Homework 4

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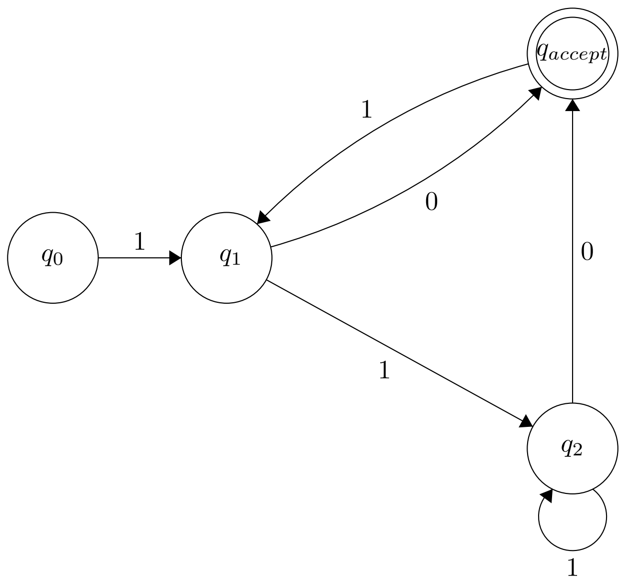
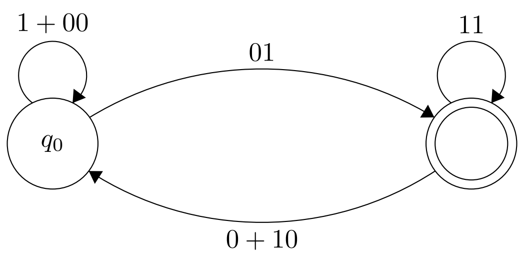
Problem 1

(a)

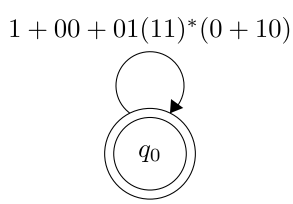


(b)

Step 1: Remove the left most node:



Step 2: Remove accept state:



Therefore, the regular expression that represents this fsm is $(1 + 00 + 01(11)^*(0 + 10))^*$

Problem 2

Proof. Assume A is non-empty, and N has k states. Then the k states in N can be chained in a line to create a unique word of length k at most. Therefore, strings in A should have lengths at most k.

Now consider a DFA D equivalent to N, we know that D can have at most 2^k states using powerset construction. From D, we can construct another DFA $D_{reverse}$ that accepts $\overline{A}$ by reversing all accepting states to non-accepting states, and vice versa. Similar to A, because $D_{reverse}$ has at most 2^k states, $\overline{A}$ can have strings with lengths of at most 2^k

$\square$

Problem 3

(a)

Proof. Assume Palindromes over $\Sigma = \{a, b, c\}$ are regular languages, then there exists a DFA that accepts this language.

Consider an arbitrary palindrome over Σ, $p = a^m b^n c^{2k} b^n a^m$, where $m, n, k \in \mathbb{N}$

Assume the DFA is in state q_{mnk} when it reads $a^m b^n c^k$. Then from state q_{mnk}, the DFA must accept upon reading $c^k b^n a^m$. Since DFA only has a finite number of states, there must exists some state $q_{m'n'k'}$ for some $m' \neq m, n' \neq n, k' \neq k$, such that $q_{mnk} = q_{m'n'k'}$. This entails that from state $q_{m'n'k'}$, the machine will accept upon reading $c^k b^n a^m$. However, $a^{m'} b^{n'} c^{k'} c^k b^n a^m$ is not a palindrome, and we arrived at contradiction.

Therefore, by contradition, palindromes over $\Sigma = \{a, b, c\}$ are not regular language. $\square$

(b)

Proof. Assume $\{10^1 10^2 10^3 \ldots 0^{n-1} 10^n 1 | n \geq 1\}$ is a regular language, then there exists a DFA that accepts it.

Consider an arbitrary string $10^1 10^2 10^3 \ldots 10^k 1, k \geq 1$. Assume the DFA is in state q_k when it reads $10^1 10^2 \ldots 0^{k-1} 1$. Then from q_k, the machine will accept upon reading $0^k 1$. Since DFA only has a finite number of states, then there must exists some $k' \neq k$ such that $q_k = q_{k'}$. This entails that in state $q_{k'}$ the machine will accept upon reading $0^k 1$. However, $10^1 10^2 \ldots 0^{k'-1} 10^k 1$ is not in the language, and we have arrived at a contradiction.

Therefore, by contradiction, $\{10^1 10^2 10^3 \ldots 0^{n-1} 10^n 1 | n \geq 1\}$ is not a regular language. $\square$