Suppose A and B are the two endpoints of a diameter in the tree. For any two points u, v in the tree, denote $\text{dis}(u, v)$ to be the length of the shortest path from u to v.

For any node x in the tree, we claim that the longest path starting from x is equal to $\max(\text{dis}(x, A), \text{dis}(x, B))$.

Proof

WLOG, assume that $\text{dis}(x, A) \geq \text{dis}(x, B)$. Let z be the first point the path from x to A intersects with diameter AB. Note that since $\text{dis}(x, z) + \text{dis}(z, A) = \text{dis}(x, A) \geq \text{dis}(x, B) = \text{dis}(x, z) + \text{dis}(z, B)$, we must have $\text{dis}(z, A) \geq \text{dis}(z, B)$.

Now assume for the sake of contradiction that one of the farthest points reachable from x is y, where $\text{dis}(x, y) > \text{dis}(x, A)$.

If the path from x to y crosses diameter AB, then it must intersect AB first at vertex z (otherwise there is clearly a cycle on the tree, which is impossible). In this case, there are two sub-cases to consider:

1. the path from x to y either overlaps with the path from z to A and doesn't overlap with the path from z to B;
2. it overlaps with the path from z to B and doesn't overlap with the path from z to A.
3. it doesn't overlap with any of the two segments that divide diameter AB.

Note that from $\text{dis}(z, y) + \text{dis}(x, z) = \text{dis}(x, y) > \text{dis}(x, A) = \text{dis}(z, A) + \text{dis}(x, z) \geq \text{dis}(x, B) = \text{dis}(z, B) + \text{dis}(x, z)$, we get $\text{dis}(z, y) > \text{dis}(z, A) > \text{dis}(z, B)$. If it is sub-case 1. that we're dealing with, then $\text{dis}(y, B) = \text{dis}(y, z) + \text{dis}(z, B) > \text{dis}(z, A) + \text{dis}(z, B) = \text{dis}(A, B)$, contradicting the definition of diameter AB; on the other hand, if it is sub-case 2. or 3. that occurs, then $\text{dis}(y, A) = \text{dis}(y, z) + \text{dis}(z, A) > \text{dis}(B, z) + \text{dis}(z, A) = \text{dis}(B, A)$, again contradicting the definition of diameter AB.

Hence, if the path from x to y crosses diameter AB, we reach a contradiction. So this cannot be the case.

We now examine the case where the path from x to y doesn't cross diameter AB. In this case, let u be the last vertex that the path from x to y and the path from x to A share. Since path x—y doesn't intersect with diameter AB, u must be a node in the middle of the path from x to z. Now, $\text{dis}(x, u) + \text{dis}(u, y) = \text{dis}(x, y) > \text{dis}(x, A) \geq \text{dis}(x, B) = \text{dis}(x, u) + \text{dis}(u, z) + \text{dis}(z, B)$ implies that $\text{dis}(u, y) > \text{dis}(u, z) + \text{dis}(z, B)$. Consider then the path from A to y, we have that $\text{dis}(A, y) = \text{dis}(A, u) + \text{dis}(u, y) > \text{dis}(A, u) + \text{dis}(z, u) + \text{dis}(z, B) = \text{dis}(A, z) + 2 \times \text{dis}(z, u) + \text{dis}(z, B) = \text{dis}(A, B) + 2 \times \text{dis}(z, u) > \text{dis}(A, B)$, where $\text{dis}(z, u) > 0$ because path x—y doesn't intersect diameter AB. This is apparently another contradiction with the definition of diameter AB.

Hence, our original assumption that there exists a vertex y with $\text{dis}(x, y) > \text{dis}(x, A)$ was false. This means that the length of the longest path starting from vertex x is given by $\text{dis}(x, A) = \max(\text{dis}(x, A), \text{dis}(x, B))$.

$\square$

Therefore, we simply have to find a diameter, then run two DFS's from the two endpoints (A, B) of this diameter, and for each node x in the tree, output $\max(\text{dis}(x, A), \text{dis}(x, B))$.