NE450 - Principles of nuclear engineering
Xenon neutron poison production

R. A. Borrelli

University of Idaho · Idaho Falls Center for Higher Education

Nuclear Engineering and Industrial Management Department

r.angelo.borrelli@gmail.com

2020.07.30

1 Mathematical models

1.1 Iodine

Rate of change of iodine -

$$\frac{dI}{dt} = \gamma_I \Sigma_F \phi_T - \lambda_I I$$
$$I(0) = 0 \tag{1}$$

1.2 Xenon

Rate of change of xenon -

$$\frac{dX}{dt} = \lambda_I I + \gamma_X \Sigma_F \phi_T - \lambda_X X - \sigma_A \phi_T X$$
$$X(0) = 0 \tag{2}$$

2 Solutions

2.1 Iodine

Apply laplace transform to eq. 1 -

$$\frac{dI}{dt} = \gamma_I \Sigma_F \phi_T - \lambda_I I$$
$$s\tilde{I} = \frac{1}{s} \gamma_I \Sigma_F \phi_T - \lambda_I \tilde{I} \tag{3}$$

Rearrange terms to obtain laplace solution -

$$\tilde{I} = \frac{1}{s(s + \lambda_I)} \gamma_I \Sigma_F \phi_T \tag{4}$$

Invert eq. 4 to obtain real time domain solution -

$$I(t) = \frac{\gamma_I \Sigma_F \phi_T}{\lambda_I} (1 - e^{-\lambda_I t}) \tag{5}$$

2.2 Xenon

Substitute the iodine solution in eq. 5 into eq. 2 -

$$\frac{dX}{dt} = \lambda_I I + \gamma_X \Sigma_F \phi_T - \lambda_X X - \sigma_A \phi_T X$$
$$\frac{dX}{dt} = \lambda_I \frac{\gamma_I \Sigma_F \phi_T}{\lambda_I}(1 - e^{-\lambda_I t}) + \gamma_X \Sigma_F \phi_T - \lambda_X X - \sigma_A \phi_T X$$

(6)

Rearrange the terms and simplify -

$$\frac{dX}{dt} = \gamma_I \Sigma_F \phi_T (1 - e^{-\lambda_I t}) + \gamma_X \Sigma_F \phi_T - \lambda_X X - \sigma_A \phi_T X$$
$$\frac{dX}{dt} = \gamma_I \Sigma_F \phi_T - \gamma_I \Sigma_F \phi_T e^{-\lambda_I t} + \gamma_X \Sigma_F \phi_T - \lambda_X X - \sigma_A \phi_T X$$

(7)

$$\frac{dX}{dt} = \gamma_I \Sigma_F \phi_T - \gamma_I \Sigma_F \phi_T e^{-\lambda_I t} + \gamma_X \Sigma_F \phi_T - (\lambda_X + \sigma_A \phi_T)X$$

(8)

Apply laplace transform to eq. 8 -

$$s\tilde{X} = \frac{1}{s}\gamma_I \Sigma_F \phi_T - \frac{1}{s + \lambda_I}\gamma_I \Sigma_F \phi_T + \frac{1}{s}\gamma_X \Sigma_F \phi_T - (\gamma_X + \sigma_A \phi_T)\tilde{X}$$

(9)

Rearrange the terms in eq. 9 to obtain the laplace solution -

$$(s + [\gamma_X + \sigma_A \phi_T])\tilde{X} = \frac{1}{s}\gamma_I \Sigma_F \phi_T - \frac{1}{s + \lambda_I}\gamma_I \Sigma_F \phi_T + \frac{1}{s}\gamma_X \Sigma_F \phi_T$$

(10)

Then -

$$\tilde{X} = +\frac{1}{s(s + [\gamma_X + \sigma_A \phi_T])}\gamma_I \Sigma_F \phi_T - \frac{1}{(s + \lambda_I)(s + [\gamma_X + \sigma_A \phi_T])}\gamma_I \Sigma_F \phi_T + \frac{1}{s(s + [\gamma_X + \sigma_A \phi_T])}\gamma_X \Sigma_F \phi_T$$

(11)

2

Invert eq. 11 to obtain real time domain solution -

$$X(t) = +\frac{\gamma_I \Sigma_F \phi_T}{\gamma_X + \sigma_A \phi_T}(1 - e^{-(\gamma_X + \sigma_A \phi_T)t}) - \frac{\gamma_I \Sigma_F \phi_T}{\gamma_X + \sigma_A \phi_T - \lambda_I}(e^{-\lambda_I t} - e^{-(\gamma_X + \sigma_A \phi_T)t}) + \frac{\gamma_X \Sigma_F \phi_T}{\gamma_X + \sigma_A \phi_T}(1 - e^{-(\gamma_X + \sigma_A \phi_T)t}) \tag{12}$$

3   Equilibrium time

The equilibrium time for xenon should be when the concentration achieves a steady state; i.e., the rate of change in the concentration is zero.

Then, compute the derivative of the xenon concentration model in eq. 12 -

$$\frac{dX}{dt} =$$
$$+ [\frac{\gamma_I \Sigma_F \phi_T}{\gamma_X + \sigma_A \phi_T}]((\gamma_X + \sigma_A \phi_T)e^{-(\gamma_X + \sigma_A \phi_T)t})$$
$$- [\frac{\gamma_I \Sigma_F \phi_T}{\gamma_X + \sigma_A \phi_T - \lambda_I}]((\gamma_X + \sigma_A \phi_T)e^{-(\gamma_X + \sigma_A \phi_T)t} - \lambda_I e^{-\lambda_I t})$$
$$+ [\frac{\gamma_X \Sigma_F \phi_T}{\gamma_X + \sigma_A \phi_T}]((\gamma_X + \sigma_A \phi_T)e^{-(\gamma_X + \sigma_A \phi_T)t}) \tag{13}$$

Multiply out all the terms -

$$\frac{dX}{dt} =$$
$$+ \gamma_I \Sigma_F \phi_T e^{-(\gamma_X + \sigma_A \phi_T)t}$$
$$- \frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)}e^{-(\gamma_X + \sigma_A \phi_T)t}$$
$$+ \frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)}e^{-\lambda_I t}$$
$$+ \gamma_X \Sigma_F \phi_T e^{-(\gamma_X + \sigma_A \phi_T)t} \tag{14}$$

Let $\frac{dX}{dt} = 0$ and group the terms by exponential -

$$(\gamma_I \Sigma_F \phi_T + \gamma_X \Sigma_F \phi_T - \frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)})e^{-(\gamma_X + \sigma_A \phi_T)t} + \frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)}e^{-\lambda_I t} = 0 \qquad (15)$$

The next series of steps is just some algebra -

$$- (\gamma_I \Sigma_F \phi_T + \gamma_X \Sigma_F \phi_T - \frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)})e^{-(\gamma_X + \sigma_A \phi_T)t} = \frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)}e^{-\lambda_I t}$$

$$(-\gamma_I \Sigma_F \phi_T - \gamma_X \Sigma_F \phi_T + \frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)})e^{-(\gamma_X + \sigma_A \phi_T)t} = \frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)}e^{-\lambda_I t} \qquad (16)$$

$$(\frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)} - \gamma_I \Sigma_F \phi_T - \gamma_X \Sigma_F \phi_T)e^{-(\gamma_X + \sigma_A \phi_T)t} = \frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)}e^{-\lambda_I t} \qquad (17)$$

Divide through eq. 17 by $e^{-\lambda_I t}$ and $(\frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)} - \gamma_I \Sigma_F \phi_T - \gamma_X \Sigma_F \phi_T)$ -

$$\frac{e^{-(\gamma_X + \sigma_A \phi_T)t}}{e^{-\lambda_I t}} = \frac{\frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)}}{(\frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)} - \gamma_I \Sigma_F \phi_T - \gamma_X \Sigma_F \phi_T)} \qquad (18)$$

Compute the exponential division -

$$e^{(\lambda_I - (\gamma_X + \sigma_A \phi_T))t} = \frac{\frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)}}{(\frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)} - \gamma_I \Sigma_F \phi_T - \gamma_X \Sigma_F \phi_T)} \qquad (19)$$

Compute the natural logarithm -

$$(\lambda_I - (\gamma_X + \sigma_A \phi_T))t = ln[\frac{\frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)}}{(\frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)} - \gamma_I \Sigma_F \phi_T - \gamma_X \Sigma_F \phi_T)}] \qquad (20)$$

Solve for time (t) -

$$t = \frac{1}{\lambda_I - (\gamma_X + \sigma_A \phi_T)} \cdot ln\left[\frac{\frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)}}{\left(\frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)} - \gamma_I \Sigma_F \phi_T - \gamma_X \Sigma_F \phi_T\right)}\right] \tag{21}$$

Simplify $ln[\cdot]$ -

$$\frac{\frac{(\gamma_I \Sigma_F \phi_T)(\lambda_I)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)} \cdot (\gamma_X + \sigma_A \phi_T - \lambda_I)}{\left(\frac{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T)}{(\gamma_X + \sigma_A \phi_T - \lambda_I)} \cdot (\gamma_X + \sigma_A \phi_T - \lambda_I) - \gamma_I \Sigma_F \phi_T(\gamma_X + \sigma_A \phi_T - \lambda_I) - \gamma_X \Sigma_F \phi_T(\gamma_X + \sigma_A \phi_T - \lambda_I)\right)} \tag{22}$$

$$\frac{\gamma_I \lambda_I \Sigma_F \phi_T}{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T) - \gamma_I \Sigma_F \phi_T(\gamma_X + \sigma_A \phi_T - \lambda_I) - \gamma_X \Sigma_F \phi_T(\gamma_X + \sigma_A \phi_T - \lambda_I)} \tag{23}$$

$$\frac{\gamma_I \lambda_I \Sigma_F \phi_T}{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T) - (\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T - \lambda_I) - (\gamma_X \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T - \lambda_I)} \tag{24}$$

$$\frac{\gamma_I \lambda_I \Sigma_F \phi_T}{(\gamma_I \Sigma_F \phi_T)((\gamma_X + \sigma_A \phi_T) - (\gamma_X + \sigma_A \phi_T - \lambda_I)) - (\gamma_X \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T - \lambda_I)} \tag{25}$$

$$\frac{\gamma_I \lambda_I \Sigma_F \phi_T}{(\gamma_I \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T - \gamma_X - \sigma_A \phi_T + \lambda_I) - (\gamma_X \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T - \lambda_I)} \tag{26}$$

$$\frac{\gamma_I \lambda_I \Sigma_F \phi_T}{\gamma_I \lambda_I \Sigma_F \phi_T - (\gamma_X \Sigma_F \phi_T)(\gamma_X + \sigma_A \phi_T - \lambda_I)} \tag{27}$$

5

$$\frac{\gamma_I \lambda_I \Sigma_F (\phi_T)}{\gamma_I \lambda_I \Sigma_F (\phi_T) - (\gamma_X \Sigma_F (\phi_T))(\gamma_X + \sigma_A \phi_T - \lambda_I)} \tag{28}$$

$$\frac{\gamma_I \lambda_I \Sigma_F}{\gamma_I \lambda_I \Sigma_F - \gamma_X \Sigma_F (\gamma_X + \sigma_A \phi_T - \lambda_I)} \tag{29}$$

Finally, substitute eq. 29 back into eq. 30 -

$$t = \frac{1}{\lambda_I - (\gamma_X + \sigma_A \phi_T)} \cdot ln[\frac{\gamma_I \lambda_I \Sigma_F}{\gamma_I \lambda_I \Sigma_F - \gamma_X \Sigma_F (\gamma_X + \sigma_A \phi_T - \lambda_I)}] \tag{30}$$