Static Assignment with Multiple Nurses Required for Each Shift

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3 Preliminary Research

3.1 Task 1.1: Static Assignment with Multiple Nurses Required for Each Shift

Parameters of the Problem

We define the following parameters:
$\mathcal{N}_j^1$ = Set of nurses to whom shift $j \in \mathcal{J}$ should be offered in Problem **S1** (where $n_1 = 1$); $\mathcal{N}_j^1 \subseteq \mathcal{N}$
$\mathcal{N}_j^2$ = Set of nurses to whom shift $j \in \mathcal{J}$ should be offered in Problem **S2** (where $n_1 \geq 1$); $\mathcal{N}_j \subseteq \mathcal{N}$
$\mathcal{N}_j \subseteq \mathcal{N}$
Y_j^1 = Optimal static assignment policy for shift j in Problem **S1**
Y_j^2 = Optimal static assignment policy for shift j in Problem **S2**
Y_j = A static assignment policy in Problem **S1** or **S2**
$\mathcal{N}_{Y_j}$ = A set of nurses to whom shift $j \in \mathcal{J}$ is offered in Problem **S1** or **S2** when the policy is Y_j;
$\mathbb{E}(\tilde{R} \mid Y_i, Y_{i+1}, \ldots, Y_n)$ = The total expected revenue obtained from nurses $i, i+1, \ldots, n$, given the policy is Y
R_1 = Total revenue obtained in Problem **S1**
R_2 = Total revenue obtained in Problem **S2**
$\Theta_{k,j}$ = Number of nurses who signed up for shift j among nurses $1, 2, \ldots, k$
$\Phi_{j,k_1,k_2}(x)$ = Probability that x nurses signed up for shift j among nurses $k_1, k_1+1, \ldots, k_2$
$\dim(A)$ = The cardinality of vector A.

3.1.1 Algorithm HH1

HH1 is a heuristic algorithm that determines a near-optimal solution to Problem **S2**. The steps of the algorithm are defined in a way that we can ensure that the output of the algorithm obtains some characteristics of the optimal solution that we have identified in Proposition 2 (and Proposition 5 that I will prove next, which is removing the first arriving nurse in Step 2.2.1). Now we discuss the intuition behind the steps of Algorithm **HH1**.

It starts with **Step 1** in which the algorithm suggests the optimal static policy in Problem **S1** as a basic solution to Problem **S2**. This is due to the fact that for each shift $j \in \mathcal{J}$, Proposition 2 indicates that the optimal set of nurses to whom shift j must be offered in Problem **S2** includes all the nurses to whom this shift is offered in Problem **S1**. Then, the algorithm may include additional nurses to $\mathcal{N}_j^1$ to the solution to improve it in the following steps, but it never excludes any of the nurses in $\mathcal{N}_j^1$ from the solution.

Next, it runs **Step 2** aiming at improving the current solution. In this step, instead of considering shift j which requires n_j nurses, we can imagine that there are n_j shifts, the same as shift j, each of which requires only one nurse. Then, for each such shift, we can find a set of nurses to whom it should be offered to maximize the revenue we can obtain from all these shifts. Then, the solution is to offer shift j to the union of all such sets.

Now, suppose that nurse k is the first nurse that arrives in the set of nurses to whom it is optimal to offer shift j when it requires only one nurse. Then, no matter how many shifts j are available, nurse k either signs up for one or leaves the platform without registering for shift j at all. Therefore, we can offer the first shift of n_j shifts to set $\mathcal{N}_j^1$. This means whether nurse k signs up for it and the expected revenue associated with this nurse is obtained, or nurse k leaves the platform without registering and it will be available for the next nurse in set $\mathcal{N}_j^1$ arriving after nurse k, and so on.

If nurse k does sign up, then shift j is still available for the other nurses with probability one, unlike the case where $n_j = 1$, and it would be available if and only if nurse k would not register for it. Thus, we can imagine that for the other $n_j - 1$ shifts, nurse k does not exist, since they cannot be offered to this nurse, and moreover, nurse k does not involve in the expected revenue obtained from the nurses in $\mathcal{N} \backslash k$.

So, in **Step 2.2** we move on to the next shift j, and again since it requires only one shift, we can find the best set of nurses to whom this shift should be offered by solving Problem **S1** as if nurse k does not exist in the problem setting. Then, in this solution there may be nurses that were not previously in the offering set of the first shift j. That means there may be one or more nurses to whom it might not be optimal to offer shift j when it requires only one shift. However, it may be optimal to offer this shift to them if it requires more than one shift.

As offering the shift to different combination of these additional nurses together with the previous set may result in different total expected revenues, then in **Step 2.3** we evaluate these different policies and find the one with the maximum expected revenue. Note that our heuristic algorithm only comes up with two alternative policies corresponding to each additional nurse found in **Step 2.2** (as discussed in Proposition 3) and compares their expected revenue with the current solution. Therefore, it is of polynomial time, and it avoids the complexity of evaluating exponentially many policies. (The numerical results show that it can find the optimal solution in 99.9 percent of the instances.) Then, to find the expected revenue, we set $n_{cur} = 2$ when we move on to the second shift of shift j, since so far, we assumed that there are two of shift j in the problem.

Then, whenever we update the current solution and add more nurses to the offering set we update n_{cur}, which is as if we used one unit of the capacity that we have for shift j (**Step 2.4**). We also, update the current solution to include the additional profitable nurses if their combination brings us more expected revenue than the current solution. Here, we only add nurses to the initial set $\mathcal{N}_j$ that was equal to $\mathcal{N}_j^1$, which guarantees that Proposition 2 holds here.

Algorithm HH1 A Heuristic Algorithm for Problem **S2**

Result: Finding a set of nurses to whom shift j ($j \in J$) should be offered.

Initialization: $\mathcal{N} = \{1, 2, \ldots, N\}$ set of all nurses, $\mathcal{V} = \{1, 2, \ldots, V\}$ set of all shifts, preference probabilities p_{ij}, commitment probabilities q_{ij}, number of nurses that shift j requires (n_j).

Step 1: Finding a solution to Problem **S2**, loop initialization

Step 1.1: Finding a solution to Problem **S2**

Set $Y_j = Y_j^1$, $\mathcal{N}_j = \mathcal{N}_j^1$

Step 1.2: Loop Initialization

Set $\mathcal{N}_{cur} = \mathcal{N}$, $n_{cur} = 2$

Step 2: Forming a loop to improve the solution to Problem **S2**

Step 2.1: Stopping Criteria

If $n_j = 1$ go to Step 2.5

If $dim(Y_j^1) \leq 1$ go to Step 2.5

If N_j^1 is empty go to Step 2.5

Step 2.2: Finding a set of potentially profitable nurses ($\mathcal{N}_{j,p}$), to whom shift j is not offered in the current solution

Step 2.2.1: Modifying the setting of Problem **S1**

Set of all nurses $\mathcal{N}_{cur} \leftarrow \mathcal{N}_{cur} \backslash \{min(\mathcal{N}_j^1)\}$

Step 2.2.2: Finding the solution to the modified Problem **S1**

Find $Y_j^1, \mathcal{N}_j^1$

Step 2.2.3: Finding $\mathcal{N}_{j,p}$

$\mathcal{N}_{j,p} \leftarrow \mathcal{N}_j^1 \backslash \mathcal{N}_j^2$

Let $\mathcal{N}_{j,p} = \{n_{1,p}, n_{2,p}, \ldots, n_{k,p}\}$, where $k = dim(\mathcal{N}_{j,p})$

Step 2.3: Finding alternative policies to Y_j and evaluate their total expected revenue

Step 2.3.1: Defining alternative policies

Step 2.3.1.1: Defining two policies associated with each nurse $n_{i,p} \in \mathcal{N}_{j,p}$

to offer shift j to all nurses in $\mathcal{N}_j$ and nurse $n_{i,p}$

to offer shift j to all nurses in $\mathcal{N}_j$ and nurses $\{n_{1,p}, n_{2,p}, \ldots, n_{i,p}\}$

Step 2.3.1.2: Compute the total expected revenue obtained from the policies found in Step 2.3.1.1 in Problem **S2** where n_j is replaced by n_{cur}

Step 2.4: Updating the solution to **S2** and the parameters

$Y_j \leftarrow$ The policy with the maximum total expected revenue among the policies found in Step 2.3.1.1 and Y_j.

$\mathcal{N}_j \leftarrow \mathcal{N}_{Y\,j}$

If $\mathcal{N}_j \cap \mathcal{N}_{j,p} \neq \emptyset$ and $n_{cur} < n_j$, set $n_{cur} \leftarrow n_{cur} + 1$

Go to Step 2

Step 2.5: Returning the best solution found to Problem **S2**

Return Y^j

Stop