

Suppose A and B are the two endpoints of a diameter in the tree. For any two points u, v in the tree, denote $\text{dis}(u, v)$ to be the length of the shortest path from u to v .

For any node x in the tree, we claim that the longest path starting from x is equal to $\max(\text{dis}(x, A), \text{dis}(x, B))$.

Proof WLOG, assume that $\text{dis}(x, A) > \text{dis}(x, B)$.

Now assume for the sake of contradiction that one of the farthest point reachable from x is y , where $\text{dis}(x, y) > \text{dis}(x, A)$. According to the algorithm used to find a diameter of the tree, we know that y is an endpoint of a diameter in the tree.

Before we continue, note that any two diameters in a tree must meet at some node. To prove this, we can first assume two diameters don't intersect, and then find a contradiction. But the contradiction is obvious. Suppose the two diameters have a common length D (common because they are diameters). Pick any two nodes on those two diameters, say points a and b , and connect them. Suppose D_a is the longer of the distances from a to the two endpoints of the first diameter, and denote D_b for b similarly. Now, we can see that there is a path of length $D_a + D_b + n$, where n is the length of the path connecting the two diameters. But $D_a \geq \lceil \frac{D}{2} \rceil$, $D_b \geq \lceil \frac{D}{2} \rceil$, so $D_a + D_b + n \geq \lceil \frac{D}{2} \rceil + \lceil \frac{D}{2} \rceil + n \geq D + n > D$. Hence, there is a path in the tree longer than the original diameters, and hence a contradiction arises.

Using this fact about any two diameters must intersect, we know that in the original problem, as we go from x to y , we must cross some points that lie on the diameter with endpoints A and B . Suppose the first such point is z . Then we have that $\text{dis}(x, y) = \text{dis}(x, z) + \text{dis}(z, y) > \text{dis}(x, A) = \text{dis}(x, z) + \text{dis}(z, A)$, which means $\text{dis}(z, y) > \text{dis}(z, A)$. Now consider the path from y to B , we have that $\text{dis}(y, B) = \text{dis}(y, z) + \text{dis}(z, B) > \text{dis}(A, z) + \text{dis}(z, B) = \text{dis}(A, B)$, contradicting the fact that A and B are the endpoints of a diameter in the tree.

Thus, such a y doesn't exist for x , and the farthest distance reachable from x is $\text{dis}(x, A)$. □

Therefore, we simply have to find a diameter, then run two DFS's from the two endpoints (A, B) of this diameter, and for each node x in the tree, output $\max(\text{dis}(x, A), \text{dis}(x, B))$.