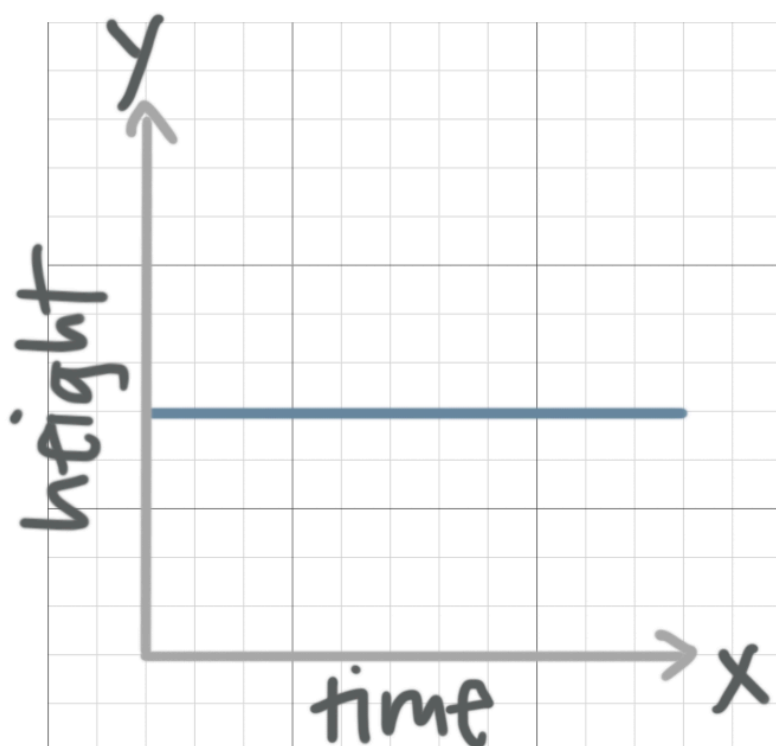


Sketching graphs from story problems

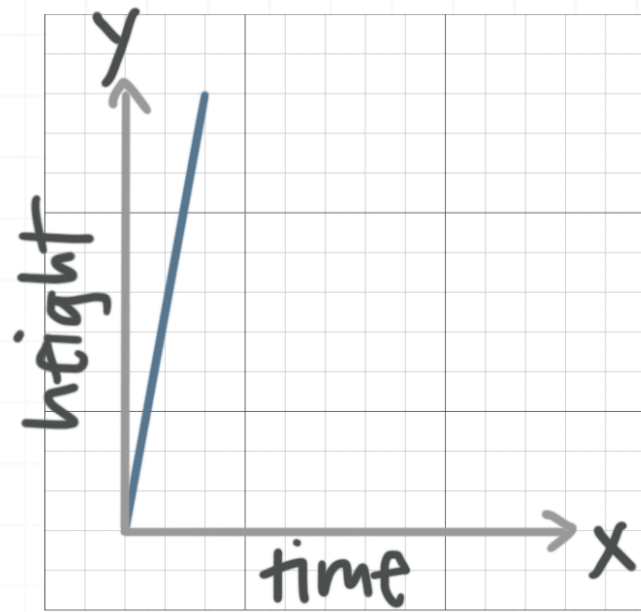
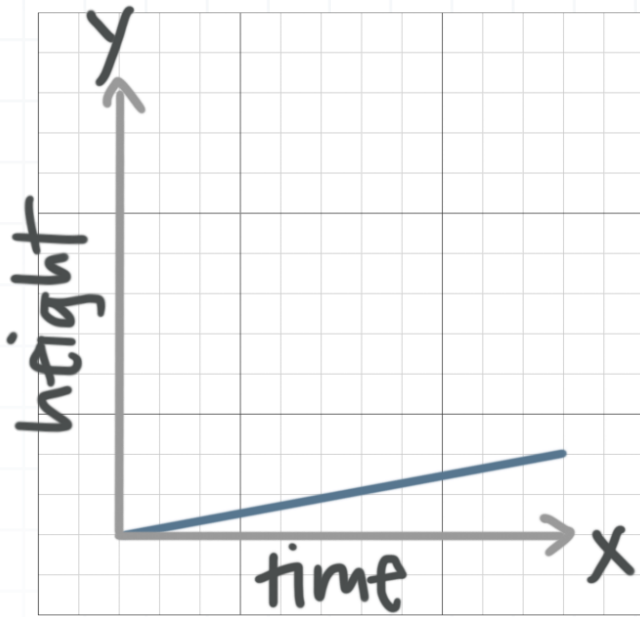
In this lesson we'll look at how we can use simple concepts to identify the graph of a piecewise function that represents a situation described in a word problem, to compose a word problem that's represented by the graph of a piecewise function, and to identify a verbal description of a situation that's represented by the graph of a piecewise function.

In a graph, a horizontal line represents variable whose value doesn't change (it stays the same). A positive slope shows a variable whose value is increasing, and a negative slope shows a variable whose value is decreasing. The steepness of the slope shows how fast the value of a variable is changing.

Let's look at some example graphs to get an idea of how this works. First, we'll consider graphs that show the height of an object above ground level as a function of time. The graph below tells us that, as time goes on, the height of the object doesn't change.

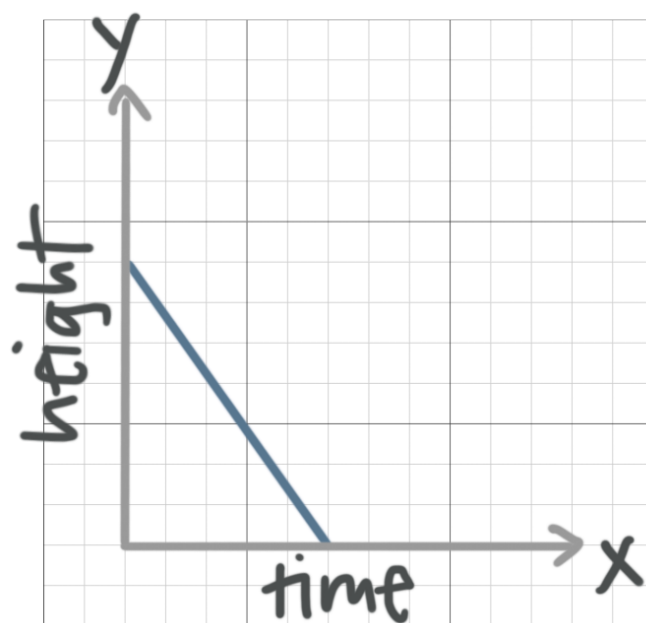
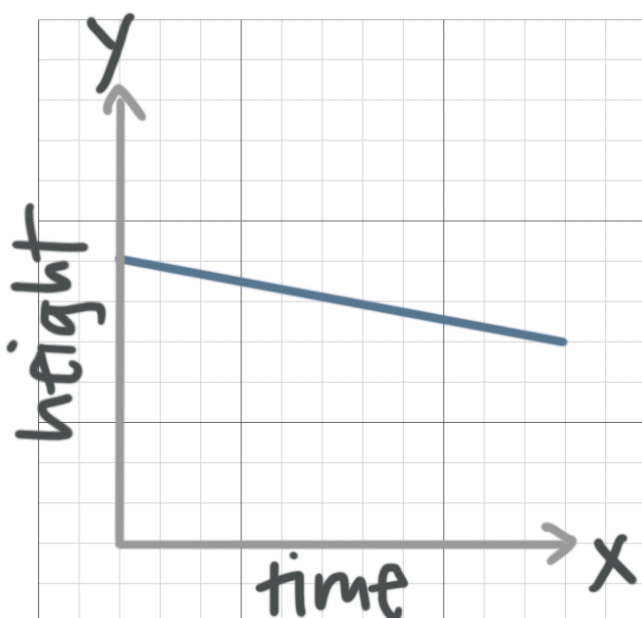


In both of the following graphs the height of the object is increasing, because the line has a positive slope.



A shallower line, like the first graph, means that height is increasing at a slower rate. A steeper line, like the second graph, means that the height is increasing at a faster rate.

In both of the following graphs the height of the object is decreasing, because the line has a negative slope.

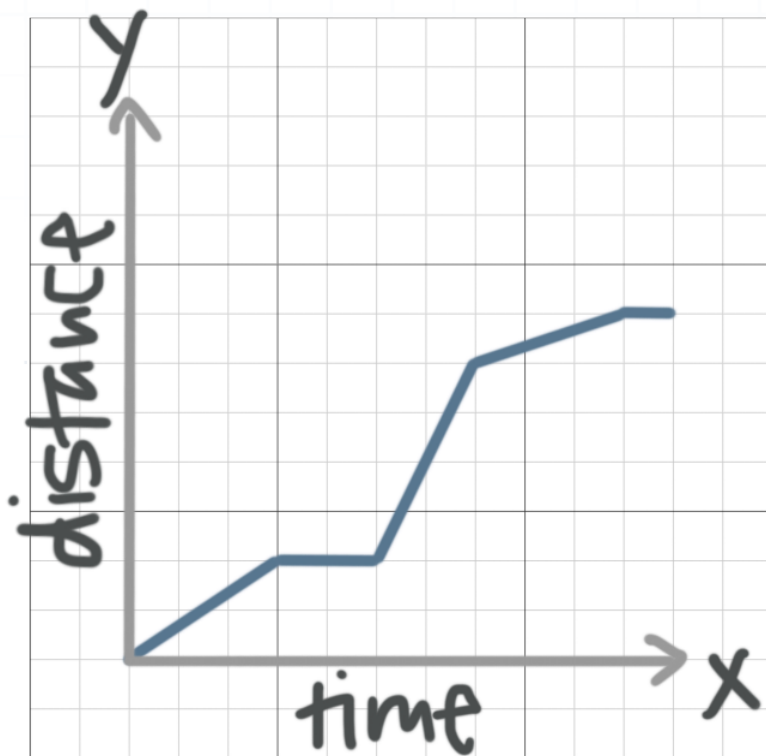


A shallower line, like the first graph, means the height is decreasing at a slower rate. A steep line, like the second graph, means that the height is decreasing at a faster rate.

Now let's look at word problems that involve either composing a story that goes with a given graph of a piecewise function or identifying the graph of a piecewise function that goes with a given story.

Example

Emily left on a trip to go to her grandmother's house in her car. The graph below shows her distance from her house as a function of time. Write a possible story to go along with the graph.



Looking at the graph, we see that the piecewise function it represents has five pieces. On three of the pieces (the ones that are represented by a line with a positive slope), Emily's distance is increasing (in one of these three



pieces increasing at a considerably faster rate than in the other two), and on the other two pieces (the ones that are represented by a horizontal line) her distance is constant (she isn't moving at all). The second piece that represents no change in distance (the piece on the far right) probably represents being at her grandmother's house. We can make up a story around the situation that's represented by the piecewise function shown in the graph. Any story that corresponds to this graph would work.

We could say something like this:

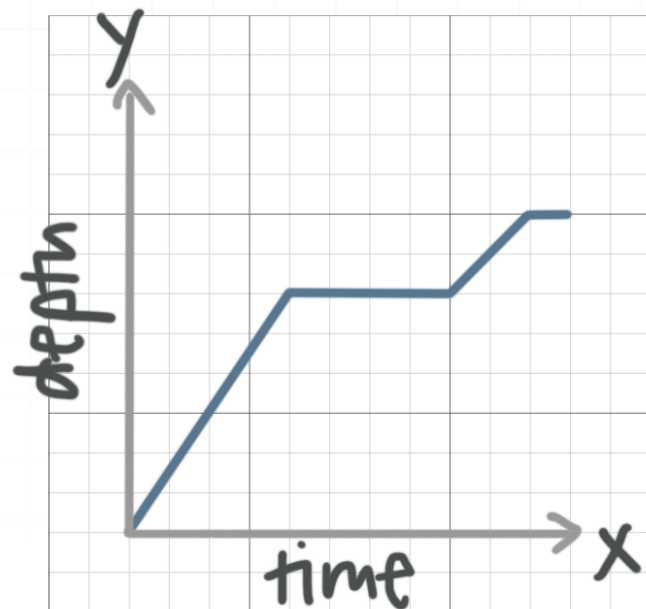
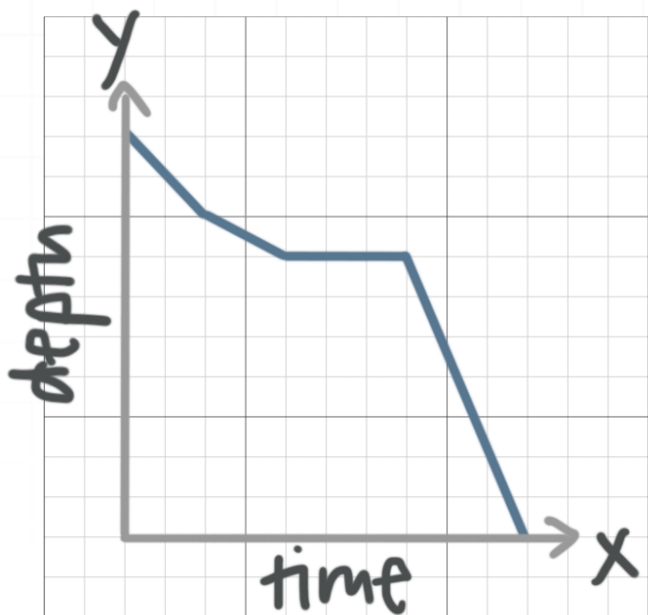
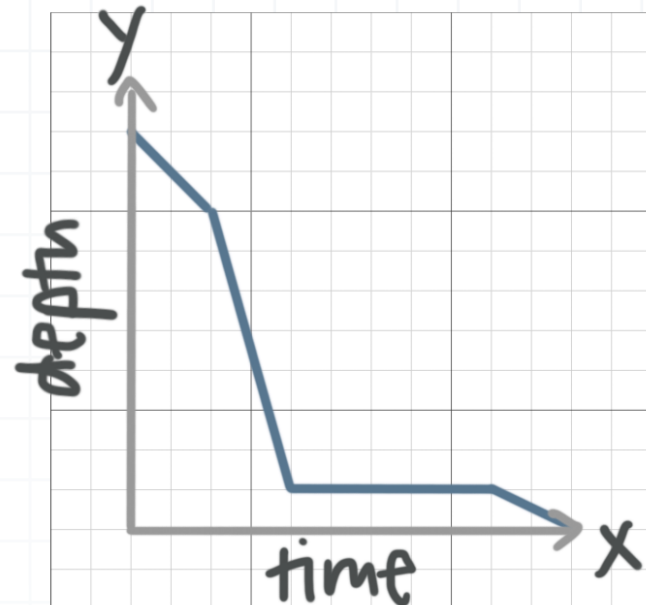
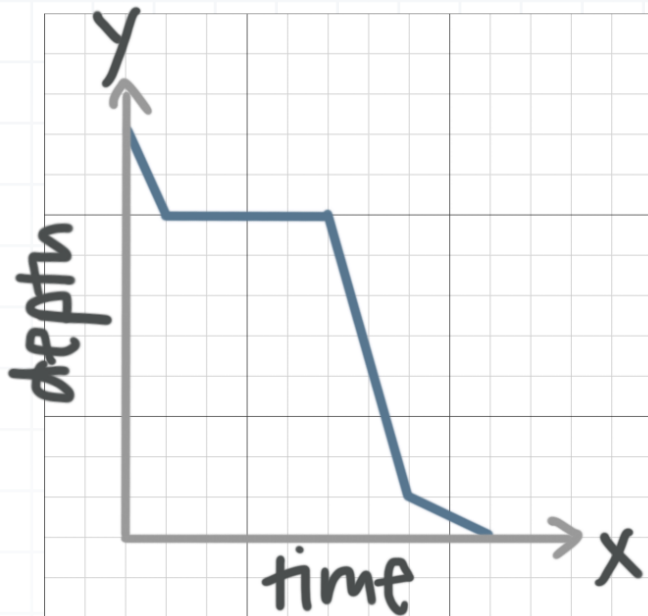
Emily left her house and drove along a local road, and then stopped for gas. After stopping for gas, she drove on a highway, where she was able to drive faster. When she got off the highway, she drove more slowly through a neighborhood, and then arrived at her grandmother's house, where she stopped.

Let's try an example we're given a story and we have to identify the graph that goes with it.

Example

A bulldozer is filling in a hole, and then it slows down and eventually breaks. After it's repaired, it fills in the hole quickly. Which graph best shows how the depth of the hole changes as time goes on?





Since the bulldozer is filling in the hole, the depth of the hole will be decreasing as time goes on, so we need a graph that has negative slopes. At first the bulldozer is working, so the slope should be steep and negative. Then the bulldozer slows down, so the slope should be less steep but still negative. While the bulldozer is being fixed, the depth of the hole remains constant. Then after the bulldozer is fixed, it fills in the hole quickly, so the slope will again be steep and negative. The only graph that fits this story is



