MATLAB ASSIGMENT-2

Emre Can Şen-21902516

Part I:

(a), (b)

(a) $x(t) = p(t)[a_1 \sin(2\pi f_1 t) + a_2 \sin(2\pi f_2 t) + \dots + a_{10} \sin(2\pi f_{10} t)]$

$\psi_k(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_k t)$ $\quad 2\int_0^1 \sin^2(2\pi f_k t) = \int_0^1 1-\cos(4\pi f_k t) = 1$, k - dimension, 10 dimension

$s_1 = \begin{bmatrix} a_1/\sqrt{2} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \; s_2 = \begin{bmatrix} 0 \\ a_2/\sqrt{2} \\ \vdots \\ 0 \end{bmatrix} \dots , s_{10} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ a_{10}/\sqrt{2} \end{bmatrix}, \; \int_0^1 x(t)\psi_k(t)\,dt = s_k = \frac{a_k}{\sqrt{2}}$

Sampling $\Rightarrow \sum_{n=1}^{1200} x[n] \sin[2\pi \frac{f_k}{1200} n] = \frac{a_k}{2}$, where $f_k \in \{30k-20, 30k-10, 30k\}$ Hz & $k = 1, 2, \dots, 10$

for $k=1 \Rightarrow f_1 \in \{10, 20, 30\}$ Hz $\Rightarrow \sum_{n=1}^{1200} \underbrace{x[n]}_{\text{known}} \sin[2\pi \underbrace{f_1}_{\to\{10,20,30\}} n] = \frac{a_1}{2} \Rightarrow \hat{a}_1 = \{a_1^{(1)}, a_1^{(2)}, a_1^{(3)}\}$

For the case where $f_1 \neq f_{true,1}$ the multiplication of sine's of $x[n]$ function with $\sin[2\pi f_1 n]$ will yield 0. So if $a_1 \neq a_{true,1}$ the result will be 0. If $f_1 = f_{true,1}$

orthogonal

$\Rightarrow \sum_{n=0}^{1200} (a_1 \sin[2\pi \frac{f_1}{1200} n] + a_2 \sin[2\pi \frac{f_2}{1200} n] + \dots + a_{10} \dots) \sin[2\pi \frac{f_1}{1200} n] = \sum_{n=0}^{1200} a_1 \sin^2[2\pi f_1 n/1200] = \frac{a_1}{2}$

For theoretical case the result is zero if we chose the incorrect frequency value, but for the simulation case the result will be very small due to approximation errors of sine values. So we will pick the frequency that maximizes the output.

Figure 1- Theoretical Solutions of part a & b

(c)

k-value	1	2	3	4	5	6	7	8	9	10
Frequency	30	50	80	120	150	160	190	230	250	300

Table 1.1- Frequency Values Estimations

k-value	1	2	3	4	5	6	7	8	9	10
a_k value	9.8769	1.9319	9.1355	1.6180	1.4142	0.6691	4.3571	0.7167	1.2941	1.1257e-13

Table 1.2- a_k Values Estimations

(d)

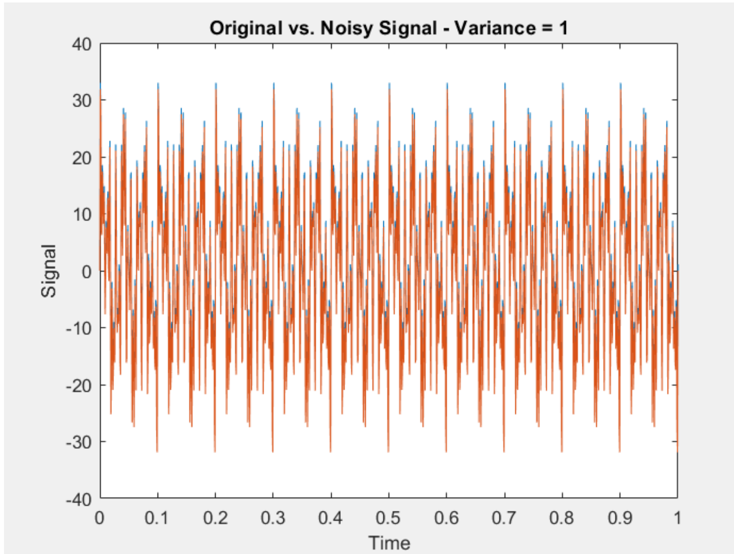


Figure 2– Original and Corrupted Signal with Variance 1

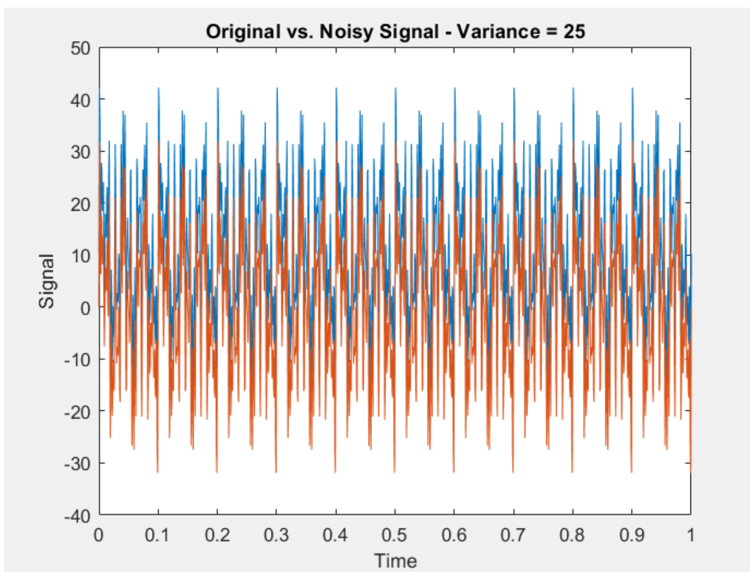


Figure 3– Original and Corrupted Signal with Variance 25

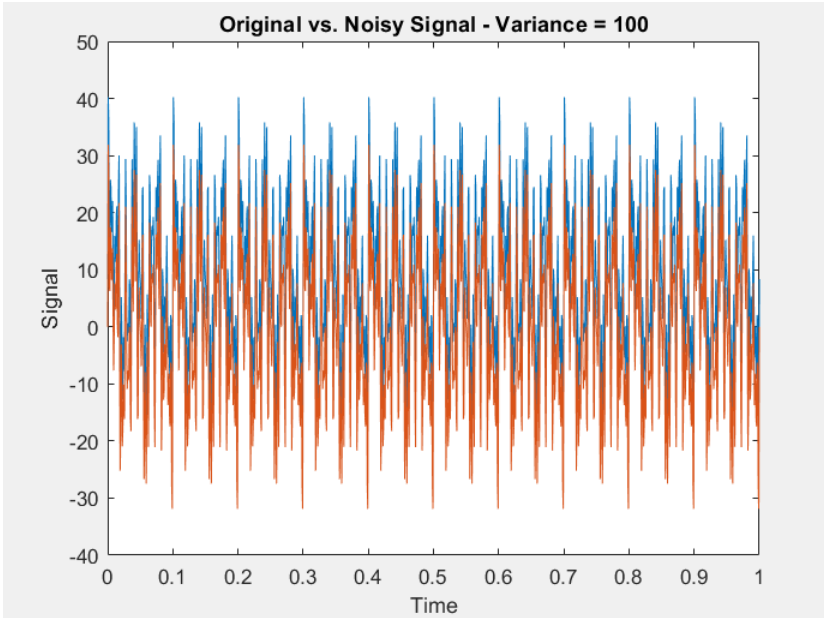


Figure 4– Original and Corrupted Signal with Variance 100

€

Variance 1:

k-value	1	2	3	4	5	6	7	8	9	10
Frequency	30	50	80	120	150	160	190	230	250	300

Table 2.1- Frequency Values Estimations for var=1

k-value	1	2	3	4	5	6	7	8	9	10
a_k value	9.8767	1.9316	9.1350	1.6173	1.4134	0.6683	4.3561	0.7156	1.2930	0.0012

Table 2.2- a_k Values Estimations for var=1

Variance 25:

k-value	1	2	3	4	5	6	7	8	9	10
Frequency	30	50	80	120	150	160	190	230	250	300

Table 3.1- Frequency Values Estimations for var=25

k-value	1	2	3	4	5	6	7	8	9	10
a_k value	9.8776	1.9330	9.1373	1.6207	1.4174	0.6725	4.3609	0.7209	1.2984	0.0045

Table 3.2- a_k Values Estimations for var=25

Variance 100:

k-value	1	2	3	4	5	6	7	8	9	10
Frequency	30	50	80	120	150	160	190	230	250	300

Table 4.1- Frequency Values Estimations for var=100

k-value	1	2	3	4	5	6	7	8	9	10
a_k value	9.8756	1.9298	9.1322	1.6133	1.4085	0.6632	4.3504	0.7092	1.2863	0.0080

Table 4.2- a_k Values Estimations for var=100

With the increase in variance, the estimation error for amplitudes also increases. For the frequencies, the estimation was able to correctly predict each frequency point. These results make sense as the estimation error for frequency prediction will be due to incorrect frequencies correlation, which will be just zero mean noise.

Part II:

(a)

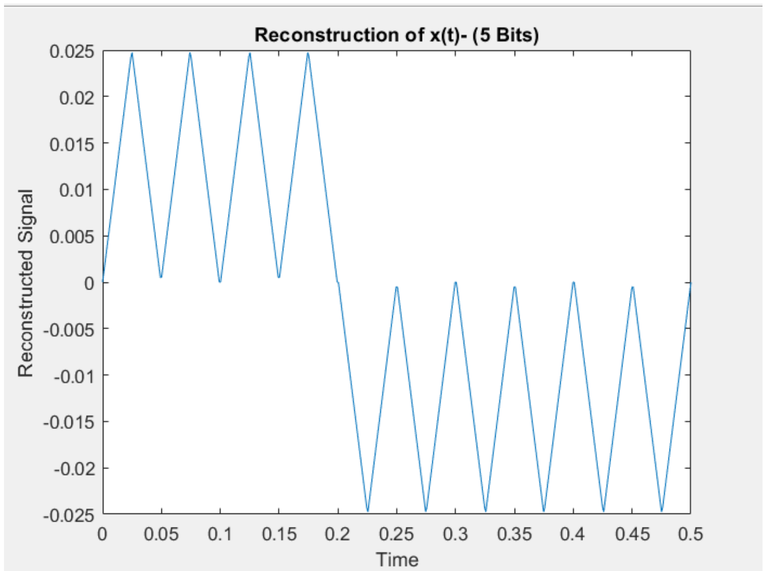


Figure 5- Reconstructed x(t) with 5 Bits

(b)

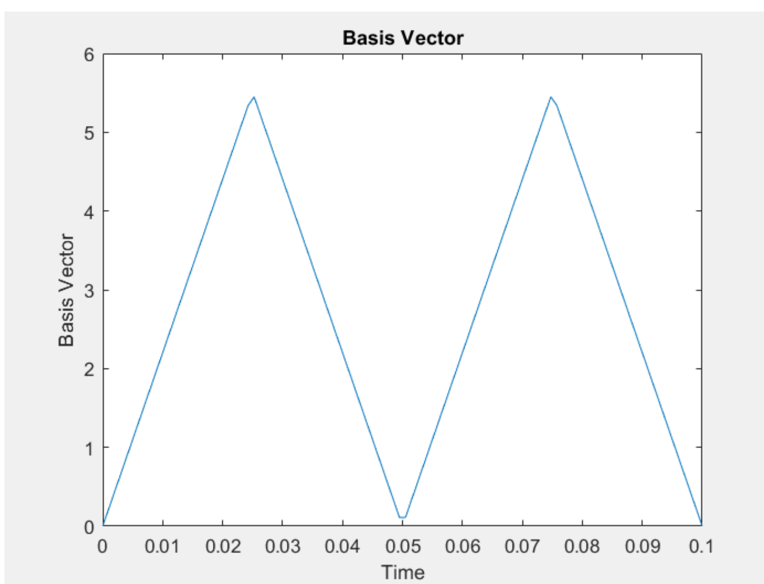


Figure 6- Positive Basis Vector

The dimension of signal space will be 1, since the two signals are negative versions of each other hence, can be expressed as -1 multiplicative of each other.

(c)

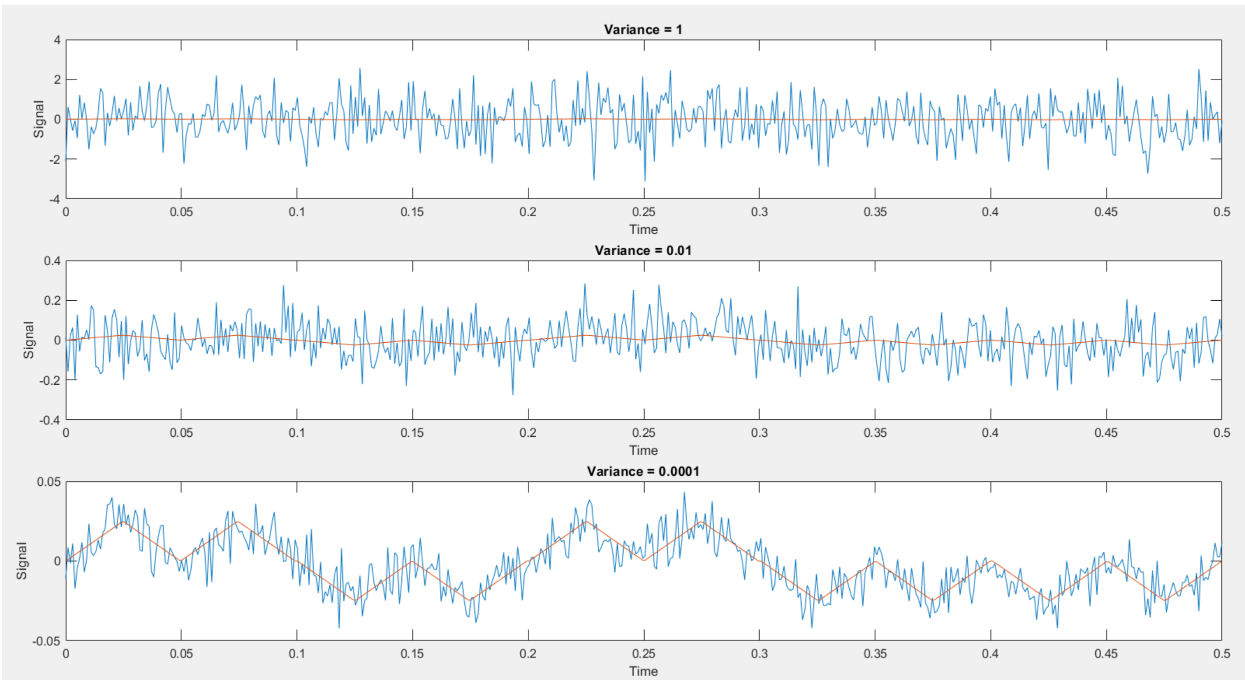


Figure 7- Original and Corrupted Signal with Variances= 1, 0.01, 0.0001

Theoretical value of SNR is found by Es/No. This holds true for ideal case where the white Gaussian noise spans the whole spectrum. This is not true for the simulation as we are bandlimited. So as the sampling rate increases, the noise will be spread across the bandwith and SNR will increase.

(d)

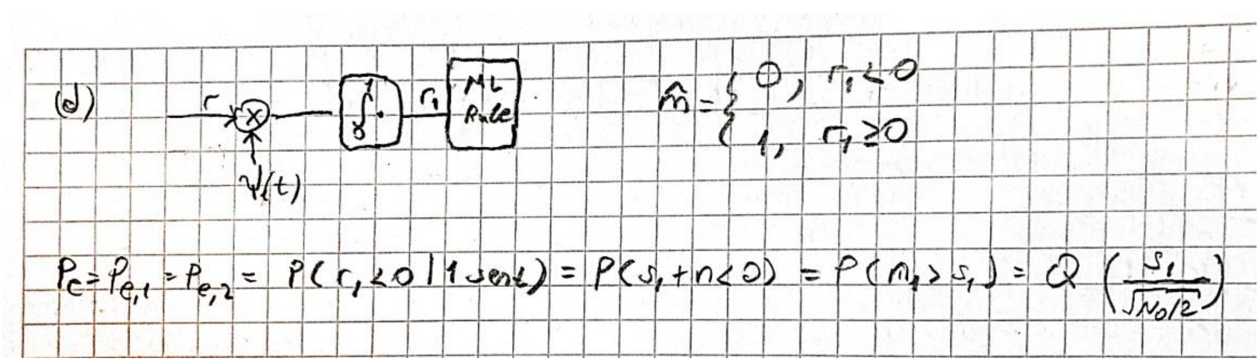


Figure 8- Theoretical Design of the Receiver and Error Probability

(e)

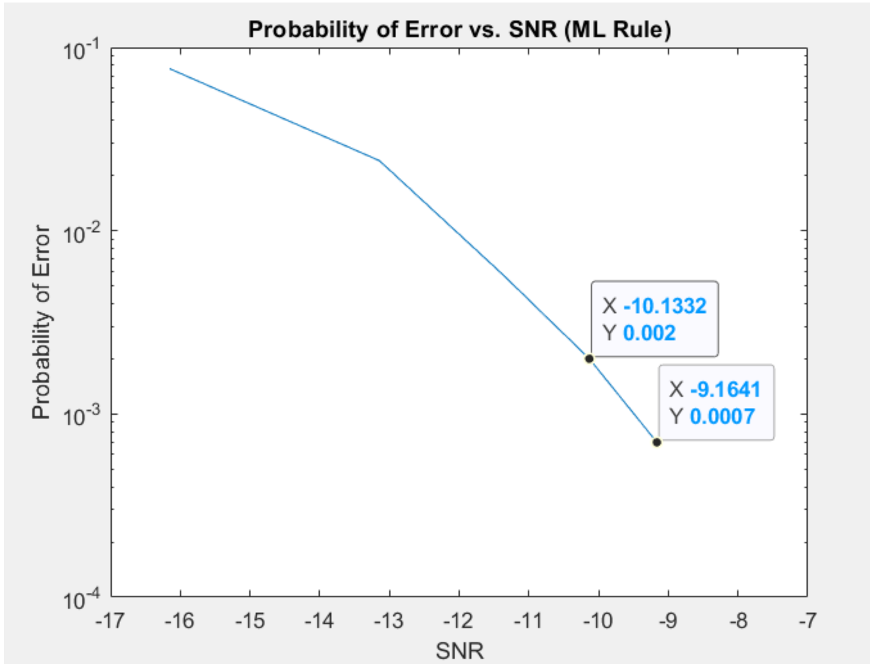


Figure 9- Simulation Probability of Error vs. SNR

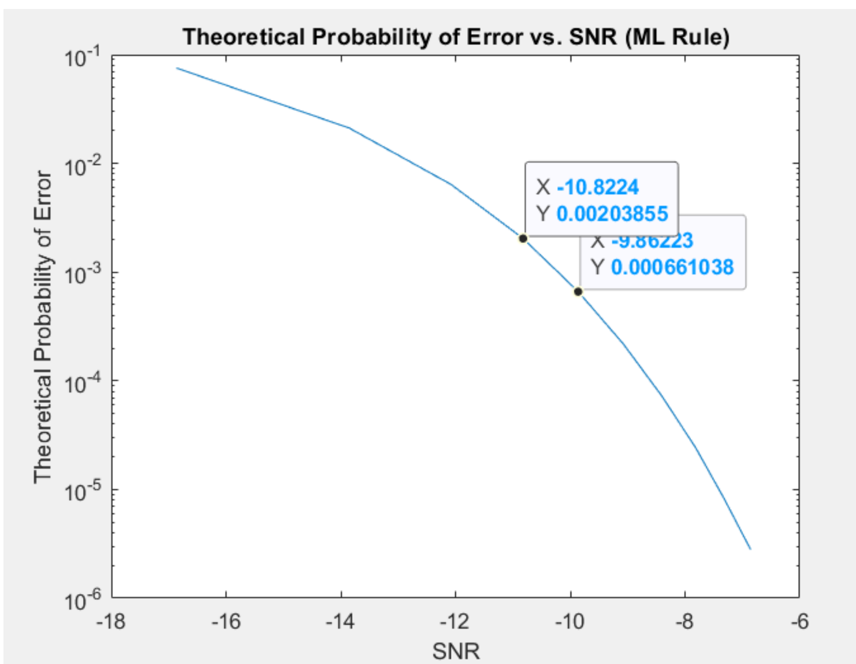


Figure 10- Theoretical Probability of Error vs. SNR

The obtained graphs look similar as expected. As it was hard to align SNR values such that both theoretical and simulation results match at the exact points, it is hard to make direct comparisons between points. However, error probabilities are similar to one another for similar SNR values.

(f)

The design of receiver stays the same except the ML Rule part. For this part, we need to use the MAP Rule. Following modifications are made:

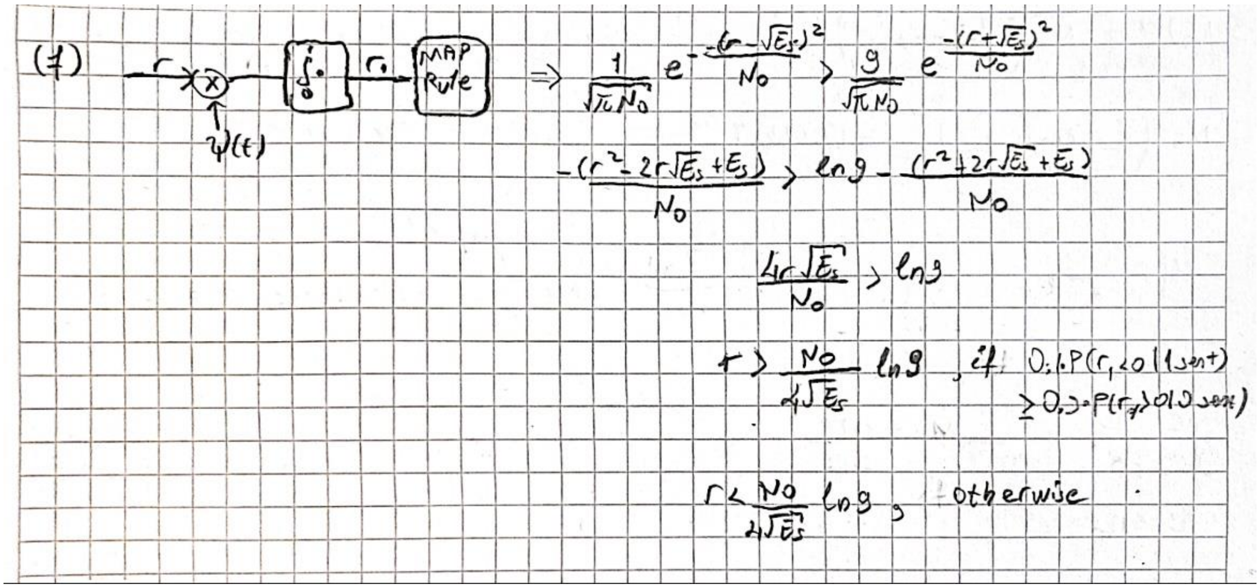


Figure 11- Theoretical Design of the Receiver and Error Probability

(g)

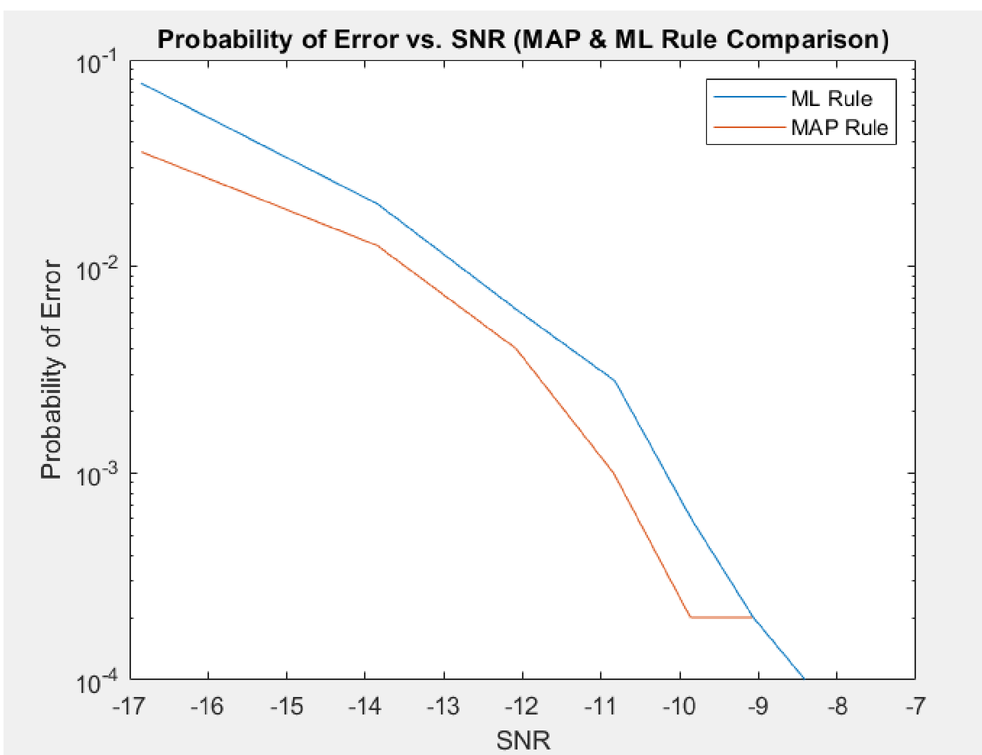
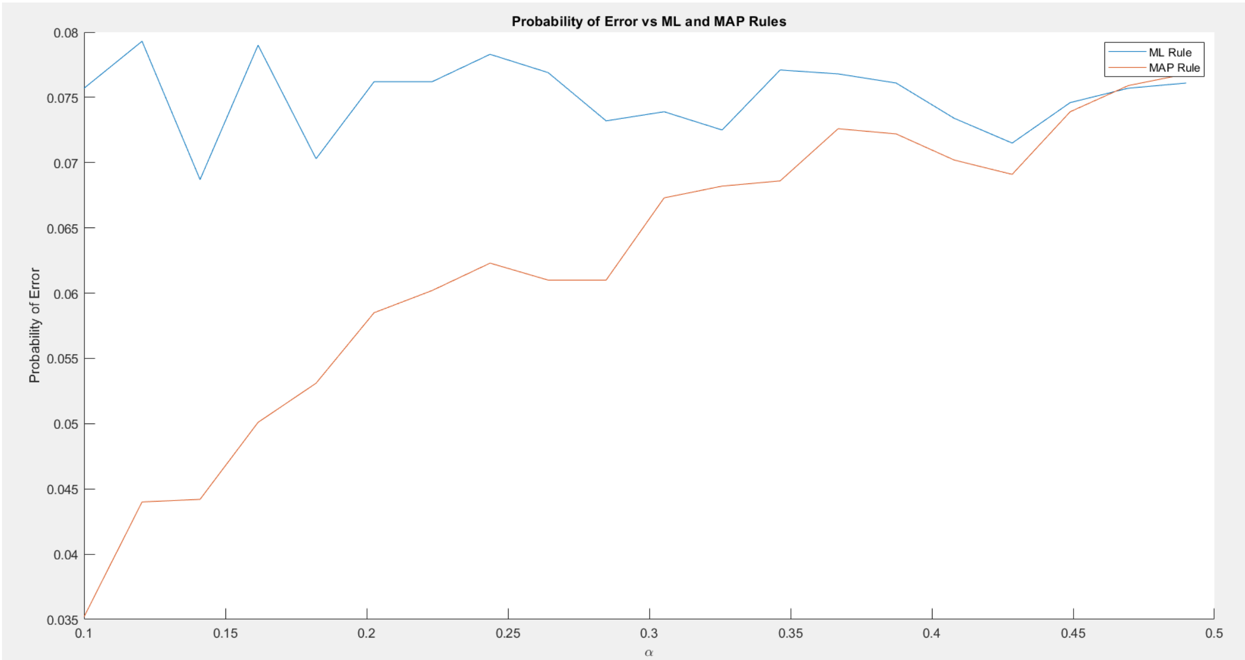


Figure 12- Probability of Error vs. SNR for MAP and ML Rule

As the MAP rule accounts for the prior distributions its error probability is better for same SNR values compared to ML rule.

(h)



These results are as expected, since at the start MAP rule performs better since it accounts for the different probabilities of priors. With the priors becoming more equally likely, the MAP rule will converge to the case of ML rule since ML rule is special case of MAP rule where each prior has the same probability. The results of the graph reflect these points.