PHY 115

Assignment 4

1.

a. Sketch the total displacement ($\Delta\vec{r}$) of the object in the xy plane. The axes must be the x-axis and the y-axis.

$\vec{r_1}$ = *(2.0m, 3.0 m)* $\boldsymbol{or}$ $2.0\ m\ \hat{x} + 3.0\ m\ \hat{y}$ (alternative representation)

New position: $\vec{r}_2$

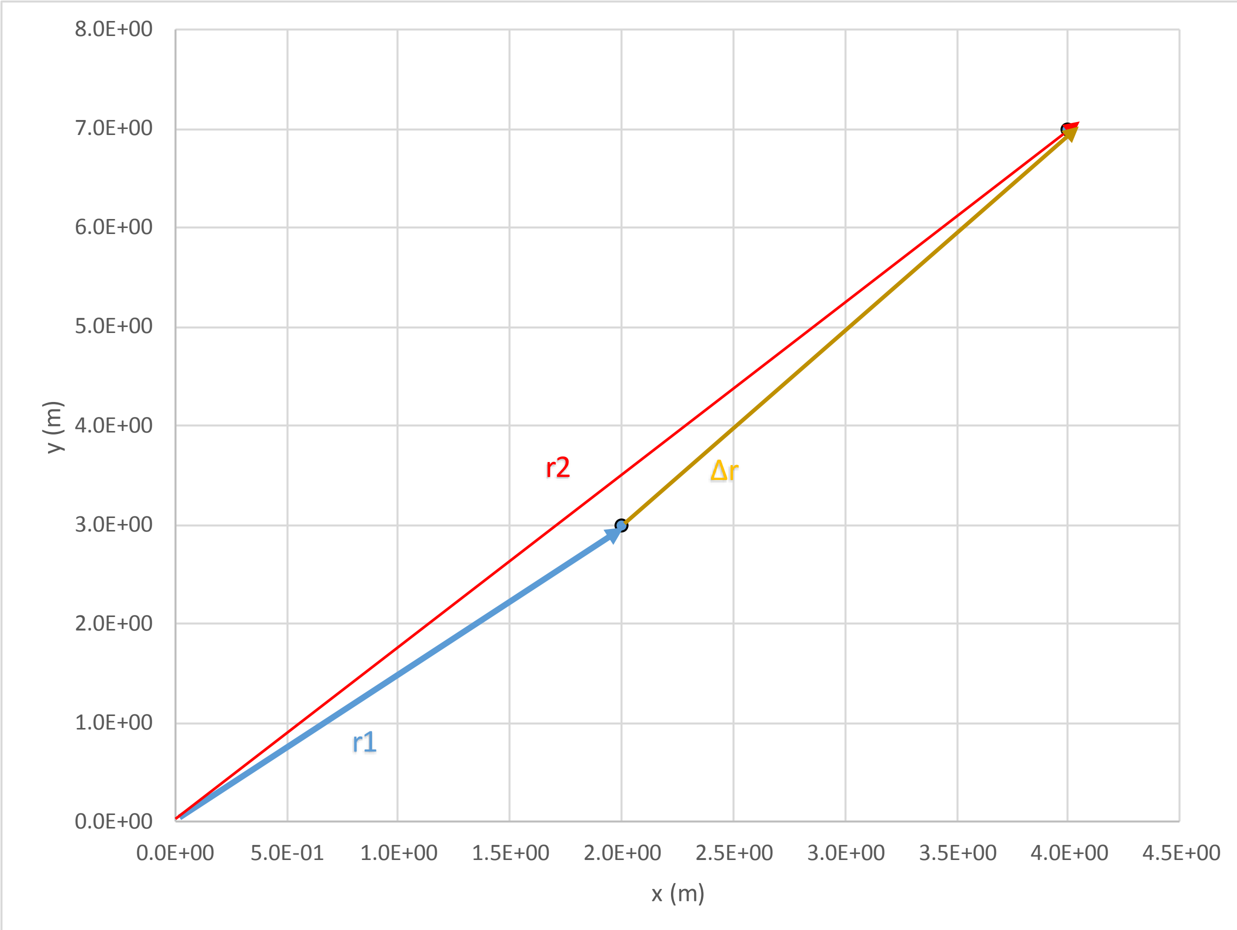
Note that both positions are with respect to the origin.

$\Delta\vec{r} = \vec{r_2} - \vec{r_1}$

= *(2.0 m, 4.0 m)* $\boldsymbol{or}$ $2.0\ m\ \hat{x} + 4.0\ m\ \hat{y}$

$\vec{r_2} = \Delta\vec{r} + \vec{r_1} = (2.0\ m, 4.0\ m) + (2.0\ m, 3.0\ m) = (4.0\ m, 7.0\ m)$ $\boldsymbol{or}$ $4.0\ m\ \hat{x} + 7.0\ m\ \hat{y}$

The graph below represents both position vectors and the displacement vector. Note that $\Delta\vec{r}$ and $\vec{r_1}$ are placed head-to-tail, and $\vec{r_2}$ is the resultant vector.



b. Express the total displacement ($\Delta\vec{r}$) in terms of its magnitude and the angle it makes with the positive x-axis.

$\Delta\vec{r} = 2.0\ m\ \hat{x} + 4.0\ m\ \hat{y}$

The magnitude $\Delta r\ is$

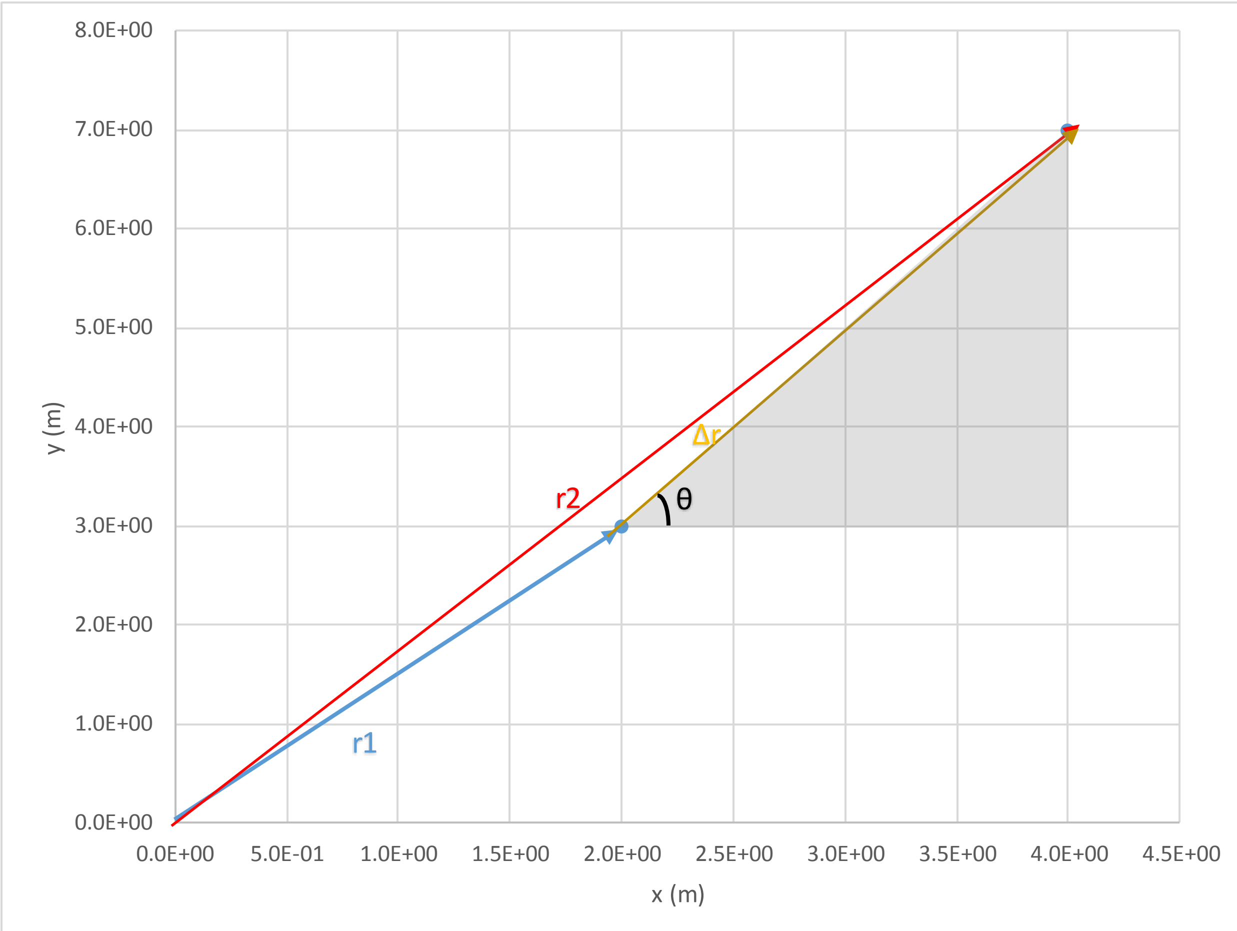
$\Delta r = \sqrt{(2.0\ m)^{\,2} + (4.0\ m)^2} = \sqrt{20\ m^2} \approx \mathbf{4.5\ m}$

The angle θ with the positive horizontal axis (see fig. below) is:

$$\tan\theta = \frac{4.0\ m}{2.0\ m} = 2.0$$

$\theta = \tan^{-1} 2.0\ \approx \mathbf{63°}$

The displacement vector $\Delta\vec{r}$ has magnitude 4.5 m and makes an angle of 63° with the positive horizontal axis.



2.

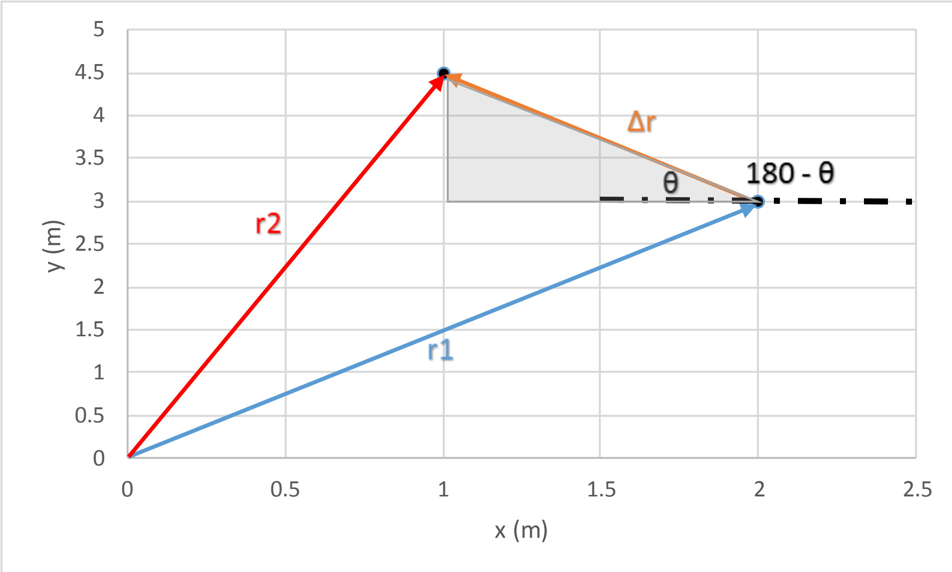
a. Express the displacement of the ship in terms of the magnitude of the displacement vector and the angle it makes with the positive x-axis.

$\Delta\vec{r} = \vec{r_2} - \vec{r_1} = (1.0\,m, 4.5\,m) - (2.0\,m, 3.0\,m) = (-1.0\,m, 1.5\,m) = -1.0\,m\,\hat{x} + 1.5\,m\,\hat{y}$

The magnitude is $\Delta r = \sqrt{(-1.0\,m)^2 + (1.5\,m)^2} \approx \mathbf{1.8\,m}$

The angle $\Delta\vec{r}$ makes with the positive x-axis is: $\theta = \tan^{-1}(1.5\,m)/(1.0\,m) \approx 56°$

A diagram should help us understand these parameters:



So the angle calculated is the angle with –x. The angle with +x is 180° - 56° = 124° ≈ $\mathbf{1.2\ x\ 10^2}$ °

b. Calculate the average velocity (direction and magnitude) of the ship, during the time interval between t_1 and t_2. The direction must be expressed in terms of the angle that the average velocity vector makes with the horizontal axis.

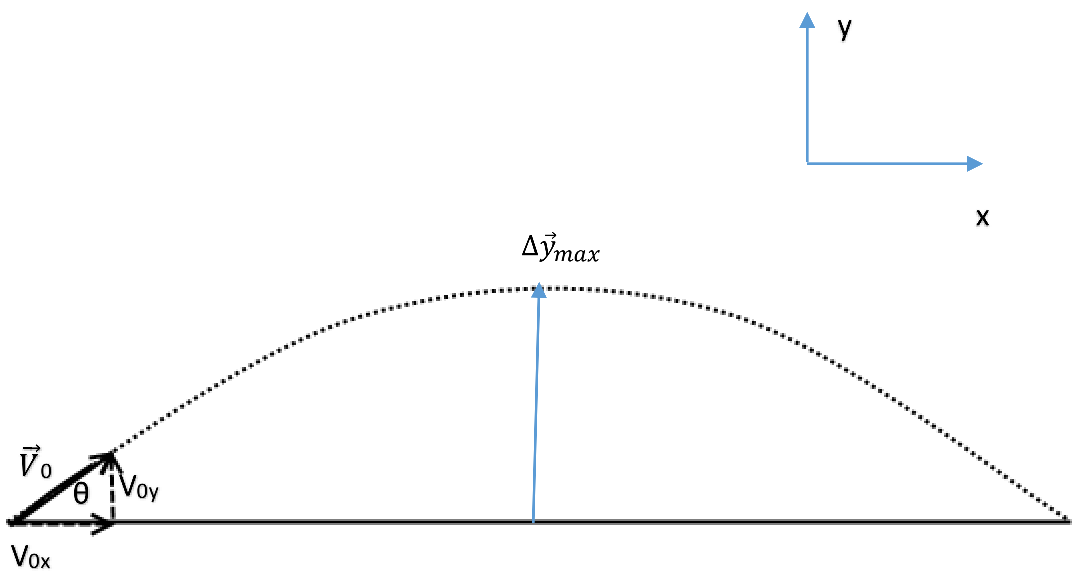
Since $\overline{v}\,(average\ velocity) = \frac{\Delta\vec{r}}{\Delta t} = \frac{-1.0\,m\,\hat{x}+1.5\,m\,\hat{y}}{2.0\,s} = -0.50\,\frac{m}{s}\,\hat{x} + 0.75\,\frac{m}{s}\hat{y}$

So the magnitude of the av. velocity is equal to $\sqrt{(-0.5\ m/s)^{\,2} + (0.75\ m/s)^2} \approx \mathbf{.90\ m/s}$

The average velocity is parallel to the displacement vector $\Delta\vec{r}$. So the direction is 124 °$\approx 1.2\ x\ 10^2$ °

Checking: $\theta = \tan^{-1}\frac{0.75\frac{m}{s}}{0.50\frac{m}{s}} \approx 56°$; The angle with +x is 180° - 56° = 124° ≈ $1.2\ x\ 10^2$ °

3. Projectile motion



Calculating the horizontal and vertical components of the initial velocity $\vec{V}_0$

$$\vec{v}_{0x} = v_0 \cos\theta \ \hat{x} = 20\frac{m}{s}\cos 40° \ \hat{x} \approx 15.32\frac{m}{s} \ \hat{x} \ (note\ that\ \vec{v}_x = \vec{v}_{0x} = constant, since\ the\ motion\ is\ uniform\ in\ the\ x - direction)$$

$$\vec{v}_{0y} = v_0 \sin\theta \ \hat{y} = 20\frac{m}{s}\sin 40° \ \hat{y} \approx 12.86\frac{m}{s} \ \hat{y}$$

a. t = ?, where t is the total flight time.

<u>**Method 1:**</u>

$$\Delta\vec{y} = 0$$

$$0 = \vec{v}_{0y}\, t + \left(\frac{1}{2}\right)\vec{g}t^2 = 12.86\frac{m}{s}\, t + \frac{1}{2}\left(-9.8\ \frac{m}{s^2}\right)t^2$$

Isolating t,

$$t = \frac{12.86}{4.9}s = 2.62\ s \ \approx \mathbf{2.6\ s}$$

Method 2:

$\vec{v}_y = \vec{v}_{0y} + \vec{g}\, t_{1/2}$, where $t_{1/2}$ is half of the total flight time (i.e., $t_{1/2}$ is the time at which the height is maximum) and $\vec{v}_y$= 0.

$t_{1/2} = \frac{12.86\frac{m}{s}}{9.8\frac{m}{s^2}} = 1.31\ s$; This is half of the total time, since the projectile goes back to the ground, so

$t = 2t_{1/2} = 2 * 1.31\ s \approx \mathbf{2.6\ s}$

Please note that this method can only be used when the projectile goes back to the ground, i.e., $\Delta\vec{y} = 0$.

b. $\Delta\vec{y}_{max}$=?, where $\Delta\vec{y}_{max}$is the max. vertical displacement vector. Below the quantities were replaced with their magnitudes and algebraic signs.

$v_y^2 = v_{0y}^2 + 2(-9.8\frac{m}{s^2})\Delta y_{max}$

$\Delta y_{max} = \frac{-v_{0y}^2}{2*-9.8\frac{m}{s^2}} = -\frac{\left(12.86\frac{m}{s}\right)^2}{\left(-19.6\frac{m}{s^2}\right)} = 8.44\ m \approx 8.4\ m\ so\ \boldsymbol{\Delta\vec{y}_{max} \approx 8.4\ m\ \hat{y}}$

c. Horizontal range means the horizontal distance, which in this case is the magnitude of the horizontal displacement. Note that, for proj. motion, the horizontal displacement is always > 0.

Horizontal range= $\Delta x = v_{0x}t = 15.32\frac{m}{s} * 2.62\ s = 40.14\ m \approx \mathbf{40\ m}$.

d. Extra-credit

$\boldsymbol{\vec{v} = ?\ at\ t = 1.0\ s}$

Since the x-component of the velocity is constant, $\vec{v}_x = 15.32\frac{m}{s}\hat{x} \approx 15\frac{m}{s}\ \hat{x}$

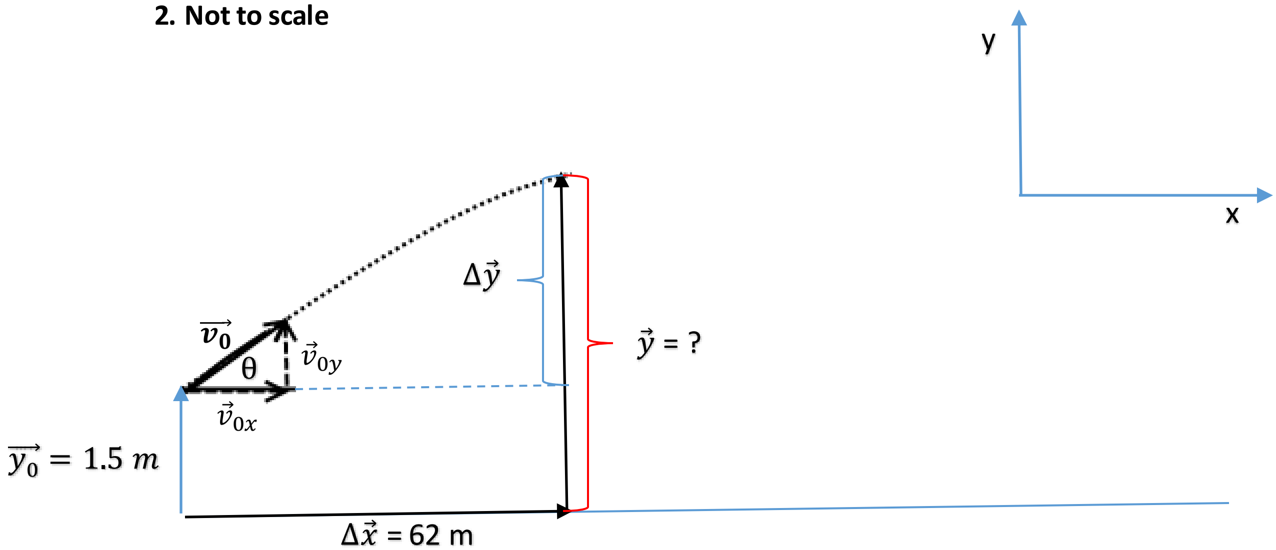
However,
$\vec{v}_y$is changing at the constant rate of -9.8 m/s^2

At t = 1.0 s,

$\vec{v}_y = \vec{v}_{0y} + \vec{g}t = 12.86\frac{m}{s}\ \hat{y} + \left(-9.8\frac{m}{s^2}\hat{y}\right)\ (1.0\ s) \approx 3.1\frac{m}{s}\hat{y}$

Therefore the velocity vector at 1.0 s is $\boldsymbol{\vec{v}} = \vec{v}_x + \vec{v}_y = \boldsymbol{15\frac{m}{s}\ \hat{x} + 3.1\frac{m}{s}\hat{y}\ or\ (15\frac{m}{s}, 3.1\frac{m}{s})}$

2. Not to scale



Since we will use the horizontal and vertical components of the initial velocity to answer a. and b. , they will be calculated now.

$$\vec{v}_{0x} = v_0 \cos\theta\ \hat{x} = 50\frac{m}{s}\cos 27°\ \hat{x}\ \approx 44.55\frac{m}{s}\hat{x}$$

$$\vec{v}_{0y} = v_0 \sin\theta\ \hat{y} = 50\frac{m}{s}\sin 27°\ \hat{y} \approx 22.70\frac{m}{s}\ \hat{y}$$

a.

t = ?

The horizontal range is $\Delta x = 64\ m = \ v_{0x}\, t = 44.55\frac{m}{s} * t$

$t = \frac{64\ m}{44.55\frac{m}{s}} = 1.39\ s\ \approx \mathbf{1.4\ s}$

b. $\vec{\boldsymbol{y}} =?$

Here we will use the time we calculated in a, but the values will be rounded only in the end of the process.

$$\Delta\vec{y} = \ \vec{v}_{0y}t + \left(\frac{1}{2}\right)\vec{g}t^2 = 22.70\ \left(\frac{m}{s}\right) * (1.39s) - 4.9\frac{m}{s^2} * (1.39s)^2 = 31.55\ m - 9.47\ m \approx 22m$$

The vertical position vector is:

$\vec{y} = \Delta\vec{y} + \vec{y}_0 = 22\ m + 1.5m = \mathbf{23.5\ m}\,\widehat{y}$, so the height is **y = 23.5 m**