

NE450 - Principles of nuclear engineering Maxwell equations

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Definition of average energy -

$$\bar{E} = \frac{\int_0^\infty n(E) E dE}{\int_0^\infty n(E) dE} \quad (1)$$

$$n(E) = n_0 \frac{2\pi}{(\pi kT)^{\frac{3}{2}}} \sqrt{E} e^{-\frac{E}{kT}} \quad (2)$$

Denominator first -

$$\int_0^\infty n(E) dE \equiv F(E) = a n_0 \int_0^\infty \sqrt{E} e^{-bE} dE \quad (3)$$

$$a \equiv \frac{2\pi}{(\pi kT)^{\frac{3}{2}}}$$

$$b \equiv \frac{1}{kT}$$

$$x = \sqrt{E}$$

$$2x dx = dE$$

$$F(E) \rightarrow F(x) = a n_0 \int_0^\infty x e^{-bx^2} 2x dx \quad (4)$$

$$\int u dv = uv - \int v du$$

$$u = x$$

$$du = dx$$

$$dv = 2x e^{-bx^2} dx$$

$$v = -\frac{1}{b} e^{-bx^2}$$

$$F(x) = a n_0 \left[-\frac{1}{b} x e^{-bx^2} + \frac{1}{b} \int_0^\infty e^{-bx^2} dx \right] \quad (5)$$

$$w = x\sqrt{b}$$

$$\frac{1}{\sqrt{b}} dw = dx$$

$$F(x) = a n_0 \left[-\frac{1}{b} x e^{-bx^2} + \frac{1}{b} \int_0^\infty e^{-w^2} \frac{1}{b} dw \right] \quad (6)$$

$$F(x) = a n_0 \left[-\frac{1}{b} x e^{-bx^2} + \frac{1}{b^{\frac{3}{2}}} \int_0^\infty e^{-w^2} dw \right] \quad (7)$$

$$F(x) = a n_0 \left[-\frac{1}{b} x e^{-bx^2} + \frac{1}{b^{\frac{3}{2}}} \frac{\sqrt{\pi}}{2} \int_0^\infty \frac{2}{\sqrt{\pi}} e^{-w^2} dw \right] \quad (8)$$

$$F(x) = a n_0 \left[-\frac{1}{b} x e^{-bx^2} + \frac{1}{b^{\frac{3}{2}}} \frac{\sqrt{\pi}}{2} \operatorname{erf}(w) \right] \quad (9)$$

$$F(x) = a n_0 \left[-\frac{1}{b} x e^{-bx^2} + \frac{1}{b^{\frac{3}{2}}} \frac{\sqrt{\pi}}{2} \operatorname{erf}(x\sqrt{b}) \right] \quad (10)$$

$$F(x) = a n_0 \left[-\frac{1}{b} x e^{-bx^2} + \frac{1}{b^{\frac{3}{2}}} \frac{\sqrt{\pi}}{2} \operatorname{erf}(x\sqrt{b}) \right] \Big|_0^\infty \quad (11)$$

$$F(E) = a n_0 \left[-\frac{1}{b} \sqrt{E} e^{-bE} + \frac{1}{b^{\frac{3}{2}}} \frac{\sqrt{\pi}}{2} \operatorname{erf}(\sqrt{bE}) \right] \Bigg|_0^\infty \quad (12)$$

$$F(E) = a n_0 \left[\left(0 + \frac{1}{b^{\frac{3}{2}}} \frac{\sqrt{\pi}}{2} 1 \right) - (0 + 0) \right] \quad (13)$$

$$F(E) = \frac{2\pi}{\sqrt{\pi^3 k^3 T^3}} n_0 \left[\frac{k^3 T^3}{2} \frac{\sqrt{\pi}}{2} \right] \quad (14)$$

$$\int_0^\infty n(E) dE = n_0 \quad (15)$$

Now numerator -

$$\int_0^\infty n(E) E dE \equiv F(E) = a n_0 \int_0^\infty \sqrt{E} E e^{-bE} dE \quad (16)$$

$$a \equiv \frac{2\pi}{(\pi kT)^{\frac{3}{2}}}$$

$$b \equiv \frac{1}{kT}$$

$$x = \sqrt{E}$$

$$2x dx = dE$$

$$F(E) \rightarrow F(x) = a n_0 \int_0^\infty x^2 e^{-bx^2} 2x dx \quad (17)$$

$$F(E) \rightarrow F(x) = a n_0 \int_0^\infty x^3 e^{-bx^2} 2x dx \quad (18)$$

$$\int u dv = uv - \int v du$$

$$u = x^3$$

$$du = 3x^2 dx$$

$$dv = 2x e^{-bx^2} dx$$

$$v = -\frac{1}{b} e^{-bx^2}$$

$$F(x) = a n_0 \left[-\frac{1}{b} x^3 e^{-bx^2} + \frac{3}{b} \int_0^\infty x^2 e^{-bx^2} dx \right] \quad (19)$$

$$F(x) = a n_0 \left[-\frac{1}{b} x^3 e^{-bx^2} + \frac{3}{b} \left(\int_0^\infty x x e^{-bx^2} dx \right) \right] \quad (20)$$

$$u = x$$

$$du = dx$$

$$dv = x e^{-bx^2} dx$$

$$v = -\frac{1}{2b} e^{-bx^2} \quad (21)$$

$$F(x) = a n_0 \left[-\frac{1}{b} x^3 e^{-bx^2} + \frac{3}{b} \left(-\frac{1}{2b} x e^{-bx^2} + \frac{1}{2b} \int_0^\infty e^{-bx^2} dx \right) \right] \quad (22)$$

$$w = x\sqrt{b}$$

$$\frac{1}{\sqrt{b}} dw = dx$$

$$F(x) = a n_0 \left[-\frac{1}{b} x^3 e^{-bx^2} + \frac{3}{b} \left(-\frac{1}{2b} x e^{-bx^2} + \frac{1}{2b} \int_0^\infty e^{-w^2} \frac{1}{\sqrt{b}} dw \right) \right] \quad (23)$$

$$F(x) = a n_0 \left[-\frac{1}{b} x^3 e^{-bx^2} + \frac{3}{b} \left(-\frac{1}{2b} x e^{-bx^2} + \frac{1}{2b^{\frac{3}{2}}} \int_0^\infty e^{-w^2} dw \right) \right] \quad (24)$$

$$F(x) = a n_0 \left[-\frac{1}{b} x^3 e^{-bx^2} - \frac{3}{2b^2} x e^{-bx^2} + \frac{3}{2b^{\frac{5}{2}}} \int_0^\infty e^{-w^2} dw \right] \quad (25)$$

$$F(x) = a n_0 \left[-\frac{1}{b} x^3 e^{-bx^2} - \frac{3}{2b^2} x e^{-bx^2} + \frac{3}{2b^{\frac{5}{2}}} \frac{\sqrt{\pi}}{2} \int_0^\infty \frac{2}{\sqrt{\pi}} e^{-w^2} dw \right] \quad (26)$$

$$F(x) = a n_0 \left[-\frac{1}{b} x^3 e^{-bx^2} - \frac{3}{2b^2} x e^{-bx^2} + \frac{3}{2b^{\frac{5}{2}}} \frac{\sqrt{\pi}}{2} \operatorname{erf}(w) \right] \quad (27)$$

$$F(x) = a n_0 \left[-\frac{1}{b} x^3 e^{-bx^2} - \frac{3}{2b^2} x e^{-bx^2} + \frac{3}{2b^{\frac{5}{2}}} \frac{\sqrt{\pi}}{2} \operatorname{erf}(x\sqrt{b}) \right] \quad (28)$$

$$F(x) = a n_0 \left[-\frac{1}{b} \sqrt{E^3} e^{-bE} - \frac{3}{2b^2} \sqrt{E} e^{-bE} + \frac{3}{2b^{\frac{5}{2}}} \frac{\sqrt{\pi}}{2} \operatorname{erf}(\sqrt{bE}) \right] \quad (29)$$

$$F(E) = a n_0 \left[-\frac{1}{b} \sqrt{E^3} e^{-bE} - \frac{3}{2b^2} \sqrt{E} e^{-bE} + \frac{3}{2b^{\frac{5}{2}}} \frac{\sqrt{\pi}}{2} \operatorname{erf}(\sqrt{bE}) \right] \Bigg|_0^\infty \quad (30)$$

$$F(E) = a n_0 \left[(0 - 0 + \frac{3}{2b^{\frac{5}{2}}} \frac{\sqrt{\pi}}{2}) - (0 - 0 + 0) \right] \quad (31)$$

$$F(E) = a n_0 \frac{3}{2b^{\frac{5}{2}}} \frac{\sqrt{\pi}}{2} \quad (32)$$

$$F(E) = \frac{2\pi}{\sqrt{\pi^3 k^3 T^3}} n_0 \frac{3\sqrt{k^5 T^5}}{2} \frac{\sqrt{\pi}}{2} \quad (33)$$

$$F(E) = \frac{2\pi}{\sqrt{\pi^3 k^3 T^3}} n_0 \frac{3\sqrt{k^5 T^5}}{2} \quad (34)$$

$$F(E) = \frac{3}{2} n_0 kT \quad (35)$$

$$\overline{E} = \frac{\int_0^\infty n(E) E dE}{\int_0^\infty n(E) dE} \quad (36)$$

$$\overline{E} = \frac{\frac{3}{2} n_0 kT}{n_0} \quad (37)$$

$$\overline{E} = \frac{3}{2} kT \quad (38)$$