

Firstly, notice that the condition $1 \leq a_i \leq k$ for each $1 \leq i \leq |a|$ can simply be ignored, as this is always true when $1 \leq x \leq k$.

Suppose that we have a two dimension array f , where $f[x][\text{len}]$ denotes the number of ways to express x as the product of “len” prime numbers, where the order of primes in the presentation matters. That is, $6 = 2 \times 3$ and $6 = 3 \times 2$ are regarded as different representations.

Finding the value of $f[x][\text{len}]$ for all useful values of x and “len” is not a difficult task.

To find these values, consider what happens after we have fixed the value of x to an integer between 1 and k inclusive. We can now iterate over the value of “len” less than or equal to n . For each pair of x and “len”, we know that there are $f[x][\text{len}]$ integer arrays a if $|a| = \text{len}$ and each value in a is greater than 1. To take into account the extra ones, note that in reality, $|a|$ can now take any value from len to n . When $|a|$ is fixed, the number of arrays a is simply given by the expression $\binom{|a|}{\text{len}} \times f[x][\text{len}]$, since there are $\binom{|a|}{\text{len}}$ ways to choose the entries in the array that are not 1, and for each of these choices, there are $f[x][\text{len}]$ ways to permute the entries.

So for each pair of x and len, the total number of arrays a is given by

$$\sum_{|a|=\text{len}}^n \left(f[x][\text{len}] \times \binom{|a|}{\text{len}} \right) = f[x][\text{len}] \times \sum_{|a|=\text{len}}^n \binom{|a|}{\text{len}} = f[x][\text{len}] \times \binom{n+1}{\text{len}+1},$$

where the Hockey-Stick combinatorial identity is applied to the last summation.

The problem is now quite solvable with the help of modular arithmetic.