SOLUTIONS

Other solutions in notebook

Answers: 101 DCDE BDAC CCAD

Spring 2016

Serial Number: **101**

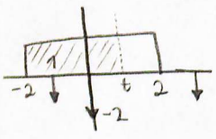
Name: SOLUTIONS

ECE 3300 SPRING 2016 (SIGNALS, SYSTEMS, AND TRANSFORMS): EXAM 1

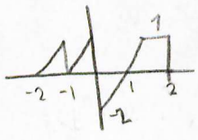
Record your name on this test; record your name, student ID, and test serial number on the scantron. Enter the test serial number in *COURSE*; you may leave *SECTION* blank. You must show your work on every problem, showing all steps on your test. Do not use scratch paper or write your work anywhere but on the test. Circle your answers on the test and bubble in the corresponding answers on your scantron. The examination lasts 50 minutes and you may use one sheet of notes (front and back); no old test questions can be on your notes. Calculator use is permitted. There is one correct answer per question. In problems asking to find coefficients A, B, C, etc., some of these coefficients may equal zero.

Question 1: Suppose $x(t) = (u(t+2) - u(t-2)) - \delta(t+1) - 2\delta(t) - \delta(t-3)$. Suppose $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$. Determine $y(1)$, $y(2)$, and $y(4)$. What is the sum of these three values? Choose the closest answer.

A: 4.
B: 5.
C: 3.
D: 1.
E: 2.



Graphically: $y(t)$



$y(1) = \emptyset$
$y(2) = 1$
$y(4) = 0$

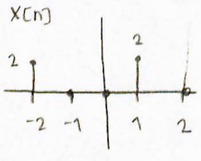
1

Non graphically:
$Y(1) = 3 - 1 - 2 = 0$
$Y(2) = 4 - 1 - 2 = 1$
$Y(4) = 4 - 1 - 2 = 0$

Question 2: Suppose $x[n] = (\overset{4}{n^2} + \overset{2}{n})(u[n+2] - u[n-2])$ and $y[n] = 2x[n-1]$. What is $y[-1] + y[1] + y[3]$? Choose the closest answer.

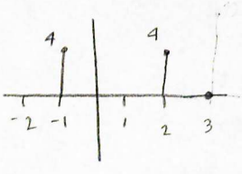
A: 12.
B: 10.
C: 4.
D: 6.
E: 8.

X[n]



Don't include final term with discrete!

Y[n]



From graph:
$Y[-1] = 4$
$Y[1] = 0$
$Y[3] = 0$

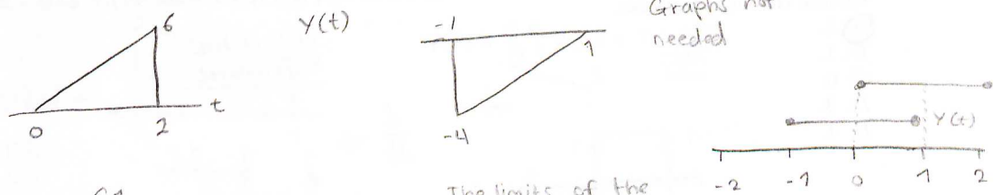
4

This can also be done w/o manipulating the graph

Question 3: Suppose $x(t) = 3t(u(t) - u(t-2))$ and $y(t) = 2(t-1)(u(t+1) - u(t-1))$. Determine the correlation $R_{x,y}$. Choose the closest answer.

A: -3.
B: -4.
C: -5.
D: -1.
E: -2.

Graphs not needed

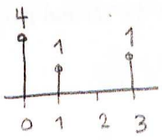


$$R_{xy} = \int_0^1 3t \cdot 2(t-1)\,dt = -1$$

The limits of the integral should be where the 2 plots overlap ONLY

Question 4: Suppose $x[n] = (n-2)^2(u[n] - u[n-4])$. Determine the difference signal $y[n] = x[n] - x[n-1]$. What is $y[0]$? $y[2]$? $y[4]$? Determine the sum of these values. Choose the closest answer.

A: 4.
B: 1.
C: 3.
D: 0.
E: 2.



$Y[0] = X[0] - X[-1] = 4 - 0 = 4$

$Y[2] = X[2] - X[1] = 0 - 1 = -1$

$Y[4] = X[4] - X[3] = 0 - 1 = -1$

2

Question 5: Suppose the periodic signals $\tilde{x}[n]$ and $\tilde{y}[n]$ each have fundamental period $N_0 = 5$ and fundamental cycles $x[n]$ and $y[n]$. Suppose these fundamental cycles have energy $E_x = 5$ and $E_y = 6$ and correlation $R_{x,y} = 3$. Determine the mean-square error of the periodic signals, $\text{MSE}_{\tilde{x},\tilde{y}}$. Choose the closest answer.

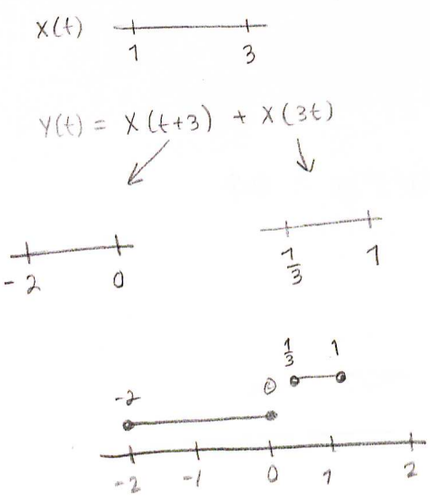
A: 2.
B: 1.
C: 5.
D: 3.
E: 4.

$P_{\tilde{x}} = \frac{E_x}{N_0} = \frac{5}{5} = 1$ $\quad P_{\tilde{y}} = \frac{E_y}{N_0} = \frac{6}{5}$ $\quad R_{\tilde{x}\tilde{y}} = \frac{R_{xy}}{N_0} = \frac{3}{5}$

$\text{MSE}_{xy} = P_{\tilde{x}} + P_{\tilde{y}} - 2\text{Re}\{R_{xy}\} = 1 + \frac{6}{5} - \frac{3}{5} = 1$

Question 6: Suppose $x(t)$ has turn-on time $t = 1$ and turn-off time $t = 3$. Suppose $y(t) = x(t+3) + x(3t)$. What is the time duration of $y(t)$? Choose the closest answer. *Hint:* The time duration is the duration of time between turn-on and turn-off times; if the signal equals zero in intermediate points in time, the time duration is unaffected.

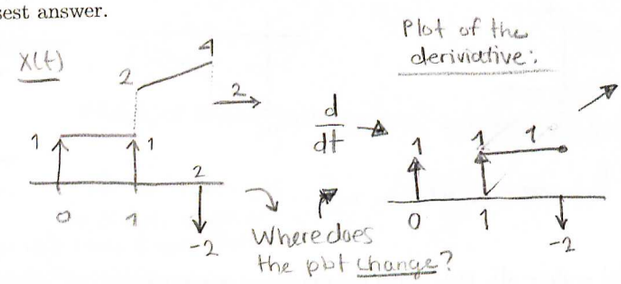
A: 5.
B: 7.
C: 11.
D: 3.
E: 9.



Time duration from -2 to $1 \rightarrow 3$

Question 7: Suppose $x(t) = 1$ if $0 < t \le 1$, $2t$ if $1 < t \le 2$, 0 if $t \le 0$, and 2 if $t > 2$. Determine $y(t) = \frac{d}{dt}x(t)$. The answer has the form $A(u(t) - u(t-1)) + B(u(t-1) - u(t-2)) + C\delta(t) + D\delta(t-1) + E\delta(t-2)$. What is $A + B + C + D + E$? Choose the closest answer.

A: 2.
B: 1.
C: 0.
D: 3.
E: 4.



$= \delta(t) + \delta(t-1) - 2\delta(t-2) + 2(u(t-1) - u(t-2))$

A 0
B 2
C 1
D 1
E −2
2

Question 8: Consider the complex-valued signal $x(t) = \frac{2}{2+e^{jt}}$. Determine $\mathrm{Re}\{x(t)\}$. The answer can be written in the form $\frac{4+A\cos(t)}{B+C\cos(t)}$. What is $A + B + C$? Choose the closest answer. *Hint:* Use $\mathrm{Re}\{x(t)\} = \frac{1}{2}x(t) + \frac{1}{2}x^*(t)$ and put the result over a common denominator.

Note: $e^{-jt} = \cos t - j\sin t$ & $e^{jt} = \cos t + j\sin t$

A: 3.
B: 7.
C: 11.
D: 9.
E: 5.

$$\underbrace{\frac{2}{2+\cos t+j\sin t}}_{\frac{1}{2}x(t)} + \underbrace{\frac{2}{2+\cos t-j\sin t}}_{\frac{1}{2}x^*(t)} = \frac{4+2\cos t}{5+4\cos t}$$

can get from calculator

A 2
B 5
C 4
11

Question 9: Simplify $\sum_{m=n-3}^{4} (\frac{1}{2})^m \delta[m+1]$. The answer has the form $Au[B + Cn]$, where $C = 1$ or -1. What is $A + B + C$? Choose the closest answer.

A: 7.
B: 6.
C: 3.
D: 5.
E: 4.

$$\sum_{m=-\infty}^{\infty} (u[m-(n-3)] - u[m-4-1])(\tfrac{1}{2})^m \delta[m+1]$$

$$\sum_{m=-\infty}^{\infty} (u[-1-(n+3)] - \cancel{u[-1-5-1]})(\tfrac{1}{2})^{-1} \delta[m+1]$$

$$(\tfrac{1}{2})^{-1}(u[-1-(n-3)]) \sum_{m=-\infty}^{\infty} \delta[m+1] \rightarrow 2u[2-n]$$

A 2
B 2
C −1
3

Question 10: Suppose $\tilde{x}(t)$ is periodic with fundamental period $T_0 = 2$ and fundamental cycle $x(t) = t^2(u(t+1) - u(t-1))$. Determine the power $P_{\tilde{x}}$. Choose the closest answer.

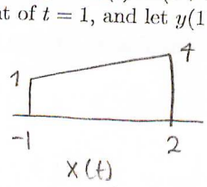
A: 0.6.
B: 1.0.
C: 0.2.
D: 0.8.
E: 0.4.

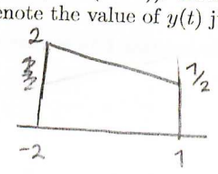
$$P_x = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} |x(\tau)|^2 d\tau$$

$$\frac{1}{2}\int_{-1}^{1} |x(\tau)|^2 d\tau \rightarrow \frac{1}{2}\int_{-1}^{1} (t^2)^2 dt = 0.2$$

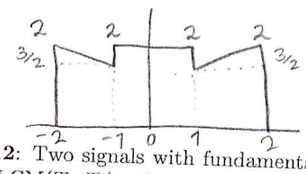
Question 11: Suppose $x(t) = (t+2)(u(t+1) - u(t-2))$. Determine $y(t) = \text{Ev}\{x(t)\}$. Let $y(1^+)$ denote the value of $y(t)$ just to the right of $t = 1$, and let $y(1^-)$ denote the value of $y(t)$ just to the left of $t = 1$. What is $y(1^+) + y(1^-)$? Choose the closest answer.

A: 3.5.
B: 1.5.
C: 2.5.
D: 5.5.
E: 4.5.





$y(t) = \frac{1}{2}\{x(t) + x(-t)\}$



$y(1^+) = 1.5$
$y(1^-) = 2$
$\Big\} = 3.5$

Question 12: Two signals with fundamental periods $T_1 = 4.84$ and $T_2 = 2.64$. If the sum of the signals has fundamental period $T = \text{LCM}(T_1, T_2)$, what is T? Choose the closest answer.

A: 12.
B: 24.
C: 18.
D: 30.
E: 6.

$\frac{1}{100}\text{LCM}(484, 264)$

$\frac{1}{100} \cdot 2904 \rightarrow 29.04 \approx 30$

Answers: 101 AAAE DDEB CCCB

(If your test is not 101, match up your questions and answers with this test.)

Fall 2016

Solutions

Serial Number: **101** Name: SOLUTIONS

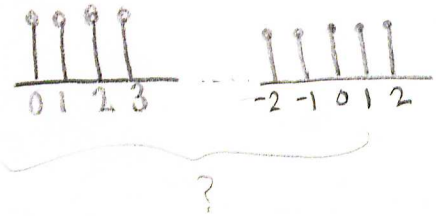
ECE 3300 Fall 2016 (Signals, Systems, and Transforms): Exam 1

Record your name on this test; record your name, student ID, and test serial number on the scantron. Enter the test serial number in *COURSE*; you may leave *SECTION* blank. You must show your work on every problem, showing all steps on your test. Do not use scratch paper or write your work anywhere but on the test. Circle your answers on the test and bubble in the corresponding answers on your scantron. The examination lasts 60 minutes and you may use one sheet of notes (front and back); no old test questions can be on your notes. Calculator use is permitted. There is one correct answer per question. In problems asking to find coefficients A, B, C, etc., some of these coefficients may equal zero.

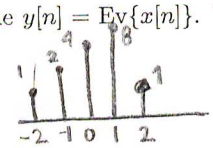
take (non) pulses to 1 less than this

Question 1: Suppose $x[n] = 4(2)^n(u[n+2] - u[n-2]) + \delta[n-2]$. Determine $y[n] = \text{Ev}\{x[n]\}$. What is $y[0]$? $y[1]$? $y[2]$? Determine the sum of these values. Choose the closest answer.

(A) 10.
B: 8.
C: 6.
D: 9.
E: 7.



$x[n] =$



$\text{Ev}\{x[n]\} = \frac{1}{2}(x[n] + x[-n])$

$y[0] = \frac{1}{2}x[0] + \frac{1}{2}x[0] = 4$

$y[1] = \frac{1}{2}x[1] + \frac{1}{2}x[-1] = 5$

$y[2] = \frac{1}{2}x[2] + \frac{1}{2}x[-2] = 1$

Σ: 10

Question 2: Simplify $\int_{t-4}^{t-1} (2-\tau)\delta(\tau-1)d\tau$. The answer can be written in the form $A(u(t-B) - u(t-C))$. What is $A+B+C$? Choose the closest answer.

(A) 8.
B: 4.
C: 2.
D: 6.
E: 10.

$\int_a^b g(\tau)d\tau = \int_{-\infty}^{\infty} \{u(\tau-a) - u(\tau-b)\} g(\tau)$

$\int_{t-4}^{t-1} (2-\tau)\delta(\tau-1)d\tau$

$a = t-4 \quad b = t-1$

$g(\tau) = (2-\tau)\delta(\tau-1)$

$I = \int_{-\infty}^{\infty} \{u(\tau-(t-4)) - u(\tau-(t-1))\}(2-\tau)\delta(\tau-1)d\tau$

$\int_{-\infty}^{\infty} \{u(1-t+4) - u(1-t+1)\}(2-1)\delta(\tau-1)d\tau$

$\{u(5-t) - u(2-t)\} \cdot 1 \cdot \cancel{\int_{-\infty}^{\infty} \delta(\tau-1)d\tau}^{\,1}$

$1(u(5-t) - u(2-t))$

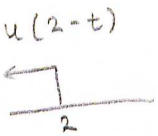
A 1
B 5
C 2

Σ: 8

NOTE

$u(5-t)$

$u(2-t)$



$$x(t) = \begin{cases} \sqrt{t+1} & -1 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Is 0 outside this range, so just use smaller range, -1 to 2

Question 3: Suppose $\tilde{x}(t)$ is periodic with fundamental period $T_0 = \frac{9}{2}$ and fundamental cycle $x(t) = \sqrt{t+1}(u(t+1) - u(t-2))$. Determine the power $P_{\tilde{x}}$. Choose the closest answer.

A: 1.0.
B: 0.6.
C: 0.8.
D: 0.2.
E: 0.4.

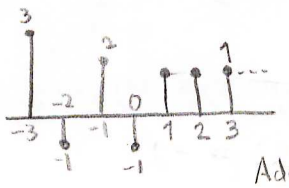
stays the same when squared

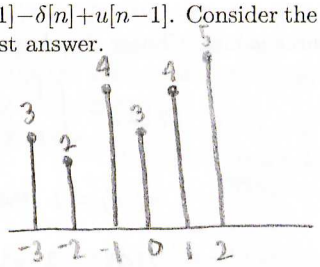
$$P_x = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} |x(t)|^* dt = \frac{1}{9/2}\int_{-\frac{9}{4}}^{\frac{9}{4}} (t+1)(u(t+1) - u(t-2))\, dt$$

$$= \frac{2}{9}\int_{-1}^{2} (t+1)\, dt = 1 \quad \text{A}$$

Question 4: Suppose $x[n] = 3\delta[n+3] - \delta[n+2] + 2\delta[n+1] - \delta[n] + u[n-1]$. Consider the summation signal $y[n] = \sum_{m=-\infty}^{n} x[m]$. At what time value n does $y[n] = 5$? Choose the closest answer.

A: 5.
B: 4.
C: 3.
D: 1.
E: 2.





Add each impulse from the left

$y[-3] = 3$
$y[-2] = 2$
$y[-1] = 4$
$y[0] = 3$
$y[1] = 4$
$! \rightarrow y[2] = 5 \qquad n = 2$

Question 5: Consider the sum of two signals with fundamental periods $T_1 = \frac{3}{8}\pi$ and $T_2 = \frac{9}{16}\pi$. If the sum of the signals has fundamental period $T = \text{LCM}(T_1, T_2)$, what is T? Choose the closest answer.

A: 6.5.
B: 5.5.
C: 7.5.
D: 3.5.
E: 4.5.

$\frac{3}{8}\pi \, \text{LCM}(1, \frac{3}{2})$

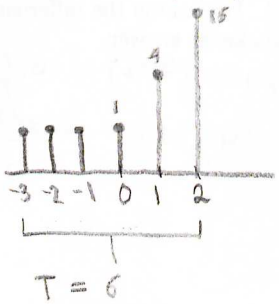
$\frac{3}{8}\pi \, \text{LCM}(\frac{2}{2}, \frac{3}{2})$

$\frac{3\pi}{16} \text{LCM}(2,3) \rightarrow \frac{18\pi}{16}$

$= 3.5$

Question 6: Suppose $x[n] = u[n+3] - u[n+2] + n^2(\underbrace{u[n] - u[n-3]}_{0 \text{ to } 2})$. Determine the time duration. Choose the closest answer.

A: 4.
B: Infinite.
C: 7.
D: 6.
E: 5.



Use that equation!

$n_{off} - n_{on} + 1$

$2 - (-3) + 1$

$2 + 3 + 1$

6 D

Method

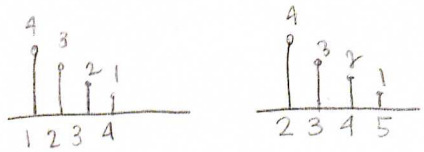
$= 4^2 + (-1) + (-1) + (-1) = 20$

range = 1 to 4

Shift right by 1

Question 7: Suppose $x[n] = (5-n)(u[n-1] - u[n-5])$ and $y[n] = x[n-1]$. Determine the mean-squared error $\text{MSE}_{x,y}$. Choose the closest answer.

A: 25.
B: 10.
C: 30.
D: 15.
E: 20.



$MSE_{xy} = E_x + E_y - 2\,\text{Re}\{R_{xy}\}$

$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 = 16 + 9 + 4 + 1 = 30$

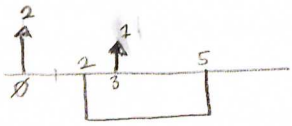
$E_y = 30$

$R_{xy} = \sum_{n=-\infty}^{\infty} x[n]y[n] = 0 + 12 + 6 + 2 + 0 = 20$

$MSE = 60 - 2(20) = 20$

Question 8: Suppose $x(t) = 2\delta(t) - 2(u(t-2) - u(t-5)) + \delta(t-3)$ and suppose $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$. What is $y(1)$? $y(2)$? $y(4)$? Determine the sum of these three values. Choose the closest answer.

A: 1.
B: 3.
C: 5.
D: 2.
E: 4.



$y(1) = \int_{-\infty}^{1} x(\tau)d\tau = 2$

$2 + 2 - 1 = 3$

$y(2) = 2$ NOTHING CHANGES FROM PREVIOUS VALUE

$y(4) = 2 + 1 + (-2 \cdot 2) = -1$

$a - jb$

Question 9: Consider the complex-valued signal $x[n] = (n+1)(u[n] - u[n-3]) - 4j(u[n-2] - u[n-4])$. Determine $|x[n]|$. The answer can be written in the form $A\delta[n] + B\delta[n-1] + C\delta[n-2] + D\delta[n-3] + E\delta[n-4]$. What is $A + B + C + D + E$? Choose the closest answer.

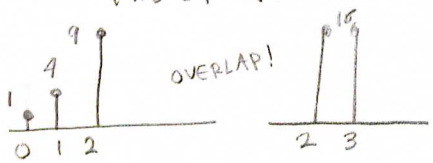
A: 14.
B: 13.
C: 12.
D: 11.
E: 10.

A1 B0 C2 D5 E4 → 1 + 2 + 5 + 4 + 0 = 12

$|x[n]| = \sqrt{x[n]\,x^*[n]}$

$|x[n]| = \sqrt{a^2 + b^2}$

$|x[n]|^2 = a^2 + b^2 = (n+1)^2(u[n+1] - u[n-3]) + 16(u[n-2] - u[n-4])$



$|x[n]|^2$

$|x[n]| = 1\delta[n] + 2\delta[n-2] + 5\delta[n-3] + 4\delta[n-4]$

Question 10: Suppose $x[n] = (\cos(\frac{\pi}{2}n) + \sin(\frac{\pi}{2}n)(u[n] - u[n-5])$. Determine the difference signal $y[n] = x[n] - x[n-1]$. What is $y[3]$? $y[4]$? Determine the sum of these values. Choose the closest answer.

A: 0.
B: −2.
C: 2.
D: −1.
E: 1.

$y[3] = x[3] - x[2] = \underbrace{\cos(3\pi/2)}_{0} + \underbrace{\sin(3\pi/2)}_{-1} - \underbrace{\cos(\pi)}_{-1} - \underbrace{\sin(\pi)}_{0} = 0$

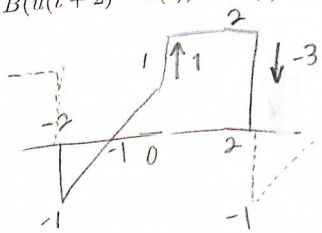
$y[4] = x[4] - x[3] = \underbrace{\cos(2\pi)}_{1} + \underbrace{\sin(2\pi)}_{0} - \underbrace{\cos(\frac{3\pi}{2})}_{0} - \underbrace{\sin(\frac{3\pi}{2})}_{-1} = 2$

$0 + 2 = 2$ C

Couldn't fully read solution for #10.

Question 11: Consider the periodic signal $\tilde{x}(t)$ with fundamental period $T_0 = 4$ and fundamental cycle $x(t) = (t+1)(u(t+2) - u(t)) + 2(u(t) - u(t-2))$. Suppose $\tilde{y}(t) = \frac{d}{dt}\tilde{x}(t)$. Determine the fundamental cycle $y(t)$. The answer has the form $A\delta(t+2) + B(u(t+2) - u(t)) + C\delta(t) + D(u(t) - u(t-2))$. What is $A + B + C + D$? Choose the closest answer.

A: 2.
B: 1.
C: −1.
D: 0.
E: −2.



$-3\delta(t+2) + 1(u(t+2) - u(t)) + 1\delta(t) + 0(u(t) - u(t-2))$

A −3
B 1
C 1
D 0

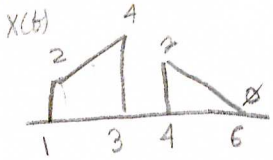
$= -1$

C

No change in this region!

Question 12: Suppose $x(t) = (t+1)(u(t-1) - u(t-3)) + (6-t)(u(t-4) - u(t-6))$ and $y(t) = x(4-2t)$. At what time does $y(t)$ equal 3? At what time does $y(t) = 1$? Determine the sum of these time values. Choose the closest answer.

A: 1.5.
B: 0.5.
C: 4.5.
D: 3.5.
E: 2.5.



$$X(t) = \begin{cases} t+1 & 1 < t < 3 \\ 6-t & 4 < t < 6 \\ 0 & \text{other} \end{cases}$$

$\frac{-3}{-2}$

$$Y(t) = X(4-2t) = \begin{cases} 4-2t+1 & 1 < 4-2t < 3 \\ 6-4+2t & 4 < 4-2t < 6 \\ 0 & \text{otherwise} \end{cases}$$

$$Y(t) = \begin{cases} 5-2t & \frac{1}{2} < t < \frac{3}{2} \\ 2+2t & -1 < t < 0 \\ 0 & \text{otherwise} \end{cases}$$

$1 = 5 - 2t$
$-4 = -2t \quad t = 2$

$1 = 2 + 2t$
$-1 = 2t$
$t = -1/2$ ✓

Not verified

couldn't fully read solution! for #12

Answers: CDDD ACDD ADBB

(An error was made on Problem 4; the intended question was to find the power for $x[n] = n(u[n+3] - u[n-3])$. The answer of A on Problem 4 was also accepted.)

Serial Number: **101**

Name: SOLUTIONS

ECE 3300 FALL 2015 (SIGNALS, SYSTEMS, AND TRANSFORMS): EXAM 1

Record your name on this test; record your name, student ID, and test serial number on the scantron. Enter the test serial number in *COURSE*; you may leave *SECTION* blank. You must show your work on every problem, showing all steps on your test. Do not use scratch paper or write your work anywhere but on the test. Circle your answers on the test and bubble in the corresponding answers on your scantron. The examination lasts 50 minutes and you may use one sheet of notes (front and back); no old test questions can be on your notes. Calculator use is permitted. There is one correct answer per question.

Question 1: The *average energy* of two signals $x(t)$ and $y(t)$ is defined as $\frac{1}{2}(E_x + E_y)$. Determine the average energy of $x(t)$ and $y(t)$ if $R_{x,y} = 3$ and $\text{MSE}_{x,y} = 2$. Choose the closest answer.

A: 1.
B: 3.
C: 4.
D: 5.
E: 2.

$2 = E_x + E_y$?

$\text{MSE}_{xy} = E_x + E_y - 2\,\text{Re}\{R_{xy}\}$

$(\text{MSE}_{xy} + 2R_{xy}) = \frac{1}{2}(E_x + E_y)$

$(\underset{2}{2} + \underset{6}{2 \cdot 3}) = \frac{1}{2}(E_x + E_y)$

$\frac{1}{2}(8) = \frac{1}{2}(E_x + E_y) \longrightarrow 4$

Question 2: Consider the complex-valued signal $x(t) = 2e^{jt} + e^{-jt}$. Determine $|x(t)|$. The answer can be written in the form $\sqrt{1 + a\cos^2(t)}$. What is a? Choose the closest answer.

A: 6.
B: 4.
C: 10.
D: 8.
E: 2.

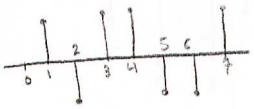
$\sqrt{(2e^{jt} + e^{-jt})(2e^{-jt} + e^{jt})} \rightarrow \sqrt{4 + 2e^{-2jt} + 2e^{jt} + 1} \rightarrow \sqrt{5 + 2(e^{-jt} + e^{jt})}$

$= \sqrt{5 + 4\cos(2t)} \rightarrow \sqrt{5 + 4(2\cos^2 t + 1)} \rightarrow \sqrt{5 + 8\cos^2 t - 4}$

$= \sqrt{1 + 8\cos^2 t} \qquad a = 8$

Question 3: Suppose $x[n] = 1$ at times $n = 1, 3, 4$, and 7, $x[n] = -1$ at times $n = 2, 5$, and 6, and $x[n] = 0$ at all other times. Suppose $y[n] = x[n] - x[n-1]$. Let A be the number of points in time that $y[n]$ equals -2 and B the number of points in time that $y[n] = -1$. What is $A + B$? Choose the closest answer.

A: 1.
B: 4.
C: 2.
D: 3.
E: 5.



n	x[n]	x[n-1]	y[n]
0	0	0	0
1	1	0	1
2	-1	1	-2
3	1	-1	2
4	1	1	0
5	-1	1	-2
6	-1	-1	0
7	1	-1	2
8	0	1	-1

A when $= -2$

2

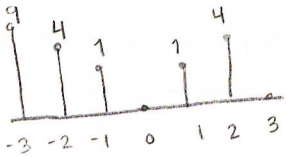
B when $= -1$

1

$\Sigma : 3$

Question 4: Suppose $\tilde{x}[n]$ is periodic with fundamental period $N_0 = 6$ and fundamental cycle $x[n] = n^2(u[n+3] - u[n-3])$. Determine the power $P_{\tilde{x}}$. Choose the closest answer.

A: 3.
B: 1.
C: 4.
D: 5.
E: 2.



$P_{\tilde{x}} = \frac{1}{N_0} \sum_{-3 \text{ to } 3} |x[n]|^2$

?

$\frac{1}{6}(9 + 4 + 1 + 1 + 4) \approx 3.2$

Question 5: Determine the fundamental period of $x(t) = \cos(255\pi t) + \cos(340\pi t)$. Choose the closest answer. *Hint:* It is equal to the LCM of the fundamental periods of the two cosines.

A: 0.025.
B: 0.015.
C: 0.005.
D: 0.01.
E: 0.02.

$\frac{2\pi t}{T_1} = 255\pi t \rightarrow T_1 = \frac{2}{255}$

$\frac{2\pi t}{T_2} = 340\pi t \rightarrow T_2 = \frac{2}{340}$

$LCM(2/255, 2/340)$

$\frac{2}{5} LCM\left(\frac{1}{51}, \frac{1}{68}\right) = \frac{2}{85} \approx 0.025$

Question 6: Suppose $x(t) = t(u(t) - u(t-1))$ and $y(t) = (u(t) - u(t-1))$. Determine the correlation coefficient $\rho_{x,y}$. The answer can be written as $\sqrt{n}/m$ where n and m are positive integers and the fraction is as simplified as possible. What is $n + m$? Choose the closest answer.

A: 11.
B: 7.
C: 5.
D: 9.
E: 13.

$R_{xy} = \int_0^1 t\,dt = 1/2$

$E_x = \int_0^1 t^2 = 1/3$

$E_y = \int_0^1 1^2 dt = 1$

$\rho_{xy} = \frac{R_{xy}}{\sqrt{E_x E_y}} = \frac{1/2}{\sqrt{1/3}} = \frac{\sqrt{3}}{2}$

$n + m$
$3 + 2$
5

Question 7: Suppose $x[n] = (n+3)(n-2)(u[n+2] - u[n-3])$. Determine $y[n] = \text{Od}\{x[n]\}$. What is $y[0] + y[1] + y[2]$? Choose the closest answer.

A: 1.
B: 2.
C: 5.
D: 3.
E: 4.

$Y[0] = \frac{1}{2}X[0] - \frac{1}{2}X[0] = 0$

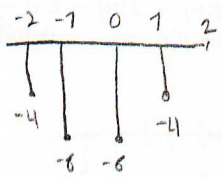
$6/2 \quad 6/2$

$Y[1] = \frac{1}{2}X[1] - \frac{1}{2}X[-1] = 1$

$-4/2 \quad +6/2$

$Y[2] = 2$

$\Sigma = 3$



Question 8: Simplify $\int_t^{\infty} \sin(\frac{\pi}{2}\tau)\delta(\tau+1)d\tau$.

A: $-u(-1+t)$.
B: $u(-1+t)$.
C: $u(-1-t)$.
D: $-u(-1-t)$.
E: 0.

$\int_{-\infty}^{\infty} u(t-a) - u(\tau - t)\, g(\tau)\, d\tau \qquad a = t \quad b = \infty$

$g(\tau) = \sin(\frac{\pi}{2}t)\,\delta(\tau+1) \rightarrow \int_{-\infty}^{\infty} (u(\tau - t) - \cancel{u(\tau - \infty)}^{\,0}) \sin(\frac{\pi}{2}\tau)\,\delta(\tau - 1)\, d\tau$

$\int_{-\infty}^{\infty} u(t-1-t) \sin(-\pi/2)\,\delta(\tau+1)\,dt$

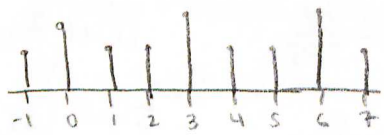
$= \underbrace{u(-1-t)} \int \delta(\tau+1)\,d\tau$

Question 9: Suppose $\tilde{x}[n]$ is periodic with fundamental period $N_0 = 3$ and fundamental cycle $x[n] = \delta[n+1] + 2\delta[n] + 2\delta[n-1]$. Suppose $\tilde{y}[n]$ is the periodic sum signal for $\tilde{x}[n]$. What is $\tilde{y}[16]$? Choose the closest answer. *Hint:* The periodic sum signal is periodic with period N_0. Thus $\tilde{y}[16] = \tilde{y}[16 - N_0] = \tilde{y}[16 - 2N_0]$ etc. If you keep shifting back you will eventually be in the time region of the fundamental cycle $y[n]$.

A: 5.
B: 3.
C: 1.
D: 2.
E: 4.

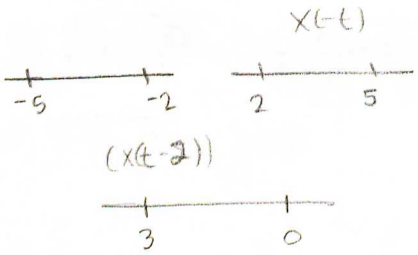
$Y[10] = Y[7] = Y[4] = Y[1] = X[1] = 5$

$N_0 = 3$



Question 10: Suppose $x(t)$ has turn-on time $t = -5$ and turn-off time $t = -2$. Suppose $y(t) = x(-t) + x(t-2)$. What is the time duration of $y(t)$? Choose the closest answer. *Hint:* The time duration is the duration of time between turn-on and turn-off times; if the signal equals zero in intermediate points in time, the time duration is unaffected.

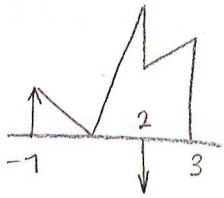
A: 10.
B: 6.
C: 9.
D: 8.
E: 7.



$Y(t) = 5 - (-3) = 8$

Question 11: Suppose $x(t) = t^2(u(t+1) - u(t-2)) + t(u(t-2) - u(t-3))$. Determine the derivative signal $y(t) = \frac{d}{dt}x(t)$. The answer has the form $A\delta(t+1) + Bt(u(t+1) - u(t-2)) + C\delta(t-2) + D(u(t-2) - u(t-3)) + E\delta(t-3)$. What is $A + B + C + D + E$? Choose the closest answer.

A: 1.
B: -1.
C: 0.
D: 2.
E: -2.

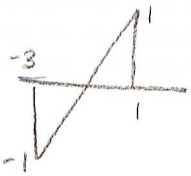


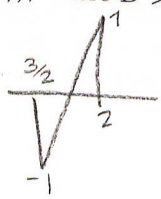
$\delta(t+1) - 2\delta(t-2) - 3\delta(t+3)$
$+ 2t\,(u(t+1) - u(t-2))$
$+ 1\,(u(t-2) - u(t-3))$

A 1
B 2
C -2
D 1
E -3
-1

Question 12: Suppose $x(t) = (2t-1)(u(t+3) - u(t-1))$ and $y(t) = x(2t)$. The signal $y(t)$ can be written as $y(t) = (At+B)(u(t-C) - u(t-D))$, where $D > C$. What is $A + B + C + D$? Choose the closest answer.

A: 3.
B: 2.
C: 1.
D: 5.
E: 4.





$y(t) = (At+B)(u(t+\frac{3}{2}) - u(t-\frac{1}{2}))$

$\frac{3A}{2} + B = -1$

$\frac{A}{2} + 3 = 1$

A4 B-1 C $-\frac{3}{2}$ D $\frac{1}{2}$

T2 S2016

Serial Number: **201**

Name: SOLUTIONS

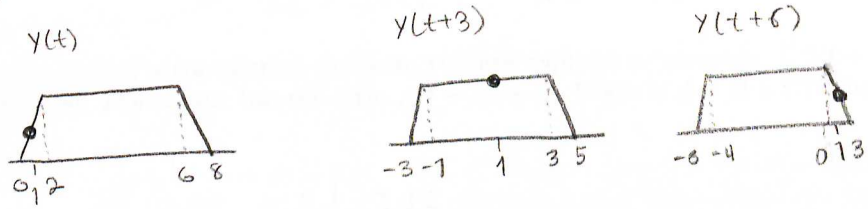
ECE 3300 SPRING 2016 (SIGNALS, SYSTEMS, AND TRANSFORMS): EXAM II

Record your name on this test; record your name, student ID, and test serial number on the scantron. Enter the test serial number in *COURSE*; you may leave *SECTION* blank. You must show your work on every problem, showing all steps on your test. Do not use scratch paper or write your work anywhere but on the test. The examination lasts 50 minutes and you may use two sheets of notes (front and back); no old test questions can be on your notes. Calculator use is permitted. There is one correct answer per question. In problems asking to find coefficients A, B, C, etc., some of these coefficients may equal zero.

Question 1: Consider a LTI system with impulse response $h(t)$ and periodic input $\tilde{x}(t)$ with $T_0 = 3$ such that the convolution of the impulse response with the fundamental cycle of the input is $[x * h](t) = t(u(t) - u(t-2)) + 2(u(t-2) - u(t-6)) + (8-t)(u(t-6) - u(t-8))$. If the output is $\tilde{y}(t)$, determine $\tilde{y}(1)$. Choose the closest answer.

$\tilde{y}(t) = \sum_{n=-\infty}^{\infty} y(t - nT_0)$

A: 4.
B: 5.
C: 3.
D: 6.
E: 7.



$\tilde{y}(1) = 1 + 2 + 1 = 4$

Question 2: Consider the system with input $x[\cdot]$ and output $y[\cdot]$ such that $y[n] = x[2n+1] - x[2n]$. This system is

A: Nonlinear because $x[2n+1]$ is not equal to $x[0]$ at $n = 0$.
B: Nonlinear because an input of $x[n] = 1$ for all n gives an output of zero for all n.
C: Linear because, for all inputs $x_1[n]$ and $x_2[n]$ and all constants a_1 and a_2, the output due to $a_1x_1[n] + a_2x_2[n]$ is $a_1y_1[n] + a_2y_2[n]$, where $y_1[n]$ is the output due to $x_1[n]$ and $y_2[n]$ is the output due to $x_2[n]$.
D: Nonlinear because, for the input $x[n] = u[n]$, doubling the input does not double the output.
E: Linear because an input of zero for all n gives an output of zero for all n.

① $a_1x_1[2n+1] + a_2x_2[2n+1] - a_1x_1[2n] - a_2x_2[2n]$

② $a_1x_1[2n+1] - a_1x_1[2n] + a_2x_2[2n+1] - a_2x_2[2n]$

Same → LINEAR

12 S2016

Test 201 Answers: ACAE DDBD DCDA

On Problem 8, E is also correct. On Problem 12, B is also correct.

Question 3: Consider an LTI system with impulse response $h(t) = \frac{1}{t^2}u(-t-1)$. This system is

A: Stable but not causal and not memoryless.
B: Causal but not stable and not memoryless.
C: Causal and stable but not memoryless.
D: Not causal, not stable, and not memoryless.
E: Causal, stable, and memoryless.

Causal: $h(t) = 0, t < 0 \rightarrow$ Causal

Memoryless: $h(t) = k\delta(t)$

Stability: $\int_{-\infty}^{\infty} |h(t)| dt < \infty \quad \int_{-\infty}^{\infty} \left| \frac{1}{t^2} u(-t-1) \right| dt$

$\int_{-\infty}^{-1} \frac{1}{t^2} dt = 1 < \infty \rightarrow$ Stable

Question 4: Consider a composite system in which System 1 is in parallel with the series combination of Systems 2 and 3. The systems have respective impulse responses $h_1(t) = (1+t)e^{-t}u(t)$, $h_2(t) = \delta(t) - e^{-t}u(t)$, and $h_3(t) = e^{-t}u(t)$. Determine the impulse response of the composite system. The answer has the form $h(t) = A\delta(t) + (B + Ct + Dt^2)e^{-t}u(t)$. Determine $A+B+C+D$. Choose the closest answer. *Hint:* The following facts may be used without proof: $e^{-t}u(t) * e^{-t}u(t) = te^{-t}u(t)$, $e^{-t}u(t) * te^{-t}u(t) = \frac{t^2}{2}e^{-t}u(t)$.

A: 4.
B: 1.
C: 3.
D: 5.
E: 2.

$[h_2 * h_3] = (\delta(t) - e^{-t}u(t)) * e^{-t}u(t)$

$= \delta(t) * e^{-t}u(t) - e^{-t}u(t) * e^{-t}u(t) - te^{-t}u(t)$

$= e^{-t}u(t)$

$h_1(t) + (h_2 * h_3) = (1+t)e^{-t}u(t) + e^{-t}u(t) - te^{-t}u(t)$

$= 2e^{-t}u(t)$ A 0 B 2 C 0 D 0 $\} = 2$

Question 5: As part of a convolution problem, you are required to compute $\int_0^t 2(t+\tau)d\tau$. Determine the answer to this integral in this case. The answer has the form $At^2 + Bt + C$. What is $A + B + C$? Choose the closest answer.

A: 1.
B: 2.
C: 5.
D: 3.
E: 4.

$2\int_0^t t\,d\tau + 2\int_0^t \tau\,d\tau$

$2t \cdot \tau \Big|_0^t + 2\frac{\tau^2}{2}\Big|_0^t = 2t^2 + t^2 = 3t^2$

A 3
B 0
C 0

Question 6: Consider the system $y(t) = tx(t-\frac{1}{2}) + (t-1)x(t-\frac{3}{2})$. Let $g(t)$ be the step response of this system. Determine $g(0) + g(1) + g(2)$. Choose the closest answer.

A: 1.
B: 2.
C: 3.
D: 4.
E: 5.

$g(t) = tu(t - 1/2) + (t-1)u(t - \frac{3}{2})$

$g(0) = 0 + (-1)u(-3/2) = 0$

$g(1) = 1u(1/2) + 0 = 1$

$g(2) = 2u(3/2) + 1u(1/2) = 3$

$= 4$

Question 7: Suppose $x(t)$ and $h(t)$ have no impulses and

$$[x * h](t) = \begin{cases} (A + Bt) & \text{if } 2 < t \leq 3 \\ C & \text{if } 3 < t \leq 4 \\ 2e^{-(t-4)} & \text{if } t > 4 \\ 0 & \text{otherwise} \end{cases}$$

Use the "checking" properties of convolution to determine A, B, and C. What is $A + B + C$? Choose the closest answer.

A: 4.
B: 0.
C: 1.
D: 2.
E: 3.

$A + 3B = C$
$A + 2B = 0$ $\longrightarrow$ $A + 3B = 2$, $A + 2B = 0$, $B = 2$

$A = -2B$
$A = -4$

@ $t = 4$ $\quad 2e^0 = C \rightarrow C = 2$

$2 + 2 - 4 = \emptyset$

Question 8: Consider the system $y[n] = x[n] - n$. This system is

A: Time-invariant because the output due to $x[n] = n - 1$ is a delay by 1 of the output due to $x[n] = n$.
B: Time-invariant because the output due to $x[n] = (-1)^{n-2}$ is a delay by 2 of the output due to $x[n] = (-1)^n$.
C: Time-invariant because, for all inputs $x[\cdot]$ and all values n_0, the output due to $x[\cdot - n_0]$ is a delay by n_0 of the output due to $x[\cdot]$.
D: Time-varying because the output due to $x[n] = n - 1$ is not a delay by 1 of the output due to $x[n] = n$.
E: Time-varying because the output due to $x[n] = (-1)^{n-2}$ is not a delay by 2 of the output due to $x[n] = (-1)^n$.

both correct

① $x[n-n_0] - n$
② $x[n-n_0] - n_0 + n$

different $\rightarrow$ TIME VARYING

Question 9: Determine the convolution of $2\delta[n] - \delta[n-1] - 2\delta[n-2] + 2\delta[n-3]$ with $\delta[n-1] - \delta[n-2] + \delta[n-3]$. The answer can be written in the form $A\delta[n-1] + B\delta[n-2] + C\delta[n-3] + D\delta[n-4] + E\delta[n-5] + F\delta[n-6]$. Determine $B + D + F$. Choose the closest answer.

A: 0.
B: 1.
C: 3.
D: 2.
E: 4.

$(h_1 * h_2) = h_1 * (\delta[n-2] - \delta[n-2] + \delta[n-3])$
$= h_1[n-2] - h_1[n-2] + h_1[n-3]$

B	D	F	
-3	3	2	= 2

It is made easier if you line up like terms

$= 2\delta[n-1] - \delta[n-2] - 2\delta[n-3] + 2\delta[n-4]$
$-(\quad -2\delta[n-2] - \delta[n-3] - 2\delta[n-4] + 2\delta[n-5])$
$+ \quad 2\delta[n-3] - \delta[n-4] - 2\delta[n-5] + 2\delta[n-6]$

Be very careful w/ this problem! It's easy to mess up a sign.

$= 2\delta[n-1] - 3\delta[n-2] + \delta[n-3] + 3\delta[n-4] - 1\delta[n-5] + 2\delta[n-6]$

Question 10: Consider the system with input $x[\cdot]$ and output $y[\cdot]$ such that $y[n] = (n+1)x[n-1] + (n-2)x[n]$. Is this system memoryless? Causal?

A: Memoryless and causal.
B: There is insufficient information to determine an answer.
C: Causal but not memoryless.
D: Neither memoryless nor causal.
E: Memoryless but not causal.

memoryless

$n = 4$ $\quad y[4] = 5x[3] + 2x[4] \rightarrow$ Not memoryless

Causal

$n = -2$ $\quad y[-2] = -1x[-3] + (-4)x[-2]$

No dependence on future $\rightarrow$ causal

Question 11: Consider the system with input $x[\cdot]$ and output $y[\cdot]$ such that $y[n] = x[n]x[n-1]x[n-2]x[n-3]$. This system is

A: Unstable because the input $x[n] = \frac{1}{n}$ for all n gives an unbounded output.
B: Stable because the input $x[n] = 1$ for all n gives a bounded output.
C: Stable because the input $x[n] = 0$ for all n gives output $y[n] = 0$ for all n.
D: Stable because $|x[n]| \leq A$ for all n implies that $|y[n]| \leq B$ for all n for $B = A^4$.
E: Unstable because the input $x[n] = n$ for all n gives an unbounded output.

$|x[n]| \leq A$
$|x[n-1]| \leq A$
$|x[n-2]| \leq A$
$|x[n-3]| \leq A$

System is stable $\Leftarrow$ $Y[n] \leq A^4$

Question 12: Consider the system defined by $y(t) = x(t+1)u(t+1)$ where $x(\cdot)$ is the input and $y(\cdot)$ is the output. This system is

both correct

A: Not invertible because the inputs $x(t) = 1$ for all t and $x(t) = u(t+2)$ give the same output.
B: Not invertible because the inputs $x(t) = 1$ for all t and $x(t) = u(t)$ give the same output.
C: Invertible because the inverse system is $x(t) = \frac{y(t-1)}{u(t)}$.
D: Invertible because the inputs $x(t) = 1$ for all t and $x(t) = -1$ for all t give different outputs.
E: Invertible because the inverse system is $x(t) = \frac{y(t-1)}{u(t-1)}$.

$x(t+1) = \frac{y(t)}{u(t+1)}$

$x(-2) = \frac{y(-2)}{u(-2+1)} = \infty \rightarrow$ Not invertible

A) $x(t) = 1$ for all t $\quad x(t) = u(t+2)$

$y(t) = u(t+2)u(t+1) = u(t+1)$

B)?

Serial Number: **201** Name: SOLUTIONS

ECE 3300 FALL 2015 (SIGNALS, SYSTEMS, AND TRANSFORMS): EXAM II

Record your name on this test; record your name, student ID, and test serial number on the scantron. Enter the test serial number in *COURSE*; you may leave *SECTION* blank. You must show your work on every problem, showing all steps on your test. Do not use scratch paper or write your work anywhere but on the test. The examination lasts 50 minutes and you may use two sheets of notes (front and back); no old test questions can be on your notes. Calculator use is permitted. There is one correct answer per question.

Question 1: Consider the system $y[n] = \cos(x[n+1] - x[n-1])$. This system is

A: Time-varying because the output due to $x[n] = (-1)^n u[n]$ is not a delay by 1 of the output due to $x[n] = (-1)^n u[n-1]$.

B: Time-invariant because, for all inputs $x[\cdot]$ and all values n_0, the output due to $x[\cdot - n_0]$ is a delay by n_0 of the output due to $x[\cdot]$.

C: Time-invariant because the output due to $x[n] = (-1)^n$ is does not vary over time (it is a constant).

D: Time-varying because the output due to $x[n] = (-1)^n u[n]$ is not a delay by 1 of the output due to $x[n] = (-1)^{n-1} u[n-1]$.

E: Time-invariant because the output due to $x[n] = (-1)^n u[n]$ is a delay by two of the output due to $x[n] = (-1)^{n-2} u[n-2]$.

① $y[n] = \cos(x[n-n_0+1] - x[n-n_0-1])$

② $y[n-n_0] = \cos(x[n-n_0+1] - x[n-n_0-1])$

Testing Answer → $x[n] = (-1)^n u(n) \rightarrow y[n] = \cos[(-1)^{n+1} u[n+1] - (-1)u($
$= \cos((-1)^n u(n+1) - (-1)^n u($

$x[n] = (-1)^{n-2} u(n-2) \rightarrow \cos((-1)^n u[n-1] - (-1)^n u(n-$

Same So time invariant

Question 2: Consider the system with input $x[\cdot]$ and output $y[\cdot]$ such that $y[n] = n^2 x[0] - x[n]$. This system is

A: Nonlinear because, for the input $x[n] = u[n]$, doubling the input does not double the output.

B: Linear because an input of zero for all n gives an output of zero for all n.

C: Nonlinear because an input of zero for all n does not give an output of zero for all n.

D: Linear because, for all inputs $x_1[n]$ and $x_2[n]$ and all constants a_1 and a_2, the output due to $a_1 x_1[n] + a_2 x_2[n]$ is $a_1 y_1[n] + a_2 y_2[n]$, where $y_1[n]$ is the output due to $x_1[n]$ and $y_2[n]$ is the output due to $x_2[n]$.

E: Nonlinear because, for the input $x[n] = 1$ for all n, the output is not linear.

See definition of linearity!

① $n^2(a_1 x_1[\cdot] + a_2 x_2[\cdot]) \rightarrow a_1 x_1[n] - a_2 x_2[n]$

② $a_1 n^2 x[0] - a_1 x_1[n] + a_2 n^2 x_2[0] - a_2 x_2[n]$

→ Same → LINEAR

Question 3: Consider the system with input $x[\cdot]$ and output $y[\cdot]$ such that $y[n] = \frac{n}{n+1} x[n]$. This system is

A: Unstable because the input $x[n] = 1$ for all n gives an unbounded output.

B: Stable because $|x[n]| \leq A$ for all n implies that $|y[n]| \leq B$ for all n for $B = A$.

C: Unstable because the input $x[n] = u[n]$ gives an unbounded output. → No

D: Unstable because the input $x[n] = n + 1$ for all n gives an unbounded output.

E: Stable because the input $x[n] = 1$ for all n gives a bounded output.

If $n = -1$ $y[n] = \infty$, so it can't be stable.

(A) $x[-1] = 1$ for all n

(D) Input not bounded because of n

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Test 201 Answers: BDAC EECD DBDA

depends on past, not future

Question 4: Consider an LTI system with impulse response $h[n] = 8\delta[n-2]$. This system is

A: Not causal, not stable, and not memoryless.
B: Stable but not causal and not memoryless.
C: Causal and stable but not memoryless.
D: Causal but not stable and not memoryless.
E: Causal, stable, and memoryless.

Memoryless: $k\delta[n] \rightarrow$ Not memoryless ×

Causality: $h[n]=0 \quad n<0 \qquad h[n]=8 \quad n=2$, $=0 \quad 0$ } causal ✓

$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$, $8 < \infty \rightarrow$ Stable ✓

Question 5: Consider the system with input $x(\cdot)$ and output $y(\cdot)$ such that $y(t) = (t-2)x(-4)x^2(t)$. Is this system memoryless? Causal?

A: Memoryless but not causal.
B: There is insufficient information to determine an answer.
C: Memoryless and causal.
D: Causal but not memoryless.
E: Neither memoryless nor causal.

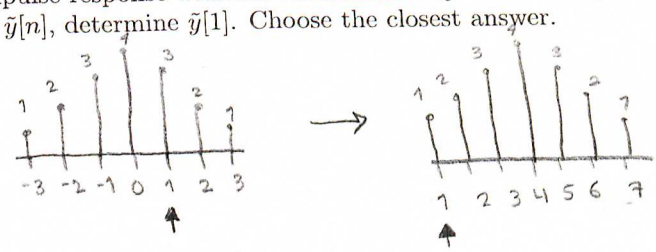
IF $t=5$ $\quad y(5) = (3) \times (-4) \times^2 (5)$ depends on past → not memoryle

IF $t=-10$ $\quad y(-10) = (-12) \times \underbrace{(-4)} \times^2 (-10)$

depends on future → not causal

Question 6: Consider a LTI system with impulse response $h[n]$ and periodic input $\tilde{x}[n]$ with $N_0 = 4$ such that the convolution of the impulse response with the fundamental cycle of the input is $(x * h)[n] = 4 - |n|$ if $-3 \le n \le 3$ and 0 otherwise. If the output is $\tilde{y}[n]$, determine $\tilde{y}[1]$. Choose the closest answer.

A: 3.
B: 6.
C: 5.
D: 7.
E: 4.



$\tilde{y}[1] = 3 + 1 = 4$

Move right & left in both directions by 4 (the period) and add all nonzero amounts

Question 7: Consider a composite system in which System 1 is in series with the parallel combination of Systems 2 and 3. The systems have respective impulse responses $h_1[n] = 2\delta[n] - \delta[n-1]$, $h_2[n] = \delta[n] + 2\delta[n-1]$, and $h_3[n] = \delta[n] - \delta[n-1] + \delta[n-2]$. Determine the impulse response of the composite system. The answer has the form $h[n] = A\delta[n] + B\delta[n-1] + C\delta[n-2] + D\delta[n-3]$. Some of these coefficients might be zero. Determine $A+B+C+D$. Choose the closest answer.

A: 3.
B: 1.
C: 4.
D: 5.
E: 2.

$(h_2 + h_3)[n] = \delta[n] + 2\delta[n-1] + \delta[n] - \delta[n-1] + \delta[n-2]$

$h'[n] = 2\delta[n] + \delta[n-1] + \delta[n-2]$

$(h_1 * h_2)[n] = h_1 * (2\delta[n] - \delta[n-1]) \rightarrow 2h'[n] - h'[n]$

$= 2(2\delta[n] + \delta[n-1] + \delta[n-2])$

$= 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-1] - \delta[n-3]$

A 4
B 0
C -1
D 1
4

Question 8: Consider the system defined by $y(t) = \begin{cases} x(t) & \text{if } -2 < x(t) < 2 \\ 2 & \text{if } x(t) \geq 2 \\ -2 & \text{if } x(t) \leq -2 \end{cases}$ where $x(\cdot)$ is the input and $y(\cdot)$ is the output. This system is

A: Invertible because the inverse system is $x(t) = \begin{cases} y(t) & \text{if } -2 < x(t) < 2 \\ 2 & \text{if } y(t) \geq 2 \\ -2 & \text{if } y(t) \leq -2 \end{cases}$.

B: Invertible because the inputs $x(t) = 3$ for all t and $x(t) = -3$ for all t give different outputs.

C: Not invertible because the inputs $x(t) = \sin(t)$ and $x(t) = 2\sin(t)$ give the same output.

D: Not invertible because the inputs $x(t) = 3$ for all t and $x(t) = 4$ for all t give the same output.

E: Invertible because the inverse system is $x(t) = \begin{cases} y(t) & \text{if } -\frac{1}{2} < x(t) < \frac{1}{2} \\ \frac{1}{2} & \text{if } y(t) \geq \frac{1}{2} \\ -\frac{1}{2} & \text{if } y(t) \leq -\frac{1}{2} \end{cases}$

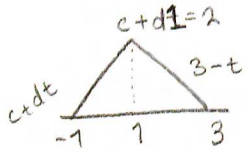
If $x(t) = 5 \rightarrow y(t) = 2$
and
If $x(t) = 10 \rightarrow y(t) = 2$
} 2 identicle values $\rightarrow$ NOT invertible

Question 9: Suppose $x(t)$ and $h(t)$ have no impulses and

$$[x * h](t) = \begin{cases} (c + dt) & \text{if } -1 < t \leq 1 \\ (3 - t) & \text{if } 1 < t \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Assume $x(t)$ is "on" from $t = -2$ to $t = 0$ and $h(t)$ is "on" from $t = a$ to $t = b$. Use the "checking" properties of convolution to determine a, b, c, and d. What is $a + b + c + d$? Choose the closest answer.

A: 8.
B: 10.
C: 9.
D: 6.
E: 7.



$x \rightarrow (-2, 0)$
$h \rightarrow (a, b)$
$y \rightarrow (a-2, 0+b) = (a-2, b)$

$c - d = 0$
$c + d = 2$
$c = 1$ $d = 1$

$a - 2 = -1$ $a = 1$
$b = 3$

$1 + 3 + 1 + 1 = 6$ D

Question 10: As part of a convolution problem, you are required to compute $\int_{-\infty}^{\infty} u(\tau - 1)u(t - \tau - 1)d\tau$ in the region $t > 2$. Determine the answer to this integral in this case. The answer has the form $At + B$. What is $A + B$? Choose the closest answer.

A: -4.
B: -1.
C: -3.
D: -5.
E: -2.

$\int_1^{t-1} d\tau = \tau \Big|_1^{t-1} = t - 1 - 1 = t - 2$

A1 B-2
-1

Question 11: Consider the system $y[n] = (n+2)x[n-2] - (n-1)x[n-1]$. Let $h[n]$ be the impulse response of this system. Determine $h[0] + h[1] + h[2]$. Choose the closest answer.

A: 3.
B: 0.
C: 1.
D: 4.
E: 2.

$h[n] = (n+2)\delta[n-2] - (n-1)\delta[n-1]$

$h[0] = (2)\delta[-2] - (-1)\delta[-1] = 0$

$h[1] = 3\delta[-1] - 0 \cdot \delta[0] = 0$

$h[2] = 4\delta[0] - 1\delta[1] = 4$

$0 + 0 + 4 = 4$

Question 12: In this problem use without proof the facts that $e^{-2t^2} * e^{-2t^2} = \frac{1}{2}\sqrt{\pi}e^{-t^2}$ and $te^{-2t^2} * e^{-2t^2} = \frac{1}{4}\sqrt{\pi}te^{-t^2}$. Determine the convolution of $te^{-2(t-1)^2}$ with e^{-2t^2}. The answer can be written in the form $\frac{1}{4}\sqrt{\pi}(At+B)e^{-(t-C)^2}$. Determine $A + B + C$. Choose the closest answer. *Hint:* Use properties. You may find it helpful to write $t = (t-1) + 1$.

A: 3.
B: 2.
C: 4.
D: 5.
E: 1.

This solution may be incorrect

$\{(t-1)+1\} e^{-2(t+1)^2}$

$= (t-1)e^{-2(t+1)^2} * e^{-2t^2} + e^{-2t^2} * e^{-2t^2}$

$= (t-1)e^{-2(t-1)^2} * e^{-2t^2}$

$= \frac{1}{4}\sqrt{\pi}(t-1) \cdot e^{-(t-1)^2}$

$\frac{1}{4}\sqrt{\pi}(At+B)e^{-(t-1)^2}$

A +1
B -1
C +1

Fall 2016
Exam 2
SOLUTIONS*

Test 201 Answers: ADDB BDEA BCAA

* Other solutions in notebook *

(There was a minor error in problem 11, which has been corrected in this version. There was a more serious error in problem 3. Credit was given or not given based on work shown on the exam. The problem has been changed in this version to match what was intended.)

Serial Number: **201**

Name:

ECE 3300 Fall 2016 (Signals, Systems, and Transforms): Exam II

Record your name on this test; record your name, student ID, and test serial number on the scantron. Enter the test serial number in *COURSE*; you may leave *SECTION* blank. You must show your work on every problem, showing all steps on your test. Do not use scratch paper or write your work anywhere but on the test. The examination lasts 60 minutes and you may use two sheets of notes (front and back); no old test questions can be on your notes. Calculator use is permitted. There is one correct answer per question. In problems asking to find coefficients A, B, C, etc., some of these coefficients may equal zero.

Question 1: Suppose $x(t)$ and $h(t)$ have no impulses and $x(t)$ is "on" from $t = 1$ to $t = 2$ and $h(t)$ is "on" from $t = 0$ to $t = 3$. Further suppose that

$$[x * h](t) = \begin{cases} A + (t-3)^2 & \text{if } \overset{1}{B} < t \leq \overset{5}{C} \\ 0 & \text{otherwise} \end{cases}$$

Use the "checking" properties of convolution to determine A, B, and C. What is $A + B + C$? Choose the closest answer.

$x(t) \Rightarrow (1,2)$

$h(t) \rightarrow (0,3)$

$(x*h)(t) \rightarrow (1,5)$

A: 2.
B: 6.
C: 8.
D: 4.
E: 10.

$A - (B-3)^2 = 0$

$A = (B-3)^2$

$= -(1-3)^2$

$= -4$

$= -4 + 1 + 5$

Question 2: Consider the system defined by $y[n] = x[n] - x[-n]$ where $x[\cdot]$ is the input and $y[\cdot]$ is the output. This system is

A: Invertible because the inputs $x[n] = \delta[n]$ and $x[n] = \delta[n-1]$ give different outputs.
B: Not invertible because the inputs $x[n] = u[n]$ and $x[n] = u[-n]$ give the same output.
C: Invertible because the inverse system is $x[n] = y[n] + y[-n]$.
D: Not invertible because the inputs $x[n] = 1$ for all n and $x[n] = 2$ for all n give the same output.
E: Not invertible because the inputs $x[n] = \delta[n]$ and $x[n] = \delta[n-1]$ give the same output.

$x[n] = 1$ for all $n \rightarrow y[n] = 0$

$x[n] = 10$ for all $n \rightarrow y[n] = 0$

Not invertible, D

E $x(n) = u(n)$

$y(n) = u(n) - u(-n)$

$x[n] = u[-n]$

$y(n) = u[-n] - u(n)$

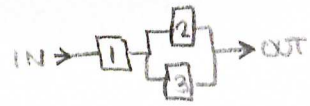
2 diff inputs provide 2 diff outputs

E $x[n] = \delta[n]$

$y[n] = \delta[n] - \delta[-n] = 0$

$x[n] = \delta(n-1]$

$y[n] = \delta[n-1] - \delta[-n-1]$



Question 3: Consider a composite system in which System 1 is in series with the parallel combination of Systems 2 and 3. The systems have respective impulse responses $h_1[n] = \delta[n]+\delta[n-2]$, $h_2[n] = \delta[n]+2\delta[n-1]$, and $h_3[n] = \delta[n]-\delta[n-2]$. Determine the impulse response of the composite system. The overall impulse response is denote $h[n]$. Determine $h[2]+h[3]+h[4]$. Choose the closest answer.

A: 0.
B: 3.
C: 4.
D: 2.
E: 1.

$S1 * (S2 + S3)$

$h_2[n] + h_3[n]$

$= \delta[n] + 2\delta[n-1] + \delta[n] - \delta[n-2]$

$= 2\delta[n] + 2\delta[n-1] - \delta[n-2]$

$(h * h')[n] = h_1[n](2\delta[n] + 2\delta[n-2] - \delta[n-2])$

$h_1 * ()$ $= 2h_1[n] + 2h_1[n-2] - h_1[n-2]$

$h[2] = 2h_1[2] + 2h_1[1] - h_1[0]$

$h_1[0] = \delta[2] + \delta[0] = 1$

$h_1[1] = \delta[1] + \delta[-1] = 0$

$h_1[0] = \delta[0] + \delta[-2] = 1$

$h[2] = 1$

$h[3] = 2h_1[3] + 2h_1[2] - h_1[1] = 0 + 2 + 0 = 2$

$h[4] = 2h_1[4] + 2h_1[4] - h_1[4] = 0 + 0 - 1$

$h[4] = -1$

$h[2] + h[3] + h[4] = 2$

Question 4: Suppose $a(t)$, $b(t)$, and $c(t)$ are signals such that $[a*a](t) = b(t)$ and $[a*b](t) = c(t)$. Determine the convolution of $a(t-1) - b(t)$ with $2a(t-2)$. The answer has the form $Ab(t-B) + Cc(t-D)$. What is $A+B+C+D$? Choose the closest answer.

A: 7.
B: 5.
C: 6.
D: 8.
E: 9.

$(a*a)(t) = b(t) \longrightarrow a(t-1) * a(t) = b(t-1)$, $a(t-1) * a(t-2) = b(t-1-2) = b(t-3)$

$(a*b)(t) = c(t) \longrightarrow (b*a)(t) = c(t)$

$b(t) * a(t-2) = c(t-2)$

$(a(t-1) - b(t)) * 2a(t-2)$

$= 2a(t-1) * a(t-2) - 2b(t) * a(t-2)$

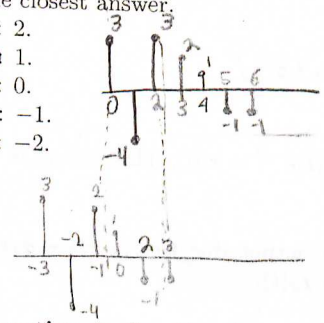
$A = 2$ $C = -2$

$B = 3$ $D = 2$

5

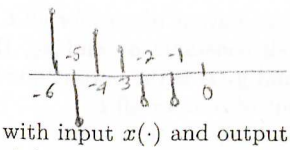
Question 5: Consider a LTI system with impulse response $h[n]$ and periodic input $\tilde{x}[n]$ with $N_0 = 3$ such that the convolution of the impulse response with the fundamental cycle of the input is $(x*h)[n] = 3\delta[n] - 4\delta[n-1] + 3\delta[n-2] + 2\delta[n-3] + \delta[n-4] - \delta[n-5] - \delta[n-6]$. If the output is $\tilde{y}[n]$, determine $\tilde{y}[0]$ and $\tilde{y}[1]$. What is the sum of these two values? Choose the closest answer.

A: 2.
B: 1.
C: 0.
D: −1.
E: −2.



$\tilde{y}[n] = \sum_{k=-\infty}^{\infty} y[n - kN_0]$

$\sum_{k=-\infty}^{\infty} y[n-3k]$



$\tilde{y}[1] = \sum_{k=-\infty}^{\infty} y[1-3k] = 4 - 3 = -3$

$\tilde{y}[0] = 3 + 2 - 1 = 4$

$-3 + 4 = \boxed{1}$

Question 6: Consider the system with input $x(\cdot)$ and output $y(\cdot)$ such that $y(t) = (\frac{t+1}{t+2})x(t)$. This system is

A: Stable because the input $x(t) = 0$ for all t gives a bounded output.
B: Unstable because the input $x(t) = u(t+1)$ gives an unbounded output. ← Not a proof
C: Stable because $|x(t)| \le A$ for all t implies that $|y(t)| \le B$ for all t for $B = A$.
D: Unstable because the input $x(t) = 1$ for all t gives an unbounded output.
E: Unstable because the input $x(t) = t+2$ for all t gives an unbounded output.

$Y(t) = \frac{t+1}{t+2} x(t)$

$X(t) = A$

$t = -2$

$y(-2) = \frac{-2+1}{-2+2}$ $A \not\leq \infty$ UNSTABLE

$y(t)$ is undefined at $t = -2$

Question 7: Consider an LTI system with impulse response $h[n] = 4\delta[n] + 5u[n-10]$. This system is

A: Not causal, not stable, and not memoryless.
B: Causal and stable but not memoryless.
C: Causal, stable, and memoryless.
D: Stable but not causal and not memoryless.
E: Causal but not stable and not memoryless.

$h[n] = k\delta[n] \rightarrow$ memoryless

$h[n] = 0 \quad n<0 \rightarrow$ Causal

$\sum_{n=-\infty}^{\infty} |h[n]| < \infty \rightarrow$ Stable

$h[n] = 4\delta[n] + 5u[n-10]$ Not mem. Causal

$\sum_{n=\infty}^{\infty} 4\delta[n] + \sum_{m=-\infty}^{\infty} 5u[n-10] = \infty$

Not Stable

Question 8: Consider the system $y[n] = 2^n(x[n] - x[n-2])x[n-1]$. Let $g[n]$ be the step response of this system. Determine $g[0] + g[1] + g[2]$. Choose the closest answer.

A: 2.
B: 1.
C: 5.
D: 3.
E: 4.

$g[n] = 2^n(u[n] - u[n-2])u[n-1]$

$g[0] = 0$

$g[1] = 2^1(\cancel{u[1]}^{1} - \cancel{u[-1]}^{0})\cancel{u[0]}^{1} = 2$

$g[2] = 0$

$= 2$

Question 9: Consider the system $y[n] = x[n-2]\cos(x[n+3])$. This system is

A: Time-varying because the output due to $x[n] = \frac{\pi}{2}u[n-1]$ is not a delay by 1 of the output due to $x[n] = \frac{\pi}{2}u[n]$.
B: Time-invariant because, for all inputs $x[\cdot]$ and all values n_0, the output due to $x[\cdot - n_0]$ is a delay by n_0 of the output due to $x[\cdot]$.
C: Time-invariant because $x[n-2]$ is a delay by 5 of $x[n+3]$.
D: Time-varying because the output due to $x[n] = u[n-1]$ is not a delay by 1 of the output due to $x[n] = u[n]$.
E: Time-varying because the output due to $x[n] = nu[n-1]$ is a delay by 1 of the output due to $x[n] = nu[n]$.

i) $x(\cdot) \rightarrow x(\cdot - n_0)$

$x[n-n_0-2]\cos(x[n-n_0+3])$

ii) $n \rightarrow n-n_0$

$x[n-n_0-2]\cos(x[n-n_0+3])$

TIME - Invariant

Question 10: Consider the system with input $x(\cdot)$ and output $y(\cdot)$ such that $y(t) = x(t)e^{-t^2} - x(0)$. This system is

A: Linear because $y(t) \leq x(t)$ for all t. → NOT A PROOF
B: Nonlinear because an input of $x(t) = e^{t^2}$ for all t gives an output of zero for all t.
C: Linear because, for all inputs $x_1(t)$ and $x_2(t)$ and all constants a_1 and a_2, the output due to $a_1x_1(t) + a_2x_2(t)$ is $a_1y_1(t) + a_2y_2(t)$, where $y_1(t)$ is the output due to $x_1(t)$ and $y_2(t)$ is the output due to $x_2(t)$.
D: Linear because an input of zero for all t gives an output of zero for all t.
E: Nonlinear because, for the input $x(t) = e^{t^2}$ for all t, doubling the input does not double the output.

i) $x(t) \rightarrow a_1x_1(t) + a_2x_2(t)$

$(a_1x_1(t) + a_2x_2(t))e^{-t^2} - (a_1x_1(0) + a_2x_2(0))$

ii) $a_1x_1e^{-t^2} - a_1x_1(0) + a_2x_2(t)e^{-t^2} - a_2x_2(0)$

Question 11: As part of a convolution problem, you are required to compute $\int_0^\infty (2te^{-\tau} + 2e^{-2\tau})d\tau$. Determine the answer to this integral in this case. The answer has the form $A + Bt + Ce^{-t} + De^{-2t}$. What is $A + B + C + D$? Choose the closest answer.

A: 3.
B: 4.
C: 2.
D: 1.
E: 5.

$\int_0^\infty (2te^{-\tau} + 2e^{-2\tau})d\tau$

$2t\int_0^\infty e^{-\tau}d\tau + 2\int_0^\infty e^{-2\tau}d\tau \rightarrow$ Calculator

LOSE POINTS IF NO "WORK"!

A 1 B 2 C 0 D 0

Question 12: Consider the system with input $x(\cdot)$ and output $y(\cdot)$ such that $y(t) = x(0) + e^{t+2}x(t)$. Is this system memoryless? Causal?

A: Neither memoryless nor causal.
B: There is insufficient information to determine an answer.
C: Memoryless but not causal.
D: Memoryless and causal.
E: Causal but not memoryless.

depends on past

$t = -2$

$y(-2) = x(0) + e^0 x(-2)$

Neither memoryless or causal

$\boxed{A}$

Serial Number: **301** Name: SOLUTIONS

ECE 3300 SPRING 2016 (SIGNALS, SYSTEMS, AND TRANSFORMS): EXAM III

Record your name on this test; record your name, student ID, and test serial number on the scantron. Enter the test serial number in *COURSE*; you may leave *SECTION* blank. You must show your work on every problem, showing all steps on your test. Do not use scratch paper or write your work anywhere but on the test. The examination lasts 50 minutes and you may use three sheets of notes (front and back); no old test questions can be on your notes. Calculator use is permitted. There is one correct answer per question. In problems asking to find coefficients A, B, C, etc., some of these coefficients may equal zero.

Question 1: Suppose $|X(j\omega)| = \begin{cases} \sqrt{1+3\omega^2} & \text{if } 0 < |\omega| \le 1 \\ 2 & \text{if } 1 < |\omega| < 2 \\ 0 & \text{otherwise} \end{cases}$.

Determine the percentage of energy in the frequency band $(0, 1)$. Choose the closest answer.

A: 70%.
B: 60%.
C: 30%.
D: 40%.
E: 50%.

$$\frac{\int_0^1 (1+3\omega^2)d\omega}{\int_0^1 (1+3\omega^2)d\omega + \int_1^2 4 d\omega} = \frac{2}{6} \times 100\% = 33.3\%$$

$1+1+4$

Question 2: Suppose $\tilde{x}(t)$ has Fourier transform $\tilde{X}(j\omega) = 4\pi\delta(\omega+\pi) - \overset{2\pi X_1}{2\pi\delta(\omega+\frac{\pi}{2})} + 4\pi\delta(\omega) - 2\pi\delta(\omega-\frac{\pi}{2}) + \overset{2\pi X_2}{4\pi\delta(\omega-\pi)}$. Determine the Fourier series coefficients x_1 and x_2 and the fundamental period T_0. What is $x_1 + x_2 + T_0$? Choose the closest answer.

$-1 \quad 2 \quad 4$

A: 4.
B: 3.
C: 5.
D: 7.
E: 6.

$$\tilde{X}(j\omega) = 2\pi \sum_{k=\infty}^{\infty} X_k \delta\left(\omega - k\frac{2\pi}{T_0}\right)$$

$$\frac{k\pi}{2} = \frac{k2\pi}{T_0} = T_0 = 4$$

$\cancel{X_1 = 2}$ $X_1 = -1$ $X_2 = 2$

Answers to Test 301: CCEA BAAD EAAC

SPRING 2016

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Question 3: Suppose $x[n]$ is a real, odd signal, and suppose $X(e^{j\omega}) = \frac{j\sin(\pi\omega)}{1-\omega^2}$. Determine $\angle X(j\omega)$ at $\omega = \frac{1}{2}$ (call this value A), $\omega = \frac{3}{2}$ (call this value B), and $\omega = \frac{5}{2}$ (call this value C). What is $A + 2B + 3C$? Choose the closest answer. *Hint:* A, B, and C should each be represented in the range $-\pi \le \angle X(j\omega) < \pi$. Also, read carefully; the problem is not asking for $A + B + C$.

A: π.
B: $\pi/2$.
C: 2π.
D: $3\pi/2$.
E: 0.

$\measuredangle X(e^{jw})$ @ $w = 1/2, 3/2, 5/2$

$\frac{X(e^{jw})}{j} > 0 \Rightarrow \measuredangle = 0$

$\quad < 0 \rightarrow \measuredangle = -$

w	$X(e^{jw})$	$\measuredangle$
$1/2$	$j\,4/3$	$\pi/2$
$3/2$	$\frac{j(-1)}{-?}$	$\pi/2$
$5/2$	$\frac{j(1)}{-?}$ jw	$-\pi/2$

$\frac{j(-1)}{-?} \leftarrow$

$\frac{\pi}{2} + 2\cdot\frac{\pi}{2} + 3\left(-\frac{\pi}{2}\right)$

Question 4: Determine the Fourier transform of $2\delta[n+2] - 2\delta[n-2]$. The answer can be written in the form $A\cos(B\omega) + Cj\sin(B\omega)$, where $B > 0$. What is $A + B + C$? Choose the closest answer.

A: 6.
B: 10.
C: 4.
D: 2.
E: 8.

$X(e^{jw}) = 2e^{2jw} - 2e^{-2jw}$

$\delta[n] \leftrightarrow 1$

$x[n-n_0] \leftrightarrow X(e^{jw})e^{-jwn_0}$

$= 2\cdot 2j\sin(2w)$
$= 4j\sin(2w)$

$\frac{e^{j\theta} - e^{-j\theta}}{2j} = \sin\theta \rightarrow e^{j\theta} - e^{-j\theta} = 2j\sin\theta$

$e^{j\theta} = \cos\theta + j\sin\theta$
$e^{-j\theta} = \cos\theta - j\sin\theta$

Question 5: Suppose $x(t)$ has Fourier transform $\frac{2+3j\omega}{(1-2j\omega)^2}$. Determine the Fourier transform of $x(t) - 2\frac{d}{dt}x(t)$. The answer has the form $\frac{A+Bj\omega}{1+Cj\omega}$. Determine $A + B + C$. Choose the closest answer.

A: 1.
B: 3.
C: 4.
D: 2.
E: 5.

$x(t) - 2\frac{d}{dt}x(t) \leftrightarrow \frac{(1-2jw)(2+3jw)}{(1-2jw)^2} = \frac{2+3jw}{1-2jw}$

$2 + 3 - 2$
$5 - 2$
3

Question 6: Suppose $\tilde{x}[n]$ has fundamental cycle $x[n]$ and fundamental period $N_0 = 6$. If the Fourier transform of the fundamental cycle is $X(e^{j\omega}) = \frac{36}{1+2\cos^2(\omega)}$, determine the Fourier series coefficients x_k. What is $x_0 + x_1$? Choose the closest answer.

A: 6.
B: 7.
C: 9.
D: 10.
E: 8.

$x_k = \frac{1}{N_0} X\left(e^{jk\frac{2\pi}{N_0}}\right) \rightarrow = \frac{1}{6}\,\frac{36}{1+2\cos^2\left(k\frac{2\pi}{6}\right)}$

Complex signal section of Ch. 2

$= \frac{6}{1+2\cos^2\left(k\frac{\pi}{6}\right)}$

$X_0 = \frac{6}{1+2} = 2$

$X_1 = \frac{6}{1+2(1/4)} = \frac{6}{3/2} = 4$

$4 + 2 = \boxed{6}$

Question 7: In this problem use without proof the fact that $\sum_{n=0}^{\infty} \frac{a^n}{n!} = e^a$ for all a, even if complex-valued. Using the definition of the Fourier transform, determine $X(e^{j\omega})$ if $x[n] = \frac{(\frac{1}{2})^n}{n!}u[n]$. The answer can be written in the form $e^{(Ae^{Bj\omega})}$. Determine $A + B$. Choose the closest answer.

A: −0.5.
B: −2.5.
C: −1.5.
D: −1.0.
E: −2.0.

$$X(e^{j\omega}) = \sum_{n=0}^{\infty} x[n]e^{-j\omega n} \qquad x[n] = \frac{(\frac{1}{2})^n}{n!}u[n] \qquad \left(\frac{1}{2}e^{-jw}\right)^n$$

$$\sum_{n=0}^{\infty} \frac{a^n}{n!} = e^{a}$$

$$X(e^{jw}) = \frac{(\frac{1}{2})^n}{n!} e^{-jwn}$$

$$= \sum_{n=0}^{\infty} \frac{(\frac{1}{2}e^{-jw})}{n!} = e^{\frac{1}{2}e^{-jw}}$$

Question 8: Determine the Fourier transform of $((\frac{1}{2})^n + (-\frac{1}{2})^n)^2 u[n]$. The answer has the form $\frac{A}{1-Be^{-j\omega}} + \frac{C}{1-De^{-j\omega}}$. What is $A + B + C + D$? Choose the closest answer. *Hint:* Expand the square.

A: 5.
B: 1.
C: 2.
D: 4.
E: 3.

$$\left(\frac{1}{2}\right)^{2n} + \left(-\frac{1}{2}\right)^{2n} + 2\left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^n \Big] u[n]$$

?

$$= 2\left(\frac{1}{4}\right)^n u[n] + 2\left(-\frac{1}{4}\right)^n u[n]$$

$$a^n u[n] = \frac{1}{1-ae^{-jw}} \qquad X(e^{jw}) = \frac{2}{1-\frac{1}{4}e^{-jw}} + \frac{2}{1+\frac{1}{4}e^{-jw}}$$

Question 9: Given that $\frac{\sqrt{2}}{2}e^{-\sqrt{2}|t|}$ has Fourier transform $\frac{2}{2+\omega^2}$, determine the Fourier transform of $\frac{\sqrt{2}}{2}te^{-\sqrt{2}|t|}$. The answer has the form $\frac{Aj\omega+B}{\omega^4+C\omega^2+D}$. Determine $A + B + C + D$. Choose the closest answer.

A: 12.
B: 8.
C: 6.
D: 10.
E: 4.

$$\frac{\sqrt{2}}{2}e^{-\sqrt{2}|t|} \longleftrightarrow \frac{2}{2+w^2}$$

$$\frac{\sqrt{2}}{2}te^{-\sqrt{2}|t|} \longleftrightarrow Y \frac{d}{dw}\left(\frac{2}{2+w^2}\right)$$

Question 10: Suppose $x(t) = 4e^{-t}u(t) - 3e^{-2t}u(t)$. Determine the magnitude of the Fourier transform. The answer has the form $|X(j\omega)| = \frac{\sqrt{A+\omega^2}}{\sqrt{B+\omega^2}\sqrt{C+\omega^2}}$. Determine $A/(B + C)$. Choose the closest answer. *Hint:* Read carefully. This is not asking for $A + B + C$.

A: 5.
B: 2.
C: 1.
D: 4.
E: 3.

$$X(jw) = \frac{4}{1+jw} - \frac{3}{2+jw}$$

$$= \frac{4(2+jw) - 3(1+jw)}{(1+jw)(2+jw)} = \frac{5+jw}{(1+jw)(2+jw)}$$

$$|X(jw)| = \frac{\sqrt{25+w^2}}{\sqrt{1+w^2}\sqrt{4+w^2}}$$

Question 11: Suppose $\tilde{x}(t)$ has period $T_0 = 2$ and Fourier series coefficients $x_2 = 2-j$, $x_{-2} = 2+j$, $x_3 = 2j$, and $x_{-3} =$ all other x_ks are zero. Determine the percentage of power in the second harmonic. Choose the closest answer.

A: 55%.
B: 75%.
C: 65%.
D: 70%.
E: 60%.

$$\frac{|X_2|^2 + |X_{-2}|^2}{|X_{-2}|^2 + |X_3|^2 + |X_2|^2 + |X_{-3}|^2}$$

$|X_2| = \sqrt{2^2+(-1)^2} = \sqrt{5}$

$|X_{-2}| = \sqrt{5}$

$|X_3| = \sqrt{0^2+2^2} = 2$

$|X_{-3}| = 2$

$$= \cancel{\frac{5+5}{5+2+5+2}} = \frac{10}{14} * 100\% \rightarrow 71.43\%$$

Question 12: Determine the inverse Fourier transform $x(t)$ if $X(j\omega) = e^{j\omega}\text{rect}(\frac{\omega}{2})$. The answer has the form $\frac{1}{\pi(t-B)}\sin(A(t-B))$. What is $A+B$? Choose the closest answer.

A: -2.
B: 1.
C: 0.
D: -1.
E: 2.

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-1}^{1} \underbrace{e^{j\omega} e^{j\omega t}}_{e^{j\omega(1+t)}}\, d\omega$$

$$\frac{1}{2\pi}\frac{1}{j(1+t)}\left(e^{j(1+t)(1)} - e^{j(t+1)(-1)}\right)$$

$$\frac{1}{2\pi}\frac{1}{j(1+t)}\, 2j\sin(1+t) = \frac{1}{\pi}\frac{\sin(1+t)}{1+t}$$

Answers to Test 301: CDEC DABD CECE

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FALL 2015

Serial Number: **301**

Name: SOLUTIONS

ECE 3300 FALL 2015 (SIGNALS, SYSTEMS, AND TRANSFORMS): EXAM III

Record your name on this test; record your name, student ID, and test serial number on the scantron. Enter the test serial number in *COURSE*; you may leave *SECTION* blank. You must show your work on every problem, showing all steps on your test. Do not use scratch paper or write your work anywhere but on the test. The examination lasts 50 minutes and you may use three sheets of notes (front and back); no old test questions can be on your notes. Calculator use is permitted. There is one correct answer per question.

Question 1: In this problem use without proof the fact that $\sum_{n=0,n \text{ even}}^{\infty} a^n = \frac{1}{1-a^2}$ if $|a| < 1$. Using the definition of the Fourier transform, determine $X(e^{j\omega})$ if $x[n] = (\frac{1}{2})^n$ if $n \geq 0$ and n is even, and $x[n] = 0$ otherwise. The answer can be written in the form $\frac{A}{B-e^{-Cj\omega}}$. Determine $A + B + C$. Choose the closest answer.

A: 9.
B: 6.
No C: 10.
D: 8.
E: 7.

$$x(n) = \begin{cases} (\frac{1}{2})^n & \text{if } n \geq 0, \text{even} \\ 0 & \text{other} \end{cases} \qquad = \frac{1}{1-\frac{1}{4}e^{-2jw}}$$

$$X(e^{jw}) = \sum_{n=\infty}^{\infty} x[n]e^{-jwn} = \sum_{n=0}^{\infty} (1/2)^n e^{-jwn} \rightarrow \sum_{n=0}^{\infty} \left(\frac{1}{2}e^{-jw}\right)^n = \frac{1}{1-(\frac{1}{2}e^{-jw})^2}$$

Question 2: Given that $x(t)$ has Fourier transform $\frac{1}{(1+\omega^2)^2}$, determine the Fourier transform of $tx(t)$. The answer has the form $\frac{A+Bj\omega}{(1+\omega^2)^C}$. What is $A + B + C$? (Some of these coefficients may equal zero.) Choose the closest answer.

A: 0.
B: -2.
C: 2.
D: -1.
E: 1.

$$x(t) \leftrightarrow \frac{1}{(1+w^2)^2}$$

$$tx(t) \leftrightarrow j\frac{d}{dw}(1+w^2)^{-2}$$

$$= j(-2)(1+w^2)^{-3}\,2w$$

$$= \frac{-4jw}{(1+w^2)^3} \qquad 0 - 4 + 3 = -1$$

Question 7: Suppose $\tilde{x}(t)$ has fundamental cycle $x(t) = 2\pi\cos(t)\text{rect}(\frac{t}{\pi})$ and fundamental period $T_0 = 2\pi$. Determine the Fourier series coefficients x_k. What is $x_0 + x_2$? Choose the closest answer. *Hint:* You may use without proof the fact that the Fourier transform of the fundamental cycle is $X(j\omega) = \frac{4\pi\cos(\pi\omega/2)}{1-\omega^2}$.

A: 1.
B: 3. ✓
C: 5.
D: 2.
E: 4.

$$X(j\omega) = \frac{4\pi\cos\left(\frac{\pi\omega}{2}\right)}{1-\omega^2}$$

$$x_0 = \frac{2\cos 0}{1} = 2$$

$$x_2 = \frac{2\cos\pi}{1-4} \quad \frac{2\cdot -1}{-3} = \frac{2}{3}$$

$$2 + \frac{2}{3} \approx 3$$

$$x_k = \frac{1}{T_0}X\left(jk\frac{2\pi}{T_0}\right) \rightarrow \frac{1}{2\pi}X(jk) \quad w = k \quad = \frac{2\cos\left(\frac{\pi k}{2}\right)}{1-k^2}$$

Question 8: Suppose $x[n] = \delta[n] - \frac{1}{2}(\frac{1}{2})^n u[n]$. Determine the magnitude of the Fourier transform. The answer has the form $|X(e^{j\omega})| = \frac{\sqrt{A - B\cos(\omega)}}{\sqrt{C - \cos(\omega)}}$. Determine $A + B + C$. Choose the closest answer.

A: 3.
B: 1.
C: 4.
D: 2.
E: 5.

Find F.T.:

$$X(e^{jw}) = 1 - \frac{\frac{1}{2}}{1-\frac{1}{2}e^{-jw}} \rightarrow \frac{1-\frac{1}{2}e^{-jw}-\frac{1}{2}}{1-\frac{1}{2}e^{-jw}} = \frac{\frac{1}{2}-\frac{1}{2}e^{-jw}}{1-\frac{1}{2}e^{-jw}}$$

$$= \frac{\frac{1}{2}(1-e^{-jw})}{1-\frac{1}{2}e^{-jw}}$$

$$|X(e^{jw})| = \frac{\frac{1}{2}\sqrt{2-2\cos\omega}}{\sqrt{\frac{5}{4}-\cos\omega}} = \frac{\sqrt{\frac{1}{2}-\frac{1}{2}\cos\omega}}{\sqrt{\frac{5}{4}-\cos\omega}}$$

$$\frac{1}{2}+\frac{1}{2}+\frac{5}{4}$$

$$1 + 1.2 = 2.2$$

Question 9: Suppose $x(t) = f(t)u(t)$ has Fourier transform $X(j\omega) = \frac{1+j\omega}{2+j\omega}$. Determine the Fourier transform of $y(t) = f(|t|)$. The answer has the form $\frac{A+B\omega^2}{C+\omega^2}$. Determine $A + B + C$. Choose the closest answer. *Hint:* Note that $y(t) = x(t) + x(-t)$. monste hint

A: 8.
B: 7.
C: 10. ✓
D: 9.
E: 6.

$$Y(jw) = \frac{1+jw}{2+jw} + \frac{1-jw}{2-jw} = \frac{(1+jw)(2-jw)+(1-jw)(2+jw)}{(2+jw)(2-jw)}$$

← All jw terms cancel!

$$= \frac{4+2w^2}{4+w^2}$$

$$4 + 2 + 4 = 10$$

Question 10: Suppose $|X(j\omega)| = \begin{cases} 3 & \text{if } 0 < |\omega| \le 1 \\ 2 & \text{if } 2 < |\omega| < 6 \\ 0 & \text{otherwise} \end{cases}$.

Determine the percentage of energy in the frequency band $(1, 5)$. Choose the closest answer.

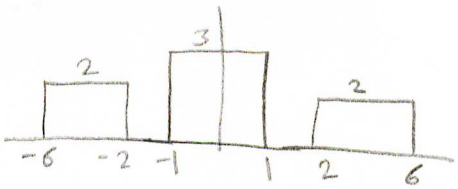
A: 10%.
B: 30%.
C: 20%.
D: 40%.
E: 50%. ✓

Formula:

$$E_x = \frac{1}{2\pi}\int_0^\infty |X(jw)|^2\,dw$$

$$E_{[1,5]} = \frac{1}{\pi}\int_1^5 |X(jw)|^2\,dw$$

$$\frac{\int_2^5 4\,dw}{\int_0^1 9\,dw + \int_2^6 4\,dw} = \frac{12}{9+16} = 48\%$$



all properties
~~all~~ table transforms

Question 3: Determine the inverse Fourier transform $x[n]$ if $X(e^{j\omega}) = (1-2e^{-j\omega})^2$. What is $x[1]+x[2]$? Choose the closest answer. *Hint:* Expand the square.

$= 1 - 4e^{-jw} + 4e^{-2jw}$

A: -2.
B: -1.
C: 1.
D: 2.
E: 0.

$\delta[n] \leftrightarrow 1$

$x[n] = \delta[n] - 4\delta[n-1] + 4\delta[n-2]$

$\uparrow$ $x(1)$ $\quad$ $\uparrow$ $x(2)$ $\quad = 4 - 4 = 0$

Question 4: Suppose $\tilde{x}(t)$ has Fourier series coefficients $x_k = \frac{1}{\sqrt{(|k|+1)(|k|+2)}}$ for all k. Determine the mean-square error between $\tilde{x}(t)$ and $\tilde{x}_2(t) = \sum_{k=-2}^{2} x_k e^{jk2\pi t/T_0}$ as a percentage of the total power $P_{\tilde{x}}$. Choose the closest answer. *Hint:* You may use without proof the fact that $P_{\tilde{x}} = \frac{3}{2}$.

NOTES: MSE equations

A: 40%.
B: 10%.
C: 30%.
D: 50%.
E: 20%.

$MSE_{\tilde{x}_1, \tilde{x}_2} = \tilde{P}_x - |X_0|^2 - 2|X_1|^2 - 2|X_2|^2$

$\frac{MSE}{P_x} * 100\% = \cancel{30}$ 30%

$= \frac{3}{2} - \frac{1}{2} - 2\left(\frac{1}{6} - \frac{1}{12}\right)$

$= \frac{1}{2}$

Question 5: Given that $\sqrt{2}e^{-\sqrt{2}|t|}$ has Fourier transform $\frac{4}{2+\omega^2}$, determine the Fourier transform of $\int_{-\infty}^{t} \sqrt{2}e^{-\sqrt{2}|\tau|}d\tau$. The answer has the form $\frac{4}{j(A+B\omega+C\omega^2+D\omega^3)} + E\pi\delta(\omega)$. (Some of these coefficients may equal zero.) Determine $A+B+C+D+E$. Choose the closest answer.

$X(jw)$

A. 7.
B: 9.
C: 6.
D: 5.
E: 8.

$\int_{-\infty}^{t} \sqrt{2}e^{-\sqrt{2}|\tau|} d\tau \longleftrightarrow \frac{1}{jw} X(jw) + \pi X(jw)\delta[w]$

$= \frac{4}{jw(2+w^2)} + 2\pi\delta[w]$

$A = 2 \quad C = 1 \quad E = 2$

Question 6: Given that, for $-1 < a < 1$, $a^n u[n]$ has Fourier transform $\frac{1}{1-ae^{-j\omega}}$, determine the Fourier transform of $x[n] = (\frac{1}{2})^{n-2}u[n-1]$. What is $X(e^{j0})$? Choose the closest answer. *Hint:* $(\frac{1}{2})^{n-2} = (\frac{1}{2})^{n-1}(\frac{1}{2})^{-1}$.

A: 4.
B: 5.
C: 3.
D: 1.
E: 2.

$= (\frac{1}{2})^{n-1} u[n-1] \times (1/2)^{-1}$

$= 2(1/2)^{n-1} u[n-1] \longleftrightarrow 2 \frac{1}{1-\frac{1}{2}e^{-jw}} e^{-jw}$

Question 11: Suppose $x(t)$ is a real, even signal, and suppose $X(j\omega) = \frac{(1-\omega^2)(4-\omega^2)}{9-\omega^2}$. At what ranges of frequencies does $\angle X(j\omega)$ equal zero?

A: $1 < |\omega| < 2$ and $|\omega| > 3$ only.
B: $1 < |\omega| < 2$ only.
C: $|\omega| < 1$ and $2 < |\omega| < 3$ only.
D: $2 < |\omega| < 3$ only.
E: $|\omega| < 1$ and $|\omega| > 3$ only.

$$\angle X(j\omega) = \begin{cases} 0 & \text{if } X(j\omega) > 0 \\ -\pi & \text{if } X(j\omega) < 0 \end{cases}$$

$x(t)$ is real + odd:

NOTES

$$\angle X(j\omega) = \begin{cases} \frac{\pi}{2} & \text{if } \frac{X(j\omega)}{j} > 0 \\ -\frac{\pi}{2} & \frac{X(j\omega)}{j} < 0 \end{cases}$$

TINY TABLE

ω	X	$\phi\omega$
$0<\omega<1$	$\frac{+}{+}\frac{+}{+} \Rightarrow X$ is $(+)$	0
$1<\omega<2$	$\frac{-}{+}\frac{+}{+} \rightarrow X$ is $(-)$	$-\pi$
$2<\omega<3$	$\frac{-}{+}\frac{-}{-} \rightarrow X$ is $(+)$	0
$\omega>3$	$\frac{-}{-}\frac{-}{-} \rightarrow X$ is $(-)$	$-\pi$

frequencies are zero at these points

Question 12: Suppose $\tilde{x}(t)$ has Fourier transform $\tilde{X}(j\omega) = \sum_{k=-6}^{6} \frac{4\pi}{k^2-2}\delta(\omega - k\pi)$. Determine the Fourier series coefficient x_2 and the fundamental period T_0. What is $x_2 + T_0$? Choose the closest answer.

Last section of chapter!

A: 4.
B: 5.
C: 6.
D: 7.
E: 3.

match up

$$\tilde{X}(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} X_k \, \delta\left(\omega - \frac{k2\pi}{T_0}\right)$$

$$k\pi = \frac{k2\pi}{T_0} \qquad T_0 = 2 \qquad \frac{4\pi}{2^2-2} = 2\pi X_2$$

$$2\pi = 2\pi X_2 \qquad X_2 = 1$$

π π π π π π

Fall 2016
SOLUTIONS

Answers to Test 301: BCBE DEDB DEDB

Serial Number: **301** Name: SOLUTIONS

ECE 3300 FALL 2016 (SIGNALS, SYSTEMS, AND TRANSFORMS): EXAM III

Record your name on this test; record your name, student ID, and test serial number on the scantron. Enter the test serial number in *COURSE*; you may leave *SECTION* blank. You must show your work on every problem, showing all steps on your test. Do not use scratch paper or write your work anywhere but on the test. Circle your answers on the test and bubble in the corresponding answers on your scantron. The examination lasts 60 minutes and you may use three sheets of notes (front and back); no old test questions can be on your notes. Calculator use is permitted. There is one correct answer per question. In problems asking to find coefficients A, B, C, etc., some of these coefficients may equal zero.

Question 1: Determine $x[n]$ if $X(e^{j\omega}) = \frac{4e^{-2j\omega}}{2+e^{-j\omega}}$. The answer has the form $A(\frac{1}{B})^{n-C}u[n-C]$. What is $A+B+C$? Choose the closest answer. *Hint:* Work from the fact that, if $-1 < a < 1$, $a^n u[n] \leftrightarrow \frac{1}{1-ae^{-j\omega}}$.

A: 10.
B: 2.
C: 8.
D: 4.
E: 6.

$$4e^{-2j\omega}\frac{1}{2+e^{-j\omega}} \Rightarrow 4e^{-2j\omega}\cdot\frac{1/2}{2/2+1/2e^{j\omega}} \Rightarrow 2e^{-2j\omega}\frac{1}{1-(-1/2)e^{-j\omega}} \rightarrow \text{Now apply properties}$$

Time Shift Property:

$$x[n-n_0] \leftrightarrow e^{-j\omega n_0}X(e^{j\omega})$$

Time Scaling as well?

$$2x[n-2] = 2\left(-\frac{1}{2}\right)^{n-2}u[n-2]$$

A 2 B -2 C 2

Σ: 2 B

Question 2: Suppose $\tilde{x}(t)$ has fundamental cycle $x(t)$ and fundamental period $T_0 = 4\pi$. If the Fourier transform of the fundamental cycle is $X(j\omega) = \frac{4\pi}{\omega^2}\sin^2(\frac{\pi\omega}{2})$, determine the Fourier series coefficient x_1. Choose the closest answer.

A: 6.
B: 8.
C: 2.
D: 10.
E: 4.

$$X_k = \frac{1}{T_0}X\left(j\frac{2\pi k}{T_0}\right)$$

$$X_1 = \frac{1}{4\pi}X\left(j\frac{2\pi\cdot 1}{4\pi}\right) = \frac{1}{4\pi}X\left(j\frac{1}{2}\right)$$

$$= \frac{1}{4\pi}\cdot\frac{4\pi}{(\frac{1}{2})^2}\sin^2\left(\frac{\pi}{2}\cdot\frac{1}{2}\right) = \frac{1}{\frac{1}{4}}\sin^2(\pi/4) = 2 \quad \underline{C}$$

Gives range

Question 3: Suppose $|X(j\omega)| = \sqrt{9-\omega^2}\text{rect}(\frac{\omega}{6})$. Determine the percentage of energy in the frequency band $(0, 1)$. Choose the closest answer. *Hint:* Recall that $\text{rect}(\frac{\omega}{2a}) = 1$ for $-a < \omega < a$ and 0 otherwise.

A: 30%.
B: 50%.
C: 40%.
D: 60%.
E: 70%.

$a = 3$

$\text{rect}(\frac{\omega}{2a}) \rightarrow -a < \omega < a$

① $E_x = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$

$= \frac{1}{2\pi}\int_{-3}^{3}(9-\omega^2)d\omega = \frac{18}{\pi}$

$E[\alpha,\beta] = \frac{1}{2\pi}\int_{-\beta}^{-\alpha}|X(j\omega)|^2 d\omega + \ldots$

ONLY FOR SYMMETRIC! ② $E_{\alpha,\beta} = \frac{1}{\pi}\int_{\alpha}^{\beta}|X(j\omega)|d\omega$

Symmetric Formula

$\frac{26/3\pi}{18/\pi} \approx \boxed{50\% \; B} \leftarrow = \frac{1}{\pi}\int_0^1(9-\omega^2)d\omega = \frac{26}{3\pi}$

Question 4: Suppose $x[n] = (\frac{1}{2})^n u[n] - (\frac{1}{3})^n u[n]$. Determine the magnitude of the Fourier transform. The answer has the form $|X(e^{j\omega})| = \frac{A}{\sqrt{B-\cos\omega}\sqrt{C-\frac{2}{3}\cos(\omega)}}$. What is $A+B+C$? Choose the closest answer.

A: 5.
B: 4.
C: 2.
D: 1.
E: 3.

$X(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}} - \frac{1}{1+\frac{1}{3}e^{-j\omega}}$

$\frac{1-\frac{1}{3}e^{-j\omega} - 1 + \frac{1}{2}e^{-j\omega}}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{3}e^{-j\omega})} \rightarrow \frac{1/6\, e^{-j\omega}}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{3}e^{-j\omega})}$

$|X(e^{j\omega})| = \frac{1/6}{|1-\frac{1}{2}e^{-j\omega}||1-\frac{1}{3}e^{-j\omega}|} = \frac{1/6}{\sqrt{1+\frac{1}{4}-2\cdot\frac{1}{2}\cos\omega}}$

$= \frac{1/6}{\sqrt{\frac{5}{4}-\cos\omega}\sqrt{\frac{10}{9}-\frac{2}{3}\cos\omega}}$ A $1/6$ B $5/4$ C $10/9$ Σ: $2.54 \approx 3$

Question 5: Suppose $x[n]$ is a real, even signal, and suppose $X(e^{j\omega}) = (\frac{1}{2} - \cos\omega)(\frac{\sqrt{3}}{2} + \cos\omega)$. For $0 < \omega < \pi$, in which of the following regions is $\angle X(e^{j\omega})$ equal to zero?

A: $\frac{\pi}{6} < \omega < \frac{2\pi}{3}$.
B: $\angle X(e^{j\omega})$ is never equal to zero over $0 < \omega < \pi$.
C: $0 < \omega < \frac{\pi}{3}$ and $\frac{5\pi}{6} < \omega < \pi$.
D: $\frac{\pi}{3} < \omega < \frac{5\pi}{6}$.
E: $0 < \omega < \frac{\pi}{6}$ and $\frac{2\pi}{3} < \omega < \pi$.

real, even signal → $\angle$ has only 2 value!

$\angle X(e^{j\omega}) = \begin{cases} 0 & X(e^{j\omega}) > 0 \\ -\pi & 0 \; \omega \end{cases}$

Expand: $\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{2}\cos\omega + \frac{1}{2}\cos\omega - \cos^2\omega > 0$

Unusual method of solving...

$\cos^2\omega + 0.366\cos\omega - 0.433\cos^2\omega < 0 \rightarrow -\frac{\pi}{3} < \omega < \frac{\pi}{6}$

$\frac{1}{2} < \cos\omega < -\frac{\sqrt{3}}{2}$

Question 6: Determine the Fourier transform of $(e^{-t} + e^{-2t})^2 u(t)$. The answer has the form $\frac{A}{B+j\omega} + \frac{C}{D+j\omega} + \frac{E}{F+j\omega}$. What is $A+B+C+D+E+F$? Choose the closest answer. *Hint:* Expand the square.

A: 14.
B: 15.
C: 12.
D: 11.
E: 13.

$(e^{-2t} + 2e^{-t}e^{-2t} + e^{-4t})u(t)$

$(e^{-2t} + 2e^{-3t} + e^{-4t})u(t)$

$\frac{1}{2+j\omega} + \frac{2}{3+j\omega} + \frac{1}{4+j\omega}$

A	B	C	D	E	F
1	2	2	3	1	4

13

E

Question 7: Suppose $\tilde{x}(t)$ has period $T_0 = 4$ and Fourier series coefficients $x_k = \frac{k^2}{|k|+4}$. One of the terms in the Fourier transform of $\tilde{x}(t)$ is $A\delta(\omega - 2\pi)$. What is A? Choose the closest answer.

A: π.
B: 3π.
C: 5π.
D: 4π.
E: 2π.

$$X(e^{j\omega}) = 2\pi \sum_{k=-\infty}^{\infty} x_k \delta\left(\omega - \frac{2\pi k}{T_0}\right)$$ ← amount of shift is determined by k

$$2\pi = \frac{2\pi k}{4}$$

$$k = 4$$

$$A\delta(\omega - 2\pi) = 2\pi x_k \delta\left(\omega - \frac{2\pi k}{T_0}\right)$$

$$A = 2\pi x_k$$

$$2\pi \cdot \frac{16}{8} = 4\pi \quad D$$

Question 8: Given that, if $a > 0$, $\sqrt{t}e^{-at}u(t) \leftrightarrow \frac{\sqrt{\pi}}{2}\frac{1}{(a+j\omega)^{3/2}}$, determine the Fourier transform of $\int_{-\infty}^{t} \sqrt{\tau}e^{-2\tau}u(\tau)d\tau$. The answer has the form $\frac{\sqrt{\pi}}{2}\frac{1}{j\omega(2+j\omega)^{3/2}} + \frac{\pi A}{B}\delta(\omega)$. What is $A + B$? Choose the closest answer. applications of properties

A: 8.
B: 7.
C: 6.
D: 9.
E: 10.

$$x(t) = \sqrt{t}e^{-2t}u(t) \leftrightarrow \frac{\sqrt{\pi}}{2} \cdot \frac{1}{(2+j\omega)^{3/2}} = X(j\omega)$$

$$\int_{-\infty}^{t} x(\tau)d\tau = \int_{-\infty}^{t} \sqrt{\tau}e^{-2\tau}d\tau \leftrightarrow \frac{1}{j\omega}X(j\omega) + \pi X(j0)\delta(\omega)$$

$$= \frac{1}{j\omega} \cdot \frac{\sqrt{\pi}}{2} \cdot \frac{1}{(2+j\omega)^{3/2}} + \pi \frac{\sqrt{\pi}}{2} \cdot \frac{1}{2^{3/2}}\delta(\omega) \rightarrow \frac{\pi^{3/2}}{2^{5/2}}\delta(\omega)$$

$A = \frac{3}{2}$

$B = 2^{5/2}$

$= 7$ B

Question 9: Suppose $p[n]$ is a sequence of numbers such that $\sum_{n=1}^{\infty} p[n]a^n = \ln(G(1-a))$, where $G(\cdot)$ is a function. Consider the signal $x[n] = (\frac{1}{2})^n p[n]u[n-1]$. Determine $X(e^{j\omega})$.

A: $\frac{\ln(G(1-e^{j\omega}))}{1-\frac{1}{2}e^{-j\omega}}$.
B: $\ln(G(1-\frac{1}{2}e^{j\omega}))$.
C: $\frac{\ln(G(1-e^{-j\omega}))}{1-\frac{1}{2}e^{-j\omega}}$.
D: $\ln(G(1-\frac{1}{2}e^{-j\omega}))$.
E: $(\frac{1}{2})^n \ln(G(1-e^{j\omega}))$.

$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$\ln\left(G\left(1 - \frac{1}{2}e^{-j\omega}\right)\right) \quad D$$

$$\sum_{-\infty}^{\infty} \left(\frac{1}{2}\right)^n p[n]\underbrace{u[n-1]}_{\text{changed sum range}}e^{-j\omega n}$$

$$\sum_{n=1}^{\infty} p[n]\left(\underbrace{\frac{1}{2}e^{-j\omega}}_{\alpha}\right)^n$$

Can now use given ID...

Question 10: Suppose $x[n]$ has Fourier transform $X(e^{j\omega})$. Determine the Fourier transform of $y[n] = x[n+2] - x[n] + x[n-2]$. The answer has the form $(A\cos(B\omega) + C)X(e^{j\omega})$. What is $A + B + C$? Choose the closest answer.

A: 2.
B: 4.
C: 5.
D: 1.
E: 3.

Time Shift Property

$$e^{2j\omega} = \cos(2\omega) + j\sin(2\omega)$$

$$e^{-2j\omega} = \cos(2\omega) - j\sin(2\omega)$$

$$x[n] \leftrightarrow X(e^{j\omega})$$

$$x[n+2] \leftrightarrow e^{j\omega 2}X(e^{j\omega})$$
$n_0 = -2$

$$x[n-2] \leftrightarrow e^{-2j\omega}X(e^{j\omega})$$
$n_0 = 2$

$$Y(e^{j\omega}) = e^{j\omega}X(e^{j\omega}) - X(e^{j\omega}) + e^{-2j\omega}X(e^{j\omega})$$

$$(e^{2j\omega} + e^{-2j\omega} - 1)X(e^{j\omega})$$

$$(2\cos(2\omega) - 1)X(e^{j\omega})$$

$$\left.\begin{matrix} A & 2 \\ B & 2 \\ C & -1 \end{matrix}\right\} = 3 \quad \boxed{E}$$

Property!

Question 11: Given that $x(t)$ has Fourier transform $\frac{\frac{1}{4}}{(1+2j\omega)^4}$, determine the Fourier transform of $tx(t)$. The answer has the form $\frac{A+jB}{(1+2j\omega)^C}$, where A, B, and C are real. What is $A+B+C$? Choose the closest answer.

A: 9.
B: 6.
C: 8.
D: 7.
E: 10.

$$tx(t) \longleftrightarrow j\frac{d}{d\omega}X(j\omega)$$

$$\frac{1}{4}j\frac{d}{d\omega}\frac{1}{(1+2j\omega)^4}$$

$$\frac{1}{4}j\cdot\frac{2}{(1+2j\omega)^5}$$

A 2
B ~~0~~
C 5
~~E~~

Σ 7 D

Question 12: Suppose $\tilde{x}(t)$ has period $T_0 = \pi$ and Fourier series coefficients $x_k = jk$ if $-4 \le k \le 4$ and $x_k = 0$ otherwise. Determine the percentage of power in the frequency band $[3, 7]$. Choose the closest answer.

A: 30%.
B: 40%.
C: 10%.
D: 50%.
E: 20%.

$$P_x = \sum_{k=-4}^{4} |jk|^2 = \sum_{-4}^{4} k^2 = 60 \text{ Total Power}$$

$[3,7] \rightarrow$ not range on k, but the frequency value!

$$\frac{jk}{T_0} = 3 \qquad \frac{2\pi k}{T_0} = 7 \qquad k = 1.5 \rightarrow k = 3.5$$

INTEGER VALUES:

2 & 3

$$P_{[3,7]} = |X_{-2}|^2 + |X_2|^2 + |X_{-3}|^2 + |X_3|^2$$

$$\frac{26}{60} = 43.33\% \approx 40\% \quad B$$

FALL 2016
SOLUTIONS

401 Answers: EEEB DEDC BACB

Serial Number: **401**

Name: SOLUTIONS
3/29/17

ECE 3300 Fall 2016 (Signals, Systems, and Transforms): Exam IV

Record your name on this test; record your name, student ID, and test serial number on the scantron. Enter the test serial number in *COURSE*; you may leave *SECTION* blank. You must show your work on every problem, showing all steps on your test. Do not use scratch paper or write your work anywhere but on the test. Circle your answers on the test and bubble in the corresponding answers on your scantron. The examination lasts 60 minutes and you may use four sheets of notes (front and back); no old test questions can be on your notes. Calculator use is permitted. There is one correct answer per question. In problems asking to find coefficients A, B, C, etc., some of these coefficients may equal zero.

Slide 66, 8.5

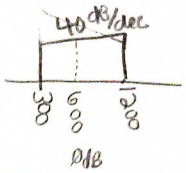
Question 1: Suppose a Bode plot of $|H(j\omega)|$ is a straight line of slope 40 dB/decade between $\omega = 300$ and $\omega = 1200$ such that $|H(j600)| = 0$ dB. What is $|H(j1200)|$? Choose the closest answer.

A: 18 dB.
B: 30 dB.
C: 24 dB.
D: 6 dB.
E: 12 dB.

$20n\log_{10}\left(\frac{\omega_2}{\omega_1}\right)$ DROP IN HEIGHT

$0 + 40\log_{10}\left(\frac{1200}{600}\right)$

12dB



Question 2: Consider a linear time-invariant system with impulse response $h(t) = (e^{-t} - e^{-3t})u(t)$. Determine the magnitude response.

A: $\frac{1}{\sqrt{1+\omega^2}} - \frac{1}{\sqrt{9+\omega^2}}$.
B: $\frac{1}{\sqrt{1+\omega^2}} + \frac{1}{\sqrt{9+\omega^2}}$.
C: $\frac{2\sqrt{4+\omega^2}}{\sqrt{1+\omega^2}\sqrt{9+\omega^2}}$.
D: $\frac{1}{\sqrt{1+\omega^2}\sqrt{9+\omega^2}}$.
E: $\frac{2}{\sqrt{1+\omega^2}\sqrt{9+\omega^2}}$.

$$H(j\omega) = \frac{1}{1+j\omega} - \frac{1}{3+j\omega} \rightarrow \frac{3+j\omega-1-j\omega}{(1+j\omega)(3+j\omega)} = \frac{2}{(1+j\omega)(3+j\omega)}$$

$$\frac{|2|}{\sqrt{1+\omega^2}\sqrt{9+\omega^2}}$$

Question 3: Consider a linear time-invariant system with $H(j\omega) = \frac{(2+j\omega)^3}{(1+j\omega)^2(3+j\omega)}$. Determine the phase response $\angle H(j\omega)$. The answer has the form $A\tan^{-1}(\frac{\omega}{B}) + C\tan^{-1}(\frac{\omega}{D}) + E\tan^{-1}(\frac{\omega}{F})$, where $B > 0$, $D > 0$, and $F > 0$. What is $A + B + C + D + E + F$? Choose the closest answer.

A: 7.
B: 8.
C: 9.
D: 10.
E: 6.

$\angle(a+j\omega) \Leftrightarrow \tan^{-1}(\frac{\omega}{a})$

$3\angle(2+j\omega) - 2\angle(1+j\omega) - \angle(3+j\omega)$

$3\tan^{-1}(\frac{\omega}{2}) - 2\tan^{-1}(\omega) - \tan^{-1}(\frac{\omega}{3})$

A 3 C -2 E -1
B 2 D 1 F 3 $\Sigma: 6$

Question 4: Consider a linear time-invariant system with impulse response $h[n] = (\frac{2}{3})^n u[n]$ and output $y[n] = (\frac{1}{3})^n u[n]$. Use Fourier techniques to determine the input $x[n]$. The answer has the form $A\delta[n] + B(\frac{1}{C})^n u[n]$. What is $A + B + C$? Choose the closest answer.

A: 5.
B: 4.
C: 1.
D: 2.
E: 3.

$H(e^{j\omega}) = \frac{1}{1-\frac{2}{3}e^{-j\omega}}$

$Y(e^{j\omega}) = \frac{1}{1-\frac{1}{3}e^{-j\omega}}$

$X(e^{j\omega}) = \frac{\frac{1}{1-\frac{1}{3}e^{-j\omega}}}{\frac{1}{1-\frac{2}{3}e^{-j\omega}}} = \frac{1-\frac{2}{3}e^{-j\omega}}{1-\frac{1}{3}e^{-j\omega}} \rightarrow \frac{2-\frac{2}{3}e^{-j\omega}-1}{1-\frac{1}{3}e^{-j\omega}}$

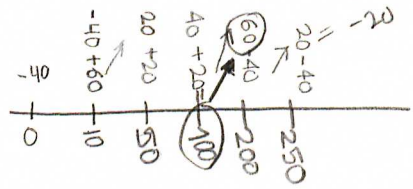
$= \frac{2-\frac{2}{3}e^{-j\omega}}{1-\frac{1}{3}e^{-j\omega}} \rightarrow \frac{2(1-\frac{1}{3}e^{-j\omega})}{1-\frac{1}{3}e^{-j\omega}} - \frac{1}{1-\frac{1}{3}e^{-j\omega}}$

$x[n] = 2\delta[n] - (\frac{1}{3})^n u[n]$

A 2
B -1
C 3

Question 5: Suppose $H(j\omega) = \frac{(10+j\omega)^3(100+j\omega)(50+j\omega)}{(j\omega)^2(200+j\omega)^2(250+j\omega)^2}$. Determine the slope of the Bode plot in the region from $\omega = 100$ to $\omega = 200$.

A: 0 dB/decade.
B: 40 dB/decade.
C: 20 dB/decade.
D: 60 dB/decade.
E: 80 dB/decade.



Question 6: Suppose $H(j\omega) = \frac{j\omega(80+j\omega)^3}{(20+j\omega)(40+j\omega)^2(160+j\omega)}$. Determine the value of the Bode approximation to $|H(j\omega)|$ at $\omega = 40$. Choose the closest answer.

A: 15 dB.
B: 9 dB.
C: 3 dB.
D: 12 dB.
E: 6 dB.

$|H(j40)| = \frac{(40)(80)^3}{(40)(40)^2(160)} = 2$

$2\log_{10}(2) = \boxed{6dB}$

NOT NATURAL LOG!

Question 7: Consider a linear time-invariant system with impulse response $h(t) = \frac{2}{\sqrt{t}}e^{-3t}u(t)$ and input $x(t) = e^{-3t}u(t)$. Determine the output $y(t)$. You may use without proof the facts that, if $a > 0$, $\sqrt{t}e^{-at}u(t)$ has Fourier transform $\frac{\sqrt{\pi}}{2}\frac{1}{(a+j\omega)^{3/2}}$ and $\frac{1}{\sqrt{t}}e^{-at}u(t)$ has Fourier transform $\frac{\sqrt{\pi}}{(a+j\omega)^{1/2}}$. Determine the output $y(t)$. The answer has the form $(A+B\sqrt{t}+Ct)e^{-Dt}u(t)$. What is $A+B+C+D$? Choose the closest answer. $Y = X \cdot H$

A: 6.
B: 10.
C: 9.
D: 7.
E: 8.

$$H(j\omega) = \frac{2\sqrt{\pi}}{(3+j\omega)^{1/2}} \qquad X(j\omega) = \frac{1}{3+j\omega}$$

$$Y(j\omega) = \frac{2\sqrt{\pi}}{(3+j\omega)^{1/2+1}} = \frac{2\sqrt{\pi}}{(3+j\omega)^{3/2}} \rightarrow 4\frac{\sqrt{\pi}}{2}\frac{1}{(3+j\omega)^{3/2}} \rightarrow \frac{1}{\sqrt{t}}e^{-3t}u(t)$$

A 0 B 4 C 0 D 3 Σ: 7

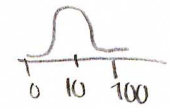
Question 8: Consider the filter with magnitude response $|H(j\omega)| = \frac{(1+j\omega)(100+j\omega)}{(10+j\omega)^2}$. Let $A = 0$ if this filer is passive and $A = 1$ if it is active. Let $B = 1$ if this filter is lowpass, $B = 2$ if highpass, $B = 3$ if bandpass, and $B = 4$ if bandstop. What is $A + B$? *Hint:* Look at the Bode plot.

A: 5.
B: 1.
C: 4.
D: 2.
E: 3.

$\omega = 0$: $|H(j0)| = \frac{|1||100|}{|100|^2} = 1$; $20\log_{10} 1 = 0$

$\omega = 10$: $|H(j10)| = \frac{|10||100|}{|10|^2} = 10$; $20\log(10) = 20$

$\omega = 100$: $|H(j100)| = \frac{|100||100|}{|100|^2} = 1$; $20\log_{10}(1) = 0$



Bandpass → B = 3

A = 1 (active)

Question 9: Consider a linear time-invariant system with $H(e^{j\omega}) = \frac{1-\frac{1}{2}e^{-j\omega}}{(1-\frac{3}{4}e^{-j\omega})^2}$. Determine group delay $D(\omega)$. What is $D(0)$? Choose the closest answer.

A: 4.
B: 5.
C: 1.
D: 3.
E: 2.

$$\angle H(e^{j\omega}) = \angle\left(1-\tfrac{1}{2}e^{-j\omega}\right) - 2\angle\left(1-\tfrac{3}{4}e^{-j\omega}\right)$$

$a = \frac{1}{2}$ $\qquad$ $a = 3/4$

$$D(\omega) = \frac{-(a\cos\omega - a^2)}{1+a^2-2a\cos\omega}$$

$$D(\omega) = \frac{-(\frac{1}{2}\cos\omega - (\frac{1}{2})^2)}{1+(\frac{1}{2})^2 - 2\cdot\frac{1}{2}\cos\omega} - 2\left\{\frac{-(\frac{3}{4}\cos\omega - (\frac{3}{4})^2)}{1+(\frac{3}{4})^2 - 2\cdot\frac{3}{4}\cos\omega}\right\}$$

$$= \frac{-(\frac{1}{2}\cos\omega - \frac{1}{4})}{5/4 - \cos\omega} - 2\left[\frac{\frac{3}{4}\cos\omega - \frac{9}{16}}{25/16 - 3/2\cos\omega}\right]$$

$D(0) = 5$ (Plug in $\omega = \emptyset$)

Question 10: Consider a linear time-invariant system with input $x(t) = 5\delta(t) - 4e^{-2t}u(t)$ and output $y(t) = e^{-t}u(t) + 4e^{-2t}u(t)$. Use Fourier techniques to determine the impulse response $h(t)$. The answer has the form $h(t) = A\delta(t) + Be^{-Ct}u(t)$. What is $A + B + C$? Choose the closest answer.

A: 2.
B: 1.
C: 5.
D: 3.
E: 4.

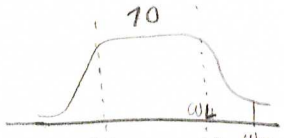
$$X(j\omega) = 5 - \frac{4}{2+j\omega} \rightarrow \frac{6+5j\omega}{2+j\omega}$$

$$Y(j\omega) = \frac{1}{1+j\omega} + \frac{4}{2+j\omega} \rightarrow \frac{2+j\omega+4+4j\omega}{(1+j\omega)(2+j\omega)} \rightarrow = \frac{6+5j\omega}{(1+j\omega)(2+j\omega)}$$

Constant but not exponential

$$H = \frac{Y}{X} = \frac{\frac{6+5j\omega}{2+j\omega}}{\frac{6+5j\omega}{(1+j\omega)(2+j\omega)}} = \frac{1}{1+j\omega}$$

A 0
B 1
C 1
2



Question 11: Consider the low-pass system with frequency response $H(j\omega) = \begin{cases} 10 & \text{if } |\omega| \leq 25 \\ 10e^{-(|\omega|-25)/10} & \text{if } |\omega| > 25 \end{cases}$. Determine the roll-off ratio. Assume the passband attenuation level is 90% of peak and the the stopband attenuation level is 10% of peak. Choose the closest answer.

ROLL OFF RATIO

A: 1.6.
B: 2.0.
C: 1.8.
D: 1.2.
E: 1.4.

Peak value of $H(j\omega) = 10$

90% of 10 is 9 10% of 1 = 1

$10e^{\frac{-(|\omega|-25)}{10}} = 9$ $10e^{\frac{-(|\omega|-25)}{10}} = 1$

$\omega_L = 26$ $\omega_H = 48$

ratio $= \frac{\omega_H}{\omega_L} = \frac{48}{26} = 1.4$

Question 12: Consider a linear time-invariant system such that $H(e^{j\omega}) = \frac{4}{5-4\cos^2(\omega)}$. If the input $\tilde{x}[n]$ is periodic with period $N_0 = 6$ and Fourier series coefficients $x_k = \frac{4}{4-|k|}$ for $-3 \leq k \leq 2$ (and x_ks are periodic with period 6) then determine the output Fourier series coefficients y_0 and y_2. What is $y_0 + y_2$? Choose the closest answer.

A: 10.
B: 6.
C: 8.
D: 2.
E: 4.

$2 \cdot \frac{2\pi}{6} = \frac{2\pi}{3}$

$y_k = x_k H(e^{jk\frac{2\pi}{N_0}})$

$y_0 = x_0 H(e^{j\cdot 0}) = \frac{4}{5-4\cos^2(0)} = \frac{4}{5-4} = 4$

$y_2 = x_2 H(e^{j2\cdot\frac{2\pi}{6}}) = \frac{4}{5-4\cos^2(\frac{2\pi}{3})} = x_2 \cdot \frac{4}{5-4(-\frac{1}{2})} \rightarrow \cancel{2} \cdot \frac{4}{5+2} = \frac{8}{7}$

$2 \cdot 1 = 2$

$4 + \frac{8}{7} = 6$ $x_2 = \frac{4}{4-2} = 2$

$2 + 4 = 6$

4

SPRING

Serial Number: 401

Name: Solutions

ECE 3300 Spring 2016 (Signals, Systems, and Transforms): Exam IV

THIS PRINTED INCORRECTLY

Record your name on this test; record your name, student ID, and test serial number on the scantron. Enter the test serial number in COURSE; you may leave SECTION blank. You must show your work on every problem, showing all steps on your test. Do not use scratch paper or write your work anywhere but on the test. The examination lasts 50 minutes and you may use four sheets of notes (front and back); no old test questions can be on your notes. Calculator use is permitted. There is one correct answer per question. In problems asking to □nd coe□cients A, B, C, etc., some of these coe□cients may equal zero.

$\delta[n] - \frac{3}{5}(\frac{1}{5})^n \quad h[n] = \frac{7}{2}(\frac{1}{2})^n u[n] - \delta[n]$

Question 1: Consider a linear time-invariant system with input x[n] = □[n] □ $\frac{3}{5}(\frac{1}{5})^n$u[n] and impulse response h[n] = $\frac{7}{2}(\frac{1}{2})^n$u[n] □ □[n]. Use Fourier techniques to determine the output y[n]. The answer has the form A□[n] + B$(\frac{1}{5})^n$u[n]. What is A + B? Choose the closest answer.

A: 2.
B: 4.
C: 3.
D: 5.
E: 1.

$Y(e^{j\omega}) = \frac{(2-e^{j\omega})}{5-e^{-j\omega}} \cdot \frac{5+e^{-j\omega}}{(2-e^{j\omega})}$

$= \frac{5+e^{-j\omega}}{5-e^{-j\omega}}$ Not all work shown

$= \frac{2}{1-\frac{1}{5}e^{j\omega}} - 1$

$= 2(\frac{1}{5})^n - \delta[n]$

$= 2$

$= -1$

$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$

$H(e^{j\omega}) = \frac{7}{2} \cdot \frac{1}{1-\frac{1}{2}e^{-j\omega}} - 1$

$= \frac{7}{2} \cdot \frac{2}{2-e^{-j\omega}}$

$H(e^{j\omega}) = \frac{5+e^{j\omega}}{2-e^{-j\omega}}$

$X(e^{j\omega}) = 1 - \frac{3}{5} \cdot \frac{1}{1-\frac{1}{5}e^{-j\omega}}$

$1 - \frac{3}{5} \cdot \frac{5}{5-e^{-j\omega}}$

$1 - \frac{3}{5-e^{-j\omega}}$

$X(e^{j\omega}) \frac{2-e^{-j\omega}}{5-e^{-j\omega}}$

Question 2: Consider a linear time-invariant system with H(j!) = $\frac{(3+j!)(□1+j!)}{(1+j!)^2}$. Determine the group delay D(!). What is D(1)? Choose the closest answer.

A: 0:9.
B: 1:2.
C: 1:5.
D: 0:3.
E: 0:6.

$D(\omega) = -\frac{d}{d\omega}\angle H(j\omega)$ $H(j\omega) = \frac{(3+j\omega)(-1+j\omega)}{(1+j\omega)^2}$

$\angle H(j\omega) = \angle(3+j\omega) + \angle(-1+j\omega) - 2\angle(1+j\omega)$

$a+j\omega$

$-\frac{a}{a^2+\omega^2}$

$D(\omega) = \frac{-3}{3^2+\omega^2} + \frac{-(-1)}{(-1)^2+\omega^2} + \frac{2(1)}{1^2+\omega^2}$

$= \frac{-3}{9+\omega^2} + \frac{3}{1+\omega^2}$

$D(1) = \frac{-3}{9+1^2} + \frac{3}{1+1^2} = 1.2 \quad \frac{6}{5}$

13

Answers: EBED EADD AEDB

Note: Problem 1 has been modi□ed here to give a valid answer. In the original test, the problem was not solvable.

Question 3: Consider a linear time-invariant system with impulse response h(t) = (1+ 2t)e□ᵗu(t) and output y(t) = te□ᵗu(t). Use Fourier techniques to determine the input x(t). The answer has the form x(t) = Ae□ᴮᵗu(t). What is A + B? Choose the closest answer.

A: 2.
B: 1.
C: 3.
D: 5.
E: 4.

$h(t) = (1+2t)e^{-t}u(t)$

$= e^{-t}u(t) + 2te^{-t}u(t)$

$H(j\omega) = \frac{1}{1+j\omega} + \frac{2}{(1+j\omega)^2}$

$H(j\omega) = \frac{3+j\omega}{(1+j\omega)^2}$

$Y(t) = te^{-t}u(t)$

$Y(j\omega) = \frac{1}{(1+j\omega)^2}$

4 ··· E

$X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} = \frac{\frac{1}{(1+j\omega)^2}}{\frac{3+j\omega}{(1+j\omega)^2}}$

$X(j\omega) = \frac{1}{3+j\omega}$

A = 1
B = 3

Question 4: Consider the □lter with magnitude response jH(e^j!)j = $\frac{2j\sin(!)j}{2□\cos^2(!)}$. Let A = 0 if this □ler is passive and A = 1 if it is active. Let B = 1 if this □lter is lowpass, B = 2 if highpass, B = 3 if bandpass, and B = 4 if bandstop. What is A + B? Hint: Evaluate jH(e^j!)j at multiples of □/2.

A: 4.
B: 1.
C: 2.
D: 3.
E: 5.

$|H(e^{j\omega})| = \frac{2\sin\omega}{2-\cos^2\omega}$

$0 \rightarrow |H(e^{j0})| = \frac{2\sin(0)}{2-\cos^2(0)} = 0$

$\frac{\pi}{2} \rightarrow |H(e^{j(\pi/2)})| = \frac{2}{2} = 1$

$\pi \rightarrow |H(e^{j\pi})| = 0$

$\frac{3\pi}{2} \rightarrow |H(e^{j\frac{3\pi}{2}})| = 1$

B = 3 $\boxed{D}$

Magnitude does not exceed 1

→ A = 0 PASSIVE

Question 5: Consider a linear time-invariant system with H(e^j!) = (1□ e□j!)(1+ 2e□j!). Determine the phase response \H(e^j!). The answer has the form tan□¹($\frac{A\sin!}{1□B\cos!}$) + tan□¹($\frac{C\sin!}{1□D\cos!}$). What is A + B + C + D? Choose the closest answer.

A: □3.
B: □4.
C: 0.
D: □1.
E: □2.

$\angle(1+ae^{-j\omega}) = \tan^{-1}\left(\frac{a\sin\omega}{1+a\cos\omega}\right)$

$H(e^{j\omega}) = (1-e^{-j\omega})(1+2e^{-j\omega})$

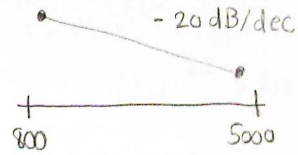
$\angle H(e^{j\omega}) = \angle(1-e^{-j\omega}) + \angle(1+2e^{-j\omega})$

$\angle H(e^{j\omega}) = \tan^{-1}\left(\frac{\sin\omega}{1-\cos\omega}\right) + \tan^{-1}\left(\frac{-2\sin\omega}{1+2\cos\omega}\right)$

$\boxed{E}$ -2

Question 6: Suppose a Bode plot of jH(j!)j is a straight line of slope □20 dB/ decade between ! = 800 and ! = 5000 such that jH(j 1000)j = 26 dB. What is jH(j 2500)j? Choose the closest answer.

A: 18 dB.
B: 9 dB.
C: 12 dB.
D: 6 dB.
E: 15 dB.

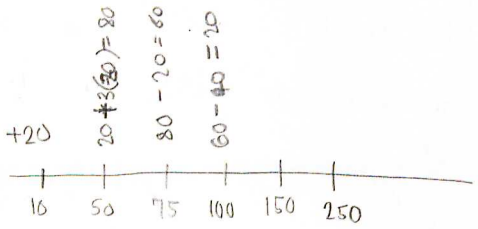


$|H(j800)| = 26dB$

$26 - 20\log_{10}\left(\frac{2500}{1000}\right) = 18.042dB = \boxed{18dB\ A}$

Question 7: Suppose $H(j\omega) = \dfrac{(10 + j\omega)(50 + j\omega)^3(150 + j\omega)}{(100 + j\omega)^2(250 + j\omega)^2(75 + j\omega)}$. Determine the slope of the Bode plot in the region from ! = 100 to ! = 150.

A: 40 dB/ decade.
B: □20 dB/ decade.
C: □40 dB/ decade.
D: 20 dB/ decade.
E: 0 dB/ decade.



D 20 dB/dec

Question 8: Suppose $H(j!) = \dfrac{20(5 + j!)^2(100 + j!)^2(250 + j!)}{(j!)^3(40 + j!)(200 + j!)}$. Determine the value of the Bode approximation to jH(j!)j at ! = 100. Choose the closest answer.

A: 31 dB.
B: 34 dB.
C: 37 dB.
D: 28 dB.
E: 25 dB.

$$|H(j100)| = \frac{|20(5+j100)^2(100+j100)^2(250+j(100))|}{(j100)^3(40+j100)(200+j100)}$$

$$= \frac{20 \cdot 100^2 \cdot 100^2 \cdot 250}{100^3 \cdot 100 \cdot 200} = 25 \rightarrow 20\log_{10}(25) = 28\,dB \quad D$$

Question 9: Consider a linear time-invariant system such that $H(j!) = \frac{2}{1+!^2}$. If the input x(t) is periodic with period T_0 = 4□ and Fourier series coe□cients $x_k = \frac{25}{4}(\frac{2}{5})^{jkj}$, then determine the output Fourier series coe□cients y_1 and y_2. What is $y_1 + y_2$? Choose the closest answer.

A: 5.
B: 2.
C: 1.
D: 4.
E: 3.

$$Y_k = X_k H\left(j\frac{k2\pi}{4\pi}\right)$$

$$Y_k = X_k H\left(\frac{1}{2}jk\right)$$

$$Y_1 = X_1 H\left(\frac{1}{2}j\right)$$

$$= \frac{25}{4} \cdot \frac{2}{5}\left(\frac{2}{1+1/4}\right) = 4$$

$$Y_1 = 4$$

$$Y_2 = X_2 H\left(j\frac{2}{2}\right)$$

$$X_2 H(j)$$

$$\frac{25}{4}\left(\frac{2}{5}\right)^2\left(\frac{2}{1^2+1^2}\right)$$

$$Y_1 = 1$$

$$Y_1 + Y_2 = 4 + 1 = 5 \quad A$$

Question 10: Consider a linear time-invariant system with impulse response h(t) = (1 + at)e□2t u(t). Determine a if the magnitude response is jH(j!)j = $\sqrt{\frac{16+!^2}{4+!^2}}$. Choose the closest answer.

A: 5.
B: 1.
C: 4.
D: 3.
E: 2.

$$h(t) = (1 + at)e^{-2t}u(t)$$

$$= e^{-2t}u(t) + ate^{-2t}u(t)$$

$$H(j\omega) = \frac{1}{2+j\omega} + \frac{a}{(2+j\omega)^2}$$

$$|H(j\omega)| = \frac{|a+2+j\omega|}{|(2+j\omega)|^2}$$

$$= \frac{\sqrt{(a+2)^2+\omega^2}}{4+\omega^2}$$

$$= \frac{\sqrt{16+\omega^2}}{4+\omega^2}$$

$$(a+2)^2 = 16$$

$$a = 2 \quad E$$

Question 11: Suppose $H(j!) = \frac{100(j!)(20+j!)}{(2+j!)^2}$. Determine the value of the Bode phase plot at ! = 2. Choose the closest answer.

A: □=4.
B: □□=2.
C: □□=4.
D: 0.
E: □=2.

$\angle H(j2) =$

$\frac{\pi}{2} + \tan^{-1}\left(\frac{2}{20}\right) - 2\tan^{-1}\left(\frac{2}{2}\right)$

$= \frac{\pi}{2} + 0 - \frac{\pi}{4} \cdot 2$

$= 0$ $\boxed{D}$

$\angle H(j\omega) = \angle 100 + \angle j\omega + \angle(20 + j\omega) - 2\angle(2 + j\omega)^2$

$= 0 + \frac{\pi}{2}\,\mathrm{sgn}(\omega) + \tan^{-1}\left(\frac{\omega}{20}\right) - 2\tan^{-1}\left(\frac{\omega}{2}\right)$

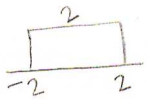
$= \begin{cases} \pi/2 & \frac{\omega}{a} > 10 \\ \pi/4 + \pi/4 \log_{10}\left(\frac{\omega}{a}\right) & 0.1 < \omega < 10 \\ 0 & 0 < \frac{\omega}{a} < 0.1 \end{cases}$

Question 12: Consider a linear time-invariant system with impulse response $h(t) = \frac{2}{\square t}\sin(2t)$ and input $x(t) = \frac{1}{\square t}\sin(t) + \frac{3}{\square t}\sin(3t)$. Determine the output y(t). The answer has the form $\frac{A}{\square t}\sin(Bt) + \frac{C}{\square t}\sin(Dt)$. What is A + B + C + D? Choose the closest answer.

A: 8.
B: 11.
C: 10.
D: 9.
E: 12.

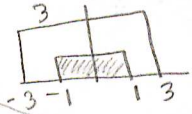
$H(j\omega)$:

$= 2\,\mathrm{rect}\left(\frac{\omega}{4}\right)$*

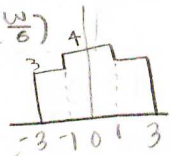


$X(j\omega)$:

$= \mathrm{rect}\left(\frac{\omega}{2}\right) + 3\,\mathrm{rect}\left(\frac{\omega}{6}\right)$



or



multiply together → $Y(j\omega) = 2\,\mathrm{rect}\left(\frac{\omega}{2}\right) + 6\,\mathrm{rect}\left(\frac{\omega}{4}\right)$

*use transform table

$y(t) = \frac{2}{\pi t}\sin(t) + \frac{6}{\pi t}\sin(2t)$

||

F2015 #1

$$H(e^{j\frac{\pi}{2}}) = \frac{1}{(1-\frac{1}{2}e^{-j\pi/2})^2} = \frac{4}{5} - \frac{2}{5}j \quad \Big\} \text{ ANSWER}$$

Equation: $y_k = x_k H(e^{jk\frac{2\pi}{N_0}})$

$$y_2 = x_2 \left[\frac{4}{5} - \frac{2}{5}j\right]$$

Given

$$= (1-\tfrac{1}{2}j)\left[\tfrac{4}{5} - \tfrac{2}{5}j\right]$$

$$= 1$$

<u>NOTE</u>: $\dfrac{1}{1+\frac{1}{2}j} = \dfrac{4}{5} - \dfrac{2}{5}j$

4 FALL

Serial Number: **401**

Name: SOLUTIONS McGee

ECE 3300 Fall 2015 (Signals, Systems, and Transforms): Exam IV

Record your name on this test; record your name, student ID, and test serial number on the scantron. Enter the test serial number in *COURSE*; you may leave *SECTION* blank. You must show your work on every problem, showing all steps on your test. Do not use scratch paper or write your work anywhere but on the test. The examination lasts 50 minutes and you may use four sheets of notes (front and back); no old test questions can be on your notes. Calculator use is permitted. There is one correct answer per question.

Question 1: Consider a linear time-invariant system such that $h[n] = (n+1)(\frac{1}{2})^n u[n]$. If the input $\tilde{x}[n]$ is periodic with period $N_0 = 8$ and if the input Fourier series coefficient $x_2 = 1 + \frac{1}{2}j$, then determine the output Fourier series coefficient y_2. In rectangular form, $y_2 = a + jb$. What is $a + b$? Choose the closest answer.

A: 0.8.
B: 1.0.
C: 0.6.
D: 0.2.
E: 0.4.

$H(e^{j\omega}) = \frac{1}{(1-\frac{1}{2}e^{j\omega})^2}$ $N_0 = 8$

$x_2 = 1 + \frac{1}{2}j$

$H(e^{j\frac{\pi}{2}}) = \frac{1}{(1-\frac{1}{2}e^{-j\frac{\pi}{2}})^2}$

$e^{-\frac{j\pi}{2}} = \cos(\frac{\pi}{2}) - j\sin($

$= \frac{1}{(1+\frac{1}{2}j)^2}$

$= \frac{1-\frac{1}{2}j}{1+\frac{1}{4}}$

$Y_k = X_k H(e^{jk2\pi/N_0})$

$= X_k H(e^{jk\frac{2\pi}{8}}$

$= X_k H(e^{jk\pi/4})$

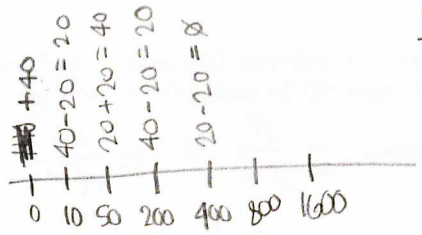
$Y_2 = X_2 H(e^{j2\frac{\pi}{4}})$

$(1+\frac{1}{2}j) X_2 H(e^{j2\frac{\pi}{4}})$

$Y_2 = \frac{4}{5} - \frac{2}{5}j$ $a = \frac{4}{5}$ $b = -2/5$ 0.4 E

Question 2: Suppose $H(j\omega) = \frac{(j\omega)^2(800+j\omega)^2(1600+j\omega)^2(50+j\omega)}{(10+j\omega)(200+j\omega)(400+j\omega)}$. Determine the slope of the Bode plot in the region from $\omega = 400$ to $\omega = 800$.

A: 40 dB/decade.
B: 0 dB/decade.
C: −20 dB/decade.
D: −40 dB/decade.
E: 20 dB/decade.



B 0dB/dec

Question 3: Suppose $H(j\omega) = \frac{2(8+j\omega)(40+j\omega)^2(200+j\omega)}{(j\omega)(20+j\omega)(80+j\omega)^2}$. Determine the value of the Bode approximation to $|H(j\omega)|$ at $\omega = 40$. Choose the closest answer.

A: 8 dB.
B: 2 dB.
C: 11 dB.
D: 5 dB.
E: 14 dB.

$|H(j40)| = \frac{|2(8+j40)(40+j40)^2(200+j40)|}{(40j)(20+j40)(80+j40)^2} \approx 2.5$

$\frac{2 \times 40 \times 40^2 \times 200}{40 \times 40 + 80^2} = 2.5$

$20\log_{10}(2.5) = 8$dB A

Answers: EBAC DDCB ADDE

Question 4: Consider a linear time-invariant system with input $x[n] = (n+3)(-\frac{1}{2})^n u[n]$ and impulse response $h[n] = -(-\frac{1}{3})^n u[n]$. Use Fourier techniques to determine the output $y[n]$. What is $y[1]$? Choose the closest answer. *Hint:* $n+3 = (n+1)+2$.

A: 1.
B: 4.
C: 3.
D: 2.
E: 5.

$(n+1+2)(-\frac{1}{2})^n u[n]$

$h[n] = -(-\frac{1}{3})^n u[n]$

$(n+1)(-\frac{1}{2})^n u[n] + 2(-\frac{1}{2})^n u[n]$

$H(e^{j\omega}) = -\frac{1}{1+\frac{1}{3}e^{-j\omega}} = -\frac{3}{3+e^{-j\omega}}$

$X(e^{j\omega}) = \frac{1}{(1+\frac{1}{2}e^{-j\omega})^2} + \frac{2}{1+\frac{1}{2}e^{-j\omega}} = \frac{3+e^{-j\omega}}{(1+\frac{1}{2}e^{-j\omega})^2}$

$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$

$Y[1] = -3(2)(-\frac{1}{2})$

$\leftarrow y[n] = -3(n+1)(-\frac{1}{2})^n u[n]$ $\quad Y(e^{j\omega}) = \frac{-3}{(1+\frac{1}{2}e^{-j\omega})^2} \leftarrow \frac{(3+e^{-j\omega})}{(1+\frac{1}{2}e^{-j\omega})^2} \cdot \frac{-3}{(3+e^{-j\omega})}$

Question 5: Consider a linear time-invariant system with $h(t) = (5t+1)e^{-t}u(t)$. Determine the group delay $D(\omega)$. The answer has the form $D(\omega) = \frac{A}{B^2+\omega^2} + \frac{C}{D^2+\omega^2}$ where $B > 0$ and $D > 0$. Determine $A+B+C+D$. Choose the closest answer.

A: 5.
B: 2.
C: 4.
D: 3.
E: 1.

$h(t) = 5te^{-t}u(t) + e^{-t}u(t)$

$H(j\omega) = \frac{5}{(1+j\omega)^2} + \frac{1}{1+j\omega} = \frac{6+j\omega}{(1+j\omega)^2}$

$\angle H(j\omega) = \angle(6+j\omega) - 2\angle(1+j\omega)$

$D(\omega) = \frac{-6}{6^2+\omega^2} - 2\frac{(-1)}{1^2+\omega^2} = \frac{-6}{6^2+\omega^2} + \frac{2}{1^2+\omega^2}$

Question 6: Consider a linear time-invariant system with $H(e^{j\omega}) = \frac{e^{-2j\omega}}{(1-\frac{1}{2}e^{-j\omega})^2}$. Determine the phase response $\angle H(e^{j\omega})$. The answer has the form $A\omega + B\tan^{-1}(\frac{\sin\omega}{C+D\cos\omega})$. What is $A+B+C+D$? Choose the closest answer.

A: -2.
B: -4.
C: -1.
D: -3.
E: -5.

$\angle H(e^{j\omega}) = \angle e^{-2j\omega} - 2\angle(1-\frac{1}{2}e^{-j\omega})$

$= -2\omega - 2\tan^{-1}\left(\frac{\frac{1}{2}\sin\omega}{1-\frac{1}{2}\cos\omega}\right)$

$= -2\omega - 2\tan^{-1}\left(\frac{\sin\omega}{2-\cos\omega}\right)$

Question 7: Consider a linear time-invariant system with impulse response $h[n] = \frac{1}{2}(\frac{1}{3})^n u[n] + \frac{1}{2}(\frac{2}{3})^n u[n]$. Determine the magnitude response $|H(e^{j\omega})|$. What is the sum of the values of the magnitude response at $\omega = 0$ and $\omega = \pi$? Choose the closest answer.

A: 2.
B: 1.
C: 3.
D: 5.
E: 4.

$H(e^{j\omega}) = \frac{1/2}{1-\frac{1}{3}e^{-j\omega}} + \frac{1/2}{1-\frac{2}{3}e^{-j\omega}}$ $\quad |1-ae^{-j\omega}| = \sqrt{1+a^2-2a\cos\omega}$

$|H(e^{j\omega})| = \frac{|1-\frac{1}{2}e^{-j\omega}|}{|1-\frac{1}{3}e^{-j\omega}||1-\frac{2}{3}e^{-j\omega}|}$

$|H(e^{j\omega})| = \frac{\sqrt{1+1/4-\cos\omega}}{\sqrt{1+\frac{1}{9}-\frac{2}{3}\cos\omega}\sqrt{1+\frac{4}{9}-\frac{4}{3}\cos\omega}}$

$= \frac{\sqrt{\frac{5}{4}-\cos\omega}}{\sqrt{(\frac{10}{9}-\frac{2}{3}\cos\omega)(\frac{13}{9}-\frac{4}{3}\cos\omega)}}$

$\omega=0$: $9/4$ $\quad \omega=\pi$: $\frac{27}{40}$

"2T"?

$\frac{27}{40} + \frac{9}{4} = 2.9 \cdot 2\pi$

≈ 3

Question 8: Consider a linear time-invariant system with input $x(t) = \frac{1}{2}\delta(t) - e^{-3t}u(t)$ and output $y(t) = e^{-3t}u(t)$. Use Fourier techniques to determine the impulse response $h(t)$. The answer has the form $h(t) = Ae^{-Bt}u(t)$. What is $A + B$? Choose the closest answer.

A: 1.
B: 3.
C: 2.
D: 5.
E: 4.

$X(j\omega) = \frac{1}{2} - \frac{1}{3+j\omega} = \frac{1+j\omega}{2(3+j\omega)}$

$Y(j\omega) = \frac{1}{3+j\omega}$

$H(j\omega) = \frac{X()}{Y()} = \frac{\frac{1}{3+j\omega}}{\frac{1+j\omega}{2(3+j\omega)}} = \frac{2}{1+j\omega}$

$h(t) = 2e^{-t}u(t)$

Question 9: Consider the filter with magnitude response $|H(j\omega)| = \frac{100|\omega|}{\omega^2+100}$. Let $A = 0$ if this filer is passive and $A = 1$ if it is active. Let $B = 1$ if this filter is lowpass, $B = 2$ if highpass, $B = 3$ if bandpass, and $B = 4$ if bandstop. What is $A + B$? *Hint:* Evaluate $|H(j\omega)|$ at $\omega = 0$, $\omega = 10$, and $\omega = \infty$.

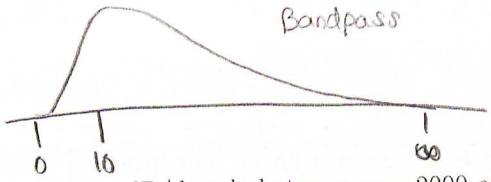
A: 4.
B: 2.
C: 5.
D: 3.
E: 1.

$\omega = 0 \rightarrow |H(j\omega)| = 0$

$\omega = 10 \rightarrow \frac{100(10)}{10^2+100} = 5$

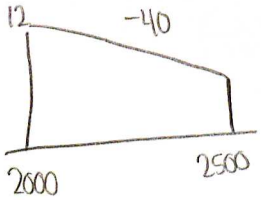
$\omega = \infty \quad \frac{100(\infty)}{\infty^2+100} = \frac{1}{\infty} = \emptyset$

$>1 \rightarrow$ ACTIVE $\rightarrow$ $A = 1$ $\quad B = 3$



Question 10: Suppose a Bode plot of $|H(j\omega)|$ is a straight line of slope -40 dB/decade between $\omega = 2000$ and $\omega = 2500$ such that $|H(j2000)| = 12$ dB. What is $|H(j2500)|$? Choose the closest answer.

A: 6 dB.
B: 4 dB.
C: 10 dB.
D: 8 dB.
E: 2 dB.



$12 - 40\log_{10}\left(\frac{2500}{2000}\right)$

$12 - 3.87$

≈ 8 dB

Question 11: Consider a linear time-invariant system with impulse response $h(t) = e^{-2t}u(t)$ and output $y(t) = e^{-2|t|}$. Determine the required input $x(t)$. The answer has the form $Ae^{Bt}u(-t)$. What is $A + B$? Choose the closest answer. *Hints:* For $a > 0$, $e^{-a|t|}$ has Fourier transform $\frac{2a}{a^2+\omega^2} = \frac{2a}{(a+j\omega)(a-j\omega)}$. Also, recall that if a signal $x(t)$ has Fourier transform $X(j\omega)$, then $x(-t)$ has Fourier transform $X(j(-\omega))$.

A: 4.
B: 3.
C: 5.
D: 6.
E: 2.

$e^{-a|t|} \rightarrow \frac{2a}{a^2+\omega^2} = \frac{2a}{(a+j\omega)(a-j\omega)}$

$H(j\omega) = \frac{1}{2+j\omega}$ $\quad y(T) = e^{-2|t|}$

$Y(e^{j\omega}) = \frac{4}{(2+j\omega)(2-j\omega)}$

$X(j\omega) = \frac{\frac{4}{(2+j\omega)(2-j\omega)}}{\frac{1}{2+j\omega}} = \frac{4}{2-j\omega}$

$4e^{2t}u(-t)$

$A = 4 \quad B = 2$

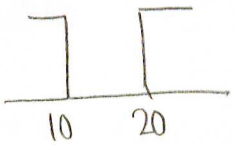
10

8

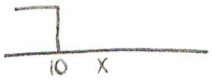
Question 12: Suppose we wish to create an ideal bandstop filter with cutoff frequencies $\omega = 10$ and $\omega = 20$. Which one of the following composite systems will accomplish this?

A: None of the other four answers will work.
B: Ideal highpass filter with cutoff $\omega = 10$ in series with ideal lowpass filter with cutoff $\omega = 20$.
C: Ideal lowpass filter with cutoff $\omega = 10$ in series with ideal highpass filter with cutoff $\omega = 20$.
D: Ideal highpass filter with cutoff $\omega = 10$ in parallel with ideal lowpass filter with cutoff $\omega = 20$.
E: Ideal lowpass filter with cutoff $\omega = 10$ in parallel with ideal highpass filter with cutoff $\omega = 20$.

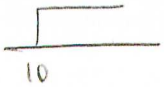
Frequency Domain
Parallel → +
Series → *



C



D



B

highpass w/ cutoff 20

Fall 2016

Answers: CDAB EEDD CBBE

Serial Number: **501**

Name: SOLUTIONS

APRIL 12, 2017

ECE 3300 FALL 2016 (SIGNALS, SYSTEMS, AND TRANSFORMS): EXAM V

Record your name on this test; record your name, student ID, and test serial number on the scantron. Enter the test serial number in *COURSE*; you may leave *SECTION* blank. You must show your work on every problem, showing all steps on your test. Do not use scratch paper or write your work anywhere but on the test. Circle your answers on the test and bubble in the corresponding answers on your scantron. The examination lasts 60 minutes and you may use five sheets of notes (front and back); no old test questions can be on your notes. Calculator use is permitted. There is one correct answer per question. In problems asking to find coefficients A, B, C, etc., some of these coefficients may equal zero.

Question 1: Suppose $x[n]$ has Z transform $X(z) = \frac{1}{(1-\frac{1}{2}z^{-1})^3(1-3z^{-1})^3}$ with ROC $\frac{1}{2} < |z| < 3$. Suppose $y[n] = x[n] + x[-n]$. Determine the ROC for $y[n]$. You do not need to determine $Y(z)$.

A: $2 < |z| < 3$.
B: $\frac{1}{3} < |z| < 3$.
C: $\frac{1}{2} < |z| < 2$.
D: $\frac{1}{2} < |z| < 3$.
E: $\frac{1}{3} < |z| < 2$.

* Time reversal property *

Replace z w/ z^{-1} in ROC

$\frac{1}{2} < |z^{-1}| < 3$

$2 > |z| > \frac{1}{3} \rightarrow$ for $x[-n]$: $\frac{1}{3} < |z| < 2$

$x[n]$: $\frac{1}{2} < |z| < 3$

overlap: $\frac{1}{2} < |z| < 2$

Question 2: Suppose $x[n] = (-\frac{1}{3})^n u[-n-1] + (-\frac{2}{3})^n u[-n-1]$. Determine the ROC for $X(z)$.

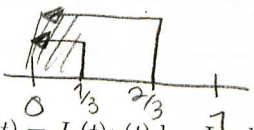
A: $-\frac{1}{3} < |z| < \frac{1}{3}$.
B: $|z| < -\frac{1}{3}$.
C: $|z| < \frac{2}{3}$.
D: $|z| < \frac{1}{3}$.
E: $|z| < -\frac{2}{3}$.

$a = -\frac{1}{3}$ | $a = -2/3$

ROC $|z| < \frac{1}{3}$ | ROC $|z| < \frac{2}{3}$

$a^n u[n]$ ROC $|z| > |a|$

$-a^n u[-n-1]$ ROC $|z| < |a|$



Overlapp $|z| < \frac{1}{3}$

Question 3: The signal $x(t) = J_0(t)u(t)$ has Laplace transform $X(s) = \frac{1}{\sqrt{1+s^2}}$ with ROC $\mathrm{Re}\{s\} > 0$. (The signal is a Bessel function but you don't need to know anything about Bessel functions to solve this problem.) Determine the inverse Laplace transform of $Y(s) = \frac{e^{-2s}}{\sqrt{1+(s+1)^2}}$ with ROC $\mathrm{Re}\{s\} > -1$. The answer has the form $x(t) = e^{A(t-B)}J_0(t-C)u(t-D)$. What is $A+B+C+D$? Choose the closest answer. *Hint:* First determine the inverse Laplace transform of $\frac{1}{\sqrt{1+(s+1)^2}}$.

A: 5.
B: 4.
C: 2.
D: 1.
E: 3.

1. Mult. by exponential property: $e^{s_0 t}x(t) \leftrightarrow X(s-s_0)$

$X(s+1) = \frac{1}{\sqrt{1+(s+1)^2}} \longleftrightarrow e^{-t}x(t)$

2. Time Shift Property $a(t-t_0) \leftrightarrow e^{-st_0}A(s)$

$Y(s) = \frac{e^{-2s}}{\sqrt{1+(s+1)^2}} = e^{-(t-2)}J_0(t-2)u(t-2)$

A -1
B 2
C 2
D 2

5

Question 4: Suppose a signal $x[n]$ has Z transform $X(z) = \frac{z}{z^3-1}$. Suppose $y[n] = nx[n]$. Determine $Y(z)$. The answer has the form $\frac{A(z)^B + C(z)^D}{(z^3-1)^2}$. What is $A+B+C+D$? Choose the closest answer. You do not need to determine the ROC.

A: 2.
B: 8.
C: 6.
D: 10.
E: 4.

$Y[n] = x[n]n$ "Multiply by n" property

$nx[n] \leftrightarrow -z\frac{d}{dz}X(z) = Y(z) \rightarrow$

$= -z\frac{d}{dz}\left(\frac{z}{z^3-1}\right)$

$= z \cdot \frac{(-2z^3-1)}{(z^3-1)^2}$

$\frac{-2z^4+z}{(z^3-1)^2}$

A2 C1
B4 D1 $\Sigma 8$

Question 5: Suppose $x(t)$ has Laplace transform $X(s) = \frac{s+1}{s^2-4s+5}$ with ROC $\text{Re}\{s\} > 2$. Determine the Laplace transform of $e^{-t}x(t)$ including the ROC. The transform has the form $\frac{As+B}{s^2+Cs+D}$ and the ROC has the form $\text{Re}\{s\} > E$. What is $A+B+C+D+E$? Choose the closest answer.

A: 5.
B: 2.
C: 1.
D: 3.
E: 4.

$e^{s_0 t}x(t) \rightarrow X(s-s_0)$ $\quad \text{Re}\{s-s_0\} > a$

$e^{-t}x(t) \leftrightarrow X(s+1)$ $\quad \text{Re}\{s+1\} > 2$

$\text{Re}\{s\} > 1$

$X(s+1) = \frac{(s+1)+1}{(s+1)^2 - 4(s+1) + 5}$

$= \frac{s+2}{s^2-2s+2}$

A1 C-2 E1
B2 D2 $\Sigma 4$

Question 6: Suppose $X(z) = \frac{1}{1+z^{-1}} + \frac{2}{1+2z^{-1}} + \frac{3}{1-z^{-1}} + \frac{4}{1-2z^{-1}}$. If the ROC is $1 < |z| < 2$, determine $x[n]$. The answer has the form $A(B)^n u[n] + C(D)^n u[n] + E(F)^n u[-n-1] + G(H)^n u[-n-1]$. What is $A+B+C+D+E+F+G+H$? Choose the closest answer.

A: -3.
B: -1.
C: -4.
D: 0.
E: -2.

$|z|>|a|$ $a^n u[n]$ $\quad |z|<|a|$ $-a^n u[-n-1]$

$a=-1$ $a=-2$ $a=1$ $a=2$

$|z| \geq 1$ $|z| \leq 2$ $|z| \leq 1$ $|z| \leq 2$

$|z|>1$ $|z|<2$ $|z|>1$ $|z|<2$

$(-1)^n u[n] - 2(-2)^n u[-n-1] + 3(1)^n u[n]$

A-1 D+1 G-4
B-1 E-2 H2 $\Sigma -2$
C3 F-2

Question 7: Suppose $X(z) = \frac{3-z^{-1}}{(1-\frac{1}{2}z^{-1})^2}$ with ROC $|z| > \frac{1}{2}$. It follows that $x[n] = (An+B)(\frac{1}{2})^n u[n]$. Determine B. Choose the closest answer. *Hint:* Note that $x[n] = (A(n+1) + (B-A))(\frac{1}{2})^n u[n]$.

A: 2.
B: 5.
C: 1.
D: 3.
E: 4.

$= \frac{1+2-z^{-1}}{(1-\frac{1}{2}z^{-1})^2} \rightarrow \frac{1}{(1-\frac{1}{2}z^{-1})^2} + \frac{2(1-\frac{1}{2}z^{-1})^2}{(1-\frac{1}{2}z^{-1})^2}$ $\quad |z|>\frac{1}{2}$

$= \frac{1}{(1-\frac{1}{2}z^{-1})^2} + \frac{2}{(1-\frac{1}{2}z^{-1})}$

$x[n] = (n+1)(\frac{1}{2})^n u[n] + 2(\frac{1}{2})^n u[n]$

$= (n+3)(\frac{1}{2})^n u[n]$

Question 8: Suppose $X(s) = \frac{4(s+2)^3}{(s+1)(s+3)^2}$ with ROC $\text{Re}\{s\} > -1$. It follows that $x(t) = A\delta(t) + Be^{-t}u(t) + (C + Dt)e^{-3t}u(t)$. What is C? Choose the closest answer.

A: -3.
B: -4.
C: -2.
D: -5.
E: -1.

$X(s) = A + \frac{B}{s+1} + \frac{C}{s+3} + \frac{D}{(s+3)^2}$

$B = (s+1)X(s)\big|_{s=-1}$

$= \frac{4(s+2)^3}{(s+3)^2}\Big|_{s=-1}$ $\boxed{B=1}$

$D = (s+3)^2 X(s)\big|_{s=-3} = \frac{4(s+2)^3}{s+1}\Big|_{s=-3}$

$\boxed{D=2}$

$\lim_{s\Rightarrow\infty} \frac{4\cdot\infty^3}{\infty\cdot\infty^2} = A + \frac{B}{\infty} + \frac{C}{\infty} + \frac{D}{\infty^2}$

$\boxed{A=4}$

$s=-2$ $\quad 0 = A - B + C + D$

$\boxed{A+C=-1}$

Question 9: Suppose the signal $x[n]$ has Z transform $X(z) = \frac{(\frac{3}{2}+z^{-1})^2}{(\frac{5}{4}-\frac{1}{2}z^{-1})^3}$. with ROC $|z| > \frac{2}{5}$. Suppose $y[n] = \sum_{m=-\infty}^{n} x[m]$. Determine $Y(z)$. What is $Y(2)$? Choose the closest answer. You do not need to determine the ROC in this problem.

Property involves $X(z) \to X(\frac{1}{z})$

A: 10.
B: 9.
C: 8.
D: 7.
E: 6.

$X[n] \leftrightarrow X(z) = \frac{(3/2 + z^{-1})^2}{(5/4 - \frac{1}{2}z^{-1})^3}$ $\quad |z| > \frac{2}{5}$

$Y(z) = \frac{1}{1-1/z} \cdot \frac{(\frac{3}{2}+\frac{1}{z})^2}{(\frac{5}{4}+\frac{1}{2}\frac{1}{z})^3}$ $\qquad Y(2) = \frac{1}{(1-1/2)} \cdot \frac{(\frac{3}{2}+\frac{1}{2})^2}{(5/4+1/4)^3} = 8$

Question 10: Suppose $x[n] = (\overset{a}{-2})^n u[-n-1] + (\overset{a}{-1})^n u[-n-1]$. Which statement is correct?

CLASSIFYING

A: The Z transform exists but the Fourier transform does not.
B: The Z and Fourier transforms exist but $X(e^{j\omega})$ can not be found by substituting $z = e^{j\omega}$ in $X(z)$.
C: The Z and Fourier transforms exist and $X(e^{j\omega})$ can be found by substituting $z = e^{j\omega}$ in $X(z)$.
D: The Fourier transform exists but the Z transform does not.
E: Neither the Z transform nor the Fourier transform exists.

$a = -2$ $\qquad a = -1$

$|z| < 2$ $\qquad |z| < 1$ $\qquad$ Overall ROC: $|z| < 1$ Borders $|z| = 1$

Question 11: Suppose $x(t) = e^{-t}\cos(2t)u(t) - 3e^{-t}\sin(2t)u(t)$. Determine $X(s)$. The answer has the form $\frac{A+Bs}{s^2+Cs+D}$. Determine $A+B+C+D$. Choose the closest answer. You do not need to determine the ROC. Properties

$a=1$ $b=2$ $\qquad a=1$ $b=2$

A: 1.
B: 3.
C: 4.
D: 2.
E: 5.

$X(s) = \frac{s+1}{(s+1)^2+4} - 3 \cdot \frac{2}{(s+1)^2+2^2}$

$= \frac{s-5}{s^2+2s+1+4}$

A-5 C2
B1 D5

Question 12: Suppose $X(z) = \frac{1-\frac{1}{3}z^{-1}}{1-\frac{2}{3}z^{-1}+\frac{4}{9}z^{-2}}$ with ROC $|z| > \frac{2}{3}$. Determine $x[n]$. The answer has the form $a^n(A\cos(bn) + B\sin(bn))u[n]$ where $0 < b < \pi$. What is $a\sin(b)$? Choose the closest answer.

A: 1/3.
B: $\sqrt{2}/3$.
C: $\sqrt{5}/3$.
D: 2/3.
E: $\sqrt{3}/3$.

$X(z)$ is combination of both functions
· denominator same for both!

$1 - \frac{2}{3}z^{-1} + \frac{4}{9}z^{-2} = 1 - 2a\cos(b)z^{-1} + a^2z^{-2}$

$a^2 = \frac{4}{9} \to a = \frac{2}{3}$

$2a\cos(b) = \frac{2}{3}$

$\cos(b) = \frac{1}{2}$

$b = \frac{\pi}{3}$

$\frac{2}{3} \cdot \sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{3}$

Serial Number: **501** Name: Solutions

ECE 3300 Fall 2015 (Signals, Systems, and Transforms): Exam V

Record your name on this test; record your name, student ID, and test serial number on the scantron. Enter the test serial number in *COURSE*; you may leave *SECTION* blank. You must show your work on every problem, showing all steps on your test. Do not use scratch paper or write your work anywhere but on the test. The examination lasts 50 minutes and you may use five sheets of notes (front and back); no old test questions can be on your notes. Calculator use is permitted. There is one correct answer per question.

Question 1: Suppose $x(t) = e^{-t}(\cos(2t) - 2\sin(2t))u(t)$. Determine $X(s)$. The answer has the form $\frac{As+B}{s^2+Cs+D}$. Determine $A+B+C+D$. Choose the closest answer. You do not need to determine the ROC.

A: 3.
B: 1.
C: 4.
D: 2.
E: 5.

$$= \frac{s+1}{(s+1)^2+2^2} - \frac{2\cdot 2}{(s+1)^2+2^2}$$

$$= \frac{s-3}{s^2+2s+5}$$

Question 2: Given that $J_0(t)u(t)$ has Laplace transform $\frac{1}{\sqrt{s^2+1}}$, and given that $\frac{d}{dt}[J_0(t)u(t)] = \delta(t) - J_1(t)u(t)$, determine the Laplace transform of $J_1(t)u(t)$. The answer has the form $A + \frac{Bs+C}{\sqrt{s^2+1}}$. Determine $A+B+C$. Choose the closest answer.

A: -2.
B: 1.
C: 2.
D: -1.
E: 0.

rearrange: $J_1(t)u(t) = \delta(t) - \frac{d}{dt}(J_0(t)u(t))$

Find $\mathcal{L}\{\}$: $1 - sX(s) \Rightarrow 1 - \frac{s}{\sqrt{s^2+1}}$

Question 3: Suppose the signal $x[n]$ has Z transform $X(z) = \frac{1}{(1+4z^{-4})^2}$. Suppose $Y(z) = \frac{1}{1+4z^{-4}}$ with the same ROC as that of $X(z)$. Express $y[n]$ as a function of $x[n]$. *Hint:* First find $Y(z)$ as a function of $X(z)$.

A: $y[n] = (-4)^n x[n]$.
B: $y[n] = (-\frac{1}{4})^n x[n]$.
C: $y[n] = [(-\frac{1}{4})^n]^4 x[n]$.
D: $y[n] = \sqrt{x[n]}$.
E: $y[n] = x[n] + 4x[n-4]$.

$$Y(z) = \frac{1}{1-4z^{-4}} = \frac{(1-4z^{-4})}{(1-4z^{-4})^2}$$

$$= \frac{1}{(1-4z^{-4})} - 4z^{-4}\frac{1}{(1-4z^{-4})^2}$$

$$Y(z) = X(z) + 4z^{-4}X(z)$$

$$y[n] = x[n] + 4x[n-4]$$

Answers: EEEC DEDC BACB

5

Fall

Question 9: Suppose $x(t)$ has Laplace transform $X(s)$ with ROC $1 < \text{Re}\{s\} < 9$ and $y(t) = (e^{2t} + e^{-t})^2 x(t)$. Determine the ROC for $Y(s)$. The answer has the form $a < \text{Re}\{s\} < b$. What is $b - a$? Choose the closest answer. *Hint:* Expand the square.

A: 4.
B: 2.
C: 10.
D: 6.
E: 8.

expand first $y(t) = (e^{4t} + 2e^{2t} \cdot e^{-t} + e^{-2t})\, x(t)$

$= e^{4t} x(t) + 2e^{2t} x(t) + e^{-2t} x(t)$

$a = 5$, $b = 7$, $b - a = 2$

$Y(s) = X(s-4) + 2X(s-1) + X(s+2)$

$1 < \text{Re}\{s-4\} < 9$ $\quad 1 < \text{Re}\{s-1\} < 9$ $\quad 1 < \text{Re}\{s+2\} < 9$

$5 < \text{Re}\{s\} < 13$ $\quad 2 < \text{Re}\{s\} < 10$ $\quad 3 < \text{Re}\{s\} < 11$ ← WRONG

$\boxed{5 < \text{Re}\{s\} < 7}$

Question 10: Suppose $x[n] = (\frac{1}{2})^{|n|} + (-\frac{1}{3})^{|n|}$. Determine the ROC for $X(z)$. *Hint:* $a^{|n|} = a^n u[n] + a^{-n} u[-n-1]$.

A: $\frac{1}{2} < |z| < 2$.
B: $\frac{1}{3} < |z| < 2$.
C: $\frac{1}{2} < |z| < 3$.
D: $-\frac{1}{3} < |z| < \frac{1}{2}$.
E: $\frac{1}{3} < |z| < 3$.

$= (\frac{1}{2})^n u(n) + (\frac{1}{2}) u[-n-1] + (-\frac{1}{3})^n u(n) + (-\frac{1}{3})^n u[-n-1]$

$= (\frac{1}{2})^n u(n) + (2^{-1})^n u[-n-1] + (\frac{1}{3})^n u[n] + (-3^{-1})^n u[-n-1]$

$= (\frac{1}{2})^n u[n] + 2^n u[-n-1] + (-\frac{1}{3})^n u[n] + (-3)^n u[-n-1]$

$|z| > \frac{1}{2}$ $\quad |z| < 2$ $\quad |z| > \frac{1}{3}$ $\quad |z| < 3$

$\frac{1}{2} < |z| < 2$

See graph below

Question 11: Suppose $x[n] = (-2)^n u[-n-1] + (-1)^n u[n] + (\frac{3}{2})^n u[-n-1]$. Which statement is correct?

A: Neither the Z transform nor the Fourier transform exists.
B: The Z and Fourier transforms exist and $X(e^{j\omega})$ can be found by substituting $z = e^{j\omega}$ in $X(z)$.
C: The Z and Fourier transforms exist but $X(e^{j\omega})$ can not be found by substituting $z = e^{j\omega}$ in $X(z)$.
D: The Z transform exists but the Fourier transform does not.
E: The Fourier transform exists but the Z transform does not.

$|z| < |-2|$ $\quad |z| > |-1|$ $\quad |z| < |\frac{3}{2}|$

2 $\quad$ 1 $\quad$ same

1 $\quad$ $\frac{3}{2}$

$|z| = 1$ → Borders $|z| = 1$ but $X(z) \neq X(e^{j\omega})|_{z=e^{j\omega}}$

Both Z & F.T.

Question 12: Suppose $X(z) = \frac{2+3z^{-1}}{1-2z^{-1}+4z^{-2}}$. For one of the ROCs the inverse Z transform has the form $(A\cos(bn) + B\sin(bn))a^n u[n]$. What is $a + b$, where $0 \le b \le \pi$ and $a > 0$? Choose the closest answer.

A: 1.
B: 3.
C: 2.
D: 5.
E: 4.

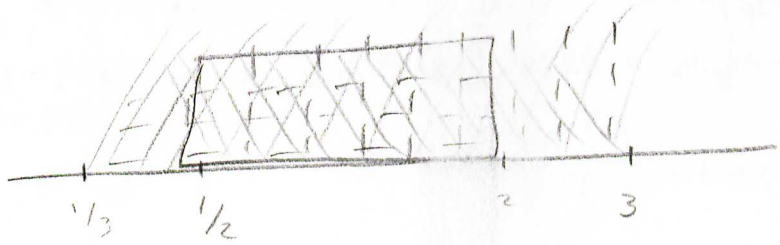
Quadratic Form: Use $1 - 2a\cos(b)z^{-1} + a^2 z^{-2}$

$a^2 = 4 \rightarrow a = 2 \Rightarrow A\cos(b) = 2$

$\cos(b) = 1/2$ $\quad b = \pi/3$

$a + b = \frac{\pi}{3} + 2$

$= 3.04 \approx$ B



Question 4: Determine the Laplace transform of $(1+t)\sin(t)u(t)$. You may use without proof the fact that $\sin(at)u(t)$ has Laplace transform $\frac{a}{s^2+a^2}$. The answer has the form $\frac{As^3+Bs^2+Cs+D}{s^4+Es^3+Fs^2+Gs+H}$. What is $A+B+C+D+E+F+G+H$? Choose the closest answer. You do not need to consider ROCs in this problem.

A: 10.
B: 6.
C: 7.
D: 9.
E: 8.

$\sin(at)u(t) \leftrightarrow \frac{a}{s^2+a^2} = \sin t\, u(t) + t\sin t\, u(t)$

$\sin t\, u(t) \leftrightarrow \frac{1}{s^2+1} X(s)$

$t\sin t\, u(t) \leftrightarrow -\frac{d}{ds} X_1(s) = \frac{2s}{(s^2+1)^2}$

$X(s) = \frac{1}{s^2+1} + \frac{2s}{(s^2+1)^2}$

$= \frac{s^2+2s+1}{s^4+2s^2+1}$

Question 5: Suppose $X(z) = 1 + \frac{1}{1-z^{-1}} + \frac{1}{1+z^{-1}} + \frac{1}{(1-z^{-1})^2} + \frac{1}{1-2z^{-1}} + \frac{1}{1+2z^{-1}}$. If the ROC is $1 < |z| < 2$, determine $x[n]$. What is $x[1]$? Choose the closest answer.

A: 3.
B: 1.
C: 0.
D: 2.
E: 4.

$a^n u[n]$, $|z| > |a|$, $|z| < |a|$

$\delta[n]$

$a=1$, $|z| \geq |1|$

$a=-1$, $|z| > |-1|$

$a=1$, $|z|>1$

$a=2$, $|z|<2$

$a=-2$, $|z|<2$

$X[1] = 2$

NOTE: $u[-2]=0$.
* unit step *

$\delta[n] + u[n] + (-1)^n u[n] + (n+1)u[n] - 2^n u[-n-1] - (-2)^n u[-n-1]$

Question 6: Suppose $X(s) = \frac{s^2}{(s+1)^3}$. The partial fraction expansion has the form $\frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)^3}$. Determine B. Choose the closest answer.

A: 2.
B: 0.
C: -1.
D: 1.
E: -2.

$C = s^2\big|_{s=-1} = 1$

$s=-1$: $\frac{1}{\cancel{8}} = \frac{A}{2} + \frac{B}{4} + \frac{1}{\cancel{8}}$

$2A + B = 0$

$A = -\frac{B}{2}$

$s=0$: $0 = A + B + 1$

$A + B = -1$

$\frac{B}{2} = -1$

$B = -2$

Question 7: Suppose a signal $x[n]$ has Z transform $\frac{1}{1+z^{-4}}$. Determine the Z transform of $y[n] = (-1)^n x[n-1]$. The answer has the form $Y(s) = \frac{Az^B}{1+Cz^{-4}}$. What is $A+B+C$? Choose the closest answer. You do not need to determine the ROC.

A: -2.
B: -4.
C: -5.
D: -1.
E: -3.

$x[n] \leftrightarrow X(z) = \frac{1}{1+z^{-4}}$

$x[n-1] \leftrightarrow z^{-1}X(z) = \frac{z^{-1}}{1+z^{-4}}$

$(-1)^n x[n-1] \leftrightarrow \left(\frac{z}{-1}\right) X\left(\frac{z}{-1}\right)$

$x[n] \leftrightarrow X\left(\frac{z}{z_0}\right)$

$= \frac{-z^{-1}}{1+(-z)^{-4}} = \frac{-z^{-1}}{1+z^{-4}}$

Σ: -1

Question 8: Suppose $X(z) = \frac{8z^{-1}(1-z^{-1})}{(1-2z^{-1})^2}$ with ROC $|z| > 2$. It follows that $x[n] = a\delta[n] + (bn+c)2^n u[n]$. What is $a+b+c$? Choose the closest answer.

A: -2.
B: 1.
C: 2.
D: 0.
E: -1.

Find partial fraction expansion

$X(z) = \frac{8p(1-p)}{(1-2p)^2} = A + \frac{B}{(1-2p)^2} = -2 + \frac{2}{(1-2z^{-1})^2}$

$B = 8p(1-p)\big|_{p=1/2} = 8(1/2)(1/2) = 2$

$p=0$: $0 = A + \frac{2}{(1-0)^2} = A = -2$

$= -2\delta[n] + 2(n+1)(2)^n u[n]$

$= -2\delta[n] + (2n+2)2^n u[n]$

Σ: C

$\boxed{\#2}$ $\frac{b}{(s+a)^2+b^2}$ $\begin{cases} e^{-at}\sin(bt)u(t) & Re\{s\} \geq Re\{a\} \\ e^{at}\sin(bt)u(t) & Re\{s\} \leq -Re\{a\} \end{cases}$

$A=1 \quad B=1 \quad C=-1 \quad D=-1 \quad E=-1$

Serial Number: **501**

Name: SOLUTIONS

ECE 3300 Spring 2016 (Signals, Systems, and Transforms): Exam V

Record your name on this test; record your name, student ID, and test serial number on the scantron. Enter the test serial number in *COURSE*; you may leave *SECTION* blank. You must show your work on every problem, showing all steps on your test. Do not use scratch paper or write your work anywhere but on the test. The examination lasts 50 minutes and you may use five sheets of notes (front and back); no old test questions can be on your notes. Calculator use is permitted. There is one correct answer per question. In problems asking to find coefficients A, B, C, etc., some of these coefficients may equal zero.

Question 1: Suppose $X(z) = \frac{4z^{-3}}{(1-z^{-1})(1-2z^{-1})}$ with ROC $|z| > 2$. It follows that $x[n] = A\delta[n] + B\delta[n-1] + Cu[n] + D2^n u[n]$. What is A? Choose the closest answer.

A: 5.
B: 3.
C: 4.
D: 2.
E: 1.

$\frac{4p^3}{(1-p)(1-2p)} = A + \frac{B}{1-p} + \frac{C}{1-2p}$

$B = \frac{4p^3}{1-2p}\Big|_{p=1} = -4$

$C = (1-2p)X(s) = \frac{4p^3}{1-p}\Big|_{p=\frac{1}{2}} = 1$

$p=0$: $\frac{4(0)^3}{(1-0)(1-2(0))} = 0 = A - \frac{4}{1-0} + \frac{1}{1-2(0)}$

$0 = A - 4 + 1$

$x[n] = -3\delta[n] - 4(1)^n u[n]$

$A = 3$

Question 2: Suppose $X(s) = \frac{1}{s+1} + \frac{1}{s-1} + \frac{1}{s^2+1}$. If the ROC is $-1 < \text{Re}\{s\} < 0$, determine $x(t)$. The answer has the form $Ae^{-t}u(Bt) + Ce^{t}u(Dt) + E\sin(t)u(Ft)$ where each of A through F are 1 or -1. What is $A+B+C+D+E+F$? Choose the closest answer.

A: -4.
B: 4.
C: -2.
D: 0.
E: 2.

$-1 < Re\{s\} < 0$ *WORK MESSED UP*

$\frac{1}{s+a}$

$Re\{s\} > -Re\{a\}$ $e^{-at}u(t)$

$Re\{s\} < -Re\{a\}$ $e^{-at}u(t)$

$-Re\{a\} = -1$, $Re\{s\} > -1$, $e^{-t}u(t)$ $\Big\}$ $a = 1$

$-Re\{a\} \Rightarrow -Re\{-1\} = 1$, $Re\{s\} < 1$, $e^{t}u(t)$ $\Big\}$ $a = -1$

Continued above

Question 3: Suppose $x(t) = 2tu(-t) + te^{t}u(-t) + te^{-t}u(t) + te^{2t}u(-t)$. Determine the ROC for $X(s)$.

A: $0 < \text{Re}\{s\} < 1$.
B: $-1 < \text{Re}\{s\} < 0$.
C: $-2 < \text{Re}\{s\} < 1$.
D: $1 < \text{Re}\{s\} < 2$.
E: $-2 < \text{Re}\{s\} < -1$.

$te^{-at}u(-t)$

$Re\{s\} < -Re\{a\}$

$Re\{s\} < 0$

$a = 1$

$Re\{s\} > -Re\{1\}$

$Re\{s\} > -1$

$a = -1$

$Re\{s\} < -Re\{-1\}$

$Re\{s\} < 1$

$a = -2$

$Re\{s\} < -Re\{-2\}$

$Re\{s\} < 2$

~~Smalle~~ Smallest terms!

$-1 < Re\{s\} < 0$ (B)

6

Answers: BCBD BEDE ADBB

F2016 #3

$x(t) = J_0(t)u(t) \longleftrightarrow \frac{1}{\sqrt{1+s^2}}$

$Y(s) = \frac{e^{-2s}}{\sqrt{1+(s+1)^2}}$

$e^{-st_0} X(s) \leftrightarrow x(t-t_0)$

$t_0 = 2$ $\quad x(t-2)$

$\frac{1}{\sqrt{1+(s+1)^2}} \longrightarrow e^{-1t} J_0(t) u(t)$

$a=1$

$x(t) = e^{-1(t-2)} J_0(t-2)u(t-2)$

A	-1		
B	2	-1	5
C	2	6	
D	2		

Spring

Question 4: Suppose $X(s) = \frac{-2-s^3}{s(s+1)^2}$. The partial fraction expansion has the form $A + \frac{B}{s} + \frac{C}{s+1} + \frac{D}{(s+1)^2}$. Determine C. Choose the closest answer.

A: 5.
B: 1.
C: 3.
D: 4.
E: 2.

$B = \left.\frac{-2-s^3}{(s+1)^2}\right|_{s=0} = \frac{-2}{1} = -2$

$D = \left.\frac{-2-s^3}{s}\right|_{s=-1} = \frac{-2+1}{-1} = 1$

$X(s) = A - \frac{2}{s} + \frac{C}{s+1} + \frac{1}{(s+1)^2}$ form

$s=1 \quad \frac{3}{1(1+1)^2} = A - 2 + \frac{C}{2} + \frac{1}{4}$

$\Rightarrow A + \frac{C}{2} = 1 \Rightarrow A + \frac{C}{3} = \frac{1}{3}$

$-\frac{3}{4} - \frac{1}{4} = A - 2 + C/2$

$-1 + \frac{C}{2} = 1 \to C = 2$

$C = 4$ D

Something something calculator

Question 5: Suppose $x[n]$ has Z transform $X(z) = \frac{2+z}{z^3-2z+8}$. Determine the Z transform of $(-\frac{1}{2})^n x[n]$. The answer has the form $\frac{1+Az}{B+Cz+Dz^2+Ez^3}$. What is $A+B+C+D+E$? Choose the closest answer. You do not need to determine an ROC in this problem.

A: -1.
B: 1.
C: 0.
D: -2.
E: 2.

$x[n] \leftrightarrow X(z) = \frac{2+z}{z^3-2z+8}$

$(-\frac{1}{2})^n x[n] \leftrightarrow X(\frac{z}{-1/2}) = X(-2z) = \frac{2-2z}{(-2z)^3+4z+8} \Rightarrow \frac{1-z}{4+2z-4z^3}$

$A = -1 \quad B = 4$

$C = 2 \quad D = 0$

$E = -4$

application of properties

Question 6: Suppose $x[n]$ has Z transform $X(z)$ with ROC $|z| < 2$, $y[n] = x[-n]$, and $w[n] = \sum_{m=-\infty}^{n} y[m]$. Determine the ROC for the Z transform of $w[n]$.

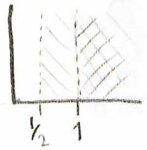
A: $|z| > 2$.
B: $|z| < \frac{1}{2}$.
C: $\frac{1}{2} < |z| < 1$.
D: $1 < |z| < 2$.
E: $|z| > 1$.

$x[n] \leftrightarrow X(z) \quad |z| < 2$

time reversal property $\quad y[n] = x[-n] \leftrightarrow X(-z) \quad |z^{-1}| < 2 \Rightarrow |z| > \frac{1}{2}$

Summation Property: $w[n] = \sum_{m=-\infty}^{n} y[m] \leftrightarrow \frac{1}{1-z^{-1}} Y(z) \quad |z| > \frac{1}{2} \cap |z| > 1$

intersect



$|z| > 1$

Question 7: Suppose $x(t) = e^{-2t}u(t) + e^{-t}u(-t)$. Which statement is correct?

A: The Fourier transform exists but the Laplace transform does not.
B: The Laplace and Fourier transforms exist but $X(j\omega)$ can not be found by substituting $s = j\omega$ in $X(s)$.
C: Neither the Laplace transform nor the Fourier transform exists.
D: The Laplace transform exists but the Fourier transform does not.
E: The Laplace and Fourier transforms exist and $X(j\omega)$ can be found by substituting $s = j\omega$ in $X(s)$.

$e^{-2t}u(t) + e^{-t}u(-t)$ $\quad -2 < \mathrm{Re}\{s\} < -1$ Excludes $\emptyset$

$a = 2 \quad a = 1$

$\mathrm{Re}\{s\} > -2 \quad \mathrm{Re}\{s\} < -1$

Laplace exists, not Fourier.

Question 8: Suppose the signal $x[n] = 3n^2(\frac{1}{3})^n u[n]$ has Z transform $X(z)$. Further suppose $Y(z) = (3z^{-2} - 1)X(z)$. Determine $y[n]$. What is $y[3]$? Choose the closest answer. *Hint:* Use properties. Do not determine $X(z)$ and $Y(z)$ but instead determine a formula for $y[n]$ in terms of $x[n]$.

A: 3.
B: 4.
C: 1.
D: 5.
E: 2.

$3z^{-2}X(z) - X(z)$

Time Shifting

$3x[n-2] - x[n]$

$y[n] = 3\left(3(n-2)^2(\frac{1}{3})^{n-2}u[n-2]\right) - 3n^2(\frac{1}{3})^n u[n]$

$y[n] = \cancel{9}^{3}(1)^2(\cancel{\tfrac{1}{3}})^{1}\cancel{u[3-2]}^{1} - 3(\cancel{3})^{2}(\cancel{\tfrac{1}{3}})^{3}\cancel{u[3]}^{1}$

$= 3 - 1 = 2$ E

7

Question 9: Suppose $x[n] = (-\sqrt{2})^n(\cos(\frac{\pi}{4}n) + \sin(\frac{\pi}{4}n))u[n]$. Determine $X(z)$. The answer has the form $\frac{1+Az^{-1}}{1+Bz^{-1}+Cz^{-2}}$. Determine $A+B+C$. Choose the closest answer. You do not need to determine the ROC.

A: 4.
B: 2.
C: 0.
D: 3.
E: 1.

$a^n \cos(bn)u[n] = (-\sqrt{2})^n \cos(\pi/4\, n)u[n] + (-\sqrt{2})^n \sin(\frac{\pi}{4}n)u[n]$

$a = -\sqrt{2}$
$b = \pi/4$

$X(z) = \frac{1 + (\sqrt{2})^1 \cos(\frac{\pi}{4}) z^{-1}}{1 - 2(-\sqrt{2})\cos(\frac{\pi}{4}) z^{-1} + (-\sqrt{2})^2 z^{-2}} = \frac{1 + \sqrt{2}\cdot\frac{1}{\sqrt{2}} z^{-1} - \sqrt{2}\cdot\frac{1}{\sqrt{2}} z^{-1}}{1 + 2\sqrt{2}\cdot\frac{1}{\sqrt{2}} z^{-1} + 2z^{-2}} = \frac{1}{1+2z^{-1}+2z^{-2}}$

4 A

$A=0 \quad B=2 \quad C=2$

Question 10: Suppose $X(s) = \frac{2s+3}{(s+2)^2+1}$ with ROC $\mathrm{Re}\{s\} > -2$. Determine $x(t)$. The answer has the form $e^{-at}(A\cos(bt) + B\sin(bt))u(t)$ where $b > 0$. What is $A+B+a+b$? Choose the closest answer.

A: 3.
B: 6.
C: 2.
D: 4.
E: 5.

$X(s) = \frac{2s+4-1}{(s+2)^2+1^2} = \frac{2(s+1)-1}{(s+2)^2+1^2} = \underbrace{\frac{2(s+2)}{(s+2)^2+1^2} - \frac{1}{(s+2)^2+1^2}}$

$e^{-at}\cos(bt)u(t)$
$e^{-at}\sin(bt)u(t)$ } use the Proper

$= 2e\cos t\, u(t) + e^{t}\sin(t)\, u(t)$

$a=2,\ A=2,\ b=1,\ B=-1$

Question 11: Suppose a signal $x(t)$ has Laplace transform $X(s) = \frac{s}{\sqrt{1+s^2}}$. Suppose $y(t) = 2tx(t)$. Determine $Y(s)$. What is $Y(0)$? Choose the closest answer. You do not need to determine the ROC.

A: 1.
B: -2.
C: -1.
D: 2.
E: 0.

$Y(t) = 2t\,x(t) \longleftrightarrow -2s\frac{d}{ds}\cdot\frac{s}{\sqrt{1+s^2}}$

$= -2\cdot\frac{1}{(s^2+1)^{3/2}}$

$x(t) \leftrightarrow X(s)$

$A\,x(t) \leftrightarrow -s\frac{d}{ds}X(s)$

$Y(0) = -\frac{2}{1} = -2$

Question 12: Given that $t\cos(t)u(t)$ has Laplace transform $\frac{s^2-1}{(s^2+1)^2}$ with ROC $\mathrm{Re}\{s\} > 0$ determine the inverse Laplace transform of $\frac{s(s^2-1)}{(s^2+1)^2}$ with ROC $\mathrm{Re}\{s\} > 0$. The answer has the form $(A+Bt)\cos(t)u(t) + (C+Dt)\sin(t)u(t)$. Determine $A+B+C+D$. Choose the closest answer.

A: 2.
B: 0.
C: 1.
D: -1.
E: -2.

$X(t) \leftrightarrow X(s)$

$\frac{d}{dt}X(t) \leftrightarrow sX(s)$

$\frac{d}{dt}(t\cos t)u(t) \leftrightarrow \frac{s(s^2-1)}{(s^2+1)^2}X(s)$

$= (\cos t - t\sin t)u(t)$

$= \cos t\, u(t) - t\sin t\, u(t)$

$A=1 \quad B=0$
$C=0 \quad D=-1$

0

8

ECE 3300
TEST 6
Fall 2016

SOLUTIONS

Answers: CDDA EDDD BBDD

Serial Number: **601** Name: Solutions

ECE 3300 Fall 2016 (Signals, Systems, and Transforms): Exam VI

Record your name on this test; record your name, student ID, and test serial number on the scantron. Enter the test serial number in *COURSE*; you may leave *SECTION* blank. You must show your work on every problem, showing all steps on your test. Do not use scratch paper or write your work anywhere but on the test. Circle your answers on the test and bubble in the corresponding answers on your scantron. The examination lasts 60 minutes and you may use six sheets of notes (front and back); no old test questions can be on your notes. Calculator use is permitted. There is one correct answer per question. In problems asking to find coefficients A, B, C, etc., some of these coefficients may equal zero.

Question 1: Consider a linear time-invariant system with impulse response $h[n] = \frac{1}{3}\delta[n] - \frac{2}{3}(2)^n u[-n-1]$. Determine $h_I[n]$, the impulse response of the causal INVERSE system. This impulse response has the form $h_I[n] = A\delta[n] + B(C)^n u[n]$. What is $A + B + C$? Choose the closest answer.

A: 5.
B: 1.
C: 2.
D: 4.
E: 3.

$H(z) = \frac{1}{3} + \frac{2}{3}\cdot\frac{1}{1-2z^{-1}} = \frac{1}{3}\left(1 + \frac{2}{1-2z^{-1}}\right) = \frac{1}{3}\left(\frac{1-2z^{-1}+2}{1-2z^{-1}}\right) = \frac{1/3(3-2z^{-1})}{1-2z^{-1}} = \frac{1-\frac{2}{3}z^{-1}}{1-2z^{-1}}$

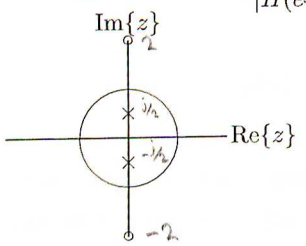
Inverse

$H_I(z) = \frac{1-2z^{-1}}{1-\frac{2}{3}z^{-1}}$ $|z| > \frac{2}{3}$ $\xrightarrow{PFE}$ $3 - \frac{2}{1-\frac{2}{3}p}$ $3\delta[n] - 2\left(\frac{2}{3}\right)^n u[n]$

A 3 B −2 C $\frac{2}{3}$

Question 2: For the system with pole-zero plot shown, there are single poles at $z = \frac{j}{2}$ and $z = -\frac{j}{2}$ and there are single zeroes at $z = 2j$ and $z = -2j$. Determine $\frac{|H(e^{j\pi/2})|}{|H(e^{j0})|}$. Choose the closest answer. *Hint:* This can be determined from the plot.

A: 0.6.
B: 0.8.
C: 0.4.
D: 1.0.
E: 0.2.



$|H(e^{j\frac{\pi}{2}})| = \frac{(1)(3)}{(1/2)(3/2)} = 4$ Zeroes

Poles

$|H(e^{j0})| = \frac{\sqrt{5}\sqrt{5}}{\sqrt{\frac{1}{4}+1}\sqrt{\frac{1}{4}+1}} = \frac{\sqrt{5}\sqrt{5}}{\sqrt{\frac{5}{4}}\sqrt{\frac{5}{4}}} = 4$ $\rightarrow$ $\frac{|H(e^{j\pi/2})|}{|H(e^{j0})|} =$ 1.0

Question 3: Suppose a causal system with transfer function $H(z)$ is such that there is a double pole at $z = -1$ and single zeroes at $z = 0$ and $z = -\frac{1}{2}$. Assume $H(1) = \frac{1}{2}$. What is $H(-2)$? Choose the closest answer.

A: 3.
B: 1.
C: 5.
D: 4.
E: 2.

$H(z) = \frac{k(z)(z+1/2)}{(z+1)^2}$ $H(-2) = ?$

$H(-2) = \frac{(4/3)(-2)(-2+1/2)}{(-2+1)^2} = 4$

$H(1) = \frac{k(1)(1+1/2)}{(1+1)^2} = \frac{1}{2}$ $k = \frac{4}{3}$

Question 4: Suppose an LTI system has impulse response $h[n] = \delta[n] - 2(2)^n u[-n-1]$ and input $x[n] = \delta[n] + (\frac{2}{3})^n u[n]$. The output has the form $y[n] = \delta[n] - A(2)^n u[-n-1]$. Determine A. Choose the closest answer.

A: 5.
B: 1.
C: 4.
D: 3.
E: 2.

$H(z) = 1 + \frac{2}{1-2z^{-1}} = \frac{3-2z^{-1}}{1-2z^{-1}}$ $X(z) = 1 + \frac{1}{1-\frac{2}{3}z^{-1}} = \frac{2-\frac{2}{3}z^{-1}}{1-\frac{2}{3}z^{-1}} = \frac{2(1-\frac{1}{3}z^{-1})}{1-\frac{2}{3}z^{-1}}$

$= \frac{2(1-\frac{1}{3}z^{-1})}{(1-\frac{2}{3}z^{-1})} \cdot \frac{3(1-\frac{2}{3}z^{-1})}{(1-2z^{-1})} = \frac{6(1-\frac{1}{3}z^{-1})}{1-2z^{-1}} \rightarrow 1 + \frac{5}{1-2z^{-1}}$ $|z| < 2$

$\delta[n] - 5(2)^n u[-n-1]$

ROC changes sign!

Question 5: Consider a linear time-invariant system with impulse response $h[n] = \delta[n] + (-1)^n(n+2)u[n]$. The input-ouput difference equation for this system can be shown to have the form

$$y[n] + Ay[n-1] + By[n-2] = Cx[n] + Dx[n-1] + Ex[n-2]$$

Determine $(C+D) - (A+B+E)$. Choose the closest answer. *Hint:* Careful - this is NOT asking for $A+B+C+D+E$.

A: 0.
B: −2.
C: 1.
D: −1.
E: 2.

$h[n] = \delta[n] + (-1)^n(n+1)u[n] + (-1)^n u[n]$

$H(z) = 1 + \frac{1}{(1+z^{-1})^2} + \frac{1}{(1+z^{-1})} = \frac{1+2z^{-1}+z^{-2}+2+z^{-1}}{1+2z^{-1}+z^{-2}} = \frac{3+3z^{-1}+z^{-2}}{1+2z^{-1}+z^{-2}}$

$Y(z)(1+Az^{-1}+Bz^{-2}) = X(z)(C+Dz^{-1}+Ez^{-2})$

$H(z) = \frac{C+Dz^{-1}+Ez^{-2}}{1+Az^{-1}+Bz^{-2}}$

A 2
B 1
C 3
D 3
E 1

Question 6: Consider two linear time-invariant systems with impulse responses $h_1(t) = \delta(t) + 6e^t u(t)$ and $h_2(t) = \delta(t)$ and suppose that the two systems are arranged in a causal feedback combination (the system with impulse response $h_2(t)$ is the one "fed back"). Determine $h(t)$ for the composite feedback system. The answer has the form $A\delta(t) + Be^{-Ct}u(t)$. Determine $A+B+C$. Choose the closest answer.

A: 1.
B: 5.
C: 3.
D: 4.
E: 2.

$H_1(s) = 1 + \frac{6}{s-1} = \frac{s+5}{s-1}$ $\quad H_2(s) = 1$

Feedback system → $= \frac{\frac{s+5}{s-1}}{1+\frac{s+5}{s-1}\cdot 1} = \frac{s+5}{2(s+2)} \xrightarrow{PFE} \frac{3}{2}\cdot\frac{1}{s+2} + \frac{1}{2}$

$H(s) = \frac{H_1(s)}{1+H_1(s)H_2(s)}$

$\frac{1}{2}\delta(t) + \frac{3}{2}e^{-2t}u(t)$

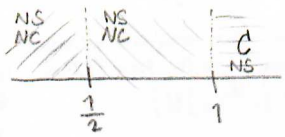
Question 7: Suppose a signal is sampled at 20% oversampling and a sampling rate of 585 ksamples/sec. Determine the maximum frequency of the pre-sampled signal. Choose the closest answer.

A: 500 kHz.
B: 750 kHz.
C: 1250 kHz.
D: 250 kHz.
E: 1000 kHz.

$100 * \frac{585 - 2f_m}{2f_m} = 20\%$ $\quad f_m = 250\text{kHz}$

Question 8: Consider a linear time-invariant system given by $H(z) = \frac{1}{1-z^{-1}} + \frac{1}{1+z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1+\frac{1}{2}z^{-1}}$. Let T denote the total number of realizations of this system. Let C denote the number of these realizations that are causal and S the number that are stable. Determine $T+C+S$.

A: 6.
B: 7.
C: 3.
D: 4.
E: 5.



Realizations ROC

1: $|z| < \frac{1}{2}$

2: $\frac{1}{2} < |z| < 1$

3 $|z| > 1$ causal

$a=1$	$a=-1$	$a=\frac{1}{2}$	$a=-\frac{1}{2}$
$\lvert z\rvert \geq 1$,	$\lvert z\rvert \geq 1$,	$\lvert z\rvert \leq \frac{1}{2}$,	$\lvert z\rvert \leq \frac{1}{2}$

$T = 3$ realizations
$C = 1$ causal systems
$S = \emptyset$ stable systems

$\Sigma: 4$

Causal: ROC on right of rightmost pole
Stable: Include $|z|=1$ not border

Question 9: Consider a linear time-invariant system with input $x(t)$ and output $y(t)$ such that $X(s) = \frac{s(s+2)}{(s+1)^2}$, $Y(s) = \frac{s}{(s+1)^2}$, and both have ROC $\text{Re}\{s\} < -1$. Determine all possible impulse responses $h(t)$. The answer has the form $Ae^{-Bt}u(t)$ (in which case determine $A+B$), $Ce^{-Dt}u(-t)$ (in which case determine $C+D$), or both (in which case determine $A+B+C+D$). Choose the closest answer.

A: 6.
B: 4.
C: 3.
D: 5.
E: 2.

$H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{s}{(s+1)^2}}{\frac{s(s+2)}{(s+1)^2}} = \frac{1}{s+2}$ $\quad e^{-2t}u(t) \to \text{Re}\{s\} > -2$ $\quad e^{-2t}u(-t) \to \text{Re}\{s\} < -2$

$\frac{1}{s+a}$ $\quad e^{-at}u(t)$ $\text{Re}\{s\} > -\text{Re}\{a\}$ $\quad e^{-at}u(-t)$ $\text{Re}\{s\} < -\text{Re}\{s\}$

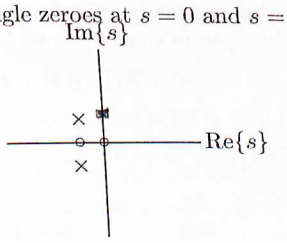
$\text{Re}\{s\} < -1$ maintained after operation!

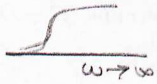
both possible!

A 1
B 2
C −1
D 2

4

Question 10: Consider the pole-zero plot shown. The zeroes are single zeroes at $s = 0$ and $s = -1$, and the poles are single poles at $s = -1 + j$ and $s = -1 - j$. This filter is





A: Allpass.
B: Highpass.
C: Bandstop.
D: Bandpass.
E: Lowpass.

$|H(j0)| = \frac{0 \cdot 1}{\cdot} = 0$

$|H(j1)| = \frac{1 \cdot \sqrt{2}}{1 \cdot \sqrt{5}} = 0.6325$

$|H(j\infty)| = \frac{\infty \cdot \infty}{\infty \cdot \infty} = 1$

Question 11: Suppose $h(t) = e^{-t}\cos(2t)u(t)$. Determine the poles and zeroes. What is the sum of the values of all the poles and zeroes? Choose the closest answer.

A: -5.
B: -2.
C: -1.
D: -3.
E: -4.

$H(s) = \frac{s+1}{(s+1)^2+4} = \frac{s+1}{s^2+2s+5}$

$s = -1$
$s = -1 \pm 2j$

$1 - 1 - 2j - 1 + 2j$
$= -3$

Question 12: Suppose a linear time-invariant system with input $x(t)$ and output $y(t)$ is governed by the differential equation $\frac{d^2}{dt^2}y(t) + \frac{d}{dt}y(t) - 2y(t) = \frac{d}{dt}x(t) + x(t)$. Determine $H(s)$. The stable realization of $H(s)$ is $h(t) = Ae^{Bt}u(t) + Ce^{Dt}u(-t)$. What is $B + D$? Choose the closest answer. *Hint:* It is not necessary to complete the partial-fraction expansion to answer this question.

A: -2.
B: 0.
C: 2.
D: -1.
E: 1.

$Y(s)(s^2 + s - 2) = X(s)(s+1)$

$H(s) = \frac{s+1}{s^2+s-2} = \frac{(s+1)}{(s-1)(s+2)} = \frac{A}{s-1} + \frac{B}{s+2}$

$Ae^{t}u(t) + Ce^{2t}u(-t)$

$B = -2$
$D = 1$
-1

Serial Number: **601** Name: SOLUTIONS

ECE 3300 FALL 2015 (SIGNALS, SYSTEMS, AND TRANSFORMS): EXAM VI

Record your name on this test; record your name, student ID, and test serial number on the scantron. Enter the test serial number in *COURSE*; you may leave *SECTION* blank. You must show your work on every problem, showing all steps on your test. Do not use scratch paper or write your work anywhere but on the test. The examination lasts 50 minutes and you may use six sheets of notes (front and back); no old test questions can be on your notes. Calculator use is permitted. There is one correct answer per question.

Question 1: Suppose an LTI system has impulse response $h[n] = \delta[n] + (-1)^n u[n]$ and input $x[n] = \delta[n] + (-\frac{1}{2})^n u[n]$. The output has the form $y[n] = A\delta[n] + B(C)^n u[n]$. Determine $A + B + C$. Choose the closest answer.

A: 4.
B: 5.
C: 1.
D: 3.
E: 2.

$H(z) = 1 + \frac{1}{1+z^{-1}} \rightarrow \frac{2+z^{-1}}{1+z^{-1}} = \frac{2(1+\frac{1}{2}z^{-1})}{1+z^{-1}}$

$X(z) = 1 + \frac{1}{1+\frac{1}{2}z^{-1}} \rightarrow \frac{2+\frac{1}{2}z^{-1}}{1+\frac{1}{2}z^{-1}}$

$Y(z) = X(z)H(z)$

$= \frac{2+\frac{1}{2}z^{-1}}{(1+\frac{1}{2}z^{-1})} \cdot \frac{2(1+\frac{1}{2}z^{-1})}{(1+z^{-1})} \rightarrow = \frac{4+z^{-1}}{1+z^{-1}}$

$= \frac{3+1+z^{-1}}{1+z^{-1}} = \frac{3}{1+z^{-1}} + \frac{1+z^{-1}}{1+z^{-1}} = 1 + \frac{3}{1+z}$

$A = 1 \quad B = 3 \quad C = -1$

$y[n] = \delta[n] + 3(-1)^n u(n)$

Question 2: Consider a linear time-invariant system with impulse response $h(t) = \delta(t) - e^{-2t}\cos(t)u(t)$. The input-ouput differential equation for this system can be shown to have the form

$$\overset{s^2}{y''(t)} + \overset{As}{Ay'(t)} + \overset{B}{By(t)} = \overset{Cs^2}{Cx''(t)} + \overset{Ds}{Dx'(t)} + \overset{E}{Ex(t)}$$

Determine $(A + B) - (C + D + E)$. Choose the closest answer.

A: 2.
B: 1.
C: 0.
D: -2.
E: -1.

$H(s) = \frac{Cs^2 + Ds + E}{s^2 + As + B}$

$h(t) = \delta(t) - e^{-2t}\cos(t)u(t)$

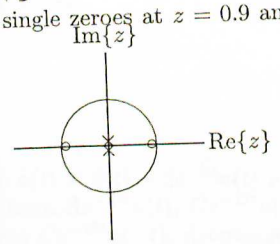
$H(s) = 1 - \frac{s+2}{(s+1)^2+1} = \frac{s^2+3s+3}{s^2+4s+5}$

$A = 4$
$B = 5$
$C = 1$
$D = 3$
$E = 3$

Question 3: Consider the pole-zero plot shown. The zeroes are single zeroes at $z = 0.9$ and $z = -0.9$, and the poles are single poles at $z = 0.1j$ and $-0.1j$. This filter is

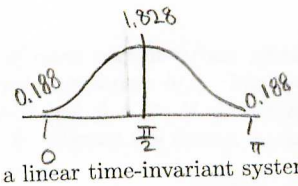
A: Bandpass.
B: Bandstop.
C: Lowpass.
D: Highpass.
E: Allpass.

$|H(e^{j\pi})| = 0.188$ already calculated!



$|H(e^{j0})| = \frac{(0.1)(1.9)}{\sqrt{1.01}\sqrt{1.01}} =$

$|H(e^{j\pi/2})| = \frac{\sqrt{1.81}\sqrt{1.81}}{0.9 * 1.1} =$



Question 4: Consider a linear time-invariant system with impulse response $h[n] = \delta[n-1] + 2^n u[n]$. Determine $H_I(z)$, the Z transform of the INVERSE system of $h[n]$. $H_I(z)$ has the form $A\frac{(1-Bz^{-1})}{(1-Cz^{-1})(1-Dz^{-1})}$. What is $(A + B) - (C + D)$? Choose the closest answer.

A: 5.
B: 1.
C: 4.
D: 3.
E: 2.

$H(z) = z^{-1} + \frac{1}{1-2z^{-1}} \rightarrow \frac{z^{-1} - 2z^{-2} + 1}{1-2z^{-1}}$

$H_I(z) = \frac{1-2z^{-1}}{1+z^{-1}-2z^{-2}} = \frac{(1-2z^{-1})}{(1-z^{-1})(1+2z^{-1})}$

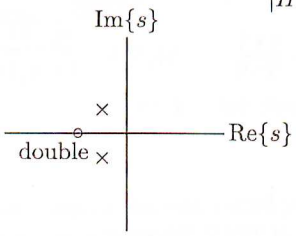
$A = 1$
$B = 2$
$C = 1$
$D = 2$

4

Answers: DAAC ACEB DDCC

Question 5: For the system with pole-zero plot shown, there are single poles at $s = -1 + j$ and $s = -1 - j$ and there is a double zero at $s = -2$. Determine $\frac{|H(j2)|}{|H(j0)|}$. Choose the closest answer. *Hint:* This can be determined from the plot.

A: 0.9.
B: 1.2.
C: 0.8.
D: 1.1.
E: 1.0.



$|H(j2)| = \frac{\sqrt{8}\cdot\sqrt{8}}{\sqrt{2}\sqrt{10}} = 1.788$

$|H(j0)| = \frac{(2)(2)}{\sqrt{2}\sqrt{2}} = 2$

$\frac{|H(j2)|}{|H(j0)|} = \frac{1.788}{2} = 0.89$

A

Question 6: Suppose $h(t) = 2e^{-t}u(t) - e^{-At}u(t)$. What should A be so that the system has a single zero at $s = -5$? Choose the closest answer.

A: 1.
B: 5.
C: 3.
D: 4.
E: 2.

$H(s) = \frac{2}{s+1} - \frac{1}{s+A} = \frac{s+2A-1}{(s+A)(s+1)} = \frac{s+5}{(s+A)(s+1)}$

$2A+1 = 5 \quad s = -5$

$A = 3$

Question 7: Suppose a linear time-invariant system with input $x[n]$ and output $y[n]$ is governed by the difference equation $y[n] - 3y[n-1] - 4y[n-2] = x[n] - 4x[n-2]$. Determine $H(z)$. The causal realization of $H(z)$ is $h[n] = A(B)^n u[n] + C(D)^n u[n] + E\delta[n]$. What is $B + D$? Choose the closest answer. *Hint:* It is not necessary to complete the partial-fraction expansion to answer this question.

A: 0.
B: 2.
C: 1.
D: 4.
E: 3.

$(1 - 3z^{-1} - 4z^{-2})Y(z) = (1 - 4z^{-2})X(z)$

$H(z) = \frac{1-4z^{-2}}{1-3z^{-1}-4z^{-2}} = \frac{1-4z^{-2}}{(1+z^{-1})(1-4z^{-1})} \xrightarrow{PFE} = 1 - \frac{3/5}{(1+z^{-1})} + \frac{3/5}{(1-4z^{-1})}$

$A = 0$
$B = 4$
$C = -1$
$D = -1$

$h[n] = \delta[n] - \frac{3}{5}(-1)^n u[n] + \frac{3}{5}(4)^n u[n]$

E

Question 8: Suppose a pre-sampled signal has maximum frequency 85 kHz and the signal is sampled at rate 200,000 samples per second. Determine the percentage oversampling. Choose the closest answer.

A: 27%.
B: 18%.
C: 15%.
D: 24%.
E: 21%.

$\frac{R_s - 2f_m}{2f_m} \quad \frac{200 - 2(85)}{2(85)} \rightarrow 17.6\%$

Question 9: Consider a linear time-invariant system with input $x(t) = \delta(t) - 3e^{-5t}u(t)$ and output $y(t) = e^{-5t}u(t)$. Determine all possible impulse responses $h(t)$. The answer has the form $Ae^{-Bt}u(t)$, $Ce^{-Dt}u(-t)$, or both. If the answer has the form $Ae^{-Bt}u(t)$, determine $A + B$. If the answer has the form $Ce^{-Dt}u(-t)$, determine $C + D$. If the answer is both, determine $A + B + C + D$. Choose the closest answer.

A: 1.
B: 5.
C: 3.
D: 4.
E: 2.

$X(s) = 1 - \frac{3}{s+5} = \frac{s+2}{s+5} \quad Re\{s\} > -5$

$Y(s) = \frac{1}{s+5} \quad Re\{s\} > -5$ (same)

$H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{1}{s+5}}{\frac{s+2}{s+5}} \quad Re\{s\} \gtrless Re\{-2\}$

$Re\{s\} \lessgtr -2$

$A = -1$
$B = 2$
$C = -1$
$D = 2$

Question 10: Consider two linear time-invariant systems with impulse responses $h_1(t) = \delta(t) + e^{-t}u(t)$ and $h_2(t) = \delta(t) - 3e^{-4t}u(t)$ and suppose that the two systems are arranged in a causal feedback combination (the system with impulse response $h_2(t)$ is the one "fed back"). Determine $H(s)$ for the composite feedback system. The answer has the form $A\frac{(s+B)(s+C)}{(s+D)(s+E)}$. Determine $(B+C)-(D+E)$. Choose the closest answer.

A: 0.
B: -1.
C: 1.
D: 2.
E: -2.

$H_1(s) = 1 + \frac{1}{s+1} = \frac{s+2}{s+1}$ $\quad H_2(s) = 1 - \frac{3}{s+4} = \frac{s+1}{s+4}$ $\quad H(s) = \frac{H_1(s)}{1+H_1(s)H_2(s)}$

$= \frac{\frac{s+2}{s+1}}{1+\frac{s+2}{s+1}\cdot\frac{s+1}{s+4}} \to \frac{1}{2}\cdot\frac{(s+2)(s+4)}{(s+1)(s+3)}$

$A = 1/2$
$B = 2$
$C = 4$ 6
$D = 1$
$E = 3$ 4

Question 11: Suppose a system with transfer function $H(z)$ is such that there is a double pole at $z = \frac{1}{2}$ and single zeroes at $z = -1+j$ and $z = -1-j$. Suppose $\sum_{n=-\infty}^{\infty} h[n] = 180$. What is $\sum_{n=-\infty}^{\infty}(-1)^n h[n]$? Choose the closest answer. *Hint:* Because $H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n}$, it follows that $\sum_{n=-\infty}^{\infty} h[n] = H(1)$ and $\sum_{n=-\infty}^{\infty}(-1)^n h[n] = H(-1)$.

A: 2.
B: 3.
C: 4.
D: 1.
E: 5.

$H(z) = \frac{k(z+1-j)(z+1+j)}{(z-1/2)^2} \to \frac{k(z^2+2z+2)}{(z-1/2)^2}$ $\quad \sum_{n=-\infty}^{\infty} h[n] = 180$ $\quad$ value of $H(1)$

$H(1) = \frac{k(1+2+2)}{1/4} = 180$
$k = 9$

$H(z) = \sum_{-\infty}^{\infty} h[n]z^{-n} \to H(-1) = \sum_{-\infty}^{\infty} h[n](-1)^n = \frac{9((-1)^2-2+2)}{(-1-1/2)^2} = 4$ (c)

Question 12: Consider a linear time-invariant system given by $H(z) = \frac{(1+2z^{-1})(1-2z^{-1})}{(1+z^{-1})(1-z^{-1})^2(1+\frac{1}{2}z^{-1})}$. Let T denote the total number of realizations of this system. Let C denote the number of these realizations that are causal and S the number that are stable. Determine $T+C+S$.

A: 7.
B: 3.
C: 4.
D: 6.
E: 5.

$H(z) = \underbrace{\frac{A}{1+z^{-1}}}_{|z|\geq|-1|} + \underbrace{\frac{B}{1-z^{-1}} + \frac{C}{(1-z^{-1})^2}}_{|z|\geq|1|} + \underbrace{\frac{D}{1+\frac{1}{2}z^{-1}}}_{|z|\geq|-1/2|}$

$|z| \leq 1$ $\quad |z| \geq 1/2$

$T = 3$
$C = 1$
$S = 0$ NONE INCLUDE 1

$|z| < 1/2$
$\frac{1}{2} < |z| < 1$
$|z| > 1$

8

5

6

Serial Number: **601** Name: SOLUTIONS

ECE 3300 SPRING 2016 (SIGNALS, SYSTEMS, AND TRANSFORMS): EXAM VI

Record your name on this test; record your name, student ID, and test serial number on the scantron. Enter the test serial number in *COURSE*; you may leave *SECTION* blank. You must show your work on every problem, showing all steps on your test. Do not use scratch paper or write your work anywhere but on the test. Circle your answers on the test and bubble in the corresponding answers on your scantron. The examination lasts 50 minutes and you may use six sheets of notes (front and back); no old test questions can be on your notes. Calculator use is permitted. There is one correct answer per question. In problems asking to find coefficients A, B, C, etc., some of these coefficients may equal zero.

Question 1: Consider a linear time-invariant system with impulse response $h(t) = (2t-1)e^{-t}u(t)$. The input-ouput differential equation for this system can be shown to have the form

$$y''(t) + Ay'(t) + By(t) = Cx''(t) + Dx'(t) + Ex(t)$$

$$s^2Y(s) + As\,Y(s) + B\,Y(s) = Cs^2X(s) + DsX(s) + EX(s)$$

Determine $A+B+C+D+E$. Choose the closest answer.

$$Y(s)(s^2 + As + B) = X(s)(Cs^2 + Ds + E)$$

A: 1.
B: 4.
C: 2.
D: 5.
E: 3.

$$H(s) = \frac{Cs^2 + Ds + E}{s^2 + As + B}$$

$$h(t) = (2t-1)e^{-t}u(t) = 2te^{-t}u(t) - e^{-t}u(t)$$

$$H(s) = \frac{2}{(s+1)^2} - \frac{1}{s+1} = \frac{1-s}{s^2+}$$

$A=2$ $B=1$ $C=0$
$D=-1$ $E=1$

Question 2: Suppose a pre-sampled signal has maximum frequency 425 kHz and the signal is sampled at rate 1,000,000 samples per second. Determine the percentage oversampling. Choose the closest answer.

A: 27%.
B: 24%.
C: 15%.
D: 18%.
E: 21%.

$$\frac{R_s - 2f_m}{2f_m} = \frac{1000 - 2(425)}{2(425)} \times 100 = 17.6\% \approx \boxed{18\%}$$

Question 3: Suppose a causal system with transfer function $H(s)$ is such that there are single poles at $s=-2$ and $s=-4$ and single zeroes at $s=-1$ and $s=-5$. Assume $H(0)=1$. What is $H(4)$? Choose the closest answer.

A: 5.5.
B: 3.5.
C: 2.5.
D: 4.5.
E: 1.5.

$$H(s) = \frac{K(s+1)(s+5)}{(s+2)(s+4)} \quad \text{zeroes / poles}$$

$$\Rightarrow \frac{k(0+1)(0+5)}{(0+2)(0+4)} = 1 \rightarrow k = \frac{8}{5}$$

$$H(s) = \frac{\frac{8}{5}(s+1)(s+5)}{(s+2)(s+4)} \qquad H(4) = \frac{\frac{8}{5}\cdot 5 \cdot 9}{6 \cdot 8} = 1.5$$

E

Answers: EDEE CABA DEDC

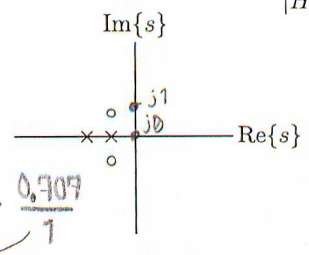
Distance between given point (j1) and zeroes (numerator) and poles denom.

$\sqrt{1^2+2^2}$

2 1 1 1

Question 4: For the system with pole-zero plot shown, there are single poles at $s = -1$ and $s = -2$ and there are single zeroes at $s = -1 + j$ and $s = -1 - j$. Determine $\frac{|H(j1)|}{|H(j0)|}$. Choose the closest answer. *Hint:* This can be determined from the plot.

A: 0.4.
B: 1.3.
C: 1.0.
D: 1.6.
E: 0.7.



$$|H(j1)| = \frac{(1)\sqrt{5}}{\sqrt{2}\cdot\sqrt{5}} = \frac{1}{\sqrt{2}} = 0.707$$

zeroes

poles

$$|H(j0)| = \frac{\sqrt{2}\cdot\sqrt{2}}{1\cdot 2} = \frac{2}{2} = 1$$

$$\frac{|H(j1)|}{|H(j0)|} = \frac{0.707}{1}$$

Question 5: Suppose a linear time-invariant system with input $x[n]$ and output $y[n]$ is governed by the difference equation $y[n] - 4y[n-1] + 4y[n-2] = x[n] - 2x[n-1] + x[n-2]$. Determine $H(z)$. The causal realization of $H(z)$ is $h[n] = (A + Bn)(C)^n u[n] + D\delta[n]$. What is C? Choose the closest answer. *Hint:* It is not necessary to complete the partial-fraction expansion to answer this question.

A: −2.
B: 1.
C: 2.
D: −1.
E: 0.

$$Y(z) - 4z^{-1}Y(z) + 4z^{-2}Y(z) = X(z) - 2z^{-1}X(z) + z^{-2}X(z)$$

$$\Rightarrow Y(z)(1 - 4z^{-1} + 4z^{-2}) = X(z)(1 - 2z^{-1} + z^{-2})$$

$$\Rightarrow H(z) = \frac{1-2z^{-1}+z^{-1}}{1-4z^{-1}+4z^{-2}} = \frac{(1-z^{-1})^2}{(1-2z^{-1})^2} = D + \frac{A}{(1-2z^{-1})} + \frac{B}{(1-2z^{-1})^2}$$

$$h[n] = D\delta[n] + A(2)^n u[n] + Bn(2)^n u[n] \qquad C = 2$$

Question 6: Consider a linear time-invariant system with input $x[n] = \delta[n] - 2(-1)^n u[n]$ and output $y[n] = (-1)^n u[n]$. Determine all possible impulse responses $h[n]$. The answer has the form $Au[n]$, $Bu[-n-1]$, or both. If the answer has the form $Au[n]$, determine A. If the answer has the form $Bu[-n-1]$, determine B. If the answer is both, determine $A + B$. Choose the closest answer.

$|z| > |-1| \rightarrow |z| > 1$

A: −1.
B: 2.
C: −2.
D: 0.
E: 1.

$$X(z) = 1 - \frac{2}{1+z^{-1}} = \frac{-(1-z^{-1})}{(1+z^{-1})}$$

$$Y[n] = (-1)^n u[n] \rightarrow Y(z) = \frac{1}{1+z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1+z^{-1}} \cdot \frac{(1+z^{-1})}{-(1+z^{-1})}$$

$$= -\frac{1}{1-z^{-1}}$$

$$-(1)^n u[n] \qquad A = -1$$

Question 7: Consider two linear time-invariant systems with impulse responses $h_1[n] = (\frac{1}{2})^n u[n]$ and $h_2[n] = \frac{1}{2}\delta[n]$ and suppose that the two systems are arranged in a causal feedback combination (the system with impulse response $h_2[n]$ is the one "fed back"). Determine $h[n]$ for the composite feedback system. The answer has the form $A(B)^n u[n]$. Determine $A + B$. Choose the closest answer. *Hint:* $\frac{c}{a-bz^{-1}} = \frac{c/a}{1-(b/a)z^{-1}}$.

A: $\frac{5}{3}$.
B: 1.
C: $\frac{1}{3}$.
D: $\frac{2}{3}$.
E: $\frac{4}{3}$.

$$H_1(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \qquad H_2(z) = \frac{1}{2}$$

$$H(z) = \frac{H_1(z)}{1 + H_1(z)H_2(z)} = \frac{\frac{1}{1-\frac{1}{2}z^{-1}}}{1 + \frac{1/2}{1-\frac{1}{2}z^{-1}}} = \frac{1}{\frac{3}{2} - \frac{1}{2}z^{-1}} \Rightarrow \frac{2}{3}\cdot\frac{1}{1-\frac{1}{3}z^{-1}}$$

$$h[n] = \frac{2}{3}\left(\frac{1}{3}\right)^n u(n)$$

$$A + B = \frac{2}{3} + \frac{1}{3} = 1$$

Distances of 3 poles to something

Question 8: Consider the pole-zero plot shown. The zeroes are single zeroes at $z = 1$, $z = \frac{1}{\sqrt{2}}(1+j)$, and $z = \frac{1}{\sqrt{2}}(1-j)$, and the pole is a triple pole at $z = 0$. This filter is

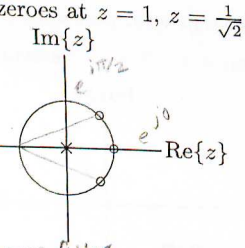
*drawn

A: Highpass.
B: Lowpass.
C: Bandpass.
D: Allpass.
E: Bandstop.

$|H(e^{j0})| = \frac{0}{\text{[illegible]}} = \emptyset$

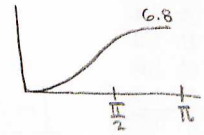
$|H(e^{j\frac{\pi}{2}})| = \frac{\sqrt{2}\cdot\sqrt{(1-\frac{1}{2})^2+(\frac{1}{\sqrt{2}})^2}\cdot\sqrt{(1-\frac{1}{2})^2+(\frac{1}{\sqrt{2}})^2}}{(1)^3}$

$= 0.999 \approx 1$



Highpass filter

$|H(e^{j\pi})| = \frac{2\sqrt{(1+\frac{1}{\sqrt{2}})^2+(\frac{1}{\sqrt{2}})^2}^{\,2}}{(1)^3}$



Question 9: Suppose an LTI system has impulse response $h(t) = 7e^{-3t}u(t) - \delta(t)$ and input $x(t) = (7+14t)e^{4t}u(-t)$. The output has the form $y(t) = Ae^{4t}u(-t) + Be^{-3t}u(t)$. Determine $A + B$. Choose the closest answer.

A: 2.
B: 5.
C: 4.
D: 3.
E: 1.

$H(s) = \frac{7}{s+3} - 1 = \frac{-(s-4)}{s+3}$ (common denominator)

$X(s) = -\left\{\frac{7}{s-4} + \frac{14}{(s-4)^2}\right\} = -\left\{\frac{7s-28+14}{(s-4)^2}\right\}$

$= \frac{-7(s-2)}{(s-4)^2}$ $Re\{s\} > -3$

$x(t) = (7+14t)e^{4t}u(-t)$

$= (-7-14t)e^{-(-4)t}u(t)$

$= -\{-7e^{-(-4t)}u(-t) - 14te^{-(-4t)}u(-t)\}$

$Y(s) = X(s)H(s) \rightarrow = \frac{7(s-2)}{(s-4)(s+3)} = \frac{2}{s-4} + \frac{5}{s+3} \rightarrow = -2e^{4t}u(-t) + 5e^{-3t}u(t)$

$A = -2$ $B = 5$

Question 10: Suppose $h[n] = -(-2)^n u[-n-1] + u[n]$. How many zeroes does $H(z)$ have? Choose the closest answer.

A: 1.
B: 3.
C: 0.
D: 4.
E: 2.

$H(z) = \frac{1}{1+2z^{-1}} + \frac{1}{1-z^{-1}} \rightarrow \frac{z}{z+2} + \frac{z}{z-1} \rightarrow \frac{2z^2+z}{(z+2)(z-1)} = \frac{z(2z+1)}{(z+2)(z-1)}$

$z = 0$ $2z+1=0$

$z = 1/2$

2 zeroes

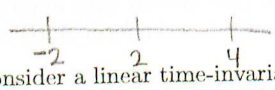
Question 11: Consider a linear time-invariant system given by $H(s) = \frac{1}{s+2} + \frac{1}{(s+2)^2} + \frac{1}{s-2} + \frac{1}{s-4}$. Let T denote the total number of realizations of this system. Let C denote the number of these realizations that are causal and S the number that are stable. Determine $T + C + S$.

Look @ ROC

A: 3.
B: 5.
C: 7.
D: 6.
E: 4.

$\frac{1}{s+2} + \frac{1}{(s+2)^2} + \frac{1}{s-2} + \frac{1}{s-4}$

$Re\{s\} \geq -2$ $Re\{s\} \geq 2$ $Re\{s\} \leq 4$



$Re\{s\} < -2$

$-2 < Re\{s\} < 2$

$2 < Re\{s\} < 4$

$Re\{s\} > 4$

Possibilities

Causal! Need ROC to the right of the rightmost pole.

$T = 4$ $C = 1$ $S = 1$ $= 6$

Causal

Stable

Where $\emptyset$ is contained in it

Something larger than largest pole

Question 12: Consider a linear time-invariant system with impulse response $h(t) = \delta(t) + 2e^{-t}\cos(t)u(t)$. Determine $h_I(t)$, the impulse response of the causal INVERSE system. This impulse response has the form $h_I(t) = A\delta(t) + (Bt+C)e^{-Dt}u(t)$. What is $A+B+C+D$? Choose the closest answer.

A: 1.
B: 4.
C: 3.
D: 5.
E: 2.

$H(s) = 1 + \frac{2(s+1)}{(s+1)^2+1} \rightarrow \frac{s^2+4s+4}{s^2+2s+2}$

$H_I(s) = \frac{s^2+2s+2}{s^2+4s+4} = \frac{s^2+2s+2}{(s+2)^2}$

(PFE) $= 1 - \frac{2}{s+2} + \frac{2}{(s+2)^2}$

$h(t) = \delta(t) - 2e^{-2t}u(t) + 2te^{-2t}u(t)$

$= \delta(t) + (2t-2)e^{-2t}u(t)$