PHYS-E0562 Nuclear Engineering, advanced course

Lecture 6 – Basics of Heat Transfer and Coolant Flow
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Motivation

Energy release in fissions

Daughter nuclei	173.70 MeV
Prompt neutrons	6.48 MeV
Delayed neutrons	0.0044 MeV
Prompt gammas	6.18 MeV
Delayed gammas	6.49 MeV
Delayed betas	6.62 MeV
Neutrinos	8.88 MeV
Total	208.35 MeV
Total minus neutrinos	199.47 MeV

The total energy release following a fission can be divided into a prompt component and a delayed component.

The prompt energy release consists of the energy that is distributed between the daughter nuclei and the prompt fission neutrons.

In addition to the prompt energy release, the daughter nuclei are typically radioactive and emit further particles or photons as they decay. The delayed neutrons, gammas and beta particles as well as the neutrinos associated with beta-decay are the result of this radioactive decay and carry a considerable amount of energy.

Motivation

Energy deposition from fissions

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The energy of the daughter nuclei as well as the energy of the delayed beta particles is deposited into the close vicinity of the fission site.

The energy of the emitted neutrons and gammas is distributed over a larger volume into the fuel, coolant and structural materials.

The energy of the neutrinos cannot be recovered and is lost.

Approximately 180 MeV of the fission energy is deposited into fuel, whereas 20 MeV is deposited directly to moderator/coolant or structural materials.

Motivation

For a nuclear reactor to produce power in steady state operation there is a simple requirement for the heat balance of the core: The amount of heat produced in unit time must be the amount of heat transferred out of core in unit time. Specifically this can be divided into two important balance requirements:

- The power deposited to fuel must equal the power transferred from the fuel rod to the coolant.
- The power transferred from fuel rods to the coolant (plus the power deposited directly to the coolant) must equal the power transferred by coolant out of the core.

Failure to fulfil the first requirement will result in an increase in fuel temperature, which can lead to the shattering of the fuel pellet or the melting of the fuel. Failure to fulfil the second requirement will lead to an increase in coolant temperature, which can result in reduced heat transfer from fuel rods to coolant.

Motivation

- How do we decide the power output of a nuclear reactor?
- The need to keep the fuel rods intact during operation is the limiting factor for the reactor power level.
- The maximum power level is based on the coolability of the fuel.

Figure 1 : Partially melted fuel rods in Reactor #2 at Three Mile Island (Image courtesy of the AP Photo Archive)

Since there are local variations in the fission power density, there are also local variations in coolant and fuel temperature. Specifically there is always some fuel rod that is at a higher temperature than any of the other rods. Identifying this *hot rod* and ensuring that its temperature is below a specified safety limit is an important aspect of reactor design and safety modelling.

Outline

First part considers solving the fuel maximum temperatures based on a known power distribution and system parameters:

- Heat transfer and temperature distribution in coolant.
- Heat transfer and temperature distribution inside fuel rod.

Second part goes a bit more in depth on the interface of the fuel rods and coolant:

- Heat transfer from cladding to coolant.

Third part applies the equations derived in the first part to a specific example:

- Worked exercise, simple thermal hydraulics solver for an average Loviisa type (VVER-440) representative fuel rod and accompanying flow channel.

Heat transfer in coolant

Overview

The core produces heat at a rate q (W). The core is cooled by a constant mass flow rate of coolant w ($\frac{\text{kg}}{\text{s}}$) entering the core at T_{in} and exiting the core at T_{out} (both K).

The coolant absorbs heat from the fuel elements, which increases the specific enthalpy of the coolant by $\Delta h = h_{\text{out}} - h_{\text{in}}$ ($\frac{\text{J}}{\text{kg}}$). The coolant finally flows out of the core, transferring the absorbed heat out of the assembly by convection.

In pressurized water reactors the coolant exits the core at a temperature that is lower than the saturation temperature at the operating pressure and the absorbed energy goes into increasing the temperature of the coolant:

$$q = w \int_{T_{\text{in}}}^{T_{\text{out}}} c_p(T) \mathrm{d}T \tag{1}$$

In boiling water reactors the coolant reaches its saturation temperature in the core, after which any addition of heat will generate vapour:

$$q = w \int_{T_{\text{in}}}^{T_{\text{sat}}} c_p(T) \mathrm{d}T + w\alpha\Delta h_{vap} \tag{2}$$

Coolant axial temperature distribution

We will derive a solution for the coolant axial temperature distribution in a flow channel: The coolant enters the flow channel at the bottom of the assembly and travels upwards. We'll consider a short segment of the flow channel with a height of dz and a flow area of A_c. The volume of this coolant region is $V_c = A_c dz$. The power production of the adjacent fuel rod is

$$dq(z) = q'''(z) A_f dz. \tag{3}$$

Here, the power density is assumed to be constant in the radial direction, which is a reasonable approximation for fresh fuel.

In the time $dt = dz/v$ it takes our coolant segment to travel the length dz some heat (de Joules) is transferred from the fuel rod to the coolant, which increases the coolant enthalpy. The coolant temperature is also increased as long as the temperature is below the saturation temperature.

Since there is no accumulation of heat in steady state conditions, the amount of heat entering the coolant corresponds to the amount of heat generated in the fuel.

$$de = dt \times dq(z) \tag{4}$$

Coolant axial temperature distribution

We'll solve the temperature distribution in subcooled conditions (i.e. below the boiling point of the coolant), separate considerations are needed for BWR's where some of the coolant is boiled into vapour.

Addition of our infinitesimal amount of power to our small volume of increases its temperature by dT based on

$$de = dt \times dq(z) = C_p(T)dT, \tag{5}$$

where C_p is the heat capacity of our coolant slice. This can be written based on the specific heat capacity and the mass of the coolant slice yielding

$$dt \times dq(z) = c_p(T)\rho_c V_c dT = c_p(T)\rho_c A_c dz dT. \tag{6}$$

Dividing by dt and remembering that $dz/dt = v$ is the velocity of the coolant flow we get

$$dq(z) = c_p \rho_c A_c v dT_b = c_p(T) w dT, \tag{7}$$

where the coolant mass flow rate $w = \rho_c A_c v$ has been substituted in.

Coolant axial temperature distribution

Finally, we'll use equation 3 for the dq to get

$$q'''(z)A_f\,dz = c_p(T)wdT, \tag{8}$$

or

$$\frac{\mathrm{d}T}{\mathrm{d}z} = \frac{q'''(z)A_f}{wc_p(T)}. \tag{9}$$

If the volumetric heat production rate $q'''(z)$ and the temperature dependent specific heat capacity of the coolant $c_p(T)$ are known, the axial temperature can be solved at least numerically. It should be noted that c_p is typically not constant over the axial length of the fuel. For example in Loviisa NPP

$$c_{p,\mathrm{in}} \approx 4930\,\frac{\mathrm{J}}{\mathrm{kg\,K}}$$
$$c_{p,\mathrm{out}} \approx 5579\,\frac{\mathrm{J}}{\mathrm{kg\,K}}.$$

Coolant axial temperature distribution

As a crude approximation we can divide the axial active length into n_z cells with a constant heat production

$$q'''_n = \frac{1}{V_{f,n}} \int_{V_{f,n}} q'''(z) \mathrm{d} V \tag{10}$$

and a constant specific heat for the coolant

$$c_{p,n} = c_p(T_{\mathrm{eff.},n}), \tag{11}$$

where the effective temperature is for example a volume average of the temperature at cell n

$$T_{\mathrm{eff.},n} = \frac{1}{V_{c,n}} \int_{V_{c,n}} T(z) \mathrm{d} V. \tag{12}$$

Coolant axial temperature distribution

With these additional approximations we get the increase in temperature over a single axial cell to be

$$\Delta T_n = T_{n,\mathrm{top}} - T_{n,\mathrm{bottom}} = \frac{q_n''' \Delta z A_f}{w c_{p,n}}. \tag{13}$$

Starting from a known inlet temperature the temperature distribution can then be solved throughout the axial length of the assembly.

We should highlight here the fact that no information is required about the temperature distribution inside the fuel rod if steady-state conditions are assumed.

Heat transfer inside fuel rod

Overview

- The heat is deposited into the fuel pellet.
- The heat conducts through the fuel pellet to the pellet outer surface.
- The heat transfer from pellet outer surface through gas gap to the cladding inner surface is a combination of conduction through gas, radiative transfer and conduction through pellet–cladding solid contacts.
- From cladding inner surface, the heat is again conducted to the cladding outer surface.
- At cladding outer surface the heat is transferred to the coolant.
- The coolant will transfer the heat by convection out of the core.

Here, we'll disregard the radiative and solid–solid conduction in gas gap and assume the heat transfer in gas gap is solely based on heat conduction through the fill gas[1].

Starting from a known coolant temperature distribution, we'll work our way inwards towards the centerline of the pellet.

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[1] Radiative transfer is mostly important in accident scenarios and solid–solid contacts only occur with increased burnup

Heat transfer inside fuel rod

Heat transfer from cladding to coolant

The heat transfer from the cladding outer surface to the coolant is governed by Newton's law of cooling

$$q''_{\mathrm{clad}} = h_{\mathrm{clad}}(T_{\mathrm{clad}} - T_{\mathrm{b}}), \tag{14}$$

where q''_{clad} is the surface heat flux at the cladding surface ($\frac{\mathrm{W}}{\mathrm{m}^2}$), h is the heat transfer coefficient ($\frac{\mathrm{W}}{\mathrm{m}^2\,\mathrm{K}}$) and T_{b} is a reference temperature for the coolant. The heat transfer coefficient is very much dependent on the thermodynamic properties and state of the coolant. This fact is discussed more in the second part of the lecture.

Temperature solution inside fuel rod

Cladding outer temperature

If we know the coolant bulk temperature and the heat transfer coefficient from cladding to coolant we can write the cladding temperature as

$$T_{\mathrm{clad,out}} = T_b + \frac{q''_{\mathrm{clad}}}{h_{\mathrm{clad}}} \tag{15}$$

The power flowing through any of the cylindrical surfaces in our fuel rod should equal the power generated inside that cylindrical surface (in steady state). Thus the heat flux $q''(R)$ is just the power generated inside that radius $P(r < R)$ divided by the area of the cylindrical surface $2\pi R \Delta z$

$$q''(R) = \frac{P(r < R)}{2\pi R \Delta z} \tag{16}$$

For our cladding outer surface ($R = R_{c,1}$) then

$$T_{\mathrm{clad,out}} = T_b + \frac{1}{h_{\mathrm{clad}}} \frac{P(r < R_{\mathrm{c},1})}{2\pi R_{\mathrm{c},1} \Delta z} = T_b + \frac{1}{h_{\mathrm{clad}}} \frac{q''' A_f \Delta z}{2\pi R_{\mathrm{c},1} \Delta z}. \tag{17}$$

Temperature solution inside fuel rod

Cladding outer temperature

This means that the axial variation of the cladding outer temperature can be calculated based on the local bulk coolant temperature and heat transfer coefficient from

$$T_{\mathrm{clad,out}}(z) = T_b(z) + \frac{1}{h_{\mathrm{clad}}(z)} \frac{q'''(z) A_f}{2\pi R_{\mathrm{c},1}}. \tag{18}$$

Again, the temperature distribution inside the fuel rod does not affect the cladding outer temperature in steady state.

Heat transfer inside fuel rod

Heat conduction

We now know the cladding outer temperature at any axial elevation and are ready to tackle the heat conduction problem inside the fuel rod to find out the fuel centerline temperature.

The differential heat conservation equation assuming only conduction is

$$\rho c_p \frac{\partial}{\partial t} T - \nabla \cdot (k \nabla T) = q''' \tag{19}$$

where ρ is the material density ($\frac{\mathrm{kg}}{\mathrm{m}^3}$), c_p is the local specific heat ($\frac{\mathrm{J}}{\mathrm{kg\,K}}$), T is temperature (K), k is the local heat conductivity ($\frac{\mathrm{W}}{\mathrm{m\,K}}$) and q''' is the volumetric heat source ($\frac{\mathrm{W}}{\mathrm{m}^3}$). For a material region with constant heat conductivity k this simplifies to

$$\rho c_p \frac{\partial}{\partial t} T - k \nabla^2 T = q''' \tag{20}$$

And in time independent (steady state) situation we have

$$-k \nabla^2 T = q''' \Rightarrow \nabla^2 T = -\frac{q'''}{k}. \tag{21}$$

Heat transfer inside fuel rod

Heat conduction

The equation

$$\nabla^2 T = -\frac{q'''}{k} \tag{22}$$

is a Poisson's equation for the temperature distribution. The volumetric heat production rate q''' and the heat conductivity k generally depend on the position.

We'll now make the approximation of dividing our fuel element into a number of annular rings (ring n between $R_{n,0}$ and $R_{n,1}$) and assuming that the heat production and the heat conductivity are constant in each ring[2], we can solve the Poisson's equation analytically in each of the rings.

$$\nabla^2 T_n = -\frac{q_n'''}{k_n} \tag{23}$$

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[2] This approximation will get better as the number of rings is increased

Heat transfer inside fuel rod

Heat conduction

Considering cylindrical fuel elements, we can move to cylindrical coordinates

$$\nabla^2 T = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}T\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}T + \frac{\partial^2}{\partial z^2}T \tag{24}$$

We'll only consider the radial heat transfer, i.e., we'll assume the situation is symmetric in the ϕ direction and the second derivative of the temperature field in the z-direction is small compared to our radial term. This leads us to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}T_n(r)\right) = -\frac{q_n'''}{k_n}. \tag{25}$$

The general solution to the equation can be obtained by simple separation of variables and integration yielding

$$T_n(r) = -\frac{q_n'''}{4k_n}r^2 + C_1^n \ln r + C_2^n. \tag{26}$$

Heat transfer inside fuel rod

Heat conduction

The constants in the equation

$$T_n(r) = -\frac{q_n'''}{4k_n} r^2 + C_1^n \ln r + C_2^n. \tag{27}$$

are defined by boundary conditions, for example:

- $T(r)$ is continuous.
- $q''(r) = -k\nabla T(r)$ (heat flux) is continuous.
- $T(0)$ is finite.
- $T(R_{\text{clad}}) = T_{\text{clad}}$.

This functional form applies both to the fuel and cladding parts of the rod as well as in the gas gap now that we have approximated away the radiative heat transfer as well as the solid–solid heat conduction between pellet and cladding.

Heat transfer inside fuel rod

Heat conduction

The temperature at the ring outer radius

$$T(R_{n,1}) = T_{n,\text{out}} \tag{28}$$

is known, as is the heat flux at the ring inner radius

$$q''(R_{n,0}) = \left(-k_n \nabla T_n(r)\right)_{r=R_{n,0}} = \frac{P(r < R_{n,0})}{2\pi R_{n,0} \Delta z} \tag{29}$$

which is equal to the power generated inside the ring inner radius divided by the ring inner surface area. This gives us two equations to solve our two unknown constants:

$$\begin{cases} -\frac{q_n'''}{4k_n} R_{n,1}^2 + C_1^n \ln R_{n,1} + C_2^n & = T_{n,\text{out}} \\ -k\left(-\frac{q_n'''}{2k_n} R_{n,0} + C_1^n \frac{1}{R_{n,0}}\right) & = \frac{P(r<R_{n,0})}{2\pi R_{n,0} \Delta z} \end{cases} \tag{30}$$

Temperature solution inside fuel rod

Temperatures in fuel pellet

Solving for the constants gives

$$C_1^n = \begin{cases} 0 & , n = 1 \\ \frac{q_n''' R_{i,0}^2}{2k_n} - \frac{P(r < R_{n,0})}{2\pi \Delta z k_n} & , n > 1 \end{cases}$$

$$C_2^n = T_{n,\mathrm{out}} + \frac{q_n'''}{4k_n} R_{n,1}^2 - C_1 \ln R_{n,1}$$

And the temperature distribution is

$$T(r) = T_{n,\mathrm{out}} + \frac{q_n'''}{4k_n}(R_{n,1}^2 - r^2) - C_1^n \ln(\frac{R_{n,1}}{r}). \quad (31)$$

This depends on the ring outer temperature $T_{n,\mathrm{out}}$ and known constants. As we know the outer temperature of the outermost ring (cladding outer surface temperature) we can now solve the temperature distribution inside the fuel rod.

Summary of the first part of the lecture

Temperature solution in coolant and fuel rods

- The coolant enters the core at a certain temperature, travels up through the flow channel and absorbs heat as it travels.
- The heat transferred from the fuel rod to the coolant as the coolant travels some axial height is equal to the heat produced in the corresponding axial slice of fuel in the time it takes for the coolant to travel that distance.

$$\frac{\mathrm{d}T}{\mathrm{d}z} = \frac{q'''(z)A_f}{wc_p(T)}$$

- Heat transfer from coolant to cladding is dependent on a heat transfer coefficient h_{clad}, which is as of yet unknown, but depends on the coolant thermodynamic state.

$$T_{\mathrm{clad,out}}(z) = T_b(z) + \frac{1}{h_{\mathrm{clad}}(z)}\frac{q'''(z)A_f}{2\pi R_{\mathrm{c},1}}.$$

Summary of the first part of the lecture

Temperature solution in coolant and fuel rods

- The heat transfer inside the fuel rod is largely based on heat conduction.
- In regions with constant heat conductivity, the differential heat conduction equation reduces to a Poisson's equation (inhomogeneous Laplace's equation).
- The analytic radial solution for the Poisson's equation in a ring with constant volumetric heat production rate q_n''' and heat conductivity k_n is

$$T_n(r) = -\frac{q_n'''}{4k_n} r^2 + C_1^n \ln r + C_2^n.$$

- The constants can be solved based on a known ring outer temperature and a known ring inner surface heat flux giving

$$C_1^n = \begin{cases} 0 & , n = 1 \\ \frac{q_n''' R_{i,0}^2}{2k_n} - \frac{P(r < R_{n,0})}{2\pi \Delta z k_n} & , n > 1 \end{cases}$$

$$C_2^n = T_{n,\mathrm{out}} + \frac{q_n'''}{4k_n} R_{n,1}^2 - C_1 \ln R_{n,1}$$

Heat transfer from cladding to coolant

Boiling regimes

- No boiling. The temperature of the cladding outer surface is below the saturation temperature. No boiling can occur.
- Local boiling (subcooled nucleate boiling). The temperature of the cladding outer surface is above the saturation temperature, but the coolant bulk temperature is below the saturation temperature. Vapour bubbles form at the cladding surface, detach and are carried away by the coolant flow. Good heat transfer from cladding to coolant.
- Bulk boiling (saturated nucleate boiling). The temperature of the cladding outer surface is above the saturation temperature and the bulk of the coolant is at the saturation temperature. Vapour bubbles are formed both in the cladding surface and in the bulk of the coolant. Very good heat transfer from cladding to coolant.
- Partial film boiling. The vapour bubbles on the cladding outer surface start to merge, forming a continuous vapour film on the cladding outer surface. In order to get to the bulk of the coolant, the heat has to conduct through the vapour. Reduced heat transfer from cladding to coolant. Risk of runaway heating of cladding.
- Film boiling. The cladding is enveloped in a vapour film, no direct heat transfer to liquid coolant possible. Very much reduced heat transfer from cladding to coolant. Risk of runaway heating of cladding imminent.

Heat transfer from cladding to coolant

Pressurized water reactors

In PWR’s the saturation temperature of the coolant is increased by keeping a high pressure in the primary circuit. Vapor bubbles are only formed on the surface of the cladding and there is no net production of steam. This kind of a boiling process is called subcooled nucleate boiling.

Increasing cladding temperature increases heat transfer until a critical heat flux is reached, after which partial film boiling starts to occur and lead to fuel damage. See: https://www.youtube.com/watch?v=4767rEtecUM

Heat transfer from cladding to coolant

Pressurized water reactors

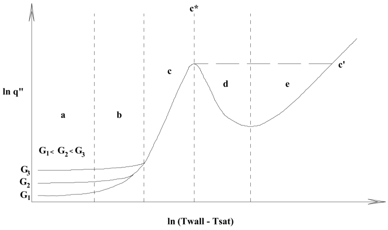

Figure 2 : Heat flux at cladding versus temperature difference between clad temperature and saturation temperature
(PWR's)(from http://www4.ncsu.edu/~doster/NE402/Text/BoilingHeatTransfer/BoilingHeatTransfer.pdf)

a) No boiling $T_{\text{clad}} < T_{\text{sat}}$.
b) Subcooled nucleate boiling based on bulk flow quality.
c) Subcooled nucleate boiling independent on bulk flow quality.
c*) Critical heat flux (CHF) and departure from nucleate boiling (DNB).
d) Partial film boiling.
e) Complete film boiling.

Heat transfer from cladding to coolant

Boiling water reactors

In BWR's the bulk temperature of the coolant increases over the saturation point of the fluid as the coolant flows upward in the channel. The coolant goes through various flow regimes as it travels through the assembly:

- Bubbly flow
- Slug flow
- Churn flow
- Annular flow

See:
https://www.youtube.com/watch?v=YV_BlnpJvao

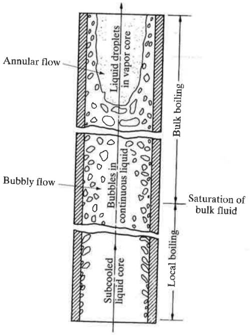

Figure 3 : Flow regimes in a vertical heated pipe. From *Lamarsh, Baratta: Introduction to Nuclear Engineering*

Increasing boiling increases the heat transfer until a critical heat flux is reached, after which channel dryout starts to occur and lead to fuel damage.

Heat transfer from cladding to coolant

Boiling water reactors

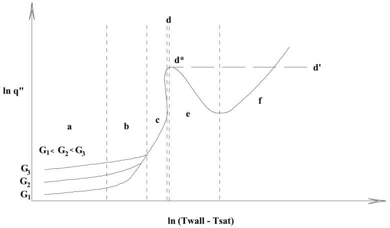

Figure 4 : Heat flux at cladding versus temperature difference between clad temperature and saturation temperature (BWR's)
(from http://www4.ncsu.edu/~doster/NE402/Text/BoilingHeatTransfer/BoilingHeatTransfer.pdf)

a) No boiling $T_{\text{clad}} < T_{\text{sat}}$.
b) Subcooled nucleate boiling based on bulk flow quality.
c) Subcooled nucleate boiling independent on bulk flow quality.
d) Saturated nucleate boiling / forced convection vaporization.
d*) Critical heat flux.
d) Partial dryout of rod.
e) Complete dryout of rod.

Heat transfer from cladding to coolant

Heat transfer coefficient

As described above, there are different flow and boiling regimes and things are generally complicated regarding the heat transfer between cladding and coolant. How is the heat transfer between cladding and coolant then calculated?

For water at pressures 35–140 bar undergoing nucleate boiling[3] (local or bulk) we have the Jens and Lottes correlation directly for the cladding temperature:

$$T_{\text{clad}} = T_{\text{sat.}} + \frac{0.79(q'')^{0.25}}{\exp\left[P/6.2\right]} \tag{32}$$

Onset of local nucleate boiling (forming of vapour bubbles on cladding outer surface):

$$T_{LB} = T_{\text{sat.}} + \left(T_{\text{clad}} - T_{\text{sat.}}\right)_{JL} - \frac{q''}{h} = T_{\text{sat.}} + \frac{0.79(q'')^{0.25}}{\exp\left[P/6.2\right]} - \frac{q''}{h} \tag{33}$$

where h is the convective heat transfer coefficient that has to be estimated with some suitable correlation typically based on the qualities of the coolant flow (Nusselt, Prandtl and Reynolds numbers).

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[3] This should cover most LWR cases in normal operation.

Summary of the second part of the lecture

Heat transfer from cladding to coolant

- In pressurized water reactors the coolant does not reach the saturation temperature (boiling point) while heating up in the core.
- The coolant thus undergoes subcooled nucleate boiling at most, where vapour bubbles form at the cladding surface ($T_{\mathrm{clad}} > T_{\mathrm{sat.}}$), dislodge themselves and carry their energy to the bulk coolant flow, cooling down.
- Increasing the heat flux (power production) increases the bubble formation at the cladding as well as the heat transfer coefficient between the cladding and the coolant until a critical heat flux (CHF) is reached.
- Further increasing the heat flux will lead to departure from nucleate boiling (DNB) and transition to partial film boiling, where parts of the fuel rod are covered in a vapour film for periods of time, leading to degradation of the heat transfer and increase in the cladding and fuel temperatures and possibly to fuel damage.

Summary of the second part of the lecture

Heat transfer from cladding to coolant

- In boiling water reactors the coolant heats up to its saturation temperature (boiling point) during its travel through the core, after which additional energy goes into vaporizing a part of the coolant.
- The coolant thus undergoes subcooled nucleate boiling at the bottom of the core and saturated nucleate boiling (bulk boiling) for the rest of the core. The vigorous boiling of the coolant increases the heat transfer greatly.
- The coolant in BWR's undergoes many different flow regimes during its ascent through the flow channel, starting from 1-phase flow and moving through bubbly flow and slug flow to churn flow and to annular flow at last.
- In annular flow, the fuel elements are covered with a liquid film, but a vapour core forms into the flow channel. Heat transfers through the liquid film to the liquid-vapour interface, where some of the remaining liquid is vaporized.
- Increasing the heat flux (power production) increases the boiling of the liquid to vapour until a critical heat flux (CHF) is reached.
- Further increasing the heat flux will lead to the rest of the liquid film boiling to vapour and leaving the top part of the fuel element completely surrounded by vapour. This leads to degradation of the heat transfer and increase in the cladding and fuel temperatures and possibly to fuel damage.

Practical exercise

Let's solve the heat transfer in a Loviisa-type (VVER-440) single fuel rod and surrounding flow channel.

Parameter	Value	SI-value
Fuel inner radius $(R_{f,0})$	0.06 cm	6×10^{-4} m
Fuel outer radius $(R_{f,1})$	0.38 cm	3.8×10^{-3} m
Clad inner radius $(R_{f,0})$	0.3865 cm	3.865×10^{-3} m
Clad outer radius $(R_{f,1})$	0.4535 cm	4.535×10^{-3} m
Fuel rod pitch (p)	1.23 cm	1.23×10^{-2} m
Fuel rod active length (L)	248 cm	2.48 m

Practical exercise

Let's solve the heat transfer in a Loviisa-type (VVER-440) single fuel rod and surrounding flow channel.

Parameter	Value	SI-value
Thermal power	1500 MW	1500×10^6 W
Coolant flow	$6 \times 7100 \frac{m^3}{h}$	$11.833 \frac{m^3}{s}$
System pressure	123 bar	12.3×10^6 Pa
Inlet temperature	265 °C	538 K
Outlet temperature	300 °C	573 K
Number of fuel assemblies	313	313
Number of rods in assembly	126	126

Practical exercise

Power distribution

We'll decide some power distribution for our fuel rod. Mean thermal power for a fuel rod is

$$P_{\mathrm{rod}} = \frac{P_{\mathrm{tot}}}{N_{\mathrm{ass.}} \times N_{\mathrm{rods/ass.}}} = \frac{1500 \times 10^6\ \mathrm{W}}{313 \times 126} \approx 22.1 \times 10^3\ \mathrm{W} \tag{34}$$

Going for a cosine-shaped power distribution in fuel (disregarding extrapolation length)

$$q'''(z) = C \cos\left[z \frac{\pi/2}{L/2}\right] = C \cos\left[z \frac{\pi}{L}\right] \tag{35}$$

we should have

$$\int_{V_{\mathrm{rod}}} q'''(z) \mathrm{d}V = \int_{-L/2}^{L/2} C \cos\left[z \frac{\pi}{L}\right] A_f \mathrm{d}z = P_{\mathrm{rod}} \tag{36}$$

which we can use to figure out our normalization

$$C = \frac{P_{\mathrm{rod}}}{2A_f} \Rightarrow q'''(z) = \frac{P_{\mathrm{rod}}}{2A_f} \cos\left[z \frac{\pi}{L}\right] \tag{37}$$

Practical exercise

Coolant temperature distribution

We'll divide the geometry to nz axial zones and solve the coolant temperature at $nz + 1$ zone boundaries. We can start from the known inlet temperature and use the previously derived equation

$$\Delta T_n = T_{n,\mathrm{top}} - T_{n,\mathrm{bottom}} = \frac{q_n''' \Delta z A_f}{w c_{p,n}}.$$

to calculate the increase in coolant temperature in each cell. We'll still need the mass flow rate w and the coolant specific heat capacity $c_{p,n}$.

Practical exercise

Cladding outer temperature distribution

We can estimate the cladding outer temperature based on the local coolant bulk temperature and Newton's cooling law

$$T_{\mathrm{clad,out}}(z) = T_b(z) + \frac{q''_{\mathrm{clad}}(z)}{h_{\mathrm{clad}}(z)} = T_b(z) + \frac{1}{h_{\mathrm{clad}}(z)} \frac{q'''(z) A_f}{2\pi R_{\mathrm{c},1}}.$$

if we only know the heat transfer coefficient $h_{\mathrm{clad}}(z)$.

Practical exercise

Radial temperature inside the fuel rod

We can divide the inner part of the fuel rod into rings with constant volumetric heat production rate and heat conductivity. We'll then solve the temperature distribution in each of the rings, starting from the outermost one based on our equation

$$T_n(r) = -\frac{q_n'''}{4k_n} r^2 + C_1^n \ln r + C_2^n,$$

where the constants are

$$C_1^n = \begin{cases} 0 & , n = 1 \\ \frac{q_n''' R_{i,0}^2}{2k_n} - \frac{q_{n,in}'' R_{n,0}}{k_n} & , n > 1 \end{cases}$$

$$C_2^n = T_{n,\mathrm{out}} + \frac{q_n'''}{4k_n} R_{n,1}^2 - C_1 \ln R_{n,1}$$

We'll have to get the heat conductivities of each ring (be it fuel, gas or cladding) from some correlation.

Practical exercise

Implementation and solution

See the `.m`-file in MyCourses