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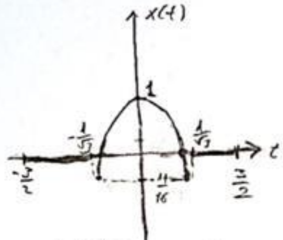
Section-2

Lab-02

Analytical Part

$T=3$, $w=1.5$ $\quad t \in \left(-\frac{3}{2}, \frac{3}{2}\right)$, $x(t)=\begin{cases} 1-3t^2, & -\frac{3}{4}<t<\frac{3}{4} \\ 0, & \text{o.w.} \end{cases}$



$$X_k=\frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-j\frac{2\pi k t}{T}}dt \Rightarrow \frac{1}{3}\int_{-3/4}^{3/4}(1-3t^2)e^{-j\frac{2\pi k}{3}t}dt$$

$$=\frac{1}{3}\left[\int_{-3/4}^{3/4} e^{-j\frac{2\pi k}{3}t}dt - 3\int_{-3/4}^{3/4} t^2 e^{-j\frac{2\pi k}{3}t}dt\right]=\frac{1}{3}\left[-\frac{3}{2\pi k j}e^{-j\frac{2\pi k}{3}t}\Big|_{-3/4}^{3/4}\right.$$

$$\left.-3\left(t^2\int_{-3/4}^{3/4} e^{-j\frac{2\pi k}{3}t}dt - \int_{-3/4}^{3/4} 2t\int e^{-j\frac{2\pi k}{3}t}dt\,dt\right)\right]$$

$$=\frac{1}{3}\left[\underbrace{-\frac{3}{\pi k}\;\frac{e^{j\frac{2\pi k}{3}\frac{3}{4}}-e^{-j\frac{2\pi k}{3}\frac{3}{4}}}{2j}}_{\frac{3}{\pi k}\sin\left(\frac{\pi k}{2}\right)} - 3\left(\underbrace{-2jt^2\,\frac{e^{j\frac{2\pi}{3}\frac{3}{4}}-e^{-j\frac{2\pi}{3}\frac{3}{4}}}{2j}}_{-2jt^2\sin\left(\frac{\pi k}{2}\right)} - \underbrace{\int_{-3/4}^{3/4} 2t\left(\frac{-3}{j2\pi k}\right)e^{-j\frac{2\pi}{3}kt}dt}_{*}\right)\right]$$

$$\Downarrow *$$

$$\frac{3}{j2\pi k}\left(2t\int_{-3/4}^{3/4} e^{-j\frac{2\pi}{3}kt}dt - \int_{-3/4}^{3/4} 2\int e^{-j\frac{2\pi}{3}kt}dt\,dt\right)$$

$$* \Rightarrow \frac{3}{j2\pi k}\left[-4tj\sin\left(\frac{\pi k}{2}\right)+\frac{3}{j\pi k}\left(-2j\sin\left(\frac{\pi k}{2}\right)\right)\right]$$

$$\frac{1}{3}\left[\frac{3}{\pi k}\sin\left(\frac{\pi k}{2}\right)+6jt^2\sin\left(\frac{\pi k}{2}\right)-\frac{9}{j2\pi k}\left[-4tj\sin\left(\frac{\pi k}{2}\right)+\frac{3}{j\pi k}\left(-2j\sin\left(\frac{\pi k}{2}\right)\right)\right]\right]$$

$$=\frac{\sin\left(\frac{\pi k}{2}\right)}{\pi k}+2jt^2\sin\left(\frac{\pi k}{2}\right)+\frac{6t}{\pi k}\sin\left(\frac{\pi k}{2}\right)+\frac{9j}{2\pi^2 k^2}\sin\left(\frac{\pi k}{2}\right)$$

- While computing cosine & sine functions, Matlab has a error margin, because the π in the Matlab is not exactly the real π.

- As k increases, the function has more points to work with so it is a better approximation of the real function. Increased k will increase the likeliness of the function. For the discontinuities, we can observe the Gibbs phenomenon.

- Part 4

a-) This operation flips the function on y-axis. But since the function looks even, it does not have much effect on graph

b-) The 't' in the func. becomes 't-to': Time shift to right by to

c-) This takes the derivative of the fnc. So it is the speed of change of the fnc.

d-) Takes the function from min point to peak & flips it, then from peak to min. and flip it again. Then readjusts the 700 points. Since this is a periodic signal, it looks like a time-shifted version.

CamScanner ile tarandı

Part 1

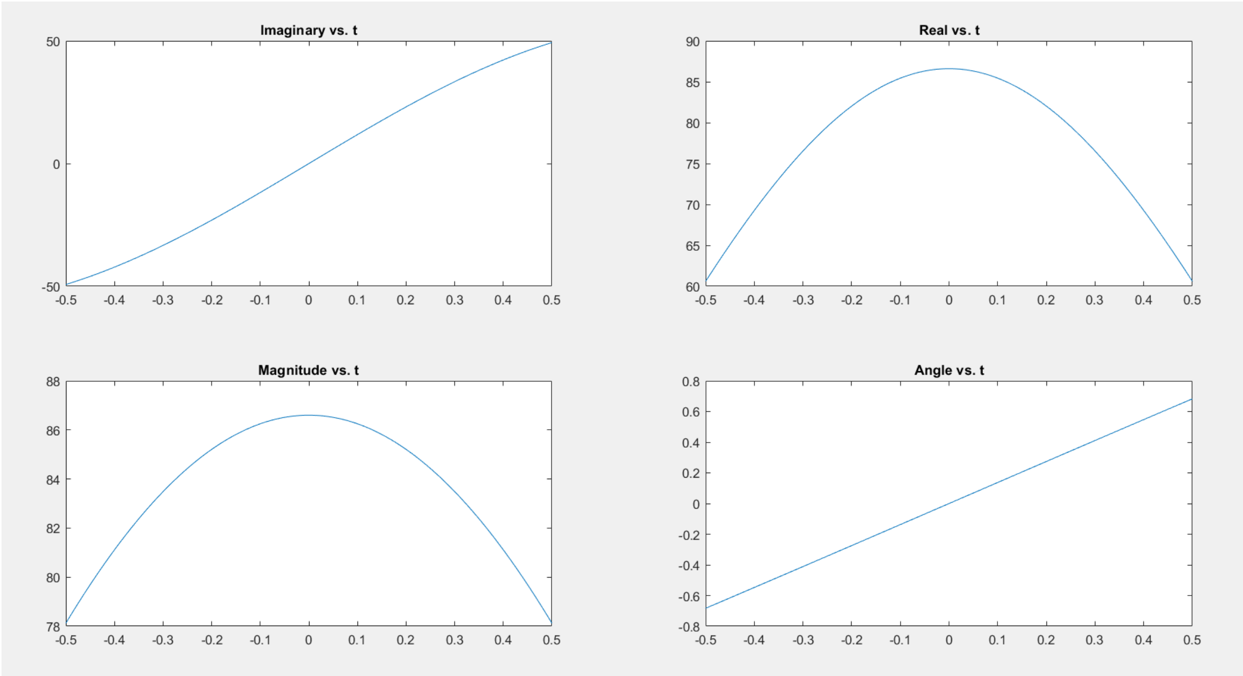


Figure 1- SUMCS Output

Part 3

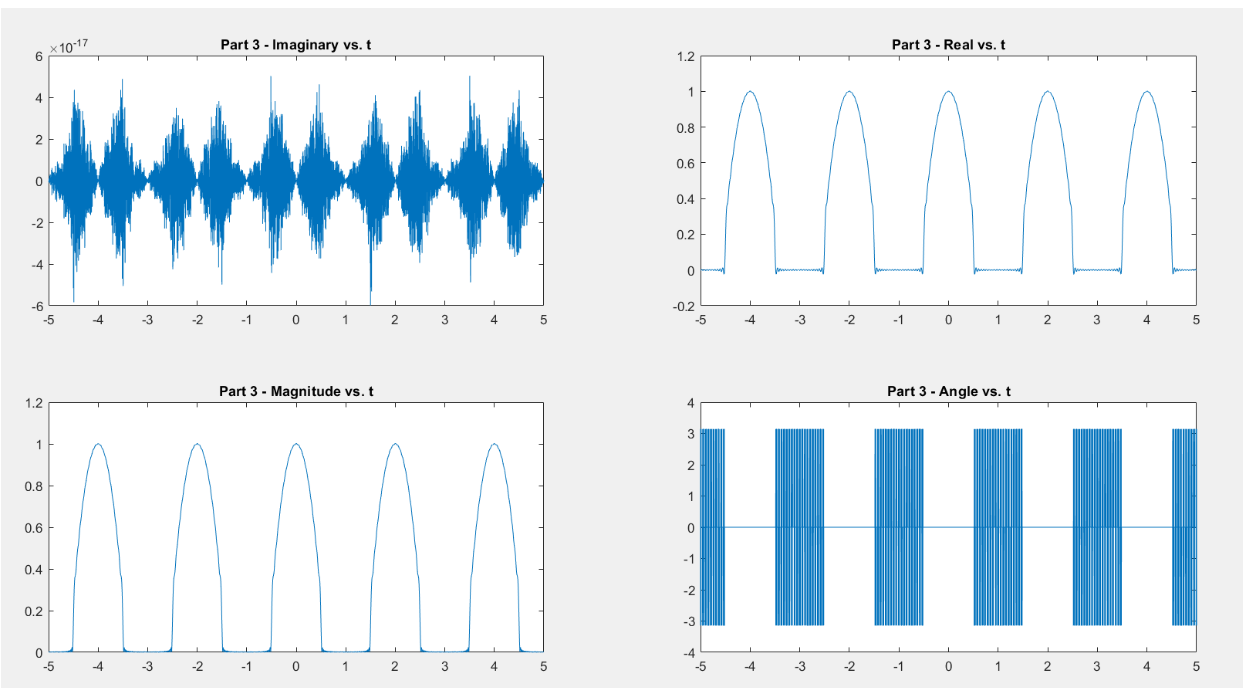


Figure 2- FSWave Output

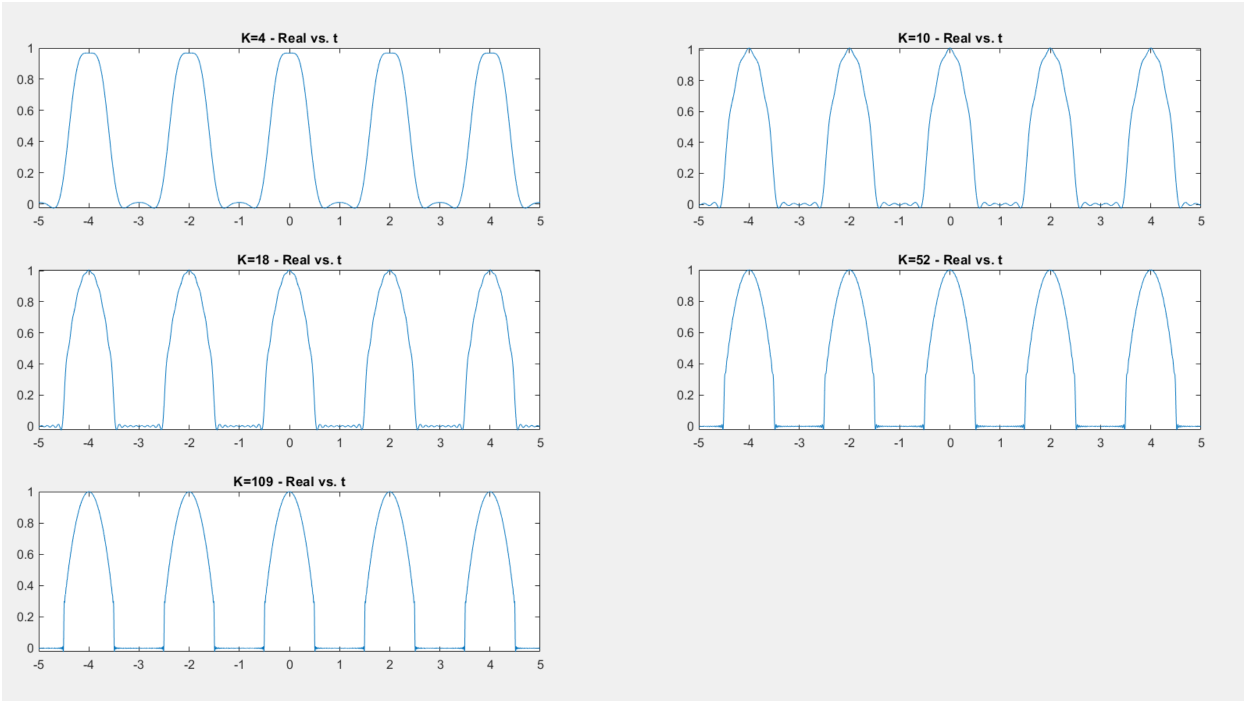


Figure 3- FSWave Output for Differing K's

Part 4

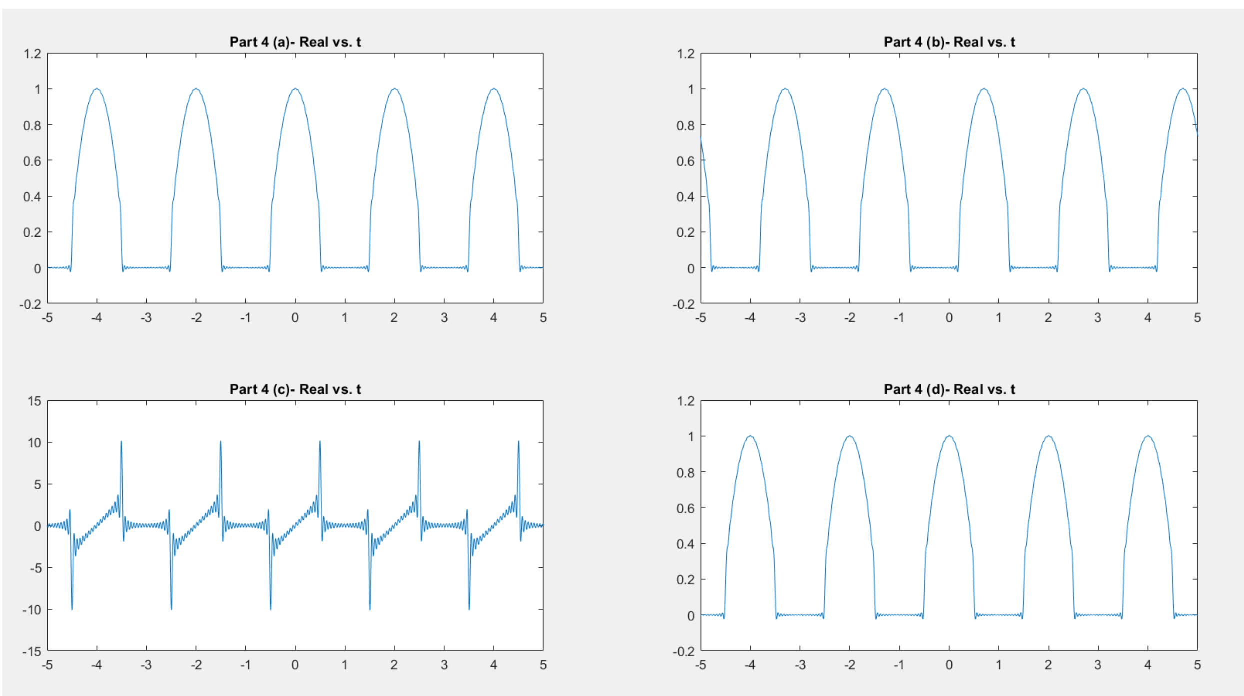


Figure 4- Part 4 Output

MATLAB Code

```
t=[-0.5:0.001:0.5];
t3=[-5:0.001:5];
t4=[-5:0.001:5];
t4b=[-5:0.001:5]-0.7;
A=1+(4).*rand(1,33);
omega=pi.*rand(1,33);
K=37;
K1=4;
K2=10;
K3=18;
K4=52;
K5=109;
K6=34;
T=2;
W=1;
x1=SUMCS(t,A,omega);
x2=FSWave(t3,K,T,W);
x3=FSWave(t4,K1,T,W);
x4=FSWave(t4,K2,T,W);
x5=FSWave(t4,K3,T,W);
x6=FSWave(t4,K4,T,W);
x7=FSWave(t4,K5,T,W);

x8=FSWave(-t4,K6,T,W);

xb=FSWave(t4b,K6,T,W);

xc=FS1Wave(t4,K6,T,W);

xd=FS2Wave(t4,K6,T,W);


figure(1)
subplot(2,2,1);
plot(t,imag(x1))
title('Imaginary vs. t')

subplot(2,2,2);
plot(t,real(x1))
title('Real vs. t')

subplot(2,2,3);
plot(t,abs(x1))
```

```
title('Magnitude vs. t')

%((imag(x1)).^2+(real(x1)).^2).^(1/2)

subplot(2,2,4);
plot(t,angle(x1))
title('Angle vs. t')


figure(2)
subplot(2,2,1);
plot(t3,imag(x2))
title('Part 2 - Imaginary vs. t')

subplot(2,2,2);
plot(t3,real(x2)),
title('Part 2 - Real vs. t')

subplot(2,2,3);
plot(t3,abs(x2))
title('Part 2 - Magnitude vs. t')

subplot(2,2,4);
plot(t3,angle(x2))
title('Part 2 - Angle vs. t')

figure(3)
subplot(3,2,1);
plot(t4,real(x3))
title('K=4 - Real vs. t')

subplot(3,2,2);
plot(t4,real(x4))
title('K=10 - Real vs. t')

subplot(3,2,3);
plot(t4,real(x5))
title('K=18 - Real vs. t')

subplot(3,2,4);
plot(t4,real(x6))
title('K=52 - Real vs. t')

subplot(3,2,5);
plot(t4,real(x7))
```

```
title('K=109 - Real vs. t')

figure(4)
subplot(2,2,1)
plot(t4,real(x8))
title('Part 4 (a)- Real vs. t')

subplot(2,2,2)
plot(t4,real(xb))
title('Part 4 (b)- Real vs. t')

subplot(2,2,3)
plot(t4,real(xc))
title('Part 4 (c)- Real vs. t')

subplot(2,2,4)
plot(t4,real(xd))
title('Part 4 (d)- Real vs. t')


function xs = SUMCS(t,A,omega)
    xs=zeros(1,length(t));
    for i=1:length(omega)
        xs= xs+A(i)*exp(1j*omega(i)*t);
    end
end

function xt= FSWave(t,K,T,W)
    k0=[-K:K];
    fun=@(x) exp(-1j*2*pi/T*k0*x).*(1-3*x.^2);
    xk=1/T*integral(fun,-W/2, W/2,'ArrayValued',true);
    xt=SUMCS(t,xk,2*pi*k0/T);
end

function xt= FS1Wave(t,K,T,W)
    k0=[-K:K];
    fun=@(x) exp(-1j*2*pi/T*k0*x).*(1-3*x.^2);
    xkz=1/T*integral(fun,-W/2, W/2,'ArrayValued',true);
    xk=(-1j*2*pi/T*k0).*xkz;
    xt=SUMCS(t,xk,2*pi*k0/T);
end

function xt= FS2Wave(t,K,T,W)
    k1=[-K:-1];
```

```
    k2=[1:K];
    k1=flip(k1);
    k2=flip(k2);
    k0=[k1 0 k2];
    fun=@(x) exp(-1j*2*pi/T*k0*x).*(1-3*x.^2);
    xk=1/T*integral(fun,-W/2, W/2,'ArrayValued',true);
    xt=SUMCS(t,xk,2*pi*k0/T);
end
```