Even, odd, or neither

We can categorize a function - according to its “behavior” - as even, odd, or neither. Each of those categories corresponds to a particular type of symmetry of the graph of a function. In fact, it’s often easiest to tell whether a function is even, odd, or neither by looking at its graph. Sometimes it’s difficult or impossible to graph a function, so there is an algebraic way to check as well.

Even functions

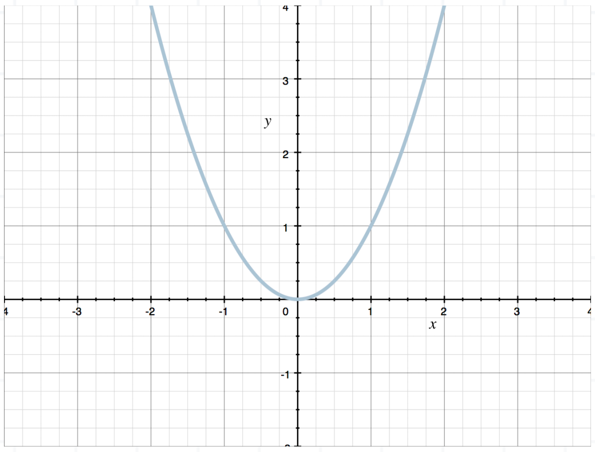
Symmetric with respect to the y-axis

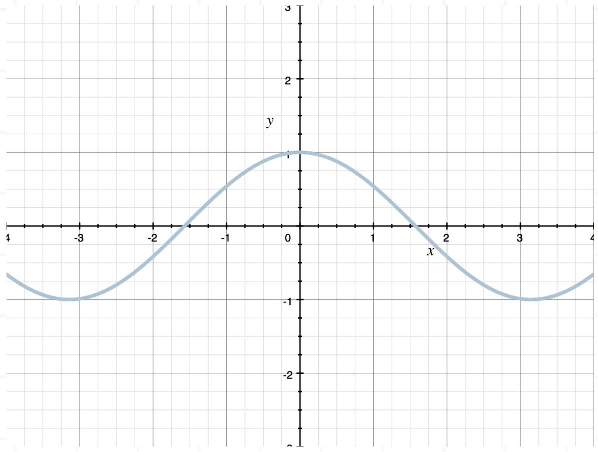
When you plug $-x$ into the expression for an even function, it will simplify to the expression for the original function. This means that it doesn’t matter whether you plug in x or $-x$, your output will be the same. So

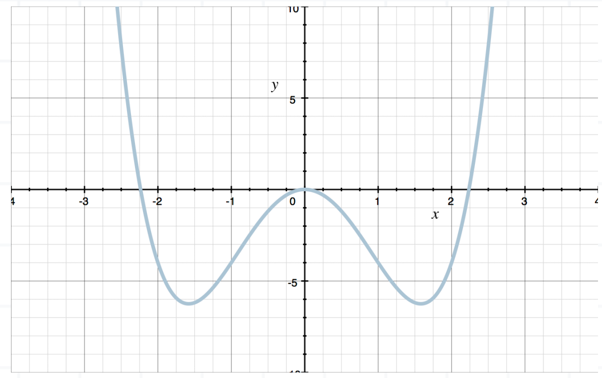
$$f(-x)=f(x)$$

What this means in terms of the graph of an even function is that the part that’s to the left of the y-axis is a mirror image of the part that’s to the right of the y-axis.

Below are graphs that are symmetric with respect to the y-axis and therefore represent even functions.







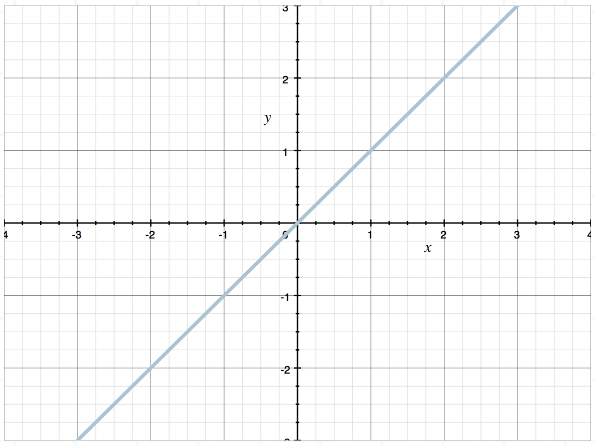
Odd functions

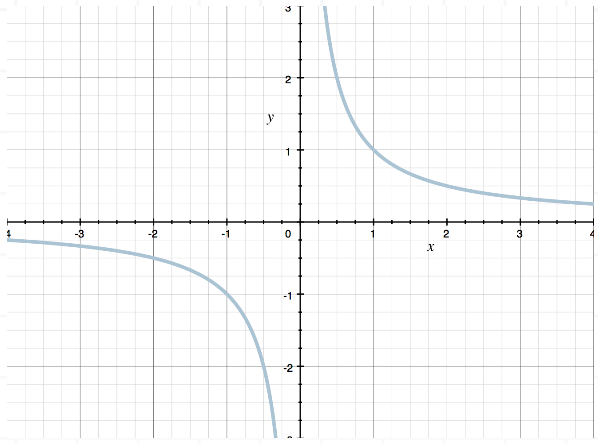
Symmetric with respect to the origin

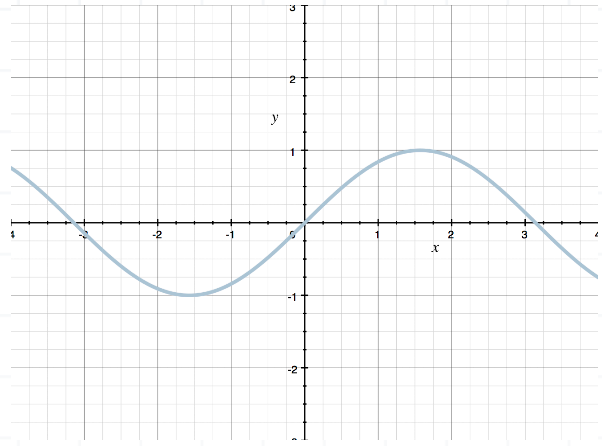
When you plug $-x$ into the expression for an odd function, it will simplify to the negative of the expression for the original function, or the expression for the original function multiplied by -1. This means that when you plug in $-x$, you'll get essentially the same output that you get when you plug in x, the only difference being that its sign will be opposite the sign of the original output. So

$$f(-x) = -f(x)$$

Below are graphs that are symmetric with respect to the origin and therefore represent odd functions. Be sure to visually compare quadrants that are diagonal from each other (quadrants I and III, and quadrants II and IV). For every first-quadrant point (x, y) in the graph of an odd function, there's a third-quadrant point of the graph with coordinates $(-x, -y)$. Similarly, for every second-quadrant point (x, y) in the graph of an odd function, there's a fourth-quadrant point of the graph with coordinates $(-x, -y)$.



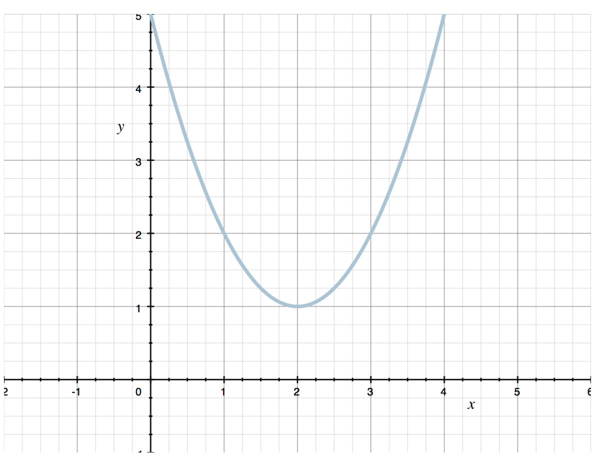


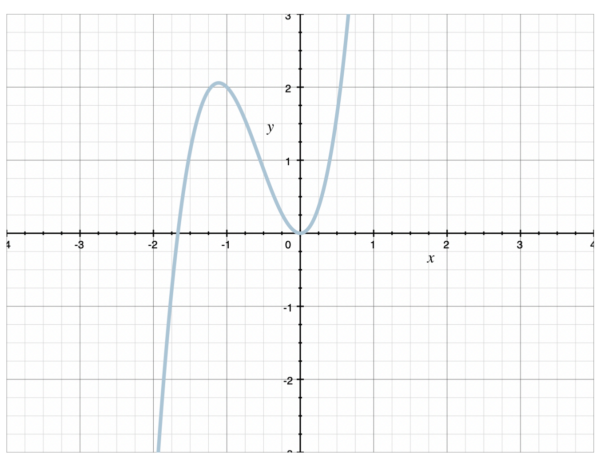


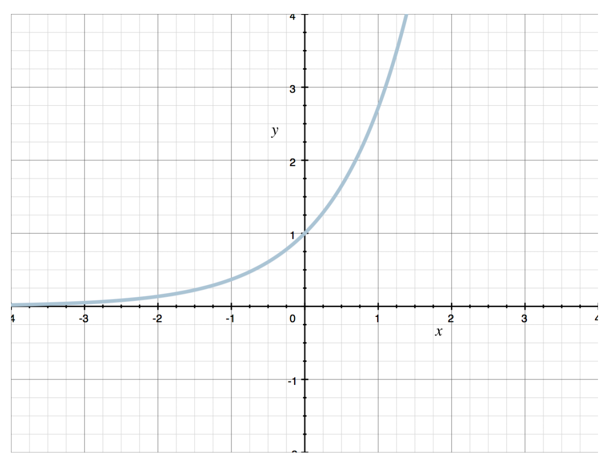
Neither even nor odd

Not symmetric with respect to the y-axis, and not symmetric with respect to the origin

The function has no symmetry. It's possible that a graph could be symmetric with respect to the x-axis, but then it wouldn't pass the Vertical Line Test and therefore wouldn't represent a function.







Example

Is the function even, odd, or neither?

$$f(x) = x^5 - 3x^3$$

To solve algebraically we need to find the expression for $f(-x)$, so we'll replace every x (in the expression for $f(x)$) with $-x$.

$$f(-x) = (-x)^5 - 3(-x)^3$$

Remember that

$$(-x)^5 = (-1x)^5 = (-1)^5x^5$$

and

$$(-x)^3 = (-1x)^3 = (-1)^3x^3$$

Raising -1 to an odd power gives -1, so

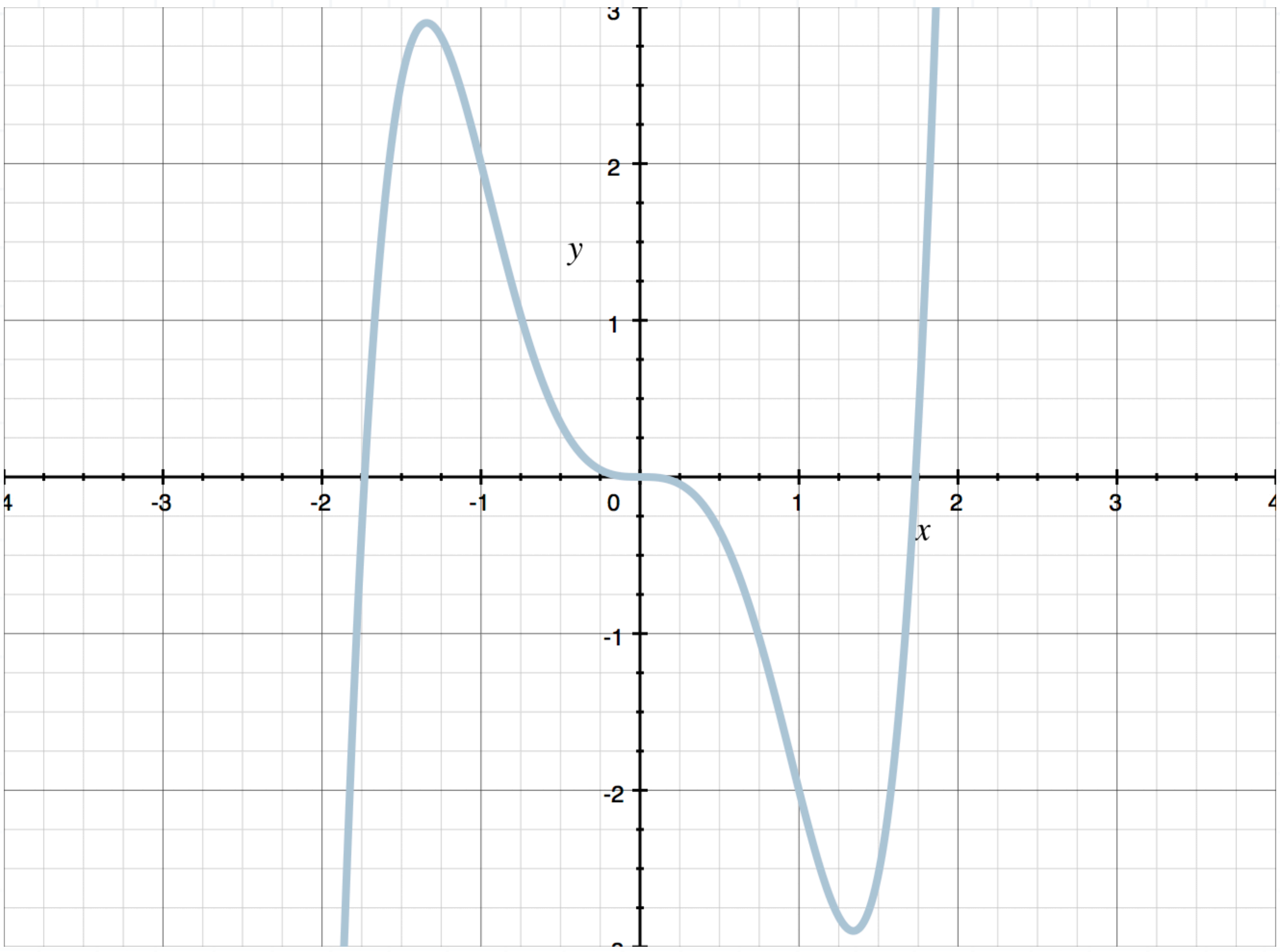
$$f(-x) = (-1)x^5 - 3(-1)x^3$$

Factor out a -1, and then simplify.

$$f(-x) = -1(x^5 - 3x^3)$$

$$f(-x) = -(x^5 - 3x^3)$$

Since $f(-x) = -f(x)$, the function is odd. We can see that the graph is symmetric with respect to the origin.



Let's try another example of even, odd, or neither.

Example

Is the function even, odd, or neither?

$$f(x) = 5x^2 - x^4$$

To solve algebraically, we need to find the expression for $f(-x)$, so we'll replace every x (in the expression for $f(x)$) with $-x$.

$$f(-x) = 5(-x)^2 - (-x)^4$$

Remember that

$$(-x)^2 = (-1x)^2 = (-1)^2x^2$$

and

$$(-x)^4 = (-1x)^4 = (-1)^4x^4$$

Raising -1 to an even power gives 1, so

$$f(-x) = 5(1)x^2 - (1)x^4$$

$$f(-x) = 5x^2 - x^4$$

Since $f(-x) = f(x)$, the function is even. We can see that the graph is symmetric with respect to the y-axis.