

# The Recipe Approximation Problem


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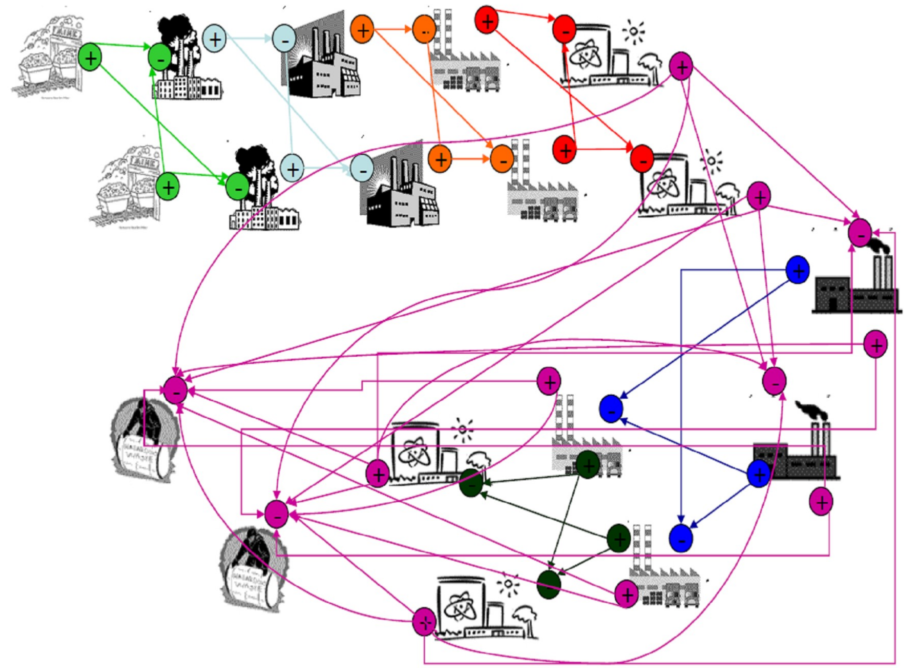
# Overview

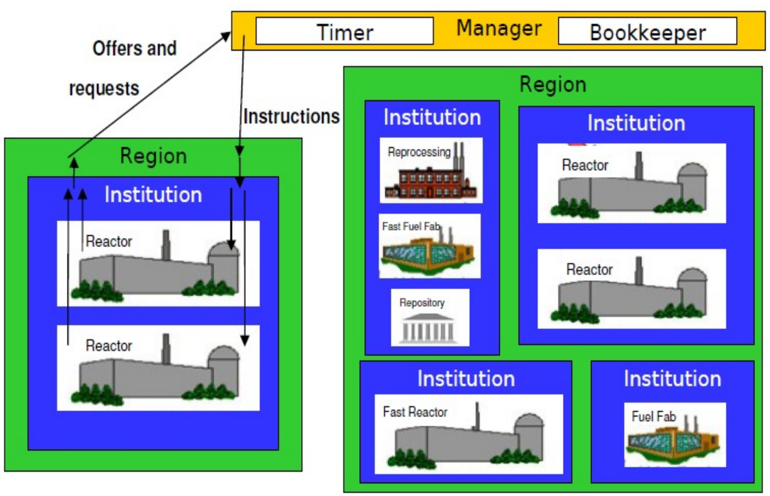
- Architecture Overview
- Recipe Approximation Problem
- Linear Approximation Formulation
- Tests
- Proposed Future Formulations

# GENIUS V2 Overview

GENIUSv2
(Global Evaluation of Nuclear Infrastructure Utilization Scenarios)

- Facilities in the nuclear industry are owned by distinct governments and institutions which buy sell and trade nuclear material.

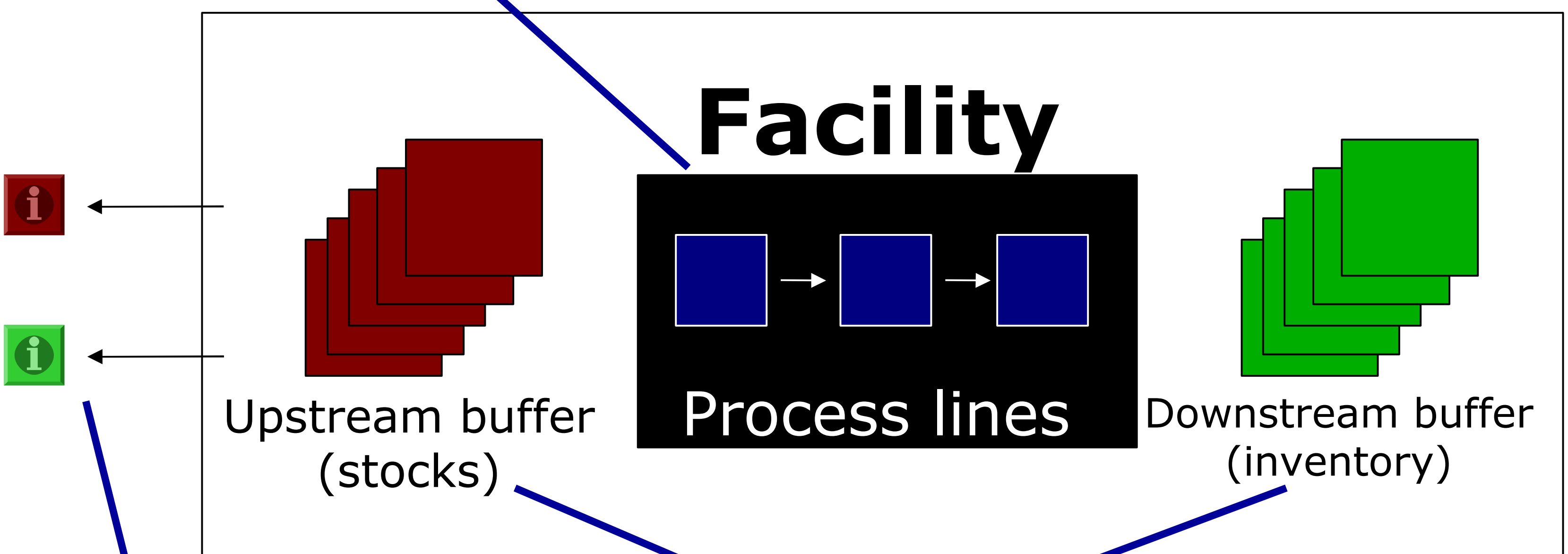





- In some closed fuel cycle simulations, spent fuel Separations and Fuel Refabrication facilities fabricate fresh thermal MOX fuel from spent UOX fuel.

# Facilities as Black Boxes with Clear Interfaces

Process lines store the material being operated upon (converted, enriched, etc.)




Messages are sent to offer or request materials or services.

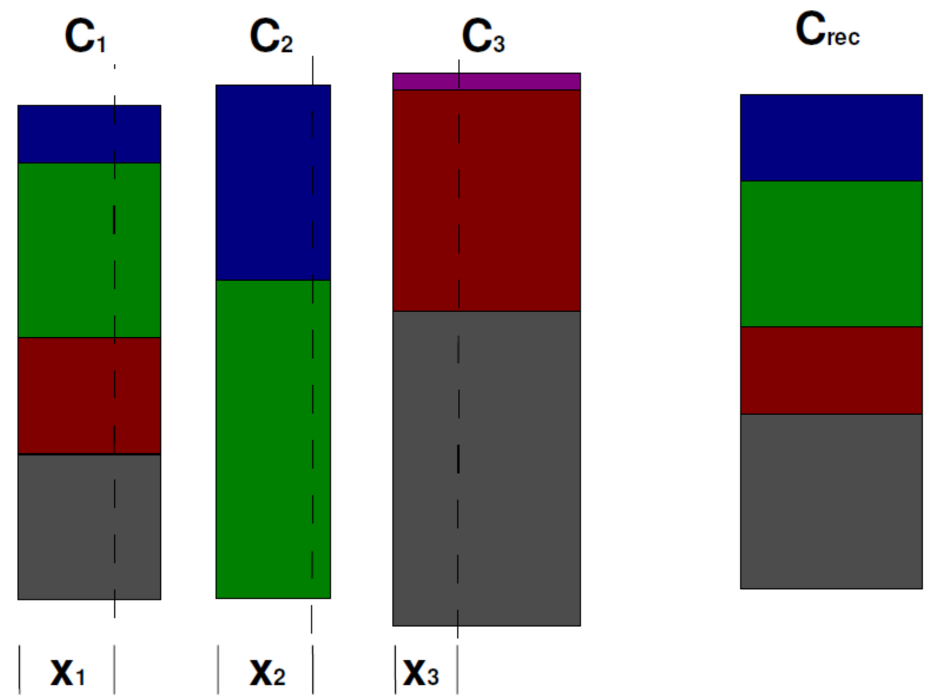
Buffers store materials waiting to be processed or sent to another facility.

# Choosing New Fuel Constituents from Available Separated Streams

Available "barrels" of recycle material

Requested fuel recipe

Must preserve stoichiometry of components after they are separated!




Choose fractions to attempt matching of target recipe w/r/t stoichiometry, total mass, and total neutronics.

# Employ linear programming to minimize recipe deviation

**Matrix component $M_{bi}$ is the mass of isotope $i$ in barrel $b$.**

***e* is vector of ones of length *B*, the number of barrels; *T* is the transpose operator.**

$$
\begin{aligned}
\min_{x,y} \quad & e^T y \\
\text{subject to} \quad & y = |Mx - r| \\
& 0 \le x_b \le 1, \ \forall b
\end{aligned}
$$

**Minimize deviation sum of isotope-wise deviation from recipe, *r*.**

**Choose a fraction, $x_b$, of each barrel to use.**

# Include Constraints to Guarantee Neutronics Performance

$$\min_{x,y} \quad \vec{c}^{\,T}\vec{y}$$

$$\text{subject to} \quad y = \left|M\vec{x} - \vec{r}\right|$$

Minimize sum of isotope-wise relative deviation from recipe, $r$.

$M_{bi}$ is the mass of isotope $i$ in barrel $b$

$$0 \le x_b \le 1, \forall b$$

Choose a fraction of each barrel, $x_b$

$$\left|\vec{w}^{\,T}\vec{x} - w_r\right| \le \varepsilon_w$$

Constrain the neutronics performance, $w$, to match the recipe within $\varepsilon_w$

$$\left|\vec{m}^{\,T}\vec{x} - m_r\right| \le \varepsilon_m$$

Constrain the total mass, $m$, to match the recipe within $\varepsilon_m$

# Normalization may Discourage Preference for Abundant Isotopes

Thought Experiment :

A neutronically unimportant isotope is very abundant in the requested recipe, while an important fissile isotope constitutes a comparatively small mass percent.

The naive formulation proposed in the previous slide preferentially selects barrels of the unimportant isotope.

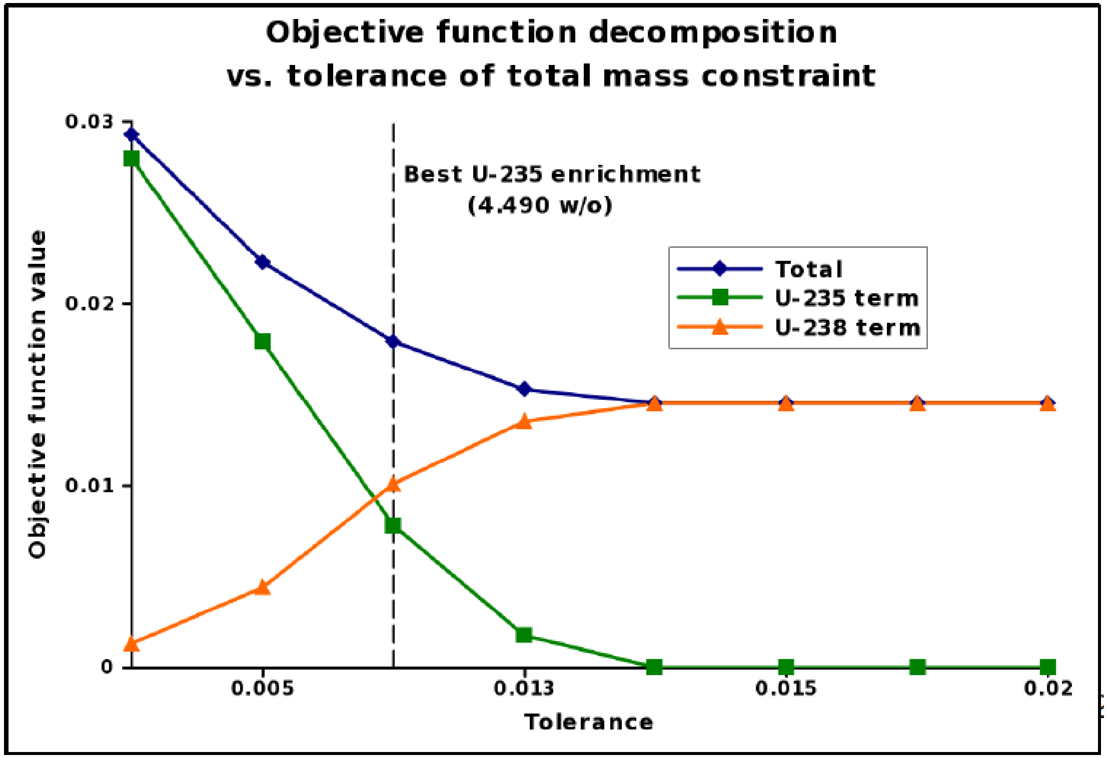
$$\begin{aligned} \min_{x,y} \quad & \vec{c}^{\,T}\vec{y} \\ \text{subject to} \quad & y = \left|M\vec{x} - \vec{r}\right| \\ & 0 \le x_b \le 1, \forall b \\ & \left|\vec{w}^{\,T}\vec{x} - w_r\right| \le \varepsilon_w \\ & \left|\vec{m}^{\,T}\vec{x} - m_r\right| \le \varepsilon_m \end{aligned}$$

$$c_i = \begin{cases} 1/r_i & \text{if } r_i \neq 0 \\ 1/m_r & \text{if } r_i = 0 \end{cases}$$

Normalization factors within the objective function discourage the algorithm from preoccupation with the most abundant isotope.

# Normalization Avoids Preoccupation with $^{238}$U

**Normalized coefficients provide incentive to match U-235 exactly, given sufficient slack in total mass constraint.**



# Closed Fuel Cycle Scenario to Test Recipe Approximation

## Parameters for thermal MOX recycle scenario

| Parameter | Value | |
|---|---|---|
| | UOX PWR | MOX PWR |
| Start year | 2010 | |
| End year | 2109 | |
| Decay | Turned off | |
| Fuel cooling delay | None | |
| Separation plant requests | All used fuel | |
| Construction + license time | 5 years | |
| Operating time, $OT$ | 50 years | |
| Capacity factor, $CF$ | 0.90 | |
| Power capacity, $P$ [MWe] | 1050 | 1050 |
| Thermal efficiency, $\eta$ | 0.34 | 0.34 |
| Cycle time, $T$ [months] | 12 | 12 |
| Fuel burnup, $Bu$ [GWd/tHM] | 51 | 46 |
| Fuel batches per core, $N$ | 5 | 5 |

## Facility deployment for thermal MOX recycle scenario

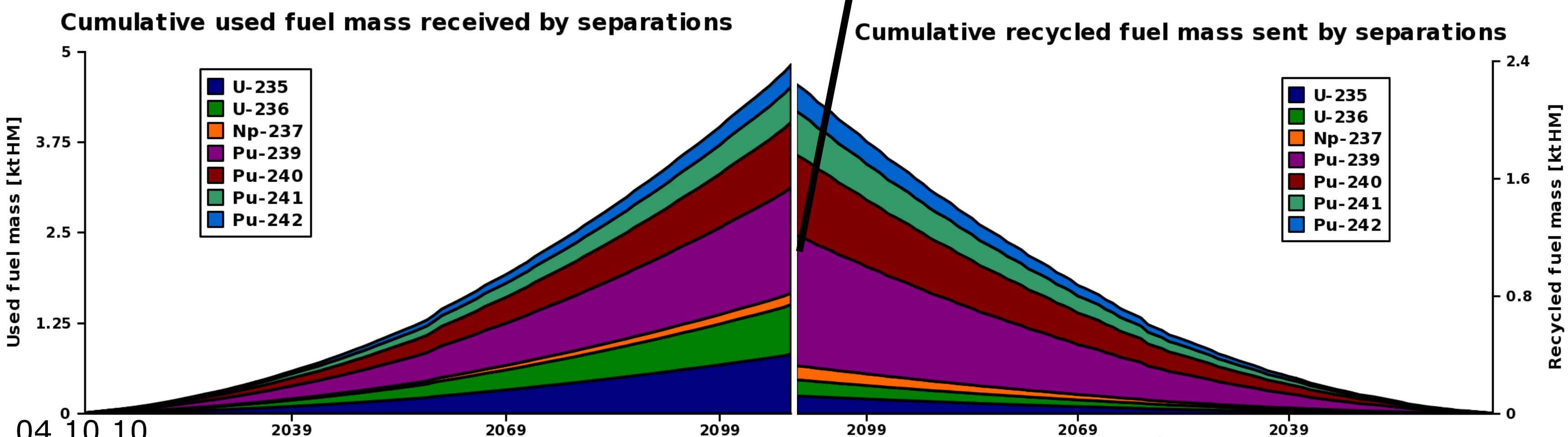
| Region | Institution | Facilities |
|---|---|---|
| 1 | 1 | 12 UOX PWRs in Jan. 2010<br>Linear growth: 850 MWe/year |
| | 2 | 1 UOX Fuel Fab |
| | 3 | 1 Separations |
| | 4 | 1 Mine/Mill |
| | 5 | 1 Conversion |
| | 6 | 1 Enrichment |
| 2 | 7 | 3 MOX PWRs in Jan. 2010<br>Linear growth: 142 MWe/year |
| | 8 | 1 MOX Fuel Fab |

# "Best Fit" Barrels Get Preference

| Stream | Isotope | Compositions [w/o] | | |
|---|---|---|---|---|
| | | Desired for fresh MOX | Available from spent UOX | Available from spent MOX |
| **Uranium** | U-232 | 0 | 4.27e-9 | 7.06e-8 |
| | U-233 | 0 | 2.14e-8 | 1.23e-6 |
| | U-234 | 2.00e-2 | 4.35e-4 | 3.16e-2 |
| | **U-235** | **0.822** | **0.756** | **0.517** |
| | U-236 | 0.613 | 0.599 | 0.608 |
| | U-238 | 98.5 | 98.6 | 98.8 |
| **Neptunium-Plutonium** | Np-237 | 5.03 | 5.25 | 4.17 |
| | Pu-238 | 2.50 | 2.45 | 5.79 |
| | **Pu-239** | **50.4** | **47.0** | **39.3** |
| | Pu-240 | 23.9 | 23.8 | 27.2 |
| | **Pu-241** | **11.2** | **14.1** | **14.1** |
| | Pu-242 | 6.99 | 7.37 | 9.48 |
| | Pu-244 | 0 | 2.45e-4 | 2.10e-4 |

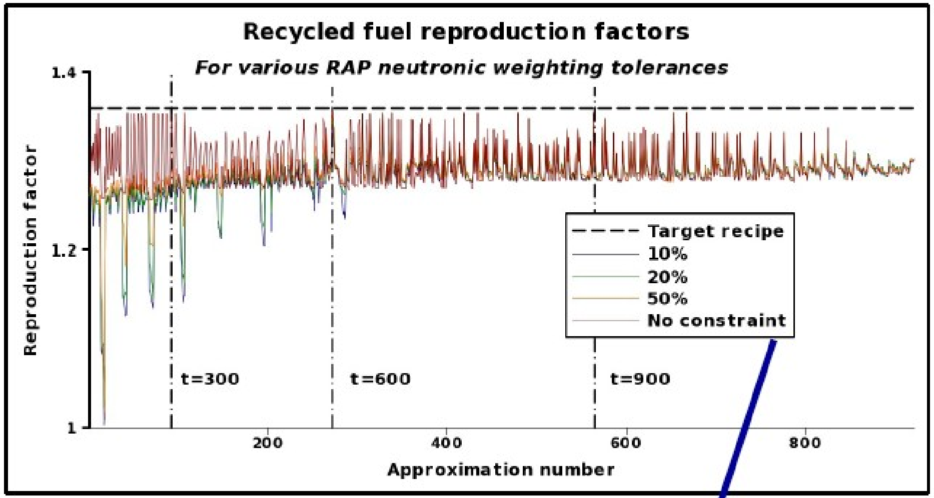
Barrels from Np-Pu stream in months with no spent MOX "pollutants" have nearly correct ratio of isotopes.

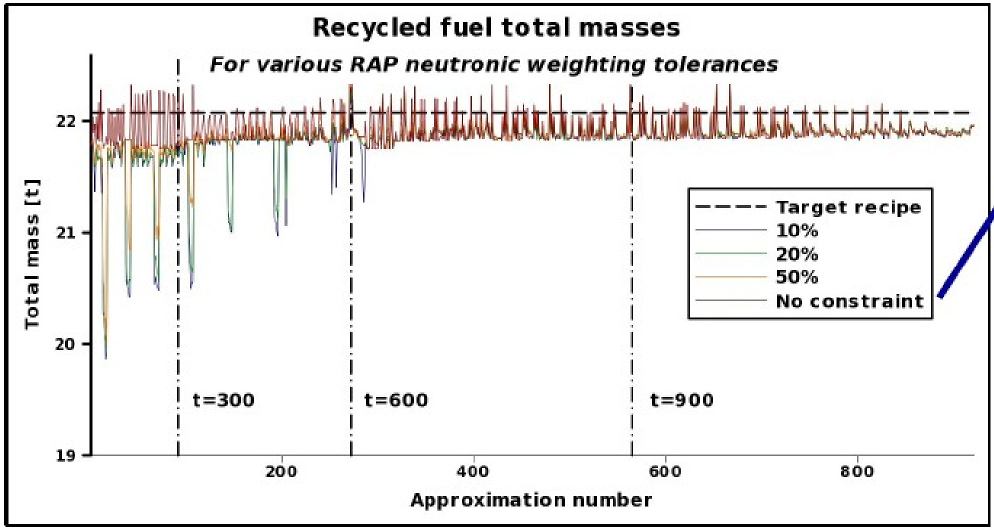
Higher fraction of available Np-Pu than available U used in approximations.



# Thermal MOX Recycle Tests: Best Results w/o Neutronics Constraint

Variability caused by number, size, and composition of available barrels.




No neutronics constraint used in scenario plotted in previous slide.

# Future Proposed Formulations

**Would like to use *η* as proxy for neutronic behavior. Unfortunately, a poor approximation results.**

**Note** $$\eta = \frac{\sum_{i=1}^{I} \nu^i \sigma_f^i n^i}{\sum_{i=1}^{I} \sigma_a^i n^i} = \frac{\sum_{i=1}^{I} \nu^i \sigma_f^i N^i / V}{\sum_{i=1}^{I} \sigma_a^i N^i / V} = \frac{\cancel{\frac{1}{V}} \sum_{i=1}^{I} \nu^i \sigma_f^i N^i}{\cancel{\frac{1}{V}} \sum_{i=1}^{I} \sigma_a^i N^i} = \frac{\sum_{i=1}^{I} \nu^i \sigma_f^i N^i}{\sum_{i=1}^{I} \sigma_a^i N^i}$$

**Rearrange neutron weight constraint for case $\varepsilon_w$=0 (strictest choice)**

$$\sum_{b=1}^{B} w_b x_b = w_r$$

**Substitute *w* = *η***

$$\sum_{b=1}^{B} \left( \frac{\sum_{i=1}^{I} \nu^i \sigma_f^i N^{i,b}}{\sum_{i=1}^{I} \sigma_a^i N^{i,b}} \right) \Bigg|_b x_b \neq \frac{\sum_{i=1}^{I} \nu^i \sigma_f^i \sum_{b=1}^{B} N^{i,b} x_b}{\sum_{i=1}^{I} \sigma_a^i \sum_{b=1}^{B} N^{i,b} x_b}$$

**The *η* of the whole does not equal a weighted sum of the *η*'s of its parts (*η* not an extrinsic property)!**

# Future Proposed Formulations

$$\begin{aligned} \min_{x,y} \quad & c^T y + e^T z \\ \text{subject to} \quad & y = |Mx - r| \\ & z = r_w - Wx \\ & 0 \le x_b \le 1, \ \forall b \\ & |mx - m_r| \le \epsilon_m \end{aligned}$$

**Dual infeasible. Also (because?) mass matching competes with neutronics matching.**

**Idea:**

$$\begin{aligned} \min_{x,y} \quad & e^T z \\ \text{subject to} \quad & z = |r_w - Wx| \\ & 0 \le x_b \le 1, \ \forall b \\ & |mx - m_r| \le \epsilon_m \end{aligned}$$

**Promising, but no preference for more reproduction than less or between degenerate, "neutronically equivalent" choices.**

# References


Bertsekas, D. P. (1998). *Network Optimization: Continuous and Discrete Models*. Athena Scientific, Nashua, NH.

Board on Energy and Environmental Systems (2008). *Review of DOE's nuclear energy research and development program*. Technical report, National Research Council,Washington, DC. Accessed 5 January 2009 from http://www.nap.edu/catalog/11998.html.

Ferris, M. C., Mangasarian, O. L., and Wright, S. J. (2008). *Linear Programming with Matlab*. MPS-SIAM Series on Optimization. Society for Industrial and Applied Mathematics, Philadelphia, PA, first edition.

Jain, R. and Wilson, P. P. H. (2006). "Transitioning to global optimization in fuel cycle system study tools." *Transactions of the American Nuclear Society*, **95**, pages 162–3.

Juchau, C. (2008). "Development of the global evaluation of nuclear infrastructure and utilization scenarios (GENIUS) nuclear fuel cycle systems analysis code." Master's thesis, Idaho State University.

Juchau, C. A. and Dunzik-Gougar, M. L. (2006). *A review of nuclear fuel cycle systems codes*. Technical report, SINEMA LDRD Project. Accessed 13 February 2007 from http://thesinema.org/.

Lisowski, P. (2007). Global Nuclear Energy Partnership. In *Global Nuclear Energy Partnership Annual Meeting*, Litchfield Park, AZ. Global Nuclear Energy Partnership.

Nuclear and Radiation Studies Board (2008). *Internationalization of the nuclear fuel cycle: Goals, strategies, and challenges* [prepublication copy]. Technical report, National Academy of Sciences, National Research Council, and Russian Acadmy of Sciences,Washington, DC. Accessed 6 January 2009 from http://www.nap.edu/catalog/12477.html.

Oliver, K. (2009). Report on GENIUS Development at Time of Handoff. Private Communication.