Calculus 1 Workbook Solutions

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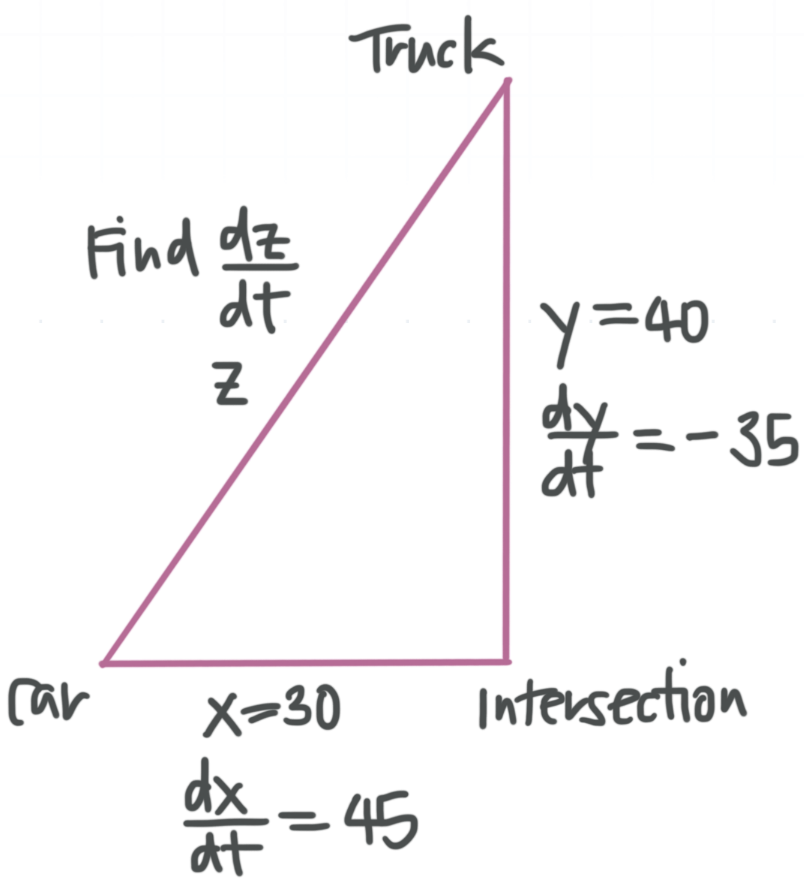
1. A truck is 40 miles north of an intersection, traveling toward the intersection at 35 mph. At the same time, another car is 30 miles west of the intersection, traveling away from the intersection at 45 mph. Is the distance between the vehicles increasing or decreasing at that moment? At what rate?

Solution:

Draw a diagram.



Use the Pythagorean theorem $x^2+y^2=z^2$, then differentiate with respect to time.

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$$

$$x\frac{dx}{dt}+y\frac{dy}{dt}=z\frac{dz}{dt}$$

Substitute what we know into the derivative, then solve for dz/dt.

$$30(45)+40(-35)=50\frac{dz}{dt}$$

$$\frac{dz}{dt}=\frac{30(45)+40(-35)}{50}$$

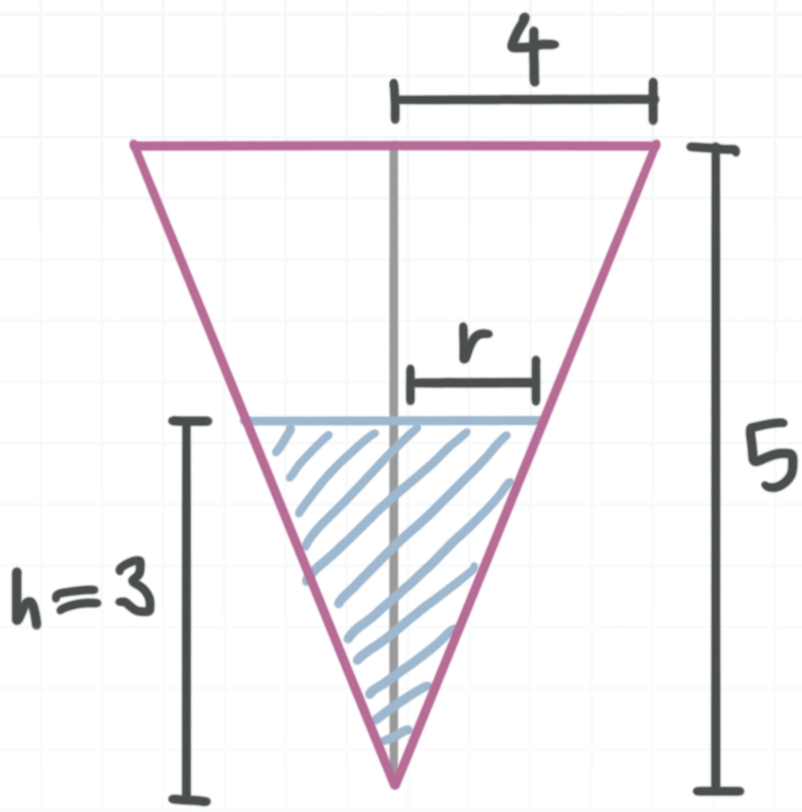
$$\frac{dz}{dt}=-\frac{50}{50}=-1$$

The distance between the two vehicles is decreasing at a rate of 1 mph.

■ 2. Water is flowing out of a cone-shaped tank at a rate of 6 cubic inches per second. If the cone has a height of 5 inches and a base radius of 4 inches, how fast is the water level falling when the water is 3 inches deep?

Solution:

Draw a diagram.



The volume of a cone is $V=(1/3)\pi r^2h$. To find the base radius of the water, we'll use similar triangles.

$$\frac{r}{3}=\frac{4}{5}$$

$$r=\frac{12}{5}$$

The volume of the cone of water is

$$V=\frac{1}{3}\pi r^2h$$

$$V=\frac{1}{3}\pi\left(\frac{12}{5}\right)^2h$$

$$V=\frac{1}{3}\pi\left(\frac{144}{25}\right)h$$

$$V = \frac{48}{25}\pi h$$

Differentiate the volume equation with respect to t.

$$\frac{dV}{dt} = \frac{48}{25}\pi \cdot \frac{dh}{dt}$$

The problem states that $dV/dt = -6$. Substitute this into the derivative equation and solve for dh/dt.

$$-6 = \frac{48}{25}\pi \cdot \frac{dh}{dt}$$

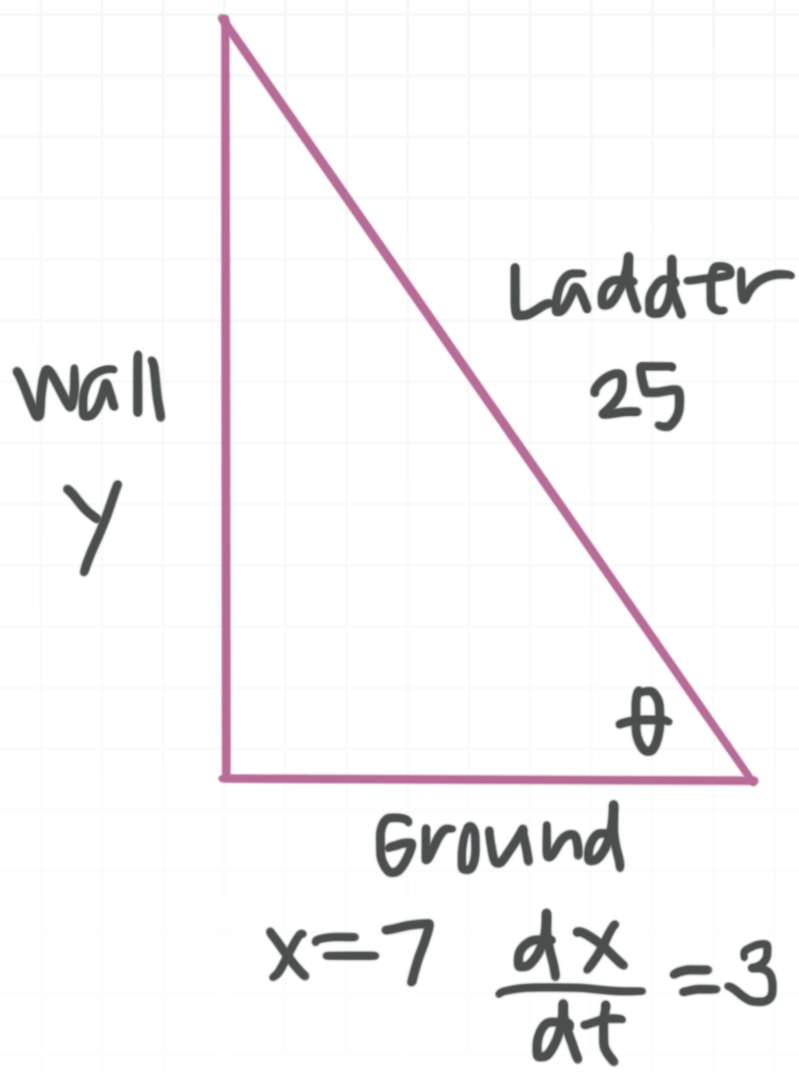
$$-6 \cdot \frac{25}{48\pi} = \frac{dh}{dt}$$

$$\frac{dh}{dt} = -\frac{25}{8\pi} \text{ inches per second}$$

■ 3. A ladder 25 feet long leans against a vertical wall of a building. If the bottom of the ladder is pulled away horizontally from the building at 3 feet per second, how fast is the angle formed by the ladder and the horizontal ground decreasing when the bottom of the ladder is 7 feet from the base of the wall?

Solution:

Draw a diagram.



Find y using the Pythagorean theorem.

$$y^2 + 7^2 = 25^2$$

$$y = 24$$

Use the cosine function, which gives the equation

$$\cos\theta = \frac{x}{25}$$

Differentiate with respect to t.

$$-\sin\theta\frac{d\theta}{dt} = \frac{1}{25}\frac{dx}{dt}$$

Substitute what we know.

$$-\frac{24}{25}\cdot\frac{d\theta}{dt}=\frac{1}{25}\cdot 3$$

$$\frac{d\theta}{dt}=\frac{3}{25}\cdot -\frac{25}{24}=-\frac{3}{24}=-\frac{1}{8}\text{ feet per second}$$

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