PHY 115

Assignment 3 numerical solutions (with additional material)

1.

$\vec{v}_0$= 20 m/s (at t = 0 s), $\vec{a} = \text{constant} = 2.5\ \text{m/s}^2$

Traveling in the positive direction.

Coord. System (1D motion):

v_0 = 20 m/s

x_0 = 0.0 m

a.

$$\vec{v} = \ \vec{v}_0 + \vec{a}t$$

After t = 1.0 s,

$$\vec{v} = \ 20\ \frac{m}{s} + \ 2.5\ \frac{m}{s^2} * 5.0\ s = 32.5 \frac{m}{s} \approx \mathbf{33}\frac{\boldsymbol{m}}{\boldsymbol{s}}$$

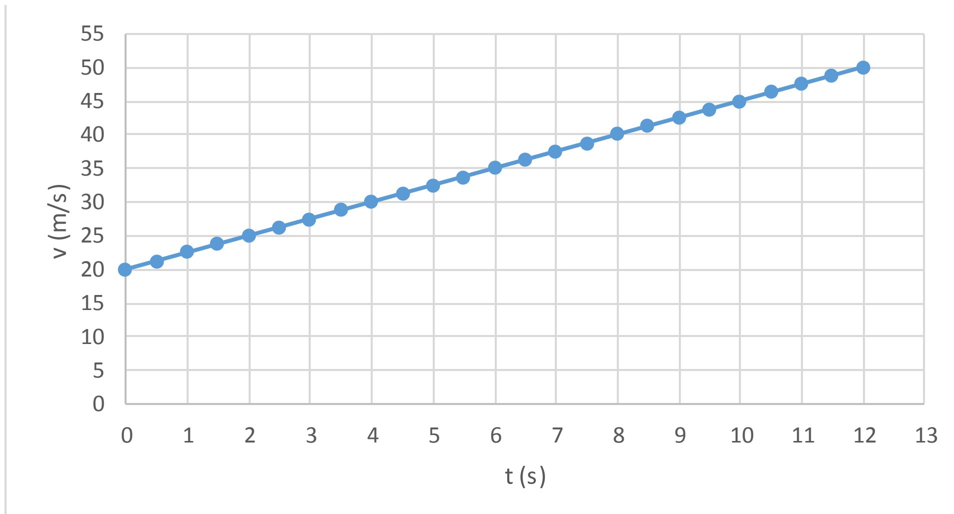
b. To calculate the additional time, now

$\vec{v}_0$= 32.5 m/s (at t = 0 s)

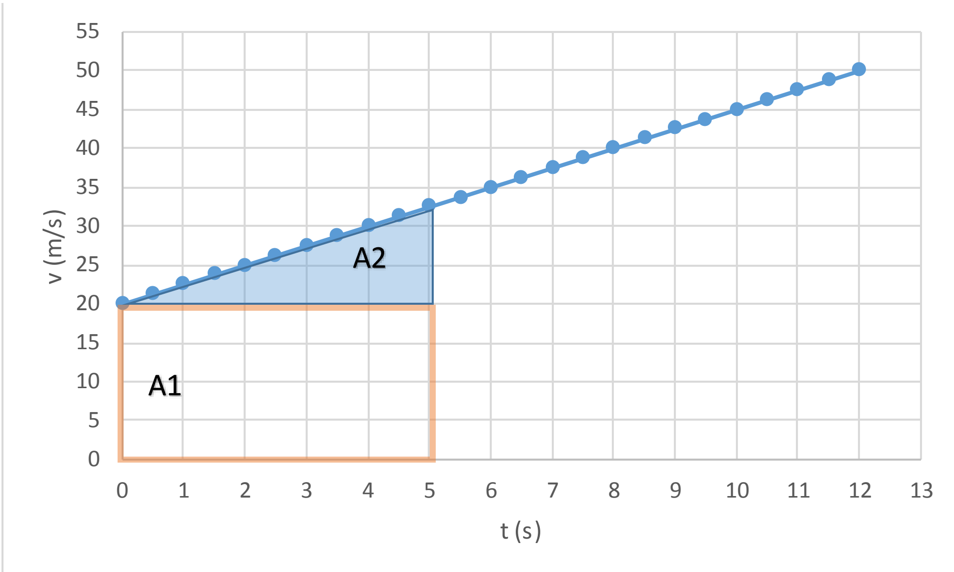
$t = \frac{v - v_0}{a} = \frac{50\frac{m}{s} - 32.5\ \frac{m}{s}}{2.5\ \frac{m}{s^2}} = \left(\frac{17.5}{2.5}\right)s = \mathbf{7.0}\ \boldsymbol{s}$ (in addition to the initial 5.0 s).

c.

t (s)	v (m/s)
0	20
0.5	21.25
1	22.5
1.5	23.75
2	25
2.5	26.25
3	27.5
3.5	28.75
4	30
4.5	31.25
5	32.5
5.5	33.75
6	35
6.5	36.25
7	37.5
7.5	38.75
8	40
8.5	41.25
9	42.5
9.5	43.75
10	45
10.5	46.25
11	47.5
11.5	48.75
12	50



d.



$Displacement = \Delta\vec{x} = area\ of\ rectangle\ (A1) + area\ of\ triangle(A2) = 20\frac{m}{s} * 5.0s + \left(\frac{1}{2}\right) * 5.0s * (32.5 - 20)\frac{m}{s} \approx 131.3\ m \approx \mathbf{1.3 \times 10^2\ m}$ (the result was rounded to 2 sig. figs).

This result can be checked trough the equation:

$\Delta\vec{x} = \vec{v_0}t + \frac{1}{2}\vec{a}t^2 = 20\frac{m}{s} * 5.0s + \left(\frac{1}{2}\right) * \left(2.5\ \frac{m}{s^2}\right) * (5.0\ s)^2 \approx \mathbf{1.3 \times 10^2\ m}$ (rounded)

e. Using

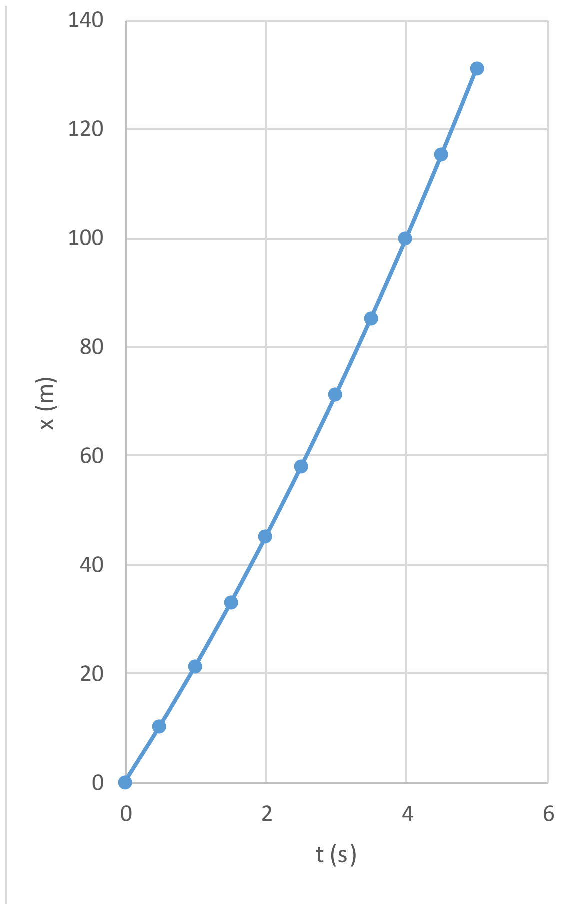
$\vec{\boldsymbol{x}} = \vec{\boldsymbol{v_o}}t + \frac{1}{2}\vec{\boldsymbol{a}}t^2$

Where $\vec{x}$ is the position vector with respect to the origin, $\vec{v}_0$= 20 m/s (at t = 0 s), $\vec{a} = \text{constant} = 2.5\ \text{m/s}^2$.

2. The question asks for 5.0 s of motion. Below is a table for 12.0 s of motion. In the graph, only 5.0 s are represented.

t (s)	v (m/s)	x (m)
0	20	0
0.5	21.25	10.3125
1	22.5	21.25
1.5	23.75	32.8125
2	25	45
2.5	26.25	57.8125
3	27.5	71.25
3.5	28.75	85.3125
4	30	100
4.5	31.25	115.3125
5	32.5	131.25
5.5	33.75	147.8125
6	35	165
6.5	36.25	182.8125
7	37.5	201.25
7.5	38.75	220.3125
8	40	240
8.5	41.25	260.3125
9	42.5	281.25
9.5	43.75	302.8125
10	45	325
10.5	46.25	347.8125
11	47.5	371.25
11.5	48.75	395.3125
12	50	420

For 5.0 s of motion:



Note that this is a half-parabola.

2. (cont.)

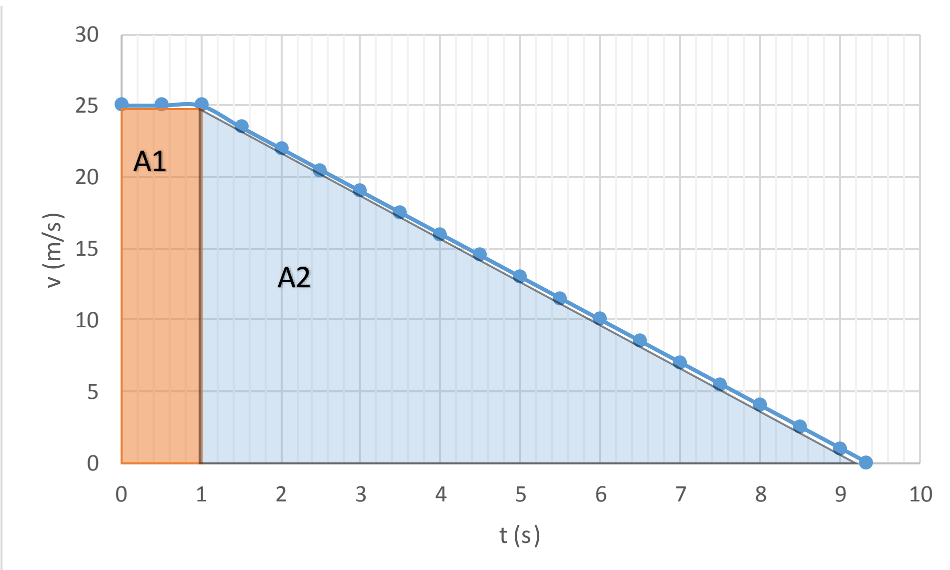
x (east)

Between t = 0.0 s and t = 1.0 s, $\vec{v}$= 25 m/s = constant.

Between t = 1.0 s and t, $\overrightarrow{v_0}$ = 25 m/s, $\vec{a} = -3.0\frac{m}{s^2}$ and the final velocity is $\vec{v}$ = 0.

a. $t = \frac{v - v_0}{a} = \frac{0\frac{m}{s} - 25\ ms}{-3.0\frac{m}{s^2}} \approx 8.3\ s$

So the total time elapsed is 1.0 s + 8.3 s = **9.3 s (approximately)**.



b.

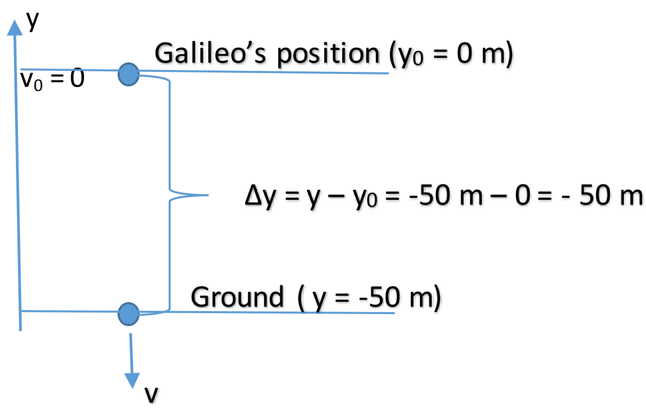
Using the area under the graph above, A1 + A2 = displacement = 25 m/s*1.0 s +(0.5*(9.3-1.0)s*25 m/s) $\approx 129\ m\ \approx$ $\mathbf{1.3\ x\ 10^2 m\ (2\ sig.figs.)}$.

You can also use the displacement equation (#2 from the lectures), combining the uniform motion (a_1 = 0 m/s^2) with the uniformly accelerated motion (**a_2** = - 3.5 m/s^2):

$\Delta\vec{x} = \overrightarrow{v_o}t_1 + \frac{1}{2}\overrightarrow{a_1}{t_1}^2\ (a_1 = 0) + \overrightarrow{v_o}t_2 +$

$+\frac{1}{2}\overrightarrow{a_2}{t_2}^2 = 25\frac{m}{s} * 1.0s + 0 + 25\frac{m}{s} * (8.3\ s) + \left(\frac{1}{2}\right) * \left(-3.0\frac{m}{s^2}\right) * (8.3\ s)^2 \approx \mathbf{1.3\ x\ 10^2 m}$

4. Since it is free fall, the acceleration is $\vec{g} = -9.8\frac{m}{s^2}$. The negative sign comes from the choice of coordinate system depicted below:



Note that, if you use the ground as y = 0, and the same coord. system shown above, you will get the same result for displacement.

a.

$$v^2 = v_0^2 + 2g\Delta y = 0.0\frac{m^2}{s^2} + 2 * \left(-9.8\frac{m}{s^2}\right) * (-50\,m) = 980\frac{m^2}{s^2}$$

$$\vec{v} = \pm\sqrt{980\frac{m^2}{s^2}} = \pm 31.3\,m/s$$

This is the mathematical result. However, we must choose the negative root according to our coordinate system depicted above. Therefore:

$$\boldsymbol{\vec{v} \approx -31\,m/s}$$

Note that the object is speeding up as it falls, and the acceleration is negative according to the coordinate system.

b. Method 1:

Use the result from a. and plug into $t = \frac{v - v_0}{a} = \frac{-31.3\frac{m}{s}}{-9.8\frac{m}{s^2}} \approx \boldsymbol{3.2\,s}$

Method 2:

OR, if you are not sure about your answer for a., you can use: $\Delta\vec{y} = \vec{v_0}t + \frac{1}{2}\vec{g}t^2$

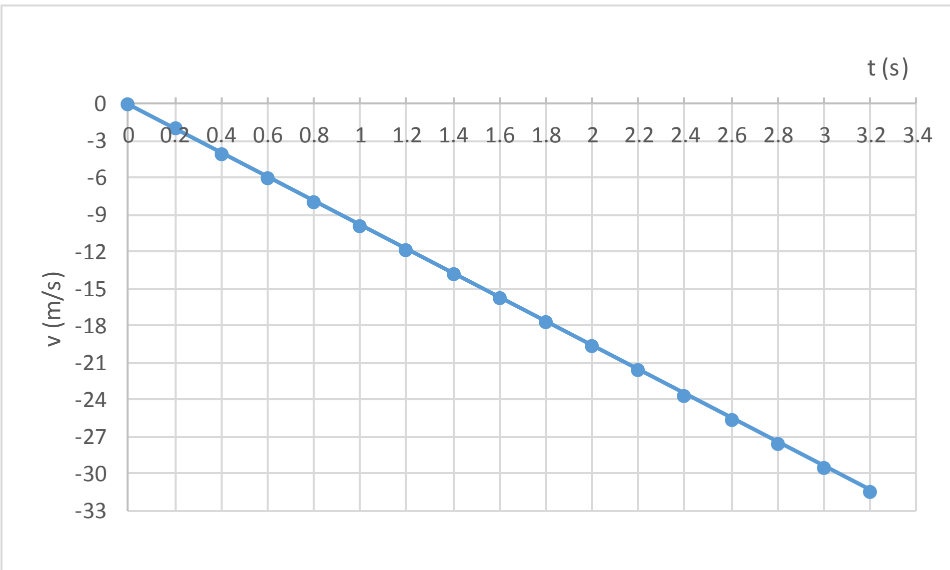
Rearranging,

$$\frac{1}{2}\vec{g}t^2 - \vec{v_o}t - \Delta\vec{y} = 0$$

Substituting with the known quantities, $-4.9\frac{m}{s^2}t^2 - 0.0t - (-50)m = 0$

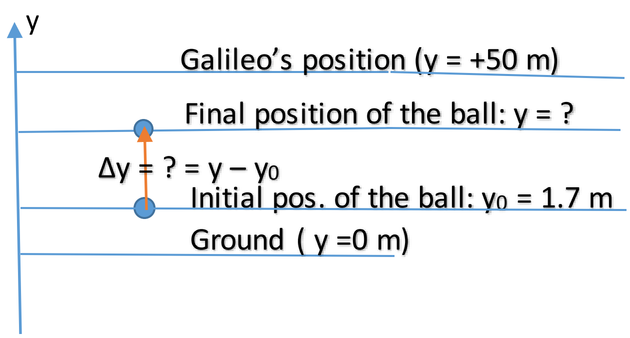
$t = \pm\sqrt{\frac{50}{4.9}}\,s$; choosing the positive root (t > 0), $t = 3.19\ s\ \approx \mathbf{3.2\ s}$

c.



d. We now use the ground as y = 0.

The figure below is **not** to scale.



$\vec{y_0} = +1.7\ m$, and the final velocity is $\vec{v} = 0.0\ m/s$ (at the peak height)

$v^2 = v_0^2 + 2g\Delta y$

Rearranging, the vertical displacement is $\Delta\,\vec{y} = \vec{y} - 1.7\ m = (v^2 - v_0^2)/2g =$

$$= \frac{0 - 225\frac{m^2}{s^2}}{2 * -9.8\frac{m}{s^2}} = 11.48\ m$$

So the final position is $\vec{y} = 11.48\ m + 1.7\ m\ \approx$ **$13\ m$ (with respect to the ground)**.

No, the ball never reaches 50 m of height.

e. It is actually impossible for Galileo to grab this ball, unless he teleports!

Here the position is always taken with respect to the ground (y = 0 m, see diagram on the previous page). The position is always positive. The displacement is positive while the ball moves upward, and negative while it moves downward.

The total displacement is: $\Delta\vec{y} = \ -1.7\ m$

Calculating the velocity immediately before the ball hits the ground:

$v^2 = v_0^2 + 2g\Delta y = 225\frac{m^2}{s^2} + 2 * \left(-9.8\frac{m}{s^2}\right) * (-1.7\ m) = 258.3\frac{m^2}{s^2}$

$\vec{v} = \ -16\ m/s$

In the graph below, in addition to the vertical position, the velocity is also depicted.