Language:

$$
\begin{array}{rcll}
e & ::= & x & \text{(variable)} \\
& \mid & \text{let } x = e_1 \text{ in } e_2 & \text{(let)} \\
& \mid & \text{transition}(x, \text{st}) & \text{(transition)} \\
& \mid & e_1 \; e_2 & \text{(application)} \\
& \mid & \lambda x.\, e & \text{(function abstraction)} \\
& \mid & \text{share } x \text{ as } x_1, x_2 \text{ in } e & \text{(share)} \\
& \mid & \text{atomic}(e) & \text{(atomic)} \\
& \mid & e_1;\; e_2 & \text{(seq)} \\
& \mid & \text{if } e \text{ then } e_1 \text{ else } e_2 & \text{(branch)}
\end{array}
$$

$(\Gamma : x \to \tau, f \to (\tau_1 \to \tau_2)$

$S ::= (\langle s_0, s_1, \ldots, s_n \rangle, <)$

$\text{cost}(s_i, s_j)$ = transition cost between s_i, s_j when $s_i \leq s_j$

$\text{cost}(s_i)$ = cost of being in state s_i

$\Delta : p \to (S, s)$ where s is the current state of the peripheral and S is the state lattice associated with the peripheral.

$\Gamma, \Delta, \vdash e : \tau \xrightarrow{C} \Delta'$

$\tau ::= b \mid \tau_1 \to \tau_2 \mid \rho$

$s_i \sqcap s_j = s_j$ if $s_i < s_j$

$\Delta' \sqcap \Delta'' = \Delta^* \to \forall p \in \Delta', \Delta'', \Delta^{*(p)} = \Delta'(p) \sqcap \Delta''(p)$

In words, the join of two peripheral contexts is a new context whose elements are the join of the current state of each peripheral in the context.

We also assume that every lattice has a unique $\top$ element that is the least upper bound of all the elements.

$$
\frac{\begin{array}{c}\Gamma, \Delta, s_{\text{max}} \vdash e_1 : \tau_1 \xrightarrow{C_1} \Delta', s'_{\text{max}} \\ C_{\text{1-max}} = \text{cost}(s_{\text{max}}) \qquad \Gamma, \Delta', s'_{\text{max}} \vdash e_2 : \tau_2 \xrightarrow{C_2} \Delta'', s''_{\text{max}} \\ C_{\text{2-max}} = \text{cost}(s'_{\text{max}}) \qquad C = C_1 + C_2 + C_{\text{1-max}} + C_{\text{2-max}}\end{array}}{\Gamma, \Delta, s_{\text{max}} \vdash e_1; e_2 : \tau_2 \xrightarrow{C} \Delta'', s''_{\text{max}}} \;\textsc{T-Seq}
$$

$$
\frac{\Gamma, \Delta, s_{\text{max}} \vdash e : \tau \xrightarrow{C_1} \Delta', s'_{\text{max}} \qquad C_{\text{max}} = \text{cost}(s'_{\text{max}}) \qquad C = C_1 + C_{\text{max}}}{\Gamma, \Delta, s_{\text{max}} \vdash \text{atomic}(e) : \tau \xrightarrow{C} \Delta', s'_{\text{max}}} \;\textsc{T-Atomic}
$$

$$
\frac{\begin{array}{c}\Gamma, \Delta \vdash e : \text{bool} \xrightarrow{C'} \Delta \qquad \Gamma, \Delta \vdash e_1 : \tau \xrightarrow{C_1} \Delta_1 \\ \Gamma, \Delta \vdash e_2 : \tau \xrightarrow{C_2} \Delta_2 \qquad C = C' + \max(C_1, C_2) \qquad \Delta' = \Delta_1 \sqcap \Delta_2\end{array}}{\Gamma, \Delta \vdash \text{if } e \text{ then } e_1 \text{ else } e_2 : \tau \xrightarrow{C} \Delta'} \;\textsc{T-Branch}
$$

$$
\frac{\Gamma, \Delta \vdash x : \rho \qquad \Delta(x) = s_i \qquad s_i \leq s \qquad C_t = \text{cost}(s_i, s)}{\Gamma, \Delta \vdash \text{transition}(x, s) : \tau \xrightarrow{C_t} \Delta[x : s]} \;\textsc{T-Transition}
$$