ORGANIZATIONAL BEHAVIOR AND HUMAN PERFORMANCE 29, 175–207 (1982)

Choice Behavior in a Sequential Decision-Making Task

JEROME R. BUSEMEYER

University of Illinois, Champaign

Many day-to-day decisions involving risk are based on a person's memory for the past sequence of outcomes. The study of sequential decision making investigates how individuals use information about the past sequence of outcomes in order to make risky decisions. In this article, a random walk decision model called the confidence strength model is evaluated which was designed to describe the cognitive processes individuals use to combine sequential information provided by event patterns with payoff information in a sequential decision-making task. Two related experiments are reported which were designed to test various assumptions of the confidence strength model. The particular task used to study sequential decision making was the differential payoff paradigm in which the decision maker was required to predict which of two uncertain events would occur on each trial. Following each choice, feedback was presented in the form of differential monetary payoffs which were contingent on the alternative–event combination occurring on that trial. Payoffs were manipulated by varying the loss for incorrect decisions associated with each alternative, and sequential information was manipulated by constructing event sequences with a fixed distribution of run lengths within each block of trials. The response measures included the proportion of trials one alternative was chosen over another and also the average choice speed for each alternative. The main theoretical result was that the confidence strength model provided a good overall fit to the pattern of results for the choice proportion measure, and also accounted for a majority of the findings obtained from the choice speed measures, although there were some minor exceptions. The confidence strength model provided a better account of the present results and previous research than several alternative choice models that were considered.

Much of the previous research in risky decision making has used a static decision task in which the decision maker is explicitly presented information about the probabilities of each outcome in a verbal or graphic

This article was based on a dissertation submitted to the University of South Carolina, in partial fulfillment of the requirements for a doctoral degree in psychology. A portion of these results was presented at the Twentieth Annual Meeting of the Psychonomic Society at Phoenix, Arizona. The research was supported by the Social & Behavioral Sciences Laboratory at the University of South Carolina. The author wishes to thank Dr. Thomas P. Cafferty for his helpful advice during the writing of this article, and special thanks are due to Dr. Thomas S. Wallsten for many helpful theoretical discussions. Finally, I wish to thank Dr. Steve Durham for providing some extremely useful guidance to the study of stochastic processes. Requests for reprints should be sent to Dr. Jerome R. Busemeyer, Department of Psychology, University of Illinois, Champaign, IL 61820.

0030-5073/82/020175-33$02.00/0

form. When a static decision task is used, the decision maker does not have to learn from past experience with the outcomes of previous decisions (cf. Rapoport & Wallsten, 1972). This feature of the static decision task becomes a problem when generalizing results to the many day-to-day decisions that repeatedly confront individuals, since explicit information concerning outcome probabilities is frequently not available and must be learned from previous experience. One important aspect of decisions based on past experience is that the decision maker will often rely on the previously experienced sequence or pattern of outcomes. The most well known example for demonstrating the importance of the previous pattern of outcomes on decision making is the gambler's fallacy, i.e., the tendency to predict a success after a consistent sequence of failures or a failure after a consistent sequence of successes. The study of sequential decision making is directly concerned with how decision makers use information about the past sequence of outcomes to make risky decisions. Thus, a sequential decision-making task may be more relevant for understanding the many day-to-day decisions that are based on past experience. Although a large body of research has investigated the effects of event patterns in probability learning (Jones, 1971; Myers, 1976) and a great deal of research has been conducted on the effects of payoffs in static decision tasks (Payne, 1973), very little is known about how these two factors are combined to affect choice behavior. The purpose of the present investigation was to study how decision makers combine sequential information provided by event patterns with payoff information to make choices in a sequential decision-making task.

The remainder of the article will be presented according to the following outline. First, the experimental paradigm used to study sequential choice behavior will be described along with a description of the variables being manipulated. Next, a random walk decision model called the confidence strength model will be presented, and evaluated in light of previous research. Finally, several predictions of the confidence strength model are described and two experiments are reported which provide tests of these predictions.

The Differential Payoff Paradigm

The payoff matrix. One of the simplest methods used to study sequential decision making is the differential payoff paradigm. In this paradigm, individuals receive a series of training trials in which they are required to *learn* to predict which of two events will occur on each trial, and they receive differential payoffs for correct and incorrect predictions. A typical choice problem can be generally represented by a payoff matrix shown below as Matrix 1. In this matrix, A_i $(i = 1,2)$ represents one of the two possible choice alternatives that can be

Matrix 1
Chance Event

$$\begin{array}{lc} & \begin{array}{cc} E_1 & E_2 \end{array} \\ \text{Response} \quad A_1 & \\ \text{Alternative} \quad A_2 & \end{array} \left[\begin{array}{cc} r_{11} & r_{12} \\ r_{21} & r_{22} \end{array}\right]$$

selected on a given trial, and E_j ($j = 1{,}2$) represents one of the two possible chance events that can be programmed to occur on a particular trial. The symbol π_j will be used to represent the marginal probability that event E_j occurs, and it is assumed that one of the events will be programmed to occur on each trial independent of the decision maker's response so that $\pi_1 + \pi_2 = 1.0$. The symbol r_{ij} represents the payoff produced by the occurrence of the alternative–event combination (A_i, E_j). Thus if A_i is chosen and E_j occurs then r_{ij} is added to the decision maker's total. Since the choice situation is meant to be a risky decision, it is assumed that $r_{11} > r_{21}$, $r_{22} > r_{12}$, and $0 < \pi_j < 1.0$.

Losses and regrets. When the marginal event probability is constant, the payoff matrix contains four payoff variables (r_{11}, r_{12}, r_{21}, r_{22}), and manipulating the value of any one of these variables results in a substantial change in choice behavior (Edwards, 1956; Katz, 1964; Myers, Reilly, & Taub, 1961; Myers & Suydam, 1964; Taub & Myers, 1961). An important question is how do individuals evaluate these four payoff variables when making choices? One classic answer to this question is that individuals compare the expected utility of each alternative by forming differences or ratios of expected utilities (Becker, Degroot, & Marschak, 1963). However, an important study by Myers, Suydam, and Gambino (1965) provides strong evidence against the expected utility rule. This study replicated earlier studies using a similar design (Katz, 1962; Myers & Katz, 1962; Myers & Sadler, 1960), but with extended training and with several levels of marginal event probability ($\pi_1 = .2, .5, .8$). Two choice problems, one called the "negative sure thing" problem and the other called the "positive sure thing" problem, were alternated in a random order throughout training. The payoff matrix for each problem is shown below as Matrices 2a and 2b. In this experiment, $r_{11} = -r_{12}$ so the subscripts were

Matrix 2a
Negative Sure Thing Problem

$$\begin{array}{cc} & \begin{array}{cc} E_1 & E_2 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \end{array} & \left[\begin{array}{cc} +r & -r \\ -1 & -1 \end{array}\right] \end{array}$$

Matrix 2b
Positive Sure Thing Problem

$$\begin{array}{cc} & \begin{array}{cc} E_1 & E_2 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \end{array} & \left[\begin{array}{cc} +r & -r \\ +1 & +1 \end{array}\right] \end{array}$$

dropped for convenience. Referring to Matrices 2a and 2b, the alternative A_1 is the risky alternative since one can win a payoff equal to either $+r$ or $-r$, where the absolute value of r is greater than one. The alternative A_2 is the sure thing since one would always lose a payoff equal to -1 when choosing the negative sure thing, and always win a payoff equal to $+1$ when choosing the positive sure thing. For convenience, the absolute value of r in Matrices 2a and 2b will be referred to as the risk magnitude.

The classic expected utility rule predicts that if an increase in risk magnitude increased the probability of choosing the risky alternative for the positive sure thing problem, then increasing the risk magnitude should also increase the probability of choosing the risky alternative for the negative sure thing problem. In other words, the direction of the effect of increasing the risk magnitude on the probability of choosing the risky alternative should be *independent* of the value of the sure thing condition (see Appendix A). The results of Myers *et al.* (1965) demonstrated a consistent violation of this independence property for all levels of π_1. More specifically, for the positive sure thing problem, increasing the risk magnitude increased the probability of choosing the risky alternative; but for the negative sure thing problem, increasing the risk magnitude decreased the probability of choosing the risky alternative. This risk $\times$ sure thing interaction effect is important since it indicates that decision makers do not evaluate the payoffs independently for each alternative as suggested by the classic expected or subjective expected utility rule. Instead, the payoffs associated with each alternative are interlocked (cf. Fishburn, 1976).

Myers *et al.* (1965) provided a simple explanation for the risk $\times$ sure thing interaction effect by proposing that individuals compare ratios of regrets produced by each alternative rather than comparing expected utilities (see also Edwards, 1956). The concept of regret refers to the perceived loss which occurs when an "incorrect" choice is made. An incorrect choice is the choice of an alternative which produces a payoff less than the maximum possible when a given event is realized, i.e., A_2 is the incorrect choice when E_1 occurs and A_1 is the incorrect choice when E_2 occurs. The loss produced by incorrectly choosing A_i when event E_j ($i \neq j$) occurs is obtained by subtracting the utility of the payoff for A_i in column j from the utility of the maximum payoff in column j. Thus a loss matrix can be constructed from the payoff matrix, and the loss matrix corresponding to Matrix 1 is shown below as Matrix 3. For convenience,

Matrix 3
Loss Matrix

$$\begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{cc} E_1 & \qquad\qquad E_2 \end{array} \\ \begin{bmatrix} 0 & L_1 = u(r_{22}) - u(r_{12}) \\ L_2 = u(r_{11}) - u(r_{21}) & 0 \end{bmatrix} \end{array}$$

the loss produced by incorrectly choosing A_2 will be referred to as the A_2 loss and will be denoted L_2, the loss produced by incorrectly choosing A_1 will be referred to as the A_1 loss and will be denoted L_1.

In order to show in a simple fashion how the concept of loss accounts for the risk × sure thing interaction effect, reconsider the payoff matrices 2a and 2b, and assume that $\pi_1 = .5$. Also assume that the probability of choosing the risky alternative, A_1, is an increasing function of the ratio L_2/L_1 and that $u(r_{ij}) = r_{ij}$. For the positive sure thing condition, increasing the risk from $r = 2$ to $r = 10$ causes the ratio L_2/L_1 to increase from 1/3 to 9/11, yielding an increase in the probability of choosing the risky alternative. For the negative sure thing problem, this same increase in risk causes the ratio L_2/L_1 to decrease from 3/1 to 11/9, yielding a decrease in the probability of choosing the risky alternative.

Run patterns. Up to this point only the payoffs and the marginal event probabilities have been considered as factors affecting choice behavior. However, it is likely that the decision maker will attempt to predict the event for the next trial based on the preceding pattern of events. Although there is a large number of possible event patterns that occur within a long sequence of trials, one particular type of event pattern that previous research has shown to be very important is a run pattern (Jones, 1971; Myers, 1976). The run pattern occurring on trial t is the number of homogeneous events that occurred prior to and including trial t. A run pattern will be symbolized in general as kE_j, where k is the number or length of the run of homogeneous events, and j is the event index. For example, $2E_1$ would represent a run of two E_1 events.

Initially the study of the effects of run patterns resulted from research designed to test stimulus–response conditioning models of probability learning. One conditioning model specifically developed for the differential payoff paradigm is the weak–strong model proposed by Myers and Atkinson (1964). This model predicts that positive recency effects should always occur. Positive recency is said to occur when the probability of choosing A_i increases as the run length of event E_j, $i = j$, increases, while negative recency is said to occur when the probability of choosing A_i decreases as the run length of event E_j, $i = j$, increases. Within the differential payoff paradigm, positive recency effects have been observed with event sequences containing a high frequency of long runs, but negative recency effects have been observed with event sequences containing a low frequency of long runs (Heuckeroth, 1969; Jones & Myers, 1966). The occurrence of negative recency effects is problematic for stimulus–response conditioning models such as the weak–strong model. Negative recency effects have also been reported in other sequential decision-making paradigms (e.g., Kubovy & Healy, 1977; Rapoport, Jones, & Kahan, 1970) which attests to the pervasiveness of these effects.

The response measures. The measure of behavior which has received the most interest in this paradigm is the proportion of trials one alternative is chosen over another, denoted $P(A_i)$. However, considerable information about the ongoing choice process can also be obtained by considering the average choice speed for each alternative, denoted $S(A_i)$. Choice speed is particularly valuable for studying the underlying choice processes in the present task for the following reason. The cognitive processes used in this task are likely to be very rapid, and highly practiced, so that they are not likely to be available for report (cf. Ericsson & Simon, 1980). However, choice speed provides a nonobtrusive method for studying the number of steps involved in the decision process (cf. Pachella, 1974). Accordingly, the primary goal of the present study was to determine the adequacy of various models in predicting choice proportion, while a secondary goal was to test predictions related to choice speed.

Previous research by Myers, Gambino, and Jones (1967) has investigated the effects of marginal event probability on choice speed in a binary prediction task. They found that both $S(A_1)$ and $S(A_2)$ increased as π_1 increased in .1 steps from .6 to .9. However, $S(A_2)$ was consistently below $S(A_1)$ at all levels of π_1. Little research is available concerning the effects of payoffs on choice speed in the differential payoff paradigm, and this question will be considered in the present study.

The Confidence Strength Model

The confidence strength model is an application of the general random walk model (Cox & Miller, 1965; Feller, 1968) that has recently been applied to a wide variety of decision-making tasks including signal detection (Link & Heath, 1975), choice reaction time (Laming, 1968; Link, 1975; Stone, 1960), memory retrieval (Ratcliff, 1978), optional stopping (Edwards, 1965), and detection of change (Stein & Rapoport, 1978) tasks. The current application extends the random walk model to the differential payoff paradigm by describing the process individuals use to integrate event pattern information with payoff information. The confidence strength model assumes that individuals perceive the task as a problem of determining whether A_1 or A_2 will be the correct choice for a given trial, i.e., a problem of correctly predicting whether E_1 or E_2 will occur on the next trial. On a given trial, evidence is accumulated over time which increases the decision maker's confidence that either A_1 or A_2 will be correct. Evidence favoring A_i increases confidence in the choice of A_i and at the same time decreases confidence in the choice of $A_{i'}$, $i' \neq i$. Once the accumulated evidence surpasses a criterion level for one alternative, then that alternative is chosen.

Figure 1 is a transition diagram for a seven-state confidence strength model, and each circle enclosing a number in the diagram represents a

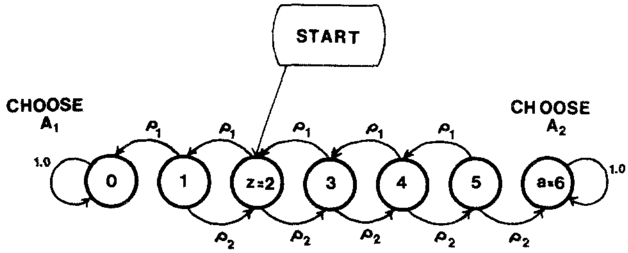

FIG. 1. Transition diagram for a seven-state confidence strength model.

different state or degree of confidence. As the degree of confidence decreases toward 0, confidence in A_1 increases and confidence in A_2 decreases; as the degree of confidence increases toward 6, confidence in A_2 increases and confidence in A_1 decreases. State 0 represents the criterion level of confidence required to choose A_1, state 6 represents the criterion level required to choose A_2, and state 2 represents the initial state of confidence existing before the decision process begins. In general, a represents the total distance in terms of the number of steps between the two boundaries, and z represents the initial state or starting point of the random walk process. In this example, $a = 6$ and $z = 2$, which indicates a slight initial bias favoring A_1 since z is closer to the A_1 boundary. The starting point is said to be unbiased whenever z is equally distant from each boundary, i.e., whenever $z = a/2$. In general, the starting point, z, and the total distance, a, are determined by the payoff matrix; but a complete description of these assumptions will be postponed until after the decision process is described in more detail.

The cognitive processes assumed to occur within a single choice trial can be described as follows. The decision maker begins with an initial degree of confidence and then attempts to increase confidence in A_1 or A_2 by making a series of independent covert predictions. If E_1 is covertly predicted then the degree of confidence in A_1 is increased by a step and the degree of confidence in A_2 is decreased by a step (e.g., referring to Fig. 1, a transition from state 2 to state 1 occurs). If E_2 is covertly predicted to occur then confidence in A_1 is decreased a step and confidence in A_2 is increased a step (e.g., referring to Fig. 1, a transition from state 2 to state 3 occurs). This covert prediction process continues until either state 0 is achieved in which case A_1 is chosen, or until state a is achieved in which case A_2 is chosen.

The covert predictions are assumed to be generated by the following memory retrieval process. After experiencing the outcomes of previous trials, the decision maker will have stored traces of the events in memory associated with contextual cues (cf. Estes, 1976a, 1976b). The generation of a covert prediction is accomplished by randomly sampling a trace from a set of traces activated by the current contextual cues. In the present task, it is assumed that run patterns operate as the most salient contextual cue for activating traces. Thus a prediction is generated by sampling a trace from the set of traces activated by the current run pattern cue (cf. Gambino & Myers, 1967; Restle, 1961). The symbol ρ_j will be used to represent the probability of sampling a trace associated with the event E_j. Referring to Fig. 1, ρ_1 represents the probability of making a transition toward A_1 and ρ_2 represents the probability of making a transition toward A_2. After extended training, it is assumed that the sampling process becomes stationary, so that ρ_j will simply be a function of the immediately preceding run pattern cue, kE_j.

The probability of choosing A_1, $P(A_1)$, equals the probability of terminating the random walk at state 0. For convenience, define $\theta = (\rho_1/\rho_2)$; then the probability of choosing A_1 is given by the following equations (cf. Feller, 1968, Chap. 14):

$$P(A_1) = \frac{\theta^a - \theta^z}{\theta^a - 1} \qquad \text{if } \rho_j \neq .5 \tag{1a}$$

$$= \frac{a - z}{a} \qquad \text{if } \rho_j = .5 \tag{1b}$$

$$= \frac{\rho_1^z}{\rho_1^z + \rho_2^z} \qquad \text{if } z = a/2. \tag{1c}$$

It is clear from the above equations that $P(A_1)$ is a decreasing function of z, the starting point; and an increasing function of ρ_1. For example, Eq. (1b) shows that when $\rho_j = .5$, then $P(A_1)$ is a negative linear function of z. When the starting point is unbiased (i.e., $z = a/2$), then Eq. (1c) indicates that $P(A_1)$ is an increasing S-shaped function of ρ_1. Equation (1c) also indicates that increasing the total distance, a, tends to amplify the effect of ρ_1 on $P(A_1)$ (i.e., increases the steepness of the S-shaped curve).

The expected number of steps to termination conditioned on the choice of A_i will be symbolized as $E(s|A_i)$. The expected number of steps to termination conditioned on the choice of A_1 was derived from the generating function for the random walk model provided by Feller (1968, Chap. 14, Eq. (4.4)):

$$E(s|A_1) = \frac{1}{\rho_1 - \rho_2}\left[\omega_1 a - \omega_2(a - z)\right] \qquad \text{if } \rho_j \neq .5 \tag{2a}$$

$$= \frac{z}{3}(2a - z) \qquad \text{if } \rho_j = .5 \tag{2b}$$

$$= \left(\frac{a}{2}\right)\left(\frac{1}{\rho_1 - \rho_2}\right)\left(\frac{\theta^z - 1}{\theta^z + 1}\right) \text{ if } \rho_j \neq .5 \text{ and } z = (a/2), \tag{2c}$$

where

$$\omega_1 = \frac{\theta^a + 1}{\theta^a - 1} \quad \text{and} \quad \omega_2 = \frac{\theta^{(a-z)} + 1}{\theta^{(a-z)} - 1}. \tag{3}$$

The expected number of steps conditioned on choosing A_2, $E(s|A_2)$, can be obtained by replacing θ and z with θ^{-1} and $(a-z)$, respectively. It is apparent from these equations that $E(s|A_1)$ is an increasing function of z, and an increasing function of a. For example, Eq. (2b) indicates that when $\rho_1 = .5$, then $E(s|A_1)$ increases according to a negatively accelerated quadratic function as z increases. Equation (2c) indicates that when $z = a/2$, then $E(s|A_1)$ increases as a increases, since the ratio $(\theta^z - 1)/(\theta^z + 1)$ increases toward 1.0 as a increases. It is natural to assume that response latency is an increasing function of the expected number of steps to termination so that $S(A_i)$ will be a decreasing function of $E(s|A_i)$.

Variable vs fixed total distance strategies. Assumptions concerning the starting point and the boundaries of the random walk process remain to be specified. There are at least two different sets of assumptions that could be employed to specify how the payoffs affect the starting point and boundaries of the random walk model. One approach suggested by the work of Audley and Pike (1965) is to assume that the magnitude of the loss produced by incorrectly choosing A_1 (i.e., the A_1 loss) determines the degree of confidence required to choose A_1; and the magnitude of the loss produced by incorrectly choosing A_2 (i.e., the A_2 loss) determines the degree of confidence required to choose A_2. In other words, the greater the loss produced by incorrectly choosing an alternative, the more confidence one requires before choosing that alternative. To make this assumption more precise, let k_1 be the number of steps or transitions between the initial state and the A_1 boundary, and let k_2 be the number of steps between the initial state and the A_2 boundary. For example, in Fig. 1, $k_1 = 2$ and $k_2 = 4$, reflecting the fact that only two steps are required to reach the A_1 bound while four steps are required to reach the A_2 bound. In sum, it is assumed that $k_1 = g_1(L_1)$ and $k_2 = g_1(L_2)$, where L_1 and L_2 are the A_1 loss and A_2 loss as defined in Matrix 3, and g_1 is an unknown weak monotonic function. This assumption implies that $a = k_1 + k_2$ and that $z = k_1$ are determined by the two independent variables k_1 and k_2.

The second approach suggested by Link and Heath (1975) is that decision makers first fix the total distance, a, and then adjust the starting point, z, toward either the A_1 bound or the A_2 bound depending on the relative magnitude of the A_2 loss to the A_1 loss. By adjusting the total distance, a, the decision maker can control the average time to make a decision and the accuracy of the decision (i.e., the percentage of correct predictions). Increases in the total distance, a, would tend to produce slower but more accurate decisions, while decreases in a would tend to produce faster and less accurate decisions. Thus control over the total distance provides control over the speed–accuracy tradeoff function (cf. Pachella, 1974). Link (1978) has analyzed a number of choice reaction time studies and demonstrated that experimenter-imposed deadlines directly affect the estimated value of the total distance, a. Following the proposal by Myers *et al.* (1965), it is assumed that the initial starting point, z, is determined by the ratio of the A_1 loss to the A_2 loss, i.e., $z = g_2(L_1/L_2)$, where g_2 is again an unknown weak monotonic function. In sum, according to this second approach, a is an independent variable determined by self-imposed or experimenter-imposed deadline pressures, and z is an independent variable determined by the ratio of the losses. This implies that $k_1 = z$ and that $k_2 = (a - z)$ are determined by the two independent variables a and z, contrary to the first assumption.

For convenience, the first version of the confidence strength model suggested by the work of Audley and Pike (1965) will be referred to as the variable total distance strategy, since a will vary across payoff conditions depending on the sum $(k_1 + k_2)$. The second version of the confidence strength model suggested by Link and Heath (1975) will be referred to as the fixed total distance strategy, since for a given response deadline, the total distance a will remain fixed across variations in payoff conditions.

One last detail that needs to be considered is the effects of marginal event probabilities, π_j, on the decision process. The marginal event probability can influence the decision process in two ways: (1) it may influence ρ_j, the probability of sampling a trace associated with E_j; and (2) it may influence the distance between the starting point and each boundary like a payoff variable. First, in many decision tasks it may not be possible for decision makers to keep track of event patterns such as run patterns. For example, Levin, Dulberg, Dooley, and Hinrichs (1972) reported that when three or more events occur within a single task, then decision makers can no longer keep track of event patterns. Under such circumstances, only general background cues will be available and ρ_j will be determined by the marginal event probability rather than being conditioned on the preceding event pattern (cf. Estes, 1976a). Second, after extended training, the marginal event probability may come to be considered as a known feature of the payoff matrix, as if it was verbally presented along with the payoff information. Decision makers may then use the a priori knowledge of π_j to adjust the starting point of the random walk similar to the use of prior probabilities in an optional stopping task (cf. Edwards, 1965). In this case an increase in π_1 would produce a corresponding decrease in z and/or an increase in $(a - z)$. Link (1975) reported results for several choice reaction time studies which supported the assumption that the marginal event probability affects the estimated distance between the starting point and each boundary.

Application of the confidence strength model to previous results. Due to the assumption that either the starting point or the boundaries of the random walk process are determined by the A_1 and A_2 losses, both versions of the confidence strength model can generate results consistent with the risk × sure thing interaction reported by Myers *et al.* (1965). For example, when $\rho_1 = .5$, both versions predict that $P(A_1)$ will be monotonically related to the loss ratio (L_2/L_1). The confidence strength model also provides a simple account of the effects of π_1 on choice speed reported by Myers *et al.* (1967). Since this study used only correct/incorrect feedback, it can be assumed that $L_1 = L_2$ in this situation. If π_j only influenced ρ_j (perhaps indirectly by influencing the distribution of run patterns), then the starting point would be unbiased. Audley and Pike (1965) have shown that under these conditions $E(s|A_i)$ is an inverted U-shaped function of ρ_j which peaks at $\rho_j = .5$, so that $E(s|A_i)$ is predicted to decrease as π_1 increases above .5. However, π_1 may also influence the distance between the starting point and the boundaries. Increasing π_1 above .5 may cause a corresponding decrease in z and/or an increase in $(a - z)$. A bias in the starting point favoring the choice of A_1 would cause $E(s|A_1)$ to be lower than $E(s|A_2)$. If it is assumed that the effect of π_1 on the starting point is relatively small compared to the effect of π_1 on ρ_j, then $E(s|A_1)$ and $E(s|A_2)$ would be expected to decrease as π_1 increased above .5 but $E(s|A_1)$ would be lower than $E(s|A_2)$, which is consistent with the pattern of results reported by Myers *et al.* (1967).

Empirical tests of the confidence strength model. The two different assumptions concerning the effects of payoffs lead to two different versions of the confidence strength model—the variable vs the fixed total distance strategies. These two strategies generate different predictions for both choice probability and choice speed measures. Differential predictions for $P(A_1)$ can be easily demonstrated by employing the logit transformation, $Y = \ln[P(A_1)/P(A_2)]$, which is a strict positive monotonic transformation of $P(A_1)$. Applying the logit transformation to both sides of Eq. (1a), and then writing the equation in terms of the independent variables k_1 and k_2 yields the following predictions for the variable total distance strategy:

$$Y = \ln\left[\theta^{k_1}/1 - \theta^{k_1}\right] + \ln\left[1 - \theta^{k_2}\right] \qquad \text{if } \theta < 1 \tag{4a}$$

$$= \ln\left[\theta^{k_1}/\theta^{k_1} - 1\right] + \ln\left[\theta^{k_2} - 1\right] \qquad \text{if } \theta > 1, \tag{4b}$$

where $k_1 = g_1(L_1)$ and $k_2 = g_1(L_2)$. Equation (4) implies that for a given value of θ, the effects of the A_1 loss and the A_2 loss are additive. In analysis of variance terms, if the A_1 loss and A_2 loss were manipulated in a factorial design, then an analysis of variance performed on the logit scores should yield no significant A_1-loss $\times$ A_2-loss interaction effect. Applying the same logit transformation to Eq. (1a) and writing the equation in terms of the independent variables a and z yields the following predictions for the fixed total distance strategy:

$$Y = \ln[\theta^z - \theta^a] - \ln[1 - \theta^z] \quad \text{if } \theta < 1 \tag{5a}$$
$$= \ln[\theta^a - \theta^z] - \ln[\theta^z - 1] \quad \text{if } \theta > 1, \tag{5b}$$

where $z = g_2(L_1/L_2)$ and a is constant across payoff conditions. Equation (5) implies that for a given value of θ, the effects of the A_1 loss and the A_2 loss are multiplicative. In analysis of variance terms, an analysis of the logit scores should yield a significant A_1-loss $\times$ A_2-loss interaction effect.

The variable and fixed total distance strategies also make differential predictions concerning the effects of the A_1 and A_2 losses on choice speed. The variable total distance strategy predicts that increasing the A_1 loss while holding the A_2 loss constant should increase both $E(s|A_1)$ and $E(s|A_2)$, although $E(s|A_1)$ will increase more than $E(s|A_2)$. Similarly, increasing the A_2 loss while holding the A_1 loss constant should again increase both $E(s|A_1)$ and $E(s|A_2)$, although $E(s|A_2)$ will now increase more than $E(s|A_1)$. In sum, increasing the loss for one alternative while holding the other loss constant will always increase the expected number of steps and therefore decrease choice speed for both alternatives. The rationale behind this prediction can be described as follows. Suppose the A_2 loss is increased while holding the A_1 loss constant. According to the variable total distance strategy, this should produce a corresponding increase in k_2, while k_1 should remain constant so that $a = (k_1 + k_2)$ increases. Increasing k_2 will increase the distance between the starting point and the A_2 boundary which will cause a corresponding increase in $E(s|A_2)$. However, $E(s|A_1)$ will also increase since the total distance, a, is an increasing function of k_2, and the expected number of steps tends to increase as the total distance increases. This effect can be seen directly by considering Eq. (2b); substituting k_1 for z, and $(k_1 + k_2)$ for a yields $E(s|A_1) = (k_1/3)\,[2(k_1 + k_2) - k_1] = (k_1/3)\,(k_1 + 2k_2)$. As can be seen from the above equation, when $\rho_j = .5$, then $E(s|A_1)$ increases linearly with increases in k_2. This prediction also holds true for Eq. (2a) since the partial derivative of Eq. (2a) with respect to k_2 is always positive.

On the other hand, the fixed total distance strategy predicts that increasing the A_1 loss holding the A_2 loss constant should increase $E(s|A_1)$ but decrease $E(s|A_2)$. Similarly, increasing the A_2 loss holding the A_1 loss constant should increase $E(s|A_2)$ and decrease $E(s|A_1)$. In sum, increasing the loss for one alternative should decrease choice speed for that alternative and increase choice speed for the other alternative. This prediction is due to the fact that, for example, increasing the A_2 loss causes the starting point to be adjusted further away from the A_2 bound and at the same time to be adjusted closer to the A_1 bound, thus keeping the total distance constant. This effect can be seen directly by considering Eq. (2b), since $E(s|A_1)$ decreases as z decreases with a held constant. This prediction also holds true for Eq. (2a) since the partial derivative of Eq. (2a) with respect to z is always positive.

Two experiments were performed which provide tests for both the choice proportion and choice speed predictions of the confidence strength model. The only difference between the two experiments was the daily training procedure. In Experiment 1, the same payoff matrix was used for an entire session, and a different payoff matrix was used each session. For Experiment 2, all payoff matrices were presented within each session, and the same payoff conditions were repeated across sessions. Since the results were very similar for the two procedures, the two experiments will be discussed simultaneously.

METHOD

Subjects

Seventeen volunteers were recruited by advertisements displayed on the University of South Carolina campus, and all participants were naive with respect to the experimental task. There were eight subjects in Experiment 1 (six males and two females), and nine in Experiment 2 (six males and three females). The subjects of Experiment 1 were required to complete 10 daily sessions, while the subjects of Experiment 2 were required to complete 18 daily sessions. One subject in Experiment 1 (S6) only completed 6 sessions. All remaining subjects completed all sessions, and verbally expressed interest in the task. The subjects were paid $2.50 per session in addition to the monetary gains or losses obtained by gambling.

Apparatus

The stimuli were displayed and the responses collected by an APL program on an IBM 370/168 time-sharing computer system communicating with a CRT (Tecktronix, model 4013). The choice latencies were measured from stimulus onset until response completion in 0.10 sec. Subjects were not aware of the fact that choice reaction time was being measured, although they were told to respond as soon as they had reached a decision on each trial. Each subject was run individually in a quiet room.

General Procedure

The first three sessions began with a review of the instructions, after which only minimal instructions were necessary. The subjects were told that they were going to play a gambling game with the computer, and that for each trial the computer would generate either an "I" or a "K" by some chance process. They were informed that although the exact sequence of events would change from one experimental session to the next, the chance process which was generating the events each session would remain the same. They were also informed that the event sequence was always generated before they began making decisions, and therefore the event sequence could not be influenced by their decisions. It should be noted, however, that the subjects were given no a priori information about how the events were being generated. Their primary task, they were told, was to make as much money as possible by predicting whether an I or a K would occur on each trial. They were told that they could win or lose money represented by points depending upon whether their prediction was correct or incorrect. Referring to the "Example Payoff Matrix" shown below, they were told that for this example, if they predicted an I and an I did occur then they would

Example Payoff Matrix

Your choice	Chance event I	K
I	4	−2
K	−1	1

win 4 points. If an I was predicted and a K occurred, then they would lose 2 points. If a K was predicted and an I occurred, then they would lose 1 point. Finally, if a K was predicted and a K occurred, then they would win 1 point. After describing the instructions, the subjects were asked to repeat the instructions to the experimenter in order to guarantee that they understood the contingencies.

Each session started with an initial stake worth $2.50, and was performed in the following manner. The subject initiated each new choice trial by depressing a "return" button, which would clear the screen and present the choice problem for that trial in the format of the "Example Payoff Matrix" shown above. Next, the subject made a prediction by typing an I or a K. If an invalid response was accidently made, the response was cancelled by the program and the participant simply typed in a new response. In addition, the subject could cancel a choice by typing the letter "C" after the undesired choice, in which case the program would ignore the old response and wait for a new choice. Choice latencies were not collected on cancellation trials. After typing the response, the program would present feedback informing the subject about the chance event programmed for that trial, and the number of points won or lost on that trial. After presenting the feedback, a new trial would be initiated which would clear the screen and present the next problem. Although the trials were self-paced, each trial took approximately 6 sec.

Construction of Event Sequences

The event sequences were designed to be representative of those in which constant event frequencies are maintained within each block of N trials. This was accomplished by constructing event sequences according to the method developed by Nicks (1959) in which the frequency of each run pattern approximates the expected frequency for a sequence generated by sampling a block of N independent observations from a Bernoulli distribution. For both experiments, the frequency of each run pattern approximated the expected frequency for a sequence generated by sampling a block of 52 independent observations from a Bernoulli distribution with equal event probabilities. The expected frequency of each run pattern for such a sequence is shown under the column labeled "Expected Frequency of Run Patterns" in Table 1. As can be seen in the table, this type of event sequence has a low frequency of long runs and a high frequency of short runs.

The method employed to construct the event sequence for each block involved using "event strings," which are different from run patterns. Consider the following event sequence: $E_1\ E_1\ E_1\ E_2\ E_2$. In terms of run patterns, $1E_1$ occurred on the first trial of this sequence, $2E_1$ occurred on

TABLE 1
EXPECTED DISTRIBUTION OF RUN PATTERNS, ACTUAL DISTRIBUTION OF RUN PATTERNS, AND DISTRIBUTION OF EVENT STRINGS FOR EACH BLOCK OF 52 TRIALS

Event	Length	Expected frequency of run patterns[a]	Actual frequency of run patterns	Frequency of event strings
E_1	6	.36	0	0
E_1	5	.73	0	0
E_1	4	1.50	1	1
E_1	3	3.06	3	2
E_1	2	6.25	7	4
E_1	1	12.75	15	8
E_2	1	12.75	15	8
E_2	2	6.25	7	4
E_2	3	3.06	3	2
E_2	4	1.50	1	1
E_2	5	.73	0	0
E_2	6	.36	0	0

[a] Run lengths greater than 6 are not shown due to their small expected frequency (less than or equal to .18).

the second trial, $3E_1$ occurred on the third trial, $1E_2$ occurred on the fourth trial, and $2E_2$ occurred on the fifth trial. Described in terms of event strings, this example sequence consists of an E_1 string of length 3 ($E_1\,E_1\,E_1$) connected to an E_2 string of length 2 ($E_2\,E_2$). Thus event strings are independent groups of homogeneous events, and more than one run pattern may be embedded within a single event string.

Each block of 52 trials was generated by sampling event strings from a fixed distribution, which is shown in Table 1 under the column labeled "Frequency of Event Strings." The event sequence was constructed by alternating a randomly selected without replacement string of E_j events with a randomly selected without replacement string of $E_{j'}$ $(j \neq j')$ events, and continuing this process until all of the strings were selected. The probability that a sequence for a particular block started with E_1 was equal to .5.

This method produced the actual frequency of run patterns for each block of 52 trials that is shown under the column labeled "Actual Frequency of Run Patterns." The eight run patterns under this column with nonzero frequencies constituted what will be called the run pattern factor in the design (i.e., the levels of the run pattern factor were $4E_1$, $3E_1$, $2E_1$, $1E_1$, $1E_2$, $2E_2$, $3E_2$, $4E_2$). Each experimental session consisted of nine blocks of event sequences with 52 trials per block. A new event sequence was generated for each block, for each session, and for each subject.

Procedure for Experiment 1

Four payoff matrices were employed in Experiment 1, which are illustrated in the four corners of Table 2. These four payoff matrices were formed by factorially combining two levels of A_2 loss with two levels of A_1 loss. The two levels of the A_2 loss represented by the pairs ($r_{11} = 1, r_{21} = -1$) and ($r_{11} = 4, r_{21} = -1$) will be referred to as the low and high A_2-loss

TABLE 2
PAYOFF MATRICES USED IN EXPERIMENTS 1 AND 2

	A_1 loss		
A_2 loss	Low	Medium	High
Low	$\begin{matrix} 1 & -1 \\ -1 & 1 \end{matrix}$	$\begin{matrix} 1 & -2 \\ -1 & 1 \end{matrix}$	$\begin{matrix} 1 & -4 \\ -1 & 1 \end{matrix}$
Medium	$\begin{matrix} 2 & -1 \\ -1 & 1 \end{matrix}$	$\begin{matrix} 2 & -2 \\ -1 & 1 \end{matrix}$	$\begin{matrix} 2 & -4 \\ -1 & 1 \end{matrix}$
High	$\begin{matrix} 4 & -1 \\ -1 & 1 \end{matrix}$	$\begin{matrix} 4 & -2 \\ -1 & 1 \end{matrix}$	$\begin{matrix} 4 & -4 \\ -1 & 1 \end{matrix}$

conditions, respectively. The two levels of the A_1 loss represented by the pairs ($r_{22} = 1, r_{12} = -1$) and ($r_{22} = 1, r_{12} = -4$) will be referred to as the low and high A_1-loss conditions, respectively.

The first two sessions of Experiment 1 were practice sessions allowing the subjects to learn the structure of the event sequence. During these first two sessions, only one payoff matrix was presented, which was formed by the low A_2-loss and low A_1-loss payoff conditions. The data from these first two sessions were not used in any of the analyses of Experiment 1. During sessions 3 through 6, all four payoff matrices were presented, with one payoff matrix per session. The order of presentation of these payoff matrices was determined by randomly selecting a 4×4 latin square, where each row of the latin square represented a different order. Two subjects were randomly assigned to each row of this latin square. The orders used for sessions 3 through 6 were then replicated for each subject during sessions 7 through 10. In Experiment 1, the subjects were shown their total accumulation of points after each choice trial. Before beginning the experiment, the subjects were informed that each point was worth one-half cent.

Procedure for Experiment 2

Nine payoff matrices were employed in Experiment 2, which are illustrated in Table 2. These nine payoff matrices were formed by combining three levels of A_2 loss with three levels of A_1 loss. The three levels of the A_2 loss represented by the three pairs ($r_{11} = 1, r_{21} = -1$), ($r_{11} = 2, r_{21} = -1$), and ($r_{11} = 4, r_{21} = -1$) will be referred to as the low, medium, and high A_2-loss conditions, respectively. The three levels of the A_1 loss represented by the pairs ($r_{22} = 1, r_{12} = -1$), ($r_{22} = 1, r_{12} = -2$), and ($r_{22} = 1, r_{12} = -4$) will be referred to as the low, medium, and high A_1-loss conditions, respectively.

One of the nine payoff matrices was presented during an entire block of trials, and a different payoff matrix was presented in each of the nine blocks, so that all nine payoff matrices were presented once each session. The order of presentation of the payoff matrices was determined by randomly selecting a 9×9 latin square, where each row of the latin square determined a new order of payoffs for a daily session. A second randomly selected 9×9 latin square was used to determine the order of presentation of the nine rows of the first latin square, and one subject was randomly assigned to each row of the second latin square. The ordering used during the first nine sessions for each subject was replicated during the second nine sessions. In Experiment 2, the subjects were shown their total accumulation of points only at the end of each block of trials. At this time they were also shown how much their total had increased since the last block. In addition, after each block of trials, a 1-min rest period was given. Before beginning the experiment, the subjects were informed that each point was worth one-fourth cent.

RESULTS

The results of both experiments will be discussed together, the tests concerning choice proportions presented first, and the tests concerning choice speed presented second. However, before describing these tests of the model, it is necessary to briefly summarize the results of preliminary analyses concerning order effects, and training effects. First, analysis of variance tests were performed to determine whether there were any reliable effects due to the order to presentation of payoff conditions. Since only one very small interaction occurred out of the 16 possible effects due to order in both experiments, the effects of payoff order were ignored. Second, an analysis of variance test for training effects was performed on the choice proportions for Experiment 2 to determine whether there were any reliable changes across training after the first 3 sessions. Since only one very small interaction occurred out of the 8 possible training effects, it was assumed that learning had stabilized after the first few practice sessions. Therefore, in all of the following analyses, $P(A_1)$ was estimated separately for each subject by collapsing the relative frequencies across the last 8 sessions in Experiment 1, and collapsing across the last 15 sessions in Experiment 2. Due to the low frequency of occurrence for the run patterns $4E_1$ and $4E_2$, these two conditions were not included in the analyses of the choice proportions.

A summary of the results for choice proportions is shown in Figs. 2 and 3, which illustrate the combined effects of payoffs and run patterns on choice proportion averaged across subjects for Experiments 1 and 2, respectively. In each figure, the abscissa indicates the run pattern, and the ordinate represents $P(A_1)$. Each curve represents one level of the A_1-loss factor, and each panel represents one level of the A_2-loss factor. The broken curves with the open circles represent the observed choice proportions, and the solid curves are predicted values that will be discussed later. These figures illustrate that for both experiments, the negative recency effect was replicated at all payoff conditions. However, the specific function relating run patterns to choice proportions changed in the direction of the bias produced by each payoff condition. Increasing and A_2 loss amplified the rate of increase in $P(A_1)$ across the run patterns, and increasing the A_1 loss attenuated the rate of increase in $P(A_1)$ across the run patterns. It should also be noted that negative recency effects were consistently obtained by each subject, and the direction of the effects of the A_1 and A_2 losses was consistent across subjects. However, the mag-

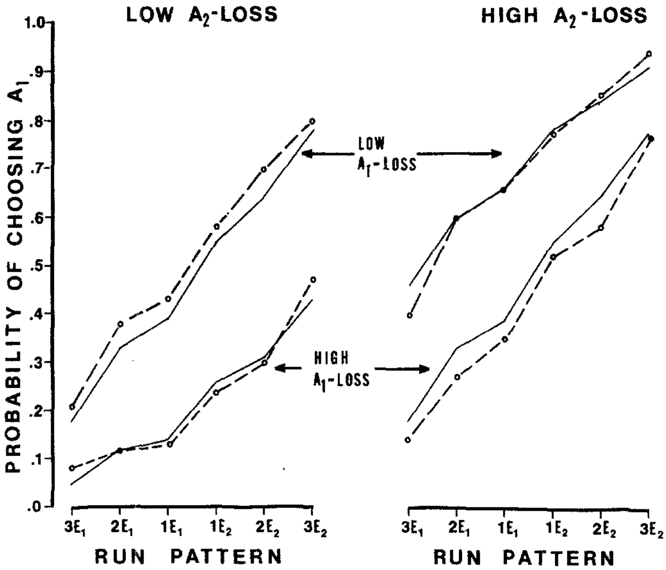

FIG. 2. The predicted and observed effect of run pattern on the proportion of A_1 choices during each payoff condition for Experiment 1.

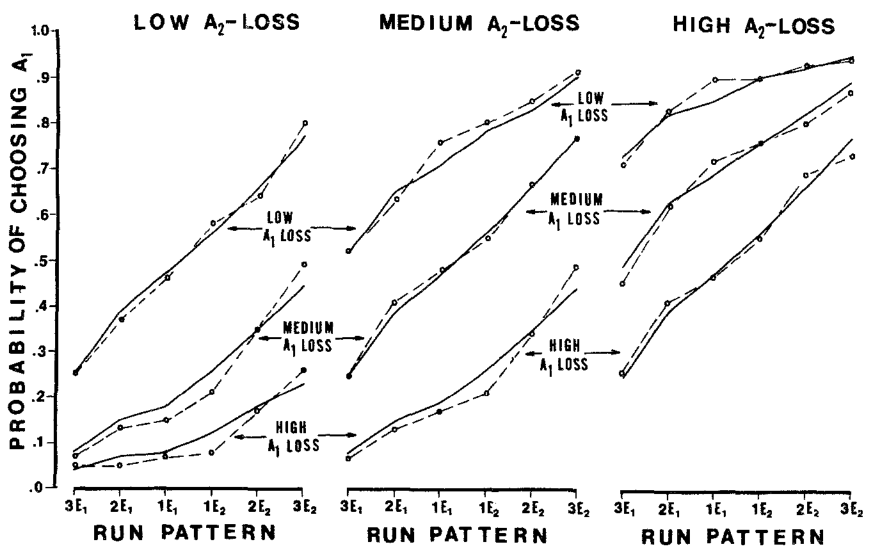

FIG. 3. The predicted and observed effect of run pattern on the proportion of A_2 choices during each payoff condition for Experiment 2.

nitudes of these effects varied across subjects. (The individual means are reported in Busemeyer, 1979.)

Tests Based on Choice Proportions

Recall that a straightforward test of the variable vs the fixed total distance model can be performed by using logit scores. The variable total distance strategy predicts that an analysis of the logit scores should yield no significant A_1-loss $\times$ A_2-loss interaction, and the fixed total distance strategy predicts that the interaction should be significant. A three-way A_2-loss $\times$ A_1-loss $\times$ Run Pattern analysis of variance was performed on the logit scores separately for each participant. Before describing these analyses, it should be noted that using sample proportions to calculate the logit scores causes the estimated logit scores to be biased. Bock and Jones (1968) have shown that the bias can be practically eliminated if the logit scores are estimated by

$$Y'_{ijk} = \ln\left[\frac{P_{ijk}(A_1) + (1/2n_{ijk})}{P_{ijk}(A_2) + (1/2n_{ijk})}\right],$$

where n_{ijk} is the sample size of P_{ijk} (A_1), and the subscripts (i,j,k) index the cell of the three-factor design.

Another problem encountered in the analysis of logit scores was that the means and variances were related. This problem can be corrected by using a weighted least-squares procedure. Therefore a weighted least-squares procedure was used to estimate the sums of squares for each effect using the weighting factor $W_{ijk} = (n_{ijk})P_{ijk}(A_1)P_{ijk}(A_2)$ (cf. Bock & Jones, 1968, p. 71).

Analysis of variance F tests were then used to test the significance of the critical A_2-loss $\times$ A_1-loss interaction effect for each individual. The mean square error for each test was estimated by using the mean square associated with the three-way A_2-loss $\times$ A_1-loss $\times$ Run Pattern interaction effect. This procedure was used because the variable total distance strategy predicts that the three-way interaction effect should be zero. Thus according to the variable total distance strategy, the mean square associated with the three-way interaction should be an unbiased estimate of error variance. This procedure is similar to a method previously used by Friedman, Carterette, and Anderson (1968, p. 450) to perform single-subject analyses which were designed to test stimulus sampling models. If the three-way interaction is nonzero, then the F tests would be biased in favor of the null hypothesis, making it difficult to reject the variable total distance strategy. Despite the fact that the F tests were possibly biased in favor of the null hypothesis, the results indicated a highly reliable A_2-loss $\times$ A_1-loss interaction. Five out of eight subjects in Experiment 1 and eight out of the nine subjects in Experiment 2 produced significant interactions at the .05 level of significance. As a further check, the choice proportions were pooled across the individuals for each experiment before being transformed into logit scores, and similar analysis of variance tests were performed on these logit scores. The A_2-loss $\times$ A_1-loss interaction from these pooled logit scores was significant for both experiments, confirming the findings of the individual analyses. Table 3 presents the F values for the A_2-loss $\times$ A_1-loss interaction tests.

The consistent occurrence of the A_1-loss $\times$ A_2-loss interaction agrees with the predictions of the fixed total distance strategy and is contrary to the variable total distance strategy. However, this test does not indicate how well the fixed total distance strategy can predict the observed pattern of results shown as the broken lines in Figs. 2 and 3. In order to answer this question, the confidence strength model was fit to the choice proportions for each subject using a nonlinear least-squares method. (A modified Gauss–Newton method developed by Marquardt was used, see Gallant, 1975.) The choice proportions were fit under the assumptions of the fixed total distance strategy: (1) the total distance a was treated as a fixed constant, (2) $z_{ij} = g_2(L_i/L_j)$ was estimated for each payoff matrix, and (3) $\theta_k = f(kE_j)$ was estimated for each of the six run patterns. The parameter a

TABLE 3
F TESTS FOR THE A_1-LOSS $\times$ A_2-LOSS INTERACTION EFFECT PERFORMED ON THE LOGIT SCORES AND R^2 GOODNESS-OF-FIT MEASURES FOR THE FIXED TOTAL DISTANCE VERSION OF THE CONFIDENCE STRENGTH MODEL[a]

Experiment 1			Experiment 2		
Subject	F value[b]	R^2	Subject	F value[c]	R^2
1	.07	.96	1	37.21**	.91
2	14.04*	.95	2	24.29**	.96
3	21.85**	.96	3	12.15**	.95
4	.62	.89	4	9.22**	.90
5	21.49**	.88	5	.41	.95
6	16.11*	.97	6	3.87*	.91
7	.14	.99	7	16.41**	.96
8	52.40**	.93	8	93.72**	.97
Pooled	30.41**		9	31.86**	.97
			Pooled	27.00**	

[a] F tests were based on an analysis of logit scores, while R^2 measures the percentage of variance in the choice proportions predicted by the model.

[b] The F tests for Experiment 1 were based on 1 and 5 degrees of freedom.

[c] The F tests for Experiment 2 were based on 4 and 20 degrees of freedom.

* Significant at the .05 level.

** Significant at the .01 level.

is not uniquely identified in this situation since any arbitrary value of a will yield the same predictions after making the appropriate adjustments[1] to θ_k and z_{ij}. Therefore, a was arbitrarily fixed at $a = 10$. Additional constraints were placed on the parameters z_{ij} in order to eliminate some unnecessary parameters. The payoff matrices in the minor diagonal of Table 2 (i.e., payoff matrices with $r_{11} = -r_{12}$) were assumed to be unbiased, and therefore z was fixed to $z = a/2 = 5$ for these three payoff matrices. Thus there was a total of 8 parameters and 24 data points for Experiment 1, and a total of 12 parameters and 54 data points for Experiment 2. The percentage of variance predicted for each subject by the fixed total distance version of the confidence strength model is shown in Table 3 under the column labeled "R^2." As can be seen in Table 3, the percentage of variance predicted is consistently high with a mean value of $\bar{R}^2 = .94$ for each experiment. Since goodness-of-fit indices such as percentage variance predicted are often insensitive to systematic deviations in pattern of fit (cf. Anderson & Shanteau, 1977; Birnbaum, 1973), a graphical analysis was also used to determine how accurately the model captured the pattern of results. The predicted choice probability for each run pattern, payoff condition, and subject was averaged across subjects for each experiment. The average predicted choice probabilities were then plotted as the solid lines in Figs. 2 and 3. Figure 2 indicates a close fit except for a constant overprediction for the low A_1-loss, low A_2-loss condition and a constant underprediction for the high A_1-loss, high A_2-loss condition (the so-called unbiased payoff conditions). Figure 3 does not show the same constant error that was observed in Fig. 2; however, a different problem in fit is apparent in Fig. 3. The predicted curves for the extreme bias conditions seem to be a bit too linear, and the problem is most obvious for the low A_2-loss panel. In sum, the predicted and observed points are fairly close (all deviations are less than .04) but there are some minor deviations between the predicted and observed patterns.

Tests Based on Choice Speed

Recall that the variable vs fixed total distance versions of the confidence strength model make different predictions concerning choice speed—the variable total distance strategy predicts that choice speed for both alternatives decreases as the magnitude of the loss (L_1 or L_2) increases; the fixed total distance strategy predicts that increasing the loss for one alternative will decrease choice speed for that alternative but increase choice speed for the other alternative. Thus the final set of

[1] Define the original set of parameters as a, z, and θ. A new set of parameters can be constructed by setting $a^* = a/c$, $z^* = z/c$, and $\theta^* = \theta^c$. The new set of parameters a^*, z^*, θ^* will produce the same set of predictions as the original set of parameters.

statistical tests to be reported was concerned with the effects of payoffs on choice speed for each alternative, averaged across subjects. The choice speed scores for each subject were calculated by first taking the reciprocals of the latencies (to stabilize the variances) and then averaging across the last 8 sessions for Experiment 1, and averaging across the last 15 sessions for Experiment 2. An A_2-loss × A_1-loss × Choice Alternative repeated-measures analysis of variance was performed to test the effects of payoffs on choice speed. Both Experiments 1 and 2 yielded a significant A_2-loss × Choice Alternative interaction (Experiment 1, $F(1,7) = 9.43$, $p < .05$; Experiment 2, $F(2,16) = 6.64$, $p < .01$), and a significant A_1-loss × Choice Alternative interaction (Experiment 1, $F(1,7) = 8.37$, $p < .05$; Experiment 2, $F(2,16) = 5.86$, $p < .05$). However, the three-way interaction was not significant.

The means for the A_2-loss × Choice Alternative and the A_1-loss × Choice Alternative interactions for Experiment 1 are plotted separately in the first and second panels of Fig. 4. Similarly, the means for these same two interactions obtained from Experiment 2 are plotted separately in the third and fourth panels of Fig. 4. In each panel, the average choice speeds are plotted on the ordinate, the solid line of each panel represents the choice speed conditional on the choice of A_1, and the broken line represents the choice speed conditional on the choice of A_2. For the first and third panels, the abscissa represents each level of the A_2 loss. Similarly, for the second and fourth panels, the abscissa represents each level of the A_1 loss.

As can be seen in Fig. 4, each panel indicates the presence of a cross-

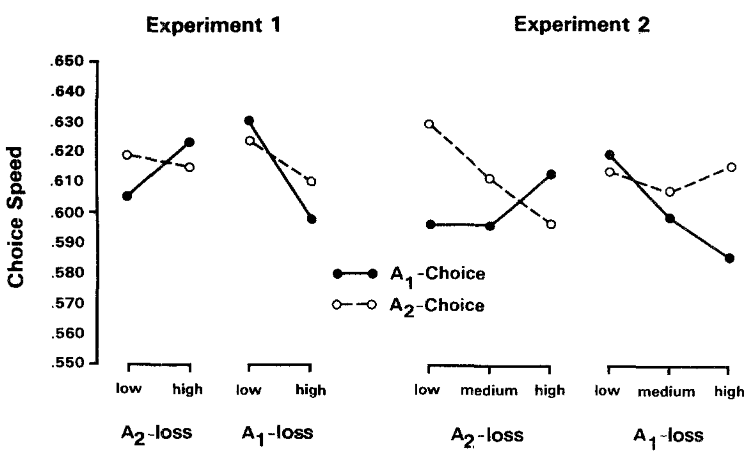

FIG. 4. The effect of payoffs on choice speed for each alternative during Experiments 1 and 2.

over interaction effect. Considering the effect of the A_2 loss, choice speed for A_1 increased and choice speed for A_2 decreased as the A_2 loss increased. However, the results for the effects of the A_1 loss are less clear. Both A_1 and A_2 choice speeds decreased as the A_1 loss increased for Experiment 1; but in Experiment 2, choice speed for A_1 decreased while choice speed for A_2 remained somewhat constant as the A_1 loss increased.

DISCUSSION

Theoretical Conclusions

The purpose of the present study was to investigate the decision process individuals use to integrate sequential information provided by event patterns with payoff information provided by a payoff matrix in order to make choices in a sequential decision-making task. A random walk model called the confidence strength model was proposed as a possible representation of this decision process. This model assumes that on a given choice trial, individuals accumulate evidence across time favoring the choice of either A_1 or A_2 by sampling traces of previous events from memory. Once the accumulated evidence surpasses a criterion level for one alternative, then that alternative is chosen. Two different versions of the confidence strength model were developed as a result of different assumptions concerning the effects of payoffs on the starting point and boundaries of the random walk process. One version called the variable total distance strategy assumed that decision makers independently adjust each bound so that as the loss for an incorrect choice increased, the amount of evidence required to choose that alternative increased. The second version called the fixed total distance strategy assumed that decision makers fix the total distance between the bounds in order to control average response speed, and then adjust the starting point between the fixed bounds according to the relative losses. The two experiments reported in this study provided straightforward tests of these two versions of the confidence strength model for both choice probability and choice speed, and the conclusions from these tests are given below.

The conclusions resulting from the tests based on choice probabilities can be summarized as follows. The variable total distance strategy predicted that an analysis based on logit scores should yield no significant A_1-loss $\times$ A_2-loss interaction, while the fixed total distance strategy predicted that this interaction would be significant. The results clearly supported the fixed total distance strategy, since a significant A_1-loss $\times$ A_2-loss interaction occurred for the majority of the subjects in both experiments. In order to determine how accurately the fixed total distance version of the confidence strength model could capture the pattern of results, a nonlinear regression procedure was used to fit the model to the observed choice proportions. The average percentage of variance predicted for each subject was 94%, which may be interpreted as a reasonably good overall fit to the individual data. However, a graphical analysis indicated two small but systematic patterns of deviation. In Experiment 1, there was a constant error for each of the so-called "unbiased" payoff conditions (payoff matrices with $r_{11} = -r_{12}$); but this deviation did not occur in Experiment 2. The constant error in Experiment 1 can easily be attributed to a violation of an overly restrictive assumption used to fit the model. In order to reduce the number of unknown parameters, it was hypothesized that the payoff matrices with $r_{11} = -r_{12}$ would be unbiased, which seemed to work for Experiment 2 but not for Experiment 1. However, the A_1 and A_2 losses associated with these payoff matrices are not identical, so that it is possible that these payoff matrices were biased. Thus if this somewhat arbitrary restriction were removed, the initial starting point for these conditions could be adjusted to account for the small constant error in prediction occurring under these payoff conditions. The second systematic deviation was only observed in Experiment 2. The predicted curves for the biased payoff conditions were a bit too linear, i.e., the observed curves were slightly more accelerated. One possible way to account for this pattern of deviations is to speculate that the total distance, a, was not fixed as hypothesized, but instead varied slightly across payoff conditions. Increasing the total distance would tend to increase the acceleration in the predicted curves. However, it should be noted that these deviations from the model were small (less than .04), and did not appear in Experiment 1. Thus, the small deviations between the predicted and observed curves in Experiment 2 cannot be considered too seriously without further research. In sum, the results of the tests concerning choice probability provide strong evidence against the variable total distance model. The fixed total distance model provided a reasonably good overall fit to the pattern of results, but there were some minor deviations that require further research.

The conclusions for the tests based on choice speed are similar to those for choice probability. The variable total distance version of the confidence strength model predicted that choice speed for both alternatives always decreases as the loss for one alternative increases, although the decrease should be greater for the alternative associated with the increased loss. The fixed total distance version predicted that increasing the loss for one alternative should decrease choice speed for that alternative, but increase choice speed for the other alternative. The pattern of results for the A_2 loss was consistently in favor of the fixed total distance strategy. However, the pattern of results for the A_1 loss was less clear—the results for Experiment 1 tended to favor the variable total distance strategy, while the results for Experiment 2 favored neither model since the A_2 choice speed remained fairly constant with increases in the A_1 loss. One can summarize these results by noting that the predictions for the variable total distance strategy were inconsistent with the effects of the A_2 loss on choice speed in both experiments. On the other hand, the predictions for the fixed total distance strategy were generally consistent with the choice speed results except for one data point, i.e., the drop in the A_2 choice speed for the high A_1-loss condition. Since this same drop was not replicated in Experiment 2, the reliability of this particular result remains questionable without further research.

In light of both the choice probability and choice speed results, the fixed total distance version of the confidence strength model provides a better representation of the decision process than the variable total distance version. The basic idea behind the fixed total distance strategy is that there are two parameters that subjects use to control the decision process—the total distance and the starting point. The present study only attempted to manipulate the starting point, z, and the covert prediction probability, ρ_j, and no attempt was made to manipulate the total distance, a. In principle the total distance, a, could be manipulated by using the appropriate experimental method. Future research needs to conducted to test this idea which factorially manipulates both the initial starting point, z, and the total distance, a, and simultaneously measures both choice proportions and decision times. One method that previous research indicates could be used to manipulate the total distance would be response time deadline procedures (see, e.g., Link, 1978).

Alternative Models of Choice

The combined results for choice probability and choice speed obtained in the present study provide strong evidence against the variable total distance version of the confidence strength model. The basic idea behind the variable total distance strategy is that the criterion bound for each alternative is determined independently by the loss produced by incorrectly choosing that alternative. One might question whether this basic idea is essentially correct, but that the random walk process describing the accumulation of evidence needs to be modified. Audley and Pike (1965) described two other processes besides the random walk process—the runs model (Audley, 1960) and the simple accumulator model (Laberge, 1962). According to the runs model, A_i will be chosen if and only if an uninterrupted run of k_i consecutive E_i events is covertly predicted. The simple accumulator process works like a horse race—an A_1 counter accumulating the number of E_1 covert predictions races against an A_2 counter accumulating the number of E_2 covert predictions. The first counter to reach its boundary, k_i, wins and is subsequently chosen. In both cases, the criterion bound, k_i, is an increasing function of the loss produced by incorrectly choosing A_i.

Neither the runs model nor the simple accumulator model generates predictions which are consistent with the major results of past research and the present study. It is fairly simple to show that the runs model, like the variable total distance random walk model, predicts that an analysis based on logit scores should yield *no* significant A_1-loss $\times$ A_2-loss interaction, which is contrary to the results of the present study.[2] The simple accumulator model does not provide a simple explanation for the choice speed results obtained by Myers *et al.* (1967). The simple accumulator model generally predicts that choice speed for an alternative is an increasing function of the probability of choosing that alternative (cf. Audley & Pike, 1965; Audley & Mercer, 1968; Pike, 1968). Contrary to this prediction, Myers *et al.* (1967) found that $S(A_2)$ increased as $P(A_2)$ decreased. (It should be noted that Pike (1968, 1973) has suggested a more complex accumulator model which assumes that the criterion bounds randomly fluctuate from trial to trial. This more complex version can account for the results of Myers *et al.*, 1967.) One common problem encountered by the variable total distance random walk model, the runs model, and the accumulator model (including the more complex version) is that all three models predict that increasing either the A_1 loss or the A_2 loss should always decrease choice speed for both alternatives. This prediction is common across all three models since they all assume that the distance between the starting point and the criterion bounds, k_i, always increases as the losses increase. This prediction is contrary to the results obtained in both experiments of the present study which found that choice speed for A_1 increased and choice speed for A_2 decreased as the A_2 loss increased. In conclusion, it appears that the basic idea behind the variable total distance strategy is wrong, since the predictions that it generates tend to fail regardless of which process is assumed for the accumulation of evidence.

It is informative to compare the predictions of the confidence strength model to those of a previously established decision model called the scanning model, which was previously developed by Estes (1960, 1962, 1972, 1976b) to describe choice behavior in the differential payoff paradigm. The basic idea of the scanning model is that on a given trial, the decision maker scans each alternative for some fixed interval of time. Once attention is focused on a particular alternative, A_i, then the decision maker anticipates the reward produced by choosing A_i. If the anticipated reward

[2] The runs model predicts (cf. Feller, 1968, p. 197) that $P(A_1) = \rho_1^{k_1-1}(1 - \rho_2^{k_2})/(\rho_1^{k_1-1} + \rho_2^{k_2-1} - \rho_1^{k_1-1}\rho_2^{k_2-1})$, so that $Y = \ln[\rho_1^{k_1-1}/1 - \rho_1^{k_1}] - \ln[\rho_2^{k_2-1}/1 - \rho_2^{k_2}]$.

is attractive enough to elicit an approach response within the interval of time that A_i is being considered, then A_i will be chosen. Otherwise, attention is redirected toward the other alternative, and this scanning process continues back and forth until an approach response is elicited.

An important feature of the scanning model is that the payoffs produced by choosing A_i (i.e., r_{i1} and r_{i2}) directly influence the anticipated reward for A_i, and therefore they directly affect the probability of an approach response, independent of the payoffs associated with the other alternative. As a result, the scanning model predicts that the direction of the effect of manipulating the payoffs associated with one alternative, while holding the payoffs associated with the other alternative constant, should be independent of the particular values being held constant. In other words, the scanning model makes the same independence prediction as the expected utility rule described earlier. As noted earlier, the risk $\times$ sure thing interaction reported by Myers *et al.* (1965) contradicts this prediction.

A second important prediction of the scanning model is that increases in the anticipated reward for one alternative, holding the payoffs associated with the other alternative constant, should always increase choice speed for either alternative (cf. Estes, 1969, 1972; also see Appendix B). This prediction results from the fact that increasing the anticipated reward for either alternative will always increase the probability of an approach response, which then decreases the average amount of scanning that occurs before an alternative is chosen. In the present experiments, the payoffs produced by choosing A_2 were held constant. An increase in the A_2 loss was produced by increasing r_{11}, which would increase the anticipated reward for A_1. Thus according to the scanning model, an increase in the A_2 loss should have always increased choice speed for either alternative. Contrary to this prediction, choice speed for A_1 increased while choice speed for A_2 decreased. In conclusion, previous research and the results of the present study do not support the scanning model as previously described by Estes (1960, 1962, 1972, 1976b). Of course, alternative assumptions concerning the definition of anticipated reward may change the nature of the choice probability and choice speed predictions. However, no straightforward way to modify the assumptions concerning anticipated reward that provides predictions consistent with both the results of Myers *et al.* (1965) and the present study is apparent at this time.

Extensions of the Confidence Strength Model

Although the confidence strength model was specifically developed for the differential payoff paradigm, it is relatively simple to extend the model to other binary choice sequential decision tasks such as the multiple-cue probability learning task used by Castellan and Edgell (1973), the observation-transfer probability learning paradigm used by Estes (1976a, 1976b), and the probabilistic categorization task used by Kubovy and Healy (1977). The model can be extended in a straightforward manner by generalizing the assumptions concerning the retrieval process. One only needs to assume that the contextual cues include available test stimuli such as verbal or graphic cues as well as event patterns and general background cues (cf. Estes, 1976a). The probe or test stimuli can activate traces associated with contextual cues that are similar to the current retrieval cue (cf. Medin & Schaffer, 1978). For example, in a multiple-cue probability learning task, the decision maker may have to decide which of two diseases are present when a particular symptom pattern is shown. The symptom pattern could serve as one possible retrieval cue, activating traces associated with similar contextual cues, and each trace would indicate whether one disease or another is the correct diagnosis. A final diagnosis would be achieved by sampling traces until a criterion level of confidence in correctness of one of the two diseases was reached. Further research with these paradigms, studying the effects of differential payoffs on choice probability and choice speed, needs to be conducted in order to compare the confidence strength model to other models such as the scanning model or the classic signal detection model which have been previously applied to these paradigms.

APPENDIX A

The difference and ratio expected utility models described by Becker *et al.* (1963) are special cases of a general class of expected utility models which can be defined as follows. Employing the same notation used earlier, $u_i = \pi_1 u(r_{i1}) + \pi_2 u(r_{i2})$ will be used to represent the expected utility of alternative A_i. Then the general model can be stated as

$$P(A_1) = F(u_1, u_2). \tag{A1}$$

The function F is required to have the following properties:

(1) F is bounded between zero and one,
(2) $\partial F/\partial u_1 > 0$,
(3) $\partial F/\partial u_2 < 0$.

This class of models predicts that for a given probability π_j, $P(A_1)$ will always be a positive function of u_1 for any particular value of u_2 held constant. Note that although u_i was defined in terms of the marginal probability π_j, one could just as easily assume that π_j was a subjective probability and the same independence property would hold.

APPENDIX B

The method used to derive the expected number of steps to termination conditioned on the choice of A_i for the scanning model (with $m \geqslant 1$ approach responses) described by Estes (1960) is presented below. The method used in the following analysis was based on the general matrix methods derived for Markov chains by Pike (1966). The transition matrix, T, for the scanning model is given as Matrix 1 in Estes (1960). This transition matrix can be rearranged into its canonical form (see Kemeny & Snell, 1960, p. 44) so that the $2m \times 2m$ submatrix Q contains only transient-state transition probabilities, and the $2m \times 2$ submatrix R contains only the transition probabilities from the transient states to the absorbing states. The $2m \times 1$ matrix O will be used to represent the initial probability vector. On the basis of these definitions, the conditioned expected number of steps can be obtained from the general equations for Markov models:

$$[P(A_1)E(s|A_1)\ P(A_2)E(s|A_2)] = O'[(I - Q)^{-1}]^2R. \tag{B1}$$

where I is an identity matrix.

For example, when $m = 1$, then

$$Q = \begin{bmatrix} 0 & 1 - P_1 \\ 1 - P_2 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}, \quad O' = [o_1\ o_2].$$

The parameter P_i $(i = 1,2)$ represents the probability of an approach response to A_i given that the individual is orientated toward A_i. In this simple case, Eq. (B1) simplifies to

$$E(s\,|A_1) = P_1[o_1(2 - d) + 2o_2(1 - P_2)]/d^2P(A_1), \tag{B2}$$

where $d = P_1 + P_2 - P_1P_2$. Substituting $o_1 = P(A_1)$ and $o_2 = P(A_2)$ (cf. Estes, 1960, 1962), then Eq. (B2) reduces to

$$E(s|A_i) = [(2 - P_i)/d] + 1. \tag{B3}$$

The partial derivative of Eq. (B3) with respect to P_i is always negative which implies that $E(s\,|A_i)$ is a decreasing function of P_i, the probability of an approach response to A_i. When m is greater than one, Eq. (B1) does not appear to reduce to a simple form. For these more complex cases, the effects of P_i on $E(s\,|A_i)$ were determined by systematically varying the parameters P_i in .1 steps from .1 to .9 for trial values $m = 2,3$. The results agreed with the analysis based on partial derivatives of Eq. (B3). Increasing P_i always decreased $E(s\,|A_i)$ and since the decrease in $E(s\,|A_i)$ was getting larger as m increased from $m = 1$ to $m = 3$, it seemed reasonable to stop at $m = 3$.

REFERENCES

Anderson, N. H., & Shanteau, J. Weak inference with linear models. *Psychological Bulletin*, 1977, **84**, 1155–1170.

Audley, R. J. A stochastic model for individual choice behavior. *Psychological Review*, 1960, **67**, 1–15.

Audley, R. J., & Mercer, A. The relation between decision time and the relative response frequency in a blue-green discrimination. *British Journal of Mathematical and Statistical Psychology*, 1968, **21**, 183–192.

Audley, R. J., & Pike, A. R. Some alternative stochastic models of choice. *British Journal of Mathematical and Statistical Psychology*, 1965, **21**, 183–192.

Becker, G. M., Degroot, M. H., & Marschak, J. Stochastic models of choice behavior. *Behavioral Science*, 1963, **8**, 41–55.

Birnbaum, M. The devil rides again: Correlation as an index of fit. *Psychological Bulletin*, 1973, **22**, 239–242.

Bock, R. D., & Jones, L. V. *The measurement and prediction of judgment and choice*. San Francisco: Holden–Day, 1968.

Busemeyer, J. R. *The combined effects of event patterns and payoffs on choice behavior in a sequential decision making task*. Unpublished doctoral dissertation, University of South Carolina, 1979.

Castellan, N. J., & Edgell, S. E. An hypothesis generation model for judgment in nonmetric multiple-cue probability learning. *Journal of Mathematical Psychology*, 1973, **10**, 204–222.

Cox, D. R., & Miller, H. D. *The theory of stochastic processes*. London: Methuen, 1965.

Edwards, W. Reward probability, amount, and information as determiners of sequential two-alternative decisions. *Journal of Experimental Psychology*, 1956, **52**, 177–188.

Edwards, W. Optimal strategies for seeking information: Models for statistics, choice reaction times, and human information processing. *Journal of Mathematical Psychology*, 1965, **2**, 312–329.

Ericsson, K. A., & Simon, H. A. Verbal reports as data. *Psychological Review*, 1980, **87**, 215–251.

Estes, W. K. A random-walk model for choice behavior. In K. J. Arrow, S. Karlin, & P. Suppes (Eds.), *Mathematical methods in the social sciences*. Stanford, Calif.: Stanford Univ. Press, 1960.

Estes, W. K. Theoretical treatments of differential reward in multiple-choice learning and two-person interactions. In J. H. Criswell, H. Solomon, & P. Suppes (Eds.), *Mathematical methods in small group processes*. Stanford, Calif.: Stanford Univ. Press, 1962.

Estes, W. K. Reinforcement in human learning. In J. Tapp (Ed.), *Reinforcement and behavior*. New York: Academic Press, 1969.

Estes, W. K. Reinforcement in human behavior. *American Scientist*, 1972, **60**, 723–729.

Estes, W. K. The cognitive side of probability learning. *Psychological Review*, 1976, **83**, 37–65. (a)

Estes, W. K. Some functions of memory in probability learning and choice behavior. In G. H. Bower (Ed.), *The psychology of learning and motivation*. New York: Academic Press, 1976. Vol. 10. (b)

Feller, W. *An introduction to probability theory and its applications*. New York: Wiley, 1968. 3rd ed., Vol. 1.

Fishburn, P. C. Binary choice probabilities between gambles: Interlocking expected utility models. *Journal of Mathematical Psychology*, 1976, **14**, 99–122.

Friedman, M. P., Carterette, E. C., & Anderson, N. H. Long term probability learning with a random schedule of reinforcement. *Journal of Experimental Psychology*, 1968, **78**, 442–455.

Gallant, A. R. Nonlinear regression. *American Statistician*, 1975, **28**, 73–82.

Gambino, B., & Myers, J. L. Role of event runs in probability learning. *Psychological Review*, 1967, **74**, 410–419.

Heuckeroth, O. Two-choice probability learning: Test of a finite Markov chain and linear operator model. *Journal of Mathematical Psychology*, 1969, **6**, 418–429.

Jones, M. R. From probability learning to sequential information processing: A critical review. *Psychological Bulletin*, 1971, **76**, 153–185.

Jones, M. R., & Myers, J. L. A comparison of two methods of event randomization in probability learning. *Journal of Experimental Psychology*, 1966, **72**, 909–911.

Katz, L. Monetary incentive and range of payoffs as determiners of risk taking. *Journal of Experimental Psychology*, 1962, **64**, 541–544.

Katz, L. The effect of differential gain and loss on sequential two-choice behavior. *Journal of Experimental Psychology*, 1964, 245–249..

Kemeny, J. G., & Snell, J. L. *Finite Markov chains*. New York: Van Nostrand, 1960.

Kubovy, M., & Healy, M. F. The decision rule in probabilistic categorization: What is it and how is it learned? *Journal of Experimental Psychology: General*, 1977, **106**, 427–440.

Laberge, D. A recruitment theory of simple behavior. *Psychometrica*, 1962, **27**, 375–396.

Laming, D. R. *Information theory of choice reaction time*. New York: Wiley, 1968.

Levin, I. P., Dulberg, C. S., Dooley, J. F., & Hinrichs, J. V. Sequential dependencies in single-item and multiple-item probability learning. *Journal of Experimental Psychology*, 1972, **93**, 262–267.

Link, S. W. The relative judgment theory of two choice reaction time. *Journal of Mathematical Psychology*, 1975, **12**, 114–136.

Link, S. W. The relative judgment theory analysis of response time deadline experiments. In N. J. Castellan, Jr. & F. Restle (Eds.), *Cognitive theory*. Hillsdale, N. J.: Lawrence Erlbaum, 1978. Vol. 3.

Link, S. W., & Heath, R. A. A sequential theory of psychological discrimination. *Psychometrica*, 1975, **40**, 77–105.

Medin, D. L., & Schaffer, M. M. Context theory of classification learning. *Psychological Review*, 1978, **85**, 207–238.

Myers, J. L. Probability learning and sequence learning. In W. K. Estes (Ed.), *Handbook of learning and cognitive processes*. Hillsdale, N. J.: Lawrence Erlbaum, 1976. Vol. 3.

Myers, J. L., & Atkinson, R. C. Choice behavior and reward structure. *Journal of Mathematical Psychology*, 1964, **1**, 170–204.

Myers, J. L., Gambino, B., & Jones, M. R. Response speeds in probability learning. *Journal of Mathematical Psychology*, 1967, **4**, 473–488.

Myers, J. L., & Katz, L. Range of payoffs and feedback in risk taking. *Psychological Reports*, 1962, **10**, 483–486.

Myers, J. L., Reilly, R. E., & Taub, H. Differential cost, gain, and frequency of reward in a sequential choice situation. *Journal of Experimental Psychology*, 1961, **62**, 357–360.

Myers, J. L., & Sadler, E. Effects of range of payoffs as a variable in risk taking. *Journal of Experimental Psychology*, 1960, **60**, 306–309.

Myers, J. L., & Suydam, M. M. Gain, cost, and event probability as determiners of choice behavior. *Psychonomic Science*, 1964, **1**, 39–40.

Myers, J. L., Suydam, M. M., & Gambino, B. Contingent gains and losses in a risk taking situation. *Journal of Mathematical Psychology*, 1965, **2**, 363–370.

Nicks, D. C. Prediction of sequential two-choice decisions from event runs. *Journal of Experimental Psychology*, 1959, **57**, 105–114.

Pachella, R. G. The interpretation of reaction time in information processing research. In B. H. Kantowitz (Ed.), *Human information processing: Tutorials in performance and cognition*. Hillsdale, N. J.: Lawrence Erlbaum, 1974.

Payne, J. W. Alternative approaches to decision making under risk: Moments versus risk dimensions. *Psychological Bulletin*, 1973, **80**, 439–453.

Pike, A. R. Stochastic models of choice behavior: Response probabilities and latencies of finite Markov chain systems. *British Journal of Mathematical and Statistical Psychology*, 1966, **19**, 15–32.

Pike, A. R. Latency and relative frequency of response in psychophysical discrimination. *British Journal of Mathematical and Statistical Psychology*, 1968, **21**, 161–182.

Pike, A. R. Response latency models for signal detection. *Psychological Review*, 1973, **80**, 53–68.

Rapoport, A., Jones, L. V., & Kahan, J. P. Gambling behavior in multiple-choice multistage betting games. *Journal of Mathematical Psychology*, 1970, **7**, 12–36.

Rapoport, A., & Wallsten, T. S. Individual choice behavior. *Annual Review of Psychology*, 1972, **23**, 131–175.

Ratcliff, R. A theory of memory retrieval. *Psychological Review*, 1978, **85**, 59–108.

Restle, F. *The psychology of judgment and choice*. New York: Wiley, 1961.

Stein, W., & Rapoport, A. A discrete time model for detection of randomly presented stimuli. *Journal of Mathematical Psychology*, 1978, **17**, 110–137.

Stone, M. Models for choice reaction time. *Psychometrica*, 1960, **25**, 251–260.

Taub, H. A., & Myers, J. L. Differential monetary gains in a tow-choice situation. *Journal of Experimental Psychology*, 1961, **61**, 559–562.

RECEIVED: July 3, 1980