Topic: Linear approximation

Question: Find the linear approximation of the function at $a = 1$.

$$f(x) = x^2$$

Answer choices:

A　　　$L(x) = 2x - 1$

B　　　$L(x) = -2x - 1$

C　　　$L(x) = 2x + 1$

D　　　$L(x) = 1 - 2x$

Solution: A

Take the derivative.

$$f(x) = x^2$$

$$f'(x) = 2x$$

Evaluate the original function at $a = 1$.

$$f(1) = 1^2$$

$$f(1) = 1$$

Evaluate the derivative at $a = 1$.

$$f'(1) = 2(1)$$

$$f'(1) = 2$$

Substitute all of these pieces into the linear approximation formula.

$$L(x) = f(a) + f'(a)(x - a)$$

$$L(x) = 1 + 2(x - 1)$$

$$L(x) = 1 + 2x - 2$$

$$L(x) = 2x - 1$$

Topic: Linear approximation

Question: Find the linear approximation of the function at $a = 0$.

$$f(x) = 3x^2 - 6x + 1$$

Answer choices:

A $L(x) = 1 + 6x$

B $L(x) = -1 - 6x$

C $L(x) = 1 - 6x$

D $L(x) = -1 + 6x$

Solution: C

Take the derivative.

$$f(x) = 3x^2 - 6x + 1$$

$$f'(x) = 6x - 6$$

Evaluate the original function at $a = 0$.

$$f(0) = 3(0)^2 - 6(0) + 1$$

$$f(0) = 1$$

Evaluate the derivative at $a = 0$.

$$f'(0) = 6(0) - 6$$

$$f'(0) = -6$$

Substitute all of these pieces into the linear approximation formula.

$$L(x) = f(a) + f'(a)(x - a)$$

$$L(x) = 1 + (-6)(x - 0)$$

$$L(x) = 1 - 6x$$

Topic: Linear approximation

Question: Find the linear approximation of the function at $a = 2$.

$$f(x) = (x + 4)^2$$

Answer choices:

A $\quad$ $L(x) = 1 + x$

B $\quad$ $L(x) = 12 + 12x$

C $\quad$ $L(x) = -12 - 12x$

D $\quad$ $L(x) = 1 - x$

Solution: B

Take the derivative.

$$f(x) = (x + 4)^2$$

$$f'(x) = 2(x + 4)(1)$$

$$f'(x) = 2x + 8$$

Evaluate the original function at $a = 2$.

$$f(2) = (2 + 4)^2$$

$$f(2) = 36$$

Evaluate the derivative at $a = 2$.

$$f'(2) = 2(2) + 8$$

$$f'(2) = 12$$

Substitute all of these pieces into the linear approximation formula.

$$L(x) = f(a) + f'(a)(x - a)$$

$$L(x) = 36 + 12(x - 2)$$

$$L(x) = 36 + 12x - 24$$

$$L(x) = 12 + 12x$$