Calculus 1 Workbook Solutions

Functions

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VERTICAL LINE TEST

1. Determine algebraically whether or not the equation represents a function.

$$(x-1)^2 + y = 3$$

Solution:

Notice that the equation can be rewritten as $y = -(x-1)^2 + 3$, which is just a transformation of $y = -x^2$. Therefore, the equation represents a function.

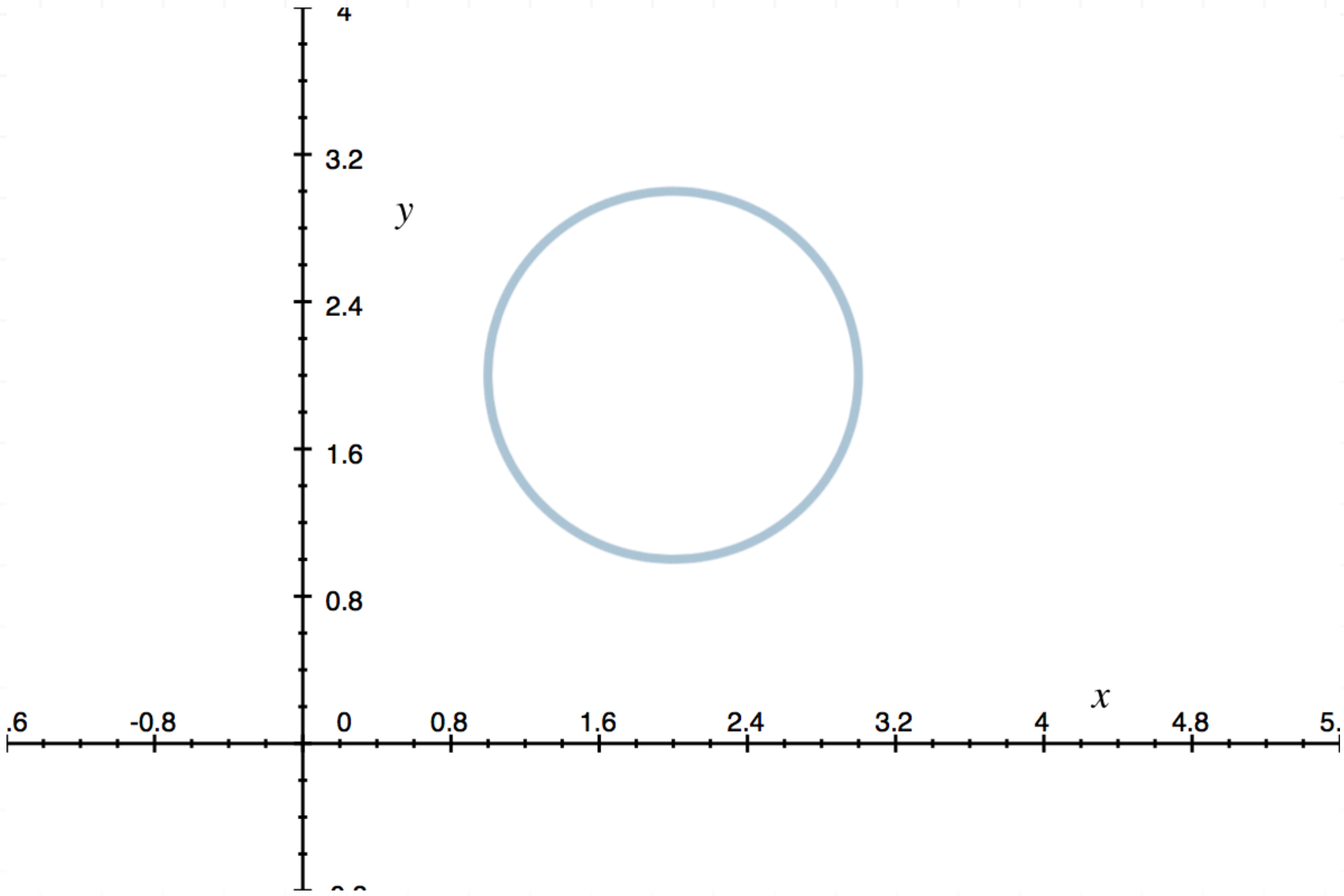
2. Fill in the blanks in the following statement using “equations,” and “functions.”

All ___________ are ___________.

Solution:

functions, equations

3. Use the Vertical Line Test to determine whether or not the graph is the graph of a function.



Solution:

The graph does not pass the Vertical Line Test, because any vertical line between the left edge of the circle and the right edge of the circle intersects the graph more than once. Therefore, the graph doesn't represent a function.

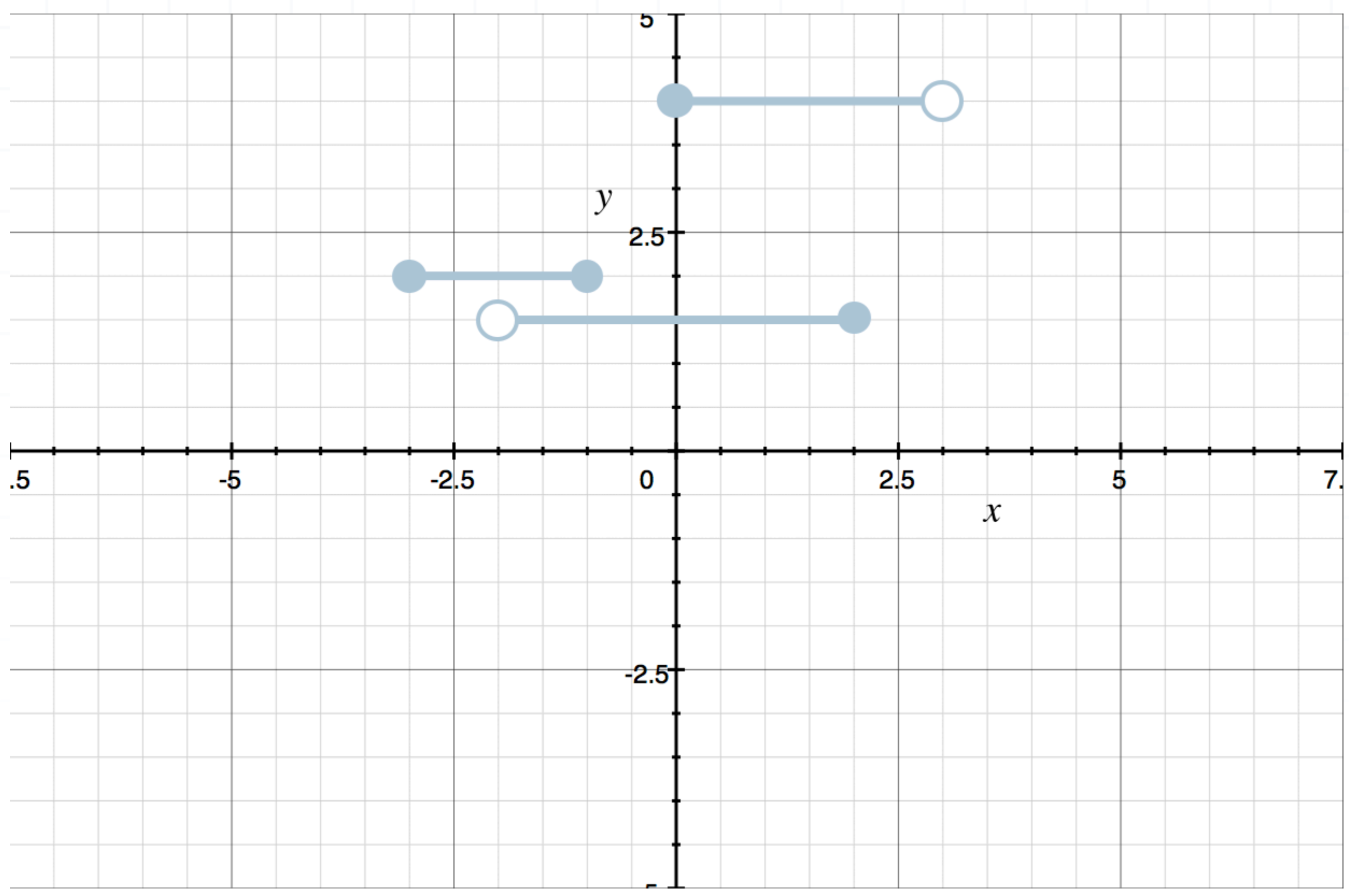
■ 4. Determine algebraically whether or not the equation represents a function.

$$y^2 = x + 1$$

Solution:

Notice that for $x = 0$, $y^2 = 1$ which gives $y = -1, 1$. So for one input x, there are two outputs for y, so the equation does not represent a function.

5. Use the Vertical Line Test to determine whether or not the graph represents a function.



Solution:

There are different vertical lines that intersect the graph more than once. An example would be $x = 0$, which intersects the graph at $y = 3/2$ and $y = 4$. So by the Vertical Line Test, the graph is not a graph of a function.

■ 6. Explain why the Vertical Line Test determines whether or not a graph represents a function.

Solution:

There are many correct answers. But they should all more or less say something like:

> "The Vertical Line Test can show whether or not a graph represents a function, because if any perfectly vertical line crosses the graph more than once, it proves that there are two output values of y for the one input value of x."

■ 7. Fill in the blanks in the following statement using: equations, functions.

Not all __________ are __________.

Solution:

equations, functions

■ 8. Determine algebraically whether or not the equation represents a function.

$$x^3 + y = 5$$

Solution:

Notice that this equation can be rewritten as $y = -x^3 + 5$, which is just a transformation of $y = -x^3$. Therefore, the equation represents a function.

DOMAIN AND RANGE

1. Find the domain of $f(x)$.

$$f(x)=\frac{3}{x(x+1)}+x^2$$

Solution:

In this function, the denominator cannot be equal to 0. The values of x that make the denominator 0 are $x=0$ and $x=-1$. So the domain of the function is all $x\neq 0,-1$, which we can write in interval notation as

$$(-\infty,-1)\cup(-1,0)\cup(0,\infty)$$

2. Find the domain and range of the given set.

$$(-1,-3),\quad (0,5),\quad (-3,6),\quad (0,-3)$$

Solution:

The domain is all the x-values and the range is all the y-values. Therefore the domain and range are

Domain: $-1,\ 0,\ -3$

Range: $-3,\ 5,\ 6$

3. Find the domain and range of $g(x)$.

$$g(x)=\frac{\sqrt{x-2}}{3}$$

Solution:

In this function, the radicand (the expression under the square root) must be 0 or positive. So $x-2\geq 0$, which tells us that $x\geq 2$. Therefore the domain of the function in interval notation is $[2,\infty)$. Since the square root function cannot be negative, the range in interval notation is $[0,\infty)$.

4. Find the domain and range of the function.

$$f(x)=\frac{2}{x}+1$$

Solution:

In this function, the denominator cannot be 0, which means $x\neq 0$. Therefore the domain of the function in interval notation is

$$(-\infty,0)\cup(0,\infty)$$

Since the term $2/x$ is never 0, $f(x)$ can never be 1. Therefore the range of the function in interval notation is

$$(-\infty,1) \cup (1,\infty)$$

5. Give an example of a function that has a domain of $[1,\infty)$.

Solution:

There are many correct answers. The simplest one is

$$f(x) = \sqrt{x-1}$$

Note that since the 1 is included in the domain, the function $f(x) = \ln(x-1)$ would not work.

6. Find the domain and range of $f(x)$.

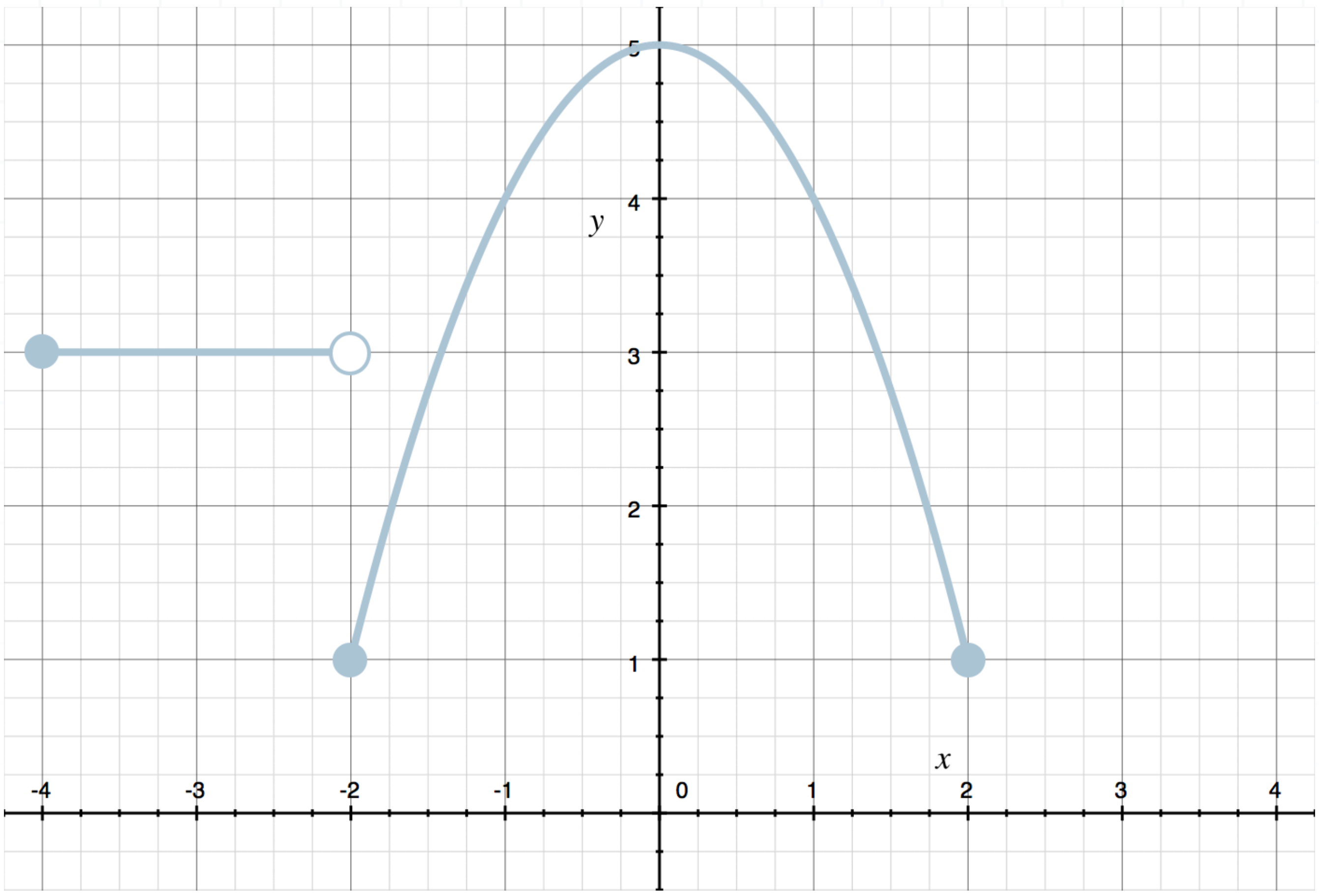
$$f(x) = \ln(x+3) + 5$$

Solution:

In this function, the input into $\ln$ must be positive. So $x + 3 > 0$, which tells us that $x > -3$. Therefore the domain in interval notation is $(-3,\infty)$. Since the range of $\ln(x)$ is $(-\infty, \infty)$, then the range of $f(x)$ is $(-\infty, \infty)$.

DOMAIN AND RANGE FROM A GRAPH

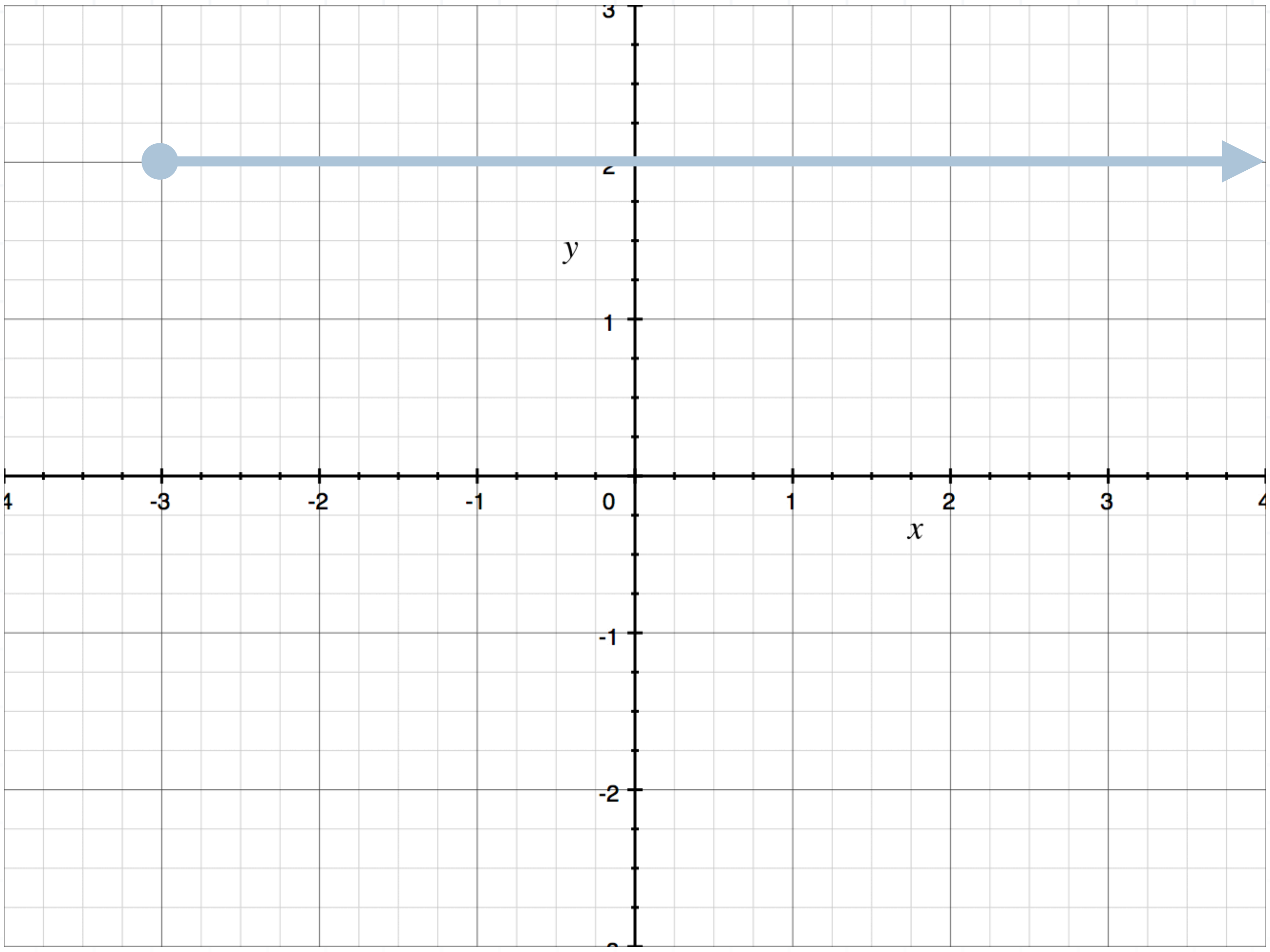
1. What is the domain and range of the function? Assume the graph does not extend beyond the graph shown.



Solution:

Solution: The domain of the function given in the graph is determined by the x-values, which are defined on the interval $[-4,2]$. The range is determined by the y-values, which are defined on the interval $[1,5]$.

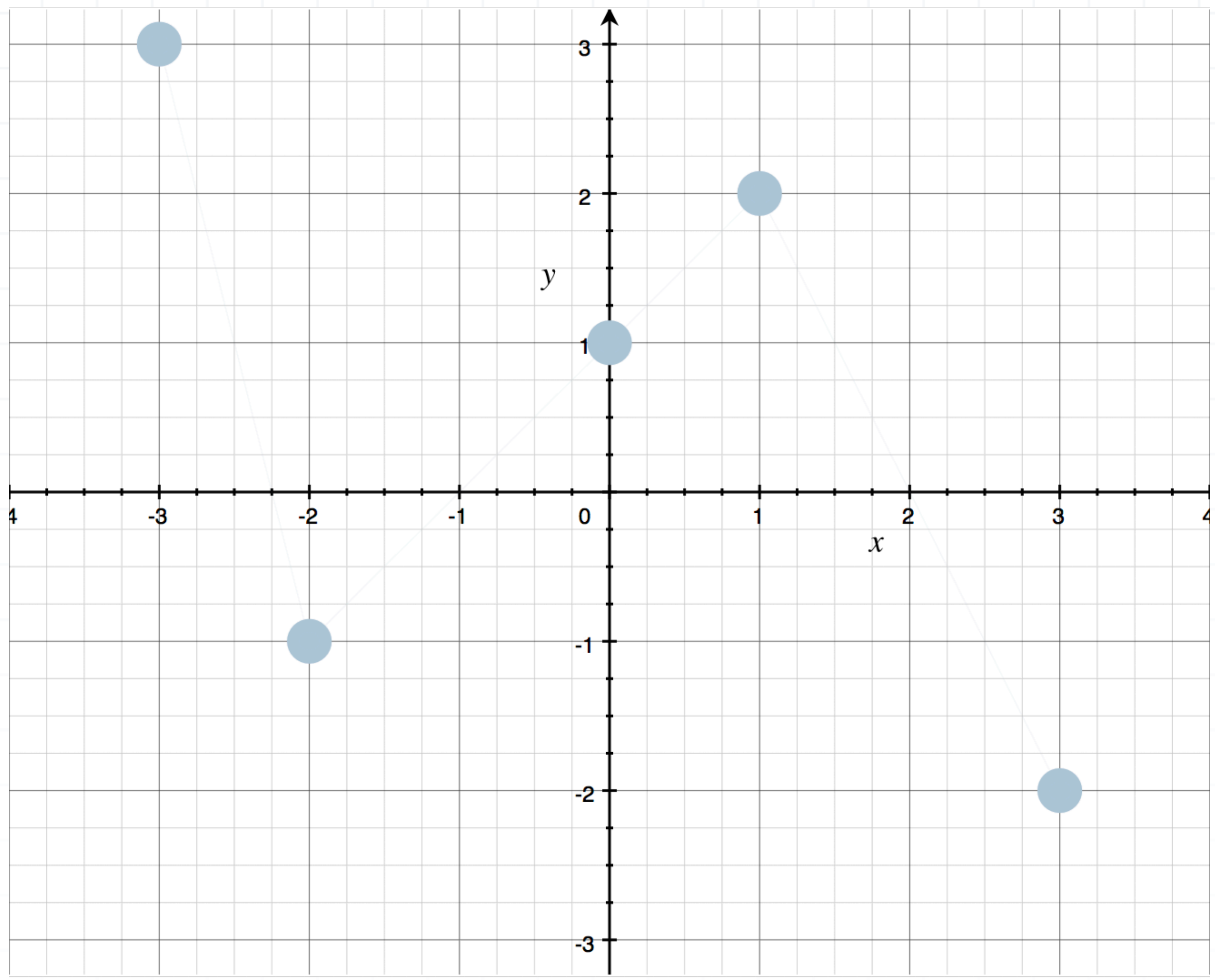
2. What is the domain and range of the function?



Solution:

The domain of the function given in the graph is determined by the x-values, which are defined by the ray on the interval $[-3,\infty)$. The range is determined by the y-values, which is only $y=2$.

■ 3. Determine the domain and range of the function.



Solution:

The domain of the function given in the graph is determined by the x-values, which are $\{-3,-2,0,1,3\}$. The range is determined by the y-values, which are $\{-2,-1,1,2,3\}$.

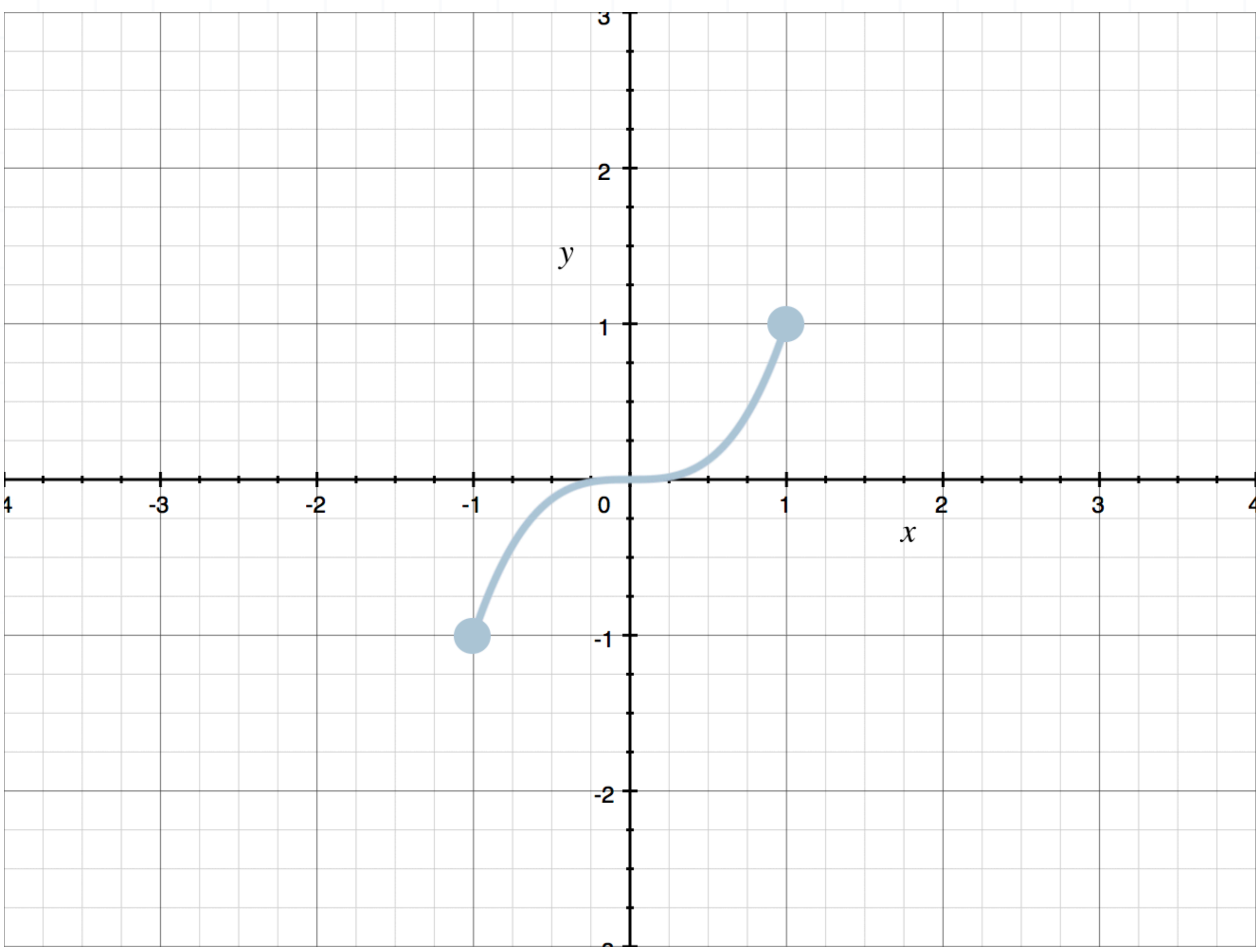
■ 4. Fill in the blanks in the following description of the domain of a graph.

"The domain is all the values of the graph from __________ to __________."

Solution:

left, right

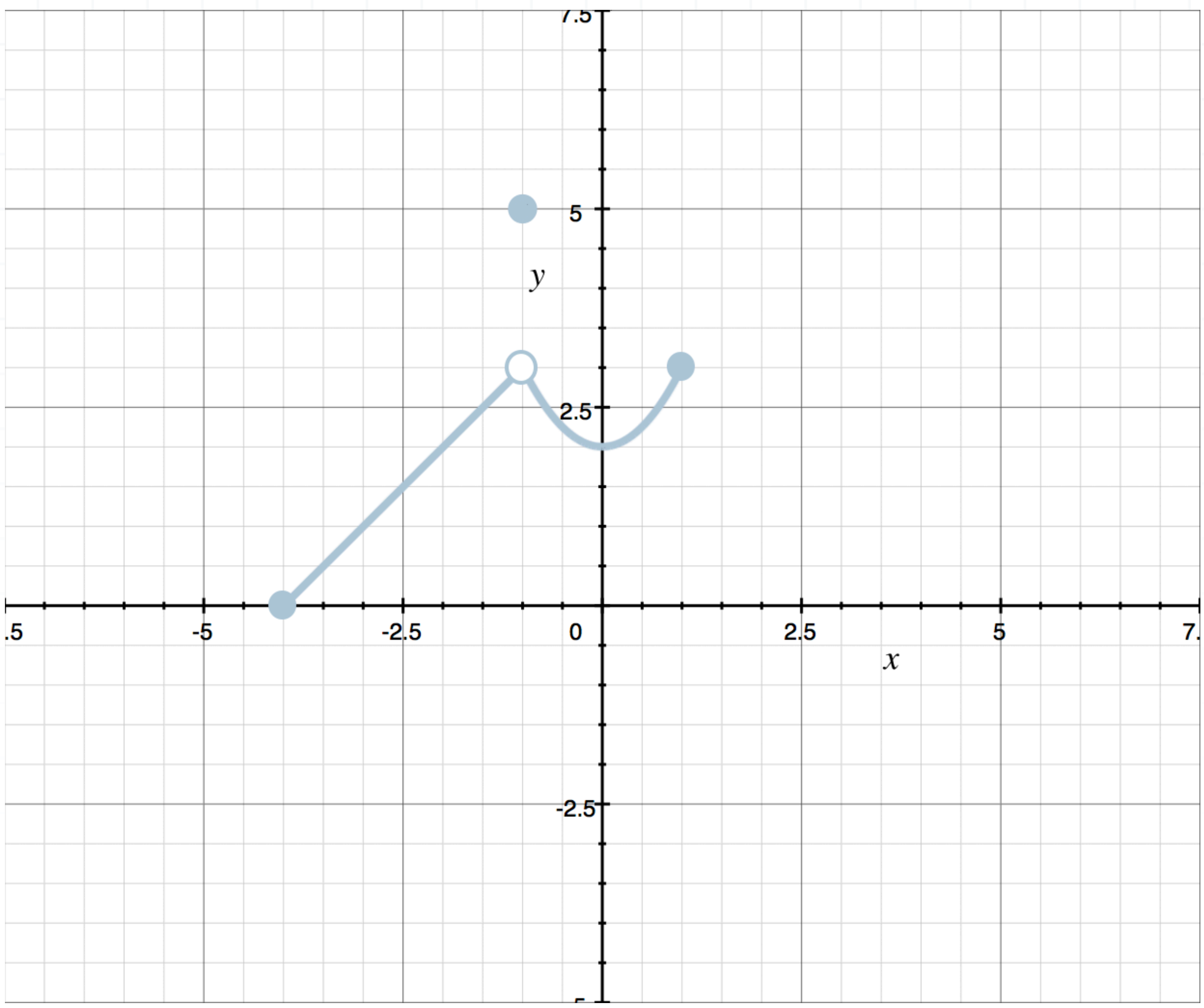
5. What is the domain and range of the function? Assume the graph does not extend beyond the graph shown.



Solution:

The domain of the function given in the graph is determined by the x-values, which are defined by the interval $[-1,1]$. The range is determined by the y-values, which are defined by the interval $[-1,1]$.

6. What is the domain and range of the function? Assume the graph does not extend beyond the graph shown.



Solution:

The domain of the function given in the graph is determined by the x-values, which are defined on the interval $[-4,1]$. The range is determined by the y-values, which are defined on the interval $[0,3]$ and at $y=5$.

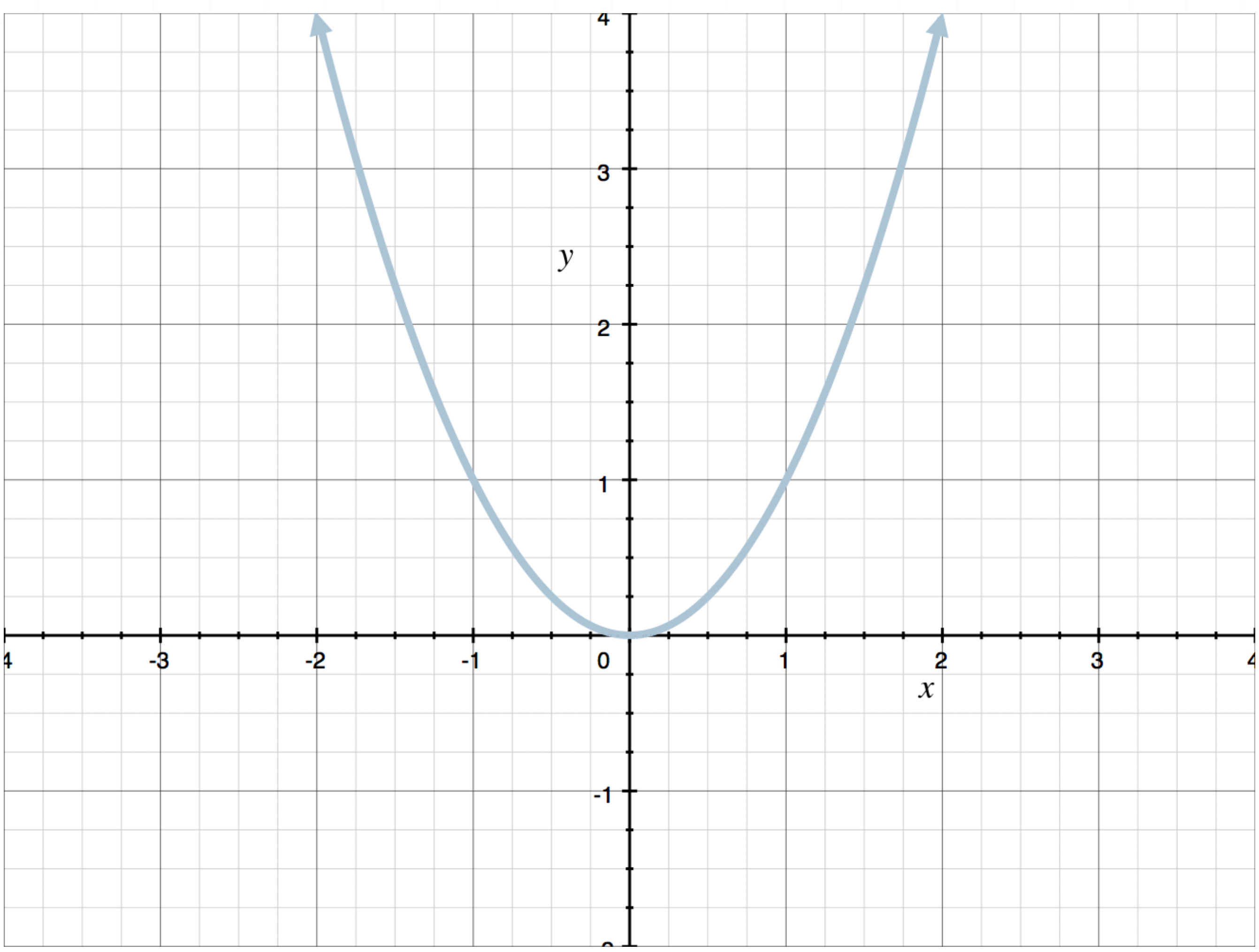
■ 7. Fill in the blanks in the following description of the range of a graph.

"The range is all the values of the graph from __________ to __________."

Solution:

down, up

■ 8. What is the domain and range of the function?



Solution:

The domain of the function given in the graph is determined by the x -values, which are defined on the interval $(-\infty, \infty)$. The range is determined by the y-values, which are defined on the interval $[0,\infty)$.

EVEN, ODD, OR NEITHER

■ 1. Is the function even, odd, or neither?

$$f(x) = -x^5 + 2x^2 - 1$$

Solution:

Substitute $-x$ for x.

$$f(-x) = -(-x)^5 + 2(-x)^2 - 1$$

$$f(-x) = x^5 + 2x^2 - 1$$

Because $f(-x) \neq f(x)$, the function is not even. To see if it's odd, we check

$$-f(x) = -(-x^5 + 2x^2 - 1) = x^5 - 2x^2 + 1$$

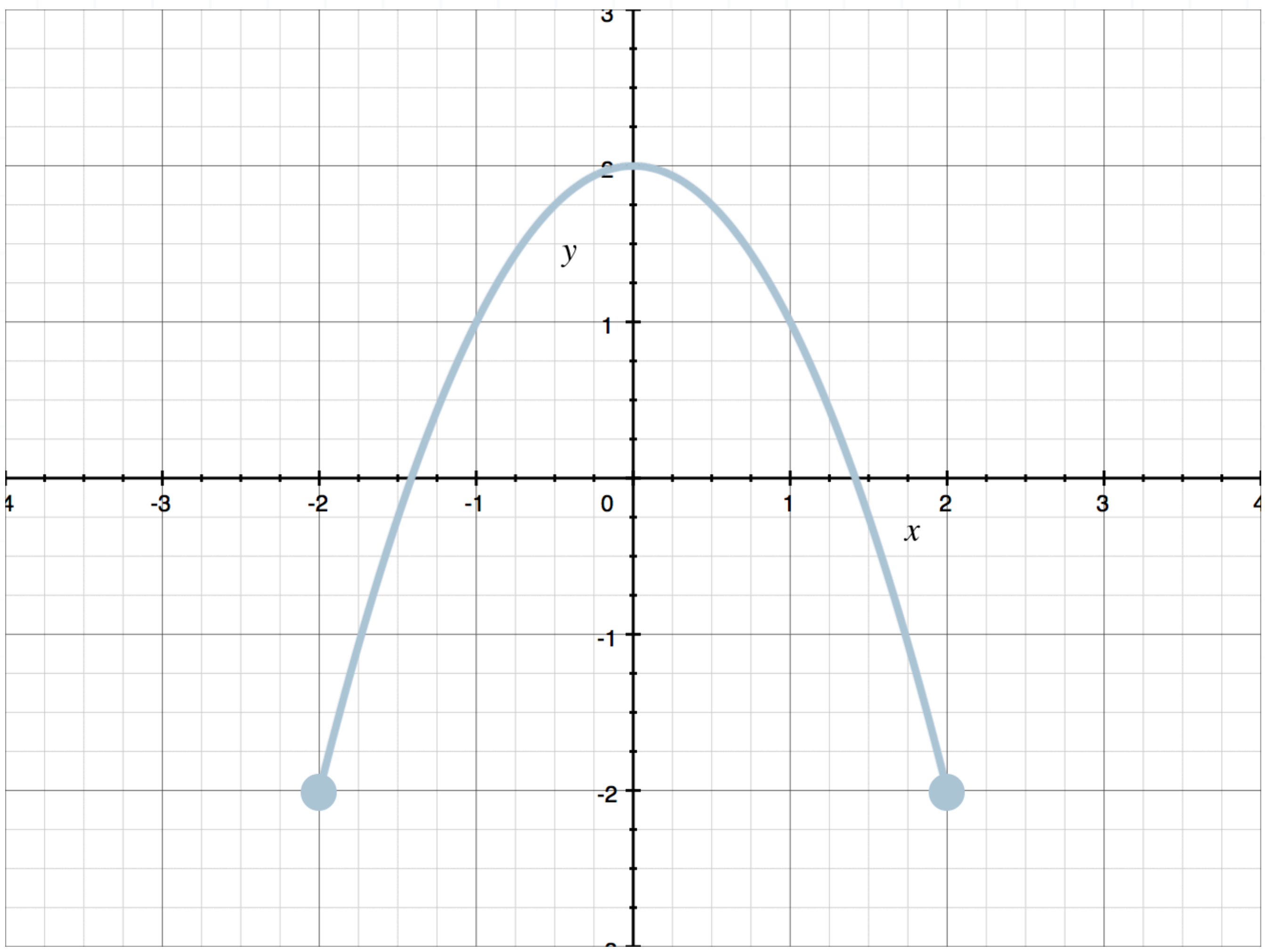
Because $f(-x) \neq -f(x)$, the function is not odd. Therefore, the function is neither even nor odd.

■ 2. Describe the symmetry of an even function, and give an example of an even function.

Solution:

An even function is symmetric about the y-axis. There are many examples of even functions, one being $f(x) = x^2$.

3. Determine if the graph is the graph of a function that is even, odd, or neither.



Solution:

Notice that the graph is symmetric about the y-axis and therefore the graph is the graph of an even function.

4. Is the function even, odd, or neither?

$$g(x) = -3x^2 + 5x^6$$

Solution:

Substitute $-x$ for x.

$$g(-x) = -3(-x)^2 + 5(-x)^6$$

$$g(-x) = -3x^2 + 5x^6$$

Because $f(-x) = f(x)$, the function is even.

5. Show that the function is neither even nor odd.

$$f(x) = x^2 - 5x + 7$$

Solution:

Substitute $-x$ for x.

$$f(-x) = (-x)^2 - 5(-x) + 7$$

$$f(-x) = x^2 + 5x + 7$$

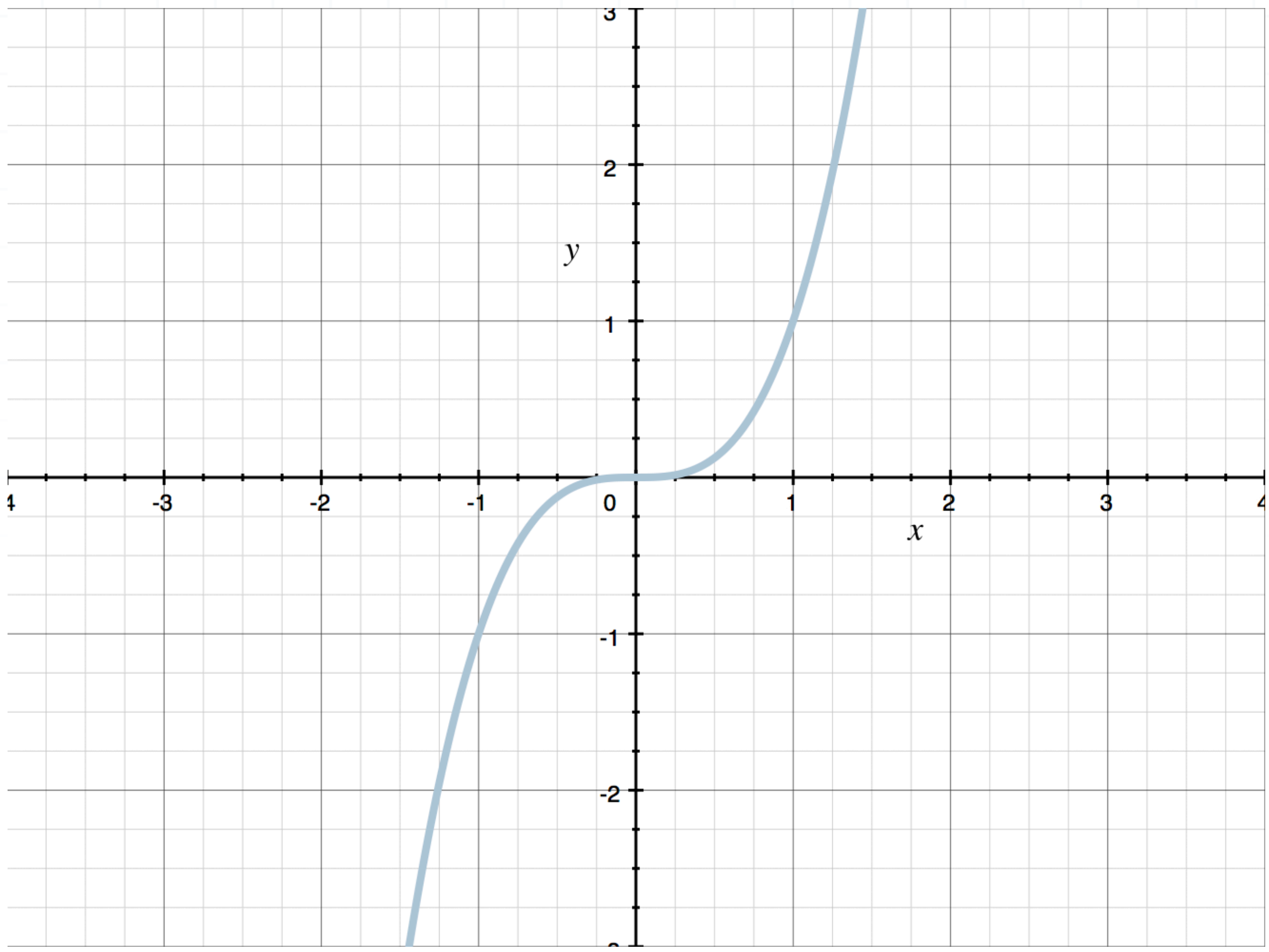
Because $f(-x) \neq f(x)$, the function is not even. To see if it's odd, we check

$$-f(x) = -(x^2 - 5x + 7)$$

$$-f(x) = -x^2 + 5x - 7$$

Because $f(-x) \neq -f(x)$, the function is not odd. Therefore, the function is neither even nor odd.

6. Determine if the graph is the graph of a function that is even, odd, or neither.



Solution:

Notice that the graph is symmetric about the origin, and therefore the graph is the graph of an odd function.

■ 7. Is the function even, odd, or neither?

$$h(x) = x^3 - 3x$$

Solution:

Substitute $-x$ for x.

$$h(-x) = (-x)^3 - 3(-x)$$

$$h(-x) = -x^3 + 3x$$

Because $f(-x) \neq f(x)$, the function is not even. To see if it's odd, we check

$$-f(x) = -(x^3 - 3x)$$

$$-f(x) = -x^3 + 3x$$

Because $f(-x) = -f(x)$, the function is odd.

■ 8. Describe the symmetry of an odd function, and give an example of an odd function.

Solution:

An odd function is symmetric about the origin. There are many correct examples of odd functions, one being $f(x) = x^3$.

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