PHY 115 Assignment 9 - Due: Wednesday, April 16

Relevant book sections: Chapter 8, sections 8.1 through 8.4, pages 231-244.

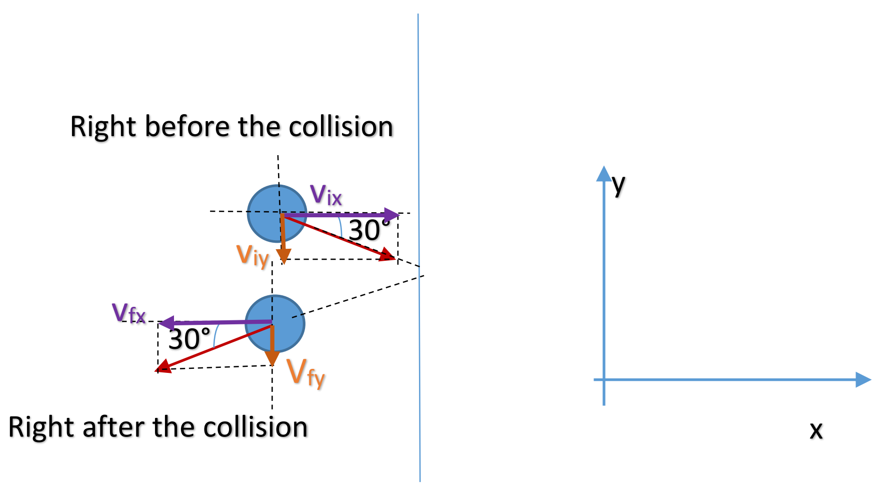
1. Digipen Zee is trying to alleviate her end-of-semester stress by throwing an elastic ball at a wall. The ball strikes the wall at an angle of 30 degrees with the positive horizontal axis, and rebounds with the same speed at 30 degrees with the negative horizontal axis (see figure). The mass of the ball is 100 grams. The speed of the ball is 25 m/s.

a. Is the kinetic energy of the ball conserved? Explain.

Yes, since the speed is not changed during this small time interval. Mass is also a constant, therefore $\Delta K = \left(\frac{1}{2}\right) m v_f^2 - \left(\frac{1}{2}\right) m v_i^2 = \left(\frac{1}{2}\right) m\left(v_f^2 - v_i^2\right) = 0$, where v_f is the final speed and v_i is the initial speed.

Note that, although the speed remains constant, the velocity is not constant. Also, the total energy of the system is constant once that there are no significant changes in the gravitational potential energy due to the short time interval.

b. Determine the change in linear momentum of the ball. Please state your answer by specifying the horizontal and vertical components of the change in linear momentum.



Using the coordinate system and the indices specified above,

$\vec{p}_{ix} = initial\ momentum\ in\ the\ x - direction = m v_{ix}\hat{x} = m v_i cos\theta \hat{x} = (1.0\ \times 10^{-1}) kg * 25\frac{m}{s} * cos30° \hat{x} \approx 2.17\ kg\ \left(\frac{m}{s}\right)\hat{x} \approx 2.2\ kg\ \left(\frac{m}{s}\right)\hat{x}$

$\vec{p}_{fx} = final\ momentum\ in\ the\ x - direction = m v_{fx}\hat{x} = m v_f cos\theta \hat{x} = -(1.0\ \times 10^{-1}) kg * 25\frac{m}{s} * cos30° \hat{x} \approx -2.17\ kg\ \left(\frac{m}{s}\right)\hat{x} \approx -2.2\ kg\ \left(\frac{m}{s}\right)\hat{x}$

$\Delta\vec{p}_x = \vec{p}_{fx} - \vec{p}_{ix} = \left[-2.2\ kg\left(\frac{m}{s}\right) - 2.2\ kg\left(\frac{m}{s}\right)\right]\hat{x} = -4.4\ kg(\frac{m}{s})\hat{x}$

$\vec{p}_{iy} = initial\ momentum\ in\ the\ y - direction = mv_{iy}\hat{y} = mv_i sin\theta\hat{y} = (1.0 \times 10^{-1})kg * 25\frac{m}{s} * sin30°\hat{y} = -1.25\ kg\ \left(\frac{m}{s}\right)\hat{y} \approx -1.3\ kg\ \left(\frac{m}{s}\right)\hat{y}$

$\vec{p}_{fy} = final\ momentum\ in\ the\ y - direction = mv_{fy}\hat{y} = mv_f sin\theta\hat{x} = -(1.0 \times 10^{-1})kg * 25\frac{m}{s} * sin30°\hat{y} = -1.25kg\ \left(\frac{m}{s}\right)\hat{y} \approx -1.3\ kg\ \left(\frac{m}{s}\right)\hat{y}$

$$\Delta\vec{p}_x = \vec{p}_{fx} - \vec{p}_{ix} = \left[-2.2\ kg\left(\frac{m}{s}\right) - 2.2\ kg\left(\frac{m}{s}\right)\right]\hat{x} = -4.4\ kg(\frac{m}{s})\hat{x}$$

$$\Delta\vec{p}_y = \vec{p}_{fy} - \vec{p}_{iy} = \left[-1.3\ kg\left(\frac{m}{s}\right) - \left(-1.3\ kg\left(\frac{m}{s}\right)\right)\right]\hat{y} = 0$$

This means that there is no momentum change in the y-direction. Therefore:

$$\boldsymbol{\Delta\vec{p} = \Delta\vec{p}_x == -4.4\ kg(\frac{m}{s})\hat{x}}$$

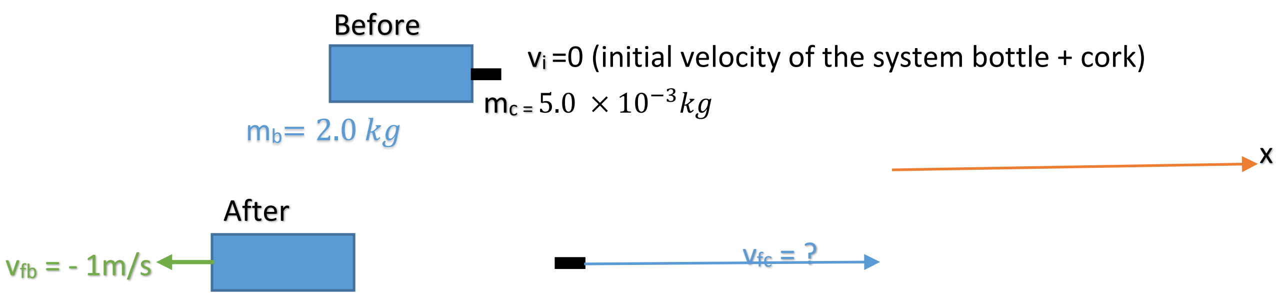
This means that the impulse (change in momentum) is a vector that has only an x-direction component, which is in agreement with the direction of the normal force from the wall, a horizontal force. This normal force causes the change in momentum in the x-direction. This concept will be explored in the next part.

c. Extra-credit (only this part is extra-credit) If the ball is in contact with the wall for approximately 3.0 x 10^{-3} s, what is the average force (direction and magnitude) of the wall on the ball?

$$Average\ force = \frac{\Delta\vec{p}}{\Delta t} = \frac{-4.4\ kg\left(\frac{m}{s}\right)\hat{x}}{3.0 \times 10^{-3}s} \approx -\mathbf{1.5 \times 10^3 N}\ (\boldsymbol{to\ the\ left})$$

2. Digipen Zee has graduated. To celebrate the event, she pops a cider bottle, holding the bottle horizontally. The cork is ejected horizontally in the positive x-direction, as the bottle recoils horizontally in the negative x-direction. The mass of the bottle (including the cider) is 2.0 kg. The mass of the cork is 5.0 grams. The bottle recoils at a speed of 1.0 m/s. What is the velocity (direction and magnitude) at which the cork is ejected from the bottle?

In the diagram below, c = cork and b = bottle



$$\overrightarrow{v_{fc}} = ?$$

Here kinetic energy is not conserved, since the initial kinetic energy of the cork and the bottle is zero, and the final kinetic energy comes from the explosion. However, <u>momentum is conserved</u> in the system bottle-cork. This happens because no external forces are present. The force of the bottle on the cork and the force of the cork on the bottle are internal to the system, and cancel each other out. The motion is purely horizontal, since all the measurements are takes immediately before and immediately after the collision.

$$\Delta\vec{p}_x = \vec{p}_{fx} - \vec{p}_{ix} = 0 \;\rightarrow\; \vec{p}_{ix} = \vec{p}_{fx}$$

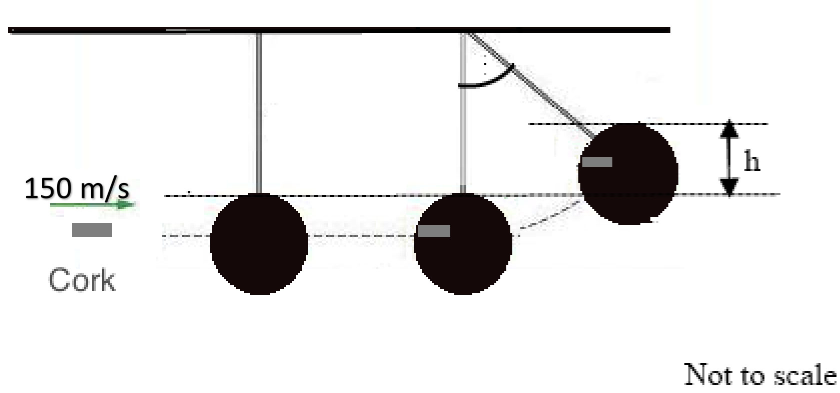
$$\vec{p}_{ix} = 0 = m_c\overrightarrow{v_{fc}} + m_b\overrightarrow{v_{fb}}$$

Isolating the final velocity of the cork $\overrightarrow{v_{fc}}$,

$$\overrightarrow{v_{fc}} = -\frac{(m_b\overrightarrow{v_{fb}})}{m_c} = -\frac{2.0\ kg*-1.0\frac{m}{s}}{5.0\ \times 10^{-3} kg} = \frac{2.0\ kg\frac{m}{s}}{5.0\ \times 10^{-3} kg} = \mathbf{4.0\ x\ 10^{2}\ \frac{m}{s}\ (in\ the\ positive\ x - direction)}$$

Note that the final speed of the cork is very large when compared to the final speed of the bottle. This is because the mass of the bullet is very small when compared to the mass of the cork.

3. The cork (mass = 5.0 grams), traveling horizontally at 150 m/s, hits a 0.50-kg rubber Kirby figurine that is hanging from the ceiling (see figure). The cork makes a completely inelastic collision with the figurine, becoming embedded in it. After the impact, the system Kirby-cork swings up to a maximum height *h.* Determine *h*. *Hint*: this is a ballistic pendulum.



Momentum is conserved in the perfectly inelastic collision:

$$v_{ic} + v_{if}(= 0) = (m_c + m_f)V_f$$

$$V_f = \frac{m_c v_{ic}}{m_c + m_f} = \frac{\left(5.0\ \times 10^{-3}\ kg * 150\frac{m}{s}\right)}{0.505\ kg} = 1.48\frac{m}{s} \approx 1.5\ m/s$$

The result above shows that the mass of the cork could have been neglected while computing the total mass.

This is the speed of the cork-figurine system as it starts rising. The final velocity (at height h) is zero. This means that the kinetic energy is completely converted into potential energy, if nonconservative forces are neglected:

$$\left(\frac{1}{2}\right) m_{total} V_f^2 = m_{total} g h$$

Dividing both sides by m_{total} and solving for *h*,

$$h = \frac{\left(\frac{1}{2}\right) V_f^2}{g} = \left(\frac{1}{2}\right) * \frac{\left(1.48\frac{m}{s}\right)^2}{9.8\frac{m}{s^2}} \approx \mathbf{1.1\ \times 10^{-1} m}$$

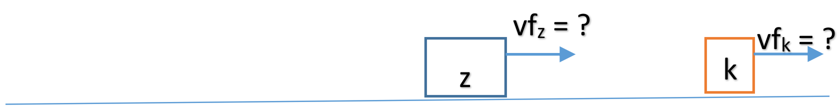
4. Digipen Zee is vacationing in NYC. She is now ice-skating at the crowded Rockefeller Center ice rink. Zee's mass is 50 kg. Neglect friction and any other nonconservative forces (meaning that all collisions are elastic) to answer parts a through c.

a. While traveling at 4.0 m/s in the positive x-direction, Zee approaches a 30-kg kid who is at rest. Zee and the kid collide. After the collision, the kid moves in the positive x-direction. What is the velocity of the kid immediately after the collision?

The next diagram refers to parts a and b.

Before the collision:

After the collision:



This is an elastic, 1-D collision with the target (the kid) at rest. Kinetic energy and momentum are conserved, and the following equation can be used to calculate the kid's velocity.

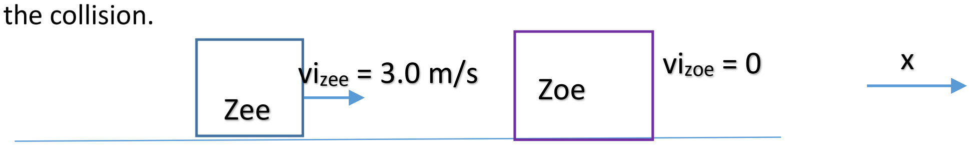
$v_{fk} = \frac{2m_z}{m_{total}} v_{iz} = \frac{2 * 50\ kg}{80\ kg}\left(4.0\frac{m}{s}\right) = \mathbf{5.0\frac{m}{s}}$ **in the positive x-direction**

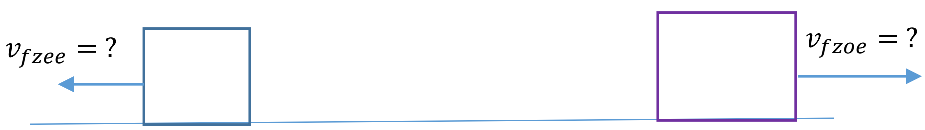
b. What is the velocity (direction and magnitude) of Zee immediately after the collision?

$v_{fz} = \frac{m_z - m_k}{m_{total}} v_{iz} = \frac{20\ kg}{80\ kg} * 4.0\frac{m}{s} = 1.0\ m/s$ **in the positive x-direction**

c. Although she is a little dizzy, Zee is still ice-skating after the collision with the kid. But, unfortunately, as she is moving in the positive x-direction at 3.0 m/s, she approaches another ice skater, who happens to be her cousin Zoe. Zoe's mass is 60 kg. Zee and Zoe collide. Zoe was at rest before the collision. What are Zee's and Zoe's velocities immediately after the collision?

Now the target (Zoe) has a greater mass than Zee. This means that Zee moves in the negative direction after the collision.



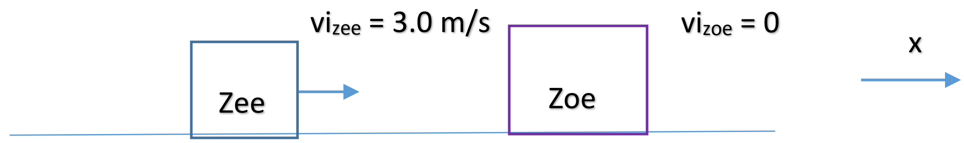


$$v_{fzoe} = \frac{2m_z}{m_{total}} v_{izee} = \frac{100\ kg}{110\ kg} * 3.0 \frac{m}{s} \approx \boldsymbol{2.7 \frac{m}{s}\ in\ the\ positive\ x - direction}$$

$$v_{fzee} = \frac{m_{zee} - m_{zoe}}{m_{total}} v_{iz} = -\frac{10\ kg}{110\ kg} * 3.0 \frac{m}{s} \approx \boldsymbol{-0.27 \frac{m}{s} (in\ the\ negative\ x - direction)}$$

d. If Zee and Zoe stuck together after the collision, what would be their final velocity (immediately after the collision)? Would that be an elastic or inelastic collision?

It's a completely inelastic collision, since they stick together. See the diagram below.



Energy is not conserved when objects stick together. However, momentum s conserved.

$$m_{zee}v_{iz} + 0 = (m_{zee} + m_{zoe})v_f$$

$$v_f = \frac{m_{zee}v_{iz}}{m_{zee} + m_{zoe}} = \frac{50\ kg * 3.0\frac{m}{s}}{110\ kg} \approx \boldsymbol{1.4\frac{m}{s}\ (in\ the\ positive\ direction)}$$