# Exponential derivatives

Exponential and logarithmic functions have their own set of derivative rules.

The exponential function we'll work with most often is $y = e^x$. The number $e$ is a special constant with a value equal to $e \approx 2.718281828459045...$ It's similar to how we use $\pi$ to represent the constant $\pi \approx 3.141592653589793...$

These kinds of exponential functions look a lot like the power functions we've seen before (like $x^3$), but there's one critical difference.

In power functions, the base is a variable, and the exponent is a constant. But in exponential functions, the base is the constant, and the exponent is the variable. So they're opposite situations.

## Exponential derivative rules

Here are the derivatives of simple exponential functions:

| **Exponential functions** | **Their derivatives** |
|---|---|
| $y = e^x$ | $y' = e^x$ |
| $y = a^x$ | $y' = a^x(\ln a)$ |

There are three important things to say about these derivative formulas.

First, we have $\ln a$ as part of the formula for the derivative of $y = a^x$. That $\ln$ value represents the **natural log function**. Similar to the way we used $\sin x$

to represent the sine function, we use $\ln x$ to represent the natural log function. So whenever we see $\ln$, we know that we're dealing with the natural log. Whatever value of $x$ we plug into $\ln x$, the natural log function will output a corresponding value. We'll talk more about logs and natural logs and their derivatives in the next lesson.

Second, there's actually no difference between the derivative formulas for $y = e^x$ and $y = a^x$. When we take the derivative of any $y = a^x$, we always need to multiply by $\ln a$. When we do that for the derivative of $y = e^x$, we get

$$y' = e^x(\ln e)$$

But $\ln e = 1$, so the derivative simplifies to

$$y' = e^x(1)$$

$$y' = e^x$$

So when the base of the exponential function is $e$, that $\ln a$ essentially disappears. But for any base other than $e$, that $\ln a$ will remain as part of the derivative.

Third, just like with trigonometric functions, we need to apply chain rule every time we take the derivative of an exponential function. With trigonometric functions, the "inside function" was the argument of the trig function. With exponential functions, the "inside function" is the exponent. In both $y = e^x$ and $y = a^x$, the exponent is $x$, and the derivative of $x$ is $1$. So when we differentiate $y = e^x$ or $y = a^x$, applying chain rule means we multiply by $1$, which of course doesn't change the value of the derivative.

But when the exponent is anything other than $x$, the derivative of the exponent will be something other than $1$, which means that applying chain rule will change the value of the derivative. For instance, the derivative of $y=e^{2x}$ is $y'=2e^{2x}$. If we wanted to show chain rule as part of the exponential derivative formulas, we'd get

| **Exponential functions** | **Their derivatives** |
|---|---|
| $y=e^{g(x)}$ | $y'=e^{g(x)}g'(x)$ |
| $y=a^{g(x)}$ | $y'=a^{g(x)}(\ln a)g'(x)$ |

Let's try an example where we differentiate an exponential function with base $a$.

### Example

Find the derivative of the exponential function.

$$y=42^{6x}$$

In this function, $a=42$ and the exponent is $6x$. We'll differentiate by applying the formula for exponential derivatives.

$$y'=a^{g(x)}(\ln a)g'(x)$$

$$y'=42^{6x}(\ln(42))(6)$$

$$y'=6(42)^{6x}\ln(42)$$

Now let's try an example with base $e$.

**Example**

Find the derivative of the exponential function.

$$y = 3e^{2x}$$

The base is $e$ and the exponent is $2x$, so the derivative is

$$y' = e^{g(x)}g'(x)$$

$$y' = 3e^{2x}(2)$$

$$y' = 6e^{2x}$$

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Let's try one final, more complex example.

**Example**

Find the derivative of the exponential function.

$$y = 8x^3 - (4^{7x})(e^{8x+1}) + 6^{2x-9}$$

We have to take the derivative one term at a time, remembering to apply product rule when we get to $(4^{7x})(e^{8x+1})$.

$$y' = 24x^2 - \left[(4^{7x})(\ln 4)(7)(e^{8x+1}) + (4^{7x})(e^{8x+1})(8)\right] + (6^{2x-9})(\ln 6)(2)$$

$$y' = 24x^2 - (7e^{8x+1})(4^{7x})(\ln 4) - (8e^{8x+1})(4^{7x}) + 2(6^{2x-9})(\ln 6)$$