

Suppose that there are  $a$  blue discs and  $b$  discs in total, so that  $a, b \in \mathbb{Z}^+ \cup \{0\}$  and  $2 \leq a \leq b$ . We wish to find the smallest value of  $b$  such that  $b > 10^{12}$  and  $P(\text{BB}) = \frac{a}{b} \times \frac{a-1}{b-1} = \frac{1}{2}$ .

Rearranging the final equation, we get

$$\begin{aligned} 2a(a-1) &= b(b-1) \\ \Leftrightarrow 2a^2 - 2a - (b^2 - b) &= 0 \\ \Leftrightarrow a &= \frac{2 \pm \sqrt{4 + 8(b^2 - b)}}{4} \\ &= \frac{2 \pm 2\sqrt{2b^2 - 2b + 1}}{4} \\ &= \frac{1 \pm \sqrt{(b-1)^2 + b^2}}{2}. \end{aligned}$$

In the context of the problem,  $a$  clearly cannot be negative, so for each value of  $b$ , we can have at most one corresponding value of  $a$  for which  $P(\text{BB}) = \frac{1}{2}$ , namely  $a = \frac{1 + \sqrt{(b-1)^2 + b^2}}{2}$ , when this equals an integer.

Hence, the problem boils down to finding the smallest value of  $b$  greater than  $10^{12}$  for which  $a = \frac{1 + \sqrt{(b-1)^2 + b^2}}{2}$  is an integer, and this corresponding value of  $a$  is the answer we want.

The expression  $\frac{1 + \sqrt{(b-1)^2 + b^2}}{2}$  is an integer if and only if  $(b-1)^2 + b^2$  is the square of an odd positive integer. According to [this post on Maths Stack Exchange](#), all such values of  $b$  could be found by the stated recursive method. Using this algorithm, we can start from  $(r, s) = (1, 1)$ , recursively find all other possible solutions of  $(r, s)$ , and for each one find the given values of  $(n, k)$  in the post's context. This  $n$  value is then equal to  $b-1$ , so each time we check when  $b = n+1$  is greater than  $10^{12}$  and odd, which can be done quite quickly with the help of a program.

Eventually, one should get an answer of  $756872327473$ .