Formal Languages and Computational Models

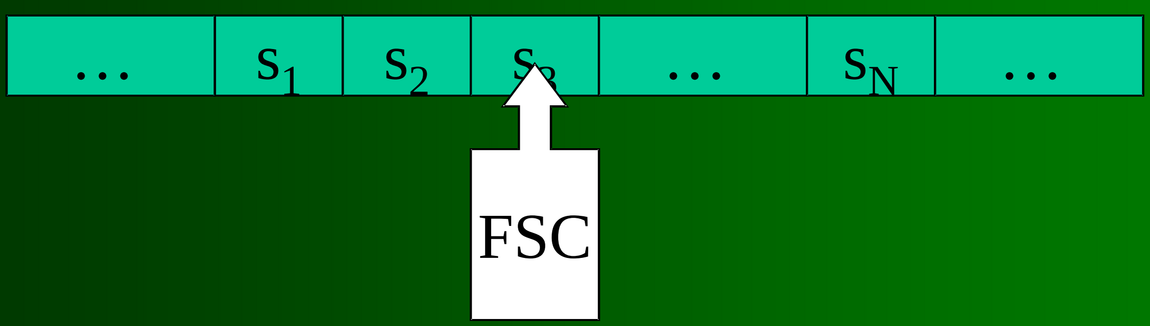
Programming Languages

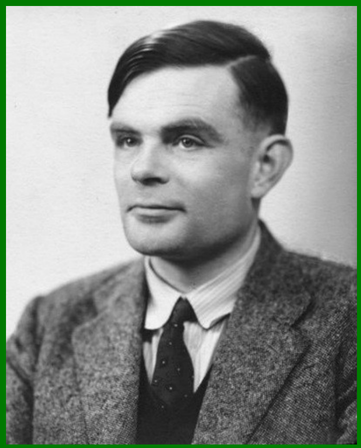
CS 214

Turing Machines

In 1936 (years before the first programmable computer), *Alan Turing* created a model for the process of computation known today as the *Turing Machine (TM)*, consisting of:

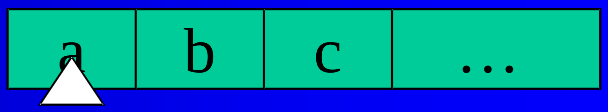
- An *I/O tape* consisting of an arbitrary number of *cells*, each able to store an arbitrary symbol;
- A *tape head* able to read/write a cell; and
- A *finite-state control* that governs movement of the head over the cells.





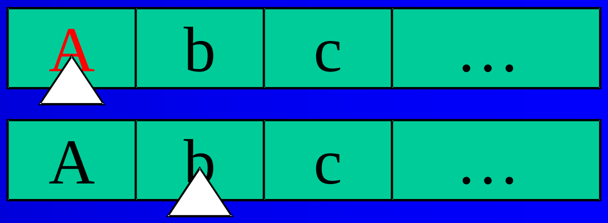
Turing Machines (ii)

Each "execution cycle", a TM reads a *symbol* from the tape.



Depending on that *symbol* and its current state, it may then:

- Write a symbol to the tape;
- Move its head left or right; and
- Change to a new state.



The finite state controller starts in state 0: the *start state*, and continues execution until it enters an *accept state*, at which point it halts and its I/O tape contains the result of the computation.

Example: TM Addition

To add two numbers *m* and *n*:

- Precond: I/O tape contains *m* ones, a zero, and *n* ones.
- Postcond: I/O tape contains *m*+*n* ones.

Our finite state controller uses these states and rules:

State 0: If *symbol* is 1 or blank: move head right; goto State 0.
If *symbol* is 0: goto State 1

State 1: Write 1; move head right; goto State 2.

State 2: If *symbol* is 1: move head right; goto State 2.
If *symbol* is blank: move head left; goto State 3

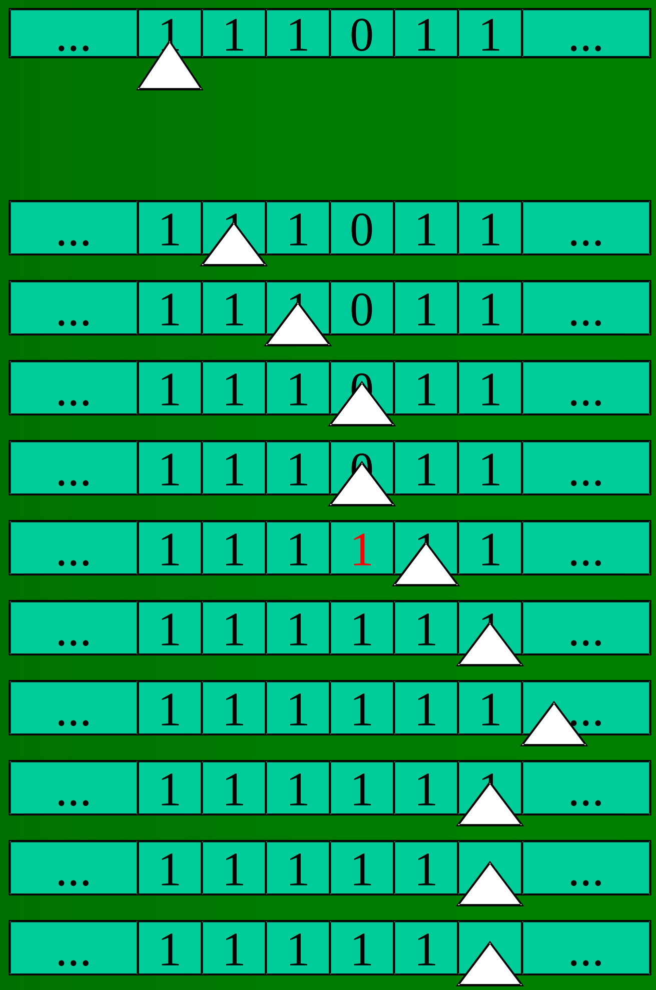
State 3: Write blank; goto State 4.

State 4: Accept.

Example: 3 + 2

To compute 3 + 2, we start with:

Step	$State_i$	Read	Write	Move	$State_j$
1	0	1	-	right	0
2	0	1	-	right	0
3	0	1	-	right	0
4	0	0	-	-	1
5	1	-	1	right	2
6	2	1	-	right	2
7	2	1	-	right	2
8	2	blank	-	left	3
9	3	-	blank	-	4
10	4	-	-	-	-



TMs and Computability

In 1931, *Kurt Godell* proved that there exist easily-described functions that cannot be computed.

In 1936, Turing proved that a TM can be built for any computable function.

He later proved that a universtal TM can be built that can perform the task of any single-function TM, implying:

- → Since it is independent of any particular hardware details, a proof about a UTM applies to <u>every</u> computer that will <u>ever</u> be built!
- → If a function *f* can be computed, then a UTM can compute *f*.
- → If a UTM cannot compute a function *g*,
  then *g cannot be computed (by any computer, ever)*.

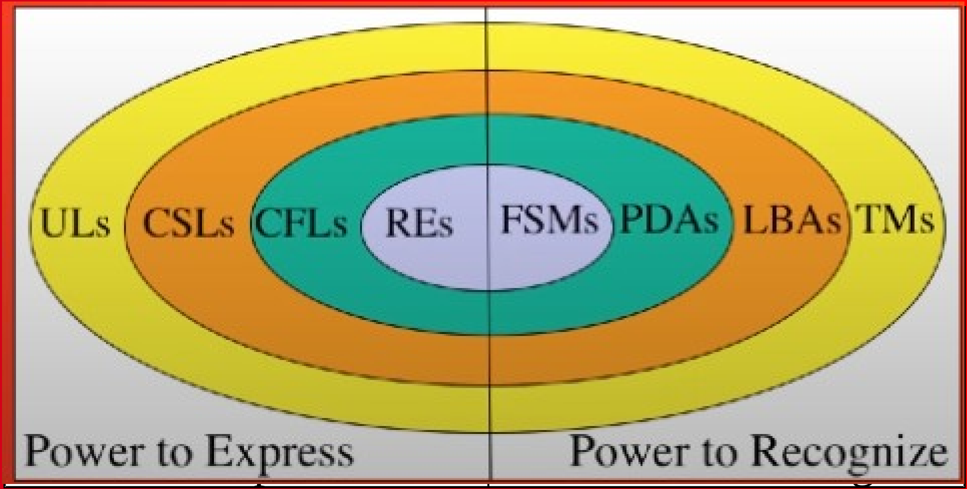
Turing proved the *Halting Problem* cannot be solved by a UTM.

The Chomsky Hierarchy

In 1956, *Noam Chomsky* classified languages as follows:

Level	Language	Recognizer
3	Regular expression (REs)	Finite stawte machine (FSM)
2	Context free (CFLs)	Pushdown automata (PDA)
1	Context sensitive (CSLs)	Linear bounded automata (LBA)
0	Unrestricted (ULs)	Turing Machine (TM)

Chomsky's categories form a *hierarchy*, organized by their power of expression (language) and power of recognition (automaton):



Chomsky and BNFs

The *Chomsky Hierarchy* specifies that:

- A TM can recognize any language able to be recognized.
- A LBA can recognize CSLs, CFLs, & REs but not ULs.
- A PDA can recognize CFLs & REs but not CSLs or ULs.
- A FSM can recognize REs but not CFLs, CSLs or ULs.

The BNF is a tool for specifying CFL syntax.

  – Programming language syntax is relatively “easy”, linquistically.

It can also be used to specify RE syntax
(but doing so is overkill -- simpler tools are available).

Different tools are needed to specify CFL and/or UL syntax.

PDAs and (BNF) Parsing

A PDA is a FSM with a stack on which it can save things...

Recall our basic parsing algorithm (for BNFs):

0. Push *S* (the starting symbol) onto a stack.
1. Get the first terminal symbol *t* from the input file.
2. Repeat the following steps:
    - a. Pop the stack into *topSymbol*;
    - b. If *topSymbol* is a nonterminal:
        - 1) Choose a production *p* of *topSymbol* based on *t*
        - 2) If *p* != ε:
            - Push *p* right-to-left onto the stack.
    - c. Else if *topSymbol* is a terminal && *topSymbol* == *t*:
        - Get the next terminal symbol *t* from the input file.
    - d. Else
        - Generate a 'parse error' message.

   while the stack is not empty.

A FSM cannot parse a CFL/ BNF because it has *no stack*.

The Random Access Machine (RAM)

Proving things about TMs was a bit clumsy…

1963: *Shepherdson and Sturgis* devise the RAM as a model that is equivalent to a TM but more convenient to use:

The RAM has four components

- A *memory*: an integer array, indexed from zero.
- A *program*: a sequence of numbered instructions.
- An *input file*.
- An *output file*.

Shepherdson and Sturgis proved a RAM can compute anything a UTM can compute, and vice versa.

The RAM Instruction Set

- `M[i] = n` → **store n at index i**
- `M[i] = M[j]` → **copy value at j to i**
- `M[i] = M[j] + M[k]` → **add and store**
- `M[i] = M[j] - M[k]` → **subtract and store**
- `M[M[j]] = M[k]` → **indirection**
- `read M[i]` → **input (destructive)**
- `write M[i]` → **output**
- `goto s` → **unconditional branch**
- `if M[i] >= 0 goto s` → **conditional branch**
- `halt` → **terminate execution**

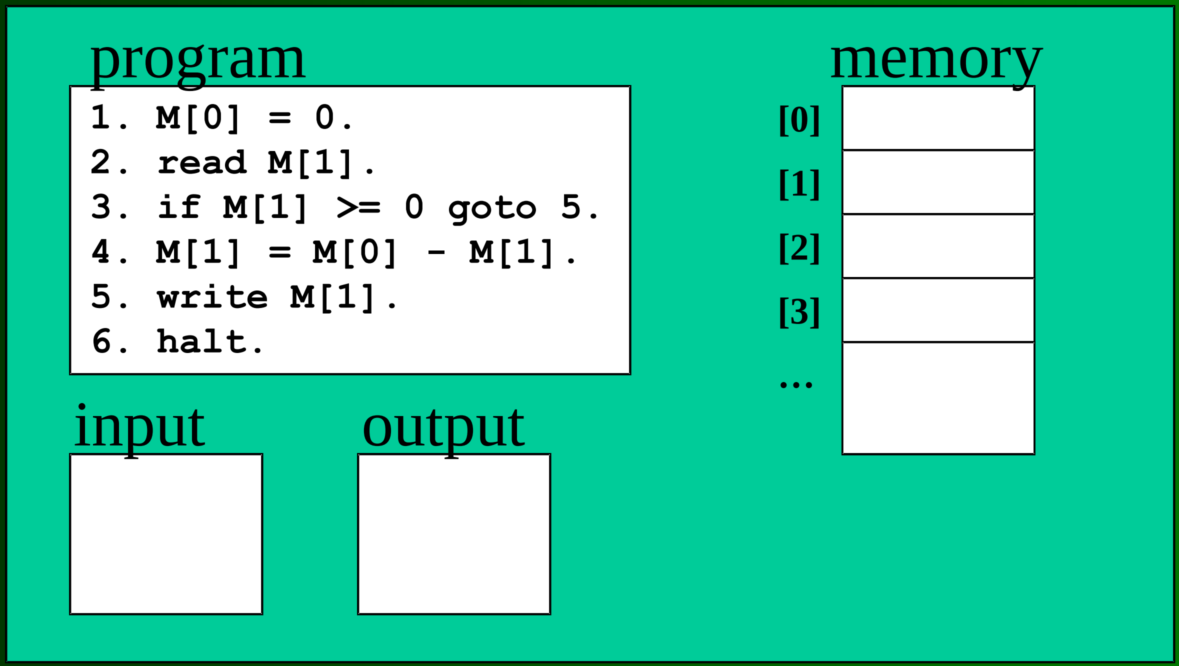
Later extensions added other operators (arithmetic, relational)

The result was quite similar to a *RISC assembly language*.

Example 1

Here is a RAM for a computation…

What does it do (try some sample inputs)?



Example 2

Here is a different RAM. What does it compute?

program

```
1. M[0] = 64.
2. M[1] = 91.
3. M[2] = 32.
4. read M[3].
5. if M[3] >= 0 goto 7.
6. goto 14.
7. M[4] = M[0] - M[3].
8. M[5] = M[3] - M[1].
9. if M[4] >= 0 goto 12.
10. if M[5] >= 0 goto 12.
11. M[3] = M[3] + M[2].
12. write M[3].
13. goto 4.
14. halt.
```

memory

[0]
[1]
[2]
[3]
[4]
[5]
…

input

output

RAM Extensions

Like a TM, a RAM can compute anything that is computable.

With these simple extensions:

- Symbolic names instead of memory locations
- multiplication and division operators
- other relational (==, !=, <, >, >=) operators
- literals within arithmetic expressions

it becomes a convenient tool for studying HLL constructs, as a "portable assembly language" to study how a compiler can translate HLL constructs.

RAM Extension Examples

Example 1 program

```
1. read val.
2. if val >= 0 goto 4.
3. val = 0 - val.
4. write val.
5. halt.
```

Example 2 program

```
1. read ch.
2. if ch < 0 goto 10.
3. lo = ch - 65.
4. hi = ch - 90
5. if lo < 0 goto 8.
6. if hi > 0 goto 8.
7. ch = ch + 32.
8. write ch.
9. goto 1.
10. halt.
```

Even with the improvements, such programs are hard to read because of their coding style (aka *spaghetti code*), just as Assembly language is harder to read than a HLL...

Summary

The Chomsky Hierarchy names four “levels” of language, plus the weakest machine able to recognize at each level:

3 Regular Expressions	→	Finite State Machine
2 Context Free Languages	→	Pushdown Automata
1 Context Sensitive Languages	→	Linear Bounded Automata
0 Unrestricted Languages	→	Turing Machine

The TM is the most powerful of the machines, able to

- recognize any language capable of being recognized.
- compute any function capable of being computed.

The RAM is a computational model that is

- as powerful as the TM
- more convenient than the TM for studying HLL constructs.

