Revenue Management Problem Second Attempt

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1 Mathematics

Given m nurses and n shifts, we have:

- $P \in \mathbb{R}^{m \times n}$, the probability that nurse i will select shift j. Note that the probabilities are all independent.
- $Q \in \mathbb{R}^{m \times n}$, the probability that nurse i will show up to shift j.
- $R \in \mathbb{R}^{m \times n}$, the revenue that is gained when nurse i covers shift j.
- $Y \in \{0, 1\}^{m \times n}$, whether or not we will show nurse i shift j.

We express the expected value of a given policy as

$$\mathbb{E}[R] = \sum_{i=1}^{m} \sum_{j=1}^{n} R_{ij} P_{ij} Q_{ij} Y_{ij} A(i, j)$$

Let $A(i, j)$ be the probability that shift j is available for nurse i. Shift j is only open if all previous nurses did not schedule the shift. A nurse k schedules shift j with probability $Y_{kj} P_{kj}$. We consider all nurses before i and find the probability as the product of each previous nurse $k < i$ not scheduling shift j. This works since all P are independent.

$$A(i, j) = \prod_{k=1}^{i-1} (1 - Y_{kj} P_{kj})$$