Homework 5

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Problem 1

To show that L is a context-free language, contruct a PDA that accepts L.
A and B are regular languages $\implies$ There exist DFAs M_A and M_B that accepts A and B respectively.
Construct the following PDA:

Algorithm 1: PDA for L

```
Initialize M_A
while M_A is not in an accept state do
    Simulate M_A
    Push an X onto the stack
Initialize M_B
while M_B is not in an accept state do
    Simulate M_B
    if Stack is empty then
        Reject
    Pop an X from the stack
if Stack is empty then
    Accept
Reject
```

This PDA correctly decides L.

Proof. Case 1: M_A pushes more X's than M_B pops $\implies$ PDA rejects.
Case 2: M_A pushes fewer X's than M_B pops $\implies$ PDA rejects.
Case 3: M_A pushes exactly the same number of X's as M_B pops $\implies$ PDA accepts.
Therefore PDA correctly decides L. $\square$

Problem 2

Prove languages are not context free using the pumping lemma for context free languages

a

Proof. Proof by contradiction: assume L is a context free language;
$\implies \exists p \geq 1$ such that every string $s \in L$ with length larger than p can be written as:

$$s = uvwxy$$

where u, v, w, x, y are substrings of s such that:

1. $|vx| \geq 1$
2. $|vwx| \leq p$
3. $\forall k \geq 0, uv^k wx^k y \in L$

For vwx there are following possible cases:

1. $vwx = a^i,\ i \leq p$, then uv^kwx^ky has at least k more a's than $s \implies uv^kwx^ky \notin L$ if $k \geq 1$
2. $vwx = a^ib^j,\ i + j \leq p$, then uv^kwx^ky has at least k more a's and b's than s but no more c's $\implies uv^kwx^ky \notin L$ if $k \geq 1$
3. $vwx = b^i,\ i \leq p$, then uv^kwx^ky has at least k more b's than $s \implies uv^kwx^ky \notin L$ if $k \geq 1$
4. $vwx = b^ic^j,\ i + j \leq p$, then uv^kwx^ky has at least k more b's and c's than s but no more a's $\implies uv^kwx^ky \notin L$ if $k \geq 1$
5. $vwx = c^i,\ i \leq p$, then uv^kwx^ky has at least k more c's than $s \implies uv^kwx^ky \notin L$ if $k \geq 1$

This contradicts the pumping lemma, therefore L is not a context free language. □

b

Proof. Proof by contradiction: assume L is a context free language;
$\implies \exists p \geq 1$ such that every string $s \in L$ with length larger than p can be written as:

$$s = uvwxy$$

where u, v, w, x, y are substrings of s such that:

1. $|vx| \geq 1$
2. $|vwx| \leq p$
3. $\forall k \geq 0, uv^kwx^ky \in L$

For vwx there are following possible cases:

1. $vwx = 0 \implies v^kwx^ky \notin L$ if $k \geq 1$ because it would have extra 0s
2. $vwx = 1 \implies v^kwx^ky \notin L$ if $k \geq 1$ because it would have extra 1s
3. $vwx = 0^i10^j, i + j + 1 \leq p \implies v^kwx^ky \notin L$ if $k \geq 1$ because of the extra 0s
4. $vwx = 0^i, i \leq p \implies v^kwx^ky \notin L$ if $k \geq 1$ because of the extra 0s

This contradicts the pumping lemma, therefore L is not a context free language. □

Problem 3

I do not know the answer to this question.