NE450 - Principles of nuclear engineering
Xenon neutron poison production

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1 Mathematical models

1.1 Iodine

Rate of change of iodine -

$$\begin{aligned} \frac{dI}{dt} &= \gamma_I \Sigma_F \phi_T - \lambda_I I \\ I(0) &= 0 \end{aligned} \tag{1}$$

1.2 Xenon

Rate of change of xenon -

$$\begin{aligned} \frac{dX}{dt} &= \lambda_I I + \gamma_X \Sigma_F \phi_T - \lambda_X X - \sigma_A \phi_T X \\ X(0) &= 0 \end{aligned} \tag{2}$$

2 Solutions

2.1 Iodine

Apply laplace transform to eq. 1 -

$$\begin{aligned} \frac{dI}{dt} &= \gamma_I \Sigma_F \phi_T - \lambda_I I \\ s\tilde{I} &= \frac{1}{s} \gamma_I \Sigma_F \phi_T - \lambda_I \tilde{I} \end{aligned} \tag{3}$$

Rearrange terms to obtain laplace solution -

$$\tilde{I} = \frac{1}{s(s + \lambda_I)} \gamma_I \Sigma_F \phi_T \tag{4}$$

Invert eq. 4 to obtain real time domain solution -

$$I(t) = \frac{\gamma_I \Sigma_F \phi_T}{\lambda_I} (1 - e^{-\lambda_I t}) \tag{5}$$

2.2 Xenon

Substitute the iodine solution in eq. 5 into eq. 2 -

$$\begin{aligned} \frac{dX}{dt} &= \lambda_I I + \gamma_X \Sigma_F \phi_T - \lambda_X X - \sigma_A \phi_T X \\ \frac{dX}{dt} &= \lambda_I \frac{\gamma_I \Sigma_F \phi_T}{\lambda_I} (1 - e^{-\lambda_I t}) + \gamma_X \Sigma_F \phi_T - \lambda_X X - \sigma_A \phi_T X \end{aligned} \tag{6}$$

Rearrange the terms and simplify -

$$\begin{aligned} \frac{dX}{dt} &= \gamma_I \Sigma_F \phi_T (1 - e^{-\lambda_I t}) + \gamma_X \Sigma_F \phi_T - \lambda_X X - \sigma_A \phi_T X \\ \frac{dX}{dt} &= \gamma_I \Sigma_F \phi_T - \gamma_I \Sigma_F \phi_T e^{-\lambda_I t} + \gamma_X \Sigma_F \phi_T - \lambda_X X - \sigma_A \phi_T X \end{aligned} \tag{7}$$

$$\frac{dX}{dt} = \gamma_I \Sigma_F \phi_T - \gamma_I \Sigma_F \phi_T e^{-\lambda_I t} + \gamma_X \Sigma_F \phi_T - (\lambda_X + \sigma_A \phi_T) X \tag{8}$$

Apply laplace transform to eq. 8 -

$$s\tilde{X} = \frac{1}{s}\gamma_I \Sigma_F \phi_T - \frac{1}{s+\lambda_I}\gamma_I \Sigma_F \phi_T + \frac{1}{s}\gamma_X \Sigma_F \phi_T - (\gamma_X + \sigma_A \phi_T)\tilde{X} \tag{9}$$

Rearrange the terms in eq. 9 to obtain the laplace solution -

$$(s + [\gamma_X + \sigma_A \phi_T])\tilde{X} = \frac{1}{s}\gamma_I \Sigma_F \phi_T - \frac{1}{s+\lambda_I}\gamma_I \Sigma_F \phi_T + \frac{1}{s}\gamma_X \Sigma_F \phi_T \tag{10}$$

$$\begin{aligned}\tilde{X} = & \\ & + \frac{1}{s(s+[\gamma_X + \sigma_A \phi_T])}\gamma_I \Sigma_F \phi_T \\ & - \frac{1}{(s+\lambda_I)(s+[\gamma_X + \sigma_A \phi_T])}\gamma_I \Sigma_F \phi_T \\ & + \frac{1}{s(s+[\gamma_X + \sigma_A \phi_T])}\gamma_X \Sigma_F \phi_T\end{aligned} \tag{11}$$

Invert eq. 11 to obtain real time domain solution -

$$\begin{aligned}X(t) = & \\ & + \frac{\gamma_I \Sigma_F \phi_T}{\gamma_X + \sigma_A \phi_T}(1 - e^{-(\gamma_X + \sigma_A \phi_T)t}) \\ & - \frac{\gamma_I \Sigma_F \phi_T}{\gamma_X + \sigma_A \phi_T - \lambda_I}(e^{-\lambda_I t} - e^{-(\gamma_X + \sigma_A \phi_T)t}) \\ & + \frac{\gamma_X \Sigma_F \phi_T}{\gamma_X + \sigma_A \phi_T}(1 - e^{-(\gamma_X + \sigma_A \phi_T)t})\end{aligned} \tag{12}$$

3 Equilibrium time

$X(t) = \frac{\gamma_I \Sigma_F \phi_T}{\gamma_X+\sigma_A \phi_T}(1-e^{-(\gamma_X+\sigma_A\phi_T)t}) - \frac{\gamma_I \Sigma_F \phi_T}{\gamma_X+\sigma_A\phi_T-\lambda_I}(e^{-\lambda_I t} - e^{-(\gamma_X+\sigma_A\phi_T)t}) + \frac{\gamma_X \Sigma_F \phi_T}{\gamma_X+\sigma_A\phi_T}(1-e^{-(\gamma_X+\sigma_A\phi_T)t})$

$\frac{dX}{dt} = [\frac{\gamma_I \Sigma_F \phi_T}{\gamma_X+\sigma_A\phi_T}]((\gamma_X+\sigma_A\phi_T)e^{-(\gamma_X+\sigma_A\phi_T)t}) - [\frac{\gamma_I \Sigma_F \phi_T}{\gamma_X+\sigma_A\phi_T-\lambda_I}]((\gamma_X+\sigma_A\phi_T)e^{-(\gamma_X+\sigma_A\phi_T)t} - \lambda_I e^{-\lambda_I t})$
$+ [\frac{\gamma_X \Sigma_F \phi_T}{\gamma_X+\sigma_A\phi_T}]((\gamma_X+\sigma_A\phi_T)e^{-(\gamma_X+\sigma_A\phi_T)t})$

$\frac{dX}{dt} = \gamma_I \Sigma_F \phi_T e^{-(\gamma_X+\sigma_A\phi_T)t}$
$- \frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X+\sigma_A\phi_T)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}e^{-(\gamma_X+\sigma_A\phi_T)t}$
$+ \frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}e^{-\lambda_I t}$
$+ \gamma_X \Sigma_F \phi_T e^{-(\gamma_X+\sigma_A\phi_T)t}$

$(\gamma_I \Sigma_F \phi_T + \gamma_X \Sigma_F \phi_T - \frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X+\sigma_A\phi_T)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)})e^{-(\gamma_X+\sigma_A\phi_T)t} + \frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}e^{-\lambda_I t} = 0$

$-(\gamma_I \Sigma_F \phi_T + \gamma_X \Sigma_F \phi_T - \frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X+\sigma_A\phi_T)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)})e^{-(\gamma_X+\sigma_A\phi_T)t} = \frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}e^{-\lambda_I t}$

$(-\gamma_I \Sigma_F \phi_T - \gamma_X \Sigma_F \phi_T + \frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X+\sigma_A\phi_T)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)})e^{-(\gamma_X+\sigma_A\phi_T)t} = \frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}e^{-\lambda_I t}$

$(\frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X+\sigma_A\phi_T)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)} - \gamma_I \Sigma_F \phi_T - \gamma_X \Sigma_F \phi_T)e^{-(\gamma_X+\sigma_A\phi_T)t} = \frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}e^{-\lambda_I t}$

$$\frac{e^{-(\gamma_X+\sigma_A\phi_T)t}}{e^{-\lambda_I t}} = \frac{\frac{(\gamma_I\Sigma_F\phi_T)(\lambda_I)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}}{(\frac{(\gamma_I\Sigma_F\phi_T)(\gamma_X+\sigma_A\phi_T)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}-\gamma_I\Sigma_F\phi_T-\gamma_X\Sigma_F\phi_T)}$$

$$e^{(\lambda_I-(\gamma_X+\sigma_A\phi_T))t} = \frac{\frac{(\gamma_I\Sigma_F\phi_T)(\lambda_I)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}}{(\frac{(\gamma_I\Sigma_F\phi_T)(\gamma_X+\sigma_A\phi_T)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}-\gamma_I\Sigma_F\phi_T-\gamma_X\Sigma_F\phi_T)}$$

$$(\lambda_I-(\gamma_X+\sigma_A\phi_T))t = ln[\frac{\frac{(\gamma_I\Sigma_F\phi_T)(\lambda_I)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}}{(\frac{(\gamma_I\Sigma_F\phi_T)(\gamma_X+\sigma_A\phi_T)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}-\gamma_I\Sigma_F\phi_T-\gamma_X\Sigma_F\phi_T)}]$$

$$t = \frac{1}{\lambda_I-(\gamma_X+\sigma_A\phi_T)} \cdot ln[\frac{\frac{(\gamma_I\Sigma_F\phi_T)(\lambda_I)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}}{(\frac{(\gamma_I\Sigma_F\phi_T)(\gamma_X+\sigma_A\phi_T)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}-\gamma_I\Sigma_F\phi_T-\gamma_X\Sigma_F\phi_T)}]$$