MAT 105 - Homework 4
Due Thursday, 2/9/16, in class.

1. Suppose A and B are disjoint $(A \cap B = \emptyset)$ and $P(A) = 0.3$ and $P(B) = 0.5$. Find the probability that

    (a) A occurs, but B does not   (b) both A and B occur   (c) A occurs or B occurs.

2. Suppose A and B are disjoint and $P(A) = 0.3$ and $P(B) = 0.5$, as in the previous problem. Find $P(A^c \cap B^c)$, where A^c is the complement of A and B^c is the complement of B. (Hint: draw a diagram.)

3. Let A and B be two *independent* events with $P(A) = 0.4$ and $P(A \cup B) = 0.64$. Compute $P(B)$.

4. Find $E[X]$ for X given by following distribution:

$$P(X = 0) = \frac{1}{6} \qquad P(X = 1) = \frac{2}{6} \qquad P(X = 2) = \frac{3}{6}.$$

5. Consider a loaded die, with 3 twice as likely to appear as each of the other five numbers on the die. Let X denote the number that appears when we roll the die once. Write the distribution of X and compute its expectation $E[X]$.

6. What is the expected sum of the numbers that appear when 2 fair dice are rolled?

7. A sample of 2 items is selected at random from a box of 20 items, out of which 4 are defective. Let X count the number of defective items in the sample.

    (a) Write the probability distribution of X. (Make sure probabilities add up to 1)

    (b) Find the expectation of X.

8. A box contains 5 red marbles and 5 blue marbles. You pick two marbles at random and receive $1.10 if the marbles are the same color and lose $1.00 if the marbles have different colors. Let X denote your winnings (negative if you lose money).

    (a) Write the probability distribution of X.

    (b) Find the expectation $E[X]$

    (c) Find the standard deviation of X.

9. A family has 3 children, each of whom is equally likely to be messy or not messy. Let A="there is at most one messy child", B="the family has both messy and not messy children". Are A and B independent?

10. Three students each have probability 1/3 of solving a problem, *independently of each other*. What is the probability that

    (a) none of them will solve the problem?   (b) at least one of them will solve the problem?