

**ECE 3170
Test 2
Session 3**

INSTRUCTIONS:

This is a closed-book, closed-notes test. You may use only the following items:

- ^{Two}~~A single~~ 3-by-5-inch cards of handwritten notes.
- A scientific calculator.
- A pencil or pen.

The following items may not be used: additional note cards, additional notes, phones, computers, portable music players, other electronic devices, and other resources of any kind.

All forms of collaboration are prohibited during this test. You may communicate only with the person administering the test, and you may not receive or give aid of any kind.

Write your name in the space provided at the top of this page and on the answer sheet (next page).

Do not separate or remove any of the pages of this test. All pages must be returned to the instructor at the conclusion of the test.

Select the best answer for each problem by drawing a circle around the letter of the correct choice on the answer sheet. Circle only one letter for each problem, and do not make any additional marks on the answer sheet.

Each problem is worth five points. Credit for a problem will be awarded only if the correct letter is circled on the answer sheet. No credit will be awarded if more than one answer is selected.

You may work each problem in the space following the problem statement, but no credit will be given for this work. There should be adequate room for all work on the front of each page, but you also may write on the back or on extra sheets provided by the instructor.

Name: Solutions

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Answer Sheet

- | | | | | | |
|-----|------------------------------------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|
| 1) | a | b | <input checked="" type="radio"/> c | d | e |
| 2) | a | b | <input type="radio"/> c | <input checked="" type="radio"/> d | e |
| 3) | a | b | c | d | <input checked="" type="radio"/> e |
| 4) | a | b | <input checked="" type="radio"/> c | d | e |
| 5) | a | b | c | <input checked="" type="radio"/> d | e |
| 6) | a | b | c | d | <input checked="" type="radio"/> e |
| 7) | a | b | c | d | <input checked="" type="radio"/> e |
| 8) | <input checked="" type="radio"/> a | b | c | d | e |
| 9) | a | b | c | d | <input checked="" type="radio"/> e |
| 10) | <input checked="" type="radio"/> a | b | c | d | e |
| 11) | a | <input checked="" type="radio"/> b | c | d | e |
| 12) | a | <input checked="" type="radio"/> b | c | d | e |
| 13) | a | b | c | <input checked="" type="radio"/> d | e |
| 14) | <input checked="" type="radio"/> a | b | c | d | e |
| 15) | a | b | c | d | <input checked="" type="radio"/> e |
| 16) | a | <input checked="" type="radio"/> b | c | d | e |
| 17) | a | <input checked="" type="radio"/> b | c | d | e |
| 18) | a | b | c | <input checked="" type="radio"/> d | e |
| 19) | a | <input checked="" type="radio"/> b | c | d | e |
| 20) | a | <input checked="" type="radio"/> b | c | d | e |

- 1) A certain electronic system fails at random times, and the length of time between successive failures is an exponential random variable. The random variable X is defined to be the number of times the system fails during one month. Which of the following types of random variables best describes X ?

a: uniform
b: exponential
c: Poisson
d: Bernoulli
e: binomial

Length of time between occurrences is exponential.
 \Rightarrow # occurrences per unit time is Poisson.

- 2) The number of interrupts received by a certain microprocessor during any interval of time is a Poisson random variable. On average, 1 interrupt occurs every minute. Which of the following is closest to the probability that exactly 3 interrupts will occur during an interval of 3 minutes?

a: 0.5
b: 0.4
c: 0.3
d: 0.2
e: 0.1

Let $X = \# \text{ interrupts in 3 minutes}$
Ave # interrupts in 3 min = $\frac{1 \text{ int.}}{\text{min.}} \times 3 \text{ min.} = 3$

$$\Rightarrow P(X=i) = \frac{a^i e^{-a}}{i!}, \text{ where } a=3$$

$$\Rightarrow P(X=3) = \frac{3^3 e^{-3}}{3!} = \frac{27 e^{-3}}{3 \cdot 2} \approx 0.224$$

- 3) Calls to a certain phone number occur at random times. The length of time between successive calls is an exponential random variable whose average is 10 minutes. Which of the following is closest to the probability that the time between two successive calls will be less than or equal to 1 minute?

a: 0.5
b: 0.4
c: 0.3
d: 0.2
e: 0.1

Let $X = \text{time between calls (min.)}$

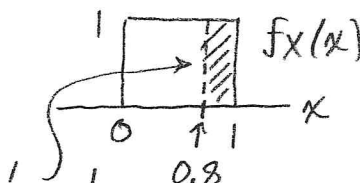
$$\Rightarrow F_X(x) = 1 - e^{-ax} \text{ for } x \geq 0$$

$$a = \frac{\text{ave. \# calls}}{\text{min.}} = \frac{1}{10}$$

$$\Rightarrow P(X \leq 1) = F_X(1) = 1 - e^{-(\frac{1}{10})(1)} = 1 - e^{-0.1} \approx 0.09516$$

- 4) The random variable X is uniformly distributed from 0 to 1. Which of the following is closest to $P(X > 0.8)$?

a: 0.4
b: 0.3
c: 0.2
d: 0.1
e: 0



$$P(X > 0.8) = \int_{0.8}^1 f_X(x) dx = (1)(0.2) = 0.2$$

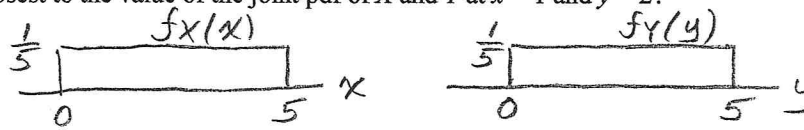
- 5) A fair, six-sided die is rolled once, and the random variable X is defined to be the number obtained on the die. Which of the following types of random variables best describes X ?

a: geometric
b: Rayleigh
c: Poisson
☒ d: discrete uniform
e: exponential

Possible values: 1, 2, 3, 4, 5, 6
(equally spaced, discrete,
equal probabilities)

- 6) Statistically independent random variables X and Y are uniformly distributed from 0 to 5. Which of the following is closest to the value of the joint pdf of X and Y at $x = 1$ and $y = 2$?

a: 0.5
b: 0.4
c: 0.3
d: 0.2
☒ e: 0.1



X and Y are S.I. $\Rightarrow f_{XY}(x, y) = f_X(x)f_Y(y)$

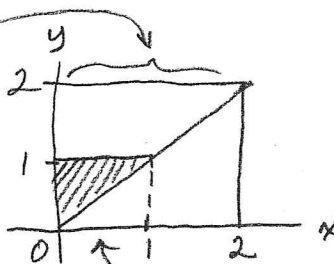
$$\Rightarrow f_{XY}(1, 2) = f_X(1)f_Y(2) = \frac{1}{5} \cdot \frac{1}{5} = \frac{1}{25} = 0.04$$

- 7) The joint probability density function of random variables X and Y is

$$f_{XY}(x, y) = \begin{cases} \frac{1}{2}, & 0 \leq x < y \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Which of the following is closest to $P(Y < 1)$?

a: 1.0
b: 0.8
c: 0.6
d: 0.4
☒ e: 0.2



$$P(Y < 1) = \iint f_{XY}(x, y) dx dy \text{ over triangle}$$

$$= \left(\frac{1}{2}\right)(\text{area}) = \left(\frac{1}{2}\right)\left(\frac{1}{2} \cdot 1 \cdot 1\right) = \frac{1}{4} = 0.25$$

- 8) For the variables in problem 7, which of the following is closest to the value of the marginal probability distribution function of X evaluated at $x = 2$?

☒ a: 1.0
b: 0.8
c: 0.6
d: 0.4
e: 0.2

$$F_X(x) = P(X \leq x)$$

$$\Rightarrow F_X(2) = P(X \leq 2) = 1$$

(Note $f_{XY}(x, y) = 0$ for $x > 2$)

- 9) The joint probability distribution function of random variables X and Y is

$$F_{XY}(x, y) = (1 - e^{-x^2/2})(1 - e^{-y})u(x)u(y).$$

Which of the following is closest to $P(X \leq 1, Y \leq 2)$?

- a: 0.7
b: 0.6
c: 0.5
d: 0.4
e: 0.3
- $P(X \leq 1, Y \leq 2) = F_{XY}(1, 2)$
 $= (1 - e^{-(1)^2/2})(1 - e^{-2}) \approx 0.3402$


- 10) If $f_{X|Y}(1|2) = 7$ and $f_Y(2) = 8$, which of the following is equal to $f_{XY}(1, 2)$?

- a: 56
b: 15
c: 8
d: 7
e: None of the above
- $f_{XY}(x, y) = f_{X|Y}(x|y)f_Y(y)$
 $\Rightarrow f_{XY}(1, 2) = f_{X|Y}(1|2)f_Y(2)$
 $= (7)(8) = 56$

- 11) The standard, Gaussian random variables X and Y are statistically independent. Which of the following is the value of their correlation coefficient (ρ)?

- a: 1
b: 0
c: 0.5
d: -1
e: None of the above
- $f_{XY}(x, y) = f_X(x)f_Y(y)$
 if and only if $\rho = 0$.

- 12) The random variable X is uniformly distributed from 0 to 1. If $Y = \frac{1}{X}$, which of the following is equal to $f_Y(3)$?

- a: 3
b: 1/9
c: 6
d: 1/3
e: None of the above
- 
 $\Rightarrow f_X(x) = 1, 0 \leq x \leq 1$
 $x = \frac{1}{y} = h(y) \Rightarrow \frac{dh(y)}{dy} = -y^{-2} = -\frac{1}{y^2}$
 $f_Y(y) = f_X[h(y)] \left| \frac{dh(y)}{dy} \right| = (1) \left| -\frac{1}{y^2} \right| = \frac{1}{y^2}$
 $\Rightarrow f_Y(3) = \frac{1}{3^2} = \frac{1}{9}$

- 13) The random variable X is uniformly distributed from 0 to 1. If $Y = 10X - 5$, which of the following is equal to $F_Y(0)$?

a: 1/10

b: 1/5

c: 1/4

d: 1/2

e: None of the above

$$\Rightarrow F_X(x) = x, 0 \leq x \leq 1$$

$$y = 10x - 5 = g(x) \leftarrow \text{Monotonically increasing}$$

$$x = \frac{y+5}{10} = h(y)$$

$$F_Y(y) = F_X[h(y)] = x|_{x=h(y)} = h(y) = \frac{y+5}{10}$$

$$\Rightarrow F_Y(0) = \frac{0+5}{10} = \frac{1}{2}$$

- 14) X is a discrete uniform random variable that may equal any value in the set $\{1, 2, 3, 4, 5\}$. If $Y = 3X + 1$, which of the following is equal to $P(Y = 10)$?

a: 1/5

b: 1/4

c: 1/3

d: 1/2

e: None of the above

$$y = 3x + 1 = g(x) \leftarrow 1 \text{ to } 16$$

$$\Rightarrow P(Y=y) = P[X=h(y)]$$

$$x = \frac{y-1}{3} = h(y)$$

$$\Rightarrow P(Y=10) = P(X = \frac{10-1}{3}) = P(X=3) = \frac{1}{5}$$

Each has probability $\frac{1}{5}$

- 15) Random variables Y_1 and Y_2 are derived from random variables X_1 and X_2 as follows:

$$Y_1 = X_1 + X_2, Y_2 = 2X_1 + X_2$$

$$y_1 = x_1 + x_2 = g_1(x_1, x_2)$$

Which of the following is equal to $J_{h_1 h_2}(y_1, y_2)$?

$$y_2 = 2x_1 + x_2 = g_2(x_1, x_2)$$

a: 1

b: 2

c: 3

d: 4

e: None of the above

$$\Rightarrow x_1 = y_2 - y_1 = h_1(y_1, y_2)$$

$$x_2 = 2y_1 - y_2 = h_2(y_1, y_2)$$

$$J_{h_1 h_2}(y_1, y_2) = \begin{vmatrix} \frac{\partial h_1}{\partial y_1} & \frac{\partial h_1}{\partial y_2} \\ \frac{\partial h_2}{\partial y_1} & \frac{\partial h_2}{\partial y_2} \end{vmatrix} = \begin{vmatrix} -1 & 1 \\ 2 & -1 \end{vmatrix} = 1 - 2 = -1$$

- 16) Random variables X_1 and X_2 are statistically independent and uniform on $[0, 1]$. Random variables Y_1 and Y_2 are derived from random variables X_1 and X_2 as follows:

$$Y_1 = X_1 + X_2, Y_2 = \frac{X_1}{X_2}$$

Which of the following is equal to the maximum possible value of Y_2 ?

a: 0

b: ∞

c: 1

d: 2

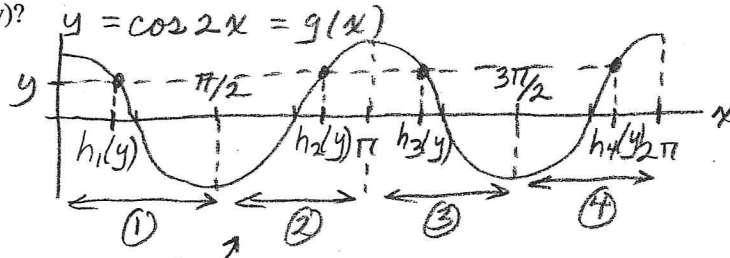
e: None of the above

$$Y_2 = \frac{X_1}{X_2}$$

$$Y_2(\max) = \frac{X_1(\max)}{X_2(\min)} = \frac{1}{0} = \infty$$

- 17) The random variable X is an angle (in units of radians) that is uniformly distributed from 0 to 2π . If $Y = \cos(2X)$, which of the following is equal to the number of different inverse functions $h(y)$ required in the computation of $f_Y(y)$?

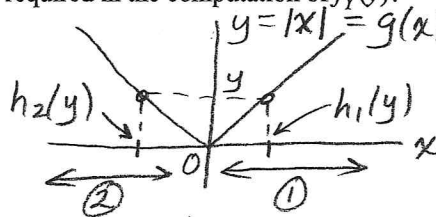
a: 5
 b: 4
 c: 3
 d: 2
 e: None of the above



$g(x)$ is 1-to-1 on 4 intervals
 \Rightarrow 4 functions $h(y)$

- 18) X is a standard, Gaussian random variable. If $Y = |X|$, which of the following is equal to the number of different inverse functions $h(y)$ required in the computation of $f_Y(y)$?

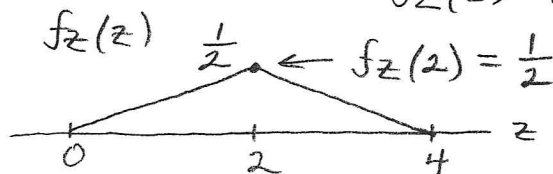
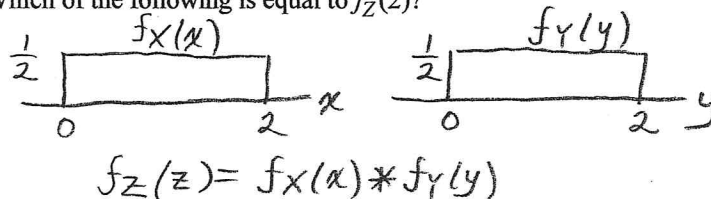
a: 5
 b: 4
 c: 3
 d: 2
 e: None of the above



$g(x)$ is 1-to-1 on 2 intervals
 \Rightarrow 2 functions $h(y)$

- 19) Random variables X and Y are statistically independent and uniform on $[0, 2]$. Random variable Z is equal to the sum of X and Y . Which of the following is equal to $f_Z(2)$?

a: 0
 b: $1/2$
 c: $3/4$
 d: 1
 e: None of the above



- 20) Random variables X and Y are statistically independent, and random variable Z is equal to the sum of X and Y . Which of the following statements must be true?

a: $f_Z(z) = f_X(z) f_Y(z)$
 b: $f_Z(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z-x) dx$
 c: $f_Z(z) = f_X(z) + f_Y(z)$
 d: It is not possible to determine $f_Z(z)$ from $f_X(x)$ and $f_Y(y)$.
 e: None of the above

See text sec. 3.4