Specifying Synatax with BNFs (Backus Naur Forms)

Programming Languages

CS 214

Sentence Structure

Consider the following “sentences”

- *there is hair in my soup*
- *there is soup in my hair*
- *is there soup in my hair*

What does each “sentence” mean?

What about this “sentence”?

- *hair soup there my is in*

→ The *order of the words* determines the meaning...

What determines which word-orders are *meaningful*?

Grammar & Syntax

Every language has a set of rules - its *grammar or syntax* - that specifies the word-sequences that form valid sentences.

The "sentence":

- *hair soup there my is in*

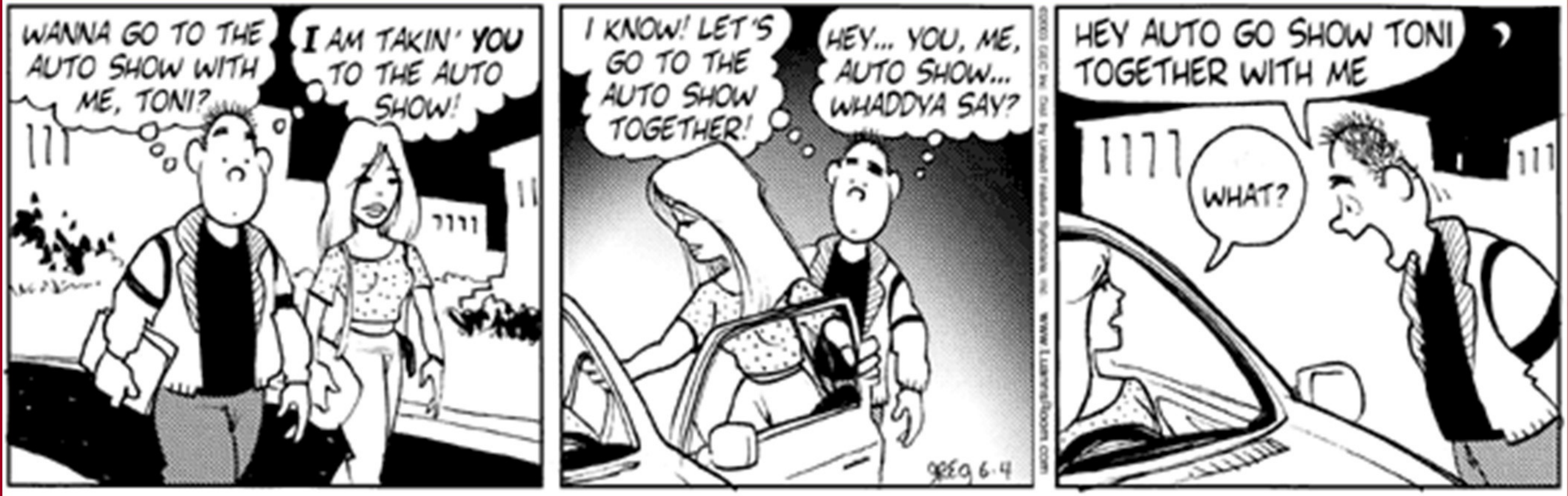
is not valid, because it violates English's grammar rules (i.e., it contains ____________________).

Grammar and syntax help us decode a sentence's meaning:

syntax were read to easier sentence would unimportant this if be

____

Syntax Matters In Real Life



Semantics

When a sentence has correct syntax, our brains can determine what the words in the sentence mean: their *semantics*.

Consider: *time flies like an arrow, fruit flies like a banana*

→ *flies* and *like* have two different meanings

We decode a word's meaning using its *surrounding context*.

Since a word's semantics (meaning) depends on its context, English is a *context sensitive language (CSL)*.

If a sentence contains syntax errors, we can't understand it, because syntax specifies what words can be adjacent, and a word's semantics depends on the words surrounding it.

Program Sentences

A program is a “sentence” in a programming language.

A program’s “meaning” depends on the order of its symbols.

To be valid, the order must obey the syntax rules of the language; for example:

```
x = y + 1;
```

is a valid statement in some languages (e.g., ___________) but not in others (e.g., ____________).

The meanings of the “words” in a program are determined by the *syntax rules* of the language in which it’s written.

Syntax Errors

Syntax rules specify those symbol-orderings that are valid, allowing a compiler to determine symbol-meanings.

```
x = y + 1;          →→ MOV y, R0
                       ADD #1, R0
                       STO R0, x
```

When a program contains syntax errors, a compiler is unable to translate it (i.e., determine its meaning):

```
y + 1 = x;          →→ ?
```

A compiler cannot determine the meaning of any "phrase" that violates the language's syntax rules.

BNF

The Backus-Naur Form is a tool for specifying the syntax of a high level language (HLL).

Example: A BNF giving the structure of C++ identifiers is:

<identifier> ::= <first_letter> <valid_sequence>

<first_letter> ::= <letter> | _

<valid_sequence> ::= <valid_symbol> <valid_sequence> | ε

<valid_symbol> ::= <letter> | <digit> | _

<letter> ::= A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | a | b | c | d | e | f | g | h | i | j | k | l | m | n | o | p | q | r | s | t | u | v | w | x | y | z

<digit> ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

A correct BNF specifies all valid "sentences", and prohibits all invalid "sentences".

Examples:

Is *R2D2* a valid sentence in our <identifier> language?

<identifier>
<first_letter> <valid_sequence>
<letter> <valid_sequence>
R <valid_sequence>
R <valid_symbol> <valid_sequence>
R <digit> <valid_sequence>
R 2 <valid_sequence>
R 2 <valid_symbol> <valid_sequence>
R 2 <letter> <valid_sequence>
R 2 D <valid_sequence>
R 2 D <valid_symbol> <valid_sequence>
R 2 D <digit> <valid_sequence>
R 2 D 2 <valid_sequence>
R 2 D 2 ε

This sequence of steps is called a *derivation*.

"Sentences" not conforming to the BNF are *invalid*.

Formal Definitions

Let Σ be a set of symbols.

- A *string over* Σ is a finite sequence of zero or more symbols from the set Σ.
- Symbols whose meaning is predefined are called *terminals*.
  - Symbols like *A*, _, *6*, etc. are terminals in our <identifier> BNF.
- Symbols whose meanings must be defined are called *non-terminals*, and are enclosed in angle-brackets (< and >).
  - Symbols like *<identifier>*, *<letter>*, etc. are non-terminals.
  - Like variables, non-terminals usually describe what they represent.
- One symbol is designated as the *starting non-terminal*.
  - The symbol *<identifier>* is the starting non-terminal in our BNF.

Formal Definitions (ii)

- Each non-terminal must be defined by a *production* (rule):

<LHS> ::= <RHS>

where: <LHS> is the nonterminal being defined,
<RHS> is a string of terminals and/or nonterminals defining <LHS>.

- Different productions defining the same non-terminal:

$<NT_i>$ ::= Def_1
$<NT_i>$ ::= Def_2
...
$<NT_i>$ ::= Def_n

can be written in shorthand using the OR (|) operator:

$<NT_i>$ ::= $Def_1 \mid Def_2 \mid \ldots \mid Def_n$

Formal Definitions (iii)

- A BNF is a quadruple: (Σ, N, P, S), where:

  Σ is the set of symbols in the BNF;

  N is the subset of Σ that are nonterminals in the language;

  P is the set of productions defining the symbols in N; and

  S is the element of N that is the starting nonterminal.

- A *derivation* is a sequence of strings, beginning with the starting nonterminal S, in which each successive string replaces a nonterminal with one of its productions, and in which the final string consists solely of terminals.

  → A derivation is sometimes called a *parse*.

Formal Definitions (iv)

- A BNF *derivation tree (or parse tree)* is a tree, such that
  - the root of the tree is the starting nonterminal (S) in the BNF;
  - the children of the root are the symbols (L to R) in a production whose <LHS> is S, the starting nonterminal;
    - each terminal child is a leaf; and
    - each nonterminal child is the root of a derivation tree for that nonterminal.
- The act of building a derivation tree for a sentence (to check its correctness) is called *parsing* that sentence.

  → The set of valid sentences in a language is the set of all sentences for which a parse tree exists!
- A *left most derivation* is a derivation built by *always expanding the left-most non-terminal* in a production.

Examples

Do parse trees (using our <identifier> BNF) exist for these?

i *a1b2* *_5_* *_$_*

Recursive Productions

Our identifier-BNF permits identifiers to have arbitrary lengths through the use of *recursive productions*:

<valid_sequence> ::= <valid_symbol> <valid_sequence> | ε

This production is recursive because the non-terminal on the <LHS> appears in the production (on the <RHS>).

The recursive production provides for *unrestricted repetition* of the non-terminal being defined, which is useful what is being defined can be appear *0 or more times*.

The ε-production provides both:

- a *base-case* for trivial instances of the non-terminal, and
- an *anchor* to terminate the recursion.

Writing BNFs: A 3-Step Process

A. Start with the non-terminal you're defining.

B. Build the productions to define the non-terminal:

1. Start with the question: What comes first*?*
2. If a construct is *optional*:
   - a. create a new nonterminal for that construct;
   - b. add a production for the non-optional case;
   - c. add an ε-production for the optional case.
3. If a construct can be repeated 0 or more times:
   - a. create a new nonterminal for that construct;
   - b. add an ε-production for the zero-reps case;
   - c. add a recursive production for the other cases.

C. For each nonterminal in the <RHS> of every production:

repeat step B until all nonterminals have been defined.

Example: C++ *if* Statement

A. Create a non-terminal for what we're defining:

<if_stmt>

B. Build a production to define it: what comes first?

1. keyword *if*
2. open parentheses
3. expression (bool)
4. close parentheses
5. statement
6. else ...

<if_stmt> ::= if (<expr>) <stmt> <else_part>

C. Repeat B for each undefined non-terminal:

<else_part> ::= else <statement> | epsilon

(We'll see how to define <expr> and <stmt> a bit later...)

A Small Problem

The C++ if statement presents a small problem: Consider…

```
if ( a < b) if (a < c) S1 else S2
```

Take a moment to build a parse tree for this “sentence”...

Ambiguities

When a "sentence" has multiple parse trees, it is *ambiguous* (i.e., it has multiple interpretations and/or meanings).

The two parses reflect different ways to associate the **`else`**:

```
if (a < b)
    if (a < c)
        S1
    else
        S2
```

```
if (a < b)
    if (a < c)
        S1
else
    S2
```

The grammar cannot resolve which is meant, so C++ uses a *semantic rule* to resolve the ambiguity:

*An **`else`** associates with the closest prior unterminated **`if`**.*

Example: C++ *do* Statement

A. Create a non-terminal for what we're defining:

<do_stmt>

B. Build a production to define it: what comes first?

1. keyword *do*
2. a statement
3. keyword *while*
4. open parentheses
5. an expression
6. close parentheses
6. a semicolon

```
<do_stmt>          ::=   do <stmt> while ( <expr>);
```

C. Repeat B for each undefined non-terminal…

<stmt> isn't too bad, so let's tackle it next.

Example: C++ <stmt>

A. Create a non-terminal for what we're defining:

<stmt>

B. Build a production to define it: what comes first?

- It depends on the kind of statement being described...
- There are seven different kinds of C++ statements, so let's introduce a new non-terminal for each one:

<stmt>	::=	<compound_stmt> | <selection_stmt> |
		<iteration_stmt> | <expression_stmt> |
		<jump_stmt> | <labeled_stmt> | <declaration_stmt>

C. Repeat B for each undefined non-terminal…

<compound_stmt>	::=	{<stmt_list>}
<stmt_list>	::=	<stmt><stmt_list> | epsilon

Example: C++ <stmt> (ii)

<selection_stmt>	::=	<if_stmt> | <switch_stmt>
<iteration_stmt>	::=	<while_stmt> | <do_stmt> | <for_stmt>
<expression_stmt>	::=	<opt_expr> ;
<opt_expr>	::=	<expr> | ε
<jump_stmt>	::=	break ; | continue ; | return <opt_expr> ; | goto <identifier> ;
<labeled_stmt>	::=	<identifier> : <stmt> | case <literal> : <stmt> | default : <stmt>
<declaration_stmt>	::=	<object_dec> | <function_dec> | <class_dec>
<object_dec>	::=	<modifier> <type> <identifier> <initializer> <more_ids> ;
<modifier>	::=	register | const | static | auto | extern | mutable | ε
<type>	::=	char | wchar_t | bool | short | int | long | signed | unsigned | float | double | <identifier> | <identifier> :: <type>
<initializer>	::=	= <expr> | ε
<more_ids>	::=	, <identifier> <initializer> <more_ids> | ε

…

Example: C++ Assignment Exprs

A. Create a non-terminal for what we're defining:

<assign_expr>

B. Build a production to define it: what comes first?

- Many things might come first (variable, pointer, array, ...) so let's introduce a non-terminal to hide the details.
- Next comes an assignment operator; then an expression:

<assign_expr> ::= <lvalue><assign_op><assign_expr> |

C. Repeat B for each undefined non-terminal…

<lvalue>	::=	____oops
<unary_op>	::=	* | epsilon
<id_suffix>	::=	[<expr>] <id_suffix> | . <lvalue> <id_suffix> | -> <lvalue> <id_suffix> | ε
<assign_op>	::=	_____ | += | -= | *= | /= | %= | <<= | >>= | &= | |= | ^=

Expressions

C++ expressions are complicated:

- Each of its 52 operators must be included
- Each of its 17 precedence levels must be included
- Associativity rules must be enforced
- The grammar/BNF must be unambiguous

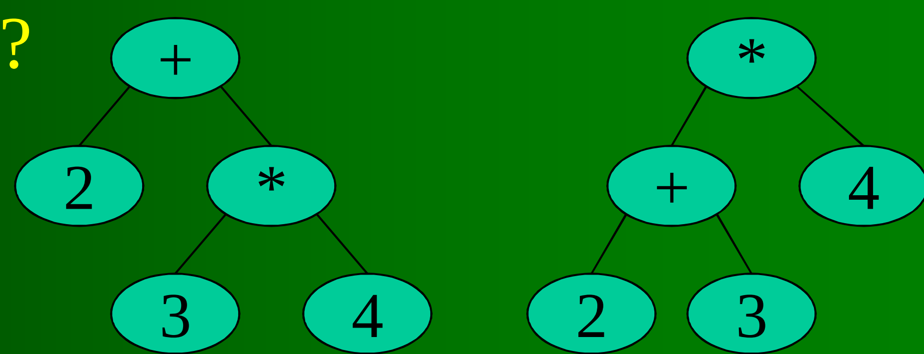
Example: For the expression: 2 + 3 * 4

our grammar must ensure that: 2 + (3 * 4) = 14

is evaluated, not: (2 + 3) * 4 = 20

Which parse tree is correct?

Our grammar must *only* generate the correct one.



Grammar and Precedence

To ensure that higher precedence operators appear lower in the parse tree, we must build *hierarchy* into our BNF:

```
<expr>          ::=  … | <add_expr> | …
<add_expr>      ::=  <mul_expr> | <add_expr> + <mul_expr> |
                       <add_expr> - <mul_expr>
<mul_expr>      ::=  <value> | <mul_expr> * <expr> |
                       <mul_expr> / <expr> | <mul_expr> % <expr>
```

Rules like these ensure that "multiply-level" operators will be applied before "add-level" operators...

```
                    <expr>
                  <add_expr>
          <add_expr> + <mul_expr>
<mul_expr>      <mul_expr> * <expr>
<value>           <value>      <add_expr>
   2                 3         <mul_expr>
                                 <value>
                                    4
```

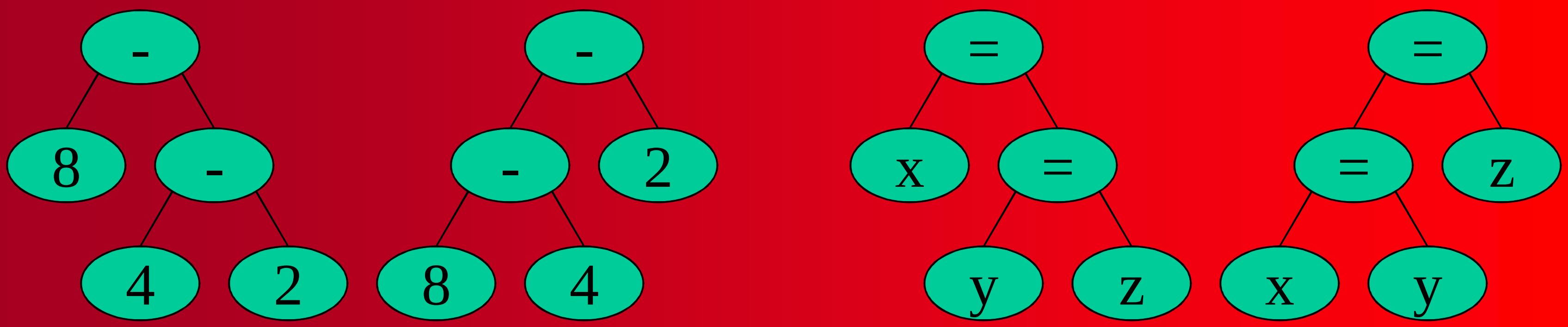
Grammar and Associativity

Associativity? gives ordering of equal-precedence operators.

Examples: 8 - 4 - 2 vs. x = y = z

- is left-associative; which is correct?

= is right-associative; which is correct?



Associativity can be built into a grammar by using left-*recursive productions* for left-associative operators; and *right-recursive productions* for right-associative operators.

Associativity Examples

Example: Since +, - are left-associative, we write:

<add_expr> ::= <mul_expr> | <add_expr> + <mul_expr> | <add_expr> - <mul_expr> |

but since = is right-associative, we write what amounts to:

<assign_expr> ::= <lvalue> <assign_op assign_expr>

These generate the correct parse trees for each expression:

<expr>
<add_expr>
<add_expr> - <mul_expr>
<add_expr> - <mul_expr> <value>
<mul_expr> <value> 2
<value> 4
8

<expr>
<assign_expr>
<lvalue> <assign_op> <assign_expr>
<identifier> = <lvalue> <assign_op> <assign_expr>
x <identifier> = <add_expr>
y <mul_expr>
<identifier>
z

C++ Expressions

<expr> ::= <assign_expr> | <expr> , <assign_expr>

<assign_expr> ::= <lvalue> <assign_op> <assign_expr> | <cond_expr>

<cond_expr> ::= <lor_expr> | <lor_expr> ? <expr> : <cond_expr>

<lor_expr> ::= <land_expr> | <lor_expr> || <land_expr>

<land_expr> ::= <bor_expr> | <land_expr> && <bor_expr>

<bor_expr> ::= <xor_expr> | <bor_expr> | <xor_expr>

<xor_expr> ::= <band_expr> | <xor_expr> ^ <band_expr>

<band_expr> ::= <equ_expr> | <band_expr> & <equ_expr>

<equ_expr> ::= <rel_expr> | <equ_expr> == <rel_expr> | <equ_expr> != <rel_expr>

<rel_expr> ::= <shft_expr> | <rel_expr> < <shft_expr> | <rel_expr> > <shft_expr>
| <rel_expr> <= <shft_expr> | <rel_expr> >= <shft_expr>

<shft_expr> ::= <add_expr> | <shft_expr> << <add_expr> |
<shft_expr> >> <add_expr>

<add_expr> ::= <mul_expr> | <add_expr> + <mul_expr> | <add_expr> - <mul_expr>

<mul_expr> ::= <ptr_expr> | <mul_expr> * <ptr_expr> | <mul_expr> / <ptr_expr> |
<mul_expr> % <ptr_expr>

<ptr_expr> ::= …

Exercise

Build a parse tree for: `a = x + y / z ;`

EBNF

The BNF is the most general tool for expressing syntax.

Another tool frequently used is the Extended BNF.

The differences are:

- EBNF terminals are distinguished from non-terminals by
  - Capitalizing the first-letter of non-terminals, AND
  - Underlining, single-quoting, or bolding terminals
    (instead of surrounding non-terminals by angle-brackets).
- Parentheses may be used to denote grouping.
- {} surround symbols that are repeated 0 or more times.
  - → No recursion!
- [] surround symbols that are optional.
  - → No ε-productions!

Examples

1. To specify a C++ block using EBNF:
   - First comes a brace, then zero or more statements, then a brace:

     Block ::= '{' {Stmt} '}'

2. To specify a C++ int literal using EBNF:
   - An optional sign, an optional base specifier, at least one digit

     Int-Literal ::= [Sign] [0x] Digit {Digit.}

     Sign ::= + | -

     Digit ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

3. To specify a C++ do statement using EBNF:
   - Keyword do, a statement, keyword while, an open-parentheses, an expression, a close-parentheses, a semicolon:

     Do-Stmt ::= do Stmt while '(' Expr ')'

Problems

1. Specify a C++ identifier using EBNF:

   Identifier ::= FirstSymbol {ValidSymbol}
   FirstSymbol ::= Letter | _
   ValidSymbol ::= FirstSymbol | Digit
   Letter = A-Z | a-z
   Digit = 0-9

2. Specify a C++ while statement using EBNF:

   WhileStmt ::= 'while' '(' Expr ')' Stmt

3. Specify a C++ if statement using EBNF:

   IfStmt ::= 'if' '(' Expr ')' Stmt ['else' Stmt]

Why use BNFs instead of EBNFs?

The recursive productions in BNFs make it easier for a compiler to parse “sentences” in the language…

Basic Parsing Algorithm:

0. Push *S* (the starting symbol) onto a stack.
1. Get the first terminal symbol *t* from the input file.
2. Repeat the following steps:
    a. Pop the stack into *topSymbol*;
    b. If *topSymbol* is a nonterminal:
        1) Choose a production *p* of *topSymbol* based on *t*
        2) If *p* != ε:
            Push *p* right-to-left onto the stack.
    c. Else if *topSymbol* is a terminal && *topSymbol* == *t*:
        Get the next terminal symbol *t* from the input file.
    d. Else
        Generate a ‘parse error’ message.

while the stack is not empty.

Example

Suppose our rules are:

<var_dec> ::= <type> id <id_list> ; <var_dec> | ε

<type> ::= int | char | float | double | …

<id_list> ::= , <id> <id-list> | ε

Let's parse the declaration: `int x, y;`

assuming that <var_dec> is our starting symbol.

stack:

<var_dec>	<type>	int	id	<id_list>	,	id	<id_list>	;	<var_dec>
	id	id	<id_list>	;	id	<id_list>	;	<var_dec>	
	<id_list>	<id_list>	;	<var_dec>	<id_list>	;	<var_dec>		
	;	;	<var_dec>		;	<var_dec>			
	<var_dec>	<var_dec>			<var_dec>				

t: int id(x) , id(y) ;

Summary

There are different ways to specify the syntax of a language. Two of them are:

- BNF
- EBNF

The EBNF is simpler and easier to use, so it is frequently used in language guids and user manuals.

The BNF's recursive- and ε-productions simplify the task of parsing, so it is more useful to compiler-writers.