Xenon Equilibrium Equations

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1 Mathematical models

1.1 Iodine

$\frac{dI}{dt} = \gamma_I \Sigma_F \phi_T - \lambda_I I$
$I(0) = 0$

1.2 Xenon

$\frac{dX}{dt} = \lambda_I I + \gamma_X \Sigma_F \phi_T - \lambda_X X - \sigma_A \phi_T X$
$X(0) = 0$

2 Solutions

2.1 Iodine

Laplace
$\frac{dI}{dt} = \gamma_I \Sigma_F \phi_T - \lambda_I I$

$s\tilde{I} = \frac{1}{s}\gamma_I \Sigma_F \phi_T - \lambda_I \tilde{I}$

Rearrange
$\tilde{I} = \frac{1}{s(s+\lambda_I)}\gamma_I \Sigma_F \phi_T$

Solve
$I(t) = \frac{\gamma_I \Sigma_F \phi_T}{\lambda_I}(1 - e^{-\lambda_I t})$

2.2 Xenon

Laplace
$\frac{dX}{dt} = \lambda_I \frac{\gamma_I \Sigma_F \phi_T}{\lambda_I}(1 - e^{-\lambda_I t}) + \gamma_X \Sigma_F \phi_T - \lambda_X X - \sigma_A \phi_T X$

$\frac{dX}{dt} = \gamma_I \Sigma_F \phi_T (1 - e^{-\lambda_I t}) + \gamma_X \Sigma_F \phi_T - \lambda_X X - \sigma_A \phi_T X$

$\frac{dX}{dt} = \gamma_I \Sigma_F \phi_T - \gamma_I \Sigma_F \phi_T e^{-\lambda_I t} + \gamma_X \Sigma_F \phi_T - \lambda_X X - \sigma_A \phi_T X$

$\frac{dX}{dt} = \gamma_I \Sigma_F \phi_T - \gamma_I \Sigma_F \phi_T e^{-\lambda_I t} + \gamma_X \Sigma_F \phi_T - (\lambda_X + \sigma_A \phi_T) X$

$s\tilde{X} = \frac{1}{s}\gamma_I \Sigma_F \phi_T - \frac{1}{s+\lambda_I}\gamma_I \Sigma_F \phi_T + \frac{1}{s}\gamma_X \Sigma_F \phi_T - (\gamma_X + \sigma_A \phi_T)\tilde{X}$

Rearrange
$(s + [\gamma_X + \sigma_A \phi_T])\tilde{X} = \frac{1}{s}\gamma_I \Sigma_F \phi_T - \frac{1}{s+\lambda_I}\gamma_I \Sigma_F \phi_T + \frac{1}{s}\gamma_X \Sigma_F \phi_T$

$\tilde{X} = \frac{1}{s(s+[\gamma_X+\sigma_A\phi_T])}\gamma_I \Sigma_F \phi_T - \frac{1}{(s+\lambda_I)(s+[\gamma_X+\sigma_A\phi_T])}\gamma_I \Sigma_F \phi_T + \frac{1}{s(s+[\gamma_X+\sigma_A\phi_T])}\gamma_X \Sigma_F \phi_T$

Solve
$X(t) = \frac{\gamma_I \Sigma_F \phi_T}{\gamma_X+\sigma_A\phi_T}(1-e^{-(\gamma_X+\sigma_A\phi_T)t}) - \frac{\gamma_I \Sigma_F \phi_T}{\gamma_X+\sigma_A\phi_T-\lambda_I}(e^{-\lambda_I t} - e^{-(\gamma_X+\sigma_A\phi_T)t}) + \frac{\gamma_X \Sigma_F \phi_T}{\gamma_X+\sigma_A\phi_T}(1-e^{-(\gamma_X+\sigma_A\phi_T)t})$

3 Equilibrium time

$$X(t) = \frac{\gamma_I\Sigma_F\phi_T}{\gamma_X+\sigma_A\phi_T}(1-e^{-(\gamma_X+\sigma_A\phi_T)t})-\frac{\gamma_I\Sigma_F\phi_T}{\gamma_X+\sigma_A\phi_T-\lambda_I}(e^{-\lambda_I t}-e^{-(\gamma_X+\sigma_A\phi_T)t})+\frac{\gamma_X\Sigma_F\phi_T}{\gamma_X+\sigma_A\phi_T}(1-e^{-(\gamma_X+\sigma_A\phi_T)t})$$

$$\frac{dX}{dt} = [\frac{\gamma_I\Sigma_F\phi_T}{\gamma_X+\sigma_A\phi_T}]((\gamma_X+\sigma_A\phi_T)e^{-(\gamma_X+\sigma_A\phi_T)t})-[\frac{\gamma_I\Sigma_F\phi_T}{\gamma_X+\sigma_A\phi_T-\lambda_I}]((\gamma_X+\sigma_A\phi_T)e^{-(\gamma_X+\sigma_A\phi_T)t}-\lambda_I e^{-\lambda_I t})$$
$$+[\frac{\gamma_X\Sigma_F\phi_T}{\gamma_X+\sigma_A\phi_T}]((\gamma_X+\sigma_A\phi_T)e^{-(\gamma_X+\sigma_A\phi_T)t})$$

$$\frac{dX}{dt} = \gamma_I\Sigma_F\phi_T e^{-(\gamma_X+\sigma_A\phi_T)t}$$
$$-\frac{(\gamma_I\Sigma_F\phi_T)(\gamma_X+\sigma_A\phi_T)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}e^{-(\gamma_X+\sigma_A\phi_T)t}$$
$$+\frac{(\gamma_I\Sigma_F\phi_T)(\lambda_I)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}e^{-\lambda_I t}$$
$$+\gamma_X\Sigma_F\phi_T e^{-(\gamma_X+\sigma_A\phi_T)t}$$

$$(\gamma_I\Sigma_F\phi_T+\gamma_X\Sigma_F\phi_T-\frac{(\gamma_I\Sigma_F\phi_T)(\gamma_X+\sigma_A\phi_T)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)})e^{-(\gamma_X+\sigma_A\phi_T)t}+\frac{(\gamma_I\Sigma_F\phi_T)(\lambda_I)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}e^{-\lambda_I t}=0$$

$$-(\gamma_I\Sigma_F\phi_T+\gamma_X\Sigma_F\phi_T-\frac{(\gamma_I\Sigma_F\phi_T)(\gamma_X+\sigma_A\phi_T)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)})e^{-(\gamma_X+\sigma_A\phi_T)t}=\frac{(\gamma_I\Sigma_F\phi_T)(\lambda_I)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}e^{-\lambda_I t}$$

$$(-\gamma_I\Sigma_F\phi_T-\gamma_X\Sigma_F\phi_T+\frac{(\gamma_I\Sigma_F\phi_T)(\gamma_X+\sigma_A\phi_T)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)})e^{-(\gamma_X+\sigma_A\phi_T)t}=\frac{(\gamma_I\Sigma_F\phi_T)(\lambda_I)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}e^{-\lambda_I t}$$

$$(\frac{(\gamma_I\Sigma_F\phi_T)(\gamma_X+\sigma_A\phi_T)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}-\gamma_I\Sigma_F\phi_T-\gamma_X\Sigma_F\phi_T)e^{-(\gamma_X+\sigma_A\phi_T)t}=\frac{(\gamma_I\Sigma_F\phi_T)(\lambda_I)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}e^{-\lambda_I t}$$

$$\frac{e^{-(\gamma_X+\sigma_A\phi_T)t}}{e^{-\lambda_I t}}=\frac{\frac{(\gamma_I\Sigma_F\phi_T)(\lambda_I)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}}{(\frac{(\gamma_I\Sigma_F\phi_T)(\gamma_X+\sigma_A\phi_T)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}-\gamma_I\Sigma_F\phi_T-\gamma_X\Sigma_F\phi_T)}$$

$$e^{(\lambda_I-(\gamma_X+\sigma_A\phi_T))t}=\frac{\frac{(\gamma_I\Sigma_F\phi_T)(\lambda_I)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}}{(\frac{(\gamma_I\Sigma_F\phi_T)(\gamma_X+\sigma_A\phi_T)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}-\gamma_I\Sigma_F\phi_T-\gamma_X\Sigma_F\phi_T)}$$

$$(\lambda_I-(\gamma_X+\sigma_A\phi_T))t=ln[\frac{\frac{(\gamma_I\Sigma_F\phi_T)(\lambda_I)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}}{(\frac{(\gamma_I\Sigma_F\phi_T)(\gamma_X+\sigma_A\phi_T)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}-\gamma_I\Sigma_F\phi_T-\gamma_X\Sigma_F\phi_T)}]$$

$$t=\frac{1}{\lambda_I-(\gamma_X+\sigma_A\phi_T)}\cdot ln[\frac{\frac{(\gamma_I\Sigma_F\phi_T)(\lambda_I)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}}{(\frac{(\gamma_I\Sigma_F\phi_T)(\gamma_X+\sigma_A\phi_T)}{(\gamma_X+\sigma_A\phi_T-\lambda_I)}-\gamma_I\Sigma_F\phi_T-\gamma_X\Sigma_F\phi_T)}]$$