CS 176
Advanced Scripting

Simple Physics

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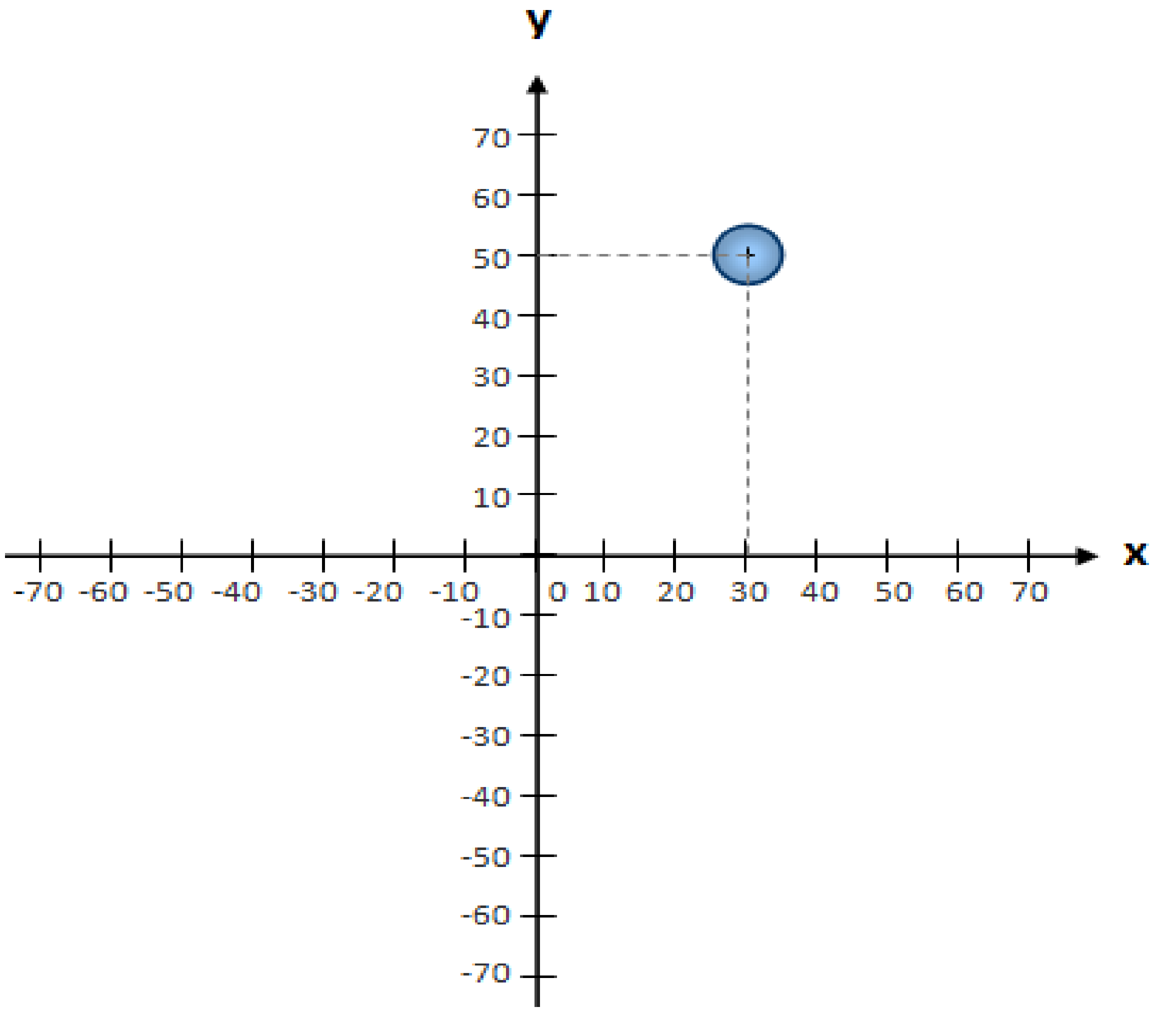
Vectors

2D Cartesian coordinate system

- A 2D Cartesian coordinate system is a system of reference for positions.
- It positions—or locates—objects in a two-dimensional space.
- It consists of an origin and two perpendicular axes coupled with a length unit such as meter, foot, or pixel.
- Any point in a 2D Cartesian coordinate system is described by a couple of numbers called components (x,y) that specify the distances from the axes.

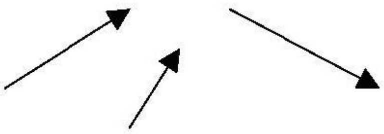
2D Cartesian coordinate system



Placing an object in a 2D Cartesian coordinate system

2-Dimensional Vectors

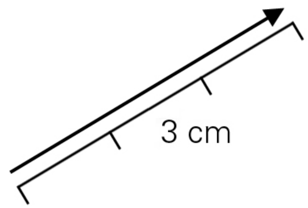
- A vector is not a point, but rather a **direction** and a **magnitude**.
- Instead of denoting a location on a 2D Cartesian coordinate system, a vector denotes a measurement of distance and direction in the dimensions of that system.
- Vectors are one of the most important concepts in mathematics, physics, and engineering.
- Additionally, they are one of the most used mathematical constructs in video game programming.



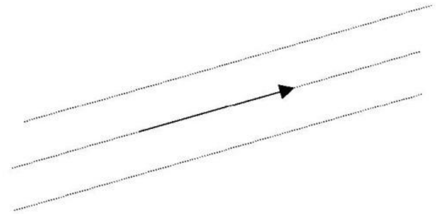
2-Dimensional Vectors

- A 2D geometric vector—simply referred to as a "vector" in the rest of this module—is defined by two essential characteristics: **magnitude** and **direction**.

- The magnitude of a vector represents its length, which is always positive.



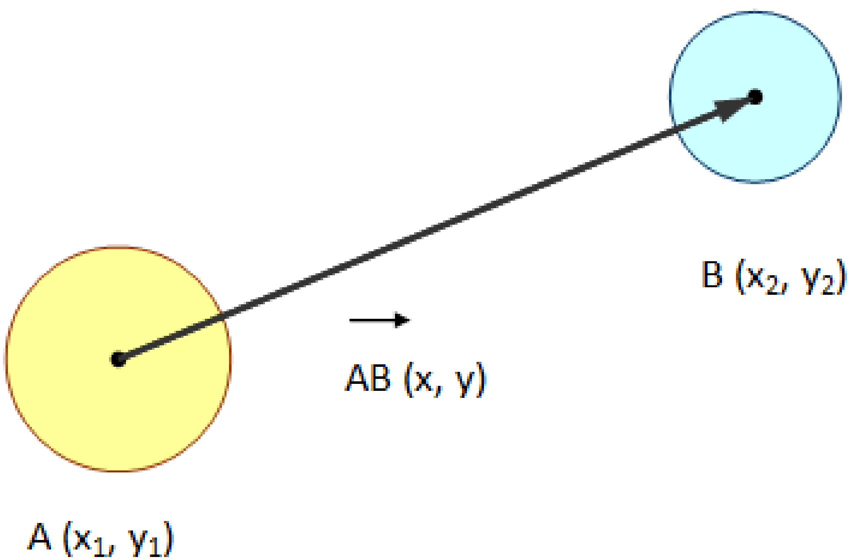
- The direction of a vector indicates how it is angled on the graph (think of the concept of slope).



Vectors & Objects

Getting the vector between two objects

- Having two objects A and B with position (x1 , y1) and (x2 , y2) respectively, we can get the vector from A to B by doing the following:



$$x = x_2 - x_1$$
$$y = y_2 - y_1$$

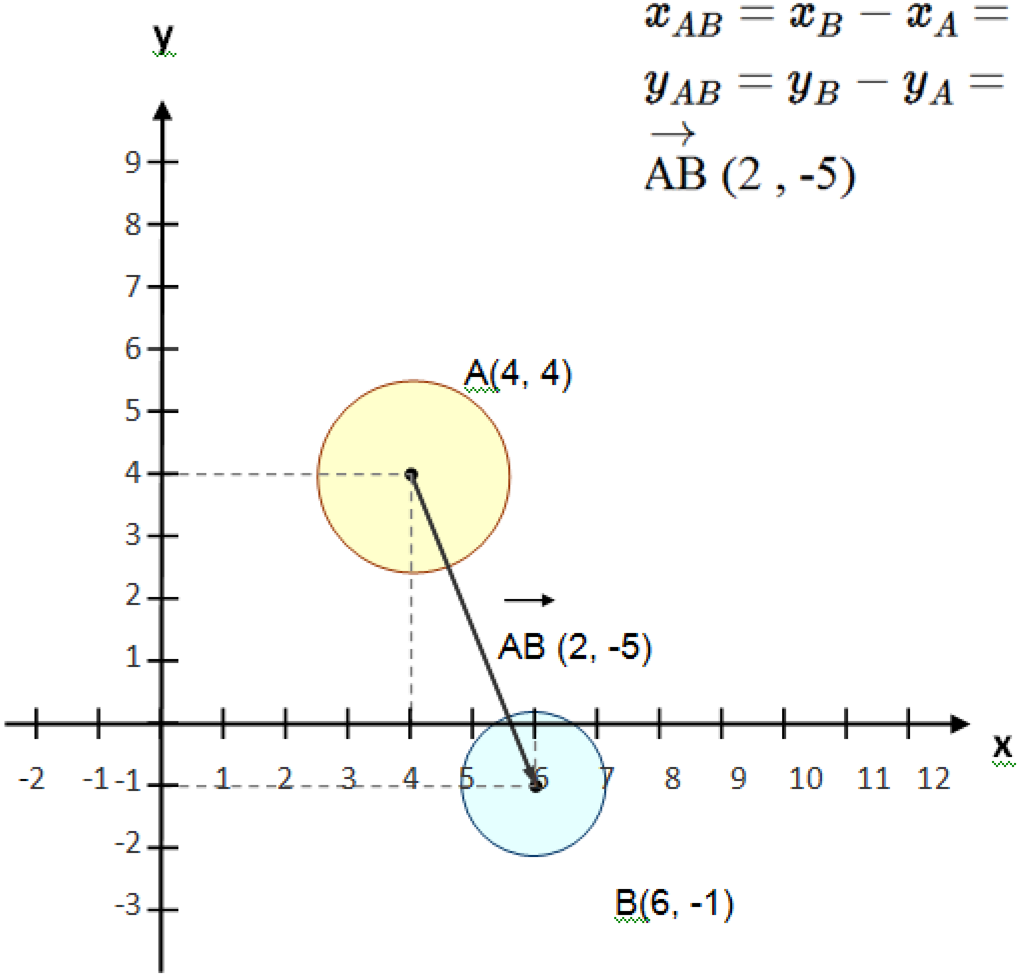
where x is the vector's x component

and y is the vector's y component

Getting the vector between two objects

Example:

$$x_{AB} = x_B - x_A = 6 - 4 = 2$$
$$y_{AB} = y_B - y_A = (-1) - 4 = (-5)$$
$$\overrightarrow{AB}\ (2\ ,\ \text{-}5)$$



Vector’s Magnitude

- The magnitude of a vector having x and y as its components is determined by plugging the components into the following formula :

$$\sqrt{x^2 + y^2}$$

- This may look familiar—it is the **distance formula**.

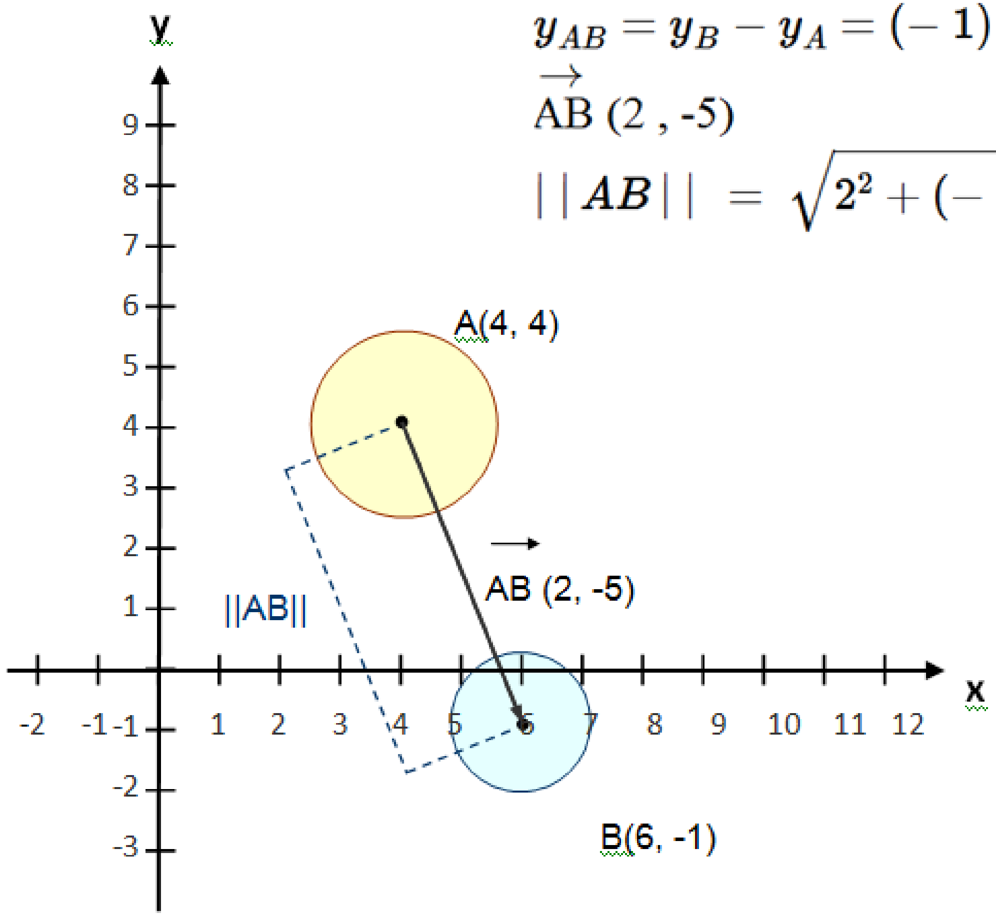
- The magnitude of any vector is denoted by the name of the vector between double vertical lines.

- Magnitude of $\overrightarrow{AB}$ is denoted by $||AB||$

Vector's Magnitude

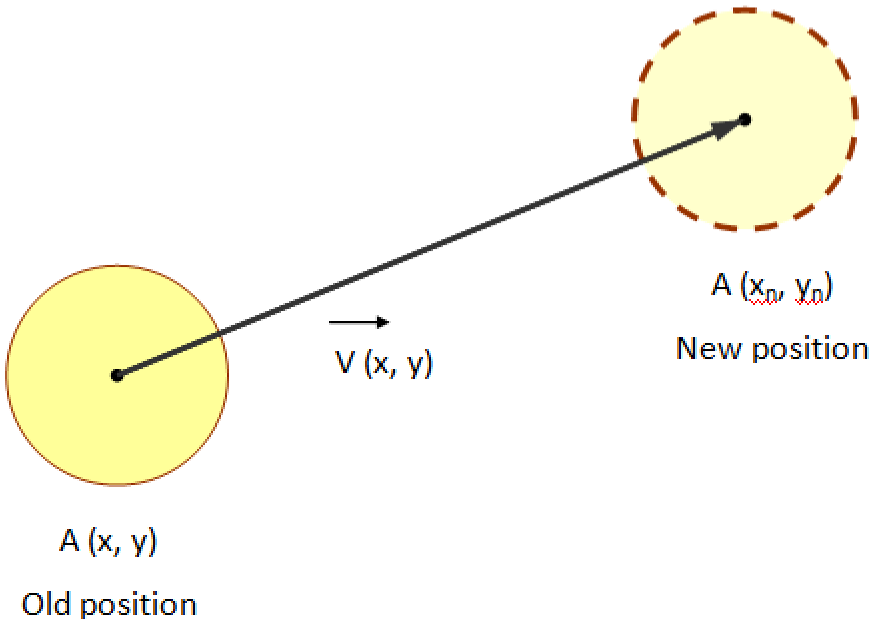
Example:

$$x_{AB} = x_B - x_A = 6 - 4 = 2$$
$$y_{AB} = y_B - y_A = (-1) - 4 = (-5)$$
$$\overrightarrow{AB}\ (2\ ,\ -5)$$
$$||AB|| = \sqrt{2^2 + (-5)^2} = \sqrt{29} = 5.385$$

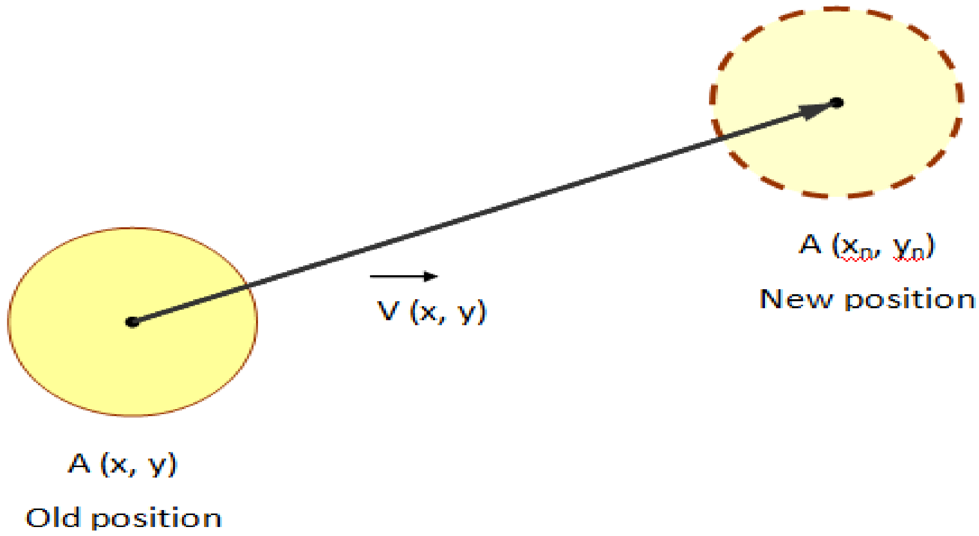


Translating an object using a vector

- In Euclidean geometry, a translation is moving every point a constant distance in a specified direction.
- It can also be interpreted as the addition of a constant vector to every point.



Translating an object using a vector



$$
\begin{aligned}
x_n &= x + x_v \\
y_n &= y + y_v
\end{aligned}
$$

where :

x *and* y *are the object's old position*

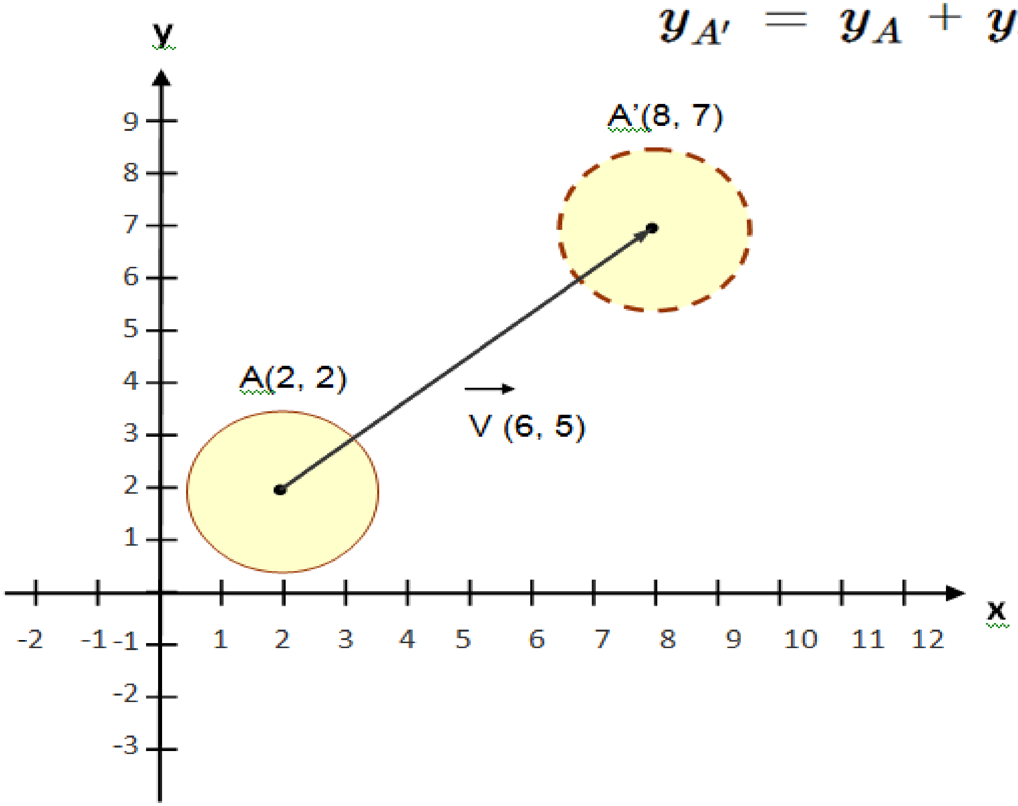
x_n *and* y_n *are the object's new position*

x_v *and* y_v *are the vector's components*

Translating an object using a vector

Example:

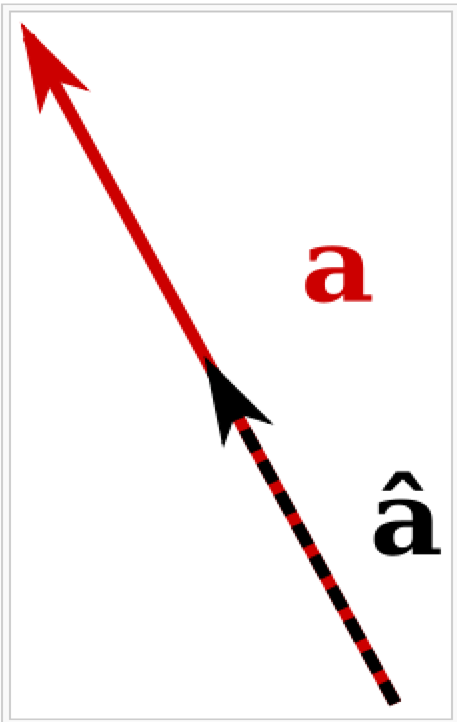
$$x_{A'} = x_A + x_v = 2 + 6 = 8$$
$$y_{A'} = y_A + y_v = 2 + 5 = 7$$



Unit Vector

- A unit vector is any vector with a length of one.
- Normally unit vectors are used simply to indicate direction.
- A vector of arbitrary length can be divided by its length to create a unit vector.
- This is known as normalizing a vector.
- A unit vector is often indicated with a hat as in â.

$$\hat{\mathbf{a}} = \frac{\mathbf{a}}{\|\mathbf{a}\|}$$



Unit Vector

<u>Example:</u>

Let's normalize the following 2D vector $\vec{V}(4 , 3)$

Magnitude (length) of V is: $||V|| = \sqrt{(4*4 + 3*3)} = \sqrt{25} = 5$

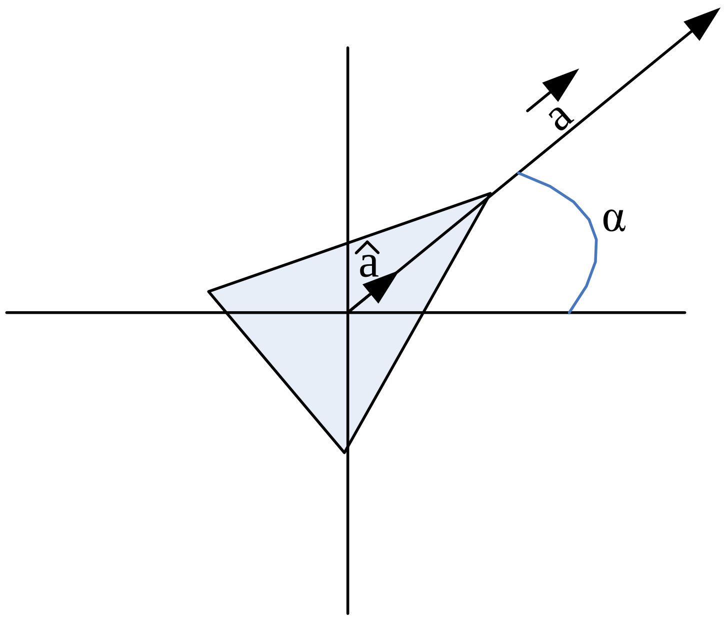
V.x = 4 / 5 and V.y = 3/5 → $\vec{V}(0.8 , 0.6)$

<u>Use the normalized vector:</u>

Object.x += V.x * iSpeed;
Object.y += V.y * iSpeed;

Where the unit vector V specifies the direction and iSpeed the magnitude (length)

Angle to Unit Vector



Use the angle to get the direction:

$$\hat{a} = (\cos\alpha\,;\sin\alpha)$$

Example:
If the object is rotated 30 degrees, we compute the vector that represents it's direction by doing the following:

a.x = cos(30) = 0.866…
a.y = sin(30) = 0.5

Note: You can be sure that ||a|| is equal to 1

Object Animation (Movement)

Physics Terms

- Velocity: The measurement of the rate and direction of change in the position of an object

$$v.x = \frac{\Delta x}{\Delta t} = \frac{meters}{\sec onds} \qquad v.y = \frac{\Delta y}{\Delta t}$$

- Acceleration: is the rate of change of velocity with time

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

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Object Animation – Frame Based

- A CS176 dynamic object will have a **position** and a **velocity**
- **Velocity** consists of a speed value (magnitude) and a direction vector (unit vector)

  Note: Having the directional vector normalized allows us to change its direction without affecting it's magnitude

- Every frame, we update the object's position

$$\vec{p} \mathrel{+}= \vec{v} \quad \text{or} \quad new\vec{P}os = \vec{v} + old\vec{P}os$$

Object Movement – Time Based

Each Frame:

- Compute time interval between previous and current frame ***(dt)***
- Compute the object's displacement within that time interval ***(v * dt)***
- Finally, compute object's new position

$$new\vec{P}os = \vec{v} * dt + curr\vec{P}os$$

Getting the velocity

- The new velocity is computed by the following:

$$new\vec{Vel} = \vec{a} * dt + curr\vec{Vel}$$

Where "a" represents the acceleration

Note: If the acceleration is zero (0) then the object is moving with constant velocity or constant speed.

- Then compute the new position

$$new\vec{Pos} = new\vec{Vel} * dt + curr\vec{Pos}$$

Getting the acceleration

- The acceleration will be computed according to all the forces applied on the object by using the following formula:

$$\sum \vec{F} = m * \vec{a}$$

Sum of all the forces = mass * acceleration

$$\vec{a} = \frac{\sum \vec{F}}{m}$$

Forces

- A force will be represented by a vector (direction and magnitude)
- That force can be applied to different positions on the object which will lead to a different result (Translation, rotation, both…)
- For the sake of simplicity, we will apply all the forces in the coming assignment to the center of the object which will lead only to a translation reaction.

To Sum it up

1. You add all the forces applied to the object
2. Compute the acceleration vector $\vec{a} = \dfrac{\sum \vec{F}}{m}$
3. Compute the new velocity vector $new\vec{Vel} = \vec{a} * dt + curr\vec{Vel}$
4. Compute the new position $new\vec{P}os = \vec{v} * dt + curr\vec{P}os$

The End ☺