(3x)(2\cos(2x))\tan(5x)e^{2x}$$

$$+\cos(3x)\sin(2x)(5\sec^2(5x))e^{2x} + \cos(3x)\sin(2x)\tan(5x)(2e^{2x})$$

$$y' = -3e^{2x}\sin(2x)\sin(3x)\tan(5x) + 2e^{2x}\cos(2x)\cos(3x)\tan(5x)$$

$$+5e^{2x}\sin(2x)\cos(3x)\sec^2(5x) + 2e^{2x}\sin(2x)\cos(3x)\tan(5x)$$

QUOTIENT RULE

■ 1. Use the quotient rule to find the derivative of the function.

$$h(x) = \frac{2x+6}{7x+5}$$

Solution:

The derivative of a function

$$h(x) = \frac{f(x)}{g(x)}$$

using the quotient rule is

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

Let

$$f(x) = 2x + 6$$

$$f'(x) = 2$$

$$g(x) = 7x + 5$$

$$g'(x) = 7$$

Then the derivative is

$$h'(x) = \frac{2 \cdot (7x+5) - (2x+6) \cdot 7}{(7x+5)^2}$$

$$h'(x) = \frac{14x + 10 - 14x - 42}{(7x+5)^2}$$

$$h'(x) = -\frac{32}{(7x+5)^2}$$

■ 2. Use the quotient rule to find the derivative of the function.

$$h(x) = \frac{5x-3}{4x-9}$$

Solution:

The derivative of a function

$$h(x) = \frac{f(x)}{g(x)}$$

using the quotient rule is

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

Let

$$f(x) = 5x - 3$$

$$f'(x) = 5$$

$$g(x) = 4x - 9$$

$$g'(x) = 4$$

Then the derivative is

$$h'(x) = \frac{5 \cdot (4x - 9) - (5x - 3) \cdot 4}{(4x - 9)^2}$$

$$h'(x) = \frac{20x - 45 - 20x + 12}{(4x - 9)^2}$$

$$h'(x) = -\frac{33}{(4x - 9)^2}$$

■ 3. Use the quotient rule to find the derivative of the function.

$$h(x) = \frac{-8x}{5x + 2}$$

Solution:

The derivative of a function

$$h(x) = \frac{f(x)}{g(x)}$$

using the quotient rule is

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

Let

$$f(x) = -8x$$

$$f'(x) = -8$$

$$g(x) = 5x + 2$$

$$g'(x) = 5$$

Then the derivative is

$$h'(x) = \frac{-8 \cdot (5x+2) - (-8x) \cdot 5}{(5x+2)^2}$$

$$h'(x) = \frac{-40x - 16 + 40x}{(5x+2)^2}$$

$$h'(x) = -\frac{16}{(5x+2)^2}$$

■ 4. Use the quotient rule to find the derivative of the function.

$$h(x) = \frac{3x^2 + 12x}{e^x}$$

Solution:

The derivative of a function

$$h(x) = \frac{f(x)}{g(x)}$$

using the quotient rule is

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

Let

$$f(x) = 3x^2 + 12x$$

$$f'(x) = 6x + 12$$

$$g(x) = e^x$$

$$g'(x) = e^x$$

Then the derivative is

$$h'(x) = \frac{(6x + 12) \cdot e^x - (3x^2 + 12x) \cdot e^x}{(e^x)^2}$$

$$h'(x) = \frac{6xe^x + 12e^x - 3x^2e^x - 12xe^x}{e^{2x}}$$

$$h'(x) = \frac{-3x^2e^x - 6xe^x + 12e^x}{e^{2x}}$$

$$h'(x) = \frac{-3x^2 - 6x + 12}{e^x}$$

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