ECE 3170 (Hubbard)
Old Test Questions

1) Given $S = \{1, 2, 3, 4, 5\}$, $A = \{1, 3, 5\}$, $B = \{2, 5\}$, and $C = \{1, 5\}$, which of the following is equal to $(A \cap C)^C \cup B$?

a: $\{1, 3, 5\}$
b: $\{2\}$
c: $\{1, 2, 4\}$
d: $\{2, 3, 4, 5\}$
e: None of the above

$A \cap C = \{1, 5\}$

$(A \cap C)^c = \{2, 3, 4\}$

$\Rightarrow (A \cap C)^c \cup B = \{2, 3, 4\} \cup \{2, 5\}$

$= \{2, 3, 4, 5\}$

2) A six-sided die is tossed three times. How many outcomes are contained in the sample space of this experiment?

a: 3
b: 18
c: 36
d: 216
e: None of the above

$S = \underbrace{\{111, 112, \ldots, 116, 121, \ldots, 666\}}_{6^3 = 216 \text{ outcomes}}$

3) A six-sided die is tossed once, and then a fair coin is tossed once if the number on the die is even and twice if the number on the die is odd. How many outcomes are contained in the sample space of this experiment?

a: 18
b: 24
c: 36
d: 64
e: None of the above

(3 even numbers) × (2 coin sides)

+ (3 odd numbers) × (4 combinations)

= 18 outcomes

4) Given the sample space $S = \{1, 2, 3, 4\}$ and events $A = \{1, 2, 3\} \in \mathcal{A}$, and $B = \{4\} \in \mathcal{A}$, where $\mathcal{A}$ is an algebra. Three of the required elements of $\mathcal{A}$ are $\{1, 2, 3, 4\}$, $\{1, 2, 3\}$ and $\{4\}$. Which of the following is another required element of $\mathcal{A}$?

a: $\{2, 3\}$
b: $\{1, 2\}$
c: $\{3, 4\}$
d: $\{1\}$
e: None of the above

$\mathcal{A} = \{S, \emptyset, A, B\}$

$S \to \{1, 2, 3, 4\}$, $A \to \{1, 2, 3\}$, $B \to \{4\}$

The only other required element is $\emptyset$.

5) A, B, and C are mutually exclusive events defined on the sample space S. If $P(A) = 0.4$ and $P(B) = 0.4$, which of the following is the maximum possible value for $P(C)$?

a: 0.1
b: 0.3
c: 0.6
d: 0.8
e: None of the above

$A \cap B = A \cap C = B \cap C = \emptyset$

$\Rightarrow \underbrace{P(A \cup B \cup C)}_{\text{Maximum} = 1} = P(A) + P(B) + P(C)$

$= 0.4 + 0.4 + P(C)$

$\Rightarrow P(C)_{max} = 0.2$

6) A and B are events defined on the sample space S. If the probability that at least one of them occurs is 0.3 and the probability that A occurs but B does not occur is 0.1, which of the following is the value of $P(B)$?

a: 0.2
b: 0.4
c: 0.6
d: 0.8
e: None of the above

$P(A \cup B) = 0.3 = P(A) + P(B) - P(A \cap B)$

$= \underbrace{[P(A) - P(A \cap B)]}_{P(A \cap B^c) = 0.1} + P(B)$

$\Rightarrow P(B) = 0.3 - 0.1 = 0.2$

7) The probability is 1% that an electronic component will fail if it is not overheated during manufacture. If it is overheated, the probability of failure is 5%. If 90% of a collection of components are not overheated and the other 10% are overheated, what is the probability that a component chosen at random from the collection will fail?

a: 0.014
b: 0.05
c: 0.1
d: 0.14
e: None of the above

Let $B = \{\text{component fails}\}$

$A = \{\text{component overheated}\}$

$P(B) = P(B|A)P(A) + P(B|A^c)P(A^c)$ (Total probability)

$= (0.05)(0.1) + (0.01)(0.9)$

$= 0.014$

8) A and B are events defined on the sample space S. If $P(A \mid B) = 0.25$ and $P(B^C) = 0.2$, what is the value of $P(A \cap B)$?

a: 0.1
b: 0.2
c: 0.25
d: 0.3
e: None of the above

$P(A \cap B) = P(A|B)P(B)$

$= (0.25)(0.8) = 0.2$

9) A and B are events defined on the sample space S. If $P(A \mid B) = 0.5$ and $P(B) = 0.4$, what is the value of $P(A \cap B)$?

a: 0.1
b: 0.04
c: 0.9
d: 0.2
e: None of the above

$$P(A \cap B) = P(A \mid B)\,P(B)$$
$$= (0.5)(0.4) = 0.2$$

10) A and B are events defined on the sample space S. If $P(A) = 0.5$, $P(B) = 0.4$, and $P(A \cap B) = 0.2$, which of the following is true?

a: A and B are statistically independent and mutually exclusive.
b: A and B are mutually exclusive but not statistically independent.
c: A and B are statistically independent but not mutually exclusive.
d: A and B are not statistically independent but may or may not be mutually exclusive.
e: None of the above.

$$\underbrace{P(A \cap B)}_{0.2} = \underbrace{P(A)}_{0.5}\,\underbrace{P(B)}_{0.4} \Rightarrow A \text{ and } B \text{ are S.I.}$$

$$P(A \cap B) \neq 0 \Rightarrow A \text{ and } B \text{ are not M.E.}$$

11) A system is composed of two statistically independent components (A and B) connected in parallel. If the probability that A fails is 0.08 and the probability that B fails is 0.12, which of the following is closest to the probability that the system does *not* fail?

a: 0.2
b: 0.4
c: 0.6
d: 0.8
e: 1.0

Let $A = \{A \text{ does not fail}\}$
$B = \{B \text{ does not fail}\}$

$$\Rightarrow P(\text{System does not fail}) = 1 - P(A^c \cap B^c)$$
$$= 1 - P(A^c)P(B^c) = 1 - (0.08)(0.12)$$
$$= 0.9904$$

12) A fair, six-sided die is tossed once, and the random variable X is defined to be the number on the die. What is the value of $F_X(3.7)$?

a: 0
b: 1/6
c: 1/3
d: 1/2
e: None of the above

$$F_X(3.7) = P(X \leq 3.7)$$
$$= P(X=1) + P(X=2) + P(X=3)$$
$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$$

13) The probability density function of a random variable X is given by the following:

$$f_X(x) = \begin{cases} 4x^3, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}.$$

Which of the following is closest to the probability that X is less than 1/2?

a: 0
b: 0.05
c: 0.1
d: 0.15
e: 0.2

$$P(X < \tfrac{1}{2}) = \int_0^{1/2} 4x^3\,dx$$

$$= x^4\Big|_0^{1/2} = (\tfrac{1}{2})^4 - (0)^4 = \frac{1}{16} = 0.0625$$

14) A discrete random variable X may equal either 0 or 1, and no other values are possible. If $P(X = 1) = 0.3$, which of the following is closest to the value of the probability mass function of X at $x = 0$?

a: 1
b: 3/4
c: 1/2
d: 1/4
e: 0

$$P(X=0) = 1 - P(X=1)$$

$$= 1 - 0.3 = 0.7$$

15) The probability density function of a random variable X is given by the following:

$$f_X(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}.$$

Which of the following is closest to the value of the cumulative distribution function of X at $x = 3/4$?

a: 1
b: 7/8
c: 3/4
d: 1/2
e: 1/4

$$F_X(x) = \int_0^x f_X(t)\,dt = \int_0^x (1)\,dt = x, \quad 0 \le x \le 1$$

$$\Rightarrow F_X(3/4) = 3/4$$

16) The random variable X is defined to be the number of heads obtained in two tosses of a fair coin. What is the value of $F_X(0)$?

a: 0
b: 1
c: 2
d: 1/2
e: None of the above

$$S = \{HH, HT, TH, TT\}$$

1 of 4 outcomes yields $X=0$

$$\Rightarrow P(X=0) = \frac{1}{4}$$

$$F_X(0) = P(X \le 0) = P(X=0) = \frac{1}{4}$$

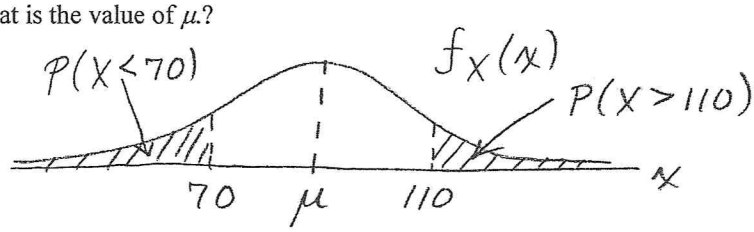
17) X is a standard Gaussian random variable. Which of the following is closest to $P(X > 2.85)$?

a: 0.5
b: 0.2
c: 0.1
d: 0.01
e: 0.001

$P(X > 2.85) = Q(2.85)$
$\cong 0.00219$
(from table)

18) X is a Gaussian random variable with mean μ and standard deviation σ. If $P(X < 70) = P(X > 110) = Q(2)$, what is the value of μ?

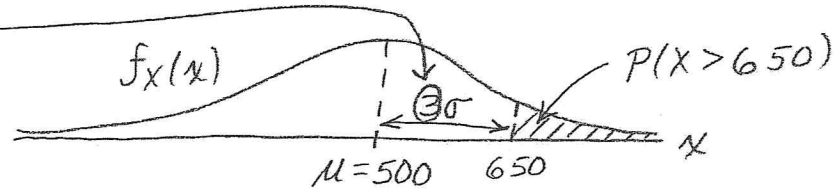
a: 2
b: 0.02275
c: 40
d: 20
e: None of the above



From symmetry (equal areas),
μ is midway between 70 and 110
$\Rightarrow \mu = 90$

19) X is a Gaussian random variable with mean $\mu = 500$ and standard deviation σ. If $P(X > 650) = Q(3)$, which of the following is closest to the value of σ?

a: 0.001
b: 50
c: 100
d: 500
e: 1000



$\Rightarrow \underbrace{650 - 500}_{150} = 3\sigma \quad \Rightarrow \sigma = 50$

20) X is a standard Gaussian random variable. Which of the following is true?

a: The variance of X is 0.
b: The mean of X is 0.
c: The standard deviation of X is 0.
d: The variance of X is 1/2.
e: None of the above

Standard Gaussian
$\Rightarrow \mu = 0, \ \sigma = 1 = \sigma^2$

Q function

	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414
0.1	.46017	.45640	.45224	.44828	.44432	.44038	.43644	.43251	.42858	.42465
0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
0.7	.24196	.23885	.23576	.23269	.22965	.22663	.22363	.22065	.21770	.21476
0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
1.0	.15866	.15625	.15386	.15150	.14917	.14686	.14457	.14231	.14007	.13786
1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08692	.08534	.08379	.08226
1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
3.0	1.35E-3	1.31E-3	1.26E-3	1.22E-3	1.18E-3	1.14E-3	1.11E-3	1.07E-3	1.04E-3	1.00E-3
3.1	9.68E-4	9.36E-4	9.04E-4	8.74E-4	8.45E-4	8.16E-4	7.89E-4	7.62E-4	7.36E-4	7.11E-4
3.2	6.87E-4	6.64E-4	6.41E-4	6.19E-4	5.98E-4	5.77E-4	5.57E-4	5.38E-4	5.19E-4	5.01E-4
3.3	4.83E-4	4.67E-4	4.50E-4	4.34E-4	4.19E-4	4.04E-4	3.90E-4	3.76E-4	3.62E-4	3.50E-4
3.4	3.37E-4	3.25E-4	3.13E-4	3.02E-4	2.91E-4	2.80E-4	2.70E-4	2.60E-4	2.51E-4	2.42E-4
3.5	2.33E-4	2.24E-4	2.16E-4	2.08E-4	2.00E-4	1.93E-4	1.85E-4	1.79E-4	1.72E-4	1.65E-4
3.6	1.59E-4	1.53E-4	1.47E-4	1.42E-4	1.36E-4	1.31E-4	1.26E-4	1.21E-4	1.17E-4	1.12E-4
3.7	1.08E-4	1.04E-4	9.96E-5	9.58E-5	9.20E-5	8.84E-5	8.50E-5	8.16E-5	7.84E-5	7.53E-5
3.8	7.24E-5	6.95E-5	6.67E-5	6.41E-5	6.15E-5	5.91E-5	5.67E-5	5.44E-5	5.22E-5	5.01E-5
3.9	4.81E-5	4.62E-5	4.43E-5	4.25E-5	4.08E-5	3.91E-5	3.75E-5	3.60E-5	3.45E-5	3.31E-5
4.0	3.17E-5	3.04E-5	2.91E-5	2.79E-5	2.67E-5	2.56E-5	2.45E-5	2.35E-5	2.25E-5	2.16E-5
4.1	2.07E-5	1.98E-5	1.90E-5	1.81E-5	1.74E-5	1.66E-5	1.59E-5	1.52E-5	1.46E-5	1.40E-5
4.2	1.34E-5	1.28E-5	1.22E-5	1.17E-5	1.12E-5	1.07E-5	1.02E-5	9.78E-6	9.35E-6	8.94E-6
4.3	8.55E-6	8.17E-6	7.81E-6	7.46E-6	7.13E-6	6.81E-6	6.51E-6	6.22E-6	5.94E-6	5.67E-6
4.4	5.42E-6	5.17E-6	4.94E-6	4.72E-6	4.50E-4	4.30E-6	4.10E-6	3.91E-6	3.74E-6	3.56E-6
4.5	3.40E-6	3.24E-6	3.09E-6	2.95E-6	2.82E-6	2.68E-6	2.56E-6	2.44E-6	2.33E-6	2.22E-6
4.6	2.11E-6	2.02E-6	1.92E-6	1.83E-6	1.74E-6	1.66E-6	1.58E-6	1.51E-6	1.44E-6	1.37E-6
4.7	1.30E-6	1.24E-6	1.18E-6	1.12E-6	1.07E-6	1.02E-6	9.69E-7	9.22E-7	8.78E-7	8.35E-7
4.8	7.94E-7	7.56E-7	7.19E-7	6.84E-7	6.50E-7	6.18E-7	5.88E-7	5.59E-7	5.31E-7	5.05E-7
4.9	4.80E-7	4.56E-7	4.33E-7	4.12E-7	3.91E-7	3.72E-7	3.53E-7	3.35E-7	3.18E-7	3.02E-7

1) A fair coin is tossed, and the random variable X is defined to be 1 if heads is obtained and 0 if tails is obtained. Which of the following types of random variables best describes X?

a: uniform
b: Poisson
c: binomial
d: Bernoulli
e: exponential

X may equal only 0 or 1

$\Rightarrow$ X is a Bernoulli R.V.

2) The number of hits on a certain website during any interval of time is a Poisson random variable. On average, 1 hit occurs every minute. Which of the following is closest to the probability that **exactly** 3 hits will be made during an interval of 3 minutes?

a: 0.8
b: 0.6
c: 0.4
d: 0.2
e: 0

Let X = # hits in 3 minutes

Ave. # hits in 3 min. $= \frac{1 \text{ hit}}{\text{min.}} \times 3 \text{ min.} = 3$

$$\Rightarrow P(X=i) = \frac{a^i e^{-a}}{i!}, \text{ where } a = 3$$

$$\Rightarrow P(X=3) = \frac{3^3 e^{-3}}{3!} = \frac{27 e^{-3}}{3 \cdot 2} \cong 0.224$$

3) A fair, six-sided die is rolled eight times. Which of the following is closest to the probability that **exactly one** "4" is obtained?

a: 0.8
b: 0.7
c: 0.6
d: 0.5
e: 0.4

Let X = # of "4"s in 8 tosses

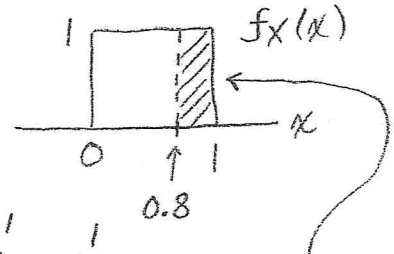
$\Rightarrow X$ is a binomial R.V. with $N = 8$ and $p = \frac{1}{6} = P(\text{"4"})$

$$\Rightarrow P(X=i) = C_i^N p^i (1-p)^{N-i}$$

$$\Rightarrow P(X=1) = \cancel{C_1^8}^{\,8} \left(\tfrac{1}{6}\right)^1 \left(\tfrac{5}{6}\right)^7 \cong 0.3721$$

4) The random variable X is uniformly distributed from 0 to 1. Which of the following is closest to $P(X > 0.8)$?

a: 0.8
b: 0.6
c: 0.4
d: 0.2
e: 0



$$P(X > 0.8) = \int_{0.8}^{1} \cancel{f_X}^{\,1}(x)\,dx = \text{Area} = 0.2$$

5) A fair, six-sided die is rolled once, and the random variable X is defined to be the number obtained on the die. Which of the following types of random variables best describes X?

a: geometric
b: Rayleigh
c: Poisson
d: discrete uniform
e: exponential

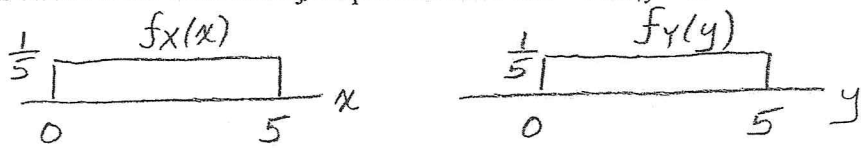
X may equal $\underbrace{1, 2, 3, 4, 5, \text{ or } 6}$ with equal probabilities

(equally spaced values)

$\Rightarrow$ X is a discrete uniform R.V.

6) Statistically independent random variables X and Y are uniformly distributed from 0 to 5. Which of the following is closest to the value of the joint pdf of X and Y at $x = 1$ and $y = 2$?

a: 0.5
b: 0.4
c: 0.3
d: 0.2
e: 0.1



X and Y are S.I. $\Rightarrow f_{XY}(x,y) = f_X(x) f_Y(y)$

$$\Rightarrow f_{XY}(1,2) = f_X(1) f_Y(2) = \frac{1}{5} \cdot \frac{1}{5}$$
$$= \frac{1}{25} = 0.04$$

7) The joint probability distribution function of random variables X and Y is

$$F_{XY}(x,y) = \left(1 - e^{-x^2/2}\right)\left(1 - e^{-y}\right)u(x)u(y).$$

Which of the following is closest to $P(X \le 1, Y \le 2)$?

a: 0.3
b: 0.4
c: 0.5
d: 0.6
e: 0.7

$$P(X \le 1, Y \le 2) = F_{XY}(1,2)$$
$$= \left(1 - e^{-(1)^2/2}\right)\left(1 - e^{-2}\right) \cong 0.3402$$

8) For the variables in problem 7, which of the following is closest to the value of the marginal probability distribution function of Y evaluated at $y = 2$?

a: 0.1
b: 0.3
c: 0.5
d: 0.7
e: 0.9

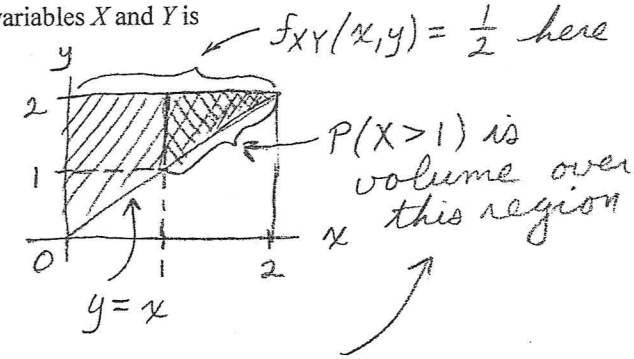
$$F_Y(y) = F_{XY}(\infty, y)$$
$$= \left(1 - \cancel{e^{-\infty^2/2}}^{\,0}\right)\left(1 - e^{-y}\right)\cancel{u(\infty)}^{\,1}u(y)$$
$$= \left(1 - e^{-y}\right)u(y)$$
$$F_Y(2) = \left(1 - e^{-2}\right)\cancel{u(2)}^{\,1} \cong 0.8647$$

9) The joint probability density function of random variables X and Y is

$$f_{XY}(x,y) = \begin{cases} \frac{1}{2}, & 0 \le x < y \le 2 \\ 0, & \text{otherwise} \end{cases}.$$

Which of the following is closest to P(X > 1)?

a: 0.2
b: 0.4
c: 0.6
d: 0.8
e: 1.0



$P(X>1)$ = volume over triangular area

$= \left(\frac{1}{2}\right)(\text{triangle area}) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\cdot 1\cdot 1\right) = \frac{1}{4} = 0.25$

10) If $f_{X|Y}(5 \mid 6) = 3$ and $f_Y(6) = 4$, which of the following is equal to $f_{XY}(5, 6)$?

a: 3
b: 4
c: 12
d: 7
e: None of the above.

$f_{XY}(x,y) = f_{X|Y}(x|y)\, f_Y(y)$

$\Rightarrow f_{XY}(5,6) = f_{X|Y}(5|6)\, f_Y(6)$

$= (3)(4) = 12$

11) The standard, Gaussian random variables X and Y are statistically independent. Which of the following is the value of their correlation coefficient (ρ)?

a: 1
b: 0
c: 0.5
d: −1
e: None of the above

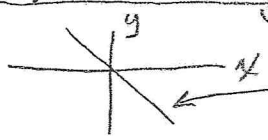
$f_{XY}(x,y) = f_X(x)\, f_Y(y)$

if and only if $\rho = 0$.

12) The random variable X is uniformly distributed from 0 to 1. If $Y = -2X$, which of the following is equal to $f_Y(-1)$?

a: 1
b: 1/2
c: −1/2
d: 1/4
e: None of the above

$\Rightarrow f_X(x) = 1,\ 0 \le x \le 1$



$y = -2x = g(x)$ is monotonically decreasing

$x = -\frac{y}{2} = h(y) \Rightarrow \frac{dh(y)}{dy} = -\frac{1}{2}$

$f_Y(y) = f_X[h(y)]\left|\frac{dh(y)}{dy}\right| = (1)\left|-\frac{1}{2}\right| = \frac{1}{2},\ -2 \le y \le 0$

$\Rightarrow f_Y(-1) = \frac{1}{2}$

13) The random variable X is uniformly distributed from 0 to 1. If $Y = 3X + 1$, which of the following is equal to $F_Y(2)$?

$\Rightarrow F_X(x) = x, \quad 0 \le x \le 1$

a: 0
b: 1/3
c: 1/2
d: 2/3
e: None of the above

$y = 3x+1 = g(x)$ is monotonically increasing

$x = \frac{y-1}{3} = h(y)$

$F_Y(y) = F_X[h(y)] = x|_{x=h(y)} = h(y) = \frac{y-1}{3}$

$\Rightarrow F_Y(2) = \frac{2-1}{3} = \frac{1}{3}$

14) X is a **discrete** uniform random variable that may equal any value in the set $\{1, 2, 3, 4, 5\}$. If $Y = 3X + 1$, which of the following is equal to $P(Y = 10)$?

a: 0
b: 1/5
c: 1/4
d: 1/3
e: None of the above

$y = 3x+1 = g(x)$ is 1-to-1

(Each has probability $= \frac{1}{5}$)

$\Rightarrow P(Y=y) = P[X = h(y)]$

$x = \frac{y-1}{3} = h(y)$

$\Rightarrow P(Y=10) = P\left(X = \frac{10-1}{3}\right) = P(X=3) = \frac{1}{5}$

15) Random variables Y_1 and Y_2 are derived from random variables X_1 and X_2 as follows:
$Y_1 = 2X_1 + 2X_2, \quad Y_2 = 2X_1 - 2X_2$
Which of the following is equal to $\mathbf{J}_{h_1 h_2}(y_1, y_2)$?

$y_1 = 2x_1 + 2x_2 = g_1(x_1, x_2)$
$y_2 = 2x_1 - 2x_2 = g_2(x_1, x_2)$

a: 8
b: −8
c: 1/8
d: −1/8
e: None of the above

$\Rightarrow x_1 = \frac{y_1 + y_2}{4} = h_1(y_1, y_2)$

$x_2 = \frac{y_1 - y_2}{4} = h_2(y_1, y_2)$

$$J_{h_1 h_2}(y_1, y_2) = \begin{vmatrix} \frac{\partial h_1}{\partial y_1} & \frac{\partial h_1}{\partial y_2} \\ \frac{\partial h_2}{\partial y_1} & \frac{\partial h_2}{\partial y_2} \end{vmatrix} = \begin{vmatrix} 1/4 & 1/4 \\ 1/4 & -1/4 \end{vmatrix} = -\frac{1}{16} - \frac{1}{16} = -\frac{1}{8}$$

16) Random variables X_1 and X_2 are statistically independent and uniform on [0,1]. Random variables Y_1 and Y_2 are derived from random variables X_1 and X_2 as follows:
$Y_1 = X_1 + X_2, \quad Y_2 = X_1 - X_2$
Which of the following is equal to the minimum possible value of Y_2?

a: −1
b: 0
c: 1
d: 2
e: None of the above

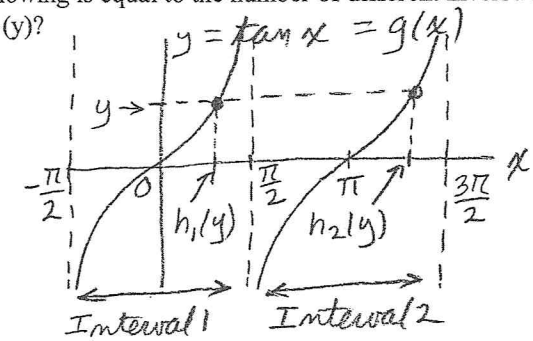
$Y_2 = X_1 - X_2$

$Y_2(\min) = \overset{0}{X_1(\min)} - \overset{1}{X_2(\max)}$

$= -1$

17) The random variable X is an angle (in units of radians) that is uniformly distributed from $-\pi/2$ to $+3\pi/2$. If $Y = \tan(X)$, which of the following is equal to the number of different inverse functions $h(y)$ required in the computation of $f_Y(y)$?

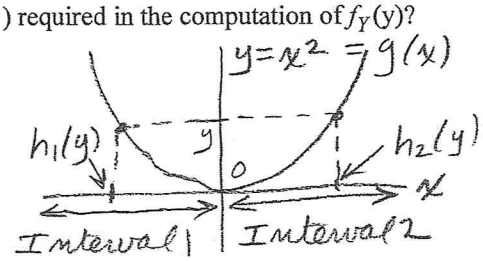
a: 1
b: 3
c: 5
d: 7
e: None of the above



g(x) is 1-to-1 on 2 different intervals
$\Rightarrow$ 2 functions h(y)

18) X is a standard, Gaussian random variable. If $Y = X^2$, which of the following is equal to the number of different inverse functions $h(y)$ required in the computation of $f_Y(y)$?

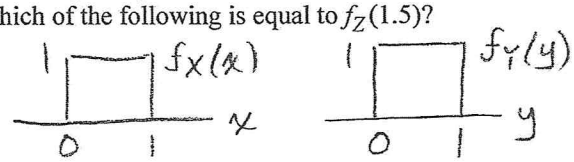
a: 0
b: 1
c: 2
d: 3
e: None of the above



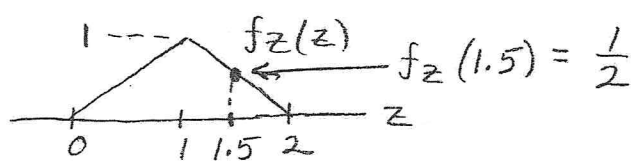
g(x) is 1-to-1 on 2 different intervals
$\Rightarrow$ 2 functions h(y)

19) Random variables X and Y are statistically independent and uniform on [0,1]. Random variable Z is equal to the sum of X and Y. Which of the following is equal to $f_Z(1.5)$?

a: 0
b: 1/8
c: 1/4
d: 1/2
e: None of the above



$f_Z(z) = f_X(x) * f_Y(y)$:



20) Random variables X and Y are statistically independent, and random variable Z is equal to the sum of X and Y. Which of the following is equal to the probability density function of Z?

a: The convolution of the pdf of X and the pdf of Y.
b: The sum of the pdf of X and the pdf of Y.
c: The product of the pdf of X and the pdf of Y.
d: The quotient of the pdf of X and the pdf of Y.
e: None of the above

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z-x)\, dx$$

$$= f_X(x) * f_Y(y)$$

1) Random variables X and Y are statistically independent and uniform on [0,1]. Random variable Z is equal to the maximum of X and Y. Which of the following is equal to $F_Z(0.5)$?

a: 0
b: 0.25
c: 0.5
d: 1.0
e: None of the above

$F_X(x) = x$, $F_Y(y) = y$ for $x, y \in [0,1]$

$Z = \max(X,Y) \Rightarrow F_Z(z) = F_X(z) F_Y(z)$
$= z \cdot z = z^2$

$\Rightarrow F_Z(0.5) = (0.5)^2 = 0.25$

2) A system is composed of 10 identical components connected in series. The time to failure of each component (in hours) is a random variable having the following probability distribution function:

$$F_X(x) = \begin{cases} 1 - e^{-ax}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}, \quad \text{where } a = 10^{-5}.$$

The failure times of the components are statistically independent. Which of the following is closest to the probability that the system will fail within 5,000 hours of operation?

a: 0
b: 0.2
c: 0.4
d: 0.6
e: 0.8

Let $X_1, \ldots, X_{10}$ = component failure times and Z = system failure time

$Z = \min(X_1, \ldots, X_{10}) \Rightarrow F_Z(z) = 1 - [1 - F_X(z)]^{10}$

$\Rightarrow F_Z(z) = 1 - [\cancel{1 - (1 - e^{-az})}]^{10} = 1 - e^{-10az}$, $a = 10^{-5}$

$P(Z \leq 5{,}000) = F_Z(5{,}000) = 1 - e^{-10(10^{-5})(5{,}000)} \cong 0.393$

3) The random variable X has the following probability density function:

$$f_X(x) = \begin{cases} 3x^2, & -1 \leq x \leq 0 \\ 0, & \text{otherwise} \end{cases},$$

Which of the following is equal to the mean of X?

a: −1
b: −3/4
c: −1/2
d: −1/4
e: None of the above

$E(X) = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_{-1}^{0} x \cdot 3x^2\,dx$

$= \frac{3}{4} x^4 \Big|_{-1}^{0} = \frac{3}{4}(0 - 1) = -\frac{3}{4}$

4) The random variable X has a mean of −5 and a mean square of 35. Which of the following is equal to the variance of X?

a: 5
b: 10
c: 30
d: 40
e: None of the above

$Var(X) = E(X^2) - [E(X)]^2$

$= 35 - (-5)^2$

$= 35 - 25 = 10$

5) The random variable X is uniform on [0,1]. Which of the following is equal to the mean square of X?

a: 0
b: 1/4
c: 1/2
d: 1
e: None of the above

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$$

$$= \int_0^1 x^2(1)\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$$

6) The random variable X is the number obtained on a single toss of a fair, six-sided die. Which of the following is closest to the value of $E(X^2)$?

a: 0
b: 5
c: 10
d: 15
e: 20

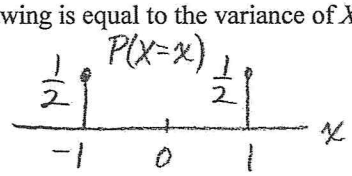
$$E(X^2) = \sum_{K=-\infty}^{\infty} x_K^2 P(X=x_K) = \sum_{K=1}^{6} K^2\left(\frac{1}{6}\right)$$

$$= \frac{1}{6}\left[1^2+2^2+3^2+4^2+5^2+6^2\right]$$

$$= \frac{1}{6}\left[1+4+9+16+25+36\right] = \frac{91}{6} \cong 15.17$$

7) The random variable X may equal the values +1 and –1 with equal likelihood, and no other values are possible. Which of the following is equal to the variance of X?

a: 0
b: 1/4
c: 1/2
d: 1
e: None of the above



← $E(X)=0$ by inspection

$$E(X^2) = \left(\tfrac{1}{2}\right)(-1)^2 + \left(\tfrac{1}{2}\right)(1)^2 = \frac{1}{2}+\frac{1}{2} = 1$$

$$Var(X) = E(X^2) - [E(X)]^2 = 1 - 0 = 1$$

8) The random variable X has the following characteristic function:

$\varphi_X(\omega) = e^{j\omega}$

Which of the following is closest to the mean of X?

a: 1
b: 2
c: 3
d: 4
e: 5

$$\frac{d\phi_X(\omega)}{d\omega} = j e^{j\omega}$$

$$E(X) = (-j)^1 \frac{d\phi_X(\omega)}{d\omega}\Big|_{\omega=0} = (-j)(j e^{j(0)})$$

$$= 1$$

9) The characteristic function of random variable X is $\varphi_X(\omega)$.

If $\left.\dfrac{d\varphi_X(\omega)}{d\omega}\right|_{\omega=0} = j5$, Which of the following is equal to the mean of X?

a: 0
b: 5
c: $j5$
d: 1
e: None of the above

$$E(X) = (-j)^1 \left.\frac{d\phi_X(\omega)}{d\omega}\right|_{\omega=0}$$

$$= (-j)(j5) = 5$$

10) The uncorrelated random variables X and Y have a correlation of 6. The mean value of X is 2. Which of the following is equal to the mean value of Y?

a: 1
b: 2
c: 3
d: 4
e: None of the above.

Uncorrelated $\Rightarrow Cov(X,Y) = 0$

$$Cov(X,Y) = \underbrace{E(XY)}_{=6} - \underbrace{E(X)}_{=2}E(Y) = 0$$

$$\Rightarrow 6 - 2E(Y) = 0 \Rightarrow E(Y) = \frac{6}{2} = 3$$

11) The random variables X and Y are uncorrelated. Which of the following must be true?

a: X and Y are orthogonal.
b: The correlation of X and Y is equal to zero.
c: The covariance of X and Y is equal to zero.
d: X and Y are statistically independent.
e: None of the above

Uncorrelated $\Leftrightarrow Cov(X,Y) = 0$

12) The random variables X and Y are related by the equation $Y + 4 = 3X$. The variance of X is 5. Which of the following is equal to the variance of Y?

a: 11
b: 15
c: 19
d: 45
e: None of the above

$$Y = 3X - 4$$

$$\Rightarrow Var(Y) = (3)^2 Var(X)$$

$$= (9)(5) = 45$$

13) The random variables X and Y are orthogonal. Which of the following must be true?

a: The covariance of X and Y is equal to zero.
b: X and Y are uncorrelated.
c: X and Y are statistically independent.
d: The correlation coefficient of X and Y is equal to zero.
e: None of the above

$$\text{Orthogonal} \iff E(XY) = 0$$

14) The random variables X and Y have a covariance of 2. The variance of X is 2, and the variance of Y is 8. Which of the following is the value of the correlation coefficient of X and Y?

a: 0
b: 1/8
c: 1/4
d: 1/2
e: None of the above

$$\rho = \frac{Cov(X,Y)}{\sqrt{Var(X)\,Var(Y)}} = \frac{2}{\sqrt{2\cdot 8}} = \frac{2}{4} = \frac{1}{2}$$

15) The random variables X_1 and X_2 are jointly Gaussian with the following mean vector and covariance matrix:

$$\mu_x = \begin{bmatrix} 2 \\ -7 \end{bmatrix} \qquad \Sigma_x = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

The random variables Y_1 and Y_2 are formed from X_1 and X_2 as follows:
$Y_1 = 2X_1 - 4X_2 - 3$, and $Y_2 = 4X_1 + 5X_2$.
Which of the following is equal to the variance of Y_2?

a: 36
b: 66
c: 32
d: −32
e: None of the above

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & -4 \\ 4 & 5 \end{bmatrix}}_{A} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \underbrace{\begin{bmatrix} -3 \\ 0 \end{bmatrix}}_{B}$$

$$\Sigma_y = A\,\Sigma_x A^T = \begin{bmatrix} 2 & -4 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ -4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -8 \\ 4 & 10 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ -4 & 5 \end{bmatrix} = \begin{bmatrix} 36 & -32 \\ -32 & 66 \end{bmatrix}$$

$\leftarrow Var(Y_2)$

16) The random variables X_1 and X_2 are jointly Gaussian with the following mean vector and covariance matrix:

$$\mu_x = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \qquad \Sigma_x = \begin{bmatrix} 3 & -2 \\ -2 & 6 \end{bmatrix}$$

The random variables Y_1 and Y_2 are formed from X_1 and X_2 as follows:
$Y_1 = X_1 + 2X_2 + 3$, and $Y_2 = 4X_1 - X_2$.
Which of the following is equal to the covariance of Y_1 and Y_2?

a: −14
b: 14
c: −19
d: 19
e: None of the above

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 2 \\ 4 & -1 \end{bmatrix}}_{A} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 3 \\ 0 \end{bmatrix}}_{B}$$

$$\Sigma_y = A\,\Sigma_x A^T = \begin{bmatrix} 1 & 2 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ -2 & 6 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 10 \\ 14 & -14 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 19 & -14 \\ -14 & 70 \end{bmatrix}$$

$\leftarrow Cov(Y_1, Y_2)$

17) The random process $X(t)$ is wide sense stationary and has autocorrelation function $R_x(t_1, t_2)$. If $R_x(0, 2) = -2$, which of the following is equal to the value of $R_x(-1, 1)$?

a: -1
b: 1
c: -2
d: 2
e: None of the above

$R_x(t_1, t_2)$ depends only on $\tau = t_2 - t_1$

$$\Rightarrow R_x(\underbrace{-1, 1}_{\tau=2}) = R_x(\underbrace{0, 2}_{\tau=2}) = -2$$

18) The ergodic random process $X(t)$ has the following autocorrelation function:

$R_x(\tau) = 10e^{-2|\tau|} + 25$

Which of the following is equal to the variance of $X(t)$?

a: 10
b: 25
c: 35
d: 100
e: None of the above

$$E[X^2(t)] = R_x(0) = 10e^{-2(0)} + 25 = 35$$

$$\{E[X(t)]\}^2 = \lim_{|\tau| \to \infty} R_x(\tau) = 10e^{-2(\infty)} + 25 = 25$$

$$Var[X(t)] = E[X^2(t)] - \{E[X(t)]\}^2 = 35 - 25 = 10$$

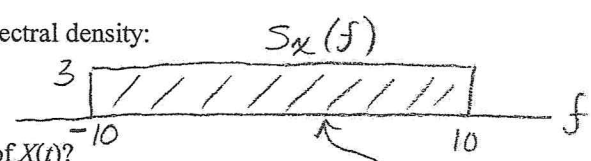
19) The random process $X(t)$ is wide sense stationary and has autocorrelation function $R_x(t_1, t_2)$. Which of the following must be true? Choose the best answer.

a: The mean of $X(t)$ is zero.
b: The mean of $X(t)$ is constant.
c: $R_x(t_1, t_2)$ depends only on $t_1 - t_2$ and not on the individual values of t_1 and t_2.
d: Both b and c
e: None of the above

WSS $\Leftrightarrow$ 1) $E[X(t)]$ = constant and 2) $R_x(t_1, t_2)$ depends only on $\tau = t_2 - t_1$.

20) The random process $X(t)$ has the following power spectral density:

$$S_x(f) = \begin{cases} 3, & |f| \le 10 \\ 0, & \text{otherwise} \end{cases}$$



Which of the following is equal to the mean square of $X(t)$?

a: 60
b: 30
c: 10
d: 3
e: None of the above

$$E[X^2(t)] = \int_{-\infty}^{\infty} S_x(f)\,df = \text{area}$$

$$= (20)(3) = 60$$