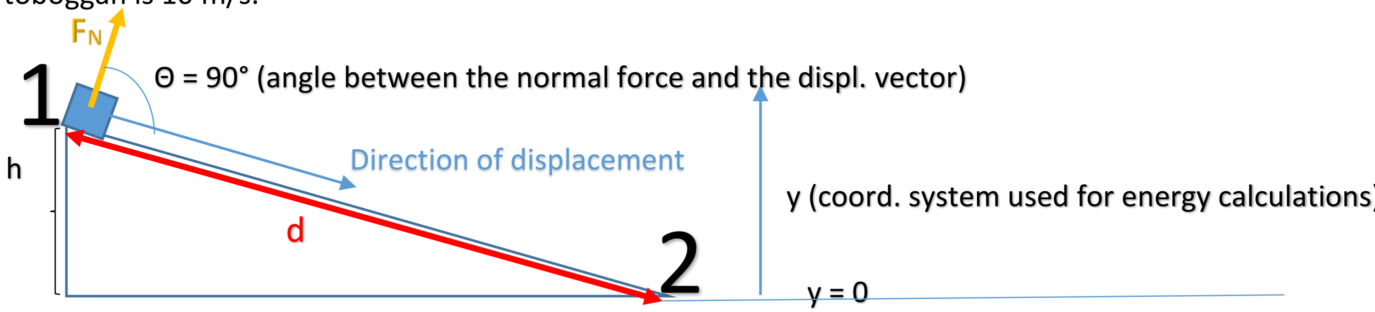
PHY 115

Assignment 8

Due: Wednesday, April 9

Relevant book sections: Chapter 7, sections 7.1 - 7.8, pages 188- 216

1. A cat of mass 4 kg is sliding down a large toboggan of (vertical) height h= 10 m. The initial speed of the cat is zero. A friction force is present, and the speed of the cat at the bottom of the toboggan is 10 m/s.



a. What is the gravitational potential energy of the cat at the top of the toboggan?

Since, according to the choice of coordinate system, y = 0 at the bottom of the toboggan,

$$U_g = gravitational\ potential\ energy = mgh = 4.0\ kg * 9.8\frac{m}{s^2} * 10m = \mathbf{392\ J\ \approx 4\ x\ 10^2 J}$$

Note that this choice of coordinate system is convenient for potential energy calculations, since U_g depends only on height.

b. What is the work done by the friction force on the cat during the entire time the cat is in contact with the toboggan? Is it positive or negative? Note: you will <u>not</u> need to determine the friction force on the cat.

If nonconservative forces <u>were not</u> present, then $U_1 + K_1 = U_2 + K_2$ (*)

Where $U_{1=} = 392\ J$ (as calculated on a), $K_1 = 0$ (cat starts from rest) and $U_2 = 0$ ($y = 0$ at point 2).

However, it is given that $v_2 = 10\ m/s$. Then we can calculate the kinetic energy at point 2:

$$K_2 = \left(\frac{1}{2}\right) mv_2^2 = \left(\frac{1}{2}\right) * 4.0kg * 100\frac{m^2}{s^2} = 200\ J$$

This result indicates that a nonconservative force (the friction force) does work on the cat. Equation * needs to be corrected to include this work:

$$U_1 + K_1 (= 0) + W_{FF} = U_2 (= 0) + K_2$$

Where W_{FF} is the work done by the friction force.

$U_1 + W_{FF} = K_2 \rightarrow W_{FF} = K_2 - U_1$

$W_{FF} = 200\,J - 392\,J = \mathbf{-192\,J \approx -2\,x\,10^2} J$

c. What is the work done by the normal force on the cat? Please explain your answer.

The work done by the normal force on the cat is zero. This is a consequence of the definition of work. The equation below gives the work done by the normal force:

$W_{FN} = F_N d cos\theta,$

Where $W_{FN} = work\ done\ by\ the\ normal\ force$

$F_N = magnitude\ of\ the\ normal\ force$

$d = magnitude\ of\ the\ displacement$ (see figure above)

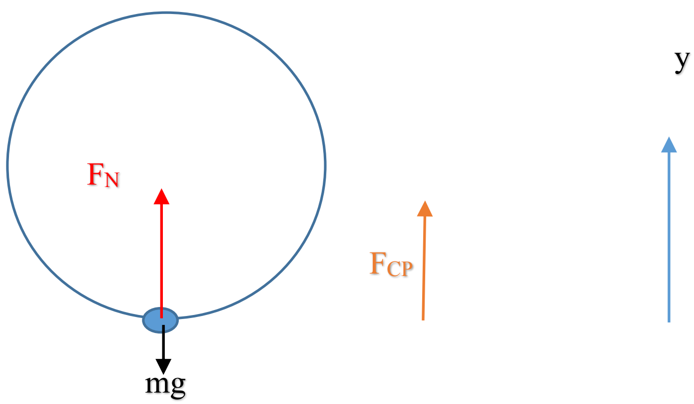
$\theta = angle\ between\ the\ normal\ force\ and\ the\ displacement\ vector$ (see figure above)

Since $\theta = 90°, \cos\theta = 0\ and\ \boldsymbol{W_{FN} = 0}$.

2. An airplane pilot, whose mass is 90 kg, wants to show off to his friend, who is watching him from the ground.

a. The pilot executes a vertical loop maneuver. In this maneuver, the airplane moves in a vertical circle of radius 2.7 km, at a constant speed of 225 m/s. What is the normal force on the pilot when the airplane is at the bottom of the loop?

At the bottom of the loop, the magnitude of the normal force (which is the apparent weight) is much larger than the magnitude of the weight, since the net force (the centripetal force) is upward:



Note that the centripetal force is drawn outside of the free-body diagram, since it is a net force (that points towards the center of the circle).

$$\vec{F}_{cp} = \vec{F}_N + m\vec{g}$$

According to the coordinate system,

$$F_{CP} = F_N - mg$$

However, we know that

$$F_{CP} = mv^2/R$$

Therefore $\frac{mv^2}{R} = F_N - mg$

Solving for the magnitude of the normal force:

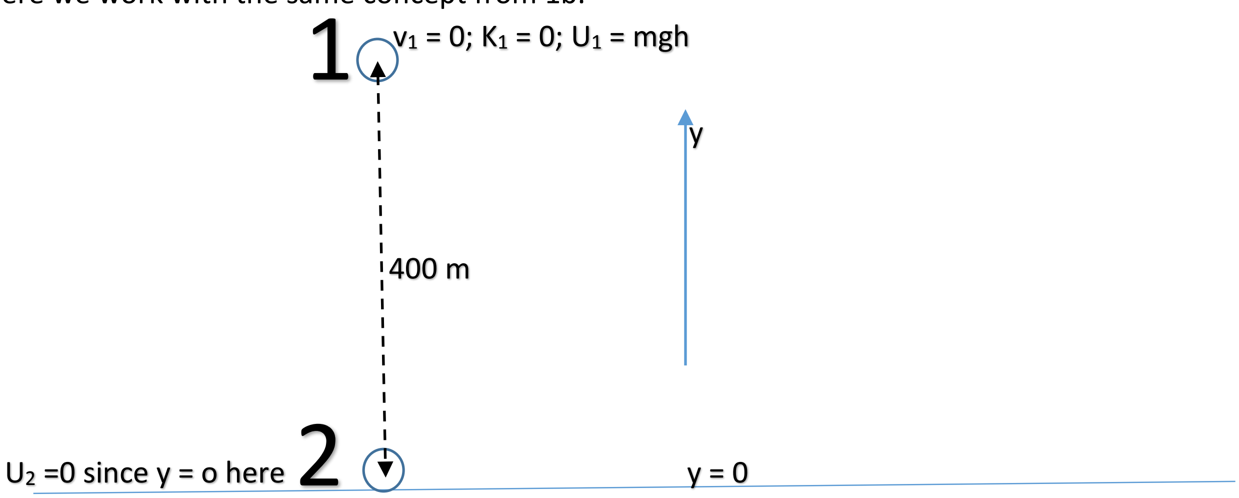
$$F_N = \frac{mv^2}{R} + mg$$

$$F_N = \frac{90\ kg * \left(225\frac{m}{s}\right)^2}{2700\ m} + 90\ \mathrm{kg} * 9.8\frac{\mathrm{m}}{\mathrm{s}^2} \approx 1688\ \mathrm{N} + 882\ \mathrm{N} = 2570\ \mathrm{N}\ \approx \mathbf{2.6\ x\ 10^3 N}$$

Note: it is a good exercise to redo the problem for when the pilot is at the top of the circular loop.

b. Unfortunately, the maneuver fails and the pilot falls (at an initial speed of zero) from the airplane. In addition to that, his parachute does not work immediately and the pilot falls 400 m, until he reaches a terminal velocity of 50 m/s. At this time, the parachute magically opens and the speed of the pilot is subsequently reduced. Calculate the work done on the pilot by the drag force during the time the parachute is not working. Neglect the mass of the parachute.

Here we work with the same concept from 1b.



$$U_1 + K_1(= 0) +\ W_{FD} = U_2(= 0) + K_2$$

Where $W_{FD} = work\ done\ by\ the\ drag\ force\ (negative\ work)$

It is important to emphasize that, unlike the friction force from problem 1, the drag force is not constant. The drag force is a function of the speed of the pilot. At 1, F_D is zero and, as the pilot falls, F_D increases until the moment the terminal velocity is attained. After this point, F_D is constant.

Although we are not determining F_D, we can determine the work done by F_D (W_{FD}) during the entire flight.

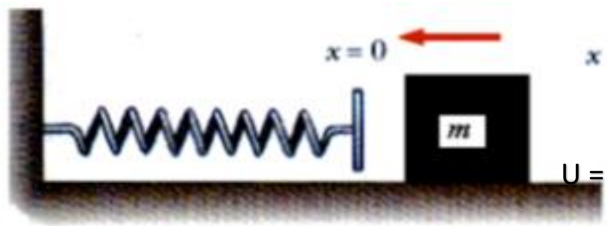
$$W_{FD} = K_2 -\ U_1 = \left(\frac{1}{2}\right) m v_2^2 - mgh = \left(\frac{1}{2}\right) * 90\ kg * \left(50\frac{m}{s}\right)^2 - 90\ kg * 9.8\frac{m}{s^2} * 400m \approx$$

$$\mathbf{-2.4\ x\ 10^5 J}$$

3. A 2.0 kg wooden box (see figure below) slides on a flat, frictionless surface, in the positive x direction (see figure on the next page). When the box is moving at 3.0 m/s, it hits a rubber screen (modeled as a spring) with a spring constant of 1.0 x 10^3 N/m. The box compresses the screen to its maximum displacement with respect to its equilibrium position (x_{max}).

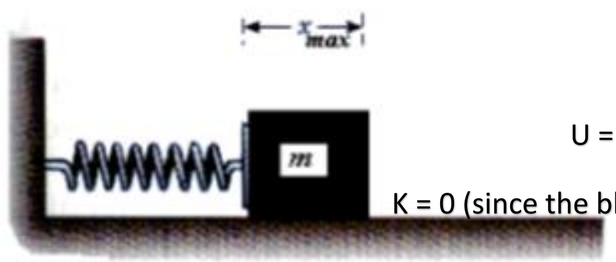
Treating the box as a particle and neglecting any nonconservative forces,

a. Determine the total energy of the box at any time.



$K = K_{max}$

$U = 0$ (since the spring is not compressed or extended)



$U = U_{max}$

$K = 0$ (since the block is not moving)

Here there is no energy transfer to the environment. The total mechanical energy of the system (the box) is conserved. This is an ideal oscillator.

This means:

$$U_{max}(when\ the\ spring\ is\ fully\ compressed) =$$
$$K_{max}\ (when\ the\ speed\ is\ maximum, and\ the\ spring\ is\ not\ compressed) =$$
$$E\ (total\ energy)$$

$$E = K_{max} = \left(\frac{1}{2}\right) m{v_{max}}^2 = \left(\frac{1}{2}\right) * 2.0\ kg * \left(3.0\frac{m}{s}\right)^2 = \mathbf{9.0\ J}$$

b. Determine x_{max}.

Using the conclusions from part a,

$$E = K_{max} = U_{max} = \left(\frac{1}{2}\right) k_s x_{max}^2$$

Where k_s is the spring constant = 1.0 x 10^3 N/m

$$x_{max}^2 = \frac{2E}{k_s} = \frac{2 * 9.0J}{1.0 * 10^3 \frac{N}{m}} = 18 \times 10^{-3} m^2 = 1.8 \times 10^{-2} m^2$$

$$x_{max} = \sqrt{1.8 \times 10^{-2} m\text{^}2} \approx \mathbf{1.34 \times 10^{-1} m \approx 13\ cm}$$

c. **Extra-credit** – Determine the speed of the box when the block is located at $x_{max}/2$.

At any time, the total energy is $E = 9.0\ J$

When the object is located at $x_{max}/2$, the initial kinetic energy K_{max} (9.0 J) is only partially converted into potential elastic energy.

$$E = \left(\frac{1}{2}\right) mv^2 + \left(\frac{1}{2}\right) k_s x^2$$

We solve for the speed *v*:

$$\left(\frac{1}{2}\right) mv^2 = E - \left(\frac{1}{2}\right) k_s x^2$$

$$v^2 = \frac{2E}{m} - \frac{k_s x^2}{m}$$

Where x = xmax/2 = 0.134 m/2 = 0.067 m

$$v = \sqrt{\frac{2E}{m} - \frac{k_s x^2}{m}} = \sqrt{\left(\frac{2 * 9.0J}{2.0kg}\right) - \left(1.0 * 10^3 \frac{N}{m}\right) * \frac{(0.067\ m)^2}{2.0kg}} = \sqrt{\left(\frac{18}{2.0}\right)\frac{m^2}{s^2} - \left(\frac{4.49}{2.0}\right)\frac{m^2}{s^2}} = \sqrt{9.0 - 2.24}\ \frac{m}{s} \approx \mathbf{2.6 \frac{m}{s}}$$