Topic Modeling - Latent Dirichlet Allocation

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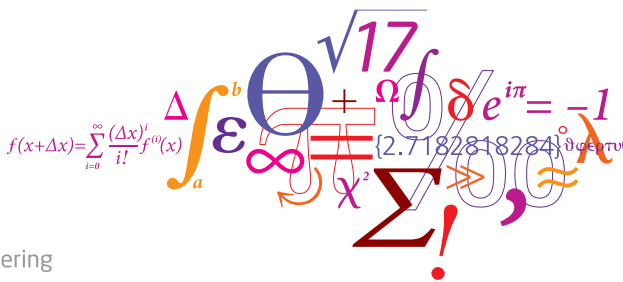


DTU Management Engineering
Department of Management Engineering

- Introduction
- In a nutshell
  - Example with traffic incidents
- Topic Modeling
- Concluding remarks

Introduction

- Sometimes some variables of our **x** are in form of human language
- Often called **unstructured** data
- How to bring it into statistical models?

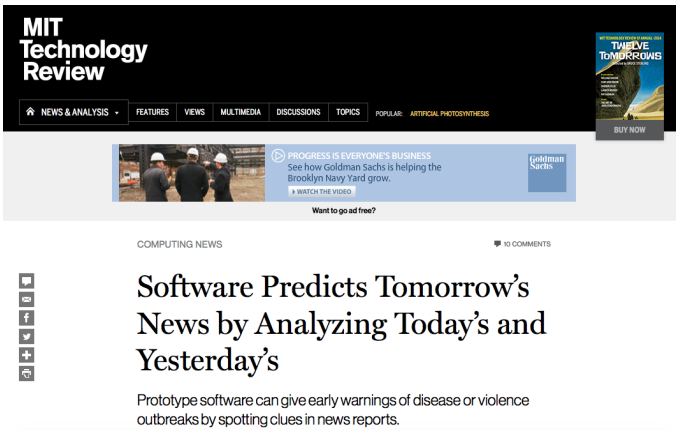
Introduction

- Our goal is to have

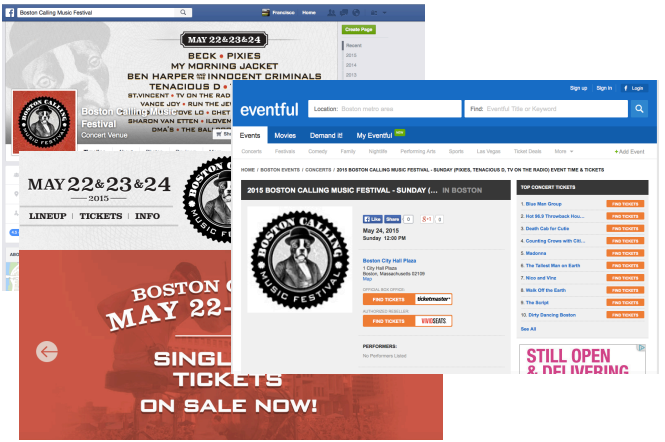
$$\hat{y} = f(\mathbf{x}_{text}, \mathbf{x}_L)$$

  where $\mathbf{x}_{text}$ are variables coming from text data, and $\mathbf{x}_L$ are non-textual variables
- we need to make $\mathbf{x}_{text}$ usable in any multivariate model discussed in this course

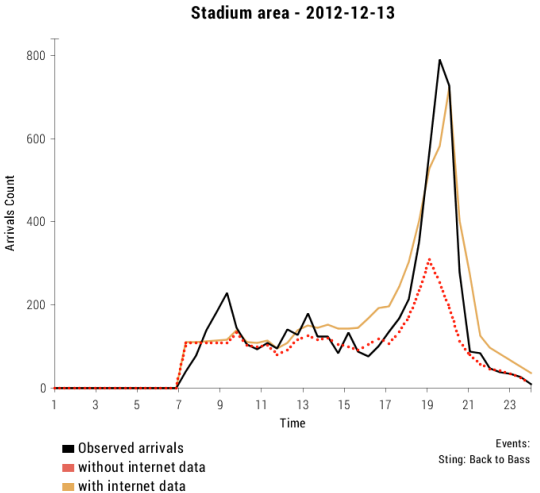
Motivation: News



Motivation: Special events



Motivation: Special events



Motivation: traffic incidents

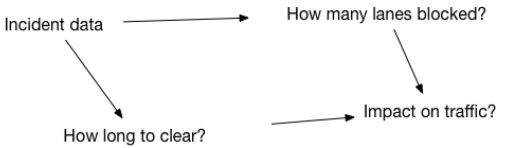
Motivation: traffic incidents

How many lanes blocked?

Impact on traffic?

How long to clear?

Motivation: traffic incidents



Motivation: traffic incidents

Motivation: traffic incidents

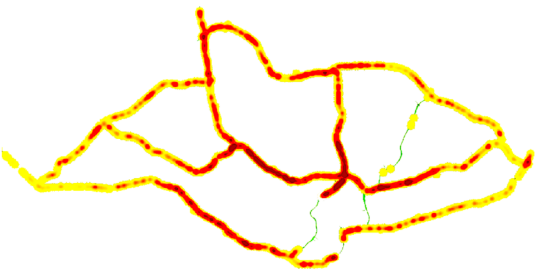
Incident report

```
Location type: 3
Lane blockage: Lane 1, Shoulder blocked
Down point: 20.32
Congestion status: 0
Queue length: 500m
Start time: 2010-08-20 22:50:01
End time: 2010-08-20 23:31:45
Number of vehicles: 2
2250hrs - TP Joe X spots an accident. car and
          bike involved.
2255hrs - Passers-by shift the bike to the
          shoulder.
2300hrs - Ambulance arrives at location. LTM
          arrives at location.
2309hrs - Ambulance conveys rider to National
          University Hospital.
2310hrs - TP arrives at location.
2311hrs - Notify by LTM the rider is seriously
          injured. The accident involves a car
          and bike.
2331hrs - TP requests RC and LTM to resume
          patrolling. All other vehicles move
          off. Shoulder clear.
```

Communications sequence

In a nutshell: an example

- **Context:** Singapore Expressways, 2010-2012



- **Objective:** Predict duration of each incident
- **Data:** Logs of all incidents, 2010-2012

In a nutshell: an example

$$y = f(\mathbf{x}_{text}, \mathbf{x}_L)$$

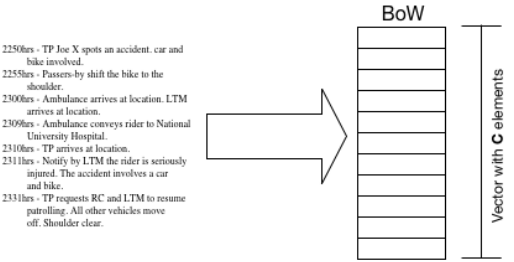
with

$$\mathbf{x}_L = \begin{bmatrix} \mathbf{x}_{expressway\_ID} \\ \mathbf{x}_{num\_block\_lanes} \\ \mathbf{x}_{time\_of\_day} \\ \mathbf{x}_{day\_of\_week} \\ \mathbf{x}_{type\_of\_day} \\ \mathbf{x}_{congestion\_status} \\ \mathbf{x}_{num\_vehicles} \end{bmatrix}$$

$$\mathbf{x}_{text} = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$$

In a nutshell: an example

- A document has a set of words, from a dictionary with size C
- How to represent it?
  - The bag-of-words (BoW) representation
  - Dimension C
  - Vector of word frequencies



BoW Representation

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2255hrs - Passers-by shift the bike to the shoulder.
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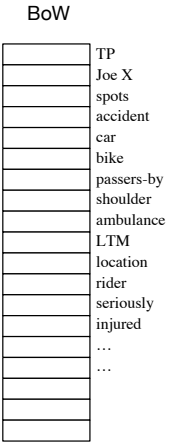

BoW Representation

2250hrs - **TP Joe X spots** an **accident**. **car** and **bike** involved.
2255hrs - **Passers-by** shift the **bike** to the **shoulder**.
2300hrs - **Ambulance** arrives at **location**. **LTM** arrives at **location**.
2309hrs - **Ambulance** conveys **rider** to National University Hospital.
2310hrs - **TP** arrives at **location**.
2311hrs - Notify by **LTM** the **rider** is **seriously injured**. The **accident** involves a **car** and **bike**.
2331hrs - **TP** requests RC and **LTM** to resume patrolling. All other vehicles move off. Shoulder clear.

Document

3	TP
1	Joe X
1	spots
2	accident
2	car
3	bike
1	passers-by
1	shoulder
2	ambulance
3	LTM
3	location
2	rider
1	seriously
1	injured
…	…
…	…
…	
…	

BoW representation - data preparation

- Replace words by their *stems* (e.g. injury, injured, injuries $\rightarrow$ injur)
- Remove words from the **stopwords** list (e.g. *and, or, that, it...*)
- The dictionary is a collection of C words (i.e. their stems)
- Calculate frequency of all words (ignore word sequence)

From text to regression

- We could use BoW frequencies directly in a model:

$$y = f(\mathbf{x}_{bow}, \mathbf{x}_L)$$

  where $\mathbf{x}_{bow}$ is the BoW vector

  - Dimensionality=C+L, can be huge (C$\simeq 10^6$ for English)!
  - Many words for same concept (e.g. synonyms, acronyms, abbreviations)
  - Many words related (e.g. “ambulance", “injury", “hospital")
  - Many words irrelevant (e.g. proper names)

- Manual work (e.g. replace synonyms with same word)?
  - VERY time consuming and prone to error!

Topic modeling

- Each document describes a set of K concepts (or **topics**), $K << C$
- Each topic is *latent*, i.e. we cannot observe it

In a nutshell: an example

Some examples of topics from our incident reports dataset

topic #1 - 0.085*tp + 0.071*convei + 0.070*ab + 0.064*tow + 0.062*hosp + 0.056*rider + 0.048*bike
topic #2 - 0.054*ir + 0.049*confirm + 0.045*congest + 0.034*traffic + 0.027*case + 0.023*soe + 0.018*heavi
topic #3 - 0.001*veh + 0.001*nch + 0.001*spot + 0.000*rc + 0.000*clear + 0.000*ltm + 0.000*1ln
topic #6 - 0.138*tow + 0.122*noinjur + 0.092*veh + 0.080*present + 0.066*clear + 0.061*owner + 0.047*nodamag
topic #11 - 0.101*assist + 0.056*taxi + 0.045*notifi + 0.032*requir + 0.029*came + 0.029*p3 + 0.027*passeng
topic #12 - 0.076*polic + 0.041*otm + 0.033*2nd + 0.033*last + 0.023*div + 0.022*tm1 + 0.016*1st
topic #13 - 0.077*damag + 0.071*call + 0.043*skid + 0.034*tp + 0.029*near + 0.028*self + 0.027*vig
topic #14 - 0.072*left + 0.057*activ + 0.050*tp + 0.045*2ln + 0.040*scdf + 0.035*open + 0.029*spill
topic #15 - 0.160*minor + 0.049*exchang + 0.047*particular + 0.032*advis + 0.029*refus + 0.025*detail + 0.013*involv2
topic #18 - 0.112*report + 0.093*rc + 0.063*tm + 0.041*btw + 0.036*lorri + 0.034*bef + 0.032*actv

Topic modeling

- Each document describes a set of K concepts (or **topics**), $K << C$
- Each topic is *latent*, i.e. we cannot observe it
- Examples of topic “labels":
  - Medical assistance
  - Significant damage
  - Low damage
  - Oil spillage
  - ...

In a nutshell: an example

Medical assistance

topic #1 - 0.085*tp + 0.071*convei + 0.070*ab + 0.064*tow + 0.062*hosp + 0.056*rider + 0.048*bike
topic #2 - 0.054*ir + 0.049*confirm + 0.045*congest + 0.034*traffic + 0.027*case + 0.023*soe + 0.018*heavi
topic #3 - 0.001*veh + 0.001*nch + 0.001*spot + 0.000*rc + 0.000*clear + 0.000*ltm + 0.000*1ln
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topic #18 - 0.112*report + 0.093*rc + 0.063*tm + 0.041*btw + 0.036*lorri + 0.034*bef + 0.032*actv

In a nutshell: an example

Significant damage

topic #1 - 0.085*tp + 0.071*convei + 0.070*ab + 0.064*tow + 0.062*hosp + 0.056*rider + 0.048*bike
topic #2 - 0.054*ir + 0.049*confirm + 0.045*congest + 0.034*traffic + 0.027*case + 0.023*soe + 0.018*heavi
topic #3 - 0.001*veh + 0.001*nch + 0.001*spot + 0.000*rc + 0.000*clear + 0.000*ltm + 0.000*1ln
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topic #18 - 0.112*report + 0.093*rc + 0.063*tm + 0.041*btw + 0.036*lorri + 0.034*bef + 0.032*actv

In a nutshell: an example

Low damage

topic #1 - 0.085*tp + 0.071*convei + 0.070*ab + 0.064*tow + 0.062*hosp + 0.056*rider + 0.048*bike
topic #2 - 0.054*ir + 0.049*confirm + 0.045*congest + 0.034*traffic + 0.027*case + 0.023*soe + 0.018*heavi
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topic #18 - 0.112*report + 0.093*rc + 0.063*tm + 0.041*btw + 0.036*lorri + 0.034*bef + 0.032*actv

In a nutshell: an example

Oil spillage

topic #1 - 0.085*tp + 0.071*convei + 0.070*ab + 0.064*tow + 0.062*hosp + 0.056*rider + 0.048*bike
topic #2 - 0.054*ir + 0.049*confirm + 0.045*congest + 0.034*traffic + 0.027*case + 0.023*soe + 0.018*heavi
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In a nutshell: an example

Some vehicle types (*taxi*, *truck/lorry*)

topic #1 - 0.085*tp + 0.071*convei + 0.070*ab + 0.064*tow + 0.062*hosp + 0.056*rider + 0.048*bike
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Topic modeling

- Can we quantify **how much** of each topic exists in a document?
  - E.g. 20% medical assistance, 50% significant damage, 30% oil spillage
  - A document would be a **linear combination** of K topics!
  - We go from C to K $\rightarrow$ **dimensionality reduction**

$$\mathbf{x}_{text} = \begin{bmatrix} \mathbf{x}_{topic\_1} \\ \mathbf{x}_{topic\_2} \\ \dots \\ \mathbf{x}_{topic\_K} \end{bmatrix}$$

Topic modeling

- Each topic:
  - Itself a vector with dimensionality C (i.e. one entry per word in dictionary)
  - Inferred from an unsupervised process (called *Latent Dirichlet Allocation*, LDA)
  - One can see a topic as a *prototypical document*

In a nutshell: an example

Topic assignment in our example text

2250hrs - TP Joe X spots an accident. car and bike involved.
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0.43	topic #1
0.07	topic #3
...	...
0.10	topic #12
0.12	topic #18

Latent Dirichlet Allocation (LDA)

Generative model without probability distributions yet

We want to generate a set of I documents ($i = 1, 2, ..., I$), each one being a sequence of words, $\mathbf{w}_i$:

1. For each topic k (k=1...K), there is a vector $\boldsymbol{\phi}_k$ of C words (dictionary)

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1. For each topic k (k=1...K), there is a vector $\boldsymbol{\phi}_k$ of C words (dictionary)
2. For each document i:
   a) We assign it a vector $\boldsymbol{\theta}_i$ of K topic proportions
   b) For each of the words, j ($j = 1, 2, ..., ||\mathbf{w}_i||$), we have
      i) A topic assignment, $z_{i,j}$ ($z_{i,j} \in \{1, 2, ..., K\}$) according to $\boldsymbol{\theta}_i$
      ii) A word, $w_{i,j}$, according to $\boldsymbol{\phi}_{z_{i,j}}$

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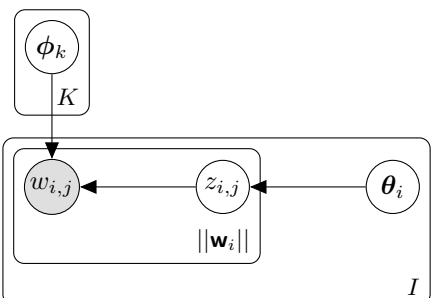


Some background

- Categorical distribution, Cat($\boldsymbol{\theta}$)
  - With A categories

$$p(x = a|\theta) = \theta_a$$

    where $\theta = [\theta_1, \theta_2, ..., \theta_A]$

$$\sum_{a=1}^{A} \theta_a = 1$$

  - When A=2, it's called *bernoulli* distribution
  - Equivalent to *multinomial* with 1 trial

Some background

- Dirichlet distribution
  - Random vector, $\boldsymbol{\theta}$, of dimension A;

$$\boldsymbol{\theta} \sim \text{Dir}(\boldsymbol{\alpha})$$

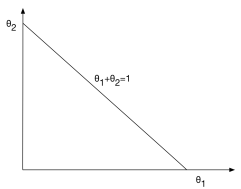
  - $\alpha_1, \alpha_2, ..., \alpha_A$, with $\alpha_i > 0$

$$p(\boldsymbol{\theta}|\boldsymbol{\alpha}) = \text{Dir}(\boldsymbol{\alpha}) = \frac{\Gamma\Big(\sum_{i=1}^{A} \alpha_i\Big)}{\prod_{i=1}^{A} \Gamma(\alpha_i)} \prod_{i=1}^{A} \theta_i^{\alpha_i - 1}$$

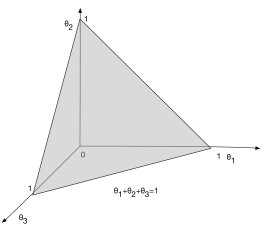
    where $\sum_{i=1}^{A} \theta_i = 1$ and $\theta_i \in [0, 1]$.
  - $\Gamma(u)$ is the gamma function, a generalization of the factorial function, such that $\Gamma(u+1) = u\Gamma(u)$

Some background - Dirichlet distribution

- It can be viewed as a probability distribution on an $A-1$ dimensional simplex



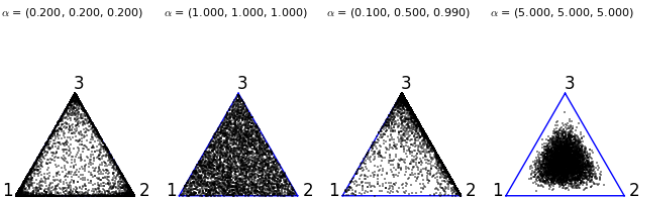
1-D Simplex (segment)



2-D Simplex (triangle)

Topic modeling

- A sample from the Dirichlet distribution with dimension A is a vector $\boldsymbol{\theta}$, such that $\sum_{i=1}^{A} \theta_i = 1$



- Perfect to generate arguments for the categorical distribution!

Playtime!

- Open "08. LDA.ipynb" notebook
- Do part 1
- Estimated time: 15 minutes

Latent Dirichlet Allocation (LDA)

1. For each topic k, there is a vector of C words, $\boldsymbol{\phi}_k$, such that $p(\boldsymbol{\phi}_k|\boldsymbol{\beta}) = \text{Dir}(\boldsymbol{\beta})$

Latent Dirichlet Allocation (LDA)

1. For each topic k, there is a vector of C words, $\boldsymbol{\phi}_k$, such that $p(\boldsymbol{\phi}_k|\boldsymbol{\beta}) = \text{Dir}(\boldsymbol{\beta})$
2. For each document i:
    a) There is a vector of K topics, $\boldsymbol{\theta}_i$, such that $p(\boldsymbol{\theta}_i|\boldsymbol{\alpha}) = \text{Dir}(\boldsymbol{\alpha})$
    b) For each of the words, j, in each document i ($j = 1, 2, ..., ||\mathbf{w}_i||$), we have
        i) A topic assignment, $z_{i,j}$ ($z_{i,j} \in \{1, 2, ..., K\}$) such that $p(z_{i,j}|\boldsymbol{\theta}_i) = \text{Cat}(\boldsymbol{\theta}_i)$
        ii) A word, $w_{i,j}$, such that $p(w_{i,j}|\boldsymbol{\phi}_{z_{i,j}}) = \text{Cat}(\boldsymbol{\phi}_{z_{i,j}})$

Latent Dirichlet Allocation (LDA)

1. For each topic k, there is a vector of C words, $\boldsymbol{\phi}_k$, such that $p(\boldsymbol{\phi}_k|\boldsymbol{\beta}) = \text{Dir}(\boldsymbol{\beta})$
2. For each document i:
   a) There is a vector of K topics, $\boldsymbol{\theta}_i$, such that $p(\boldsymbol{\theta}_i|\boldsymbol{\alpha}) = \text{Dir}(\boldsymbol{\alpha})$
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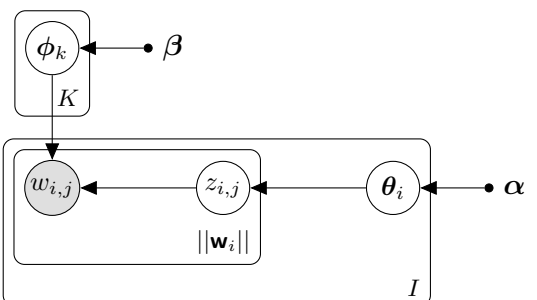


Playtime!

- Open "08. LDA.ipynb" notebook
- Do part 2
- Estimated time: 30 minutes

Topic modeling - LDA

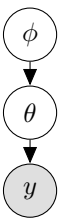
Joint distribution:

$$p(\mathbf{W}, \mathbf{Z}, \boldsymbol{\Theta}, \boldsymbol{\Phi} | \boldsymbol{\alpha}, \boldsymbol{\beta}) = \left(\prod_{i=1}^{I} p(\boldsymbol{\theta}_i | \boldsymbol{\alpha}) \prod_{j=1}^{||\mathbf{w}_i||} p(w_{i,j} | \boldsymbol{\phi}_{\mathbf{z}_{i,j}}) \, p(\mathbf{z}_{i,j} | \boldsymbol{\theta}_i) \right) \prod_{k=1}^{K} p(\boldsymbol{\phi}_k | \boldsymbol{\beta})$$

- No analytical solution
- Common implementations with Gibbs sampling or Variational Bayes

Discrete Latent Variables in STAN

- In its present version, one cannot explicitly use discrete latent variables in STAN
- Limitation with its MCMC inference process (explained in upcoming class)
- For example



$$\phi \sim \text{Beta}(2, 1)$$

$$\theta \sim \text{Bernoulli}(\phi)$$

$$y \sim \mathcal{N}(k\theta, 10)$$

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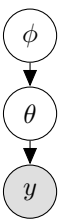

$$\phi \sim \text{Beta}(2, 1)$$

$$\theta \sim \text{Bernoulli}(\phi)$$

$$y \sim \mathcal{N}(k\theta, 10)$$

$$p(y, \theta, \phi) = p(y|\theta)\, p(\theta|\phi)\, p(\phi)$$

Discrete Latent Variables in STAN

- It would allow, however

$$p(y, \phi)$$

Discrete Latent Variables in STAN

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$$p(y, \phi) = \sum_{\theta} p(y|\theta)p(\theta|\phi)p(\phi)$$

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- We "get rid" of the latent variable, but use its distribution correctly
- How do we do this in STAN?

Discrete Latent Variables in STAN

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- Marginalization of the latent variable
- We "get rid" of the latent variable, but use its distribution correctly
- How do we do this in STAN?
- STAN takes care of the priors (because all their parameters are known!)
- So we need to focus on the likelihood

$$L(y) = p(y|\phi) = \sum_{\theta} p(y|\theta)p(\theta|\phi)$$

Discrete Latent Variables in STAN

- Notice that:

$$\theta \sim \text{Bernoulli}(\phi) \implies p(\theta|\phi) = \phi^{\theta}(1-\phi)^{(1-\theta)}$$

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$$= \phi\mathcal{N}(y|k, 10) + (1-\phi)\mathcal{N}(y|0, 10)$$

- STAN uses **log-probabilities**, so we should have:

$$LL(y) = \log\left(\phi\mathcal{N}(y|k, 10) + (1-\phi)\mathcal{N}(y|0, 10)\right)$$

Discrete Latent Variables in STAN

$$LL(y) = \log\Big(\phi\mathcal{N}(y|k,10) + (1-\phi)\mathcal{N}(y|0,10)\Big)$$

- In fact, you'll need to use the function *log_sum_exp(vector **v**): real*
  - Receives a vector **v** of real values
  - Transforms each element, $\mathbf{v}_i$, of the vector with $e^{\mathbf{v}_i}$
  - Calculates the "log sum" to all elements $\mathbf{v}_i$

Discrete Latent Variables in STAN

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  - Receives a vector **v** of real values
  - Transforms each element, $\mathbf{v}_i$, of the vector with $e^{\mathbf{v}_i}$
  - Calculates the "log sum" to all elements $\mathbf{v}_i$
  - So, to use it, we need to apply $\log(\mathbf{v}_i)$ at each element
  - The model from above in STAN:

```
model = {
    phi ~ beta(2,1); // prior

    // likelihood
    real gamma[2];
    gamma[1] = log(phi) + normal_lpdf(y| k, 10);
    gamma[2] = log(1-phi) + normal_lpdf(y| 0, 10);
    target += log_sum_exp(gamma);
}
```

Playtime!

- Open "08. LDA.ipynb" notebook
- Do part 3
- Estimated time: 1 hour

Back to the nutshell: our example

$$y = f(\mathbf{x}_{text}, \mathbf{x}_L)$$

with

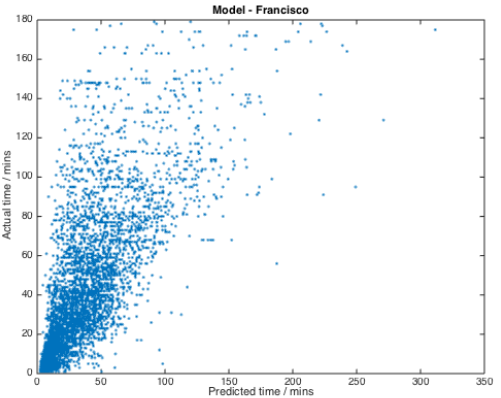
$$\mathbf{x}_L = \begin{bmatrix} \mathbf{x}_{expressway\_ID} \\ \mathbf{x}_{num\_block\_lanes} \\ \mathbf{x}_{time\_of\_day} \\ \mathbf{x}_{day\_of\_week} \\ \mathbf{x}_{type\_of\_day} \\ \mathbf{x}_{congestion\_status} \\ \mathbf{x}_{num\_vehicles} \end{bmatrix}$$

$$\mathbf{x}_{text} = \begin{bmatrix} \mathbf{x}_{topic\_1} \\ \mathbf{x}_{topic\_2} \\ \dots \\ \mathbf{x}_{topic\_K} \end{bmatrix}$$

Incident Duration Prediction

- Data:
  - 10000+ incidents
  - Each report modeled using 25 topics
  - Original information on time of day, location etc.
- Goal:
  - regression model for incident duration
- An accurate prediction for the duration is important for modelling the impact on traffic

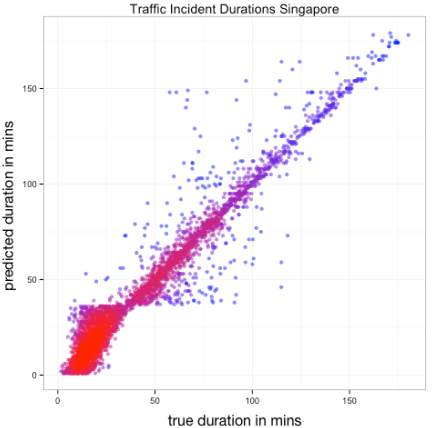
Non-parametric model (R^2=0.461)



- Predicted vs. Actual durations for test data.
- Without textual data, $R^2 = 0.230$

Incident Duration Prediction (R^2=0.603)

A slightly more complicated (non-parametric) model...



Concluding remarks

- In practice (e.g. sklearn), documents for LDA use BoW representation
  - Mechanics is exactly the same
  - We only mention so you know how to do it :-)
- LDA is super-optimized today - Variational approximation algorithm

Concluding remarks

- Topic modeling falls in the general family of *mixed membership* models
  - Each document is a mixture of components
- Similarities with Principal Components Analysis (PCA), Factor Analysis...
- What you saw was **unsupervised**...
  - What would be the supervised version?

Reference material

Introduction to Probabilistic Topic Models.
David Blei, 2011 (pp 1-8)
www.cs.princeton.edu/~blei/papers/Blei2011.pdf

More detail:
Probabilistic Topic Models, Mark Steyvers and Tom Griffiths
In T. Landauer, D McNamara, S. Dennis, and W. Kintsch (eds), Latent Semantic Analysis: A Road to Meaning. Laurence Erlbaum
psiexp.ss.uci.edu/research/papers/SteyversGriffithsLSABookFormatted.pdf

Appendix

- Another example: Demand prediction for special events

Demand prediction for special events

- **Context:**
  - Singapore November 2011- February 2012
  - 3 venues (Singapore Indoor Stadium, Kallang Theatre, Singapore Expo)
- **Objective:**
  - Predict number of public transport arrivals (in 30 min blocks)
  - Useful for operations planning (e.g. reallocate drivers)
- **Data:**
  - EZLink (tap-in/tap-out) data, November 2011-February 2012
  - 1500+ Event data from Eventful.org, Facebook, Google (600+ for selected venues)

Demand prediction for special events

- Model variables
  - Time of day (discrete and continuous), day of week, type of day
  - Time to next event/time since event started (in 30 min slots)
  - Number of Google hits, Facebook likes, Wikipedia page (dummy)
  - Topics from textual description ($K = 25$)
- Regression model(s)
  - Linear regression
  - Kernel method (Gaussian Processes)

Demand prediction for special events