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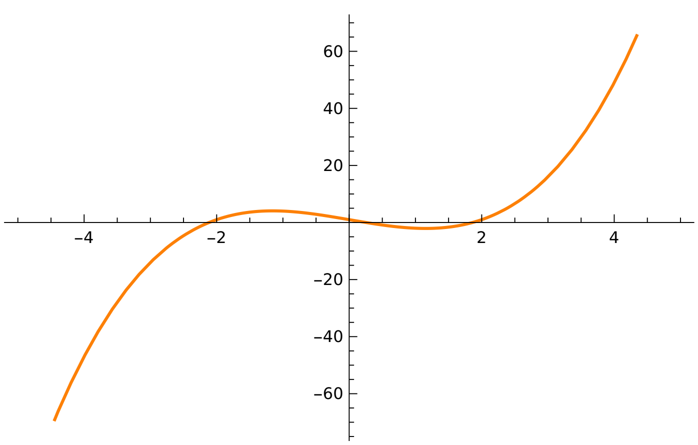
1. Bisection Method

(a)

In[3]:=
```
bisectionMethod[a0_, b0_, n_, f_] := Module[{},
a = N[a0];
b = N[b0];
m = (a + b) / 2.0;
If[f[a] * f[b] > 0,
Print["Bisection method can not be applied as f(a).f(b)>0"];
Return[]];
For[i = 1, i ≤ n, i++,
If[Sign[f[a]] == Sign[f[m]], a = m, b = m];
m = (a + b) / 2.0;
Print["After iteration :", i, "Root=", m];
];
Print["Accuracy=", Abs[(b - a) / 2.0]];
Print["Approximate root of the equation is=",
NumberForm[m, 8]]];
f1[x_] = x^3 - 4 * x + 1;
Plot[f1[x], {x, -5, 5}]
bisectionMethod[1, 2, 5, f1]
```

Out[5]=



After iteration : 1 Root= 1.75

After iteration : 2 Root= 1.875

After iteration : 3 Root= 1.8125

After iteration : 4 Root= 1.84375

After iteration : 5 Root= 1.85938

Accuracy= 0.015625

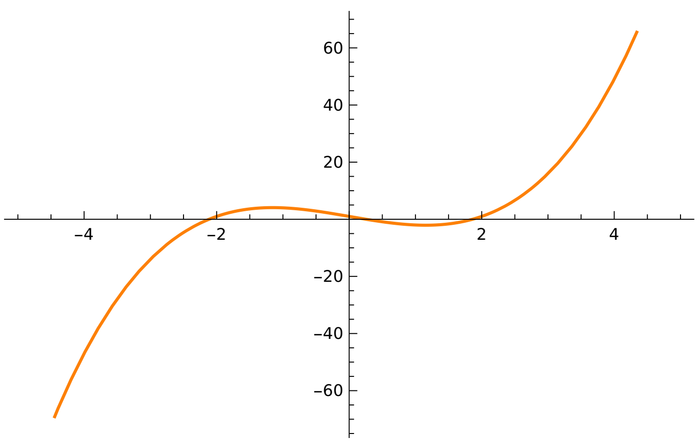
Approximate root of the equation is= 1.859375

(b)

In[7]:=

```
bisectionMethodEps[a0_, b0_, eps_, f_] := Module[{},
a = N[a0];
b = N[b0];
m = (a + b) / 2.0;
If[f[a] * f[b] > 0,
Print["Bisection method can not be applied as f(a).f(b)>0"];
Return[]];
For[i = 1, Abs[b - a] > eps && i < 100 000, i++,
If[Sign[f[a]] == Sign[f[m]], a = m, b = m];
m = (a + b) / 2.0;
Print["After iteration :", i, "Root=", m];
];
Print["Accuracy=", Abs[(b - a) / 2.0]];
Print["Approximate root of the equation is=", m]];
f1[x_] = x^3 - 4 * x + 1;
Plot[f1[x], {x, -5, 5}]
bisectionMethodEps[1, 2, 0.0001, f1]
```

Out[9]=



After iteration : 1 Root= 1.75

After iteration : 2 Root= 1.875

After iteration : 3 Root= 1.8125

After iteration : 4 Root= 1.84375

After iteration : 5 Root= 1.85938

After iteration : 6 Root= 1.86719

After iteration : 7 Root= 1.86328

After iteration : 8 Root= 1.86133

After iteration : 9 Root= 1.86035

After iteration : 10 Root= 1.86084

After iteration : 11 Root= 1.8606

After iteration : 12 Root= 1.86072

After iteration : 13 Root= 1.86078

After iteration : 14 Root= 1.86081

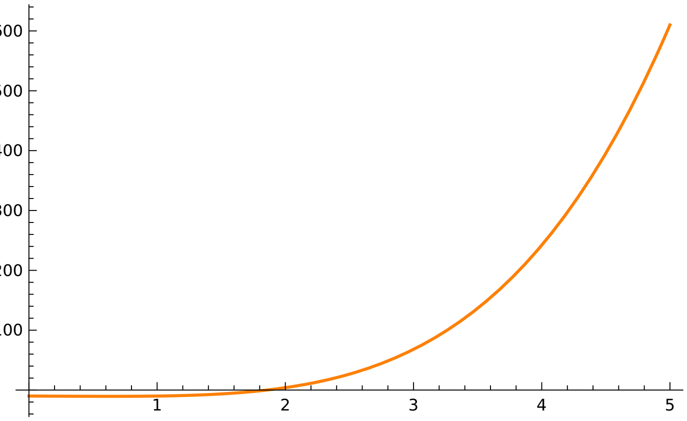
Accuracy= 0.0000305176

Approximate root of the equation is= 1.86081

(c)

In[54]:=
```
bisectionMethodEps[a0_, b0_, eps_, f_] := Module[{},
a = N[a0];
b = N[b0];
m = (a + b)/2.0;
If[f[a]*f[b] > 0,
Print["Bisection method can not be applied as f(a).f(b)>0"];
Return[]];
For[i = 1, Abs[b - a] > eps && i < 100000, i++,
If[Sign[f[a]] == Sign[f[m]], a = m, b = m];
m = (a + b)/2.0;
Print["After iteration :", i, "Root=", m];
];
Print["Accuracy=", Abs[(b - a)/2.0]];
Print["Approximate root of the equation is=", m]];
f1[x_] = x^4 - x - 10;
Plot[f1[x], {x, 0, 5}]
bisectionMethodEps[1, 2, 0.0001, f1]
```

Out[56]=



After iteration :1Root=1.75

After iteration :2Root=1.875

After iteration :3Root=1.8125

After iteration :4Root=1.84375

After iteration :5Root=1.85938

After iteration :6Root=1.85156

After iteration :7Root=1.85547

After iteration :8Root=1.85742

After iteration :9Root=1.85645

After iteration :10Root=1.85596

After iteration :11Root=1.85571

After iteration :12Root=1.85559

After iteration :13Root=1.85553

After iteration :14Root=1.85556

Accuracy=0.0000305176

Approximate root of the equation is=1.85556

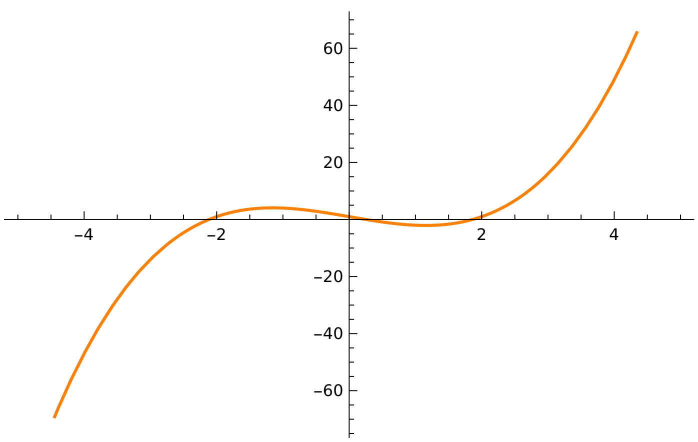
2. Secant Method

(a)

In[11]:=

```
secantMethod[a0_, b0_, n_, f_] := Module[{},
a = N[a0];
b = N[b0];
m = b - (((b - a)*f[b])/(f[b] - f[a]));
For[i = 1, i ≤ n, i++,
m = b - (((b - a)*f[b])/(f[b] - f[a]));
a = b;
b = m;
Print["After iteration :", i, "Root=",
NumberForm[m, 8]];
];
Print["Approximate root of the equation is=",
NumberForm[m, 8]];
];
f1[x_] = x^3 - 4*x + 1;
Plot[f1[x], {x, -5, 5}]
secantMethod[1, 2, 5, f1]
```

Out[13]=



After iteration : 1 Root= 1.6666667

After iteration : 2 Root= 1.8363636

After iteration : 3 Root= 1.8656906

After iteration : 4 Root= 1.8607001

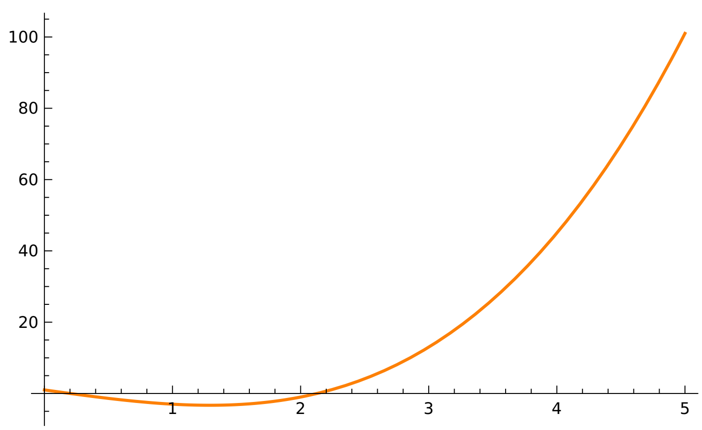
After iteration : 5 Root= 1.8608054

Approximate root of the equation is= 1.8608054

(b)

In[5]:=
```
secantMethod[a0_, b0_, n_, f_] := Module[{},
a = N[a0];
b = N[b0];
m = b - (((b - a) * f[b]) / (f[b] - f[a]));
For[i = 1, i ≤ n, i++,
m = b - (((b - a) * f[b]) / (f[b] - f[a]));
a = b;
b = m;
Print["After iteration :", i, "Root=",
NumberForm[m, 8]];
];
Print["Approximate root of the equation is=",
NumberForm[m, 8]];
];
f1[x_] = x^3 - 5 * x + 1;
Plot[f1[x], {x, 0, 5}]
secantMethod[1, 2, 5, f1]
```

Out[7]=




```
After iteration :1Root=2.5
After iteration :2Root=2.097561
After iteration :3Root=2.1213395
After iteration :4Root=2.1285851
After iteration :5Root=2.1284182
Approximate root of the equation is=2.1284182
```

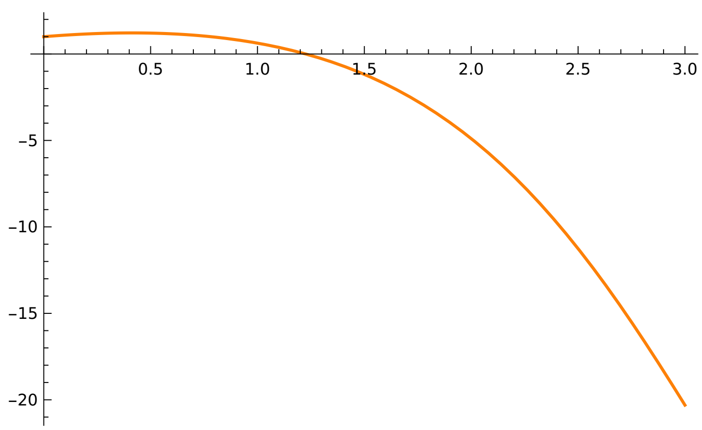
(c)

In[9]:=
```
secantMethod[a0_, b0_, n_, f_] := Module[{},
a = N[a0];
b = N[b0];
m = b - (((b - a) * f[b]) / (f[b] - f[a]));
For[i = 1, i ≤ n, i++,
m = b - (((b - a) * f[b]) / (f[b] - f[a]));
a = b;
b = m;
Print["After iteration :", i, "Root=",
NumberForm[m, 8]];
];
Print["Approximate root of the equation is=",
NumberForm[m, 8]];
];
f1[x_] = Exp[x] * Cos[x] - x * Sin[x]
Plot[f1[x], {x, 0, 3}]
secantMethod[1, 2, 5, f1]
```

Out[10]=

$e^{x} \cos(x) - x \sin(x)$

Out[11]=



After iteration :1Root=1.1136119

After iteration :2Root=1.1719943

After iteration :3Root=1.231126

After iteration :4Root=1.22512

After iteration :5Root=1.2253924

Approximate root of the equation is=1.2253924

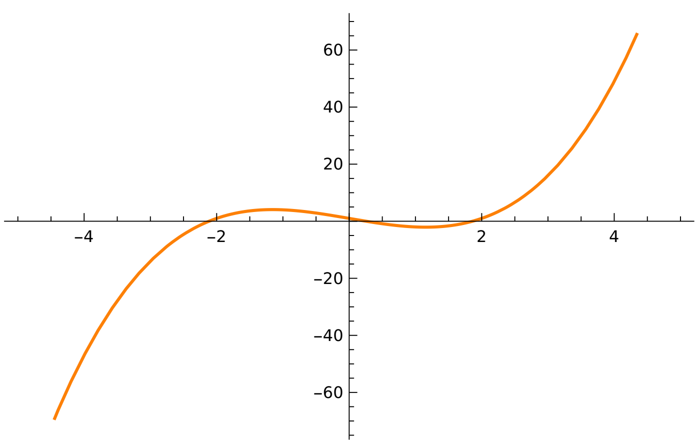
3.Regula-Falsi Method

(a)

In[15]:=

```
regulaFalsiMethod[a0_, b0_, n_, f_] := Module[{},
a = N[a0];
b = N[b0];
m = b - (((b - a) * f[b]) / (f[b] - f[a]));
If[f[a] * f[b] > 0,
Print["Regula Falsi method can not be applied as f(a).f(b)>0"];
Return[]];
For[i = 1, i ≤ n, i++,
If[Sign[f[a]] == Sign[f[m]], a = m, b = m];
m = b - (((b - a) * f[b]) / (f[b] - f[a]));
Print["After iteration :", i, "Root=", m];
];
Print["Accuracy=", Abs[(b - a) / 2.0]];
Print["Approximate root of the equation is=",
NumberForm[m, 8]]];
f1[x_] = x^3 - 4 * x + 1;
Plot[f1[x], {x, -5, 5}]
regulaFalsiMethod[1, 2, 5, f1]
```

Out[17]=



After iteration : 1 Root= 1.83636

After iteration : 2 Root= 1.85805

After iteration : 3 Root= 1.8605

After iteration : 4 Root= 1.86077

After iteration : 5 Root= 1.8608

Accuracy= 0.069614

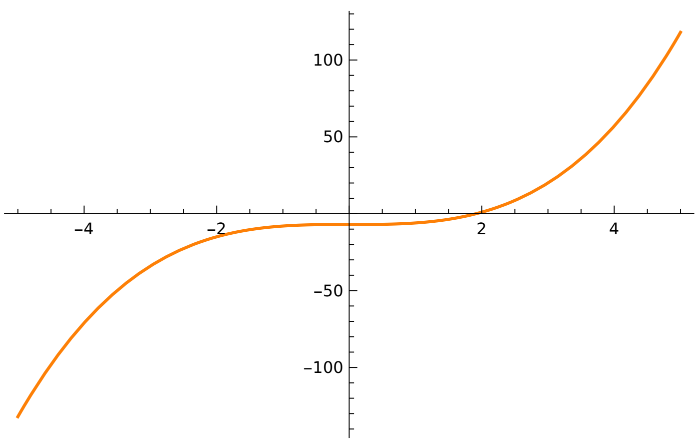
Approximate root of the equation is= 1.8608021

(b)

In[13]:=

```
regulaFalsiMethod[a0_, b0_, n_, f_] := Module[{},
a = N[a0];
b = N[b0];
m = b – (((b – a) * f[b]) / (f[b] – f[a]));
If[f[a] * f[b] > 0,
Print["Regula Falsi method can not be applied as f(a).f(b)>0"];
Return[]];
For[i = 1, i ≤ n, i++,
If[Sign[f[a]] == Sign[f[m]], a = m, b = m];
m = b – (((b – a) * f[b]) / (f[b] – f[a]));
Print["After iteration :", i, "Root=", m];
];
Print["Accuracy=", Abs[(b – a) / 2.0]];
Print["Approximate root of the equation is=",
NumberForm[m, 8]]];
f1[x_] = x^3 – 7;
Plot[f1[x], {x, –5, 5}]
regulaFalsiMethod[1, 2, 5, f1]
```

Out[15]=



After iteration :1Root=1.91042

After iteration :2Root=1.91282

After iteration :3Root=1.91293

After iteration :4Root=1.91293

After iteration :5Root=1.91293

Accuracy=0.0435345

Approximate root of the equation is=1.9129312

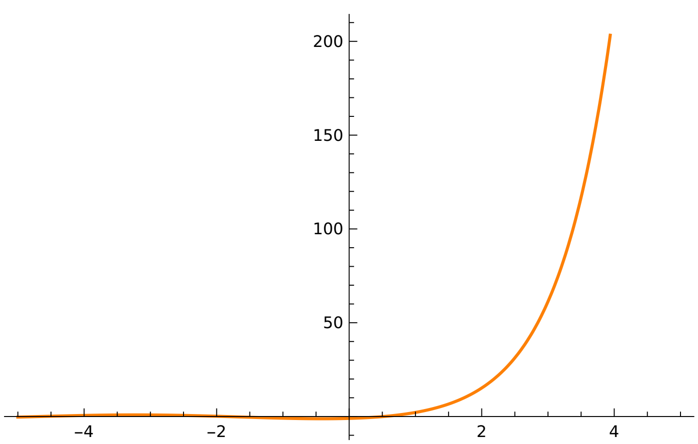
(c)

In[17]:=
```
regulaFalsiMethod[a0_, b0_, n_, f_] := Module[{},
a = N[a0];
b = N[b0];
m = b - (((b - a) * f[b]) / (f[b] - f[a]));
If[f[a] * f[b] > 0,
Print["Regula Falsi method can not be applied as f(a).f(b)>0"];
Return[]];
For[i = 1, i ≤ n, i++,
If[Sign[f[a]] == Sign[f[m]], a = m, b = m];
m = b - (((b - a) * f[b]) / (f[b] - f[a]));
Print["After iteration :", i, "Root=", m];
];
Print["Accuracy=", Abs[(b - a) / 2.0]];
Print["Approximate root of the equation is=",
NumberForm[m, 8]]];
f1[x_] = x * Exp[x] - Cos[x]
Plot[f1[x], {x, -5, 5}]
regulaFalsiMethod[0, 1, 5, f1]
```

Out[18]=

$e^{x}\,x - \cos(x)$

Out[19]=



After iteration :1Root=0.446728

After iteration :2Root=0.494015

After iteration :3Root=0.509946

After iteration :4Root=0.515201

After iteration :5Root=0.516922

Accuracy=0.242399

Approximate root of the equation is=0.51692221

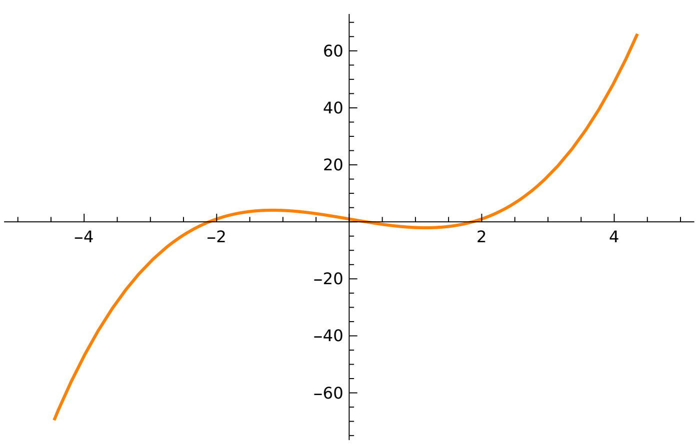
4. Newton-Raphson Method

(a)

In[19]:=

```
newtonRaphsonMethod[p0_, n_, f_] := Module[{},
p = N[p0];
For[
i = 1, i ≤ n, i++,
dp = f '[p];
If[dp == 0,
Print["newton raphson method can't be applied"];
Break[]];
p1 = p − f[p] / dp;
p = p1;
Print["After iteration :", i, "Root=",
NumberForm[p, 8]];
];
Print["Approximate root of the equation is=",
NumberForm[p, 8]]];
f1[x_] = x^3 − 4 * x + 1;
Plot[f1[x], {x, −5, 5}]
newtonRaphsonMethod[1, 5, f1]
```

Out[21]=



After iteration : 1 Root $= -1.$

After iteration : 2 Root $= 3.$

After iteration : 3 Root $= 2.3043478$

After iteration : 4 Root $= 1.9674895$

After iteration : 5 Root $= 1.8694705$

Approximate root of the equation is $= 1.8694705$

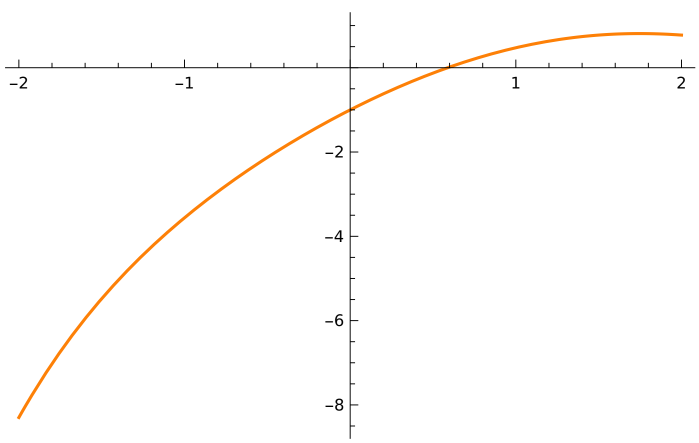
(b)

In[21]:=
```
newtonRaphsonMethod[p0_, n_, f_] := Module[{},
p = N[p0];
For[
i = 1, i ≤ n, i++,
dp = f'[p];
If[dp == 0,
Print["newton raphson method can't be applied"];
Break[]];
p1 = p - f[p]/dp;
p = p1;
Print["After iteration :", i, "Root=",
NumberForm[p, 8]];
];
Print["Approximate root of the equation is=",
NumberForm[p, 8]]];
f1[x_] = Sin[x] - Exp[-x]
Plot[f1[x], {x, -2, 2}]
newtonRaphsonMethod[0, 5, f1]
```

Out[22]=

$\sin(x) - e^{-x}$

Out[23]=



After iteration :1Root=0.5

After iteration :2Root=0.58564382

After iteration :3Root=0.58852941

After iteration :4Root=0.58853274

After iteration :5Root=0.58853274

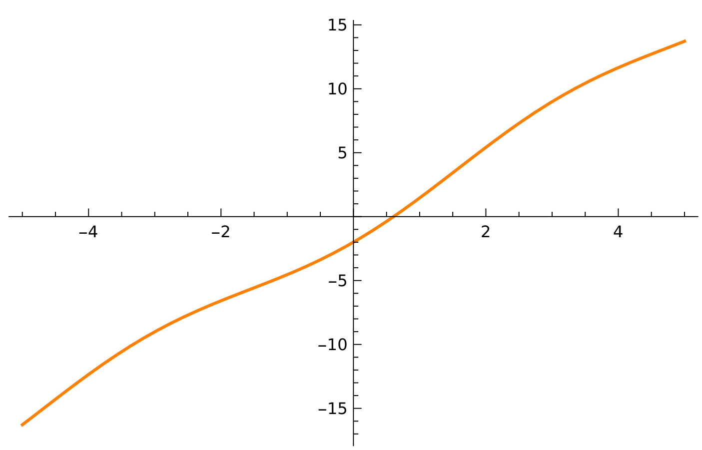
Approximate root of the equation is=0.58853274

(c)

In[25]:=

```
newtonRaphsonMethod[p0_, n_, f_] := Module[{},
p = N[p0];
For[
i = 1, i ≤ n, i++,
dp = f'[p];
If[dp == 0,
Print["newton raphson method can't be applied"];
Break[]];
p1 = p - f[p]/dp;
p = p1;
Print["After iteration :", i, "Root=",
NumberForm[p, 8]];
];
Print["Approximate root of the equation is=",
NumberForm[p, 8]]];
f1[x_] = 3*x - Cos[x] - 1;
Plot[f1[x], {x, -5, 5}]
newtonRaphsonMethod[0, 5, f1]
```

Out[27]=



After iteration :1Root=0.66666667

After iteration :2Root=0.60749285

After iteration :3Root=0.60710167

After iteration :4Root=0.60710165

After iteration :5Root=0.60710165

Approximate root of the equation is=0.60710165

5. Gaussian elimination Method (a)

In[34]:=

```
Solve [{4 x1 + 2 x2 - x3 == 1, 2 x1 + 4 x2 + x3 == -1, -x + x2 + 4 x3 == 1}, {x1, x2, x3}]
A = {{4, 2, -1}, {2, 4, 1}, {-1, 1, 4}};
b = {1, -1, 1};
aug = {{4, 2, -1, 1}, {2, 4, 1, -1}, {-1, 1, 4, 1}};
RowReduce[aug]
Solve[{x1 == 5/6, x2 == -5/6, x3 == 2}, {x1, x2, x3}]
A = {{1, 2, 3}, {2, -1, -1}, {1, 1, -1}};
b = {1, 0, 2};
aug = {{1, 2, 3, 1}, {2, -1, -1, 0}, {1, 1, -1, 2}};
RowReduce[aug]
Solve[{x1 == 4/13, x2 == 15/13, x3 == -7/13}, {x1, x2, x3}]
```

Out[34]=

$$\left\{\left\{x1 \to \frac{5+x}{7},\ x2 \to \frac{1}{7}(-5-x),\ x3 \to \frac{1}{7}(3+2x)\right\}\right\}$$

Out[38]=

$$\left\{\left\{1, 0, 0, \frac{5}{6}\right\}, \left\{0, 1, 0, -\frac{5}{6}\right\}, \left\{0, 0, 1, \frac{2}{3}\right\}\right\}$$

Out[39]=

$$\left\{\left\{x1 \to \frac{5}{6},\ x2 \to -\frac{5}{6},\ x3 \to 2\right\}\right\}$$

Out[43]=

$$\left\{\left\{1, 0, 0, \frac{4}{13}\right\}, \left\{0, 1, 0, \frac{15}{13}\right\}, \left\{0, 0, 1, -\frac{7}{13}\right\}\right\}$$

Out[44]=

$$\left\{\left\{x1 \to \frac{4}{13},\ x2 \to \frac{15}{13},\ x3 \to -\frac{7}{13}\right\}\right\}$$

(b)

```
In[1]:= Solve[{x1 + 3 x2 - 2 x3 == 5, 3 x1 + 5 x2 + 6 x3 == 7, 2 x + 4 x2 + 3 x3 == 8}, {x1, x2, x3}]
        A = {{1, 3, -2}, {3, 5, 6}, {2, 4, 3}};
        b = {5, 7, 8};
        aug = {{1, 3, -2, 5}, {3, 5, 6, 7}, {2, 4, 3, 8}};
        MatrixForm[aug]
        RowReduce[aug]
        Solve[{x1 == -15, x2 == 8, x3 == 2}, {x1, x2, x3}]
```

Out[1]= $\left\{\left\{x1 \to \frac{1}{15}\left(14\,x - 15\right),\ x2 \to -\frac{2}{5}\,(x - 5),\ x3 \to -\frac{2\,x}{15}\right\}\right\}$

Out[5]//MatrixForm=

$$\begin{pmatrix} 1 & 3 & -2 & 5 \\ 3 & 5 & 6 & 7 \\ 2 & 4 & 3 & 8 \end{pmatrix}$$

Out[6]= $\begin{pmatrix} 1 & 0 & 0 & -15 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 2 \end{pmatrix}$

Out[7]= $\{\{x1 \to -15,\ x2 \to 8,\ x3 \to 2\}\}$

(c)

In[8]:=
```
Solve[{2 x1 + x2 + x3 == 10, 3 x1 + 2 x2 + 3 x3 == 18, x + 4 x2 + 9 x3 == 16}, {x1, x2, x3}]
A = {{2, 1, 1}, {3, 2, 3}, {1, 4, 9}};
b = {10, 18, 16};
aug = {{2, 1, 1, 10}, {3, 2, 3, 18}, {1, 4, 9, 16}};
MatrixForm[aug]
RowReduce[aug]
Solve[{x1 == 7, x2 == -9, x3 == 5}, {x1, x2, x3}]
```

Out[8]= $\left\{\left\{x1 \to \frac{x + 14}{3},\ x2 \to -x - 2,\ x3 \to \frac{x + 8}{3}\right\}\right\}$

Out[12]//MatrixForm=

$$\begin{pmatrix} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{pmatrix}$$

Out[13]=

$$\begin{pmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 5 \end{pmatrix}$$

Out[14]=

$\{\{x1 \to 7,\ x2 \to -9,\ x3 \to 5\}\}$

6. Gauss-Jordan Method

(a)

In[115]:=

```
A = {{4, 2, -1}, {2, 4, 1}, {-1, 1, 4}};
b = {1, 2, 3};
dimA = Dimensions[A]
m = dimA[[1]]
n = dimA[[2]]
Det[A]
Y = Inverse[A]
X = Y.b
A = {{4, 2, -1}, {2, 4, 1}, {-1, 1, 4}};
b = {1, 2, 3};
gaussJordan[A_, b_] := Module[{dimA, m, n, Y, X},
dimA = Dimensions[A];
m = dimA[[1]];
n = dimA[[2]];
If[m ≠ n, Print ("Jordan Method Cannot be applied"), Return[]];
If[Det [A] == 0, Print["Jordan Method is not Applicable"]; Return[]];
Y = Inverse[A];
    X = b.Y]
```

Out[117]=

{3, 3}

Out[118]=

3

Out[119]=

3

Out[120]=

36

Out[121]=

$$\begin{pmatrix} \frac{5}{12} & -\frac{1}{4} & \frac{1}{6} \\ -\frac{1}{4} & \frac{5}{12} & -\frac{1}{6} \\ \frac{1}{6} & -\frac{1}{6} & \frac{1}{3} \end{pmatrix}$$

Out[122]=

$$\left\{\frac{5}{12}, \frac{1}{12}, \frac{5}{6}\right\}$$

(b)

In[15]:=

```
A = {{1, 3, -2}, {3, 5, 6}, {2, 4, 3}};
b = {5, 7, 8};
dimA = Dimensions[A]
m = dimA[[1]]
n = dimA[[2]]
Det[A]
Y = Inverse[A]
X = Y.b
A = {{1, 3, -2}, {3, 5, 6}, {2, 4, 3}};
b = {5, 7, 8};
gaussJordan[A_, b_] := Module[{dimA, m, n, Y, X}, dimA = Dimensions[A]; m = dimA[[1]]; n = dimA[[2]];
If[m ≠ n, Print ("Jordan Method Cannot be applied"), Return[]];
If[Det [A] == 0, Print["Jordan Method is not Applicable"]; Return[]];
Y = Inverse[A];
    X = b.Y]
```

Out[17]=

{3, 3}

Out[18]=

3

Out[19]=

3

Out[20]=

−4

Out[21]=

$$\begin{pmatrix} \frac{9}{4} & \frac{17}{4} & -7 \\ -\frac{3}{4} & -\frac{7}{4} & 3 \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{pmatrix}$$

Out[22]=

{−15, 8, 2}

(c)

In[26]:=

```
A = {{2, 1, 1}, {3, 2, 3}, {1, 4, 9}};
b = {10, 18, 16};
dimA = Dimensions[A]
m = dimA[[1]]
n = dimA[[2]]
Det[A]
Y = Inverse[A]
X = Y.b
A = {{2, 1, 1}, {3, 2, 3}, {1, 4, 9}};
b = {10, 18, 16};
gaussJordan[A_, b_] := Module[{dimA, m, n, Y, X}, dimA = Dimensions[A]; m = dimA[[1]]; n = dimA[[2]];
If[m ≠ n, Print ("Jordan Method Cannot be applied"), Return[]];
If[Det [A] == 0, Print["Jordan Method is not Applicable"]; Return[]];
Y = Inverse[A];
    X = b.Y]
```

Out[28]=

{3, 3}

Out[29]=

3

Out[30]=

3

Out[31]=

-2

Out[32]=

$$\begin{pmatrix} -3 & \frac{5}{2} & -\frac{1}{2} \\ 12 & -\frac{17}{2} & \frac{3}{2} \\ -5 & \frac{7}{2} & -\frac{1}{2} \end{pmatrix}$$

Out[33]=

{7, −9, 5}

7. Gauss-Jacobi Method

(a)

In[130]:=

```
A = {{4, 2, -1}, {2, 4, 1}, {-1, 1, 4}};
B = {1, 2, 3};
X0 = {0, 0, 0}
dimA = Dimensions[A];
m = dimA[[1]];
n = dimA[[2]];
If[m ≠ n,
Print["Gauss Jacobi Method Cannot be applied"],
Return];
D1 = DiagonalMatrix[Diagonal[A]]
L = LowerTriangularize[A] // MatrixForm
U = UpperTriangularize[A] // MatrixForm
For[i = 1, i ≤ 10, i++, X1 = LinearSolve[D1, -(L + U).X0 + B];
Print["X", i, "=", X1]]
Simplify[X1]
```

Out[132]=

{0, 0, 0}

Out[137]=

$$\begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

Out[138]//MatrixForm=

$$\begin{pmatrix} 4 & 0 & 0 \\ 2 & 4 & 0 \\ -1 & 1 & 4 \end{pmatrix}$$

Out[139]//MatrixForm=

$$\begin{pmatrix} 4 & 2 & -1 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{pmatrix}$$

$$\text{X1}=\left\{\frac{1}{4}\left(1-\left(\begin{pmatrix} 4 & 0 & 0 \\ 2 & 4 & 0 \\ -1 & 1 & 4 \end{pmatrix}+\begin{pmatrix} 4 & 2 & -1 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{pmatrix}\right).\{0,0,0\}\right),\right.$$

$$\left.\frac{1}{4}\left(2-\left(\begin{pmatrix} 4 & 0 & 0 \\ 2 & 4 & 0 \\ -1 & 1 & 4 \end{pmatrix}+\begin{pmatrix} 4 & 2 & -1 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{pmatrix}\right).\{0,0,0\}\right),\ \frac{1}{4}\left(3-\left(\begin{pmatrix} 4 & 0 & 0 \\ 2 & 4 & 0 \\ -1 & 1 & 4 \end{pmatrix}+\begin{pmatrix} 4 & 2 & -1 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{pmatrix}\right).\{0,0,0\}\right)\right\}$$

$$X2=\left\{\frac{1}{4}\left(1-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right),\right.$$

$$\left.\frac{1}{4}\left(2-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right),\ \frac{1}{4}\left(3-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right)\right\}$$

$$X3=\left\{\frac{1}{4}\left(1-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right),\right.$$

$$\left.\frac{1}{4}\left(2-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right),\ \frac{1}{4}\left(3-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right)\right\}$$

$$X4=\left\{\frac{1}{4}\left(1-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right),\right.$$

$$\left.\frac{1}{4}\left(2-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right),\ \frac{1}{4}\left(3-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right)\right\}$$

$$X5=\left\{\frac{1}{4}\left(1-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right),\right.$$

$$\left.\frac{1}{4}\left(2-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right),\ \frac{1}{4}\left(3-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right)\right\}$$

$$X6=\left\{\frac{1}{4}\left(1-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right),\right.$$

$$\left.\frac{1}{4}\left(2-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right),\ \frac{1}{4}\left(3-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right)\right\}$$

$$X7=\left\{\frac{1}{4}\left(1-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right),\right.$$

$$\left.\frac{1}{4}\left(2-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right),\ \frac{1}{4}\left(3-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right)\right\}$$

$$X8=\left\{\frac{1}{4}\left(1-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right),\right.$$

$$\left.\frac{1}{4}\left(2-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right),\ \frac{1}{4}\left(3-\left(\left(\begin{pmatrix}4&0&0\\2&4&0\\-1&1&4\end{pmatrix}+\begin{pmatrix}4&2&-1\\0&4&1\\0&0&4\end{pmatrix}\right)\right).\{0,0,0\}\right)\right\}$$

$$X9=\left\{\frac{1}{4}\left(1-\left(\left(\begin{array}{ccc}4&0&0\\2&4&0\\-1&1&4\end{array}\right)+\left(\begin{array}{ccc}4&2&-1\\0&4&1\\0&0&4\end{array}\right)\right).\{0,0,0\}\right),\right.$$
$$\left.\frac{1}{4}\left(2-\left(\left(\begin{array}{ccc}4&0&0\\2&4&0\\-1&1&4\end{array}\right)+\left(\begin{array}{ccc}4&2&-1\\0&4&1\\0&0&4\end{array}\right)\right).\{0,0,0\}\right),\frac{1}{4}\left(3-\left(\left(\begin{array}{ccc}4&0&0\\2&4&0\\-1&1&4\end{array}\right)+\left(\begin{array}{ccc}4&2&-1\\0&4&1\\0&0&4\end{array}\right)\right).\{0,0,0\}\right)\right\}$$

$$X10=\left\{\frac{1}{4}\left(1-\left(\left(\begin{array}{ccc}4&0&0\\2&4&0\\-1&1&4\end{array}\right)+\left(\begin{array}{ccc}4&2&-1\\0&4&1\\0&0&4\end{array}\right)\right).\{0,0,0\}\right),\right.$$
$$\left.\frac{1}{4}\left(2-\left(\left(\begin{array}{ccc}4&0&0\\2&4&0\\-1&1&4\end{array}\right)+\left(\begin{array}{ccc}4&2&-1\\0&4&1\\0&0&4\end{array}\right)\right).\{0,0,0\}\right),\frac{1}{4}\left(3-\left(\left(\begin{array}{ccc}4&0&0\\2&4&0\\-1&1&4\end{array}\right)+\left(\begin{array}{ccc}4&2&-1\\0&4&1\\0&0&4\end{array}\right)\right).\{0,0,0\}\right)\right\}$$

Out[141]=

$$\left\{\frac{1}{4}\left(1-\left(\left(\begin{array}{ccc}4&0&0\\2&4&0\\-1&1&4\end{array}\right)+\left(\begin{array}{ccc}4&2&-1\\0&4&1\\0&0&4\end{array}\right)\right).\{0,0,0\}\right),\right.$$
$$\left.\frac{1}{4}\left(2-\left(\left(\begin{array}{ccc}4&0&0\\2&4&0\\-1&1&4\end{array}\right)+\left(\begin{array}{ccc}4&2&-1\\0&4&1\\0&0&4\end{array}\right)\right).\{0,0,0\}\right),\frac{1}{4}\left(3-\left(\left(\begin{array}{ccc}4&0&0\\2&4&0\\-1&1&4\end{array}\right)+\left(\begin{array}{ccc}4&2&-1\\0&4&1\\0&0&4\end{array}\right)\right).\{0,0,0\}\right)\right\}$$

(b)

In[96]:=
```
A = {{2, -1, 0}, {-1, 2, -1}, {0, -1, 2}};
B = {7, 1, 1};
X0 = {0, 0, 0}
dimA = Dimensions[A];
m = dimA[[1]];
n = dimA[[2]];
If[m ≠ n,
Print["Gauss Jacobi Method Cannot be applied"],
Return];
D1 = DiagonalMatrix[Diagonal[A]]
L = LowerTriangularize[A] // MatrixForm
U = UpperTriangularize[A] // MatrixForm
For[i = 1, i ≤ 10, i++, X1 = LinearSolve[D1, -(L + U).X0 + B];
Print["X", i, "=", X1]]
```

Out[98]=

{0, 0, 0}

Out[103]=

$$\left(\begin{array}{ccc}2&0&0\\0&2&0\\0&0&2\end{array}\right)$$

Out[104]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}$$

Out[105]//MatrixForm=

$$\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\text{X1}=\left\{\frac{1}{2}\left(7-\left(\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}\right).\{0,0,0\}\right),\right.$$
$$\left.\frac{1}{2}\left(1-\left(\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}\right).\{0,0,0\}\right),\frac{1}{2}\left(1-\left(\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}\right).\{0,0,0\}\right)\right\}$$

$$\text{X2}=\left\{\frac{1}{2}\left(7-\left(\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}\right).\{0,0,0\}\right),\right.$$
$$\left.\frac{1}{2}\left(1-\left(\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}\right).\{0,0,0\}\right),\frac{1}{2}\left(1-\left(\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}\right).\{0,0,0\}\right)\right\}$$

$$\text{X3}=\left\{\frac{1}{2}\left(7-\left(\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}\right).\{0,0,0\}\right),\right.$$
$$\left.\frac{1}{2}\left(1-\left(\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}\right).\{0,0,0\}\right),\frac{1}{2}\left(1-\left(\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}\right).\{0,0,0\}\right)\right\}$$

$$\text{X4}=\left\{\frac{1}{2}\left(7-\left(\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}\right).\{0,0,0\}\right),\right.$$
$$\left.\frac{1}{2}\left(1-\left(\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}\right).\{0,0,0\}\right),\frac{1}{2}\left(1-\left(\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}\right).\{0,0,0\}\right)\right\}$$

$$\text{X5}=\left\{\frac{1}{2}\left(7-\left(\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}\right).\{0,0,0\}\right),\right.$$
$$\left.\frac{1}{2}\left(1-\left(\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}\right).\{0,0,0\}\right),\frac{1}{2}\left(1-\left(\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}\right).\{0,0,0\}\right)\right\}$$

$$\text{X6}=\left\{\frac{1}{2}\left(7-\left(\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}\right).\{0,0,0\}\right),\right.$$
$$\left.\frac{1}{2}\left(1-\left(\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}\right).\{0,0,0\}\right),\frac{1}{2}\left(1-\left(\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}+\begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}\right).\{0,0,0\}\right)\right\}$$

$$\text{X7}=\left\{\frac{1}{2}\left(7-\left(\begin{pmatrix}2 & -1 & 0\\ 0 & 2 & -1\\ 0 & 0 & 2\end{pmatrix}+\begin{pmatrix}2 & 0 & 0\\ -1 & 2 & 0\\ 0 & -1 & 2\end{pmatrix}\right).\{0,0,0\}\right),\right.$$

$$\left.\frac{1}{2}\left(1-\left(\begin{pmatrix}2 & -1 & 0\\ 0 & 2 & -1\\ 0 & 0 & 2\end{pmatrix}+\begin{pmatrix}2 & 0 & 0\\ -1 & 2 & 0\\ 0 & -1 & 2\end{pmatrix}\right).\{0,0,0\}\right), \frac{1}{2}\left(1-\left(\begin{pmatrix}2 & -1 & 0\\ 0 & 2 & -1\\ 0 & 0 & 2\end{pmatrix}+\begin{pmatrix}2 & 0 & 0\\ -1 & 2 & 0\\ 0 & -1 & 2\end{pmatrix}\right).\{0,0,0\}\right)\right\}$$

$$\text{X8}=\left\{\frac{1}{2}\left(7-\left(\begin{pmatrix}2 & -1 & 0\\ 0 & 2 & -1\\ 0 & 0 & 2\end{pmatrix}+\begin{pmatrix}2 & 0 & 0\\ -1 & 2 & 0\\ 0 & -1 & 2\end{pmatrix}\right).\{0,0,0\}\right),\right.$$

$$\left.\frac{1}{2}\left(1-\left(\begin{pmatrix}2 & -1 & 0\\ 0 & 2 & -1\\ 0 & 0 & 2\end{pmatrix}+\begin{pmatrix}2 & 0 & 0\\ -1 & 2 & 0\\ 0 & -1 & 2\end{pmatrix}\right).\{0,0,0\}\right), \frac{1}{2}\left(1-\left(\begin{pmatrix}2 & -1 & 0\\ 0 & 2 & -1\\ 0 & 0 & 2\end{pmatrix}+\begin{pmatrix}2 & 0 & 0\\ -1 & 2 & 0\\ 0 & -1 & 2\end{pmatrix}\right).\{0,0,0\}\right)\right\}$$

$$\text{X9}=\left\{\frac{1}{2}\left(7-\left(\begin{pmatrix}2 & -1 & 0\\ 0 & 2 & -1\\ 0 & 0 & 2\end{pmatrix}+\begin{pmatrix}2 & 0 & 0\\ -1 & 2 & 0\\ 0 & -1 & 2\end{pmatrix}\right).\{0,0,0\}\right),\right.$$

$$\left.\frac{1}{2}\left(1-\left(\begin{pmatrix}2 & -1 & 0\\ 0 & 2 & -1\\ 0 & 0 & 2\end{pmatrix}+\begin{pmatrix}2 & 0 & 0\\ -1 & 2 & 0\\ 0 & -1 & 2\end{pmatrix}\right).\{0,0,0\}\right), \frac{1}{2}\left(1-\left(\begin{pmatrix}2 & -1 & 0\\ 0 & 2 & -1\\ 0 & 0 & 2\end{pmatrix}+\begin{pmatrix}2 & 0 & 0\\ -1 & 2 & 0\\ 0 & -1 & 2\end{pmatrix}\right).\{0,0,0\}\right)\right\}$$

$$\text{X10}=\left\{\frac{1}{2}\left(7-\left(\begin{pmatrix}2 & -1 & 0\\ 0 & 2 & -1\\ 0 & 0 & 2\end{pmatrix}+\begin{pmatrix}2 & 0 & 0\\ -1 & 2 & 0\\ 0 & -1 & 2\end{pmatrix}\right).\{0,0,0\}\right),\right.$$

$$\left.\frac{1}{2}\left(1-\left(\begin{pmatrix}2 & -1 & 0\\ 0 & 2 & -1\\ 0 & 0 & 2\end{pmatrix}+\begin{pmatrix}2 & 0 & 0\\ -1 & 2 & 0\\ 0 & -1 & 2\end{pmatrix}\right).\{0,0,0\}\right), \frac{1}{2}\left(1-\left(\begin{pmatrix}2 & -1 & 0\\ 0 & 2 & -1\\ 0 & 0 & 2\end{pmatrix}+\begin{pmatrix}2 & 0 & 0\\ -1 & 2 & 0\\ 0 & -1 & 2\end{pmatrix}\right).\{0,0,0\}\right)\right\}$$

8. Gauss-Seidel Method

(a)

In[96]:=
```
A = {{27, 6, -1}, {6, 15, 2}, {1, 1, 54}};
B = {85, 72, 110};
D1 = DiagonalMatrix[Diagonal[A]];
L1 = LowerTriangularize[A, -1];
U1 = UpperTriangularize[A, 1];
X0 = {0, 0, 0};
For[i = 0, i < 10 , i++ ,
X1 = LinearSolve[D1 + L1, -U1.X0 + B];
Print[N[X1]];
   X0 = X1];
```

```
{3.14815, 3.54074, 1.91317}
{2.43217, 3.57204, 1.92585}
{2.42569, 3.57294, 1.92595}
{2.42549, 3.57301, 1.92595}
{2.42548, 3.57302, 1.92595}
{2.42548, 3.57302, 1.92595}
{2.42548, 3.57302, 1.92595}
{2.42548, 3.57302, 1.92595}
{2.42548, 3.57302, 1.92595}
{2.42548, 3.57302, 1.92595}
```

(b)

```
In[37]:= A = {{6, 1, 1}, {4, 8, 3}, {5, 4, -10}};
B = {105, 155, 65};
D1 = DiagonalMatrix[Diagonal[A]];
L1 = LowerTriangularize[A, -1];
U1 = UpperTriangularize[A, 1];
X0 = {0, 0, 0};
For[i = 0, i < 10 , i++ ,
X1 = LinearSolve[D1 + L1, -U1.X0 + B];
Print[N[X1]];
    X0 = X1];
```

```
{17.5, 10.625, 6.5}
{14.6458, 9.61458, 4.66875}
{15.1194, 10.0645, 5.08552}
{14.975, 9.98043, 4.97967}
{15.0066, 10.0043, 5.00504}
{14.9984, 9.99889, 4.99878}
{15.0004, 10.0003, 5.0003}
{14.9999, 9.99993, 4.99993}
{15., 10., 5.00002}
{15., 10., 5.}
```

(c)

```
In[44]:= A = {{3, 4, 8}, {1, 20, 1}, {25, 1, -5}};
B = {7, -16, 19};
D1 = DiagonalMatrix[Diagonal[A]];
L1 = LowerTriangularize[A, -1];
U1 = UpperTriangularize[A, 1];
X0 = {0, 0, 0};
For[i = 0, i < 10 , i++ ,
X1 = LinearSolve[D1 + L1, -U1.X0 + B];
Print[N[X1]];
    X0 = X1];
```

{2.33333, −0.916667, 7.68333}

{−16.9333, −0.3375, −88.5342}

{238.874, −8.31701, 1188.91}

{−3157., 97.6046, −15 769.3}

{41 923.6, −1308.52, 209 353.}

$\{-556\,527., 17\,357.9, -2.77916\times10^{6}\}$

$\{7.38796\times10^{6}, -230\,441., 3.68937\times10^{7}\}$

$\{-9.8076\times10^{7}, 3.05911\times10^{6}, -4.89768\times10^{8}\}$

$\{1.30197\times10^{9}, -4.06101\times10^{7}, 6.50173\times10^{9}\}$

$\{-1.72838\times10^{10}, 5.39103\times10^{8}, -8.63112\times10^{10}\}$

9. Lagrange Polynomial

(a)

In[109]:=

```
nodes = {0, 1, 2, 3, 4}
values = {1, 2, 4, 8, 16}
P[x_] = lagrangePolynomial[nodes, values]
lagrangePolynomial[nodes_, values_] := Module[
{xi, fi, n, m, poly},
xi = nodes;
fi = values;
n = Length[xi];
m = Length[fi];
If[n ≠ m, Print["List of points and function values are not of the same size"];
        Return[];];
For[i = 1, i ≤ n, i++, L[i_, x_] := (Product[(x - xi[[j]]) / (xi[[i]] - xi[[j]]), {j, 1, i - 1}]) *
(Product[(x - xi[[j]]) / (xi[[i]] - xi[[j]]), {j, i + 1, n}]);];
poly[x_] = Sum[(L[k, x] * fi[[k]]), {k, 1, n}];
Return[poly[x]]];
Simplify[P[x]]
Integrate[Exp[-x^2], {x, 0, 2}] // N
```

Out[109]=

{0, 1, 2, 3, 4}

Out[110]=

{1, 2, 4, 8, 16}

Out[111]=

$$\frac{1}{24}(1-x)(2-x)(3-x)(4-x)+\frac{1}{3}(2-x)(3-x)x(4-x)+$$
$$(3-x)(x-1)x(4-x)+\frac{4}{3}(x-2)(x-1)x(4-x)+\frac{2}{3}(x-3)(x-2)(x-1)x$$

Out[113]=

$$\frac{1}{24}\left(x^4-2x^3+11x^2+14x+24\right)$$

Out[114]=

0.882081

(b)

In[91]:=
```
nodes = {0, 0.5, 1, 3.5}
values = {1, 2, 1, 0}
P[x_] = lagrangePolynomial[nodes, values]
lagrangePolynomial[nodes_, values_] := Module[
{xi, fi, n, m, poly},
xi = nodes;
fi = values;
n = Length[xi];
m = Length[fi];
If[n ≠ m, Print["List of points and function values are not of the same size"];
        Return[];];
For[i = 1, i ≤ n, i++, L[i_, x_] := (Product[(x – xi[[j]]) / (xi[[i]] – xi[[j]]),  {j, 1, i – 1}]) *
(Product[(x – xi[[j]]) / (xi[[i]] – xi[[j]]),  {j, i + 1, n}]);];
poly[x_] = Sum[(L[k, x] * fi[[k]]), {k, 1, n}];
Return[poly[x]]];
Simplify[P[x]]
```

Out[91]=

{0, 0.5, 1, 3.5}

Out[92]=

{1, 2, 1, 0}

Out[105]=

lagrangePolynomial({0, 0.5, 1, 3.5}, {1, 2, 1, 0})

Out[107]=

$1.29524\, x^3 - 5.94286\, x^2 + 4.64762\, x + 1.$

10. Trapezoid rule

(a)

In[13]:=
```
trapezoidal[a_, b_, f_] := (b – a) * (f[a] * f[b]) / 2;
f[x_] := Exp[–x^2];
trapezoidal[0, 2, f] // N
```

Out[15]=

0.0183156

(b)

In[16]:=
```
trapezoidal[a_, b_, f_] := (b - a) * (f[a] * f[b]) / 2;
f1[x_] := -x^3 + 2 * x^2 - 3 * x + 13;
trapezoidal[0, 20, f1] // N
```

Out[18]=
```
-942 110.
```

(c)

In[19]:=
```
trapezoidal[a_, b_, f_] := (b - a) * (f[a] * f[b]) / 2;
f2[x_] := (1 + x)^(1/2);
trapezoidal[0, 9, f2] // N
```

Out[21]=
```
14.2302
```

---

11. Trapezoid Composite

(a)

In[129]:=
```
trapezoidalComposite[a_, b_, f_, n_] := ((b - a) / (2 * n)) * (f[a] + f[b] + 2 * Sum[f[a + (j * (b - a))], {j, 1, n - 1}]);
f[x_] := Exp[-x^2];
trapezoidalComposite[0, 2, f, 4] // N
```

Out[131]=
```
0.263737
```

(b)

In[25]:=
```
trapezoidalComposite[a_, b_, f_, n_] := ((b - a) / (2 * n)) * (f[a] + f[b] + 2 * Sum[f[a + (j * (b - a))], {j, 1, n - 1}]);
f1[x_] := (1 + x)^(1/2);
trapezoidalComposite[0, 9, f1, 9] // N
```

Out[27]=
```
51.7191
```

(c)

In[28]:=
```
trapezoidalComposite[a_, b_, f_, n_] := ((b - a) / (2 * n)) * (f[a] + f[b] + 2 * Sum[f[a + (j * (b - a))], {j, 1, n - 1}]);
f2[x_] := 1 / x^3;
trapezoidalComposite[1, 9, f2, 9] // N
```

Out[30]=

0.446564

12. Simpson's rule

(a)

In[109]:=
```
simpson[a_, b_, f_] := (b - a) * (f[a] + f[b] + 4 * f[(a + b) / 2]) / 6;
f[x_] := Exp[-x^2];
simpson[0, 2, f] // N
Integrate[f[x], {x, 0, 2}] // N
```

Out[111]=

0.829944

Out[112]=

0.882081

(b)

In[117]:=
```
simpson[a_, b_, f_] := (b - a) * (f[a] + f[b] + 4 * f[(a + b) / 2]) / 6;
f1[x_] := 1 / x;
simpson[1, 2, f1] // N
Integrate[f1[x], {x, 1, 2}] // N
```

Out[119]=

0.694444

Out[120]=

0.693147

(c)

In[113]:=

```
simpson[a_, b_, f_] := (b - a) * (f[a] + f[b] + 4 * f[(a + b) / 2]) / 6;
f2[x_] := x^2 + 2 * x + 1;
simpson[1, 2, f2] // N
Integrate[f2[x], {x, 1, 2}] // N
```

Out[115]=

6.33333

Out[116]=

6.33333

13. Euler Methods

(a)

In[78]:=

```
eulerMethodN[a0_, b0_, f_, n_, alpha_] := Module[{},
a = N[a0];
b = N[b0];
h = (b - a) / n;
ti = Table[a + (j - 1) * h, {j, 1, n + 1}];
wi = Table[0, {n + 1}];
wi[[1]] = alpha;
For[i = 1, i ≤ n, i++,
wi[[i + 1]] = wi[[i]] + h * f[ti[[i]], wi[[i]]];
Print["i=", i, ", ti=", ti[[i]], ", wi=", wi[[i]]]];];
f[t_, x_] = x - t^2 + 1;
eulerMethodN[0, 2, f, 10, 0.5]
```

i=1, ti=0., wi=0.5

i=2, ti=0.2, wi=0.8

i=3, ti=0.4, wi=1.152

i=4, ti=0.6, wi=1.5504

i=5, ti=0.8, wi=1.98848

i=6, ti=1., wi=2.45818

i=7, ti=1.2, wi=2.94981

i=8, ti=1.4, wi=3.45177

i=9, ti=1.6, wi=3.95013

i=10, ti=1.8, wi=4.42815

(b)

In[81]:=
```
eulerMethod[a0_, b0_, f_, n_, alpha_] := Module[{},
a = N[a0];
b = N[b0];
h = (b − a) / n;
ti = Table[a + (j − 1) * h, {j, 1, n + 1}];
wi = Table[0, {n + 1}];
wi[[1]] = alpha;
od = {{0, ti[[1]], wi[[1]]}};
For[i = 1, i ≤ n, i++,
wi[[i + 1]] = wi[[i]] + h * f[ti[[i]], wi[[i]]];
od = Append[od, {1, N[ti[[i + 1]]], N[wi[[i + 1]]]}]];
Print[NumberForm[TableForm[od, TableHeadings → {None, {"i", "ti", "wi"}}], 8]];];
f[t_, x_] = x − t^2 + 1;
eulerMethod[0, 2, f, 10, 0.5]
```

i	ti	wi
0	0.	0.5
1	0.2	0.8
1	0.4	1.152
1	0.6	1.5504
1	0.8	1.98848
1	1.	2.458176
1	1.2	2.9498112
1	1.4	3.4517734
1	1.6	3.9501281
1	1.8	4.4281538
1	2.	4.8657845