Topic: Limits at infinity and horizontal asymptotes

Question: Evaluate the limit.

$$\lim_{x\to\infty} \frac{6}{4x^2}$$

Answer choices:

A $\frac{3}{2}$

B 0

C ∞

D 1

Solution: B

To find the limit as $x \to \infty$, we'll look at the highest-degree terms in both the numerator and denominator.

The highest-degree term in the numerator is 6, which has a degree of 0. The highest-degree term in the denominator is $4x^2$, which has a degree of 2. Therefore,

N < D: $0 < 2$

When the degree of the numerator is less that the degree of the denominator, the function has a horizontal asymptote at $y = 0$.

So, as $x \to \infty$, the limit is 0.

Topic: Limits at infinity and horizontal asymptotes

Question: Evaluate the limit.

$$\lim_{x\to\infty} \frac{6x^3+2x^2-x+1}{8x^3-1}$$

Answer choices:

A -1

B ∞

C $\frac{3}{4}$

D $\frac{3}{8}$

Solution: C

To find the limit as $x \to \infty$, we'll look at the highest-degree terms in both the numerator and denominator.

The highest-degree term in the numerator is $6x^3$, which has a degree of 3. The highest-degree term in the denominator is $8x^3$, which has a degree of 3. Therefore,

N = D: $3 = 3$

When the degree of the numerator is equal to the degree of the denominator, the function has a horizontal asymptote given by the ratio of the coefficients on those highest-degree terms.

$$\frac{6x^3 + 2x^2 - x + 1}{8x^3 - 1}$$

$$\frac{6x^3}{8x^3}$$

$$\frac{6}{8}$$

$$\frac{3}{4}$$

So, as $x \to \infty$, the limit is $3/4$.

Topic: Limits at infinity and horizontal asymptotes

Question: Evaluate the limit.

$$\lim_{x\to\infty} \frac{x^2+1}{5x^4+2x^2}$$

Answer choices:

A $\sqrt{5}$

B $\frac{\sqrt{5}}{5}$

C ∞

D 0

Solution: D

To find the limit as $x \to \infty$, we'll look at the highest-degree terms in both the numerator and denominator.

The highest-degree term in the numerator is x^2, which has a degree of 2. The highest-degree term in the denominator is $5x^4$, which has a degree of 4. Therefore,

N < D: $2 < 4$

When the degree of the numerator is less that the degree of the denominator, the function has a horizontal asymptote at $y = 0$.

So, as $x \to \infty$, the limit is 0.