

NE450 - Principles of nuclear engineering

Xenon neutron poison production

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1 Mathematical models

1.1 Iodine

$$\begin{aligned}\frac{dI}{dt} &= \gamma_I \Sigma_F \phi_T - \lambda_I I \\ I(0) &= 0\end{aligned}\tag{1}$$

1.2 Xenon

$$\begin{aligned}\frac{dX}{dt} &= \lambda_I I + \gamma_X \Sigma_F \phi_T - \lambda_X X - \sigma_A \phi_T X \\ X(0) &= 0\end{aligned}\tag{2}$$

2 Solutions

2.1 Iodine

Laplace

$$\frac{dI}{dt} = \gamma_I \Sigma_F \phi_T - \lambda_I I$$

$$s\tilde{I} = \frac{1}{s} \gamma_I \Sigma_F \phi_T - \lambda_I \tilde{I}$$

Rearrange

$$\tilde{I} = \frac{1}{s(s+\lambda_I)} \gamma_I \Sigma_F \phi_T$$

Solve

$$I(t) = \frac{\gamma_I \Sigma_F \phi_T}{\lambda_I} (1 - e^{-\lambda_I t})$$

2.2 Xenon

Laplace

$$\frac{dX}{dt} = \lambda_I \frac{\gamma_I \Sigma_F \phi_T}{\lambda_I} (1 - e^{-\lambda_I t}) + \gamma_X \Sigma_F \phi_T - \lambda_X X - \sigma_A \phi_T X$$

$$\frac{dX}{dt} = \gamma_I \Sigma_F \phi_T (1 - e^{-\lambda_I t}) + \gamma_X \Sigma_F \phi_T - \lambda_X X - \sigma_A \phi_T X$$

$$\frac{dX}{dt} = \gamma_I \Sigma_F \phi_T - \gamma_I \Sigma_F \phi_T e^{-\lambda_I t} + \gamma_X \Sigma_F \phi_T - \lambda_X X - \sigma_A \phi_T X$$

$$\frac{dX}{dt} = \gamma_I \Sigma_F \phi_T - \gamma_I \Sigma_F \phi_T e^{-\lambda_I t} + \gamma_X \Sigma_F \phi_T - (\lambda_X + \sigma_A \phi_T) X$$

$$s\tilde{X} = \frac{1}{s} \gamma_I \Sigma_F \phi_T - \frac{1}{s+\lambda_I} \gamma_I \Sigma_F \phi_T + \frac{1}{s} \gamma_X \Sigma_F \phi_T - (\gamma_X + \sigma_A \phi_T) \tilde{X}$$

Rearrange

$$(s + [\gamma_X + \sigma_A \phi_T]) \tilde{X} = \frac{1}{s} \gamma_I \Sigma_F \phi_T - \frac{1}{s+\lambda_I} \gamma_I \Sigma_F \phi_T + \frac{1}{s} \gamma_X \Sigma_F \phi_T$$

$$\tilde{X} = \frac{1}{s(s+[\gamma_X + \sigma_A \phi_T])} \gamma_I \Sigma_F \phi_T - \frac{1}{(s+\lambda_I)(s+[\gamma_X + \sigma_A \phi_T])} \gamma_I \Sigma_F \phi_T + \frac{1}{s(s+[\gamma_X + \sigma_A \phi_T])} \gamma_X \Sigma_F \phi_T$$

Solve

$$X(t) = \frac{\gamma_I \Sigma_F \phi_T}{\gamma_X + \sigma_A \phi_T} (1 - e^{-(\gamma_X + \sigma_A \phi_T)t}) - \frac{\gamma_I \Sigma_F \phi_T}{\gamma_X + \sigma_A \phi_T - \lambda_I} (e^{-\lambda_I t} - e^{-(\gamma_X + \sigma_A \phi_T)t}) + \frac{\gamma_X \Sigma_F \phi_T}{\gamma_X + \sigma_A \phi_T} (1 - e^{-(\gamma_X + \sigma_A \phi_T)t})$$

3 Equilibrium time

$$X(t) = \frac{\gamma_I \Sigma_F \phi_T}{\gamma_X + \sigma_A \phi_T} (1 - e^{-(\gamma_X + \sigma_A \phi_T)t}) - \frac{\gamma_I \Sigma_F \phi_T}{\gamma_X + \sigma_A \phi_T - \lambda_I} (e^{-\lambda_I t} - e^{-(\gamma_X + \sigma_A \phi_T)t}) + \frac{\gamma_X \Sigma_F \phi_T}{\gamma_X + \sigma_A \phi_T} (1 - e^{-(\gamma_X + \sigma_A \phi_T)t})$$

$$\frac{dX}{dt} = \left[\frac{\gamma_I \Sigma_F \phi_T}{\gamma_X + \sigma_A \phi_T} \right] ((\gamma_X + \sigma_A \phi_T) e^{-(\gamma_X + \sigma_A \phi_T)t}) - \left[\frac{\gamma_I \Sigma_F \phi_T}{\gamma_X + \sigma_A \phi_T - \lambda_I} \right] ((\gamma_X + \sigma_A \phi_T) e^{-(\gamma_X + \sigma_A \phi_T)t} - \lambda_I e^{-\lambda_I t}) + \left[\frac{\gamma_X \Sigma_F \phi_T}{\gamma_X + \sigma_A \phi_T} \right] ((\gamma_X + \sigma_A \phi_T) e^{-(\gamma_X + \sigma_A \phi_T)t})$$

$$\begin{aligned} \frac{dX}{dt} &= \gamma_I \Sigma_F \phi_T e^{-(\gamma_X + \sigma_A \phi_T)t} \\ &- \frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)} e^{-(\gamma_X + \sigma_A \phi_T)t} \\ &+ \frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)} e^{-\lambda_I t} \\ &+ \gamma_X \Sigma_F \phi_T e^{-(\gamma_X + \sigma_A \phi_T)t} \end{aligned}$$

$$(\gamma_I \Sigma_F \phi_T + \gamma_X \Sigma_F \phi_T - \frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)}) e^{-(\gamma_X + \sigma_A \phi_T)t} + \frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)} e^{-\lambda_I t} = 0$$

$$-(\gamma_I \Sigma_F \phi_T + \gamma_X \Sigma_F \phi_T - \frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)}) e^{-(\gamma_X + \sigma_A \phi_T)t} = \frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)} e^{-\lambda_I t}$$

$$(-\gamma_I \Sigma_F \phi_T - \gamma_X \Sigma_F \phi_T + \frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)}) e^{-(\gamma_X + \sigma_A \phi_T)t} = \frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)} e^{-\lambda_I t}$$

$$(\frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)} - \gamma_I \Sigma_F \phi_T - \gamma_X \Sigma_F \phi_T) e^{-(\gamma_X + \sigma_A \phi_T)t} = \frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)} e^{-\lambda_I t}$$

$$\frac{e^{-(\gamma_X + \sigma_A \phi_T)t}}{e^{-\lambda_I t}} = \frac{\frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)}}{(\frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)} - \gamma_I \Sigma_F \phi_T - \gamma_X \Sigma_F \phi_T)}$$

$$e^{(\lambda_I - (\gamma_X + \sigma_A \phi_T))t} = \frac{\frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)}}{(\frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)} - \gamma_I \Sigma_F \phi_T - \gamma_X \Sigma_F \phi_T)}$$

$$(\lambda_I - (\gamma_X + \sigma_A \phi_T))t = \ln \left[\frac{\frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)}}{(\frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)} - \gamma_I \Sigma_F \phi_T - \gamma_X \Sigma_F \phi_T)} \right]$$

$$t = \frac{1}{\lambda_I - (\gamma_X + \sigma_A \phi_T)} \cdot \ln \left[\frac{\frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)}}{(\frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)} - \gamma_I \Sigma_F \phi_T - \gamma_X \Sigma_F \phi_T)} \right]$$