PHY 115

Assignment 7

Due: Wednesday, April 2.

Relevant book sections: Chapter 7, sections 7.1 – 7.3, pages 188-200.

Section 7.5, Pages 203-206.

1.

a. A meteor is moving horizontally in vacuum at a constant speed *v*. Its kinetic energy is 4.5×10^7 J. At what speed, in terms of *v*, would it be moving if it had 9.0×10^7 J of kinetic energy while following a horizontal path?

No dissipating forces (such as drag or friction) are present. The kinetic energy is:

$$K = \left(\frac{1}{2}\right) m v^2 = 4.5 \times 10^7 \, J$$

Note that energy is a scalar.

We do not know the mass or the speed. So our result is going to be in terms of the speed. The mass is canceled out (shown below). Let the new kinetic energy be *K'*:

$$K' = \left(\frac{1}{2}\right) m v'^2 = 9.0 \times 10^7 \, J = 2K = m v^2$$

So

$$\left(\frac{1}{2}\right) m v'^2 = m v^2$$

Dividing by m,

$$v'^2 = 2v^2$$

We take the square root from both sides,

$$\boldsymbol{v' = \sqrt{2}\, v \approx 1.4\, v}$$

Note that v' > v, since K' > K.

b. What would be the meteor's kinetic energy if it moved horizontally at a constant speed of *v*/2?

Now there is a new kinetic energy associated with the new speed v/2. This kinetic energy (K'') should be smaller than K.

$$K'' = \left(\frac{1}{2}\right) m v''^2 = \left(\frac{1}{2}\right) m \left(\frac{v}{2}\right)^2 = \left(\frac{1}{4}\right)\left(\frac{1}{2}\right) m v^2 = \frac{K}{4} = \frac{4.5 \times 10^7 J}{4} \approx \boldsymbol{1.1 \times 10^7 \, J}$$

Meaning that, if we divide speed by 2, the kinetic energy is divided by 4.

2. Digipen Cee is a 50-kg ninja. She is able to jump from a 10-m high roof, and walk away without a limp. As Cee falls, she thinks about conservation of energy.

To answer the following questions, neglect air resistance and treat Cee as a particle.

a. How long is Cee's flight? Assume that her initial speed is zero.

$t = ?$

Here we do not use energy equations, since the total mechanical energy is constant in time.

However, the acceleration is constant (free fall), so we can use a kinematic equation:

$$\Delta\vec{y} = \vec{v}_0 t + \left(\frac{1}{2}\right)\vec{g}t^2$$

Where

$\vec{v}_0 = 0, \Delta\vec{y} = -10\ m$

$$-10\ m = \left(\frac{1}{2}\right)\left(-9.8\frac{m}{s^2}\right)t^2$$

$$t = \sqrt{\frac{20}{9.8}}\ s \approx 1.43\ s\ \approx \mathbf{1.4\ s}$$

b. What is Cee's kinetic energy immediately before she hits the ground? Please use conservation of energy to solve this question. Of course, you can check your result by using a kinematic equation.

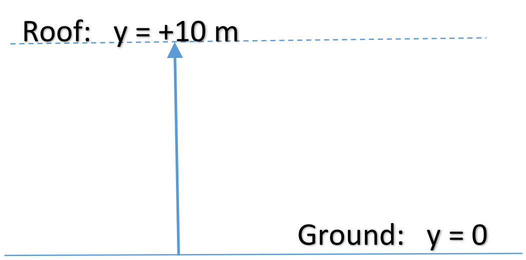
The total mechanical energy *E* is constant, since we are neglecting the drag force.

E = K + U = kinetic energy + potential energy

$$K_{roof} + U_{roof} = K_{ground} + U_{ground}$$

But since $\vec{v}_0 = 0, K_{roof} = \left(\frac{1}{2}\right)mv_0^2 = 0$

Using the coord. system below for U_{roof}:



$$U_{roof} = mgh = 50\ kg * 9.8\frac{m}{s^2} * 10m = 4.9\ \times 10^3 J = E = total\ energy$$

This potential energy will be completely converted into kinetic energy at the ground:

$4.9 \times 10^3 J = U_{ground} + K_{ground} = mg * (0) + K_{ground} = 0 + K_{ground}$

So $\boldsymbol{K_{ground} = 4.9 \times 10^3 J}$

<u>Checking this result (not required for this part, but used on c):</u>

The speed *v* at the ground is calculated using the kinematic equation: $v^2 = v_0^2 + 2\vec{g}.\overrightarrow{\Delta y} =$

$$= 0 + 2 * \left(-9.8\frac{m}{s^2}\right) * (-10m) = 196\frac{m^2}{s^2}$$

$$v = \sqrt{196\frac{m^2}{s^2}} = 14\frac{m}{s}$$

So the kinetic energy at the ground is: $K_{ground} = \left(\frac{1}{2}\right)mv^2 = \left(\frac{1}{2}\right) * 50\,kg * 196\frac{m^2}{s^2} = \boldsymbol{4.9 \times 10^3 J}$

As expected.

c. What is Cee's speed immediately before she hits the ground?

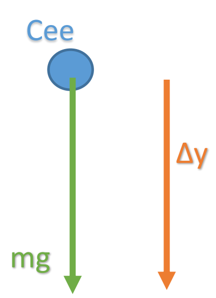
Using the result obtained above,

$\boldsymbol{v = 14\frac{m}{s}}$ immediately before she hits the ground.

d. What is the total work done by the gravitational force during the flight? Is it positive or negative?

$$W = F_g \Delta y cos\theta = mg\Delta y cos\theta = 50kg * \left(9.8\frac{m}{s^2}\right) * (10\,m) * \cos(0°) = \boldsymbol{+4.9 \times 10^3 J}$$

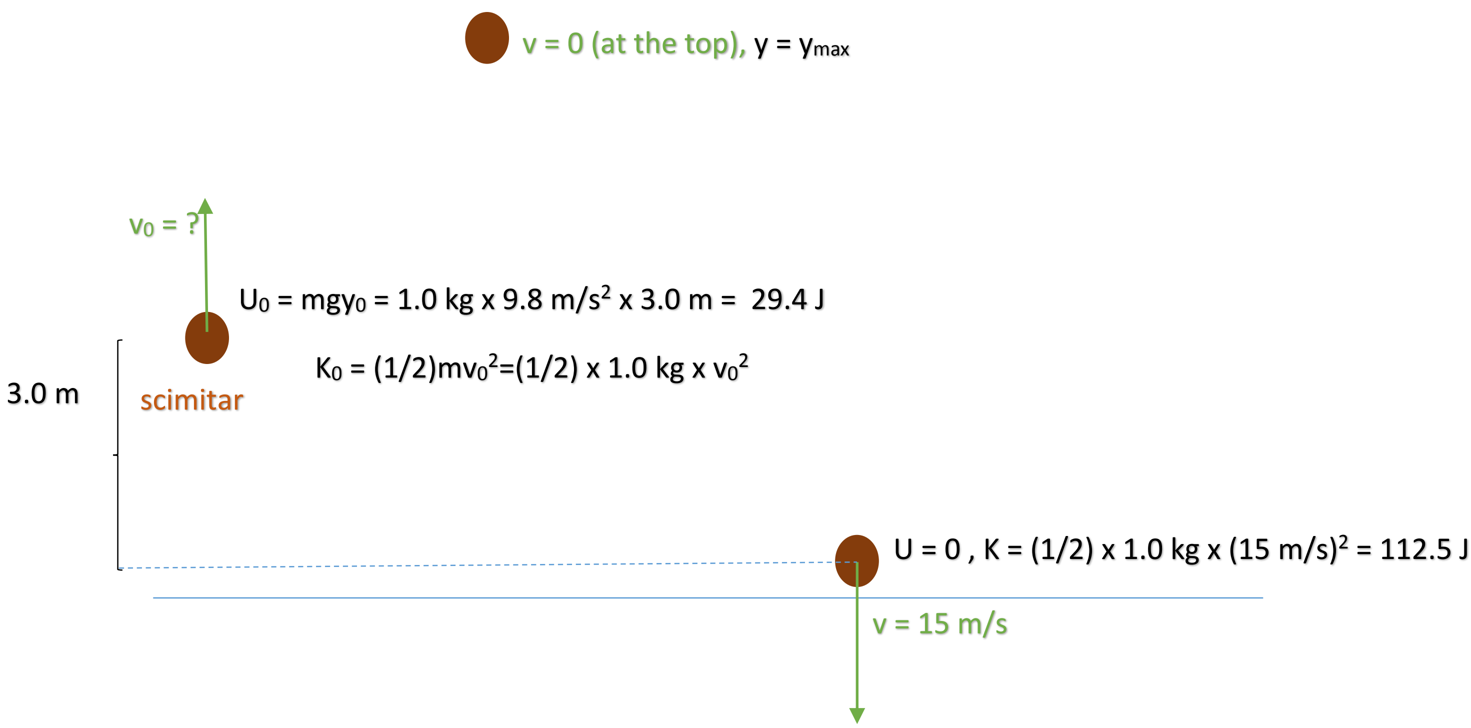
Note that in the equations above we only use **<u>magnitudes</u>** and that θ is the angle between the gravitational force $\vec{F}_g$ and the displacement $\Delta\vec{y}$. This angle is 0 degrees as shown below:



So the gravitational force does positive work on Cee.

3. After landing, Cee throws a scimitar straight upwards, from a height of 3.0 m above the ground. The mass of the scimitar is 1.0 kg. The scimitar moves vertically upwards, and lands on the ground (but Cee is not hit by the scimitar, as she has teleported somewhere far away as the scimitar fell). To answer parts a through c, neglect air resistance and treat the scimitar as a particle.

a. How fast must Cee throw the scimitar so that it will hit the ground at a speed of 15 m/s? Please use conservation of energy to solve this question. Again, you can check your result by using a kinematic equation.



Immediately before reaching the ground, the potential energy of the scimitar is zero (see figure), and the kinetic energy is equal to the total energy: $E = 112.5\,J\ at\ any\ time\ during\ the\ flight.$

$$E = U_0 + K_0 = 29.4\,J + \left(\frac{1}{2}\right) * 1.0\,kg * v_0^2 = K = 112.5\,J$$

So $v_0^2 = \frac{2*112.5\,J}{1.0\,kg} - \frac{2*29.4\,J}{1.0\,kg} = (225 - 58.8)\frac{m^2}{s^2} = 166.2\frac{m^2}{s^2}$

$$v_0 = \sqrt{166.2\frac{m^2}{s^2}} \approx 12.89\frac{m}{s} \approx \mathbf{13}\frac{\mathbf{m}}{\mathbf{s}}$$

b. What is the maximum height (relative to the ground) of the scimitar?

Since the total energy is equal to 112.5 J and the kinetic energy at the top is equal to zero (see figure),

$$mgy_{max} = 1.0\,kg * 9.8\frac{m}{s^2} * y_{max} = 112.5\,J$$

$$y_{max} = \frac{112.5\,J}{1.0\,kg * 9.8\frac{m}{s^2}} \approx 11.5\,m \approx \mathbf{12\,m}$$

c. What is the speed of the scimitar at 4.0 m from the ground?

At 4.0 m from the ground, both K and U are **not** zero.

$$E = K + U = \left(\frac{1}{2}\right)mv^2 + mgy$$

$$\left(\frac{1}{2}\right)mv^2 = E - mgy$$

$$v^2 = \frac{2(E - mgy)}{m} = \frac{2 * \left(112.5\,J - 1.0\,kg * 9.8\frac{m}{s^2} * 4.0\,m\right)}{1.0\,kg} = 2 * \frac{112.5\,J - 39.2\,J}{1.0\,kg}$$

$$v^2 = 146.6\frac{m^2}{s^2}$$

$v \approx \mathbf{12\frac{m}{s}}$ at a height of 4.0 m. Note that it is the same speed no matter if the scimitar is moving up or down.

d. Extra-credit: if the drag force did -15 J of work (the drag force does negative work) on the scimitar during the entire flight, what would be the speed of the scimitar immediately before it hit the ground?

The drag force does negative work because its direction is opposite to the direction of the displacement (see 1d).

$$E = K + U = \left(\frac{1}{2}\right)mv^2 + 0\ (U = 0\ at\ the\ ground) = 112.5\,J - 15\,J = 97.5\,J$$

$$v^2 = \frac{2 * 97.5\,J}{1.0\,kg} = 195\ \frac{m^2}{s^2}$$

$$v = 13.96\ \frac{m}{s} \approx \mathbf{14\ m/s}$$

Note that this speed is smaller than the previous 15 m/s, because some of the energy was dissipated due to the work done by the drag force.