IEOR165_Project

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Introduction This project seeks to explore the linear coefficients that best fit the relationship between the various features of wine, and the quality score it is given by experts. The features we'll be exploring will be from the following list:

- Fixed Acidity
- Volatile Acidity
- Citric Acid
- Residual Sugar
- Chlorides
- Free sulfur dioxide
- Total sulfur dioxide
- Density
- pH
- Sulphates
- Alcohol concentration

The report is broken into 5 sections: 1. Clening the data and describig it through scatterplots 2. An analysis of the results using Ordinary Least Squares 3. An analysis of the results using Ridge Regression (L2 Normalization) 4. An analysis of the results using LASSO Regression (L1 Normalization) 5. An analysis of the results using Elastic Net (L1 + L2 Normalization) 6. Presentation of findings in a table format.

Data Cleaning and Scatterplot For some basic clarity, we'll illustrate some scatterplots of the various features of the wine with the quality

Some things to note

- since the quality is effectively a categorical ordinal data, the values for qualiy are jittered in the scatterplots to give a better illustration of density (otherwise they would be on top of each other)
- there seems to be a slight positive correlation between quality and the following features, which would let us expect their coefficients to be positive
  - fixed acidity
  - citric acid
  - sulphates
  - alcohol content
- a slight negative correlation between quality and the following features, which would let us expect their coefficients to be negative
  - volatile acidity

    - chlorides
    - total sulphur dioxide
    - density
    - pH
- no meaningful correlation between quality and the following, which would let us expect thier coefficients to be close to zero
    - residual sugar
    - free sulphur dioxide

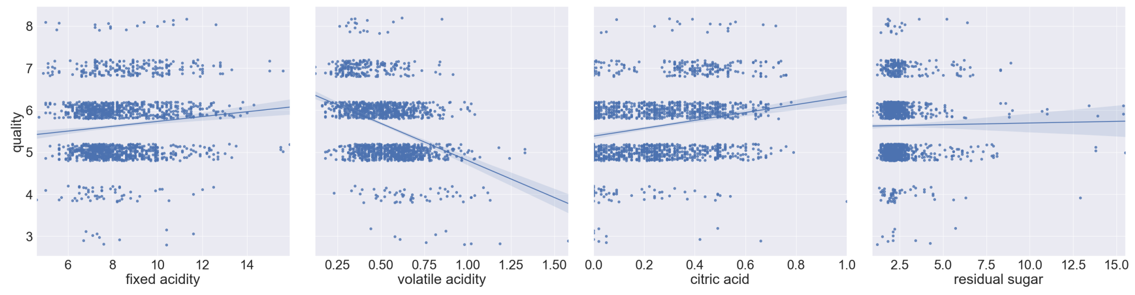
[1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline
import seaborn as sns
```

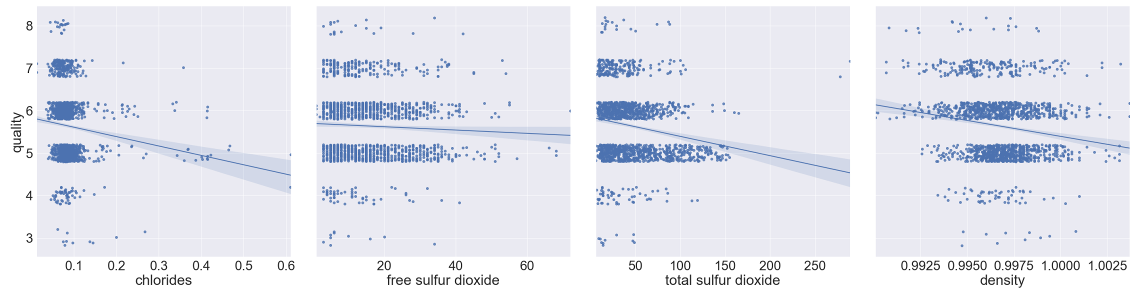
[2]:
```
headers = ["fixed acidity", "volatile acidity", "citric acid",
           "residual sugar", "chlorides", "free sulfur dioxide",
           "total sulfur dioxide", "density", "pH",
           "sulphates","alcohol","quality"]
wine = pd.read_csv('wine-quality-red.csv', header = None, skiprows = 1, sep = ';
 ↪', names = headers)
#wine = (wine-wine.min())/(wine.max()-wine.min())
# this last line normalizes the data onto a [0,1] mode, but
#display((wine.max(), wine.min()),
wine.head()
```

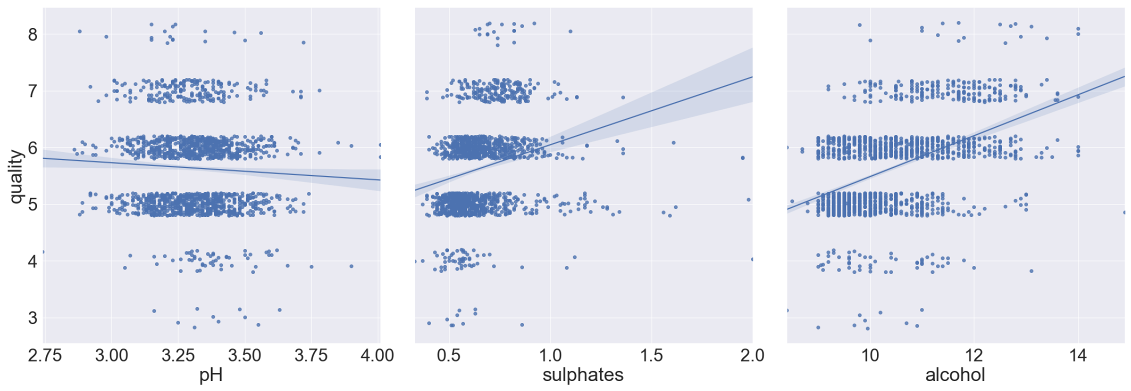
[2]:
```
   fixed acidity  volatile acidity  citric acid  residual sugar  chlorides  \
0            7.4              0.70         0.00             1.9      0.076
1            7.8              0.88         0.00             2.6      0.098
2            7.8              0.76         0.04             2.3      0.092
3           11.2              0.28         0.56             1.9      0.075
4            7.4              0.70         0.00             1.9      0.076

   free sulfur dioxide  total sulfur dioxide  density    pH  sulphates  \
0                 11.0                  34.0   0.9978  3.51       0.56
1                 25.0                  67.0   0.9968  3.20       0.68
2                 15.0                  54.0   0.9970  3.26       0.65
3                 17.0                  60.0   0.9980  3.16       0.58
4                 11.0                  34.0   0.9978  3.51       0.56

   alcohol  quality
0      9.4        5
1      9.8        5
2      9.8        5
3      9.8        6
4      9.4        5
```

```
[3]: sns.set(font_scale=3)
     a = sns.PairGrid(wine, y_vars=["quality"],
                      x_vars=["fixed acidity", "volatile acidity", "citric acid",␣
      ↪"residual sugar"],
                      height=10)
     b = sns.PairGrid(wine, y_vars=["quality"],
                      x_vars=["chlorides", "free sulfur dioxide", "total sulfur␣
      ↪dioxide", "density"],
                      height=10)
     c = sns.PairGrid(wine, y_vars=["quality"],
                      x_vars=["pH", "sulphates","alcohol"],
                      height=10)
     a.map(sns.regplot, y_jitter=0.2) #using a regplot bc scatterplot y_jiggle is␣
      ↪broken
     b.map(sns.regplot, y_jitter=0.2)
     c.map(sns.regplot, y_jitter=0.2)
```

```
[3]: <seaborn.axisgrid.PairGrid at 0x118a90e90>
```







Test / Training Split To begin, we will start by splitting our data into training and test samples. To do this we'll be using the train_test_split method from the sklearn.model_selection library. We will use the random seed 47 to be able to regenerate this data at a later date if we need to verify our results

```
[4]: from sklearn.model_selection import train_test_split
     np.random.seed(47)

     X = wine.drop(['quality'], axis=1)
     Y = pd.Series(wine['quality'])

     X_train, X_test, Y_train, Y_test = train_test_split(X, Y, test_size=0.1)
```

Root Mean Squared Error In order to assess how much error a prediction has compared to its actual results, we'll define a simple function called RMSE which assess the accuracy of a predicted set of wine qualities against the true set of wine qualities. This function takes two arrays, one of predicted and one of actual values as inputs.

```
[5]: def rmse(actual_y, predicted_y):
         return np.sqrt( sum((actual_y - predicted_y)**2) / len(predicted_y) )
```

Ordinary Least Squares (OLS) Regression

In the following section, we use Ordinary Least Squares regression to find coefficient matrices that can be used to predict the wine quality. To do this, we use the sklearn.linear_model.LinearRegression() method (documentation on https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LinearRegression.html), which the site describes as:

Ordinary least squares Linear Regression. LinearRegression fits a linear model with coefficients w = (w1, …, wp) to minimize the residual sum of squares between the observed targets in the dataset, and the targets predicted by the linear approximation.

There are no tuning parameters related to Ordinary Least Squares, so cross-validation is not necessary in this case:

[6]:
```
from sklearn.linear_model import LinearRegression

OLS_model = LinearRegression(normalize=True)
OLS_model.fit(X = X_train, y = Y_train)
display(OLS_model.coef_,
        OLS_model.intercept_) #intercept is the constant
```

```
array([ 2.38691870e-02, -1.04951484e+00, -1.68018124e-01,  1.48270614e-02,
       -1.61581005e+00,  5.20196398e-03, -3.27828674e-03, -1.10834974e+01,
       -4.08774853e-01,  9.44402334e-01,  3.04678999e-01])
```

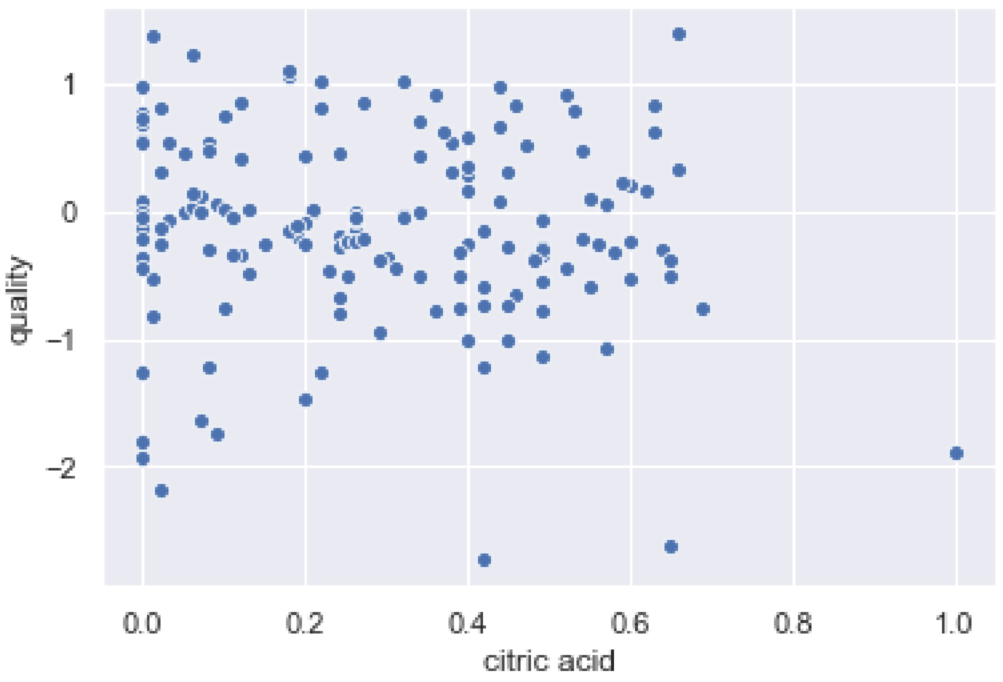
```
14.826190192089786
```

To illustrate the accuracy of this linear regression model, we'll assess both the RMSE of the created training model's ability to predict the qualiy of wine on the original training set and test set. Obviously, training error is lower than test error, but in general both values are well within a range of 1 and are fairly decent predcitions.

The graph below shows the absolute difference in predicted quality vs actual quality for the test data set, as a function of citric acid, Citric acid was chosen becasue it has a fairly even distribution and would be easier to observe. Generally, it seems like there are quite a few points dramatically underpredicting, and far fewer points dramatically overpredicting.

[7]:
```
OLS_train_prediction = OLS_model.predict(X_train)
OLS_test_prediction = OLS_model.predict(X_test)

OLS_train_rmse = rmse(Y_train, OLS_train_prediction)
OLS_test_rmse = rmse(Y_test, OLS_test_prediction)

OLS_accuracy = Y_test - OLS_test_prediction
sns.set(font_scale=1, )
sns.scatterplot(x=X_test['citric acid'], y=OLS_accuracy); #chose to use citric␣
 ↪acid as the x
                                                          #axis since it showed the␣
 ↪most even distribution of points

print('OLS Training Error: ', OLS_train_rmse)
print('OLS Test Error:', OLS_test_rmse)
```

```
OLS Training Error:  0.6344050984347456
OLS Test Error: 0.7452052680314947
```



Ridge Regression

In the following section, we use ridge regression to create a linear model that predic-cts wine quality using the features of wine. We'll be using the Ridge method from the sklearn.linear_model library, documentation for which can be found at: https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.Ridge.html#sklearn.linear_model.Ridge

Additionaly, we'll be using the KFold method from the sklearn.model_selection library to assess cross-validation error. The documentation for KFold can be found here: https://scikit-learn.org/stable/modules/generated/sklearn.model_selection.KFold.html

For this function, the μ tuning parameter is referred to as alpha in the Ridge regression function. To find the optimal tuning paramter value, we'll first define a function that computes cross validation error called compute_cv_error (imported from the UC Berkeley Data 100 curriculum). This method is return the average cross-validatin error of a given model on a specific X_train and Y_train data set.

[8]:
```
from sklearn.model_selection import KFold

def compute_CV_error(model, X_train, Y_train):
    kf = KFold(n_splits=5)
    validation_errors = []

    for train_idx, valid_idx in kf.split(X_train):
        # split the data
        split_X_train, split_X_valid = X_train.iloc[train_idx], X_train.
↪iloc[valid_idx]
        split_Y_train, split_Y_valid = Y_train.iloc[train_idx], Y_train.
↪iloc[valid_idx]

        # Fit the model on the training split
        model.fit(X = split_X_train, y = split_Y_train)
        Y_prediction = model.predict(split_X_valid)

        # Compute the RMSE on the validation split
        error = rmse(split_Y_valid, Y_prediction)

        validation_errors.append(error)

    return np.mean(validation_errors)
```

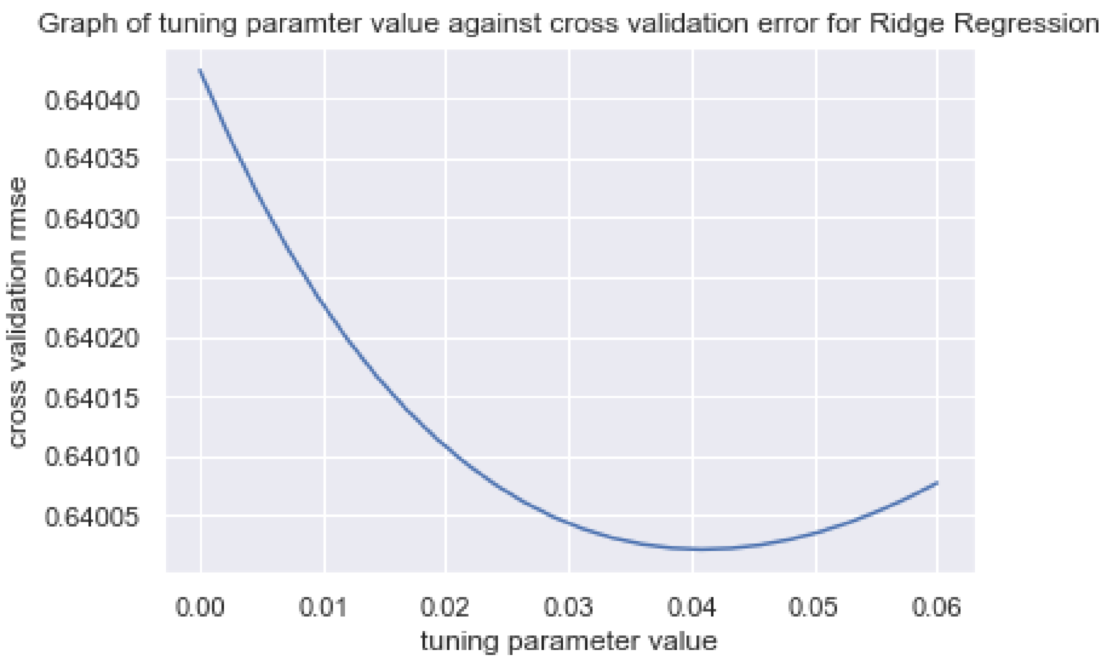
We'll run this method iteratively over multiple ridge regression models with different tuning paramter μ until we get the one with lowest cross validation error.

[9]:
```
from sklearn.linear_model import Ridge

errors = []
for i in np.linspace(0,0.06,26):
    model = Ridge(alpha = i, normalize=True)

    # compute the cross validation error
    error = compute_CV_error(model, X_train, Y_train)
    i = i.round(3)
    print(f"Alpha value: {i} \t Cross Validation Error: {error}")
    errors.append(error)

best_err = min(errors)

print(f'\nBEST ERROR VALUE IS {best_err}')
```

```
Alpha value: 0.0         Cross Validation Error: 0.6404235314757651
Alpha value: 0.002       Cross Validation Error: 0.6403676116415922
Alpha value: 0.005       Cross Validation Error: 0.6403176514317692
Alpha value: 0.007       Cross Validation Error: 0.6402730412585299
Alpha value: 0.01        Cross Validation Error: 0.6402332709303055
Alpha value: 0.012       Cross Validation Error: 0.6401979094712142
Alpha value: 0.014       Cross Validation Error: 0.6401665896448321
Alpha value: 0.017       Cross Validation Error: 0.6401389959641258
Alpha value: 0.019       Cross Validation Error: 0.6401148553134337
Alpha value: 0.022       Cross Validation Error: 0.640093929547736
Alpha value: 0.024       Cross Validation Error: 0.6400760096031334
Alpha value: 0.026       Cross Validation Error: 0.6400609107727568
Alpha value: 0.029       Cross Validation Error: 0.6400484688890753
Alpha value: 0.031       Cross Validation Error: 0.6400385372167833
Alpha value: 0.034       Cross Validation Error: 0.640030983906964
Alpha value: 0.036       Cross Validation Error: 0.6400256898977597
Alpha value: 0.038       Cross Validation Error: 0.640022547172667
Alpha value: 0.041       Cross Validation Error: 0.6400214573071022
Alpha value: 0.043       Cross Validation Error: 0.6400223302487655
Alpha value: 0.046       Cross Validation Error: 0.6400250832887323
Alpha value: 0.048       Cross Validation Error: 0.6400296401890035
Alpha value: 0.05        Cross Validation Error: 0.6400359304390907
Alpha value: 0.053       Cross Validation Error: 0.6400438886195701
Alpha value: 0.055       Cross Validation Error: 0.6400534538547309
Alpha value: 0.058       Cross Validation Error: 0.6400645693398072
Alpha value: 0.06        Cross Validation Error: 0.6400771819309071

BEST ERROR VALUE IS 0.6400214573071022
```

[10]:
```python
ridge = sns.lineplot(x=np.linspace(0,0.06,26), y=errors)
ridge.set_xlabel('tuning parameter value')
ridge.set_ylabel('cross validation rmse')
ridge.set_title('Graph of tuning paramter value against cross validation error␣
 ↪for Ridge Regression')
```

[10]:
```
Text(0.5, 1.0, 'Graph of tuning paramter value against cross validation error
for Ridge Regression')
```



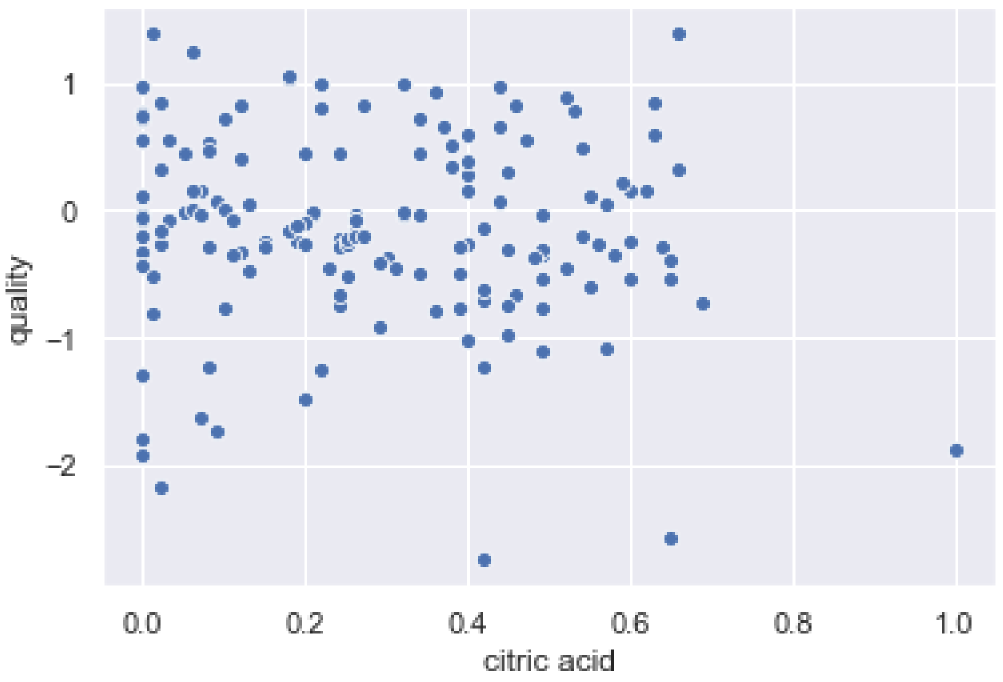
From the above graph and computed results, we see that the tuning parameter that produces the lowest cross validation error is at 0.041. Well use 0.041 as our tuning paramter for our overall ridge regression model for the entire dataset, and find the optiaml table of coefficients for our model for ridge regression and a tuning paramter of 0.041.

[11]:
```
Ridge_model = Ridge(alpha = 0.041, normalize=True)
Ridge_model.fit(X = X_train, y = Y_train)
display(Ridge_model.coef_,
        Ridge_model.intercept_) #intercept is the constant
```

```
array([ 3.14266009e-02, -9.96344821e-01, -8.84210705e-02,  1.84913959e-02,
       -1.58541597e+00,  4.52869527e-03, -3.10096729e-03, -2.26597901e+01,
       -3.06091137e-01,  9.27324408e-01,  2.81858005e-01])


26.15182248149363
```

[12]:
```
Ridge_train_prediction = Ridge_model.predict(X_train)
Ridge_test_prediction = Ridge_model.predict(X_test)

Ridge_train_rmse = rmse(Y_train, Ridge_train_prediction)
Ridge_test_rmse = rmse(Y_test, Ridge_test_prediction)

Ridge_accuracy = Y_test - Ridge_test_prediction
sns.set(font_scale=1)
sns.scatterplot(x=X_test['citric acid'], y=Ridge_accuracy); #chose to use␣
 ↪citric acid as the x
                                                            #axis since it showed the␣
 ↪most even distribution of points

print('Ridge Training Error: ', Ridge_train_rmse)
print('Ridge Test Error:', Ridge_test_rmse)
```

```
Ridge Training Error:  0.6347186608086561
Ridge Test Error: 0.7412040417792838
```



LASSO Regression

In the following section, we use lasso regression to create a linear model that predicts wine quality using the features of wine. We'll be using the Lasso method from the sklearn.linear_model library, documentation for which can be found here: https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.Lasso.html#sklearn.linear_model.Lasso

Everything else in this section will be identical to the methodology used in the Ridge Regression section. We'll use the same compute_cv_error method, and almost identical for loop to calculate best Lasso regression parameter μ.

Sincere there is no need to redefine compute_cv_error, we'll run the for loop and generate the graph of error against data right away.

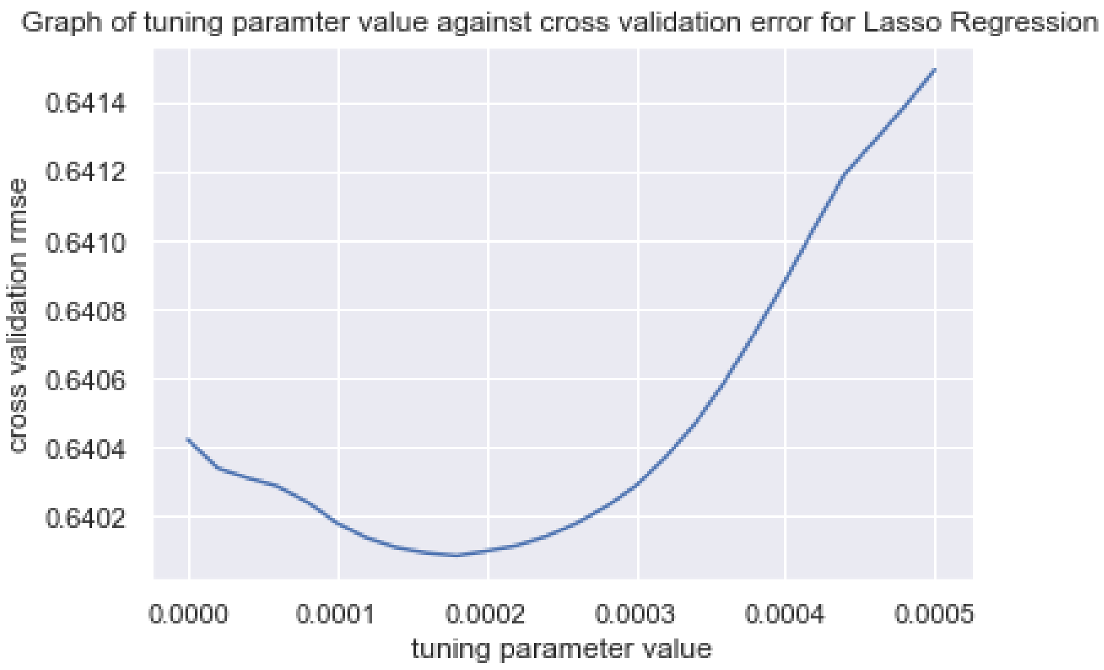
EDIT: The method also advised us against using Lasso regression, as it threw many errors indicating that the function doesn't converge properly. Never the less, to avoid cluttering the page with errors, I imported a library to ignore the errors so that we can illustrate what the effect of implementing LASSO regression was.

[13]:
```
import warnings
warnings.filterwarnings('ignore')

from sklearn.linear_model import Lasso

lasso_errors = []
for i in np.linspace(0,0.0005,26):
    model = Lasso(alpha = i, normalize=True)
    # compute the cross validation error
    error = compute_CV_error(model, X_train, Y_train)
    i = i.round(7)
    print(f"Alpha value: {i} \t Cross Validation Error: {error}")
    lasso_errors.append(error)

best_err = min(lasso_errors)

print(f'\nBEST ERROR VALUE IS {best_err}')
```

```
Alpha value: 0.0         Cross Validation Error: 0.6404235314757651
Alpha value: 2e-05       Cross Validation Error: 0.6403405670438754
Alpha value: 4e-05       Cross Validation Error: 0.6403113025218531
Alpha value: 6e-05       Cross Validation Error: 0.6402874561474643
Alpha value: 8e-05       Cross Validation Error: 0.6402419144817226
Alpha value: 0.0001      Cross Validation Error: 0.6401808124198474
Alpha value: 0.00012     Cross Validation Error: 0.6401372610218108
Alpha value: 0.00014     Cross Validation Error: 0.6401095761606023
Alpha value: 0.00016     Cross Validation Error: 0.6400943762559467
Alpha value: 0.00018     Cross Validation Error: 0.6400862245664147
Alpha value: 0.0002      Cross Validation Error: 0.6400997934492825
Alpha value: 0.00022     Cross Validation Error: 0.6401150596989582
Alpha value: 0.00024     Cross Validation Error: 0.6401417029875566
Alpha value: 0.00026     Cross Validation Error: 0.6401795218805453
Alpha value: 0.00028     Cross Validation Error: 0.6402278379529085
Alpha value: 0.0003      Cross Validation Error: 0.6402882625939659
Alpha value: 0.00032     Cross Validation Error: 0.6403721000673442
Alpha value: 0.00034     Cross Validation Error: 0.640469936542312
Alpha value: 0.00036     Cross Validation Error: 0.6405938914368517
Alpha value: 0.00038     Cross Validation Error: 0.6407339605383389
Alpha value: 0.0004      Cross Validation Error: 0.6408825542627445
Alpha value: 0.00042     Cross Validation Error: 0.6410401654173192
Alpha value: 0.00044     Cross Validation Error: 0.6411929691022384
Alpha value: 0.00046     Cross Validation Error: 0.6412893950968735
Alpha value: 0.00048     Cross Validation Error: 0.6413882043954977
Alpha value: 0.0005      Cross Validation Error: 0.64149410291968

BEST ERROR VALUE IS 0.6400862245664147
```

[14]:
```
lasso = sns.lineplot(x=np.linspace(0,0.0005,26), y=lasso_errors, )
lasso.set_xlabel('tuning parameter value')
lasso.set_ylabel('cross validation rmse')
lasso.set_title('Graph of tuning paramter value against cross validation error␣
 ↪for Lasso Regression')
```

```
[14]: Text(0.5, 1.0, 'Graph of tuning paramter value against cross validation error
      for Lasso Regression')
```



From the above graph and computed results, we see that the tuning parameter that produces the lowest cross validation error for Lasso regression is 0.00018.

We will execute the same code used to generate the coefficients of the Lasso regression with tuning parameter 0.00018. We notice that the coefficients for citric acid and density have been pushed to zero.

```
[15]: Lasso_model = Lasso(alpha = 0.00018, normalize=True)
      Lasso_model.fit(X = X_train, y = Y_train)
      display(Lasso_model.coef_,
              Lasso_model.intercept_) #note that these values are idnetical to the
                                      #values produced by the OLS prediction model
```

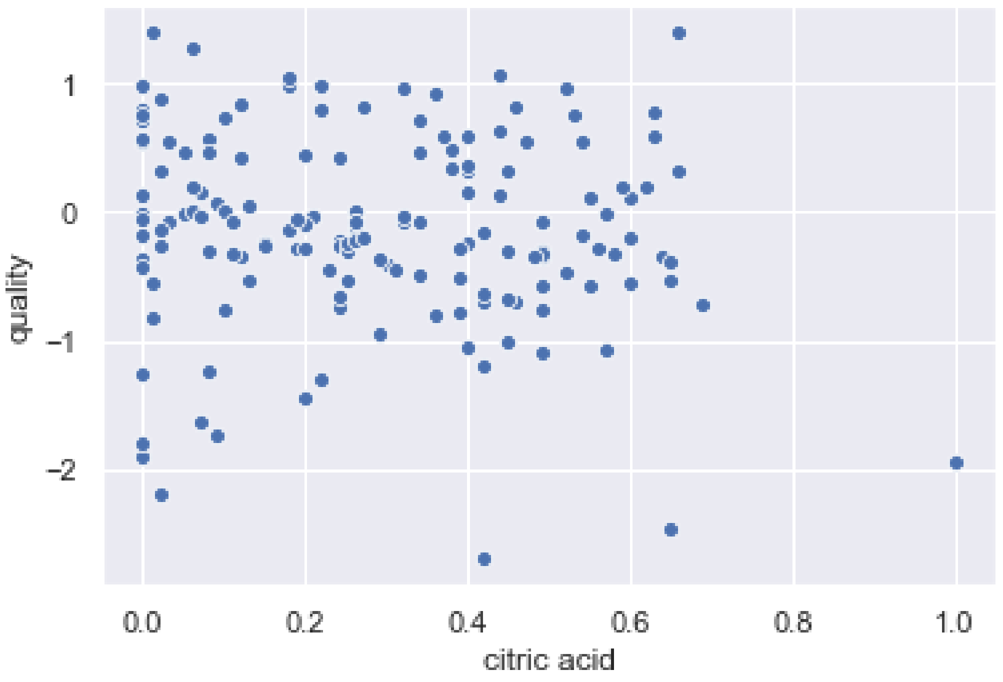
```
array([ 0.00418636, -0.98961588, -0.        ,  0.00435781, -1.49653343,
        0.00364269, -0.00282405, -0.        , -0.37537327,  0.87170008,
        0.30809713])
```

```
3.7868343920641454
```

```
[16]: Lasso_train_prediction = Lasso_model.predict(X_train)
      Lasso_test_prediction = Lasso_model.predict(X_test)
```

```
Lasso_train_rmse = rmse(Y_train, Lasso_train_prediction)
Lasso_test_rmse = rmse(Y_test, Lasso_test_prediction)

Lasso_accuracy = Y_test - Lasso_test_prediction
sns.set(font_scale=1)
sns.scatterplot(x=X_test['citric acid'], y=Lasso_accuracy); #chose to use␣
 ↪citric acid as the x
                                                           #axis since it showed the␣
 ↪most even distribution of points

print('LASSO Training Error: ', Lasso_train_rmse)
print('LASSO Test Error:', Lasso_test_rmse)
```

```
LASSO Training Error:  0.6351835876893493
LASSO Test Error: 0.7446134294028067
```



Elastic Net Regression

In the following section, we use elastic net regression to create a linear model that predicts wine quality using the features of wine. We'll be using the ElasticNet method from the sklearn.linear_model library found on https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.ElasticNet.html#sklearn.linear_model.ElasticNet.

We will be using the same compute_cv_error function, but our for loop will instead have two sections, to adjust the two different tuning parameters of an Elastic Net regression: *alpha* is the value of L1 + L2 ratio multipliers added togehter, while $l1 - ratio$ is the percentage of that ratio which is comprised of L1.

For example, an ElasticNet Model with $alpha = 2$, and $li - ratio = 0.75$ is the same as having an L1 coefficient of 1.5 and L2 coefficient of 0.5.

Sincere there is no need to redefine compute_cv_error, we'll jump right into the altered for loop for ElasticNet regression:

```
[17]: from sklearn.linear_model import ElasticNet

elastic_errors = []
for i in np.linspace(0.00003,0.00004,5):
    for j in np.linspace(0.000000000005,0.00000000002,5):
        model = ElasticNet(alpha = i, l1_ratio=j, normalize=True)

        # compute the cross validation error
        error = compute_CV_error(model, X_train, Y_train)
        i = i.round(7)
        j = j.round(14)
        print(f"Alpha value: {i} \t L1_Ratio: {j} \t \t Cross Validation Error:␣
 ↪{error}")
        elastic_errors.append(error)

best_err = min(elastic_errors)

print(f'\nBEST ERROR VALUE IS {best_err}')
```

```
Alpha value: 3e-05       L1_Ratio: 5e-12                 Cross Validation Error:
0.6400287252529583
Alpha value: 3e-05       L1_Ratio: 8.75e-12              Cross Validation Error:
0.6400287252529591
Alpha value: 3e-05       L1_Ratio: 1.25e-11              Cross Validation Error:
0.6400287252529593
Alpha value: 3e-05       L1_Ratio: 1.625e-11             Cross Validation Error:
0.64002872525296
Alpha value: 3e-05       L1_Ratio: 2e-11                 Cross Validation Error:
0.6400287252529604
Alpha value: 3.25e-05    L1_Ratio: 5e-12                 Cross Validation Error:
0.6400236645544309
Alpha value: 3.25e-05    L1_Ratio: 8.75e-12              Cross Validation Error:
0.6400236645544312
Alpha value: 3.25e-05    L1_Ratio: 1.25e-11              Cross Validation Error:
0.640023664554431
Alpha value: 3.25e-05    L1_Ratio: 1.625e-11             Cross Validation Error:
0.6400236645544313
Alpha value: 3.25e-05    L1_Ratio: 2e-11                 Cross Validation Error:
0.6400236645544314
Alpha value: 3.5e-05     L1_Ratio: 5e-12                 Cross Validation Error:
0.6400216129437519
```

```
Alpha value: 3.5e-05     L1_Ratio: 8.75e-12             Cross Validation Error:
0.6400216129437519
Alpha value: 3.5e-05     L1_Ratio: 1.25e-11             Cross Validation Error:
0.6400216129437518
Alpha value: 3.5e-05     L1_Ratio: 1.625e-11            Cross Validation Error:
0.640021612943752
Alpha value: 3.5e-05     L1_Ratio: 2e-11                Cross Validation Error:
0.640021612943752
Alpha value: 3.75e-05    L1_Ratio: 5e-12                Cross Validation Error:
0.6400224106863277
Alpha value: 3.75e-05    L1_Ratio: 8.75e-12             Cross Validation Error:
0.6400224106863274
Alpha value: 3.75e-05    L1_Ratio: 1.25e-11             Cross Validation Error:
0.6400224106863268
Alpha value: 3.75e-05    L1_Ratio: 1.625e-11            Cross Validation Error:
0.6400224106863263
Alpha value: 3.75e-05    L1_Ratio: 2e-11                Cross Validation Error:
0.640022410686326
Alpha value: 4e-05       L1_Ratio: 5e-12                Cross Validation Error:
0.6400259136912206
Alpha value: 4e-05       L1_Ratio: 8.75e-12             Cross Validation Error:
0.6400259136912201
Alpha value: 4e-05       L1_Ratio: 1.25e-11             Cross Validation Error:
0.6400259136912194
Alpha value: 4e-05       L1_Ratio: 1.625e-11            Cross Validation Error:
0.6400259136912186
Alpha value: 4e-05       L1_Ratio: 2e-11                Cross Validation Error:
0.6400259136912186

BEST ERROR VALUE IS 0.6400216129437518
```

The best cross-validated error value occurs at the row: Alpha value: 3.5e-05 Li_Ratio: 1.25e-11 Cross Validation Error: 0.6400216129437518

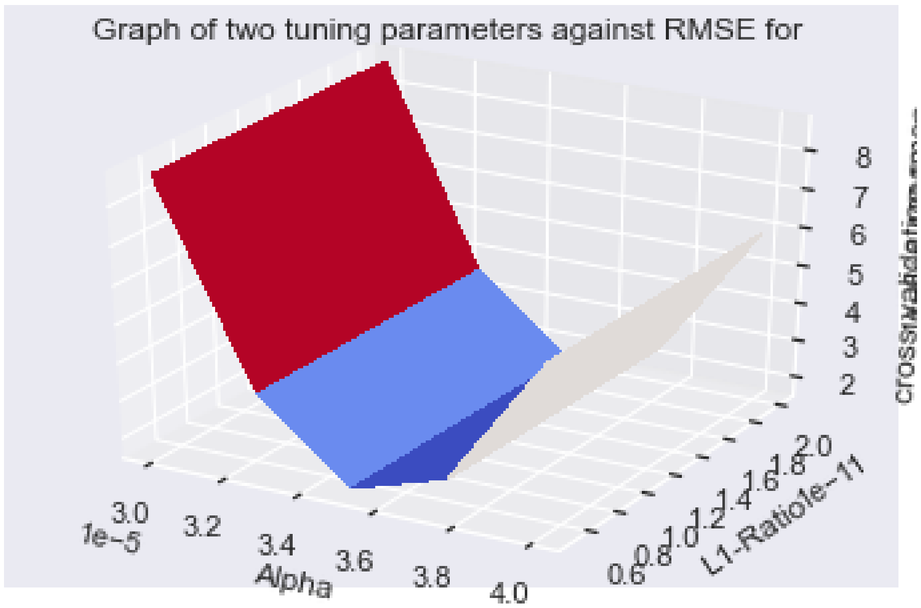
with an alpha value of $3.5 * 10^{-5}$ and an L1 ratio of $1.25 * 10^{-11}$, creating a cross validation error of 0.6400216129437518. It should be noted that at this order of magnitude, it's very difficult to conclusively say that this difference in cross validation error is significantly meaningful, especially when compared to the limitations of the system.

Below is a 3d rendering of the cross validation error against the 2 tuning parameters of alpha value and L1-ratio:

[18]:
```python
from mpl_toolkits.mplot3d import Axes3D
from matplotlib import cm

fig = plt.figure()
mat_size = 5
elastic = fig.gca(projection='3d')
X = np.linspace(0.00003,0.00004,mat_size)
```

```
Y = np.linspace(0.000000000005,0.00000000002,mat_size)
X, Y = np.meshgrid(X, Y)
Z = np.empty(shape=[mat_size, mat_size])
for i in range(mat_size):
    for j in range(mat_size):
        Z[i][j] = elastic_errors[(j*mat_size) + i];
surf = elastic.plot_surface(X, Y, Z, rstride=1, cstride=1, cmap=cm.coolwarm,
                       linewidth=0, antialiased=False)
elastic.set_xlabel('Alpha')
elastic.set_ylabel('L1-Ratio')
elastic.set_zlabel('cross validation rmse')
elastic.set_title('Graph of two tuning parameters against RMSE for ')
```

[18]:
```
Text(0.5, 0.92, 'Graph of two tuning parameters against RMSE for ')
```



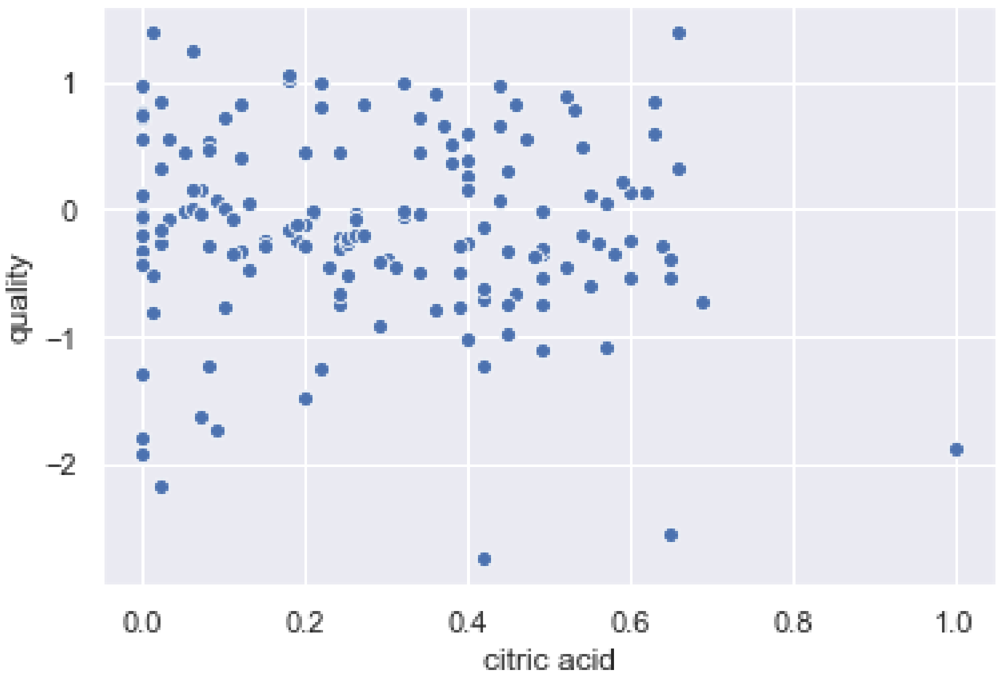
[19]:
```
Elastic_model = ElasticNet(alpha = 0.000035, l1_ratio=0.0000000000125, ␣
↪normalize=True)
Elastic_model.fit(X = X_train, y = Y_train)
display(Elastic_model.coef_,
        Elastic_model.intercept_) #intercept is the constant
```

```
array([ 3.19881132e-02, -9.86127645e-01, -7.28542201e-02,  1.88374776e-02,
       -1.57886811e+00,  4.39974418e-03, -3.06673754e-03, -2.41004609e+01,
       -2.91481938e-01,  9.21893455e-01,  2.78049541e-01])
```

```
27.567410129802752
```

[20]:
```
Elastic_train_prediction = Elastic_model.predict(X_train)
Elastic_test_prediction = Elastic_model.predict(X_test)

Elastic_train_rmse = rmse(Y_train, Elastic_train_prediction)
Elastic_test_rmse = rmse(Y_test, Elastic_test_prediction)

Elastic_accuracy = Y_test - Elastic_test_prediction
sns.set(font_scale=1)
sns.scatterplot(x=X_test['citric acid'], y=Elastic_accuracy); #chose to use␣
 ↪citric acid as the x
                                                        #axis since it showed the␣
 ↪most even distribution of points

print('Elastic Net Training Error: ', Elastic_train_rmse)
print('Elastic Net Test Error:', Elastic_test_rmse)
```

```
Elastic Net Training Error:  0.6348507491654413
Elastic Net Test Error: 0.7404494774555821
```



[21]:
```
ela = np.array([Elastic_train_rmse, Elastic_test_rmse, 0.64002])
rig = np.array([Ridge_train_rmse, Ridge_test_rmse, 0.64002])
las = np.array([Lasso_train_rmse, Lasso_test_rmse, 0.64009])
OLS = np.array([OLS_train_rmse, OLS_test_rmse, 0])
err = np.array(['Training Error', 'Test Error', 'Min CV Error'])
```

[22]:
```
Errors = pd.DataFrame({'Type of Error': err,'OLS': OLS, 'Ridge': rig, 'LASSO':␣
 ↪las, 'Elastic': ela})
Errors = Errors.set_index('Type of Error')
#There is 0 Min CV Error because I did'nt carry out CV analysis on OLS
```

[23]:
```
labels = np.insert(headers[:-1], 0, 'intercept')
OLS_coef = np.insert(OLS_model.coef_, 0, OLS_model.intercept_)
Ridge_coef = np.insert(Ridge_model.coef_, 0, Ridge_model.intercept_)
Lasso_coef = np.insert(Lasso_model.coef_, 0, Lasso_model.intercept_)
Elast_coef = np.insert(Elastic_model.coef_, 0, Elastic_model.intercept_)
```

[24]:
```
Model_coef = pd.DataFrame({'Label': labels,'OLS': OLS_coef, 'Ridge':␣
 ↪Ridge_coef, 'LASSO': Lasso_coef, 'Elastic': Elast_coef})
Model_coef = Model_coef.set_index('Label')
```

Visualization of Findings

Side by side comparison of all the final training and test errors (rounded to the 5th decimal), as well as lowest cross validation error produced by each regression method, and a table showing the final coefficients for each of the models. Below we have the final coefficients for each of the models

[25]:
```
display(Errors, Model_coef)
```

```
                     OLS     Ridge     LASSO   Elastic
Type of Error
Training Error  0.634405  0.634719  0.635184  0.634851
Test Error      0.745205  0.741204  0.744613  0.740449
Min CV Error    0.000000  0.640020  0.640090  0.640020
```

```
                            OLS      Ridge     LASSO    Elastic
Label
intercept             14.826190  26.151822  3.786834  27.567410
fixed acidity          0.023869   0.031427  0.004186   0.031988
volatile acidity      -1.049515  -0.996345 -0.989616  -0.986128
citric acid           -0.168018  -0.088421 -0.000000  -0.072854
residual sugar         0.014827   0.018491  0.004358   0.018837
chlorides             -1.615810  -1.585416 -1.496533  -1.578868
free sulfur dioxide    0.005202   0.004529  0.003643   0.004400
total sulfur dioxide  -0.003278  -0.003101 -0.002824  -0.003067
density              -11.083497 -22.659790 -0.000000 -24.100461
pH                    -0.408775  -0.306091 -0.375373  -0.291482
sulphates              0.944402   0.927324  0.871700   0.921893
alcohol                0.304679   0.281858  0.308097   0.278050
```

[26]:
```
!jupyter nbconvert IEOR165_Project.ipynb --to pdf
```

```
[NbConvertApp] Converting notebook IEOR165_Project.ipynb to pdf
```

```
[NbConvertApp] Support files will be in IEOR165_Project_files/
[NbConvertApp] Making directory ./IEOR165_Project_files
[NbConvertApp] Making directory ./IEOR165_Project_files
[NbConvertApp] Making directory ./IEOR165_Project_files
[NbConvertApp] Making directory ./IEOR165_Project_files
[NbConvertApp] Making directory ./IEOR165_Project_files
[NbConvertApp] Making directory ./IEOR165_Project_files
[NbConvertApp] Making directory ./IEOR165_Project_files
[NbConvertApp] Making directory ./IEOR165_Project_files
[NbConvertApp] Making directory ./IEOR165_Project_files
[NbConvertApp] Making directory ./IEOR165_Project_files
[NbConvertApp] Writing 78982 bytes to ./notebook.tex
[NbConvertApp] Building PDF
[NbConvertApp] Running xelatex 3 times: ['xelatex', './notebook.tex', '-quiet']
[NbConvertApp] Running bibtex 1 time: ['bibtex', './notebook']
[NbConvertApp] WARNING | bibtex had problems, most likely because there were no
citations
[NbConvertApp] PDF successfully created
[NbConvertApp] Writing 772085 bytes to IEOR165_Project.pdf
```

```
[ ]:
```