Homework 10

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Problem 1

Need to prove: $\overline{A} \in$ **co-NP** $\wedge \forall L \in$ **co-NP** $L \leq_P \overline{A}$

Proof. Assume language A is NP-complete $\implies \forall L \in NP, L \leq_P A \ \wedge A \in NP$.
Then we have $\overline{A} \in$ **co-NP** and a verifier `V_a(w, c)` that runs in polynomial time.
Let B be an arbitrary language form **co-NP**. Then $\overline{B} \in NP$ and $\overline{B} \leq_P A$.
Then there exists a verifier `V_b_complement(w, c)` that verifies $\overline{B}$ in polynomial time.

The verifier for B works by inverting the output of `V_b_complement(w, c)`:

```
function V_b(w,c){
    if (V_b_complement(w,c) accepts){
        reject
    } else {
        accept
    }
}
```

Therefore $B \leq_P \overline{B}$. Similarly, $A \leq_P \overline{A}$.
Therefore $B \leq_P \overline{B} \leq_P A \leq_P \overline{A}$.
$\therefore \forall L \in$ **co-NP** $L \leq_P \overline{A}$

$\overline{A} \in$ **co-NP** $\wedge \forall L \in$ **co-NP** $L \leq_P \overline{A} \implies \overline{A}$ is **co-NP**-complete □

Problem 2

Proof. Assume a language L is NP-complete and PSPACE-complete.
Therefore $\forall A \in NP, A \leq_P L \wedge \forall B \in PSPACE, B \leq_P L$
Therefore $\forall A \in NP, B \in PSPACE, A \leq_P B$ and $B \leq_P A$
Therefore NP = PSPACE. □

Problem 3

Need to prove:

1. $A_{LBA} \in$ PSPACE

   Proof. □

2. $\forall L \in$ PSPACE, $L \leq_P A_{LBA}$

   Proof. □