PHYS133 – Lab report

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Abstract

Radioactive sources decay randomly over time, with a fixed probability. The half life describes the time taken for a sample to halve its count rate. The count rates over a short period of time can be modelled by statistical distributions, most commonly Poisson, in order to model the spread of decays and calculate the Poisson standard deviation from this. The exact values of standard dev. and count rate for our source, U-238, were used to test the Poisson's fit and determine the 2 values of standard deviation. Here it was shown a count rate $R = 1.08 \pm 0.06$ Bq and $\sigma_{hist} = 1.71$. Our Poisson parameter was $m = 1.80$. This is close to our standard deviation within the histogram, suggesting a good fit to the data, suggesting the beta decay of U-238 is random. This is reflective with the generally accepted model of radioactive decays, which appear random over single time intervals. The use of random distributions such as Poisson can model this data over a set interval.

1 Introduction

Unstable elements undergo radioactive decay over their lifetime in order to reach stability. This occurs through 3 types of decay, alpha, beta and gamma, represented by their Greek alphabet symbols, α, β, γ. These describe emission of ${}^{4}_{2}$He, ${}^{0}_{-1}$e, and Gamma waves respectively from the nucleus of the unstable element, with a combination of these decays present until a stable state is reached. These decays can occur over a long period of time, and to measure and compare this we investigate an elements half life of decay, which describes the time it takes for the count rate, R, of a substance to fall to half its initial value. These decays are random within an arbitrary short time period due to the fixed probability of decay. Due to the long half lives (e.g. the half life of ${}^{4}_{2}$He is $\cdots$ REF!!) they are hard to accurately measure. Instead, the probability of decay can be modelled from several repeated smaller time periods. This can be modelled statistically to determine the averages specific to that element's decay. Statistical models such as Poisson are especially effective as they describe a discrete rate (here of decay) and have a flexible distribution that can be fit to a spread of data, making it the ideal choice to test for the count rate of a substance in a small time period.

This experiment aimed to investigate the random nature of beta decay, and test how it can be modelled by a Poisson distribution, equating the means and calculating the respective standard deviations. The difference between these as well as the overall shape of the distribution provide quantitative and qualitative analysis on the goodness of fit of the Poisson in this case. This is further extended with the inclusion of Chi squared goodness of fit tests, to determine whether the fit is statistically significant. Here it was shown a count rate $R = 1.08 \pm 0.06$ Bq and $\sigma_{hist} = 1.71$. Our Poisson parameter was $m = 1.80$. Our Chi squared value was – suggesting the model is an appropriate fit for our data. These standard deviations are close together suggesting the model is a good fit however ...

2 background/Theory

The concept of half life is only briefly discussed in this report, however it can be helpful to have a basic understanding of the concept. The half life is defined as,

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} \tag{1}$$

Where $t_{\frac{1}{2}}$ is the time for the number of particles to fall to half of it's original value, and λ is the decay constant, the probability per decay. This relationship can be obtained from treating the decay as exponential,

$$N = N_0 e^{-\lambda t}$$

for N the number of particles and the t time with λ as above.
These elements are in a initial state of instability, and they decay to a more stable state. The nature of the instability and the structure of the atomic nuclei determines the type of the decay. For example, elements with a larger number of neutrons compared to protons generally decay via β^- decay, hence a neutron will decay into a proton with the release of an electron and electron antineutrino.

In order to calculate the count rate, R,

$$R = \frac{N}{T} \tag{2}$$

Where R is the count rate, T is the total time interval, N is the total counts. This follows fairly intuitively from the definition. The Poisson distribution will be used to model our data. The defining equation for the distribution is,

$$P(N) = \frac{e^{-m} m^n}{N!} \tag{3}$$

Where P is the probability of obtaining N counts, and m is the mean. As the Poisson is a random variable distribution, the mean, m and standard deviation $\sigma_{Poisson}$ is defined by,

$$\sigma = \sqrt{m} \tag{4}$$

We will use this to compare against the mean of the raw data according to the histogram. This is computed as the main of the whole data array.

3 Experimental method

Before taking readings of the radioactive source, the background count was first measured. To do this, the number of decayed nuclei were measured by a digital counter, connected up to a Geiger-Müller tube, in an interval of 100 seconds.

From this, the background counts per second, or count rate in Becquerel (Bq) was calculated. This can be later considered when working out the average count rate in our data.
In order to test the random fluctuations, the ^{238}U source was carefully placed at a distance from

Figure 1: "U-238 source placed with tweezers on blue holding tray, distance d from the Geiger-Müller tube which is connected to the shielded housing and electronic counter. This takes 100 * 3 s readings of counts from the β decay of the source"

Figure 2: "Figure 2 illustrates the Poisson curve, fit ontop of the histogram with associated error bars. The Possion's parameter was calculated as, and $\sigma_{hist} =$"

the Geiger-Müller tube such that the count rate would be <10 for any 3 second interval. In testing this was roughly 8cm. Due to the hypothesised nature of the counts in 3s, this doesn't need to be extensively measured as the probability for larger count rates falls quickly towards zero regardless of distance. Care should always be taken when handling radioactive sources.
The count rate in 3s was then measured 100 times, ensuring the timer and counter was reset each measurement and the source was kept at uniform distance from the detector. This was achieved by placing the source on one of the adjustable shelves in the protective housing, which ensured the source was shielded when working with it for longer time intervals, as well as providing a stand for the Geiger-Müller tube. The data was collected raw and processed accordingly by computer software.

4 Results and analysis/discussion

The data was plotted on a histogram (fig. 2) with integer bins, and error bars calculated based on the square root of the individual bin frequencies. This is due to the random error in decay measurements.
To analyse the data, the software calculated mean of our data, 3.25, and using equation (4), the histogram's standard deviation was calculated, $\sigma_{hist} = 1.8(2.sf.)$. Following this, SciPy's Poisson and fit functions were used to fit a Poisson curve (based on eq.3) on top of the histogram. This gave a Poisson curve, mean ===, and by eq. 4, standard deviation, $\sigma_{Poisson}$ and the curve was then smoothed from Poisson's integer inputs. It can be estimated qualitatively that the curve fits fairly well to the overall shape, however further investigation of the Poisson parameter compared to the histogram mean reveals,

5 Conclusions

6 References

7 Appendices

Calculating R,

$$R = \frac{N}{T} \pm \frac{\sqrt{N}}{T}$$

Where R is the count rate, T is the total time interval, N is the total counts. With values $N = (32.5 \pm 2) * 10^1$, $T = 3 * 100[s]$, gives

$$R = 1.08 \pm 0.06$$

Bq

Count	**Frequency**
0	1
1	14
2	25
3	18
4	20
5	11
6	6
7	4
8	1

Table 1: Binned data