Topic: Value that makes two tangent lines parallel

Question: What is the value of a such that the tangent lines of $f(x)$ at $x = a$ and $x = a + 1$ are parallel?

$$f(x) = x^3 + x^2 + x - 1$$

Answer choices:

A $a = -\frac{1}{2}$

B $a = \frac{1}{2}$

C $a = -\frac{5}{6}$

D $a = \frac{5}{6}$

Solution: C

Start by finding the derivative of $f(x)$.

$$f'(x) = 3x^2 + 2x + 1$$

Now we'll plug both $x = a$ and $x = a + 1$ into the derivative.

$$f'(a) = 3a^2 + 2a + 1$$

$$f'(a + 1) = 3(a + 1)^2 + 2(a + 1) + 1$$

These represent the slope of each tangent line, so we'll set them equal to one another.

$$3a^2 + 2a + 1 = 3(a + 1)^2 + 2(a + 1) + 1$$

$$3a^2 + 2a + 1 = 3(a^2 + 2a + 1) + 2a + 2 + 1$$

$$3a^2 + 2a + 1 = 3a^2 + 6a + 3 + 2a + 2 + 1$$

Collect like terms and solve for a.

$$2a + 1 = 6a + 3 + 2a + 2 + 1$$

$$1 = 6a + 3 + 2 + 1$$

$$0 = 6a + 3 + 2$$

$$0 = 6a + 5$$

$$6a = -5$$

$$a = -\frac{5}{6}$$

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Question: What is the value of a such that the tangent lines of $f(x)$ at $x = a$ and $x = a + 1$ are parallel?

$$f(x) = 2x^3 - x^2 + x + 12$$

Answer choices:

A $a = \frac{1}{3}$

B $a = -\frac{1}{3}$

C $a = \frac{1}{2}$

D $a = -\frac{1}{2}$

Solution: B

Start by finding the derivative of $f(x)$.

$$f'(x) = 6x^2 - 2x + 1$$

Now we'll plug both $x = a$ and $x = a + 1$ into the derivative.

$$f'(a) = 6a^2 - 2a + 1$$

$$f'(a + 1) = 6(a + 1)^2 - 2(a + 1) + 1$$

These represent the slope of each tangent line, so we'll set them equal to one another.

$$6a^2 - 2a + 1 = 6(a + 1)^2 - 2(a + 1) + 1$$

$$6a^2 - 2a + 1 = 6(a^2 + 2a + 1) - 2a - 2 + 1$$

$$6a^2 - 2a + 1 = 6a^2 + 12a + 6 - 2a - 2 + 1$$

Collect like terms and solve for a.

$$-2a + 1 = 12a + 6 - 2a - 2 + 1$$

$$1 = 12a + 6 - 2 + 1$$

$$0 = 12a + 6 - 2$$

$$0 = 12a + 4$$

$$12a = -4$$

$$a = -\frac{1}{3}$$

Topic: Value that makes two tangent lines parallel

Question: What is the value of a such that the tangent lines of $f(x)$ at $x = a$ and $x = a + 2$ are parallel?

$$f(x) = x^3 + 3x^2 - x - 5$$

Answer choices:

A $a = \frac{1}{2}$

B $a = -\frac{1}{2}$

C $a = 2$

D $a = -2$

Solution: D

Start by finding the derivative of $f(x)$.

$$f'(x) = 3x^2 + 6x - 1$$

Now we'll plug both $x = a$ and $x = a + 2$ into the derivative.

$$f'(a) = 3a^2 + 6a - 1$$

$$f'(a + 2) = 3(a + 2)^2 + 6(a + 2) - 1$$

These represent the slope of each tangent line, so we'll set them equal to one another.

$$3a^2 + 6a - 1 = 3(a + 2)^2 + 6(a + 2) - 1$$

$$3a^2 + 6a - 1 = 3(a^2 + 4a + 4) + 6a + 12 - 1$$

$$3a^2 + 6a - 1 = 3a^2 + 12a + 12 + 6a + 12 - 1$$

Collect like terms and solve for a.

$$6a - 1 = 12a + 12 + 6a + 12 - 1$$

$$-1 = 12a + 12 + 12 - 1$$

$$0 = 12a + 12 + 12$$

$$0 = 12a + 24$$

$$12a = -24$$

$$a = -2$$