PHY 115

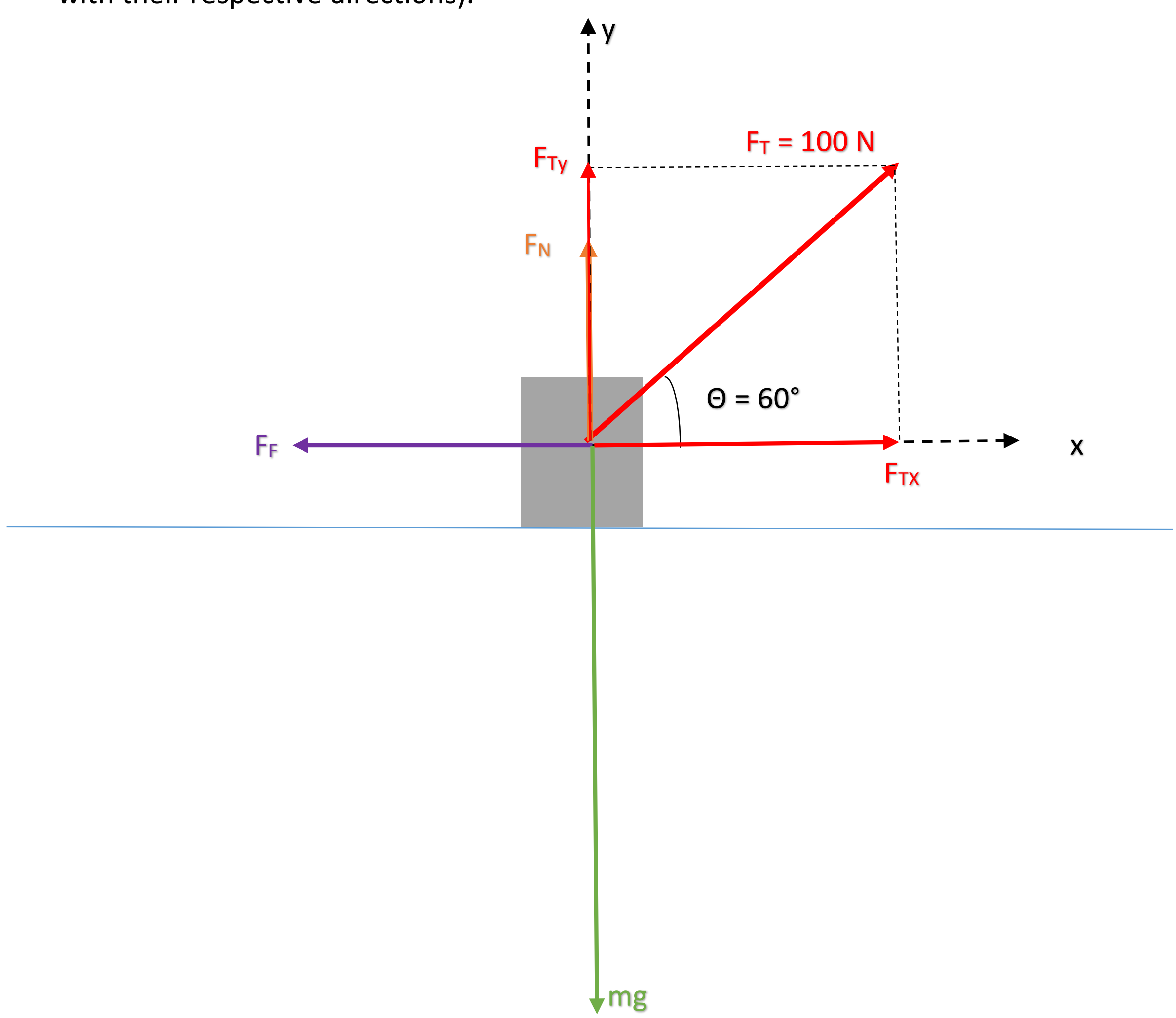
Assignment 6

Solutions

1. Digipen Dee is trying to move a tungsten* brick that is at rest on the floor by means of a rope that is attached to the brick. Dee (not represented in the figure below) pulls the rope at an angle of 60° with the positive horizontal axis, as she tries to move the brick horizontally. The magnitude of the tension force is 100 N.

The mass of the brick is 20 kg. Dee pulls onto the rope until the tungsten brick is <u>on the verge of moving</u>.

a. Draw a qualitative free-body diagram with all the forces on the brick (qualitative means you do not need to calculate the magnitudes here, but you do need to identify and label all the forces with their respective directions).



The labels refers to the magnitudes of the forces, while the directions are displayed in the diagram.

$\vec{F}_F$ = static friction force

$\vec{F}_{Tx}$ = the x-component of the tension force = $\overrightarrow{F_{Tx}} = F_T \cos\theta\,\hat{x}$

$\vec{F}_{Ty}$ = the y-component of the tension force $\overrightarrow{F_{Ty}} = F_T \sin\theta\,\hat{y}$

$m\vec{g}$ = gravitational force (weight)

$\vec{F}_N$ = normal force

Note that the magnitude of the normal force is smaller than the magnitude of the gravitational force.

b. Determine the normal force exerted by the ground on the brick. $\vec{F}_N = ?$

The brick remains at rest. That means: $a_x = a_y = 0$.

Since $a_y = 0$, $\sum \vec{F}y = 0$

This equation can be rewritten as: $\vec{F}_N + m\vec{g} + \vec{F}_{Ty} = 0$

So $F_N - mg + F_{Ty} = 0$

$$F_N = mg - F_{Ty} = 20\ kg * \left(9.8\frac{m}{s^2}\right) - 100N\ sin60° = 196N - 86.6N = 109.4\ N$$

But the normal force is a vector, so $\vec{F}_N = 109.4\ N\ \hat{y} \approx \mathbf{1.1\ \times 10^2 N\ \hat{y}}$ **(up)**

Note that the magnitude of the normal force is smaller than the magnitude of the weight (which is $196N$).

c. Determine the static friction force on the brick. $\vec{F}_F = ?$

The static friction force $\vec{F}_F$ is being balanced by the x-component of the tension force.

$$\sum \vec{F}x = m\vec{a}_x = 0$$

$$\vec{F}_{Tx} + \vec{F}_F = 0$$

$$F_{Tx} - F_F = 0$$

$$F_F = F_{Tx} = F_T \cos\theta = 100\ N \cos 60° = 50\ N$$

Note that this is just the magnitude of $\overrightarrow{F_F}$.

$$\boldsymbol{\vec{F}_F = -50\ N\ \hat{x}}$$

d. Determine the static friction coefficient. $\mu_S = ?$

Since the brick is on the verge of moving, the magnitude of $\vec{F}_F$ and the magnitude of the normal force $\vec{F}_N$ are related by the following equation:

$$F_F = \mu_S F_N$$

Where μ_S is the static coefficient of friction.

$$\mu_S = \frac{F_F}{F_N} = \frac{50\ N}{109\ N} \approx \mathbf{0.46}$$

Note that the last digit depends on how you rounded the quantities. If you used the same equations and found a coefficient of 0.45, your result is also acceptable.

d. Would it be easier to pull the rope at an angle of 50 degrees with the horizontal axis? Explain.

Intuitively, it would be **harder**, because the smaller the angle with the horizontal axis, the larger the normal force and therefore the larger the friction force.

The equation above can be rewritten as

$$\mu_S = \frac{F_F}{F_N} = (F_T \cos\theta)\,/(mg - F_T \sin\theta)$$

(See parts b and c).

We're trying to understand if the friction force will be smaller or larger at an angle of 50 degrees. Since the friction force depends on the tension force, first we need to determine F_T.

$$F_T \cos\theta = mg\mu_S - \mu_S\ F_T \sin\theta$$

$$F_T\ (\cos\theta + \mu_S\ \sin\theta) = mg\mu_S$$

$$F_T = \frac{mg\mu_S}{(\cos\theta + \mu_S\ \sin\theta)}$$

(You can use the equation above to get F_T = 100N for the values that were given in the problem, so as you test the equation that was just obtained).

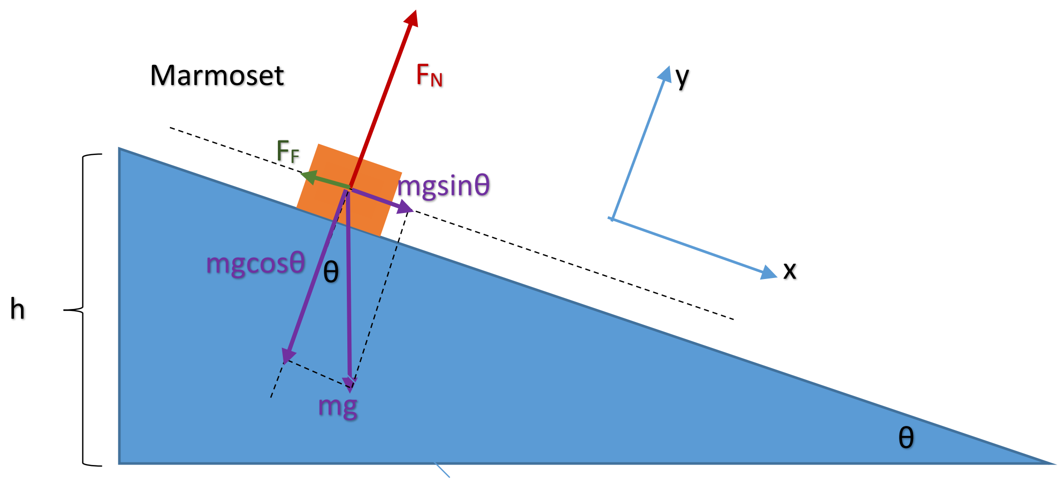
μ_S, m and g are constants. Remember that μ_S depends only on the kinds of surfaces that interact. Using θ = 50° in the equation above,

$$F_T = \frac{196\ N * (0.46)}{0.64 + 0.46 * 0.77} \approx 91\ N$$

So $F_F = F_T \cos\theta = 91\ N \cos 50° \approx \mathbf{58\ N}$

We have just shown mathematically that the friction force will have a larger magnitude if the angle with +x is 50 degrees, so it is, indeed, harder to pull the rope at this angle.

2. A 250 g marmoset is sliding on a playground slide. The kinetic friction coefficient is 0.200. The marmoset's initial speed is zero. The incline angle of the slide is 19°.



a. Draw a qualitative free-body diagram with all the forces on the marmoset.

See figure above.

$\vec{F}_F$ = kinetic friction force (the marmoset is sliding)

$m\vec{g}$ = gravitational force (weight). The weight was split into the horizontal and vertical components:

$mg \sin \theta \, \hat{x}$ = the x-component of the weight force

$mg \sin \theta \, \hat{y}$ = the y-component of the weight force

$\vec{F}_N$ = normal force

b. Determine the normal force on the marmoset. $\vec{F}_N = ?$

Since $a_y = 0$, $\sum \vec{F}y = 0$

$\vec{F}_N = mgcos\theta\ \hat{y} = 0.250kg * 9.8\frac{m}{s^2} * \cos 19°\ \hat{y} \approx 2.32\ N\ \hat{y} \approx \mathbf{2.3\ N\ \hat{y}}$

c. What is the acceleration of the marmoset down the slide?

The acceleration down the slide ($\vec{a}_x$) is calculated by using Newton's 2nd Law in the x-direction:

$$\sum \vec{F}x = m\vec{a}_x = mgsin\theta - F_F = mgsin\theta - \mu_k F_N\ (in\ the\ x - direction)$$

$$= 0.250\ kg * 9.80\frac{m}{s^2} * \sin 19° - 0.200 * 2.32\ N \approx 0.80N - 0.464N = 0.336\ N$$

So $\vec{a}_x = \frac{0.336\ N}{0.250\ kg}\hat{x} = 1.344\frac{m}{s^2}\hat{x} \approx \mathbf{1.3\ \frac{m}{s^2}\hat{x}}$

This acceleration is significantly smaller than the acceleration with no friction.

d. The height h of the slide is equal to 1.5 m. What is the velocity of the marmoset at the bottom of the slide (immediately before the marmoset loses contact with the slide)? *Hint:* you will need to calculate the length of the slide. Treat the marmoset as a particle.

It is said that the initial speed is zero. The acceleration down the slide is constant, so we can resource to our old friend: $v_x^2 = v_{0x}^2 + 2\vec{a}_x\ .\Delta\vec{x}$

The question is: $\vec{v}_x = ?$

$\Delta\vec{x}$ is the displacement vector down the ramp. We can calculate its magnitude, which is the hypotenuse of the triangle above:

$$\sin\theta = \frac{opp}{hyp} = \frac{h}{\Delta x} = \frac{1.5\ m}{\Delta x} = 0.326$$

$$\Delta x = \frac{1.5\ m}{0.326} \approx 4.60\ m$$

Substituting this value and the other predetermined quantities on the kinematic equation above,

$$v_x^2 = v_{0x}^2 + 2\vec{a}_x\ .\Delta\vec{x} = 0 + 2 * 1.34\frac{m}{s^2} * 4.60\ m = 12.33\frac{m^2}{s^2}$$

$$\vec{v}_x = \sqrt{12.33\frac{m^2}{s^2}}\hat{x} \approx \mathbf{3.5\frac{m}{s}\hat{x}}$$

3. A GTO is moving at a constant speed of 25 miles per hour, in a circular motion around a corner. The radius of this circular motion is 60 ft. The curve is flat (i.e., the road is not banked).

a. What is the acceleration of the GTO, in m/s^2?

$\vec{a}_{cp} = ?$

The direction of the centripetal acceleration (which is the only acceleration present, since the GTO is moving at a constant speed) is towards the center of the circle.

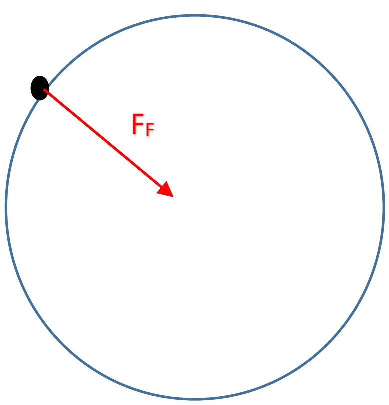
The magnitude of the centripetal acceleration is:

$$a_{cp} = \frac{v^2}{R} = \frac{\left(25\left(\frac{mi}{h}\right) * \frac{1\,h}{3600\,s} * \frac{1600\,m}{1\,mi}\right)^2}{\left(60\,ft * \frac{1\,m}{3.28\,ft}\right)} = \frac{123.5\frac{m^2}{s^2}}{18.3\,m} \approx 6.75\frac{m}{s^2}$$

The centripetal acceleration is: $\boldsymbol{\vec{a}_{cp} \approx 6.8\frac{m}{s^2}, towards\ the\ center\ of\ the\ circle}$

b. What is the static friction coefficient between the tires and the road? Hint: this calculation does not depend on the mass of the car. $\mu_S = ?$

The diagram below represents the only horizontal force, which is the static friction force.



The static friction force provides the centripetal force.

$F_F = \mu_S F_N = m a_{cp}$ (*)

Since the curve is flat, the GTO describes a section of a horizontal circle. Moreover, the magnitude of the normal force is equal to the magnitude of the weight:

$F_N = mg$ (**)

Subst. (**) on (*):

$\mu_S mg = ma_{cp}$

Dividing both sides by m and rearranging,

$$\mu_S = \frac{a_{cp}}{g} = \frac{6.8\,\frac{m}{s^2}}{9.8\,\frac{m}{s^2}} \approx \mathbf{0.69}$$

The pilot of the GTO is an astronaut who was trained to withstand an acceleration of 9 g's before passing out.

c. What is the maximum speed (in m/s) that the pilot can go around this corner, assuming that the GTO can handle it?

$v_{max} = ?$, where v_{max} is the maximum speed the car can turn for this exceptionally skilled driver.

The maximum magnitude of the acceleration would be $a_{max} = 9*g = 9*(9.8\ m/s^2) = 88.2\ m/s^2$ (*)

This acceleration is centripetal, so the free-body diagram looks like the one displayed on part b.

The equation for the centripetal acceleration magnitude is:

$a_{cp} = \frac{v_{max}^2}{R}$ $(**)$, where the subscript *max* was used to identify the maximum speed allowed for this driver.

Therefore, the centripetal acceleration is equal to the max. acceleration. Using (*) and (**):

$$a_{cp} = \frac{v_{max}^2}{R} = \frac{v_{max}^2}{18.3\ m} = 88.2\,\frac{m}{s^2}$$

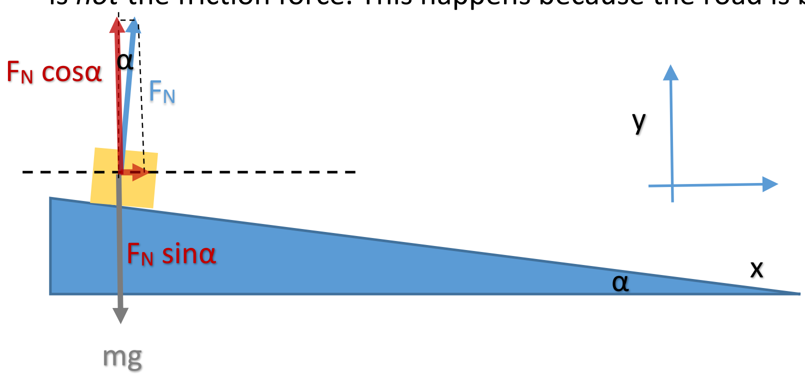
So $v_{max}^2 = 1614\,\frac{m^2}{s^2}$

Taking the square root,

$v_{max} \approx \mathbf{40\ m/s}$

4. Extra-credit: calculate the speed of the GTO (in m/s) around the same corner of problem 3 for a road that is banked at an angle of 10 degrees with respect to the horizontal axis. The radius of the circular motion is 60 ft. Assume that the road is frictionless. Hint: example 6.4, page 167.

$v = ?$ Now the road is frictionless. This means that the force that provides the centripetal force is *not* the friction force. This happens because the road is banked. See the diagram below.



Where α = 10°. Note that this diagram is similar to the one on page 167.

So the answer to the question *what force, or force component provides the centripetal force?* It is the **F_N sin α**, which is the only force that points in the direction of the centripetal acceleration, x. So, in other words, it is the horizontal component of the normal force that provides the centripetal force. Note that here we chose this coordinate system so as the centripetal acceleration is parallel to the x-axis.

$F_N \sin\alpha = \frac{mv^2}{R}$ (*)

But $F_N \cos\alpha = mg\ (see\ figure\ above), so\ F_N = \frac{mg}{\cos\alpha}$ (**)

Subst. (**) on (*),

$$\frac{mg}{\cos\alpha}\sin\alpha = \frac{mv^2}{R}$$

But $\frac{\sin\alpha}{\cos\alpha} = \tan\alpha$

So $mg\tan\alpha = \frac{mv^2}{R}$

Dividing both sides by *m* and isolating *v*,

$$v = \sqrt{Rg\tan\alpha} = \sqrt{18.3\ m * 9.8\frac{m}{s^2} * \tan 10°} = \sqrt{31.62\frac{m^2}{s^2}} \approx \mathbf{5.6\ m/s}$$

Note that this result does not depend on the mass of the car.