Theorem 0.1 *(Intersecting Diameters)* Any two diameters in a tree must intersect one another.

Proof To prove this, we can first assume two diameters don't intersect, and then find a contradiction. But the contradiction is obvious. Suppose the two diameters have a common length D (common because they are diameters). Pick any two nodes on those two diameters, say points a and b, and connect them. Suppose D_a is the longer of the distances from a to the two endpoints of the first diameter, and denote D_b for b similarly. Now, we can see that there is a path of length $D_a + D_b + n$, where n is the length of the path connecting the two diameters. But $D_a \geq \lceil \frac{D}{2} \rceil$, $D_b \geq \lceil \frac{D}{2} \rceil$, so $D_a + D_b + n \geq \lceil \frac{D}{2} \rceil + \lceil \frac{D}{2} \rceil + n \geq D + n > D$. Hence, there is a path in the tree longer than the original diameters, and hence a contradiction arises. $\square$