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#### 1 Общее

• Собственное вращение на угол  $\varphi$  с центром вращения в начале координат:

```
x' = x \cos \varphi - y \sin \varphiy' = x \sin \varphi + y \cos \varphi
```

- Расстояние между точками по сфере: L = R ·  $\arccos(\cos\theta_1 \cdot \cos\theta_2 + \sin\theta_1 \cdot \sin\theta_2 \cdot \cos(\varphi_1 \varphi_2))$  где  $\theta$  широты (от  $-\pi$  до  $\pi$ ),  $\varphi$  долготы (от  $-\pi$  до  $\pi$ )
- Объем шарового сегмента:  $V = \pi h^2 (R \frac{1}{3}h)$ , где h высота от вершины сектора до секущей плоскости
- Площадь поверхности шарового сегмента:  $S=2\pi Rh,$  где h высота
- Код Грея:  $g_n = n \oplus \frac{n}{2}$
- Числа Фибоначчи:  $F_0=0, F_1=1, F_n=\frac{(\frac{1+\sqrt{5}}{2})^n-(\frac{1-\sqrt{5}}{2})^n}{\sqrt{5}}$
- Sum-xor property:  $a+b=a\oplus b+2(a\&b), a+b=a|b+a\&b, a\oplus b=a|b-a\&b$

- Число граней в планарном графе<br/>(с учётом бесконечной): R=2-V+E
- Сумма арифметической прогрессии:  $S_n = \frac{n(a_1 + a_n)}{2}$
- Сумма геометрической прогрессии:  $S_n = \frac{b_1(q^n-1)}{q-1}$
- Определители матриц

$$\left| \begin{array}{cc} a & b \\ c & d \end{array} \right| = ad - bc$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1b_1c_1 + a_3b_1c_2 + a_2b_3c_1 - a_3b_2c_1 - a_1b_3c_2 - a_2b_1c_3$$

 $\Delta = \sum_{j=1}^n (-1)^{j+1} \cdot a_{1,j} \cdot \bar{M}^1_j, \ \bar{M}^1_j$ — определитель матрицы, полученной вычеркиванием 1 строки и j стоблца.

• Метод Крамера.  $\det A \neq 0 \implies$  единственное решение. Иначе 0 или  $\infty$ . Решения:  $x_i = \frac{\Delta_i}{\Delta}$ . В  $\Delta_i$  столбец коэффициентов при соответствующей неизвестной заменяется столбцом свободных членов системы.

## 2 Коды

#### 2.1 Basic setup

```
#include <bits/stdc++.h>
using namespace std;
#define sz(x) (int) ((x).size())
#define all(x) (x).begin(), (x).end()
#define rall(x) (x).rbegin(), (x).rend()
typedef long long 11;
typedef __int128 int128;
typedef pair<int, int> pii;
typedef pair<11, ll> pll;
const char en = '\n';
const int INF = 1e9 + 7;
const 11 INFLL = 1e18;
mt19937 rnd(chrono::high_resolution_clock::now().time_since_epoch().count
      ());
template<class T>
istream &operator>>(istream &is, vector<T> &a) {
   for (auto &i : a) {
         is >> i;
     return is;
#ifdef LOCAL
#include "debug.h"
#define debug(...) 42
#endif
void solve() {
#ifdef LOCAL
    freopen("input.txt", "r", stdin);
    ios_base::sync_with_stdio(0);
     cin.tie(0);
     solve():
     return 0;
```

#### 2.2 Бесполезное

#### Санитайзеры:

```
set(CMAKE_CXX_FLAGS "${CMAKE_CXX_FLAGS} -Wall -Wshadow -g -fsanitize= undefined -fsanitize=bounds -fsanitize=address -D_GLIBCXX_DEBUG")

-Wall -Wextra -pedantic -Wformat=2 -Wfloat-equal -Wconversion -Wlogical-op -Wshift-overflow=2 -Wduplicated-cond -Wcast-qual -Wcast-align -Werror
```

#### Прагмы:

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC target("sse,sse2,sse3,sse4)
#pragma GCC target("popcnt,abm,mmx,avx,avx2,tune=native")
#pragma GCC optimize("unroll-loops")
#pragma GCC optimize("fast-math")
#pragma GCC optimize("section-anchors")
#pragma GCC optimize("profile-values")
#pragma GCC optimize("profile-reorder-functions")
#pragma GCC optimize("tracer")
#pragma GCC optimize("vpt")
#pragma GCC optimize("rename-registers")
#pragma GCC optimize("move-loop-invariants")
#pragma GCC optimize("unswitch-loops")
#pragma GCC optimize("function-sections")
#pragma GCC optimize("data-sections")
#pragma GCC optimize("branch-target-load-optimize")
#pragma GCC optimize("branch-target-load-optimize2")
#pragma GCC optimize("branch-target-load-optimize2")
#pragma GCC optimize("branch-target-load-optimize2")
#pragma GCC optimize("branch-target-load-optimize2")
```

#### Встроенный декартач:

#### Atomic hashset, hashmap:

#### Перебор всех подмасок и надмасок:

```
for (int submask = mask;; submask = (submask - 1) & mask) {
    // use submask
    if (submask == 0) break;
}
for (int upmask = mask;; upmask = (upmask + 1) | mask) {
    // use upmask
    if (upmask == maxmask) break;
}
```

#### 2.3 Мосты

#### 2.4 Точки сочленения

```
void dfs(int v, int p = -1) {
    used[v] = true;
    tin[v] = fup[v] = timer++;
    int children = 0;
    for (auto to : g[v]) {
        if (to == p) {
            continue;
        }
        if (used[to]) {
            fup[v] = min(fup[v], tin[to]);
        } else {
            dfs(to, v);
            fup[v] = min(fup[v], fup[to]);
            if (fup[to] >= tin[v] && p != -1) {
                IS_CUTPOINT(v);
            }
            ++children;
        }
        if (p == -1 && children > 1) {
            IS_CUTPOINT(v);
        }
}
```

## 2.5 DCP (TheEvilBird)

```
struct Query {
    char type;
    int v. u:
     Query(char type) : type(type) {}
     Query(char type, int v, int u) : type(type), v(v), u(u) {}
};
struct DCP {
    int n, k, ans; // n - vertex, k - queries
     vector<int> par, rk;
     vector<pair<pii, int>> hist;
     // 0 - par, 1 - rk, 2 - ans;
    int qL, qR;
    pii edge;
     vector<vector<pii>>> tree;
    vector<Query> qs;
    DCP(int _n, int _k) {
         n = ans = _n;
         par.resize(n);
         rk.resize(n, 1);
for (int i = 0; i < n; ++i) par[i] = i;
         tree.assign(4 * k, vector<pii>());
    int dsu_get(int v) {
         while (par[v] != v) v = par[v];
    void dsu_unite(int a, int b) {
         a = dsu_get(a);
         b = dsu_get(b);
         if (a == b) return;
         if (rk[a] > rk[b]) swap(a, b);
hist.emplace_back((pii){0, a}, par[a]);
         hist.emplace_back((pii){2, -1}, ans);
par[a] = b;
         if (rk[a] == rk[b]) {
             hist.emplace_back((pii){1, b}, rk[b]);
             ++rk[b];
    void dsu_unite(pii e) {
         dsu_unite(e.first, e.second);
    void cancel(pair<pii, int> &el) {
         int &type = el.first.first;
int &id = el.first.second;
         int &val = el.second;
         if (type == 0) {
   par[id] = val;
} else if (type == 1) {
             rk[id] = val;
         } else if (type == 2) {
             ans = val;
    void add_edge(int _qL, int _qR, pii e) {// [L, R]
```

```
qL = _qL;
        qR = _qR + 1;
edge = e;
         add_edge_tree(1, 0, k);
    }
    void add_edge_tree(int v, int 1, int r) {
         if (qL \le 1 \&\& r \le qR) {
             tree[v].emplace_back(edge);
             return:
         int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
         if (qL < m) add_edge_tree(vL, 1, m);</pre>
         if (m < qR) add_edge_tree(vR, m, r);</pre>
    void go(vector<Query> &_qs) {
         go_tree(1, 0, k);
    void go_tree(int v, int 1, int r) {
        int siz = sz(hist);
for (auto &e : tree[v]) {
            dsu_unite(e);
         if (1 + 1 == r) {
             if (qs[1].type == '?') {
                 cout << ans << en:</pre>
        } else {
             int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
             go_tree(vL, 1, m);
             go_tree(vR, m, r);
        while (sz(hist) > siz) {
             cancel(hist.back());
             hist.pop_back();
    }
};
void solve() {
    int n, k;
    cin >> n >> k;
    DCP dcp(n, k);
    set<pair<pii, int>> edges;
    vector<Query> qs;
for (int i = 0; i < k; ++i) {</pre>
        char tp;
        cin >> tp;
if (tp == '?') {
            qs.emplace_back(tp);
        } else {
            int v, u;
             cin >> v >> u;
             --v;
             if (v > u) swap(v, u);
             qs.emplace_back(tp, v, u);
if (tp == '+') {
                 edges.emplace((pii){v, u}, i);
                 auto it = edges.lower_bound({(pii){v, u}, 0});
                 dcp.add_edge(it->second, i, it->first);
                 edges.erase(it);
            }
        }
    for (auto &e : edges) {
         dcp.add_edge(e.second, k - 1, e.first);
    if (k) dcp.go(qs);
```

#### 2.6 MaxFlow (TheEvilBird)

```
struct MaxFlow {
    struct Edge {
        ll flow, cap;
        int to, id;
        Edge() {}

        Edge(ll flow, ll cap, int to, int id) : flow(flow), cap(cap), to(
        to), id(id) {}
    };

int n;
    vector<vector<Edge>> g;
    vector<int> d, head, used;
    ll max_cap;
    int s, t;
```

```
MaxFlow() {}
MaxFlow(int _n) {
    n = n:
    g.resize(n);
void add_edge(int from, int to, 11 cap) {
    g[from].emplace_back(0, cap, to, sz(g[to]));
g[to].emplace_back(0, 0, from, sz(g[from]) - 1);
bool bfs() {
    d.assign(n, INF);
    d[s] = 0;
    queue<int> q;
    q.push(s):
    while (!q.empty()) {
   int v = q.front();
         q.pop();
         for (auto e : g[v]) {
   if (d[e.to] == INF && e.cap - e.flow >= max_cap) {
      d[e.to] = d[v] + 1;
                  q.push(e.to);
    }
    return d[t] != INF;
}
11 dfs(int v, 11 cur_flow) {
    if (v == t) {
        return cur_flow;
    for (; head[v] < sz(g[v]); ++head[v]) {
         auto &e = g[v][head[v]];
         if (e.cap - e.flow >= \max_{x \in \mathbb{R}} d[v] + 1 == d[e.to]) {
             11 new_flow = dfs(e.to, min(cur_flow, e.cap - e.flow));
              if (new_flow) {
                  e.flow += new_flow;
                  g[e.to][e.id].flow -= new_flow;
                  return new_flow;
    return 0;
11 find_max_flow(int _s, int _t) {
    for (int k = 30; k >= 0; --k) {
    max_cap = (1 << k);
         while (bfs()) {
             head.assign(n, 0);
             11 flow = 0;
                  flow = dfs(s, INFLL);
                  res += flow;
             } while (flow);
11 dfs_const_flow(int v, 11 cur_flow) {
    used[v] = 1;
if (v == t) {
        return cur_flow;
    for (auto &e : g[v]) {
         if (!used[e.to] && e.cap - e.flow > 0) {
             11 new_flow = dfs_const_flow(e.to, min(cur_flow, e.cap -
 e.flow));
             if (new_flow) {
                  e.flow += new_flow;
                  g[e.to][e.id].flow -= new_flow;
                  return new_flow;
        }
    return 0;
bool find_const_flow(int _s, int _t, 11 F) {
    s = _s;
t = _t;
    ll res = 0, flow = 0;
    \max_{cap} = F;
         used.assign(n, 0);
         flow = dfs_const_flow(s, INF);
         res += flow;
    } while (flow && res < F);</pre>
    return res == F;
```

```
ll get_edge_flow(int v, int id) {
    return g[v][id].flow;
}
};
```

# return ans; } };

## 2.7 MinCostMaxFlow (TheEvilBird)

```
struct MinCostMaxFlow {
    struct Edge {
        ll flow, cap, price;
        int to, id;
        Edge() {}
        Edge(ll flow, ll cap, ll price, int to, int id) : flow(flow), cap
     (cap), price(price), to(to), id(id) {}
   int n;
    int s, t;
    11 ans;
    vector<vector<Edge>> g;
    vector<int> d;
    vector<ll> add f:
    vector<pii> par;
    MinCostMaxFlow() {}
   MinCostMaxFlow(int _n) {
        n = n:
        g.resize(n);
    void add_edge(int from, int to, 11 cap, 11 price) {
        g[from].emplace_back(0, cap, price, to, sz(g[to]));
        g[to].emplace_back(0, 0, -price, from, sz(g[from]) - 1);
    11 get_edge_flow(int v, int id) {
        return g[v][id].flow;
    void FB() {
        d.assign(n, INF);
        add_f.assign(n, 0);
        par.assign(n, {-1, -1});
        d[s] = 0;
add_f[0] = INF;
        queue<int> q;
        q.push(s);
        vector<int> used(n, 0);
        used[s] = 1;
        while (!q.empty()) {
            int v = q.front();
            q.pop();
used[v] = 0;
            for (int i = 0; i < sz(g[v]); ++i) {
                 auto &e = g[v][i];
                 if (e.flow < e.cap && d[e.to] > d[v] + e.price) {
    d[e.to] = d[v] + e.price;
                     add_f[e.to] = min(add_f[v], e.cap - e.flow);
par[e.to] = {v, i};
                     if (!used[e.to]) {
                         q.push(e.to);
                          used[e.to] = 1;
                     }
                }
            }
        }
    void push_flow(11 flow) {
        int cur = t;
        while (cur != s) {
            int prev = par[cur].first, id = par[cur].second;
            g[prev][id].flow += flow;
            g[cur][g[prev][id].id].flow -= flow;
             ans += g[prev][id].price * flow;
            cur = prev;
        }
   7
    11 min_cost_max_flow(int _s, int _t) {
        ans = 0;
        s = _s;
t = _t;
        while (true) {
            FB();
            11 flow = add_f[t];
            if (flow == 0) {
                 break:
            push_flow(flow);
```

## 2.8 Эйлеров цикл

```
vector<vector<int>> g;
vector<bool> used;
vector<int> ed, tour;

void dfs(int v) {
    while (!g[v].empty()) {
        int u = g[v].back();
        g[v].pop_back();
        if (used[u]) {
            continue;
        }
        used[u] = 1;
        dfs(ed[u] ^ v);
    }
    tour.push_back(v);
}
```

## 2.9 Kyh

```
vector<vector<int>> g;
vector<int> used, mt;
int timer = 1;
bool dfs(int v) {
    if (used[v] == timer) {
        return false:
    used[v] = timer;
    for (auto u : g[v]) {
        if (mt[u] == -1) {
            mt[u] = v:
            return true;
    for (auto u : g[v]) {
        if (dfs(mt[u])) {
            mt[u] = v;
            return true;
    return false;
for (int i = 0; i < n; ++i) {
    if (dfs(i)) {
        ++timer:
```

Вершинное покрытие графа — множество вершин, что каждое ребро графа инцидентно хотя бы одной вершине из множества.

Пусть M — макс. парсоч. Мысленно ориентируем ребра графа: ребра из M проведем из правой доли в левую, остальные — из левой в правую, после чего запустим обход в глубину из всех вершин левой доли, не включенных в M. Граф разбился на несколько множеств:  $L^+, L^-, R^+, R^-$ , где «плюсовые» множества — это множества посещенных в процессе обхода вершин. Тогда  $V_{min} = L^- \cup R^+$ .

Независимое множество вершин — множество вершин, что никакая пара вершин не соединена ребром. Дополнение минимального вершинного покрытия является максимальным независимым множеством.

Покрытие дага путями: n-matching

## 2.10 HLD (TheEvilBird)

```
struct HLD {
    // insert SegTree code
    struct SegTree {};

int n, T;
    SegTree st;
    vector<vector<int>> tree;
    vector(int> par, siz, tin, tout, head;
    ll ans;
```

```
HLD(int _n) {
         n = _n;
tree.resize(n);
         par.resize(n, -1);
         siz.resize(n. 0):
         tin.resize(n);
         tout.resize(n);
         head.resize(n);
         st = SegTree(n);
    void add_edge(int v, int u) {
         tree[v].emplace_back(u);
         tree[u].emplace_back(v);
    void build(int v = 0) {
         dfs siz(v, v):
         T = 0;
         head[v] = v;
         dfs_hld(v, v);
    void dfs_siz(int v, int p) {
         par[v] = p;
siz[v] = 1;
         for (auto &u : tree[v]) {
             if (u != p) {
                  dfs_siz(u, v);
siz[v] += siz[u];
         for (int i = 0; i < sz(tree[v]); ++i) {
             int x = tree[v][0], u = tree[v][i];
if (x == p || siz[u] > siz[x]) {
                  swap(tree[v][0], tree[v][i]);
         }
    }
    void dfs_hld(int v, int p) {
   tin[v] = T++;
         for (auto u : tree[v]) {
             if (u == p) {
                  continue;
              if (u == tree[v][0]) {
                  head[u] = head[v];
              } else {
                  head[u] = u;
              dfs_hld(u, v);
         tout[v] = T;
     void update(int v, int val) {
         st.update_segment(tin[v], tin[v], val);
    bool is_anc(int v, int u) {
    return tin[v] <= tin[u] && tout[u] <= tout[v];</pre>
    void go\_up(int \&v, int u) {
         while (!is\_anc(head[v], u)) {
             ans = max(ans, st.get(tin[head[v]], tin[v]));
v = par[head[v]];
    11 get(int v, int u) {// max on path
    ans = -INFLL;
         go_up(v, u);
go_up(u, v);
         if (!is_anc(v, u)) {
              swap(v, u);
         ans = max(ans, st.get(tin[v], tin[u]));
         return ans:
    }
};
```

#### 2.11 Dominator tree (TheEvilBird)

```
struct Edge {
   int from, to, id;

Edge() = default;
   Edge(int from, int to, int id) : from(from), to(to), id(id) {}
};

struct DSU {
   int n;
   vector<int> par;
```

```
vector<pii> mn;
    DSU() = default;
    DSU(int n) : n(n) {
         par.resize(n);
         mn.resize(n);
         init();
    }
    void init() {
   for (int i = 0; i < n; ++i) {</pre>
             par[i] = i;
             mn[i] = {INF, i};
    }
    int get(int v) {
   if (par[v] == v) {
             return v;
         int p = get(par[v]);
  mn[v] = min(mn[v], mn[par[v]]);
if (mn[par[v]].first < mn[v].first) {</pre>
             mn[v] = mn[par[v]];
         par[v] = p;
         return p;
    void unite(int a, int b) {
        par[a] = b;
};
struct DominatorTree {
    int n:
    vector<Edge> edges;
     vector<int> sdom, idom, tin, order, par, used, dp;
     vector<vector<int>>> g, rg, queries;
    DSU dsu_sdom, dsu_idom;
    DominatorTree() = default:
    DominatorTree(int n) : n(n), dsu_sdom(n), dsu_idom(n) {
         sdom.resize(n, INF); // semi-dominator
         idom.resize(n, INF); // immediate dominator
         tin.resize(n, -1);
         par.resize(n);
         used.resize(n, 0);
         dp.resize(n, INF);
         g.resize(n);
         rg.resize(n);
         queries.resize(n);
    7
    void add_edge(int from, int to) {
         edges.emplace_back(from, to, sz(edges));
    void dfs(int v) {
         tin[v] = sz(order);
         order.emplace back(v):
         for (auto i: g[v]) {
              const auto &e = edges[i];
if (tin[e.to] == -1) {
                  par[e.to] = v;
                  dfs(e.to);
             }
         }
    void dfs_idom(int v) {
         used[v] = 1;
         for (auto i: g[v]) {
              const auto &e = edges[i];
              if (!used[e.to]) {
                  dfs_idom(e.to);
             }
         for (auto u: queries[v]) {
    dsu_idom.get(u);
              dp[u] = dsu_idom.mn[u].second;
         dsu_idom.mn[v] = {sdom[v], v};
for (auto i: g[v]) {
              const auto &e = edges[i];
if (par[e.to] == v) {
                  dsu_idom.unite(e.to, v);
    void build(int s) {
         for (int i = 0; i < sz(edges); ++i) {
              g[edges[i].from].emplace_back(i);
              rg[edges[i].to].emplace_back(i);
         // reorder vertex
         dfs(s);
```

```
// build sdom
     for (int _ = sz(order) - 1; _ >= 0; --_) {
   int v = order[_];
          if (v == s) {
               continue;
          for (auto i: rg[v]) {
               const auto &e = edges[i];
if (tin[e.from] == -1) {
                    continue;
               if (tin[e.from] < tin[v]) {</pre>
                    sdom[v] = min(sdom[v], tin[e.from]);
               7
               else {
                    int u = dsu_sdom.get(e.from);
sdom[v] = min(sdom[v], dsu_sdom.mn[e.from].first);
          }
          dsu_sdom.mn[v] = {sdom[v], v};
          for (auto i: g[v]) {
    const auto &e = edges[i];
               if (v == par[e.to]) {
                    dsu_sdom.unite(e.to, v);
          }
     }
     // build queries for idoms
for (int i = 0; i < n; ++i) {
   if (i == s || sdom[i] == INF || tin[i] == -1) {</pre>
               continue;
          queries[order[sdom[i]]].emplace_back(i);
     dfs_idom(s);
     // build idom
     idom[s] = tin[s];
     for (auto v: order) {
   if (v == s) {
               continue;
          if (v == dp[v]) {
               idom[v] = sdom[v];
          else {
               idom[v] = idom[dp[v]];
}
int get_idom(int v) {
     return (idom[v] == INF ? -1 : order[idom[v]]);
```

#### 2.12 Link-Cut (TheEvilBird)

```
struct Node {
    Node *ch[2] = {nullptr, nullptr};
    Node *par = nullptr;
bool rev = false;
    int val, mn;
    int siz = 1;
    Node() {}
    Node(int val) : val(val), mn(val) {}
};
typedef Node *pnode;
int get_siz(pnode v) {
    return (v == nullptr ? 0 : v->siz);
int get_min(pnode v) {
    return (v == nullptr ? INF : v->mn);
void update(pnode v) {
    v->siz = 1 + get_siz(v->ch[0]) + get_siz(v->ch[1]);
    v->mn = min(v->val, min(get_min(v->ch[0]), get_min(v->ch[1])));
}
void push(pnode v) {
   if (v == nullptr || !v->rev) {
    if (v->ch[0] != nullptr) {
         v->ch[0]->rev ^= 1;
    if (v->ch[1] != nullptr) {
```

```
v->ch[1]->rev ^= 1;
    swap(v->ch[0], v->ch[1]);
    v \rightarrow rev = 0:
}
bool is_root(pnode v) {
    return (v->par == nullptr ||
             (v->par->ch[0] != v && v->par->ch[1] != v));
int child_num(pnode v) {
   return (v->par->ch[1] == v);
void attach(pnode v, pnode p, int num) {
    if (v != nullptr) {
        v \rightarrow par = p:
    if (p != nullptr) {
        p->ch[num] = v;
    }
void rotate(pnode v) {
    int num = child_num(v);
pnode p = v->par, vb = v->ch[num ^ 1];
    pnode g = (p == nullptr ? nullptr : p->par);
if (g != nullptr) {
        if (!is_root(p)) {
            g->ch[child_num(p)] = v;
    }
    v->par = g;
    attach(p, v, num ^ 1);
    attach(vb, p, num);
update(p);
    update(v);
void splay(pnode v) {
    vector<pnode> st;
    pnode cur = v;
    st.emplace_back(cur);
    while (!is_root(cur)) {
        cur = cur->par;
        st.emplace_back(cur);
    for (int i = sz(st) - 1; i \ge 0; --i) {
        push(st[i]);
    while (!is_root(v)) {
        if (!is_root(v->par)) {
            if (child_num(v) == child_num(v->par)) {
                 rotate(v->par);
             } else {
                rotate(v);
             7
        rotate(v);
    }
void expose(pnode v) {
    splay(v);
    v->ch[1] = nullptr;
    update(v);
    while (v->par != nullptr) {
        splay(v->par);
         attach(v, v->par, 1);
         update(v->par);
        splay(v);
    }
void make_root(pnode v) {
    expose(v);
    v->rev ^= 1;
void link(pnode v, pnode u) {
   make_root(v);
    make_root(u);
    u->par = v;
void cut(pnode v, pnode u) {
   make_root(v);
    make_root(u);
    push(u);
    u->ch[1] = nullptr;
    v->par = nullptr;
bool is_connected(pnode v, pnode u) {
    make_root(v);
    make_root(u);
    if (is_root(v) && u != v) {
```

```
return false;
} else {
    return true;
}

int get_min(pnode v, pnode u) {
    make_root(v);
    make_root(u);
    return get_min(u);
}
```

## 2.13 Личао (FedShat)

```
struct LiChao {// max
       11 k = 0, b = -INFLL;
       Line() = default:
       Line(11 k, 11 b) : k(k), b(b){};
        11 operator()(11 x) {
           return k * x + b;
   }:
   struct Node {
        Node *1 = nullptr, *r = nullptr;
        Line cur;
        Node() = default;
   Node *root = nullptr;
   int n = 1e9 + 1;
   void make node(Node *&v) {
       if (v == nullptr) {
           v = new Node();
   }
   void add(Node *&v, int 1, int r, Line cur) {
       make_node(v);
        int m = (1 + r) / 2;
        if (cur(m) > v->cur(m)) {
           swap(cur, v->cur);
        if (1 + 1 == r) {
           return:
        if (cur(1) > v->cur(1)) {
           add(v->1, 1, m, cur);
        } else {
           add(v->r, m, r, cur);
   }
   void add(Line cur) {
        add(root, 0, n, cur);
   11 get(Node *v, int 1, int r, int x) {
       if (v == nullptr) {
           return -INFLL;
        11 ans = v->cur(x);
       if (1 + 1 == r) {
           return ans:
        int m = (1 + r) / 2;
        if (x < m) {
           ans = \max(ans, get(v->1, 1, m, x));
        } else {
           ans = max(ans, get(v->r, m, r, x));
        return ans;
   }
   11 get(int x) {
        return get(root, 0, n, x);
};
```

# 2.14 Segment Tree (TheEvilBird)

```
struct SegTree {
    static const ll off = 0;
    struct Node {
```

```
11 val = 0;
    11 push = off;
    Node() {}
    Node(l1 val) : val(val) {}
    Node operator+(const Node &other) const {
         return Node(val + other.val);
    void operator+=(const Node &other) {
        val += other.val;
    void use_push(int len = 1) {
        val += push * (11) (1en);
    void update_push(11 pushed) {
        push += pushed;
int n, qL, qR;
Node ans;
vector<Node> tree;
vector<11> a;
SegTree() {}
SegTree(int _n) {
    n = _n;
    tree.assign(4 * n, 0);
void update_vertex(int v, int 1, int r) \{// [l, r)
    int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
    push(vL, 1, m);
    push(vR, m, r);
    tree[v] = tree[vL] + tree[vR];
void push(int v, int 1, int r) {// [l, r)
  if (tree[v].push == off) return;
  int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
  tree[v].use_push(r - 1);
         tree[vL].update_push(tree[v].push);
        tree[vR].update_push(tree[v].push);
    tree[v].push = off;
void build(vector<11> &_a) {
    build_tree(1, 0, n);
void build_tree(int v, int 1, int r) \{// [l, r)
    if (l + 1 == r) {
tree[v] = Node(a[l]);
        return;
    int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
    build_tree(vL, 1, m);
    build_tree(vR, m, r);
    update_vertex(v, 1, r);
void update_segment(int _qL, int _qR, ll _val) {// [_qL, _qR]
    qL = _qL;
qR = _qR + 1;
    val = _val;
    update_segment_tree(1, 0, n);
void update_segment_tree(int v, int l, int r) {// [l, r)
    push(v, 1, r);
if (qL <= 1 && r <= qR) {
        tree[v].update_push(val);
        push(v, 1, r);
         return:
    int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
    if (qL < m) update_segment_tree(vL, 1, m);</pre>
    if (m < qR) update_segment_tree(vR, m, r);</pre>
    update_vertex(v, 1, r);
11 get(int _qL, int _qR) {// [_qL, _qR]
    qL = _qL;
qR = _qR + 1;
    ans = Node();
    get_tree(1, 0, n);
```

```
return ans.val;
}

void get_tree(int v, int 1, int r) {// [l, r)
    push(v, 1, r);
    if (qL <= 1 && r <= qR) {
        ans = ans + tree[v];
        return;
    }
    int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
    if (qL < m) get_tree(vL, 1, m);
    if (m < qR) get_tree(vR, m, r);
};</pre>
```

## 2.15 Segment Tree Down (TheEvilBird)

```
struct SegTreeDown {
    struct Node {
        11 val = 0;
        Node() {}
        Node(ll val) : val(val) {}
        Node operator+(const Node &other) const {
            return Node(val + other.val);
        void operator+=(const Node &other) {
            val += other.val;
        }
    };
    vector<Node> tree;
    SegTreeDown(int _n) {
        tree.assign(2 * n, Node());
    void build(vector<ll> &a) {
        for (int i = 0; i < n; ++i) {
    tree[i + n] = Node(a[i]);</pre>
        for (int i = n - 1; i \ge 1; --i) {
             tree[i] = tree[2 * i] + tree[2 * i + 1];
    void update(int i, ll val) {
        tree[i] = val;
        while (i != 0) {
            tree[i] = tree[2 * i] + tree[2 * i + 1];
            i /= 2;
    }
    11 get(int 1, int r) \{// [l, r)
        r += n;
        Node ans;
        while (1 \le r) {
            if (1 % 2 == 1) {
                ans += tree[1];
                 ++1:
             if (r \% 2 == 0) {
                ans += tree[r];
                 --r;
            1 /= 2;
            r /= 2;
        return ans.val;
    }
};
```

#### 2.16 Segment Tree Beats (TheEvilBird)

```
struct SegTree {
    struct Node {
        11 max, sec_max;
        int cnt_max;
        11 min, sec_min;
        int cnt_min;
        11 sum;
        11 push_add;
}
```

```
11 push_eq;
}:
int n, qL, qR;
11 val, ans;
vector<Node> tree;
vector<11> a;
SegTree(int _n) {
    tree.assign(4 * n, {0, -INFLL, 1, 0, INFLL, 1, 0, 0, -1});
void update_vertex(int v, int 1, int r) {
   int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
    tree[v].sum = tree[vL].sum + tree[vR].sum;
    tree[v].max = max(tree[vL].max, tree[vR].max);
    tree[v].sec_max = max(tree[vL].sec_max, tree[vR].sec_max);
    tree[v].cnt_max = 0;
    if (tree[vL].max == tree[v].max) {
        tree[v].cnt_max += tree[vL].cnt_max;
    } else {
        tree[v].sec_max = max(tree[v].sec_max, tree[vL].max);
    if (tree[vR].max == tree[v].max) {
        tree[v].cnt_max += tree[vR].cnt_max;
      else {
        tree[v].sec_max = max(tree[v].sec_max, tree[vR].max);
    tree[v].min = min(tree[vL].min, tree[vR].min);
    tree[v].sec_min = min(tree[vL].sec_min, tree[vR].sec_min);
    tree[v].cnt_min = 0;
    if (tree[vL].min == tree[v].min) {
         tree[v].cnt_min += tree[vL].cnt_min;
        tree[v].sec_min = min(tree[v].sec_min, tree[vL].min);
    if (tree[vR].min == tree[v].min) {
        tree[v].cnt_min += tree[vR].cnt_min;
        tree[v].sec_min = min(tree[v].sec_min, tree[vR].min);
}
void recalc_eq(int v, int 1, int r, 11 cur) {
   tree[v].max = tree[v].min = tree[v].push_eq = cur;
    tree[v].sec_max = -INFLL;
    tree[v].sec_min = INFLL;
    tree[v].cnt_max = tree[v].cnt_min = r - 1;
tree[v].sum = cur * (11) (r - 1);
    tree[v].push_add = 0;
void recalc_add(int v, int 1, int r, 11 cur) {
    if (tree[v].min == tree[v].max) {
        recalc_eq(v, 1, r, tree[v].max + cur);
        return:
    tree[v].max += cur;
    if (tree[v].sec_max != -INFLL) {
        tree[v].sec_max += cur;
    tree[v].min += cur;
    if (tree[v].sec_min != INFLL) {
        tree[v].sec_min += cur;
    tree[v].sum += (11) (r - 1) * cur;
    tree[v].push_add += cur;
void recalc_min(int v, int 1, int r, 11 cur) {
    if (tree[v].min >= cur) {
        recalc_eq(v, 1, r, cur);
        return:
    if (tree[v].max <= cur) return;</pre>
    if (tree[v].sec_min == tree[v].max) {
         tree[v].sec_min = cur;
    tree[v].max = cur;
void recalc_max(int v, int 1, int r, ll cur) {
   if (tree[v].max <= cur) {</pre>
        recalc_eq(v, 1, r, cur);
        return;
    if (tree[v].min >= cur) return;
    if (tree[v].sec_max == tree[v].min) {
        tree[v].sec_max = cur;
```

```
tree[v].sum += (11) (tree[v].max - cur) *
                        (11) (tree[v].cnt_max);
     tree[v].min = cur;
}
void push(int v, int 1, int r) {
     if (1 + 1 == r) return;
     int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
     if (tree[v].push_eq != -1) {
          recalc_eq(vL, 1, m, tree[v].push_eq);
recalc_eq(vR, m, r, tree[v].push_eq);
          tree[v].push_eq = -1;
     recalc_add(vL, 1, m, tree[v].push_add);
recalc_add(vR, m, r, tree[v].push_add);
tree[v].push_add = 0;
     recalc_min(vL, 1, m, tree[v].max);
     recalc_min(vR, m, r, tree[v].max);
     recalc_max(vL, 1, m, tree[v].min);
     recalc_max(vR, m, r, tree[v].min);
void build(vector<11> &_a) {
     a = _a;
build_tree(1, 0, n);
void build_tree(int v, int 1, int r) {
     if (1 + 1 == r) {
    tree[v] = {a[1], -INFLL, 1, a[1],
                        INFLL, 1, a[1], 0, -1};
          return;
     int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
     build_tree(vL, 1, m);
     build_tree(vR, m, r);
     update_vertex(v, 1, r);
void update_segment_min(int _qL, int _qR, 11 _val) {
     qL = _qL;
qR = _qR + 1;
val = _val;
     update_segment_min_tree(1, 0, n);
void update_segment_min_tree(int v, int 1, int r) {
     if (tree[v].max <= val) return;
if (qL <= 1 && r <= qR && tree[v].sec_max < val) {</pre>
          recalc_min(v, 1, r, val);
          return;
     push(v, 1, r);
     int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
     if (qL < m) update_segment_min_tree(vL, 1, m);
if (m < qR) update_segment_min_tree(vR, m, r);</pre>
     update_vertex(v, 1, r);
void update_segment_max(int _qL, int _qR, 11 _val) {
     qL = _qL;
qR = _qR + 1;
val = _val;
     update_segment_max_tree(1, 0, n);
void update_segment_max_tree(int v, int 1, int r) {
   if (tree[v].min >= val) return;
     if (qL <= 1 && r <= qR && tree[v].sec_min > val) {
          recalc_max(v, 1, r, val);
          return;
     }
     push(v, 1, r);
     int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
if (qL < m) update_segment_max_tree(vL, 1, m);
if (m < qR) update_segment_max_tree(vR, m, r);
     update_vertex(v, 1, r);
void update_segment_add(int _qL, int _qR, ll _val) {
     qL = _qL;
qR = _qR + 1;
val = _val;
     update_segment_add_tree(1, 0, n);
void update_segment_add_tree(int v, int 1, int r) {
   if (qL <= 1 && r <= qR) {</pre>
          recalc_add(v, 1, r, val);
     push(v, 1, r);
     int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
```

```
if (qL < m) update_segment_add_tree(vL, 1, m);</pre>
          if (m < qR) update_segment_add_tree(vR, m, r);</pre>
          update_vertex(v, 1, r);
     11 get(int _qL, int _qR) {
         qL = _qL;
qR = _qR + 1;
          ans = 0;
          get_tree(1, 0, n);
         return ans;
     void get_tree(int v, int 1, int r) {
   if (qL <= 1 && r <= qR) {
      ans += tree[v].sum;
}</pre>
               return;
          push(v, 1, r);
          int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
          if (qL < m) get_tree(vL, 1, m);</pre>
          if (m < qR) get_tree(vR, m, r);</pre>
     void print_all() {
         print_all_tree(1, 0, n);
     void print_all_tree(int v, int 1, int r) {
         if (1 + 1 == r) {
               cout << tree[v].sum << en;</pre>
              return:
         push(v, 1, r);
int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
         print_all_tree(vL, 1, m);
print_all_tree(vR, m, r);
};
```

## 2.17 Persistent Segment Tree (Sweezyk)

```
struct Node {
    Node *1, *r;
    int mx;
    Node() {
        1 = r = nullptr;
    }
};
const int N = 1e7 + 4e6;
const int LG = 20;
Node *nodes[N];
int ptr;
Node *new_node() {
    return nodes[ptr++];
Node *get_left(Node *t) {
   if (t && t->l) return t->l;
    return nullptr;
Node *get_right(Node *t) {
    if (t && t->r) return t->r;
    return nullptr;
int get_max(Node *t) {
    if (!t) return 0;
    return t->mx;
void update(int i, int val, Node *t, Node *old_t, int lx, int rx) {
    if (1x + 1 == rx) {
        t->mx = max(t->mx, val);
        return;
    int m = (1x + rx) / 2;
    if (i < m) {
         t->1 = new_Node();
         if (old_t && old_t->1) {
              t->1->mx = old_t->1->mx;
         t->r = get_right(old_t);
        update(i, val, t->1, get_left(old_t), lx, m);
    } else {
        t->r = new_Node();
         if (old_t && old_t->r) {
              t \rightarrow r \rightarrow mx = old_t \rightarrow r \rightarrow mx;
         t->1 = get_left(old_t);
```

```
update(i, val, t->r, get_right(old_t), m, rx);
}
t->mx = max(get_max(get_left(t)), get_max(get_right(t)));
};
int get(int l, int r, Node *t, int lx, int rx) {
    if (!t || lx >= r || rx <= l) return -1;
    if (lx >= l && rx <= r) return t->mx;
    int m = (lx + rx) / 2;
    return max(get(l, r, t->l, lx, m), get(l, r, t->r, m, rx));
}
```

#### 2.18 Fenwick (TheEvilBird)

```
struct Fenwick {
    int n:
    vector<11> f:
   Fenwick(int _n) {
        n = _n;
        f.assign(n + 1, 0);
   void update(int x, ll delta) {
        for (int i = x; i <= n; i += i & -i) {
            f[i] += delta;
   }
   11 get_sum(int x) {
        11 s = 0;
for (int i = x; i > 0; i -= i & -i) {
           s += f[i];
        return s;
   11 get(int L, int R) {// [L, R]
        return get_sum(R) - get_sum(L - 1);
   }
};
```

## 2.19 Sparse table (TheEvilBird)

```
// usage:
    auto fun = [8] (int i, int j) { return min(i, j); };
     SparseTable<int, decltype(fun)> st(a, fun);
// SparseTable<int> st(a, [\theta](int i, int j) \{ return min(i, j); \});
template <typename T, class F = function<T(const T&, const T&)>>
class SparseTable {
    vector<int> lg;
    vector<vector<T>> mat;
    F func;
    SparseTable(const vector<T>& a, const F& f) : func(f) {
        n = static_cast<int>(a.size());
        lg.resize(n + 1);
for (int i = 2; i <= n; ++i) {
   lg[i] = lg[i / 2] + 1;</pre>
        mat.resize(lg[n] + 1);
        mat[0] = a;
         for (int j = 1; j \le lg[n]; j++) {
             mat[j].resize(n - (1 << j) + 1);
for (int i = 0; i <= n - (1 << j); i++) {</pre>
                 mat[j][i] = func(mat[j - 1][i], mat[j - 1][i + (1 << (j -
       1))]);
    }
    T get(int from, int to) const {
         assert(0 <= from && from <= to && to <= n - 1);
         int 1 = lg[to - from + 1];
         return func(mat[1][from], mat[1][to - (1 << 1) + 1]);
    }
};
```

```
void push(Node *t) {
    if (t == nullptr) return;
    int p = t->push;
if (p == 0) return;
    if (t->1 != nullptr) {
        t->1->cnt += p;
        t->1->push += p;
    if (t->r != nullptr) {
         t->r->cnt += p;
         t->r->push += p;
    t->push = 0;
int size(Node *t) {
    return (t ? t->size : 0);
void update(Node *t) {
    if (t == nullptr) return;
    t \rightarrow size = size(t \rightarrow 1) + size(t \rightarrow r) + 1;
pair<Node *, Node *> split(Node *t, int k) {
    if (t == nullptr) return {nullptr, nullptr};
    if (k == 0) return {nullptr, t};
    push(t);
     if (size(t->1) + 1 <= k) {
         auto [l, r] = split(t->r, k - size(t->l) - 1);
         update(t);
        return {t, r};
    } else {
        auto [1, r] = split(t->1, k);
         update(t);
         return {1, t};
    }
Node *merge(Node *tl, Node *tr) {
    if (t1 == nullptr) return tr;
    if (tr == nullptr) return tl;
    push(t1);
     push(tr);
    if (t1->v > tr->v) {
        tl->r = merge(tl->r, tr);
         update(t1);
         return tl;
    } else {
        tr->1 = merge(t1, tr->1);
         update(tr);
        return tr;
void dfs(Node *t) {
    if (t == nullptr) return;
    push(t);
    dfs(t->1);
     cout << t->x << ' ' << t->cnt << '\n';
    dfs(t->r);
void solve() {
    int n, m;
    cin >> n >> m;
    Node *root = nullptr;
for (int i = 1; i <= n; i++) {
   Node *add = new Node(i);</pre>
        root = merge(root, add);
    for (int i = 0; i < m; i++) {
         int 1, r;
         cin >> 1 >> r;
        auto [L, R] = split(root, r);
auto [L1, L2] = split(L, 1 - 1);
         L2->push = 1;
         L2->cnt += 1;
         root = merge(L2, merge(L1, R));
    dfs(root);
```

#### 2.20 Treap (Sweezyk)

```
struct Node {
  int x, y, size, push, cnt;
  Node *1, *r;

  Node(int val) : x(val), y(rng()), size(1), push(0), cnt(0), l(nullptr
    ), r(nullptr) {}
```

# 2.21 Extended GCD (Sweezyk)

```
template<typename T>
T extgcd(T a, T b, T &x, T &y) {
   if (a == 0) {
      x = 0;
      y = 1;
      return b;
}
```

```
T p = b / a;
T g = extgcd(b - p * a, a, y, x);
x -= p * y;
    return g;
template<typename T>
bool diophantine(T a, T b, T c, T &x, T &y, T &g) {
   if (a == 0 && b == 0) {
      if (c == 0) {
              x = y = g = 0;
              return true;
          }
          return false;
    if (a == 0) {
          if (c % b == 0) {
              x = 0;
              y = c / b;
              g = abs(b);
              return true;
         return false;
     if (b == 0) {
         if (c \% a == 0) {
              x = c / a;
              y = 0;
g = abs(a);
              return true;
          return false;
    }
    g = extgcd(a, b, x, y);
if (c % g != 0) {
    return false;
    T dx = c / a;
     c = dx * a;
    T dy = c / b;
    c -= dy * b;
x = dx + (T) ((__int128) x * (c / g) % b);
    y = dy + (T) ((_int128) y * (c / g) % a);
     return true;
     // |x|, |y| \le max(|a|, |b|, |c|) [tested]
bool crt(long long k1, long long m1, long long k2, long long m2, long
      long &k, long long &m) {
     k1 %= m1;
     if (k1 < 0) k1 += m1;
     k2 \% = m2:
     if (k2 < 0) k2 += m2;
    long long x, y, g;
    if (!diophantine(m1, -m2, k2 - k1, x, y, g)) {
         return false;
    long long dx = m2 / g;
long long delta = x / dx - (x % dx < 0);
k = m1 * (x - dx * delta) + k1;
    m = m1 / g * m2;
assert(0 <= k && k < m);
    return true;
}
```

#### 2.22 FFT (FedShat)

```
constexpr int P = 998244353;
using i64 = long long;
// assume -P \ll x \ll 2P
int norm(int x) {
    if (x < 0) {
        x += P;
    if (x \ge P) {
        x -= P;
    }
    return x;
}
template<class T>
T power(T a, int b) {
    T res = 1;
for (; b; b /= 2, a *= a) {
    if (b % 2) {
             res *= a;
        }
    }
    return res;
struct Z {
```

```
Z(int x = 0) : x(norm(x)) {}
     int val() const {
         return x:
    Z operator-() const {
         return Z(norm(P - x));
    Z inv() const {
         assert(x != 0);
         return power(*this, P - 2);
     Z &operator*=(const Z &rhs) {
         x = i64(x) * rhs.x % P;
         return *this:
    Z &operator+=(const Z &rhs) {
         x = norm(x + rhs.x);
         return *this;
    Z &operator-=(const Z &rhs) {
         x = norm(x - rhs.x);
         return *this:
    Z &operator/=(const Z &rhs) {
         return *this *= rhs.inv();
    friend Z operator*(const Z &lhs, const Z &rhs) {
         Z res = lhs:
         res *= rhs;
         return res;
     friend Z operator+(const Z &lhs, const Z &rhs) {
         Z res = lhs;
res += rhs;
         return res:
    friend Z operator-(const Z &lhs, const Z &rhs) {
         res -= rhs;
         return res;
    friend Z operator/(const Z &lhs, const Z &rhs) {
         Z res = lhs;
         res /= rhs;
         return res;
    }
}:
std::vector<int> rev;
std::vector<Z> roots{0, 1};
void dft(std::vector<Z> &a) {
    int n = a.size();
    if (int(rev.size()) != n) {
         int k = __builtin_ctz(n) - 1;
         rev.resize(n);
         for (int i = 0; i < n; i++) {
    rev[i] = rev[i >> 1] >> 1 | (i & 1) << k;
    }
    for (int i = 0; i < n; i++) {
   if (rev[i] < i) {</pre>
              std::swap(a[i], a[rev[i]]);
     if (int(roots.size()) < n) {</pre>
         int k = __builtin_ctz(roots.size());
         roots.resize(n);
         while ((1 << k) < n) {
              Z = power(Z(3), (P - 1) >> (k + 1));
for (int i = 1 << (k - 1); i < (1 << k); i++) {
                  roots[2 * i] = roots[i];
                  roots[2 * i + 1] = roots[i] * e;
              k++:
         }
    for (int k = 1; k < n; k *= 2) {
   for (int i = 0; i < n; i += 2 * k) {
             for (int j = 0; j < k; j++) {
   Z u = a[i + j];
   Z v = a[i + j + k] * roots[k + j];
   a[i + j] = u + v;
   a[i + j + k] = u - v;
   }
void idft(std::vector<Z> &a) {
    int n = a.size();
     std::reverse(a.begin() + 1, a.end());
     Z inv = (1 - P) / n;
    for (int i = 0; i < n; i++) {
    a[i] *= inv;</pre>
```

```
}
```

## 2.23 KTO (FedShat)

```
struct Eq \{// x = a \pmod{m}
    11 a, m;
    Eq(){};
    Eq(11 a, 11 m) : a(a), m(m){};
};
11 binpow(11 a, 11 n, 11 m) {
   if (n == 0) {
        return 1;
    if (n \% 2 == 0) {
        int128_t b = binpow(a, n / 2, m);
return (b * b) % m;
    int128_t x = binpow(a, n - 1, m);
    return (a * x) % m;
11 binpow(11 a, 11 n) {
   if (n == 0) {
        return 1;
    if (n \% 2 == 0) {
        11 b = binpow(a, n / 2);
        return b * b;
    return a * binpow(a, n - 1);
}
ll phi(ll a) {
    11 d = 2, k = a;
map<11, int> cnt;
    while (d * d \le a) {
        if (k % d == 0) {
            ++cnt[d];
        } else {
            ++d:
    if (k != 1) {
         ++cnt[k];
    11 ans = 1;
    for (auto i : cnt) {
        ans *= binpow(i.first, i.second - 1) * (i.first - 1);
    return ans;
}
11 gcd(11 a, 11 b) {
    return std::gcd(abs(a), abs(b));
}
Eq solve(Eq ai, Eq bi) {
    if (ai.m == -1 || bi.m == -1) {
        return {0, -1};
    ll a = ai.m, b = bi.m, c = ai.a - bi.a;
    11 d = ::gcd(a, b);
    a /= d;
b /= d;
    if (c % d != 0) {
        return {0, -1};
    c /= d;
    11 x = (((int128_t) -c * (int128_t) binpow(a, phi(b) - 1, b)) % b + b
    x = ((int128_t) ai.m * (int128_t) x + ai.a) % lcm(ai.m, bi.m);
    return {x, lcm(ai.m, bi.m)}:
```

#### 2.24 Обратные по простому модулю

Пусть дан простой модуль m. Для каждого числа из [1, m-1] найти обратное к нему.

```
r[1] = 1;
for (int i = 2; i < m; ++i) {
    r[i] = (m - (m / i) * r[m % i] % m) % m;
}</pre>
```

## 2.25 Обратные факториалы

```
int inv(int a, int m) {
    if (a == 1)
        return 1;
    return (1 - inv(m % a, a) * m) / a + m;
}

{
    f[0] = 1;
    for (int i = 1; i < N; i++) {
        f[i] = i * f[i - 1] % mod;
    }

    r[N - 1] = inv(f[N - 1]) for (int i = N - 1; i >= 1; i--) {
        r[i - 1] = r[i] * i % mod;
    }
}
```

## 2.26 Faycc

```
const double eps = 1e-9;
int Gauss(vector<vector<double>> a, vector<double> &ans) {
     int n = (int)a.size(), m = (int)a[0].size() - 1;
     vector<int> pos(m, -1);
     double det = 1; int rank = 0;
     for(int col = 0, row = 0; col < m && row < n; ++col) {
          int mx = row;
          for (int i = row; i < n; i++) {
               if (fabs(a[i][col]) > fabs(a[mx][col])) { mx = i; }
          if (fabs(a[mx][col]) < eps) { det = 0; continue; }
for (int i = col; i <= m; i++) {
    swap(a[row][i], a[mx][i]);</pre>
          if (row != mx) { det = -det; }
          det *= a[row][col];
         det *= a[row][coi],
pos[col] = row;
for (int i = 0; i < n; i++) {
    if (i != row && fabs(a[i][col]) > eps) {
        double c = a[i][col] / a[row][col];
        for (int j = col; j <= m; j++) {
            a[i][j] -= a[row][j] * c;
        }
}</pre>
               }
          ++row: ++rank:
     ans.assign(m, 0);
     for(int i = 0; i < m; i++) {
          if (pos[i] != -1) { ans[i] = a[pos[i]][m] / a[pos[i]][i]; }
     for(int i = 0; i < n; i++) {
          double sum = 0;
for (int j = 0; j < m; j++) {
               sum += ans[j] * a[i][j];
          if(fabs(sum - a[i][m]) > eps) {
                return -1; //no solution
     for (int i = 0; i < m; i++) {
          if (pos[i] == -1) {
                return 2; //infinte solutions
     return 1; //unique solution
```

#### Бинарный

```
//n = number of equations, m = number of variables
int Gauss(int n, int m, vector<bitset<N>> a, bitset<N> &ans) {
    //reversing for lexocgraphically largest solution for (int i = 0; i < n; i++) {
          bitset<N> tmp;
          for (int j = 0; j < m; j++) tmp[j] = a[i][m - j - 1];
          tmp[m] = a[i][m];
          a[i] = tmp;
     int rank = 0, det = 1:
     rink = 0, det = 1;
vector(int> pos(N, -1);
for(int col = 0, row = 0; col < m && row < n; ++col) {</pre>
          int mx = row;
          for(int i = row; i < n; ++i) if(a[i][col]) { mx = i; break; }
if(!a[mx][col]) { det = 0; continue; }</pre>
          swap(a[mx], a[row]);
          if (row != mx) { det = (det == 0 ? 0 : 1); }
          det &= a[row][col];
          pos[col] = row;
           //forward elimination
          for (int i = row + 1; i < n; ++i) {
   if (i != row && a[i][col]) {
      a[i] ^= a[row];
}</pre>
```

```
}
         ++row, ++rank;
    ans.reset():
    //backward substituition
    for (int i = m - 1; i \ge 0; i--) {
         if (pos[i] == -1) {
             ans[i] = true;
         } else {
             int k = pos[i];
             for (int j = i + 1; j < m; j++) if (a[k][j]) ans[i] = ans[i]
        ans[j];
             ans[i] = ans[i] ^ a[k][m];
        }
    }
    for (int i = rank; i < n; ++i) {
   if (a[i][m]) {</pre>
            return -1; //no solution
    //reversing again beacuse we reversed earlier
    bitset<N> tmp;
for (int j = 0; j < m; j++) {</pre>
        tmp[j] = ans[m - j - 1];
    ans = tmp;
    int free_var = 0;
    for(int i = 0; i < m; ++i) {
    if (pos[i] == -1) {
             free_var++;
    }
    return free_var; //has solution
}
```

## 2.27 Быстрая факторизация (FedShat)

```
11 binpow(11 a, 11 n, 11 mod) {
   if (n == 0) {
    if (n % 2 == 0) {
   int128_t b = binpow(a, n / 2, mod);
        return (b * b) % mod;
    return (((int128_t) a) * binpow(a, n - 1, mod)) % mod;
}
constexpr int N = 1e7;
vector<int> pr, lp;
   if (n <= N) {
        return binary_search(all(pr), n);
    int iter = 60:
    int s = 0;
    11 d = n - 1;
while (d % 2 == 0) {
        d /= 2;
        ++s;
    auto test = [&](11 a) {
        if (binpow(a, d, n) == 1) {
            return true;
        11 _2r = 1;
for (int r = 0; r < s; ++r) {
            auto tmp = binpow(binpow(a, d, n), _2r, n);
if (tmp == n - 1) {
            }
            <u>2r</u> *= 2;
        return false;
    for (int _ = 0; _ < iter; ++_) {
         11 a = uniform_int_distribution<11>(1, n - 1)(rnd);
        if (!test(a)) {
             return false;
    return true;
}
11 f(11 x, 11 n) {
    return ((int128_t) x * (int128_t) x + (int128_t) 2) % n;
11 pollard(11 n) {
    11 a = uniform_int_distribution<11>(0, n - 1)(rnd);
    11 x = a, y = a, d = 1;
constexpr int iter = 5e4;
    for (int _ = 0; _ < iter; ++_) {
```

```
x = f(f(x, n), n);
       y = f(y, n);
d = gcd(abs(x - y), n);
       if (d != 1 && d != n) {
            break:
   if (d == 1 || d == n) {
       pollard(n);
   7
   return d;
vector<11> res;
void factor(ll n) {
   if (n \le N) {
       while (true) {
           if (lp[n] == 0) {
            res.push_back(lp[n]);
            n /= lp[n];
       return;
    if (prime(n)) {
        res.push_back(n);
       return:
    11 d = pollard(n);
    factor(n / d);
    factor(d);
void solve() {
   11 n;
    lp.resize(N + 1);
    for (int i = 2; i <= N; ++i) {
       if (lp[i] == 0) {
    lp[i] = i;
           pr.push_back(i);
        for (int j = 0; j < (int) pr.size() && pr[j] <= lp[i] && i * pr[j
     ] <= N; ++j) {
            lp[i * pr[j]] = pr[j];
   factor(n);
```

#### 2.28 Префикс-функция

#### **2.29 Z**-функция

```
vector<int> z_function(string &s) {
   int n = (int) s.length();
   vector<int> z(n);
   for (int i = 1, 1 = 0, r = 0; i < n; ++i) {
        if (i <= r) {
            z[i] = min(r - i + 1, z[i - 1]);
        }
        while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
            ++z[i];
        }
        if (i + z[i] - 1 > r) {
            l = i;
            r = i + z[i] - 1;
        }
        return z;
}
```

## 2.30 Суфмас (TheEvilBird)

```
int mod(int x, int m) {
    if (x < 0) x += m;
    if (x >= m) x -= m;
    return x;
}
vector<int> sufix array(string s) {
    int n = sz(s);
    vector<int> arr(n), narr(n), head(n), c(n), nc(n);
for (int i = 0; i < n; ++i) {</pre>
        arr[i] = i;
    sort(all(arr), [&](int x, int y) {
        return s[x] < s[y];</pre>
    int cl = 0;
    c[arr[0]] = cl;
    head[0] = 0;
for (int i = 1; i < n; ++i) {
         if (s[arr[i]] != s[arr[i - 1]]) {
            head[++c1] = i;
         c[arr[i]] = cl;
    for (int k = 1; k < n && cl < n; k *= 2) {
        for (int i = 0; i < n; i++) {
    int j = mod(arr[i] - k, n);
             narr[head[c[j]]++] = j;
         c1 = 0:
         head[0] = 0;
         pii prev = {c[narr[0]], c[mod(narr[0] + k, n)]};
         for (int i = 1; i < n; i++) {
             pii cur = {c[narr[i]], c[mod(narr[i] + k, n)]};
             if (cur != prev) {
    head[++cl] = i;
             nc[narr[i]] = cl;
             prev = cur;
         }
         swap(arr, narr);
         swap(c, nc);
     // returns sufix array without $
    return vector<int>(arr.begin() + 1, arr.end());
}
vector<int> build_lcp(string &s, vector<int> &suf) {
  int n = sz(s);
    vector<int> lcp(n - 1), order(n);
    for (int i = 0; i < n; ++i) {
         order[suf[i]] = i;
    int 1 = 0:
    for (int i = 0; i < n; ++i) {
         int id = order[i];
         if (id + 1 == n) {
             1 = 0;
             continue;
         int j = suf[id + 1];
         if (1) --1;
         while (\max(i + 1, j + 1) < n \&\&
                s[i + 1] == s[j + 1]) {
             ++1:
         lcp[id] = 1;
    return lcp;
```

# 2.31 Суфавтомат (TheEvilBird)

```
struct Node {
    int go[26];
    int suf, prev, term, len;

    Node() {
        for (auto &i : go) {
            i = -1;
        }
        len = 0;
        suf = -1;
        prev = -1;
        term = 0;
    }
};

vector<Node> automat;
int add(int a, int ch) {
```

```
int b = sz(automat);
automat.emplace_back();
automat[b].prev = a;
automat[b].suf = 0;
automat[b].len = automat[a].len + 1;
for (; a != -1; a = automat[a].suf) {
    if (automat[a].go[ch] == -1) {
         automat[a].go[ch] = b;
         continue;
    int c = automat[a].go[ch];
if (automat[c].prev == a) {
         automat[b].suf = c;
         break;
    int d = sz(automat);
    automat.emplace_back();
automat[d].suf = automat[c].suf;
     automat[d].len = automat[a].len + 1;
     automat[c].suf = d;
     automat[b].suf = d;
    automat[d].prev = a;
for (int i = 0; i < 26; ++i) {</pre>
         automat[d].go[i] = automat[c].go[i];
     for (; a != -1 && automat[a].go[ch] == c; a = automat[a].suf) {
         automat[a].go[ch] = d;
    break:
// returns id of the added vertex
return b:
```

## 2.32 Axo-Корасик (Sweezyk)

```
struct Node {
    int par;
    int par_c;
    int go[26];
    int link;
    int super;
    int cnt:
const int N = 1e6 + 5;
int ptr = 1
Node trie[N];
void add(string s) {
    int cur = 1;
    for (auto &q : s) {
        int c = q - 'a';
        if (trie[cur].go[c]) {
            cur = trie[cur].go[c];
        } else {
            trie[cur].go[c] = ptr;
            trie[ptr].par = cur;
            trie[ptr].par_c = c;
            cur = ptr;
        }
    trie[cur].cnt++;
void build() {
    queue<int> q;
    q.push(1);
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        if (v != 1) {
            if (trie[v].par == 1) {
                trie[v].link = trie[v].super = 1;
                trie[v].link = trie[trie[trie[v].par].link].go[trie[v].
                trie[v].super = (trie[trie[v].link].cnt ? trie[v].link :
     trie[trie[v].link].super);
        trie[v].cnt += trie[trie[v].link].cnt;
        for (int c = 0; c < 26; c++) {
            if (trie[v].go[c]) {
                q.push(trie[v].go[c]);
            } else {
                if (v == 1) {
                    trie[v].go[c] = 1;
                } else {
                    trie[v].go[c] = trie[trie[v].link].go[c];
                }
```

```
}
trie[1].link = trie[1].super = trie[1].par = 1;
build();
```

#### 2.33 Манакер

```
vector<int> d1(n):
int 1 = 0, r = -1;
for (int i = 0; i < n; ++i) {
  int k = (i > r ? 1 : min(d1[1 + r - i], r - i + 1));
     while (i + k < n &  i - k >= 0 &  s[i + k] == s[i - k]) {
    d1[i] = k;
    if (i + k - 1 > r) {
 1 = i - k + 1;
         r = i + k - 1;
}
vector<int> d2(n);
1 = 0:
r = -1;
for (int i = 0; i < n; ++i) {
    int k = (i > r ? 0 : min(d2[1 + r - i + 1], r - i + 1));
    while (i + k < n \&\& i - k - 1 >= 0 \&\& s[i + k] == s[i - k - 1]) {
         ++k;
    }
    d2[i] = k;
if (i + k - 1 > r) {
  l = i - k;
    }
}
```

## 2.34 CHT (FedShat)

```
struct Line {
    11 k = 0, b = -INFLL;
    Line() = default;
   Line(11 k, 11 b) : k(k), b(b){};
    ld operator()(ld x) {
        return k * x + b;
};
bool operator<(Line a, Line b) {</pre>
   return a.k < b.k | | (a.k == b.k && a.b > b.b);
bool operator<(pair<Line, ld> a, pair<Line, ld> b) {
   return a.second < b.second;</pre>
7
struct CHT {
   vector<pair<Line, ld>> convex;
   void add(Line a) {
       while (!convex.empty() && a(convex.back().second) > convex.back()
     .first(convex.back().second)) {
            convex.pop_back();
        ld xn = -INFLL;
        if (!convex.empty()) {
            xn = (a.b - convex.back().first.b + 0.0) / (convex.back().
     first.k - a.k);
        convex.push_back({a, xn});
    CHT(vector<Line> lines) {
        sort(all(lines)):
        for (int i = 0; i < (int) lines.size();) {
            int j = i;
            while (j < (int) lines.size() && lines[i].k == lines[j].k) {</pre>
                ++j;
            }
            add(lines[i]);
            i = j;
        }
   }
   ld get(ld x) {
        auto it = upper_bound(all(convex), pair(Line(), x)) - convex.
     begin():
        return convex[it - 1].first(x);
};
```

## 2.35 Дебаг Туриста

```
template<typename A, typename B>
string to_string(pair<A, B> p);
template<typename A, typename B, typename C>
string to_string(tuple<A, B, C> p);
template<typename A, typename B, typename C, typename D>
string to_string(tuple<A, B, C, D> p);
string to_string(const string &s) {
   return '"' + s + '"';
string to_string(const char *s) {
    return to_string((string) s);
string to_string(bool b) {
   return (b ? "true" : "false");
string to_string(vector<bool> v) {
    bool first = true;
string res = "{";
    for (int i = 0; i < static_cast<int>(v.size()); i++) {
   if (!first) {
             res += ",
         first = false;
         res += to_string(v[i]);
    res += "}";
    return res;
template<size_t N>
string to_string(bitset<N> v) {
    string res = "";
     for (size_t i = 0; i < N; i++) {
        res += static_cast<char>('0' + v[i]);
     return res;
template<typename A>
string to_string(A v) {
    bool first = true;
string res = "{";
    for (const auto &x : v) {
        if (!first) {
             res += ", ";
         first = false;
         res += to_string(x);
    res += "}":
    return res;
template<typename A, typename B>
string to_string(pair<A, B> p) {
   return "(" + to_string(p.first) + ", " + to_string(p.second) + ")";
template<typename A, typename B, typename C>
string to_string(tuple<A, B, C> p) {
    return "(" + to_string(get<0>(p)) + ", " + to_string(get<1>(p)) + ",
       " + to_string(get<2>(p)) + ")";
template<typename A, typename B, typename C, typename D>
string to_string(tuple<A, B, C, D> p) {
   return "(" + to_string(get<0>(p)) + ", " + to_string(get<1>(p)) + ","
       " + to_string(get<2>(p)) + ", " + to_string(get<3>(p)) + ")";
void debug_out() { cerr << endl; }</pre>
template<typename Head, typename... Tail>
void debug_out(Head H, Tail... T) {
   cerr << " " << to_string(H);</pre>
    debug_out(T...);
}
#ifdef LOCAL
#define debug(...) cerr << "[" << #__VA_ARGS__ << "]:", debug_out(
      __VA_ARGS__)
#define debug(...) 42
```

# 2.36 Геометрия (TheEvilBird)

```
const long double PI = 3.1415926535897932384626433832795;
const long double EPS = 1e-7;
 * Rotate relative to the origin, a - rotation angle:
 * new_x = x cos(a) - y sin(a)
* new_y = x sin(a) + y cos(a)
 * new_x = x cos(a) + y sin(a)
 * new_y = x sin(a) - y cos(a)
 * rad = degree * PI / 180
 * degree = rad * 180 / PI
template<typename T>
struct point_t {
    T x, y;
    point_t() : x(0), y(0) {}
    point_t(T _x, T _y) : x(_x), y(_y) {}
    T len_sq() const {
        return x * x + y * y;
    ld len() const {
        return sqrtl(len_sq());
    point_t operator*(T k) const {
        return \{x * k, y * k\};
    void operator*=(11 k) {
        x *= k;
        v *= k:
    point_t operator+(const point_t<T> &other) const {
        return {x + other.x, y + other.y};
    point_t operator-(const point_t<T> &other) const {
        return {x - other.x, y - other.y};
    bool operator==(const point_t<T> &other) const {
        return x == other.x && y == other.y;
    bool operator!=(const point_t<T> &other) const {
        return !(*this == other);
    T operator*(const point_t<T> &other) const {// dot product
        return x * other.x + y * other.y;
    T operator%(const point_t<T> &other) const {// cross product
        return x * other.y - y * other.x;
};
using Point = point_t<11>;
using PointLD = point_t<1d>;
template<typename T>
istream &operator>>(istream &is, point_t<T> &vec) {
    is >> vec.x >> vec.y;
    return is;
}
template<typename T>
ostream &operator<<(ostream &os, const point_t<T> &vec) {
    os << vec.x << ' ' << vec.y;
    return os;
}
typedef vector<Point> Polygon;
bool cmp_vectors(Point a, Point b) {
   if ((a.y < 0 | | (a.y == 0 \&\& a.x < 0)) \&\&
        (b.y > 0 \mid | (b.y == 0 \&\& b.x > 0))) {
        return true;
    if ((b.y < 0 \mid | (b.y == 0 \&\& b.x < 0)) \&\&
        (a.y > 0 \mid | (a.y == 0 \&\& a.x > 0))) {
        return false:
    return (a % b > 0 || (a % b == 0 && a.len_sq() < b.len_sq()));
int get_sign(ld x) {
```

```
if (x < -EPS) return -1;
    if (EPS < x) return 1;
    return 0;
Polygon build_convex_hull(Polygon &a) {
     int n = sz(a);
    for (int i = 1; i < n; ++i) {
        if ((a[i].y < a[0].y) \mid | (a[i].y == a[0].y && a[i].x < a[0].x))
      swap(a[0], a[i]);
    sort(a.begin() + 1, a.end(), [&](Point A, Point B) {
        Point oa = A - a[0], ob = B - a[0];
if ((oa % ob) == 0) return oa.len_sq() < ob.len_sq();
        return (oa % ob) > 0;
    }):
    Polvgon hull = {a[0]};
    for (int i = 1; i < n; ++i) {
        while (sz(hull) >= 2) {
             Point ab = hull[sz(hull) - 1] - hull[sz(hull) - 2], bp = a[i]
       - hull[sz(hull) - 1];
             if ((ab % bp) <= 0) {
                 hull.pop_back();
             } else {
                 break;
             }
        hull.emplace_back(a[i]);
    return hull;
11 area_of_polygon(Polygon &poly) {
    11 res = 0;
int n = sz(poly);
    for (int i = 2; i < n; ++i) {
   Point ab = poly[i - 1] - poly[0], ac = poly[i] - poly[0];
         res += (ab % ac);
     // don't forget to divide the result by 2!
    return res;
ld perimeter_of_polygon(Polygon &poly) {
    1d res = 0;
    int n = sz(poly);
    for (int i = 0; i < n; ++i) {
        Point v = poly[(i + 1 == n ? 0 : i + 1)] - poly[i];
        res += v.len();
    return res:
}
11 diameter_of_polygon(Polygon &poly) {
    int n = sz(poly), x = 1;
    11 ans = 0;
    for (int i = 0; i < n; ++i) {
        int j = (i + 1) \% n;
         while (true) {
     Point ac = poly[x] - poly[i], ab = poly[j] - poly[i], cd =
poly[(x + 1) % n] - poly[x];
    ans = max(ans, ac.len_sq());
             if ((ab % cd) <= 0) {
                  break;
             }
             x++;
             x \% = n;
    // don't forget to extract root!
    return ans;
template<typename T>
ld angle(const point_t<T> &a, const point_t<T> &b) {
    return fabsl(atan2(a % b, a * b) / PI * 180);
}
template<typename T>
ld angle_rad(const point_t<T> &a, const point_t<T> &b) {
   return (atan2(a % b, a * b));
template<typename T>
ld angle_rad(const point_t<T> &a) {
    return atan2(a.y, a.x);
template<typename T>
point_t < ld > rotate(const point_t < T > &a, ld alpha) {
    return \{a.x * cos(alpha) - a.y * sin(alpha), a.x * sin(alpha) + a.y *
       cos(alpha)};
ld from_point_to_line(const Point &p, const Point &a, const Point &b) {
    // point p, line ab
Point ba = a - b, ap = p - a;
return fabs((ba % ap) / ba.len());
```

```
}
ld from_point_to_ray(const Point &p, const Point &a, const Point &b) { //
    point p, ray ab
Point ba = a - b, ap = p - a, ab = b - a;
if ((ab * ap) < 0) {</pre>
         return ap.len();
    } else {
        return fabs((ba % ap) / ba.len());
    7
}
ld from_point_to_segment(const Point &p, const Point &a, const Point &b)
      { // point p, segment ab }
    Point ab = b - a, ap = p - a, bp = p - b, ba = a - b; if ((ab * ap) < 0) {
         return fabsl(ap.len());
    } else if ((ab * bp) > 0) {
        return fabsl(bp.len());
         return fabsl((ba % ap) / ba.len());
    }
}
bool point_on_line(const Point &p, const Point &a, const Point &b) { //
    point p, line ab
Point ab = b - a, ap = p - a;
    return (ab % ap) == 0;
}
bool point_on_ray(const Point &p, const Point &a, const Point &b) { //
    point p, ray ab
Point ab = b - a, ap = p - a;
    return ((ab % ap) == 0 && (ab * ap) > 0) || (a == p);
}
bool point_on_segment(const Point &p, const Point &a, const Point &b) {
      // point p, segment ab
    Point ap = p - a, bp = p - b, ab = b - a;
return (ap * bp) <= 0 && (ap % ab) == 0;
}
bool point_in_angle(const Point &p, const Point &a, const Point &o, const
    Point &b) { // point p, angle aob
Point oa = a - o, ob = b - o, op = p - o;
    if ((oa % ob) < 0) swap(oa, ob);
    return ((oa % op) >= 0 && (ob % op) <= 0);
}
bool segment_intersection(const Point &a, const Point &b, const Point &c,
       const Point &d) { // segment ab, segment, cd
    Point ab = b - a, cd = d - c, ac = c - a, ad = d - a, cb = b - c, ca
      = a - c;
    if (get_sign((ab % ac)) * get_sign((ab % ad)) <= 0 && get_sign((cd %</pre>
      ca)) * get_sign((cd % cb)) <= 0) {
    ll x1 = max(min(a.x, b.x), min(c.x, d.x)), x2 = min(max(a.x, b.x))
      , max(c.x, d.x));
         11 y1 = max(min(a.y, b.y), min(c.y, d.y)), y2 = min(max(a.y, b.y))
      , max(c.y, d.y));
         return (x1 <= x2 && y1 <= y2);
    7
    return false;
}
bool lines_intersection(const Point &a, const Point &b, const Point &c,
      const Point &d) { // line ab, line cd
    Point ab = b - a, cd = d - c;
return ((ab % cd) != 0 || (ab % (c - a) == 0));
bool line_ray_intersection(const Point &a, const Point &b, const Point &c
        const Point &d) { // line ab, ray cd
    if (!lines_intersection(a, b, c, d)) {
        return 0;
    Point ab = b - a, dp = d + (a - c);
if (get_sign(ab % (dp - a)) * get_sign(ab % (c - a)) <= 0) {
         return 1;
    return 0:
}
Point get_inf(const Point &a, const Point &b) { // returns inf point on
      ray ab
    if (a.x == b.x) {
         if (a.y < b.y) {
             return {a.x, INFLL};
         } else {
             return {a.x, -INFLL};
         }
    if (a.x < b.x) {
        if (a.v < b.v) {
             return {INFLL, INFLL};
         } else if (a.y == b.y) {
             return {INFLL, a.y};
         } else {
             return {INFLL, -INFLL};
```

```
// a.x > b.x
     if (a.y < b.y) {
         return {-INFLL, INFLL}:
    } else if (a.y == b.y) {
         return {-INFLL, a.y};
    } else {
         return {-INFLL, -INFLL};
    7
}
bool rays_intersection(const Point &a, const Point &b, const Point &c,
       const Point &d) { // ray ab, ray cd
     if (line_ray_intersection(a, b, c, d) && line_ray_intersection(c, d,
      a, b)) {
         Point bp = get_inf(a, b), dp = get_inf(c, d);
11 x1 = max(min(a.x, bp.x), min(c.x, dp.x)), x2 = min(max(a.x, bp.x), min(c.x, dp.x))
       .x), max(c.x, dp.x));
         ll y1 = max(min(a.y, bp.y), min(c.y, dp.y)), y2 = min(max(a.y, bp.y))
       .y), max(c.y, dp.y));
         return (x1 <= x2 && y1 <= y2);
    7
    return 0;
int point_in_polygon(const Point &p, const Polygon &poly) {
     // 0 - outside, 1 - inside, 2 - border;
     int n = sz(poly);
     if (point_in_angle(p, poly[n - 1], poly[0], poly[1])) {
         int l = 1, r = n - 1;
         while (r - 1 > 1) {
              int md = (1 + r) / 2;
              if (point_in_angle(p, poly[md], poly[0], poly[1])) {
                  r = md;
              } else {
                  1 = md;
              }
         if (point_in_angle(p, poly[r], poly[0], poly[1]) &&
              point_in_angle(p, poly[0], poly[1], poly[1]) &&
point_in_angle(p, poly[0], poly[1], poly[0])) {
  if (point_on_segment(p, poly[1], poly[r]) ||
     point_on_segment(p, poly[1], poly[r]) ||
     point_on_segment(p, poly[(1 ? 1 - 1 : n - 1)], poly[1])
      \Pi
                   point_on_segment(p, poly[r], poly[(r + 1 == n ? 0 : r +
      1)])) {
                  return 2:
              } else {
                  return 1:
         } else {
             return 0;
         7
    } else {
         return 0;
int point_in_nonconvex_polygon(const Point &p, const Polygon &poly) {
     // 0 - outside, 1 - inside, 2 - border;
     int n = sz(poly);
     for (int i = 0; i < n; ++i) {
         if (point_on_segment(p, poly[i], poly[(i + 1) % n])) {
              return 2;
         }
     1d s = 0.0;
     for (int i = 0; i < n; ++i) {
         Point pa = poly[i] - p, pb = poly[(i + 1 == n ? 0 : i + 1)] - p;
         s += angle_rad(pa, pb);
    if (s \ge PI \mid | s \le -PI) {
         return 1;
     } else {
         return 0;
    7
Polygon minkowski_sum(Polygon &a, Polygon &b) {
       a[0], b[0]: y - max, y1 = y2 => x - max. Against clockwise
     int n = sz(a), m = sz(b);
     assert(n >= 3 && m >= 3);
     Point high_a = a[0], high_b = b[0];
    Polygon va(n), vb(m);
for (int i = 0; i < n; ++i) {</pre>
         va[i] = a[(i + 1) % n] - a[i];
     for (int i = 0; i < m; ++i) {
         vb[i] = b[(i + 1) \% m] - b[i];
      sort(all(va), cmp_vectors);
    sort(all(vb), cmp_vectors);
Polygon vc(sz(va) + sz(vb));
    merge(all(va), all(vb), vc.begin(), cmp_vectors);
Point high_c(high_a.x + high_b.x, high_a.y + high_b.y);
     Polygon c(sz(vc) + 1);
     c[0] = high_c;
```

```
for (int i = 0; i < sz(c) - 1; ++i) {
        c[i + 1] = c[i] + vc[i];
    return c:
}
ld from_polygon_to_polygon(Polygon a, Polygon b) {
    for (auto &i : b) {
        i *= -1;
    }
    int pos = 0;
    for (int i = 1; i < sz(b); ++i) {
        if ((b[i].y > b[pos].y) ||
(b[i].y == b[pos].y && b[i].x > b[pos].x)) {
        }
    rotate(b.begin(), b.begin() + pos, b.end());
    Polygon c = minkowski_sum(a, b);
    int n = sz(c);
    Point p(0, 0);
    1d ans = 1e20;
    for (int i = 0: i < n - 1: ++i) {
        ans = min(ans, from_point_to_segment(p, c[i], c[i + 1]));
}
11 diameter_of_polygon_minkowski(Polygon &a) {
    Polygon ra = a;
    for (auto &i : ra) {
    int pos = 0;
    for (int i = 1; i < sz(a); ++i) {
        if ((a[i].y > a[pos].y) ||
    (a[i].y == a[pos].y && a[i].x > a[pos].x)) {
            pos = i;
        }
    }
    rotate(a.begin(), a.begin() + pos, a.end());
    pos = 0:
    for (int i = 1; i < sz(a); ++i) {
        if ((ra[i].y > ra[pos].y) ||
             (ra[i].y == ra[pos].y \&\& ra[i].x > ra[pos].x)) {
        }
    rotate(ra.begin(), ra.begin() + pos, ra.end());
    Polygon c = minkowski_sum(a, ra);
    int n = sz(c);
    11 ans = 0;
    for (int i = 0; i < n; ++i) {
        ans = max(ans, c[i].len_sq());
    // don't forget to extract root!
    return ans;
}
ld from_segment_to_segment(const Point &a, const Point &b, const Point &c, const Point &d) { // segment ab, segment cd
    if (segment_intersection(a, b, c, d)) {
        return 0;
    return min({from_point_to_segment(a, c, d), from_point_to_segment(b,
     c, d),
                 from_point_to_segment(c, a, b), from_point_to_segment(d,
     a, b)});
}
bool segment_line_intersection(const Point &a, const Point &b, const
    Point &c, const Point &d) { // segment ab, line cd Point cd = d - c;
    if (get_sign(cd % (a - c)) * get_sign(cd % (b - c)) <= 0) {
        return 1;
    return 0;
}
bool ray segment intersection(const Point &a. const Point &b. const Point
      &c, const Point &d) { // ray ab, segment cd
    if (line_ray_intersection(c, d, a, b) && segment_line_intersection(c,
      d, a, b)) {
        Point bp = get_inf(a, b);
        11 x1 = max(min(a.x, bp.x), min(c.x, d.x)), x2 = min(max(a.x, bp.x))
     x), max(c.x, d.x));
        11 y1 = max(min(a.y, bp.y), min(c.y, d.y)), y2 = min(max(a.y, bp.
     y), max(c.y, d.y));
        return (x1 <= x2 && y1 <= y2);
    }
    return 0:
}
ld from_segment_to_ray(const Point &a, const Point &b, const Point &c,
      const Point &d) { // segment ab, ray cd
    if (ray_segment_intersection(c, d, a, b)) {
        return 0;
```

```
return min({from_point_to_ray(a, c, d),
                 from_point_to_ray(b, c, d),
                from_point_to_segment(c, a, b)});
}
ld from_segment_to_line(const Point &a, const Point &b, const Point &c,
     const Point &d) { // segment ab, line cd
    Point cd = d - c;
    if (get_sign(cd % (a - c)) * get_sign(cd % (b - c)) <= 0) {
        return 0:
    return min(from_point_to_line(a, c, d), from_point_to_line(b, c, d));
ld from_ray_to_ray(const Point &a, const Point &b, const Point &c, const
     Point &d) { // ray ab, ray cd
    if (rays_intersection(a, b, c, d)) {
        return 0:
    return min(from_point_to_ray(a, c, d), from_point_to_ray(c, a, b));
ld from_ray_to_line(const Point &a, const Point &b, const Point &c, const
      Point &d) { // ray ab, line cd
    if (line_ray_intersection(c, d, a, b)) {
        return 0;
    7
    return from_point_to_line(a, c, d);
}
ld from_line_to_line(const Point &a, const Point &b, const Point &c,
      const Point &d) { // line ab, line cd
    if (lines_intersection(a, b, c, d)) {
        return 0;
    return from_point_to_line(a, c, d);
pii tangent_from_point(const Point &p, const Polygon &poly) {
      returns id of tangent point: {left tangent, right tangent} (maybe {
     right, left}, but i believe no)
2**20 ~= 1e6, 2**17 ~= 1e5
    int n = sz(poly);
    int i_min = 0;
    int i_{max} = 0;
    for (int k = 17; k \ge 0; --k) {
             int 1 = (i_min + (1 << k)) % n;
            int r = ((i_min - (1 << k)) \% n + n) \% n;
            i_min = min({1, r, i_min}, [&poly, &p](int i, int j) {
   return ((poly[i] - p) % (poly[j] - p)) < 0 ||
        (((poly[i] - p) % (poly[j] - p)) == 0 && (poly[i]</pre>
      - p).len_sq() < (poly[j] - p).len_sq());
            }):
        }
             int 1 = (i_max + (1 << k)) % n;
             int r = ((i_max - (1 << k)) % n + n) % n;
      });
        }
    7
    return {i_min, i_max};
template<typename T>
struct circle_t {
    point_t<T> c;
    circle_t() {}
    circle_t(point_t<T> _c, T _r) : c(_c), r(_r) {}
    int point_in(const point_t<T> &a) {
        if (is_same<T, 11>()) {
     11 dist = (a - c).len_sq();
             if (dist > r * r) {
                return 0;
             } else if (dist < r * r) {</pre>
                return 1;
            } else {
                return 2:
        } else {
            ld dist = (a - c).len();
             if (dist > r + EPS) {
                return 0:
             } else if (dist + EPS < r) {
                return 1;
             } else {
                return 2;
        }
```

```
vector<PointLD> tangent_from_point(const point_t<T> &a) {
        PointLD p(a.x, a.y);
PointLD vec(c.x - p.x, c.y - p.y);
        ld dist = vec.len();
if (dist + EPS < r) {</pre>
             return {};
         if (abs(r - dist) < EPS) {
             return {p};
         vec.x /= dist;
         vec.y /= dist;
ld k = sqrtl(dist * dist - r * r);
         ld alpha = atan2(r, k);
         PointLD t1 = p + rotate(vec, alpha) * k, t2 = p + rotate(vec, -
      alpha) * k;
        return {t1, t2};
    }
};
template<typename T>
istream &operator>>(istream &is, circle_t<T> &c) {
    is >> c.c >> c.r;
    return is;
```

## 2.37 Стрессы (TheEvilBird)

```
# files: generators.py gen.py stress.py
# -----
import random
import heapq
def gen_num(L: int, R: int):
   Generates a number between {\it L} and {\it R}.
   return random.randint(L, R)
# -----
\# abcdefghijklmnopqrstuvwxyz
def gen_string_abc(LEN: int, ALPH_LEN: int = 26):
   Generates a string of length LEN using the first ALPH_LEN lowercase
   letters of the alphabet.
   abc = "abcdefghijklmnopqrstuvwxyz"
   s = abc[:ALPH_LEN]
   res = "
   for i in range(LEN):
       res += random.choice(s)
   return res
def gen_string_any_aplh(LEN: int, ALPH: str):
   Generates a string of length LEN using ALPH as the alphabet.
   # ALPH_LEN = len(ALPH)
   for i in range(LEN):
      kek = 1
       res += random.choice(ALPH)
   return res
# -----
def gen_tree(N: int):
   Generates a tree with N vertices.
   edges = []
   for i in range(2, N + 1):
       v = gen_num(1, i - 1)
       edges.append((v, i))
   return edges
def gen_DAG(N: int, M: int):
   Generates a directed acyclic graph with N vertices and M edges.
   edges = []
   # for i in range(1, N):
        if len(edges) == M:
           break
         v = gen\_num(i + 1, N)
         edges.append((i, v))
   while len(edges) < M:
```

```
v = gen_num(1, N - 1)
        u = gen_num(v + 1, N)
        edges.append((v, u))
    return edges
def gen_graph(N: int, M: int):
    Generates a graph with N vertices and M edges.
    edges_set = set()
    for i in range(M):
          u = 0, 0
         while (v, u) in edges_set or v == u:
            v, u = gen_num(1, N), gen_num(1, N)
        v, u = min(v, u), max(v, u)
edges_set.add((v, u))
    return list(edges_set)
def gen_multigraph(N: int, M: int):
    Generates a multigraph with N vertices and M edges.
    edges = []
    for i in range(M):
        v, u = -1, 0
        while v == -1:
             v, u = gen_num(1, N), gen_num(1, N)
v, u = min(v, u), max(v, u)
        edges.append((v, u))
    return edges
def gen_directed_graph(N: int, M: int):
    Generates a directed graph with N vertices and M edges.
    edges_set = set()
    for i in range(M):
        v, u = 0, 0
        while (v, u) in edges_set or v == u:
    v, u = gen_num(1, N), gen_num(1, N)
edges_set.add((v, u))
    return list(edges_set)
def gen_connected_directed_graph(N: int, M: int):
    Generates a directed connected graph with N vertices and M edges.
    edges_set = set(gen_tree(N))
    for i in range(M - (N - 1)):
        \mathbf{v}, \mathbf{u} = 0, 0
        while (v, u) in edges_set or v == u:
    v, u = gen_num(1, N), gen_num(1, N)
edges_set.add((v, u))
    return list(edges_set)
def gen_connected_graph(N: int, M: int):
    Generates a connected graph with N vertices and M edges.
    edges_set = set(gen_tree(N))
    for i in range(M - (N - 1)):
        \mathbf{v}, \mathbf{u} = 0, 0
         while (v, u) in edges_set or v == u:
        v, u = gen_num(1, N), gen_num(1, N)
v, u = min(v, u), max(v, u)
edges_set.add((v, u))
    return list(edges_set)
def gen_connected_multigraph(N: int, M: int):
    Generates a connected multigraph with N vertices and M edges.
    edges = gen_tree(N)
    for i in range(M - (N - 1)):
        v, u = 0, 0
        while v == u:
            v, u = gen_num(1, N), gen_num(1, N)
        edges.append((v, u))
    return edges
# -----
def gen_perm(N: int, FIR: int = 1):
    Generates a permutation of length N with \min element FIR.
    arr = [FIR + i for i in range(N)]
    # arr = arr[1:]
    random.shuffle(arr)
    return arr
def gen_array(N: int, L: int, R: int):
    Generates an array of length N with elements between L and R.
    arr = [gen_num(L, R) for i in range(N)]
```

```
def gen_array_pairs(N: int, L: int, R: int):
    Generates an array of pairs of length N with elements between L and R
    arr = [(gen_num(L, R), gen_num(L, R)) for i in range(N)]
    return arr
def gen_array_pairs(N: int, L1: int, R1: int, L2: int, R2: int):
    Generates an array of pairs of length N with the first elements of
     each pair between L1 and R1 and between L2 and R2 for the second
    element.
    arr = [(gen_num(L1, R1), gen_num(L2, R2)) for i in range(N)]
   return arr
# -----
def gen_tree_ivanq(N: int):
    Generates a tree with N vertices. Code by IvanQ.
    code = [random.randint(1, N) for _ in range(N - 2)]
     histogram = [0] * (N + 1) 
    unused = list(set(range(1, N + 1)) - set(code))
   heapq.heapify(unused) for u in code:
       histogram[u] += 1
    res = []
   for \mathbf{u} in \mathsf{code}:
       v = heapq.heappop(unused)
       res.append((v, u))
histogram[u] -= 1
        if histogram[u] == 0:
   heapq.heappush(unused, u)
return res + [tuple(unused)]
# -----
import sys
from generators import *
SEED = 228
if len(sys.argv) > 1:
    SEED = int(sys.argv[1])
random.seed(SEED)
# print(gen_num(1, 666))
# -----
import os
import sys
os.system("g++ -std=c++17 smart.cpp -o smart")
# os.system("g++ -std=c++17 -g -fsanitize=undefined -fsanitize=bounds -
     fsanitize=address -D_GLIBCXX_DEBUG smart.cpp -o smart")
os.system("g++ -std=c++17 stupid.cpp -o stupid")
def print_testcase():
    test = open("test.txt").read().strip()
   print(test)
   print("=====
    ans1 = open("smart.out").read().strip()
    ans2 = open("stupid.out").read().strip()
   print(ans1)
   print("==
   print(ans2)
   print("====="")
while True:
   os.system(f"python3 gen.py {i} > test.txt")
   r1 = os.system("./smart < test.txt > smart.out")
   r2 = os.system("./stupid < test.txt > stupid.out")
    if r1 + r2 != 0:
        print(f"Runtime! {i}")
        print_testcase()
        exit(0)
   ans1 = open("smart.out").read()
   ans2 = open("stupid.out").read()
    if ans1 != ans2:
       print(f"POPALCYA! {i}\n")
        print_testcase()
        exit(0)
    print(f"OK: {i}")
    i += 1
```

Задача	Саша	Дима	Федя	О чём
A				
В				
С				
D				
Б				
E				
F				
G				
Н				
I				
1				
J				
K				
L				
M				
N				
IN				
О				
P				