

### **Project description**

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#### **Outline**



- Rules
- Important dates
- Peer feedback and project evaluation
- Common mistakes, misconceptions and advice
- Great projects from last year
- Take-away ideas

#### **Rules**



- Topic is **free** (creativity highly encouraged!)
- No constraints on the dataset
- If in doubt, talk with us!
- We also provide some suggestions for projects





- 31 March Milestone report (in groups, mandatory)
  - Project outline (1-2 pages):
    - Dataset and research question(s)
    - ullet First draft of model(s) that you plan to try out (PGM + generative process)
  - Initial notebook (descriptive stats, data preparation)
  - Save notebook as PDF and append to project outline.
  - Submit one PDF to the peer feedback platform (we will provide more details soon).



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- 22 May Final delivery (in groups)
  - Fully self-explanatory notebook
  - 5-10 page report (incl. figures and tables)

#### Peer feedback and project evaluatione



- Peer feedback
  - We will use peergrade.io to organise the peer feedback.
  - Each group makes one submission.
  - Everyone provides feedback on two milestone reports (individually)
  - Provide fair, respectful, caring and constructive feedback to your peers.
  - Consider the grading rubric for the final evaluation of the project when giving feedback.
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- Project evaluation
  - We provide a detailed grading rubric on DTU Learn. Please read it carefully.



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  - ullet Consider for example: Intelligence o Course grade o Recommendation letter
  - If course grade is always observed, then it blocks the path between intelligence and recommendation letter. Therefore, "Intelligence  $\rightarrow$  Course grade" and "Course grade  $\rightarrow$  Recommendation letter" become two independent models.
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- Don't just think about what variables depend on other variables, and their distribution types. Think also of how you should model those dependencies (e.g. how to condition the parameters of a Beta distribution on another variable?)



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  more complex towards the idealised/conceived PGM. Plus, try to have some
  baseline models for comparison whenever possible
- Use ancestral sampling to generate artificial data, and run inference on the model using Pyro/STAN to see if it is able to recover the true values/parameters that were used to generate the data. This is a great way of guarantying that the model is correctly implemented, and that inference is working correctly

# 2019 proj.: Analysis group membership in Social Networks I



- Graph data of human interaction is called social networks. The interaction could for example being friends on Facebook or talking to each other.
- The project inferred groups in social networks based on Game Of Thrones. Take a look at the data here https://github.com/mathbeveridge/gameofthrones
- Tried different models. Results below is from a Mixed membership stochastic block model - MMSBM.

# 2019 proj.: Analysis group membership in Social Networks II



- **1** For n = 1...N
  - **1** draw  $\pi_n \sim \mathsf{Dirichlet}(\boldsymbol{\alpha})$
- **2** For  $k, k' = \{1, 2, \dots, K\} \times \{1, 2, \dots, K\}$  where  $k \neq k'$ 
  - **1** draw  $\lambda_{k,k'} \sim \mathsf{Gamma}(a,b)$
- $\bigcirc$  For k in 1..K
  - **1** draw  $\lambda_{k,k} \sim \mathsf{Gamma}(c,d)$
- **4** For all pairs  $i,j \in \{1,2,...N\} \times \{1,2,...N\}$  where  $i \neq j$

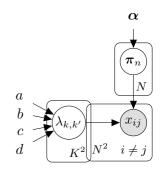


Figure: PGM for Mixed membership stochastic block model

## 2019 proj.: Analysis group membership in Social Networks III





Figure: Each node is a GoT character from season 5. Colors indicate one of 5 inferred groups.

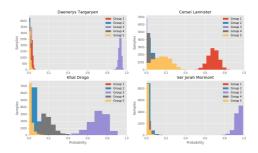


Figure: The histogram shows posterior probabilities of belonging to one of 5 groups for 4 main characters of GoT

## 2019 project: Bayesian semi-parametric soccer analysis



- Apply the time-to-event analysis methods to analyze the intensity with which goal scoring occurs in soccer matches
- Tried different time-to-event models from the class of Cox proportional hazards models and extend these to incorporate non-linear effects by use of the highly flexible Gaussian processes from the Bayesian non-parametrics toolbox
- ullet The hazard in the i-th match at time interval j is

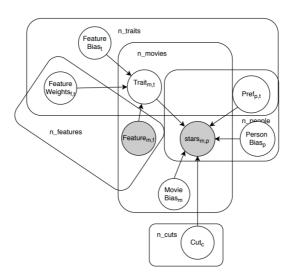
$$\lambda_{ij} = \lambda_j \exp(\beta_1 * skill\_gap + \beta_2(t) * x\_goal)$$

- Baseline of Cox model is a GP:  $\log \lambda_0 \sim \mathcal{GP}(\mathbf{0}, K(t, t'))$
- Implemented everything in STAN and did a very good experimental evaluation



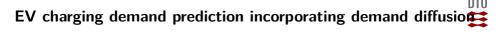
## 2019 project: Movie recommendations - MovieLens dataset

 Inspired by the example in Bishop's "Model-based Machine Learning" book, but with a good mix of creativity, lots of experimentation and insightful discussions





- Demand diffusion: Propagation of customers to different charging stations when the capacity of the desired station is met
- Bayesian autoregressive model with latent propagation process to predict EV charging demand
- Comparison with two other models: Without censoring or propagation variable



- **1** Draw propagation probability  $p_{i,j} \sim \mathsf{Dirichlet}(\alpha)$  with  $\alpha$  being a proximity measure
- 2 For each charging station i:
  - **1** Draw transition coefficient  $\beta_i \sim N\left(0,1\right)$
  - **2** Draw standard deviation  $\sigma_i \sim HalfCauchy(0.1)$
  - **3** Draw demand for initial time step  $y_i^0 \sim N(\mu, 1)$
  - **4** Calculate censored demand  $y_i^{*0} = \min(Cap_i, y_i^0)$
- **3** Draw coefficients  $\tau \sim N\left(\mathbf{0},\mathbf{1}\right)$
- **4** For each charging station i:
  - 1 For each time step t:
    - 1 Draw non-censored demand:

$$\mathbf{y_i^t} \sim N\left(\beta_i y_i^{t-1} + \tau X_i^t + \sum_{j!=i} p_{i,j} (y_j^{t-1} - y_j^{*t-1}), \sigma_i^2\right)$$

**6** Calculate censored demand  ${y_i}^{*t} = \min(Cap_i, y_i^t)$ 

#### Kaggle datasets

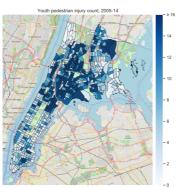




- Find a cool/crazy dataset on Kaggle e.g. Avocado Prices (https://www.kaggle.com/neuromusic/avocado-prices)
- Formulate a research question(s) e.g. how do the temporal dynamics of price change across regions
- Propose a PGM to model the data based on domain knowledge and assumptions of research question - e.g. hierarchical temporal model
- Fit model, extract results, revise model and data, propose model extensions, formulate new research questions... and iterate...

### **Bayesian Spatial Count Models**





ullet For each area i, the observed count  $y_i$  follows a Negative Binomial distribution

$$y_i \sim \mathsf{NB}(r, p_i),$$
 (likelihood) 
$$p_i = \frac{\exp(\psi_i)}{1 + \psi_i},$$
 (link function) 
$$\psi = \boldsymbol{\beta}^T \mathbf{X}_i + \phi_i,$$
 (linear function of inputs  $\mathbf{X}_i$ )

where  $\mathbf{X}_i$  are the characteristics of the area and  $\phi_i$  is a correlated error term

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  - Neural networks?
  - Hierarchical modelling?

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- Research topic 1: How to have non-linear dependencies on inputs X<sub>i</sub>?
  - Neural networks?
  - Hierarchical modelling?
- Research topic 2: How to capture spatial correlations between areas?
  - Capture correlations through the noise term?<sup>1</sup>

$$\exp(\tau \mathbf{W}) \boldsymbol{\phi} \sim \mathcal{N}(0, \sigma^2 \mathbf{I})$$

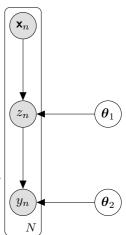
Neural networks? E.g., convolutions or graph convolution neural nets?

<sup>&</sup>lt;sup>1</sup>Reference: https://arxiv.org/abs/2007.03681

# Do Bayesian Neural Networks Need To Be Fully Stochastic?



- Bayesian neural network is a very active area of research
- Generative process
  - **1** Draw NN parameters  $\theta \sim \mathcal{N}(\theta | \mathbf{0}, \tau \mathbf{I})$
  - **2** For the  $n^{th}$  observation
    - a Draw target  $y_n \sim \mathcal{N}(y_n|f_{\mathsf{nnet}}(\mathbf{u}_n), \sigma^2)$
- But do they need to be fully stochastic<sup>2</sup>?
- Research Ideas:
  - ... can we only keep a fraction of  $\theta_1$  stochastic?
  - ... how do these work with time series?
  - ... analyse the Bayesian approach to neural networks.



<sup>&</sup>lt;sup>2</sup>https://arxiv.org/pdf/2211.06291.pdf