Содержание

-	Общее					
2	Код	цы	1			
	2.1	Basic setup	1			
	2.2	Бесполезное	2			
	2.3	Мосты	2			
	2.4	Точки сочленения	2			
	2.5	DCP (TheEvilBird)	2			
	2.6	MaxFlow (TheEvilBird)	3			
	2.7	MinCostMaxFlow (TheEvilBird)	4			
	2.8	Эйлеров цикл	4			
	2.9	Кун	4			
	2.10	HLD (TheEvilBird)	4			
	2.11	Dominator tree (TheEvilBird)	5			
	2.12	Link-Cut (TheEvilBird)	6			
	2.13	Личао (FedShat)	7			
	2.14	Segment Tree (TheEvilBird)	7			
	2.15	Segment Tree Down (TheEvilBird)	8			
	2.16	Segment Tree Beats (TheEvilBird)	8			
	2.17	Persistent Segment Tree (Sweezyk)	9			
	2.18	Fenwick (TheEvilBird)	10			
	2.19	Sparse table	10			
	2.20	Treap (Sweezyk)	10			
	2.21	Extended GCD (Sweezyk)	10			
	2.22	FFT (FedShat)	11			
	2.23	KTO (FedShat)	12			
	2.24	Обратные по простому модулю	12			
	2.25	Обратные факториалы	12			
	2.26	Γaycc	12			
		Быстрая факторизация (FedShat)	13			
	2.28	Префикс-функция	13			
		Z-функция	13			
	2.30	Суфмас (TheEvilBird)	14			
	2.31	Суфавтомат (TheEvilBird)	14			
	2.32	Axo-Корасик (Sweezyk)	14			
	2.33	Манакер	15			
		CHT (FedShat)	15			
		Дебаг Туриста	15			
		Геометрия (TheEvilBird)	16			
	2.37	Стрессы (TheEvilBird)	19			

1 Общее

• Собственное вращение на угол φ с центром вращения в начале координат:

$$x' = x \cos \varphi - y \sin \varphi$$
$$y' = x \sin \varphi + y \cos \varphi$$

- Расстояние между точками по сфере: L = R · $\arccos(\cos\theta_1 \cdot \cos\theta_2 + \sin\theta_1 \cdot \sin\theta_2 \cdot \cos(\varphi_1 \varphi_2))$ где θ широты (от $-\pi$ до π), φ долготы (от $-\pi$ до π)
- Объем шарового сегмента: $V = \pi h^2 (R \frac{1}{3}h)$, где h высота от вершины сектора до секущей плоскости
- Площадь поверхности шарового сегмента: $S=2\pi Rh,$ где h высота
- Код Грея: $g_n = n \oplus \frac{n}{2}$
- Числа Фибоначчи: $F_0=0, F_1=1, F_n=\frac{(\frac{1+\sqrt{5}}{2})^n-(\frac{1-\sqrt{5}}{2})^n}{\sqrt{5}}$
- Sum-xor property: $a+b=a\oplus b+2(a\&b), a+b=a|b+a\&b, a\oplus b=a|b-a\&b$

- Число граней в планарном графе
(с учётом бесконечной): R=2-V+E
- Сумма арифметической прогрессии: $S_n = \frac{n(a_1 + a_n)}{2}$
- Сумма геометрической прогрессии: $S_n = \frac{b_1(q^n-1)}{q-1}$
- Определители матриц

$$\left| \begin{array}{cc} a & b \\ c & d \end{array} \right| = ad - bc$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 b_1 c_1 + a_3 b_1 c_2 + a_2 b_3 c_1 - a_3 b_2 c_1 - a_1 b_3 c_2 - a_2 b_1 c_3$$

 $\Delta = \sum_{j=1}^n (-1)^{j+1} \cdot a_{1,j} \cdot \bar{M}^1_j, \ \bar{M}^1_j$ — определитель матрицы, полученной вычеркиванием 1 строки и j стоблца.

• Метод Крамера. $\det A \neq 0 \implies$ единственное решение. Иначе 0 или ∞ . Решения: $x_i = \frac{\Delta_i}{\Delta}$. В Δ_i столбец коэффициентов при соответствующей неизвестной заменяется столбцом свободных членов системы.

2 Коды

2.1 Basic setup

```
#include <bits/stdc++.h>
using namespace std;
#define sz(x) (int) ((x).size())
#define all(x) (x).begin(), (x).end()
#define rall(x) (x).rbegin(), (x).rend()
typedef long long 11;
typedef __int128 int128;
typedef pair<int, int> pii;
typedef pair<11, ll> pll;
const char en = '\n';
const int INF = 1e9 + 7;
const 11 INFLL = 1e18;
mt19937 rnd(chrono::high_resolution_clock::now().time_since_epoch().count
      ());
template<class T>
istream &operator>>(istream &is, vector<T> &a) {
   for (auto &i : a) {
         is >> i;
     return is;
#ifdef LOCAL
#include "debug.h"
#define debug(...) 42
#endif
void solve() {
#ifdef LOCAL
    freopen("input.txt", "r", stdin);
    ios_base::sync_with_stdio(0);
     cin.tie(0);
     solve():
     return 0;
```

2.2 Бесполезное

Санитайзеры:

```
set(CMAKE_CXX_FLAGS "${CMAKE_CXX_FLAGS} -Wall -Wshadow -g -fsanitize= undefined -fsanitize=bounds -fsanitize=address -D_GLIBCXX_DEBUG")

-Wall -Wextra -pedantic -Wformat=2 -Wfloat-equal -Wconversion -Wlogical-op -Wshift-overflow=2 -Wduplicated-cond -Wcast-qual -Wcast-align -Werror
```

Прагмы:

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC target("sse,sse2,sse3,sse4)
#pragma GCC target("popcnt,abm,mmx,avx,avx2,tune=native")
#pragma GCC optimize("unroll-loops")
#pragma GCC optimize("fast-math")
#pragma GCC optimize("section-anchors")
#pragma GCC optimize("profile-values")
#pragma GCC optimize("profile-reorder-functions")
#pragma GCC optimize("tracer")
#pragma GCC optimize("vpt")
#pragma GCC optimize("rename-registers")
#pragma GCC optimize("move-loop-invariants")
#pragma GCC optimize("unswitch-loops")
#pragma GCC optimize("function-sections")
#pragma GCC optimize("data-sections")
#pragma GCC optimize("branch-target-load-optimize")
#pragma GCC optimize("branch-target-load-optimize2")
#pragma GCC optimize("btr-bb-exclusive")
```

Встроенный декартач:

Atomic hashset, hashmap:

Перебор всех подмасок и надмасок:

```
for (int submask = mask;; submask = (submask - 1) & mask) {
    // use submask
    if (submask == 0) break;
}
for (int upmask = mask;; upmask = (upmask + 1) | mask) {
    // use upmask
    if (upmask == maxmask) break;
}
```

2.3 Мосты

2.4 Точки сочленения

```
void dfs(int v, int p = -1) {
    used[v] = true;
    tin[v] = fup[v] = timer++;
    int children = 0;
    for (auto to : g[v]) {
        if (to == p) {
            continue;
        }
        if (used[to]) {
            fup[v] = min(fup[v], tin[to]);
        } else {
            dfs(to, v);
            fup[v] = min(fup[v], fup[to]);
            if (fup[to] >= tin[v] && p != -1) {
                IS_CUTPOINT(v);
            }
            ++children;
        }
        if (p == -1 && children > 1) {
            IS_CUTPOINT(v);
        }
}
```

2.5 DCP (TheEvilBird)

```
struct Query {
    char type;
    int v. u:
     Query(char type) : type(type) {}
     Query(char type, int v, int u) : type(type), v(v), u(u) {}
};
struct DCP {
    int n, k, ans; // n - vertex, k - queries
     vector<int> par, rk;
     vector<pair<pii, int>> hist;
     // 0 - par, 1 - rk, 2 - ans;
    int qL, qR;
    pii edge;
     vector<vector<pii>>> tree;
    vector<Query> qs;
    DCP(int _n, int _k) {
         n = ans = _n;
         par.resize(n);
         rk.resize(n, 1);
for (int i = 0; i < n; ++i) par[i] = i;
         tree.assign(4 * k, vector<pii>());
    int dsu_get(int v) {
         while (par[v] != v) v = par[v];
    void dsu_unite(int a, int b) {
         a = dsu_get(a);
         b = dsu_get(b);
         if (a == b) return;
         if (rk[a] > rk[b]) swap(a, b);
hist.emplace_back((pii){0, a}, par[a]);
         hist.emplace_back((pii){2, -1}, ans);
par[a] = b;
         if (rk[a] == rk[b]) {
             hist.emplace_back((pii){1, b}, rk[b]);
             ++rk[b];
    void dsu_unite(pii e) {
         dsu_unite(e.first, e.second);
    void cancel(pair<pii, int> &el) {
         int &type = el.first.first;
int &id = el.first.second;
         int &val = el.second;
         if (type == 0) {
   par[id] = val;
} else if (type == 1) {
             rk[id] = val;
         } else if (type == 2) {
             ans = val;
    void add_edge(int _qL, int _qR, pii e) {// [L, R]
```

```
qL = _qL;
        qR = _qR + 1;
edge = e;
        add_edge_tree(1, 0, k);
    }
    void add_edge_tree(int v, int 1, int r) {
        if (qL \le 1 \&\& r \le qR) {
             tree[v].emplace_back(edge);
            return:
        int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
        if (qL < m) add_edge_tree(vL, 1, m);</pre>
        if (m < qR) add_edge_tree(vR, m, r);</pre>
    void go(vector<Query> &_qs) {
        go_tree(1, 0, k);
    void go_tree(int v, int 1, int r) {
        int siz = sz(hist);
for (auto &e : tree[v]) {
            dsu_unite(e);
        if (1 + 1 == r) {
             if (qs[1].type == '?') {
                 cout << ans << en:
        } else {
             int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
             go_tree(vL, 1, m);
             go_tree(vR, m, r);
        while (sz(hist) > siz) {
             cancel(hist.back());
            hist.pop_back();
    }
};
void solve() {
    int n, k;
    cin >> n >> k;
    DCP dcp(n, k);
    set<pair<pii, int>> edges;
    vector<Query> qs;
for (int i = 0; i < k; ++i) {</pre>
        char tp;
        cin >> tp;
if (tp == '?') {
            qs.emplace_back(tp);
        } else {
            int v, u;
            cin >> v >> u;
             --v;
            if (v > u) swap(v, u);
            qs.emplace_back(tp, v, u);
if (tp == '+') {
                 edges.emplace((pii){v, u}, i);
                 auto it = edges.lower_bound({(pii){v, u}, 0});
                 dcp.add_edge(it->second, i, it->first);
                 edges.erase(it);
            }
        }
    for (auto &e : edges) {
        dcp.add_edge(e.second, k - 1, e.first);
    if (k) dcp.go(qs);
```

2.6 MaxFlow (TheEvilBird)

```
struct MaxFlow {
    struct Edge {
        ll flow, cap;
        int to, id;
        Edge() {}

        Edge(ll flow, ll cap, int to, int id) : flow(flow), cap(cap), to(
        to), id(id) {}
    };

int n;
    vector<vector<Edge>> g;
    vector<int> d, head, used;
    ll max_cap;
    int s, t;
```

```
MaxFlow() {}
MaxFlow(int _n) {
    n = n:
    g.resize(n);
void add_edge(int from, int to, ll cap) {
    g[from].emplace_back(0, cap, to, sz(g[to]));
g[to].emplace_back(0, 0, from, sz(g[from]) - 1);
bool bfs() {
    d.assign(n, INF);
    d[s] = 0;
    queue<int> q;
    q.push(s):
    while (!q.empty()) {
   int v = q.front();
         q.pop();
         for (auto e : g[v]) {
   if (d[e.to] == INF && e.cap - e.flow >= max_cap) {
      d[e.to] = d[v] + 1;
                  q.push(e.to);
    }
    return d[t] != INF;
}
11 dfs(int v, 11 cur_flow) {
    if (v == t) {
        return cur_flow;
    for (; head[v] < sz(g[v]); ++head[v]) {
         auto &e = g[v][head[v]];
         if (e.cap - e.flow >= \max_{x \in \mathbb{R}} d[v] + 1 == d[e.to]) {
             11 new_flow = dfs(e.to, min(cur_flow, e.cap - e.flow));
              if (new_flow) {
                  e.flow += new_flow;
                  g[e.to][e.id].flow -= new_flow;
                  return new_flow;
    return 0;
11 find_max_flow(int _s, int _t) {
    for (int k = 30; k >= 0; --k) {
    max_cap = (1 << k);
         while (bfs()) {
             head.assign(n, 0);
             11 flow = 0;
                  flow = dfs(s, INFLL);
                  res += flow;
             } while (flow);
11 dfs_const_flow(int v, 11 cur_flow) {
    used[v] = 1;
if (v == t) {
        return cur_flow;
    for (auto &e : g[v]) {
         if (!used[e.to] && e.cap - e.flow > 0) {
             11 new_flow = dfs_const_flow(e.to, min(cur_flow, e.cap -
 e.flow));
             if (new_flow) {
                  e.flow += new_flow;
                  g[e.to][e.id].flow -= new_flow;
                  return new_flow;
        }
    return 0;
bool find_const_flow(int _s, int _t, 11 F) {
    s = _s;
t = _t;
    ll res = 0, flow = 0;
    \max_{cap} = F;
         used.assign(n, 0);
         flow = dfs_const_flow(s, INF);
         res += flow;
    } while (flow && res < F);</pre>
    return res == F;
```

```
ll get_edge_flow(int v, int id) {
    return g[v][id].flow;
}
};
```

return ans; } };

2.7 MinCostMaxFlow (TheEvilBird)

```
struct MinCostMaxFlow {
    struct Edge {
        ll flow, cap, price;
        int to, id;
        Edge() {}
        Edge(ll flow, ll cap, ll price, int to, int id) : flow(flow), cap
     (cap), price(price), to(to), id(id) {}
   int n;
    int s, t;
    11 ans;
    vector<vector<Edge>> g;
    vector<int> d;
    vector<ll> add f:
    vector<pii> par;
    MinCostMaxFlow() {}
   MinCostMaxFlow(int _n) {
        n = n:
        g.resize(n);
    void add_edge(int from, int to, 11 cap, 11 price) {
        g[from].emplace_back(0, cap, price, to, sz(g[to]));
        g[to].emplace_back(0, 0, -price, from, sz(g[from]) - 1);
    11 get_edge_flow(int v, int id) {
        return g[v][id].flow;
    void FB() {
        d.assign(n, INF);
        add_f.assign(n, 0);
        par.assign(n, {-1, -1});
        d[s] = 0;
add_f[0] = INF;
        queue<int> q;
        q.push(s);
        vector<int> used(n, 0);
        used[s] = 1;
        while (!q.empty()) {
            int v = q.front();
            q.pop();
used[v] = 0;
            for (int i = 0; i < sz(g[v]); ++i) {
                 auto &e = g[v][i];
                 if (e.flow < e.cap && d[e.to] > d[v] + e.price) {
    d[e.to] = d[v] + e.price;
                     add_f[e.to] = min(add_f[v], e.cap - e.flow);
par[e.to] = {v, i};
                     if (!used[e.to]) {
                         q.push(e.to);
                          used[e.to] = 1;
                     }
                }
            }
        }
    void push_flow(11 flow) {
        int cur = t;
        while (cur != s) {
            int prev = par[cur].first, id = par[cur].second;
            g[prev][id].flow += flow;
            g[cur][g[prev][id].id].flow -= flow;
             ans += g[prev][id].price * flow;
            cur = prev;
        }
   7
    11 min_cost_max_flow(int _s, int _t) {
        ans = 0;
        s = _s;
t = _t;
        while (true) {
            FB();
            11 flow = add_f[t];
            if (flow == 0) {
                 break:
            push_flow(flow);
```

2.8 Эйлеров цикл

```
vector<vector<int>> g;
vector<bool> used;
vector<int> ed, tour;

void dfs(int v) {
    while (!g[v].empty()) {
        int u = g[v].back();
        g[v].pop_back();
        if (used[u]) {
            continue;
        }
        used[u] = 1;
        dfs(ed[u] ^ v);
    }
    tour.push_back(v);
}
```

2.9 Kyh

```
vector<vector<int>> g;
vector<int> used, mt;
int timer = 1;
bool dfs(int v) {
    if (used[v] == timer) {
        return false:
    used[v] = timer;
    for (auto u : g[v]) {
        if (mt[u] == -1) {
            mt[u] = v:
            return true;
    for (auto u : g[v]) {
        if (dfs(mt[u])) {
            mt[u] = v;
            return true;
    return false;
for (int i = 0; i < n; ++i) {
    if (dfs(i)) {
        ++timer:
```

Вершинное покрытие графа — множество вершин, что каждое ребро графа инцидентно хотя бы одной вершине из множества.

Пусть M — макс. парсоч. Мысленно ориентируем ребра графа: ребра из M проведем из правой доли в левую, остальные — из левой в правую, после чего запустим обход в глубину из всех вершин левой доли, не включенных в M. Граф разбился на несколько множеств: L^+, L^-, R^+, R^- , где «плюсовые» множества — это множества посещенных в процессе обхода вершин. Тогда $V_{min} = L^- \cup R^+$.

Независимое множество вершин — множество вершин, что никакая пара вершин не соединена ребром. Дополнение минимального вершинного покрытия является максимальным независимым множеством.

Покрытие дага путями: n-matching

2.10 HLD (TheEvilBird)

```
struct HLD {
    // insert SegTree code
    struct SegTree {};

int n, T;
    SegTree st;
    vector<vector<int>> tree;
    vector(int> par, siz, tin, tout, head;
    ll ans;
```

```
HLD(int _n) {
         n = _n;
tree.resize(n);
         par.resize(n, -1);
         siz.resize(n. 0):
         tin.resize(n);
         tout.resize(n);
         head.resize(n);
         st = SegTree(n);
    void add_edge(int v, int u) {
         tree[v].emplace_back(u);
         tree[u].emplace_back(v);
    void build(int v = 0) {
         dfs siz(v, v):
         T = 0;
         head[v] = v;
         dfs_hld(v, v);
    void dfs_siz(int v, int p) {
         par[v] = p;
siz[v] = 1;
         for (auto &u : tree[v]) {
             if (u != p) {
                  dfs_siz(u, v);
siz[v] += siz[u];
         for (int i = 0; i < sz(tree[v]); ++i) {
             int x = tree[v][0], u = tree[v][i];
if (x == p || siz[u] > siz[x]) {
                  swap(tree[v][0], tree[v][i]);
         }
    }
    void dfs_hld(int v, int p) {
   tin[v] = T++;
         for (auto u : tree[v]) {
             if (u == p) {
                  continue;
              if (u == tree[v][0]) {
                  head[u] = head[v];
              } else {
                  head[u] = u;
              dfs_hld(u, v);
         tout[v] = T;
     void update(int v, int val) {
         st.update_segment(tin[v], tin[v], val);
    bool is_anc(int v, int u) {
    return tin[v] <= tin[u] && tout[u] <= tout[v];</pre>
    void go\_up(int \&v, int u) {
         while (!is\_anc(head[v], u)) {
             ans = max(ans, st.get(tin[head[v]], tin[v]));
v = par[head[v]];
    11 get(int v, int u) {// max on path
    ans = -INFLL;
         go_up(v, u);
go_up(u, v);
         if (!is_anc(v, u)) {
              swap(v, u);
         ans = max(ans, st.get(tin[v], tin[u]));
         return ans:
    }
};
```

2.11 Dominator tree (TheEvilBird)

```
struct Edge {
   int from, to, id;

Edge() = default;
   Edge(int from, int to, int id) : from(from), to(to), id(id) {}
};

struct DSU {
   int n;
   vector<int> par;
```

```
vector<pii> mn;
    DSU() = default;
    DSU(int n) : n(n) {
         par.resize(n);
         mn.resize(n);
         init();
    }
    void init() {
   for (int i = 0; i < n; ++i) {</pre>
             par[i] = i;
             mn[i] = {INF, i};
    }
    int get(int v) {
   if (par[v] == v) {
             return v;
         int p = get(par[v]);
  mn[v] = min(mn[v], mn[par[v]]);
if (mn[par[v]].first < mn[v].first) {</pre>
             mn[v] = mn[par[v]];
         par[v] = p;
         return p;
    void unite(int a, int b) {
        par[a] = b;
};
struct DominatorTree {
    int n:
    vector<Edge> edges;
     vector<int> sdom, idom, tin, order, par, used, dp;
     vector<vector<int>>> g, rg, queries;
    DSU dsu_sdom, dsu_idom;
    DominatorTree() = default:
    DominatorTree(int n) : n(n), dsu_sdom(n), dsu_idom(n) {
         sdom.resize(n, INF); // semi-dominator
         idom.resize(n, INF); // immediate dominator
         tin.resize(n, -1);
         par.resize(n);
         used.resize(n, 0);
         dp.resize(n, INF);
         g.resize(n);
         rg.resize(n);
         queries.resize(n);
    7
    void add_edge(int from, int to) {
         edges.emplace_back(from, to, sz(edges));
    void dfs(int v) {
         tin[v] = sz(order);
         order.emplace back(v):
         for (auto i: g[v]) {
              const auto &e = edges[i];
if (tin[e.to] == -1) {
                  par[e.to] = v;
                  dfs(e.to);
             }
         }
    void dfs_idom(int v) {
         used[v] = 1;
         for (auto i: g[v]) {
              const auto &e = edges[i];
              if (!used[e.to]) {
                  dfs_idom(e.to);
             }
         for (auto u: queries[v]) {
    dsu_idom.get(u);
              dp[u] = dsu_idom.mn[u].second;
         dsu_idom.mn[v] = {sdom[v], v};
for (auto i: g[v]) {
              const auto &e = edges[i];
if (par[e.to] == v) {
                  dsu_idom.unite(e.to, v);
    void build(int s) {
         for (int i = 0; i < sz(edges); ++i) {
              g[edges[i].from].emplace_back(i);
              rg[edges[i].to].emplace_back(i);
         // reorder vertex
         dfs(s);
```

```
// build sdom
     for (int _ = sz(order) - 1; _ >= 0; --_) {
   int v = order[_];
          if (v == s) {
               continue;
          for (auto i: rg[v]) {
               const auto &e = edges[i];
if (tin[e.from] == -1) {
                    continue;
               if (tin[e.from] < tin[v]) {</pre>
                    sdom[v] = min(sdom[v], tin[e.from]);
               7
               else {
                    int u = dsu_sdom.get(e.from);
sdom[v] = min(sdom[v], dsu_sdom.mn[e.from].first);
          }
          dsu_sdom.mn[v] = {sdom[v], v};
          for (auto i: g[v]) {
    const auto &e = edges[i];
               if (v == par[e.to]) {
                    dsu_sdom.unite(e.to, v);
          }
     }
     // build queries for idoms
for (int i = 0; i < n; ++i) {
   if (i == s || sdom[i] == INF || tin[i] == -1) {</pre>
               continue;
          queries[order[sdom[i]]].emplace_back(i);
     dfs_idom(s);
     // build idom
     idom[s] = tin[s];
     for (auto v: order) {
   if (v == s) {
               continue;
          if (v == dp[v]) {
               idom[v] = sdom[v];
          else {
               idom[v] = idom[dp[v]];
}
int get_idom(int v) {
     return (idom[v] == INF ? -1 : order[idom[v]]);
```

2.12 Link-Cut (TheEvilBird)

```
struct Node {
    Node *ch[2] = {nullptr, nullptr};
    Node *par = nullptr;
bool rev = false;
    int val, mn;
    int siz = 1;
    Node() {}
    Node(int val) : val(val), mn(val) {}
};
typedef Node *pnode;
int get_siz(pnode v) {
    return (v == nullptr ? 0 : v->siz);
int get_min(pnode v) {
    return (v == nullptr ? INF : v->mn);
void update(pnode v) {
    v->siz = 1 + get_siz(v->ch[0]) + get_siz(v->ch[1]);
    v->mn = min(v->val, min(get_min(v->ch[0]), get_min(v->ch[1])));
}
void push(pnode v) {
   if (v == nullptr || !v->rev) {
    if (v->ch[0] != nullptr) {
         v->ch[0]->rev ^= 1;
    if (v->ch[1] != nullptr) {
```

```
v->ch[1]->rev ^= 1;
    swap(v->ch[0], v->ch[1]);
    v \rightarrow rev = 0:
}
bool is_root(pnode v) {
    return (v->par == nullptr ||
             (v->par->ch[0] != v && v->par->ch[1] != v));
int child_num(pnode v) {
   return (v->par->ch[1] == v);
void attach(pnode v, pnode p, int num) {
    if (v != nullptr) {
        v \rightarrow par = p:
    if (p != nullptr) {
        p->ch[num] = v;
    }
void rotate(pnode v) {
    int num = child_num(v);
pnode p = v->par, vb = v->ch[num ^ 1];
    pnode g = (p == nullptr ? nullptr : p->par);
if (g != nullptr) {
        if (!is_root(p)) {
            g->ch[child_num(p)] = v;
    }
    v->par = g;
    attach(p, v, num ^ 1);
    attach(vb, p, num);
update(p);
    update(v);
void splay(pnode v) {
    vector<pnode> st;
    pnode cur = v;
    st.emplace_back(cur);
    while (!is_root(cur)) {
        cur = cur->par;
        st.emplace_back(cur);
    for (int i = sz(st) - 1; i \ge 0; --i) {
        push(st[i]);
    while (!is_root(v)) {
        if (!is_root(v->par)) {
            if (child_num(v) == child_num(v->par)) {
                 rotate(v->par);
             } else {
                rotate(v);
             7
        rotate(v);
    }
void expose(pnode v) {
    splay(v);
    v->ch[1] = nullptr;
    update(v);
    while (v->par != nullptr) {
        splay(v->par);
         attach(v, v->par, 1);
         update(v->par);
        splay(v);
    }
void make_root(pnode v) {
    expose(v);
    v->rev ^= 1;
void link(pnode v, pnode u) {
   make_root(v);
    make_root(u);
    u->par = v;
void cut(pnode v, pnode u) {
   make_root(v);
    make_root(u);
    push(u);
    u->ch[1] = nullptr;
    v->par = nullptr;
bool is_connected(pnode v, pnode u) {
    make_root(v);
    make_root(u);
    if (is_root(v) && u != v) {
```

```
return false;
} else {
    return true;
}

int get_min(pnode v, pnode u) {
    make_root(v);
    make_root(u);
    return get_min(u);
}
```

2.13 Личао (FedShat)

```
struct LiChao {// max
       11 k = 0, b = -INFLL;
       Line() = default:
       Line(11 k, 11 b) : k(k), b(b){};
        11 operator()(11 x) {
           return k * x + b;
   }:
   struct Node {
        Node *1 = nullptr, *r = nullptr;
        Line cur;
        Node() = default;
   Node *root = nullptr;
   int n = 1e9 + 1;
   void make node(Node *&v) {
       if (v == nullptr) {
           v = new Node();
   }
   void add(Node *&v, int 1, int r, Line cur) {
       make_node(v);
        int m = (1 + r) / 2;
        if (cur(m) > v->cur(m)) {
           swap(cur, v->cur);
        if (1 + 1 == r) {
           return:
        if (cur(1) > v->cur(1)) {
           add(v->1, 1, m, cur);
        } else {
           add(v->r, m, r, cur);
   }
   void add(Line cur) {
        add(root, 0, n, cur);
   11 get(Node *v, int 1, int r, int x) {
       if (v == nullptr) {
           return -INFLL;
        11 ans = v->cur(x);
       if (1 + 1 == r) {
           return ans:
        int m = (1 + r) / 2;
        if (x < m) {
           ans = max(ans, get(v->1, 1, m, x));
        } else {
           ans = max(ans, get(v->r, m, r, x));
        return ans;
   }
   11 get(int x) {
        return get(root, 0, n, x);
};
```

2.14 Segment Tree (TheEvilBird)

```
struct SegTree {
    static const ll off = 0;
    struct Node {
```

```
11 val = 0;
    11 push = off;
    Node() {}
    Node(l1 val) : val(val) {}
    Node operator+(const Node &other) const {
         return Node(val + other.val);
    void operator+=(const Node &other) {
        val += other.val;
    void use_push(int len = 1) {
        val += push * (11) (1en);
    void update_push(11 pushed) {
        push += pushed;
int n, qL, qR;
Node ans;
vector<Node> tree;
vector<11> a;
SegTree() {}
SegTree(int _n) {
    n = _n;
    tree.assign(4 * n, 0);
void update_vertex(int v, int 1, int r) \{// [l, r)
    int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
    push(vL, 1, m);
    push(vR, m, r);
    tree[v] = tree[vL] + tree[vR];
void push(int v, int 1, int r) {// [l, r)
  if (tree[v].push == off) return;
  int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
  tree[v].use_push(r - 1);
         tree[vL].update_push(tree[v].push);
        tree[vR].update_push(tree[v].push);
    tree[v].push = off;
void build(vector<11> &_a) {
    build_tree(1, 0, n);
void build_tree(int v, int 1, int r) \{// [l, r)
    if (l + 1 == r) {
tree[v] = Node(a[l]);
        return;
    int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
    build_tree(vL, 1, m);
    build_tree(vR, m, r);
    update_vertex(v, 1, r);
void update_segment(int _qL, int _qR, ll _val) {// [_qL, _qR]
    qL = _qL;
qR = _qR + 1;
    val = _val;
    update_segment_tree(1, 0, n);
void update_segment_tree(int v, int l, int r) {// [l, r)
    push(v, 1, r);
if (qL <= 1 && r <= qR) {
        tree[v].update_push(val);
        push(v, 1, r);
         return:
    int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
    if (qL < m) update_segment_tree(vL, 1, m);</pre>
    if (m < qR) update_segment_tree(vR, m, r);</pre>
    update_vertex(v, 1, r);
11 get(int _qL, int _qR) {// [_qL, _qR]
    qL = _qL;
qR = _qR + 1;
    ans = Node();
    get_tree(1, 0, n);
```

```
return ans.val;
}

void get_tree(int v, int 1, int r) {// [l, r)
    push(v, 1, r);
    if (qL <= 1 && r <= qR) {
        ans = ans + tree[v];
        return;
    }
    int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
    if (qL < m) get_tree(vL, 1, m);
    if (m < qR) get_tree(vR, m, r);
};</pre>
```

2.15 Segment Tree Down (TheEvilBird)

```
struct SegTreeDown {
    struct Node {
        11 val = 0;
        Node() {}
        Node(ll val) : val(val) {}
        Node operator+(const Node &other) const {
            return Node(val + other.val);
        void operator+=(const Node &other) {
            val += other.val;
        }
    };
    vector<Node> tree;
    SegTreeDown(int _n) {
        tree.assign(2 * n, Node());
    void build(vector<ll> &a) {
        for (int i = 0; i < n; ++i) {
    tree[i + n] = Node(a[i]);</pre>
        for (int i = n - 1; i \ge 1; --i) {
             tree[i] = tree[2 * i] + tree[2 * i + 1];
    void update(int i, ll val) {
        tree[i] = val;
        while (i != 0) {
            tree[i] = tree[2 * i] + tree[2 * i + 1];
            i /= 2;
    }
    11 get(int 1, int r) \{// [l, r)
        r += n;
        Node ans;
        while (1 \le r) {
            if (1 % 2 == 1) {
                ans += tree[1];
                 ++1:
             if (r \% 2 == 0) {
                ans += tree[r];
                 --r;
            1 /= 2;
            r /= 2;
        return ans.val;
    }
};
```

2.16 Segment Tree Beats (TheEvilBird)

```
struct SegTree {
    struct Node {
        11 max, sec_max;
        int cnt_max;
        11 min, sec_min;
        int cnt_min;
        11 sum;
        11 push_add;
}
```

```
11 push_eq;
}:
int n, qL, qR;
11 val, ans;
vector<Node> tree;
vector<11> a;
SegTree(int _n) {
    tree.assign(4 * n, {0, -INFLL, 1, 0, INFLL, 1, 0, 0, -1});
void update_vertex(int v, int 1, int r) {
   int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
    tree[v].sum = tree[vL].sum + tree[vR].sum;
    tree[v].max = max(tree[vL].max, tree[vR].max);
    tree[v].sec_max = max(tree[vL].sec_max, tree[vR].sec_max);
    tree[v].cnt_max = 0;
    if (tree[vL].max == tree[v].max) {
        tree[v].cnt_max += tree[vL].cnt_max;
    } else {
        tree[v].sec_max = max(tree[v].sec_max, tree[vL].max);
    if (tree[vR].max == tree[v].max) {
        tree[v].cnt_max += tree[vR].cnt_max;
      else {
        tree[v].sec_max = max(tree[v].sec_max, tree[vR].max);
    tree[v].min = min(tree[vL].min, tree[vR].min);
    tree[v].sec_min = min(tree[vL].sec_min, tree[vR].sec_min);
    tree[v].cnt_min = 0;
    if (tree[vL].min == tree[v].min) {
         tree[v].cnt_min += tree[vL].cnt_min;
        tree[v].sec_min = min(tree[v].sec_min, tree[vL].min);
    if (tree[vR].min == tree[v].min) {
        tree[v].cnt_min += tree[vR].cnt_min;
        tree[v].sec_min = min(tree[v].sec_min, tree[vR].min);
}
void recalc_eq(int v, int 1, int r, 11 cur) {
   tree[v].max = tree[v].min = tree[v].push_eq = cur;
    tree[v].sec_max = -INFLL;
    tree[v].sec_min = INFLL;
    tree[v].cnt_max = tree[v].cnt_min = r - 1;
tree[v].sum = cur * (11) (r - 1);
    tree[v].push_add = 0;
void recalc_add(int v, int 1, int r, 11 cur) {
    if (tree[v].min == tree[v].max) {
        recalc_eq(v, 1, r, tree[v].max + cur);
        return:
    tree[v].max += cur;
    if (tree[v].sec_max != -INFLL) {
        tree[v].sec_max += cur;
    tree[v].min += cur;
    if (tree[v].sec_min != INFLL) {
        tree[v].sec_min += cur;
    tree[v].sum += (11) (r - 1) * cur;
    tree[v].push_add += cur;
void recalc_min(int v, int 1, int r, 11 cur) {
    if (tree[v].min >= cur) {
        recalc_eq(v, 1, r, cur);
        return:
    if (tree[v].max <= cur) return;</pre>
    if (tree[v].sec_min == tree[v].max) {
         tree[v].sec_min = cur;
    tree[v].max = cur;
void recalc_max(int v, int 1, int r, ll cur) {
   if (tree[v].max <= cur) {</pre>
        recalc_eq(v, 1, r, cur);
        return;
    if (tree[v].min >= cur) return;
    if (tree[v].sec_max == tree[v].min) {
        tree[v].sec_max = cur;
```

```
tree[v].sum += (11) (tree[v].max - cur) *
                        (11) (tree[v].cnt_max);
     tree[v].min = cur;
}
void push(int v, int 1, int r) {
     if (1 + 1 == r) return;
     int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
     if (tree[v].push_eq != -1) {
          recalc_eq(vL, 1, m, tree[v].push_eq);
recalc_eq(vR, m, r, tree[v].push_eq);
          tree[v].push_eq = -1;
     recalc_add(vL, 1, m, tree[v].push_add);
recalc_add(vR, m, r, tree[v].push_add);
tree[v].push_add = 0;
     recalc_min(vL, 1, m, tree[v].max);
     recalc_min(vR, m, r, tree[v].max);
     recalc_max(vL, 1, m, tree[v].min);
     recalc_max(vR, m, r, tree[v].min);
void build(vector<11> &_a) {
     a = _a;
build_tree(1, 0, n);
void build_tree(int v, int 1, int r) {
     if (1 + 1 == r) {
    tree[v] = {a[1], -INFLL, 1, a[1],
                        INFLL, 1, a[1], 0, -1};
          return;
     int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
     build_tree(vL, 1, m);
     build_tree(vR, m, r);
     update_vertex(v, 1, r);
void update_segment_min(int _qL, int _qR, 11 _val) {
     qL = _qL;
qR = _qR + 1;
val = _val;
     update_segment_min_tree(1, 0, n);
void update_segment_min_tree(int v, int 1, int r) {
     if (tree[v].max <= val) return;
if (qL <= 1 && r <= qR && tree[v].sec_max < val) {</pre>
          recalc_min(v, 1, r, val);
          return;
     push(v, 1, r);
     int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
     if (qL < m) update_segment_min_tree(vL, 1, m);
if (m < qR) update_segment_min_tree(vR, m, r);</pre>
     update_vertex(v, 1, r);
void update_segment_max(int _qL, int _qR, 11 _val) {
     qL = _qL;
qR = _qR + 1;
val = _val;
     update_segment_max_tree(1, 0, n);
void update_segment_max_tree(int v, int 1, int r) {
   if (tree[v].min >= val) return;
     if (qL <= 1 && r <= qR && tree[v].sec_min > val) {
          recalc_max(v, 1, r, val);
          return;
     }
     push(v, 1, r);
     int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
if (qL < m) update_segment_max_tree(vL, 1, m);
if (m < qR) update_segment_max_tree(vR, m, r);
     update_vertex(v, 1, r);
void update_segment_add(int _qL, int _qR, ll _val) {
     qL = _qL;
qR = _qR + 1;
val = _val;
     update_segment_add_tree(1, 0, n);
void update_segment_add_tree(int v, int 1, int r) {
   if (qL <= 1 && r <= qR) {</pre>
          recalc_add(v, 1, r, val);
     push(v, 1, r);
     int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
```

```
if (qL < m) update_segment_add_tree(vL, 1, m);</pre>
          if (m < qR) update_segment_add_tree(vR, m, r);</pre>
          update_vertex(v, 1, r);
     11 get(int _qL, int _qR) {
         qL = _qL;
qR = _qR + 1;
          ans = 0;
          get_tree(1, 0, n);
         return ans;
     void get_tree(int v, int 1, int r) {
   if (qL <= 1 && r <= qR) {
      ans += tree[v].sum;
}</pre>
               return;
          push(v, 1, r);
          int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
          if (qL < m) get_tree(vL, 1, m);</pre>
          if (m < qR) get_tree(vR, m, r);</pre>
     void print_all() {
         print_all_tree(1, 0, n);
     void print_all_tree(int v, int 1, int r) {
         if (1 + 1 == r) {
               cout << tree[v].sum << en;</pre>
              return:
         push(v, 1, r);
int m = (1 + r) / 2, vL = 2 * v, vR = vL + 1;
         print_all_tree(vL, 1, m);
print_all_tree(vR, m, r);
};
```

2.17 Persistent Segment Tree (Sweezyk)

```
struct Node {
    Node *1, *r;
    int mx;
    Node() {
        1 = r = nullptr;
    }
};
const int N = 1e7 + 4e6;
const int LG = 20;
Node *nodes[N];
int ptr;
Node *new_node() {
    return nodes[ptr++];
Node *get_left(Node *t) {
   if (t && t->l) return t->l;
    return nullptr;
Node *get_right(Node *t) {
    if (t && t->r) return t->r;
    return nullptr;
int get_max(Node *t) {
    if (!t) return 0;
    return t->mx;
void update(int i, int val, Node *t, Node *old_t, int lx, int rx) {
    if (1x + 1 == rx) {
        t->mx = max(t->mx, val);
        return;
    int m = (1x + rx) / 2;
    if (i < m) {
         t->1 = new_Node();
         if (old_t && old_t->1) {
              t->1->mx = old_t->1->mx;
         t->r = get_right(old_t);
        update(i, val, t->1, get_left(old_t), lx, m);
    } else {
        t->r = new_Node();
         if (old_t && old_t->r) {
              t \rightarrow r \rightarrow mx = old_t \rightarrow r \rightarrow mx;
         t->1 = get_left(old_t);
```

```
update(i, val, t->r, get_right(old_t), m, rx);
}
t->mx = max(get_max(get_left(t)), get_max(get_right(t)));
};
int get(int l, int r, Node *t, int lx, int rx) {
    if (!t || lx >= r || rx <= l) return -1;
    if (lx >= l && rx <= r) return t->mx;
    int m = (lx + rx) / 2;
    return max(get(l, r, t->l, lx, m), get(l, r, t->r, m, rx));
}
```

2.18 Fenwick (TheEvilBird)

```
struct Fenwick {
    int n:
    vector<11> f:
   Fenwick(int _n) {
        n = _n;
        f.assign(n + 1, 0);
   void update(int x, ll delta) {
        for (int i = x; i <= n; i += i & -i) {
           f[i] += delta;
   }
   11 get_sum(int x) {
        11 s = 0;
for (int i = x; i > 0; i -= i & -i) {
           s += f[i];
        return s;
   11 get(int L, int R) {// [L, R]
        return get_sum(R) - get_sum(L - 1);
   }
};
```

2.19 Sparse table

```
// usage:
    auto fun = [8](int i, int j) { return min(i, j); };
     SparseTable<int, decltype(fun)> st(a, fun);
// SparseTable<int> st(a, [\theta](int i, int j) \{ return min(i, j); \});
template <typename T, class F = function<T(const T&, const T&)>>
class SparseTable {
    vector<int> lg;
    vector<vector<T>> mat;
    F func;
    SparseTable(const vector<T>& a, const F& f) : func(f) {
        n = static_cast<int>(a.size());
        lg.resize(n + 1);
for (int i = 2; i <= n; ++i) {
   lg[i] = lg[i / 2] + 1;</pre>
        mat.resize(lg[n] + 1);
        mat[0] = a;
         for (int j = 1; j \le lg[n]; j++) {
             mat[j].resize(n - (1 << j) + 1);
for (int i = 0; i <= n - (1 << j); i++) {</pre>
                 mat[j][i] = func(mat[j - 1][i], mat[j - 1][i + (1 << (j -
       1))]);
    T get(int from, int to) const {
         assert(0 <= from && from <= to && to <= n - 1);
         int 1 = lg[to - from + 1];
         return func(mat[1][from], mat[1][to - (1 << 1) + 1]);
    }
};
```

```
void push(Node *t) {
    if (t == nullptr) return;
    int p = t->push;
if (p == 0) return;
    if (t->1 != nullptr) {
        t->1->cnt += p;
        t->1->push += p;
    if (t->r != nullptr) {
         t->r->cnt += p;
         t->r->push += p;
    t->push = 0;
int size(Node *t) {
    return (t ? t->size : 0);
void update(Node *t) {
    if (t == nullptr) return;
    t \rightarrow size = size(t \rightarrow 1) + size(t \rightarrow r) + 1;
pair<Node *, Node *> split(Node *t, int k) {
    if (t == nullptr) return {nullptr, nullptr};
    if (k == 0) return {nullptr, t};
    push(t);
     if (size(t->1) + 1 <= k) {
         auto [l, r] = split(t->r, k - size(t->l) - 1);
         update(t);
        return {t, r};
    } else {
        auto [1, r] = split(t->1, k);
         update(t);
         return {1, t};
    }
Node *merge(Node *tl, Node *tr) {
    if (t1 == nullptr) return tr;
    if (tr == nullptr) return tl;
    push(t1);
     push(tr);
    if (t1->v > tr->v) {
        tl->r = merge(tl->r, tr);
         update(t1);
         return tl;
    } else {
        tr->1 = merge(t1, tr->1);
         update(tr);
        return tr;
void dfs(Node *t) {
    if (t == nullptr) return;
    push(t);
    dfs(t->1);
     cout << t->x << ' ' << t->cnt << '\n';
    dfs(t->r);
void solve() {
    int n, m;
    cin >> n >> m;
    Node *root = nullptr;
for (int i = 1; i <= n; i++) {
   Node *add = new Node(i);</pre>
        root = merge(root, add);
    for (int i = 0; i < m; i++) {
         int 1, r;
         cin >> 1 >> r;
        auto [L, R] = split(root, r);
auto [L1, L2] = split(L, 1 - 1);
         L2->push = 1;
         L2->cnt += 1;
         root = merge(L2, merge(L1, R));
    dfs(root);
```

2.20 Treap (Sweezyk)

```
struct Node {
  int x, y, size, push, cnt;
  Node *1, *r;

  Node(int val) : x(val), y(rng()), size(1), push(0), cnt(0), l(nullptr
    ), r(nullptr) {}
```

2.21 Extended GCD (Sweezyk)

```
template<typename T>
T extgcd(T a, T b, T &x, T &y) {
   if (a == 0) {
      x = 0;
      y = 1;
      return b;
}
```

```
T p = b / a;
T g = extgcd(b - p * a, a, y, x);
x -= p * y;
    return g;
template<typename T>
bool diophantine(T a, T b, T c, T &x, T &y, T &g) {
   if (a == 0 && b == 0) {
      if (c == 0) {
              x = y = g = 0;
              return true;
          }
          return false;
    if (a == 0) {
          if (c % b == 0) {
              x = 0;
              y = c / b;
              g = abs(b);
              return true;
         return false;
     if (b == 0) {
         if (c \% a == 0) {
              x = c / a;
              y = 0;
g = abs(a);
              return true;
          return false;
    }
    g = extgcd(a, b, x, y);
if (c % g != 0) {
    return false;
    T dx = c / a;
     c = dx * a;
    T dy = c / b;
    c -= dy * b;
x = dx + (T) ((__int128) x * (c / g) % b);
    y = dy + (T) ((_int128) y * (c / g) % a);
     return true;
     // |x|, |y| \le max(|a|, |b|, |c|) [tested]
bool crt(long long k1, long long m1, long long k2, long long m2, long
      long &k, long long &m) {
     k1 %= m1;
     if (k1 < 0) k1 += m1;
     k2 \% = m2:
     if (k2 < 0) k2 += m2;
    long long x, y, g;
    if (!diophantine(m1, -m2, k2 - k1, x, y, g)) {
         return false;
    long long dx = m2 / g;
long long delta = x / dx - (x % dx < 0);
k = m1 * (x - dx * delta) + k1;
    m = m1 / g * m2;
assert(0 <= k && k < m);
    return true;
}
```

2.22 FFT (FedShat)

```
constexpr int P = 998244353;
using i64 = long long;
// assume -P \ll x \ll 2P
int norm(int x) {
    if (x < 0) {
        x += P;
    if (x \ge P) {
        x -= P;
    }
    return x;
}
template<class T>
T power(T a, int b) {
    T res = 1;
for (; b; b /= 2, a *= a) {
    if (b % 2) {
             res *= a;
        }
    }
    return res;
struct Z {
```

```
Z(int x = 0) : x(norm(x)) {}
     int val() const {
         return x:
    Z operator-() const {
         return Z(norm(P - x));
    Z inv() const {
         assert(x != 0);
         return power(*this, P - 2);
     Z &operator*=(const Z &rhs) {
         x = i64(x) * rhs.x % P;
         return *this:
    Z &operator+=(const Z &rhs) {
         x = norm(x + rhs.x);
         return *this;
    Z &operator-=(const Z &rhs) {
         x = norm(x - rhs.x);
         return *this:
    Z &operator/=(const Z &rhs) {
         return *this *= rhs.inv();
    friend Z operator*(const Z &lhs, const Z &rhs) {
         Z res = lhs:
         res *= rhs;
         return res;
     friend Z operator+(const Z &lhs, const Z &rhs) {
         Z res = lhs;
res += rhs;
         return res:
    friend Z operator-(const Z &lhs, const Z &rhs) {
         res -= rhs;
         return res;
    friend Z operator/(const Z &lhs, const Z &rhs) {
         Z res = lhs;
         res /= rhs;
         return res;
    }
}:
std::vector<int> rev;
std::vector<Z> roots{0, 1};
void dft(std::vector<Z> &a) {
    int n = a.size();
    if (int(rev.size()) != n) {
         int k = __builtin_ctz(n) - 1;
         rev.resize(n);
         for (int i = 0; i < n; i++) {
    rev[i] = rev[i >> 1] >> 1 | (i & 1) << k;
    }
    for (int i = 0; i < n; i++) {
   if (rev[i] < i) {</pre>
              std::swap(a[i], a[rev[i]]);
     if (int(roots.size()) < n) {</pre>
         int k = __builtin_ctz(roots.size());
         roots.resize(n);
         while ((1 << k) < n) {
              Z = power(Z(3), (P - 1) >> (k + 1));
for (int i = 1 << (k - 1); i < (1 << k); i++) {
                  roots[2 * i] = roots[i];
                  roots[2 * i + 1] = roots[i] * e;
              k++:
         }
    for (int k = 1; k < n; k *= 2) {
   for (int i = 0; i < n; i += 2 * k) {
             for (int j = 0; j < k; j++) {
   Z u = a[i + j];
   Z v = a[i + j + k] * roots[k + j];
   a[i + j] = u + v;
   a[i + j + k] = u - v;
   }
void idft(std::vector<Z> &a) {
    int n = a.size();
     std::reverse(a.begin() + 1, a.end());
     Z inv = (1 - P) / n;
    for (int i = 0; i < n; i++) {
    a[i] *= inv;</pre>
```

```
}
```

2.23 KTO (FedShat)

```
struct Eq \{// x = a \pmod{m}
    11 a, m;
    Eq(){};
    Eq(11 a, 11 m) : a(a), m(m){};
};
11 binpow(11 a, 11 n, 11 m) {
   if (n == 0) {
        return 1;
    if (n \% 2 == 0) {
        int128_t b = binpow(a, n / 2, m);
return (b * b) % m;
    int128_t x = binpow(a, n - 1, m);
    return (a * x) % m;
11 binpow(11 a, 11 n) {
   if (n == 0) {
        return 1;
    if (n \% 2 == 0) {
        11 b = binpow(a, n / 2);
        return b * b;
    return a * binpow(a, n - 1);
}
ll phi(ll a) {
    11 d = 2, k = a;
map<11, int> cnt;
    while (d * d \le a) {
        if (k % d == 0) {
            ++cnt[d];
        } else {
            ++d:
    if (k != 1) {
         ++cnt[k];
    11 ans = 1;
    for (auto i : cnt) {
        ans *= binpow(i.first, i.second - 1) * (i.first - 1);
    return ans;
7
11 gcd(11 a, 11 b) {
    return std::gcd(abs(a), abs(b));
}
Eq solve(Eq ai, Eq bi) {
    if (ai.m == -1 || bi.m == -1) {
        return {0, -1};
    ll a = ai.m, b = bi.m, c = ai.a - bi.a;
    11 d = ::gcd(a, b);
    a /= d;
b /= d;
    if (c % d != 0) {
        return {0, -1};
    c /= d;
    11 x = (((int128_t) -c * (int128_t) binpow(a, phi(b) - 1, b)) % b + b
    x = ((int128_t) ai.m * (int128_t) x + ai.a) % lcm(ai.m, bi.m);
    return {x, lcm(ai.m, bi.m)}:
```

2.24 Обратные по простому модулю

Пусть дан простой модуль m. Для каждого числа из [1, m-1] найти обратное к нему.

```
r[1] = 1;
for (int i = 2; i < m; ++i) {
    r[i] = (m - (m / i) * r[m % i] % m) % m;
}</pre>
```

2.25 Обратные факториалы

```
int inv(int a, int m) {
    if (a == 1)
        return 1;
    return (1 - inv(m % a, a) * m) / a + m;
}

{
    f[0] = 1;
    for (int i = 1; i < N; i++) {
        f[i] = i * f[i - 1] % mod;
    }

    r[N - 1] = inv(f[N - 1]) for (int i = N - 1; i >= 1; i--) {
        r[i - 1] = r[i] * i % mod;
    }
}
```

2.26 Faycc

```
const double eps = 1e-9;
int Gauss(vector<vector<double>> a, vector<double> &ans) {
     int n = (int)a.size(), m = (int)a[0].size() - 1;
     vector<int> pos(m, -1);
     double det = 1; int rank = 0;
     for(int col = 0, row = 0; col < m && row < n; ++col) {
          int mx = row;
          for (int i = row; i < n; i++) {
               if (fabs(a[i][col]) > fabs(a[mx][col])) { mx = i; }
          if (fabs(a[mx][col]) < eps) { det = 0; continue; }
for (int i = col; i <= m; i++) {
    swap(a[row][i], a[mx][i]);</pre>
          if (row != mx) { det = -det; }
          det *= a[row][col];
         det *= a[row][coi],
pos[col] = row;
for (int i = 0; i < n; i++) {
    if (i != row && fabs(a[i][col]) > eps) {
        double c = a[i][col] / a[row][col];
        for (int j = col; j <= m; j++) {
            a[i][j] -= a[row][j] * c;
        }
}</pre>
               }
          ++row: ++rank:
     ans.assign(m, 0);
     for(int i = 0; i < m; i++) {
          if (pos[i] != -1) { ans[i] = a[pos[i]][m] / a[pos[i]][i]; }
     for(int i = 0; i < n; i++) {
          double sum = 0;
for (int j = 0; j < m; j++) {
               sum += ans[j] * a[i][j];
          if(fabs(sum - a[i][m]) > eps) {
                return -1; //no solution
     for (int i = 0; i < m; i++) {
          if (pos[i] == -1) {
                return 2; //infinte solutions
     return 1; //unique solution
```

Бинарный

```
//n = number of equations, m = number of variables
int Gauss(int n, int m, vector<bitset<N>> a, bitset<N> &ans) {
    //reversing for lexocgraphically largest solution for (int i = 0; i < n; i++) {
          bitset<N> tmp;
          for (int j = 0; j < m; j++) tmp[j] = a[i][m - j - 1];
          tmp[m] = a[i][m];
          a[i] = tmp;
     int rank = 0, det = 1:
     rink = 0, det = 1;
vector(int> pos(N, -1);
for(int col = 0, row = 0; col < m && row < n; ++col) {</pre>
          int mx = row;
          for(int i = row; i < n; ++i) if(a[i][col]) { mx = i; break; }
if(!a[mx][col]) { det = 0; continue; }</pre>
          swap(a[mx], a[row]);
          if (row != mx) { det = (det == 0 ? 0 : 1); }
          det &= a[row][col];
          pos[col] = row;
           //forward elimination
          for (int i = row + 1; i < n; ++i) {
   if (i != row && a[i][col]) {
      a[i] ^= a[row];
}</pre>
```

```
}
         ++row, ++rank;
    ans.reset():
    //backward substituition
    for (int i = m - 1; i \ge 0; i--) {
         if (pos[i] == -1) {
             ans[i] = true;
         } else {
             int k = pos[i];
             for (int j = i + 1; j < m; j++) if (a[k][j]) ans[i] = ans[i]
        ans[j];
             ans[i] = ans[i] ^ a[k][m];
        }
    }
    for (int i = rank; i < n; ++i) {
   if (a[i][m]) {</pre>
            return -1; //no solution
    //reversing again beacuse we reversed earlier
    bitset<N> tmp;
for (int j = 0; j < m; j++) {</pre>
        tmp[j] = ans[m - j - 1];
    ans = tmp;
    int free_var = 0;
    for(int i = 0; i < m; ++i) {
    if (pos[i] == -1) {
             free_var++;
    }
    return free_var; //has solution
}
```

2.27 Быстрая факторизация (FedShat)

```
11 binpow(11 a, 11 n, 11 mod) {
   if (n == 0) {
    if (n % 2 == 0) {
   int128_t b = binpow(a, n / 2, mod);
        return (b * b) % mod;
    return (((int128_t) a) * binpow(a, n - 1, mod)) % mod;
}
constexpr int N = 1e7;
vector<int> pr, lp;
   if (n <= N) {
        return binary_search(all(pr), n);
    int iter = 60:
    int s = 0;
    11 d = n - 1;
while (d % 2 == 0) {
        d /= 2;
        ++s;
    auto test = [&](11 a) {
        if (binpow(a, d, n) == 1) {
            return true;
        11 _2r = 1;
for (int r = 0; r < s; ++r) {
            auto tmp = binpow(binpow(a, d, n), _2r, n);
if (tmp == n - 1) {
            }
            <u>2r</u> *= 2;
        return false;
    for (int _ = 0; _ < iter; ++_) {
         11 a = uniform_int_distribution<11>(1, n - 1)(rnd);
        if (!test(a)) {
             return false;
    return true;
}
11 f(11 x, 11 n) {
    return ((int128_t) x * (int128_t) x + (int128_t) 2) % n;
11 pollard(11 n) {
    11 a = uniform_int_distribution<11>(0, n - 1)(rnd);
    11 x = a, y = a, d = 1;
constexpr int iter = 5e4;
    for (int _ = 0; _ < iter; ++_) {
```

```
x = f(f(x, n), n);
       y = f(y, n);
d = gcd(abs(x - y), n);
       if (d != 1 && d != n) {
            break:
   if (d == 1 || d == n) {
       pollard(n);
   7
   return d;
vector<11> res;
void factor(ll n) {
   if (n \le N) {
       while (true) {
           if (lp[n] == 0) {
            res.push_back(lp[n]);
            n /= lp[n];
       return;
    if (prime(n)) {
        res.push_back(n);
       return:
    11 d = pollard(n);
    factor(n / d);
    factor(d);
void solve() {
   11 n;
    lp.resize(N + 1);
    for (int i = 2; i <= N; ++i) {
       if (lp[i] == 0) {
    lp[i] = i;
           pr.push_back(i);
        for (int j = 0; j < (int) pr.size() && pr[j] <= lp[i] && i * pr[j
     ] <= N; ++j) {
            lp[i * pr[j]] = pr[j];
   factor(n);
```

2.28 Префикс-функция

2.29 Z-функция

```
vector<int> z_function(string &s) {
   int n = (int) s.length();
   vector<int> z(n);
   for (int i = 1, 1 = 0, r = 0; i < n; ++i) {
        if (i <= r) {
            z[i] = min(r - i + 1, z[i - 1]);
        }
        while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
            ++z[i];
        }
        if (i + z[i] - 1 > r) {
            l = i;
            r = i + z[i] - 1;
        }
        return z;
}
```

2.30 Суфмас (TheEvilBird)

```
int mod(int x, int m) {
    if (x < 0) x += m;
    if (x >= m) x -= m;
    return x;
}
vector<int> sufix array(string s) {
    int n = sz(s);
    vector<int> arr(n), narr(n), head(n), c(n), nc(n);
for (int i = 0; i < n; ++i) {</pre>
        arr[i] = i;
    sort(all(arr), [&](int x, int y) {
        return s[x] < s[y];</pre>
    int cl = 0;
    c[arr[0]] = cl;
    head[0] = 0;
for (int i = 1; i < n; ++i) {
         if (s[arr[i]] != s[arr[i - 1]]) {
            head[++c1] = i;
         c[arr[i]] = cl;
    for (int k = 1; k < n && cl < n; k *= 2) {
        for (int i = 0; i < n; i++) {
    int j = mod(arr[i] - k, n);
             narr[head[c[j]]++] = j;
         c1 = 0:
         head[0] = 0;
         pii prev = {c[narr[0]], c[mod(narr[0] + k, n)]};
         for (int i = 1; i < n; i++) {
             pii cur = {c[narr[i]], c[mod(narr[i] + k, n)]};
             if (cur != prev) {
    head[++cl] = i;
             nc[narr[i]] = cl;
             prev = cur;
         }
         swap(arr, narr);
         swap(c, nc);
     // returns sufix array without $
    return vector<int>(arr.begin() + 1, arr.end());
}
vector<int> build_lcp(string &s, vector<int> &suf) {
  int n = sz(s);
    vector<int> lcp(n - 1), order(n);
    for (int i = 0; i < n; ++i) {
         order[suf[i]] = i;
    int 1 = 0:
    for (int i = 0; i < n; ++i) {
         int id = order[i];
         if (id + 1 == n) {
             1 = 0;
             continue;
         int j = suf[id + 1];
         if (1) --1;
         while (\max(i + 1, j + 1) < n \&\&
                s[i + 1] == s[j + 1]) {
             ++1:
         lcp[id] = 1;
    return lcp;
```

2.31 Суфавтомат (TheEvilBird)

```
struct Node {
    int go[26];
    int suf, prev, term, len;

    Node() {
        for (auto &i : go) {
            i = -1;
        }
        len = 0;
        suf = -1;
        prev = -1;
        term = 0;
    }
};

vector<Node> automat;
int add(int a, int ch) {
```

```
int b = sz(automat);
automat.emplace_back();
automat[b].prev = a;
automat[b].suf = 0;
automat[b].len = automat[a].len + 1;
for (; a != -1; a = automat[a].suf) {
    if (automat[a].go[ch] == -1) {
         automat[a].go[ch] = b;
         continue;
    int c = automat[a].go[ch];
if (automat[c].prev == a) {
         automat[b].suf = c;
         break;
    int d = sz(automat);
    automat.emplace_back();
automat[d].suf = automat[c].suf;
     automat[d].len = automat[a].len + 1;
     automat[c].suf = d;
     automat[b].suf = d;
    automat[d].prev = a;
for (int i = 0; i < 26; ++i) {</pre>
         automat[d].go[i] = automat[c].go[i];
     for (; a != -1 && automat[a].go[ch] == c; a = automat[a].suf) {
         automat[a].go[ch] = d;
    break:
// returns id of the added vertex
return b:
```

2.32 Axo-Корасик (Sweezyk)

```
struct Node {
    int par;
    int par_c;
    int go[26];
    int link;
    int super;
    int cnt:
const int N = 1e6 + 5;
int ptr = 1
Node trie[N];
void add(string s) {
    int cur = 1;
    for (auto &q : s) {
        int c = q - 'a';
        if (trie[cur].go[c]) {
            cur = trie[cur].go[c];
        } else {
            trie[cur].go[c] = ptr;
            trie[ptr].par = cur;
            trie[ptr].par_c = c;
            cur = ptr;
        }
    trie[cur].cnt++;
void build() {
    queue<int> q;
    q.push(1);
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        if (v != 1) {
            if (trie[v].par == 1) {
                trie[v].link = trie[v].super = 1;
                trie[v].link = trie[trie[trie[v].par].link].go[trie[v].
                trie[v].super = (trie[trie[v].link].cnt ? trie[v].link :
     trie[trie[v].link].super);
        trie[v].cnt += trie[trie[v].link].cnt;
        for (int c = 0; c < 26; c++) {
            if (trie[v].go[c]) {
                q.push(trie[v].go[c]);
            } else {
                if (v == 1) {
                    trie[v].go[c] = 1;
                } else {
                    trie[v].go[c] = trie[trie[v].link].go[c];
                }
```

```
}
trie[1].link = trie[1].super = trie[1].par = 1;
build();
```

2.33 Манакер

```
vector<int> d1(n):
int 1 = 0, r = -1;
for (int i = 0; i < n; ++i) {
  int k = (i > r ? 1 : min(d1[1 + r - i], r - i + 1));
     while (i + k < n &  i - k >= 0 &  s[i + k] == s[i - k]) {
    d1[i] = k;
    if (i + k - 1 > r) {
 1 = i - k + 1;
         r = i + k - 1;
}
vector<int> d2(n);
1 = 0:
r = -1;
for (int i = 0; i < n; ++i) {
    int k = (i > r ? 0 : min(d2[1 + r - i + 1], r - i + 1));
    while (i + k < n \&\& i - k - 1 >= 0 \&\& s[i + k] == s[i - k - 1]) {
         ++k;
    }
    d2[i] = k;
if (i + k - 1 > r) {
  l = i - k;
    }
}
```

2.34 CHT (FedShat)

```
struct Line {
    11 k = 0, b = -INFLL;
    Line() = default;
   Line(11 k, 11 b) : k(k), b(b){};
    ld operator()(ld x) {
        return k * x + b;
};
bool operator<(Line a, Line b) {</pre>
   return a.k < b.k | | (a.k == b.k && a.b > b.b);
bool operator<(pair<Line, ld> a, pair<Line, ld> b) {
   return a.second < b.second;</pre>
7
struct CHT {
   vector<pair<Line, ld>> convex;
   void add(Line a) {
       while (!convex.empty() && a(convex.back().second) > convex.back()
     .first(convex.back().second)) {
            convex.pop_back();
        ld xn = -INFLL;
        if (!convex.empty()) {
            xn = (a.b - convex.back().first.b + 0.0) / (convex.back().
     first.k - a.k);
        convex.push_back({a, xn});
    CHT(vector<Line> lines) {
        sort(all(lines)):
        for (int i = 0; i < (int) lines.size();) {
            int j = i;
            while (j < (int) lines.size() && lines[i].k == lines[j].k) {</pre>
                ++j;
            }
            add(lines[i]);
            i = j;
        }
   }
   ld get(ld x) {
        auto it = upper_bound(all(convex), pair(Line(), x)) - convex.
     begin():
        return convex[it - 1].first(x);
};
```

2.35 Дебаг Туриста

```
template<typename A, typename B>
string to_string(pair<A, B> p);
template<typename A, typename B, typename C>
string to_string(tuple<A, B, C> p);
template<typename A, typename B, typename C, typename D>
string to_string(tuple<A, B, C, D> p);
string to_string(const string &s) {
   return '"' + s + '"';
string to_string(const char *s) {
    return to_string((string) s);
string to_string(bool b) {
   return (b ? "true" : "false");
string to_string(vector<bool> v) {
    bool first = true;
string res = "{";
    for (int i = 0; i < static_cast<int>(v.size()); i++) {
   if (!first) {
             res += ",
         first = false;
         res += to_string(v[i]);
    res += "}";
    return res;
template<size_t N>
string to_string(bitset<N> v) {
    string res = "";
     for (size_t i = 0; i < N; i++) {
        res += static_cast<char>('0' + v[i]);
     return res;
template<typename A>
string to_string(A v) {
    bool first = true;
string res = "{";
    for (const auto &x : v) {
        if (!first) {
             res += ", ";
         first = false;
         res += to_string(x);
    res += "}":
    return res;
template<typename A, typename B>
string to_string(pair<A, B> p) {
   return "(" + to_string(p.first) + ", " + to_string(p.second) + ")";
template<typename A, typename B, typename C>
string to_string(tuple<A, B, C> p) {
    return "(" + to_string(get<0>(p)) + ", " + to_string(get<1>(p)) + ",
       " + to_string(get<2>(p)) + ")";
template<typename A, typename B, typename C, typename D>
string to_string(tuple<A, B, C, D> p) {
   return "(" + to_string(get<0>(p)) + ", " + to_string(get<1>(p)) + ","
       " + to_string(get<2>(p)) + ", " + to_string(get<3>(p)) + ")";
void debug_out() { cerr << endl; }</pre>
template<typename Head, typename... Tail>
void debug_out(Head H, Tail... T) {
   cerr << " " << to_string(H);</pre>
    debug_out(T...);
}
#ifdef LOCAL
#define debug(...) cerr << "[" << #__VA_ARGS__ << "]:", debug_out(
      __VA_ARGS__)
#define debug(...) 42
```

2.36 Геометрия (TheEvilBird)

```
const long double PI = 3.1415926535897932384626433832795;
const long double EPS = 1e-7;
 * Rotate relative to the origin, a - rotation angle:
 * new_x = x cos(a) - y sin(a)
* new_y = x sin(a) + y cos(a)
 * new_x = x cos(a) + y sin(a)
 * new_y = x sin(a) - y cos(a)
 * rad = degree * PI / 180
 * degree = rad * 180 / PI
template<typename T>
struct point_t {
    T x, y;
    point_t() : x(0), y(0) {}
    point_t(T _x, T _y) : x(_x), y(_y) {}
    T len_sq() const {
        return x * x + y * y;
    ld len() const {
        return sqrtl(len_sq());
    point_t operator*(T k) const {
        return \{x * k, y * k\};
    void operator*=(T k) {
        x *= k;
        v *= k:
    point_t operator+(const point_t<T> &other) const {
        return {x + other.x, y + other.y};
    point_t operator-(const point_t<T> &other) const {
        return {x - other.x, y - other.y};
    bool operator==(const point_t<T> &other) const {
        return x == other.x && y == other.y;
    bool operator!=(const point_t<T> &other) const {
        return !(*this == other);
    T operator*(const point_t<T> &other) const {// dot product
        return x * other.x + y * other.y;
    T operator%(const point_t<T> &other) const {// cross product
        return x * other.y - y * other.x;
};
using Point = point_t<11>;
using PointLD = point_t<1d>;
template<typename T>
istream &operator>>(istream &is, point_t<T> &vec) {
    is >> vec.x >> vec.y;
    return is;
}
template<typename T>
ostream &operator<<(ostream &os, const point_t<T> &vec) {
    os << vec.x << ' ' << vec.y;
    return os;
}
typedef vector<Point> Polygon;
bool cmp_vectors(Point a, Point b) {
   if ((a.y < 0 | | (a.y == 0 \&\& a.x < 0)) \&\&
        (b.y > 0 \mid | (b.y == 0 \&\& b.x > 0))) {
        return true;
    if ((b.y < 0 \mid | (b.y == 0 \&\& b.x < 0)) \&\&
        (a.y > 0 \mid | (a.y == 0 \&\& a.x > 0))) {
        return false:
    return (a % b > 0 || (a % b == 0 && a.len_sq() < b.len_sq()));
int get_sign(ld x) {
```

```
if (x < -EPS) return -1;
    if (EPS < x) return 1;
    return 0;
Polygon build_convex_hull(Polygon &a) {
     int n = sz(a);
    for (int i = 1; i < n; ++i) {
        if ((a[i].y < a[0].y) \mid | (a[i].y == a[0].y && a[i].x < a[0].x))
      swap(a[0], a[i]);
    sort(a.begin() + 1, a.end(), [&](Point A, Point B) {
        Point oa = A - a[0], ob = B - a[0];
if ((oa % ob) == 0) return oa.len_sq() < ob.len_sq();
        return (oa % ob) > 0;
    }):
    Polvgon hull = {a[0]};
    for (int i = 1; i < n; ++i) {
        while (sz(hull) >= 2) {
             Point ab = hull[sz(hull) - 1] - hull[sz(hull) - 2], bp = a[i]
       - hull[sz(hull) - 1];
             if ((ab % bp) <= 0) {
                 hull.pop_back();
             } else {
                 break;
             }
        hull.emplace_back(a[i]);
    return hull;
11 area_of_polygon(Polygon &poly) {
    11 res = 0;
int n = sz(poly);
    for (int i = 2; i < n; ++i) {
   Point ab = poly[i - 1] - poly[0], ac = poly[i] - poly[0];
         res += (ab % ac);
     // don't forget to divide the result by 2!
    return res;
ld perimeter_of_polygon(Polygon &poly) {
    1d res = 0;
    int n = sz(poly);
    for (int i = 0; i < n; ++i) {
        Point v = poly[(i + 1 == n ? 0 : i + 1)] - poly[i];
        res += v.len();
    return res:
}
11 diameter_of_polygon(Polygon &poly) {
    int n = sz(poly), x = 1;
    11 ans = 0;
    for (int i = 0; i < n; ++i) {
        int j = (i + 1) \% n;
         while (true) {
     Point ac = poly[x] - poly[i], ab = poly[j] - poly[i], cd =
poly[(x + 1) % n] - poly[x];
    ans = max(ans, ac.len_sq());
             if ((ab % cd) <= 0) {
                  break;
             }
             x++;
             x \% = n;
    // don't forget to extract root!
    return ans;
template<typename T>
ld angle(const point_t<T> &a, const point_t<T> &b) {
    return fabsl(atan2(a % b, a * b) / PI * 180);
}
template<typename T>
ld angle_rad(const point_t<T> &a, const point_t<T> &b) {
   return (atan2(a % b, a * b));
template<typename T>
ld angle_rad(const point_t<T> &a) {
    return atan2(a.y, a.x);
template<typename T>
point_t < ld > rotate(const point_t < T > &a, ld alpha) {
    return \{a.x * cos(alpha) - a.y * sin(alpha), a.x * sin(alpha) + a.y *
       cos(alpha)};
ld from_point_to_line(const Point &p, const Point &a, const Point &b) {
    // point p, line ab
Point ba = a - b, ap = p - a;
return fabs((ba % ap) / ba.len());
```

```
}
ld from_point_to_ray(const Point &p, const Point &a, const Point &b) { //
    point p, ray ab
Point ba = a - b, ap = p - a, ab = b - a;
if ((ab * ap) < 0) {</pre>
         return ap.len();
    } else {
        return fabs((ba % ap) / ba.len());
    7-
}
ld from_point_to_segment(const Point &p, const Point &a, const Point &b)
      { // point p, segment ab }
    Point ab = b - a, ap = p - a, bp = p - b, ba = a - b; if ((ab * ap) < 0) {
         return fabsl(ap.len());
    } else if ((ab * bp) > 0) {
        return fabsl(bp.len());
         return fabsl((ba % ap) / ba.len());
    }
}
bool point_on_line(const Point &p, const Point &a, const Point &b) { //
    point p, line ab
Point ab = b - a, ap = p - a;
    return (ab % ap) == 0;
}
bool point_on_ray(const Point &p, const Point &a, const Point &b) { //
    point p, ray ab
Point ab = b - a, ap = p - a;
    return ((ab % ap) == 0 && (ab * ap) > 0) || (a == p);
}
bool point_on_segment(const Point &p, const Point &a, const Point &b) {
      // point p, segment ab
    Point ap = p - a, bp = p - b, ab = b - a;
return (ap * bp) <= 0 && (ap % ab) == 0;
}
bool point_in_angle(const Point &p, const Point &a, const Point &o, const
    Point &b) { // point p, angle aob
Point oa = a - o, ob = b - o, op = p - o;
    if ((oa % ob) < 0) swap(oa, ob);
    return ((oa % op) >= 0 && (ob % op) <= 0);
}
bool segment_intersection(const Point &a, const Point &b, const Point &c,
       const Point &d) { // segment ab, segment, cd
    Point ab = b - a, cd = d - c, ac = c - a, ad = d - a, cb = b - c, ca
      = a - c;
    if (get_sign((ab % ac)) * get_sign((ab % ad)) <= 0 && get_sign((cd %</pre>
      ca)) * get_sign((cd % cb)) <= 0) {
    ll x1 = max(min(a.x, b.x), min(c.x, d.x)), x2 = min(max(a.x, b.x))
      , max(c.x, d.x));
         11 y1 = max(min(a.y, b.y), min(c.y, d.y)), y2 = min(max(a.y, b.y))
      , max(c.y, d.y));
         return (x1 <= x2 && y1 <= y2);
    7
    return false;
}
bool lines_intersection(const Point &a, const Point &b, const Point &c,
      const Point &d) { // line ab, line cd
    Point ab = b - a, cd = d - c;
return ((ab % cd) != 0 || (ab % (c - a) == 0));
bool line_ray_intersection(const Point &a, const Point &b, const Point &c
        const Point &d) { // line ab, ray cd
    if (!lines_intersection(a, b, c, d)) {
        return 0;
    Point ab = b - a, dp = d + (a - c);
if (get_sign(ab % (dp - a)) * get_sign(ab % (c - a)) <= 0) {
         return 1;
    return 0:
}
Point get_inf(const Point &a, const Point &b) { // returns inf point on
      ray ab
    if (a.x == b.x) {
         if (a.y < b.y) {
             return {a.x, INFLL};
         } else {
             return {a.x, -INFLL};
         }
    if (a.x < b.x) {
        if (a.v < b.v) {
             return {INFLL, INFLL};
         } else if (a.y == b.y) {
             return {INFLL, a.y};
         } else {
             return {INFLL, -INFLL};
```

```
// a.x > b.x
     if (a.y < b.y) {
         return {-INFLL, INFLL}:
    } else if (a.y == b.y) {
         return {-INFLL, a.y};
    } else {
         return {-INFLL, -INFLL};
    7
}
bool rays_intersection(const Point &a, const Point &b, const Point &c,
       const Point &d) { // ray ab, ray cd
     if (line_ray_intersection(a, b, c, d) && line_ray_intersection(c, d,
      a, b)) {
         Point bp = get_inf(a, b), dp = get_inf(c, d);
11 x1 = max(min(a.x, bp.x), min(c.x, dp.x)), x2 = min(max(a.x, bp.x), min(c.x, dp.x))
       .x), max(c.x, dp.x));
         ll y1 = max(min(a.y, bp.y), min(c.y, dp.y)), y2 = min(max(a.y, bp.y))
       .y), max(c.y, dp.y));
         return (x1 <= x2 && y1 <= y2);
    7
    return 0;
int point_in_polygon(const Point &p, const Polygon &poly) {
     // 0 - outside, 1 - inside, 2 - border;
     int n = sz(poly);
     if (point_in_angle(p, poly[n - 1], poly[0], poly[1])) {
         int l = 1, r = n - 1;
         while (r - 1 > 1) {
              int md = (1 + r) / 2;
              if (point_in_angle(p, poly[md], poly[0], poly[1])) {
                  r = md;
              } else {
                  1 = md;
              }
         if (point_in_angle(p, poly[r], poly[0], poly[1]) &&
              point_in_angle(p, poly[0], poly[1], poly[1]) &&
point_in_angle(p, poly[0], poly[1], poly[0])) {
  if (point_on_segment(p, poly[1], poly[r]) ||
     point_on_segment(p, poly[1], poly[r]) ||
     point_on_segment(p, poly[(1 ? 1 - 1 : n - 1)], poly[1])
      \Pi
                   point_on_segment(p, poly[r], poly[(r + 1 == n ? 0 : r +
      1)])) {
                  return 2:
              } else {
                  return 1:
         } else {
             return 0;
         7
    } else {
         return 0;
int point_in_nonconvex_polygon(const Point &p, const Polygon &poly) {
     // 0 - outside, 1 - inside, 2 - border;
     int n = sz(poly);
     for (int i = 0; i < n; ++i) {
         if (point_on_segment(p, poly[i], poly[(i + 1) % n])) {
              return 2;
         }
     1d s = 0.0;
     for (int i = 0; i < n; ++i) {
         Point pa = poly[i] - p, pb = poly[(i + 1 == n ? 0 : i + 1)] - p;
         s += angle_rad(pa, pb);
    if (s \ge PI \mid | s \le -PI) {
         return 1;
     } else {
         return 0;
    7
Polygon minkowski_sum(Polygon &a, Polygon &b) {
       a[0], b[0]: y - max, y1 = y2 => x - max. Against clockwise
     int n = sz(a), m = sz(b);
     assert(n >= 3 && m >= 3);
     Point high_a = a[0], high_b = b[0];
    Polygon va(n), vb(m);
for (int i = 0; i < n; ++i) {</pre>
         va[i] = a[(i + 1) % n] - a[i];
     for (int i = 0; i < m; ++i) {
         vb[i] = b[(i + 1) \% m] - b[i];
      sort(all(va), cmp_vectors);
    sort(all(vb), cmp_vectors);
Polygon vc(sz(va) + sz(vb));
    merge(all(va), all(vb), vc.begin(), cmp_vectors);
Point high_c(high_a.x + high_b.x, high_a.y + high_b.y);
     Polygon c(sz(vc) + 1);
     c[0] = high_c;
```

```
for (int i = 0; i < sz(c) - 1; ++i) {
        c[i + 1] = c[i] + vc[i];
    return c:
}
ld from_polygon_to_polygon(Polygon a, Polygon b) {
    for (auto &i : b) {
        i *= -1;
    }
    int pos = 0;
    for (int i = 1; i < sz(b); ++i) {
        if ((b[i].y > b[pos].y) ||
(b[i].y == b[pos].y && b[i].x > b[pos].x)) {
        }
    rotate(b.begin(), b.begin() + pos, b.end());
    Polygon c = minkowski_sum(a, b);
    int n = sz(c);
    Point p(0, 0);
    1d ans = 1e20;
    for (int i = 0: i < n - 1: ++i) {
        ans = min(ans, from_point_to_segment(p, c[i], c[i + 1]));
}
11 diameter_of_polygon_minkowski(Polygon &a) {
    Polygon ra = a;
    for (auto &i : ra) {
    int pos = 0;
    for (int i = 1; i < sz(a); ++i) {
        if ((a[i].y > a[pos].y) ||
    (a[i].y == a[pos].y && a[i].x > a[pos].x)) {
            pos = i;
        }
    }
    rotate(a.begin(), a.begin() + pos, a.end());
    pos = 0:
    for (int i = 1; i < sz(a); ++i) {
        if ((ra[i].y > ra[pos].y) ||
             (ra[i].y == ra[pos].y \&\& ra[i].x > ra[pos].x)) {
        }
    rotate(ra.begin(), ra.begin() + pos, ra.end());
    Polygon c = minkowski_sum(a, ra);
    int n = sz(c);
    11 ans = 0;
    for (int i = 0; i < n; ++i) {
        ans = max(ans, c[i].len_sq());
    // don't forget to extract root!
    return ans;
}
ld from_segment_to_segment(const Point &a, const Point &b, const Point &c, const Point &d) { // segment ab, segment cd
    if (segment_intersection(a, b, c, d)) {
        return 0;
    return min({from_point_to_segment(a, c, d), from_point_to_segment(b,
     c, d),
                 from_point_to_segment(c, a, b), from_point_to_segment(d,
     a, b)});
}
bool segment_line_intersection(const Point &a, const Point &b, const
    Point &c, const Point &d) { // segment ab, line cd Point cd = d - c;
    if (get_sign(cd % (a - c)) * get_sign(cd % (b - c)) <= 0) {
        return 1;
    return 0;
}
bool ray segment intersection(const Point &a. const Point &b. const Point
      &c, const Point &d) { // ray ab, segment cd
    if (line_ray_intersection(c, d, a, b) && segment_line_intersection(c,
      d, a, b)) {
        Point bp = get_inf(a, b);
        11 x1 = max(min(a.x, bp.x), min(c.x, d.x)), x2 = min(max(a.x, bp.x))
     x), max(c.x, d.x));
        11 y1 = max(min(a.y, bp.y), min(c.y, d.y)), y2 = min(max(a.y, bp.
     y), max(c.y, d.y));
        return (x1 <= x2 && y1 <= y2);
    }
    return 0:
}
ld from_segment_to_ray(const Point &a, const Point &b, const Point &c,
      const Point &d) { // segment ab, ray cd
    if (ray_segment_intersection(c, d, a, b)) {
        return 0;
```

```
return min({from_point_to_ray(a, c, d),
                 from_point_to_ray(b, c, d),
                from_point_to_segment(c, a, b)});
}
ld from_segment_to_line(const Point &a, const Point &b, const Point &c,
     const Point &d) { // segment ab, line cd
    Point cd = d - c;
    if (get_sign(cd % (a - c)) * get_sign(cd % (b - c)) <= 0) {
        return 0:
    return min(from_point_to_line(a, c, d), from_point_to_line(b, c, d));
ld from_ray_to_ray(const Point &a, const Point &b, const Point &c, const
     Point &d) { // ray ab, ray cd
    if (rays_intersection(a, b, c, d)) {
        return 0:
    return min(from_point_to_ray(a, c, d), from_point_to_ray(c, a, b));
ld from_ray_to_line(const Point &a, const Point &b, const Point &c, const
      Point &d) { // ray ab, line cd
    if (line_ray_intersection(c, d, a, b)) {
        return 0;
    7
    return from_point_to_line(a, c, d);
}
ld from_line_to_line(const Point &a, const Point &b, const Point &c,
      const Point &d) { // line ab, line cd
    if (lines_intersection(a, b, c, d)) {
        return 0;
    return from_point_to_line(a, c, d);
pii tangent_from_point(const Point &p, const Polygon &poly) {
      returns id of tangent point: {left tangent, right tangent} (maybe {
     right, left}, but i believe no)
2**20 ~= 1e6, 2**17 ~= 1e5
    int n = sz(poly);
    int i_min = 0;
    int i_{max} = 0;
    for (int k = 17; k \ge 0; --k) {
             int 1 = (i_min + (1 << k)) % n;
            int r = ((i_min - (1 << k)) \% n + n) \% n;
            i_min = min({1, r, i_min}, [&poly, &p](int i, int j) {
   return ((poly[i] - p) % (poly[j] - p)) < 0 ||
        (((poly[i] - p) % (poly[j] - p)) == 0 && (poly[i]</pre>
      - p).len_sq() < (poly[j] - p).len_sq());
            }):
        }
             int 1 = (i_max + (1 << k)) % n;
             int r = ((i_max - (1 << k)) % n + n) % n;
      });
        }
    7
    return {i_min, i_max};
template<typename T>
struct circle_t {
    point_t<T> c;
    circle_t() {}
    circle_t(point_t<T>_c, T_r) : c(_c), r(_r) {}
    int point_in(const point_t<T> &a) {
        if (is_same<T, 11>()) {
     11 dist = (a - c).len_sq();
             if (dist > r * r) {
                return 0;
             } else if (dist < r * r) {</pre>
                return 1;
            } else {
                return 2:
        } else {
            ld dist = (a - c).len();
             if (dist > r + EPS) {
                return 0:
             } else if (dist + EPS < r) {
                return 1;
             } else {
                return 2;
        }
```

```
vector<PointLD> tangent_from_point(const point_t<T> &a) {
        PointLD p(a.x, a.y);
PointLD vec(c.x - p.x, c.y - p.y);
        ld dist = vec.len();
if (dist + EPS < r) {</pre>
             return {};
         if (abs(r - dist) < EPS) {
             return {p};
         vec.x /= dist;
         vec.y /= dist;
ld k = sqrtl(dist * dist - r * r);
         ld alpha = atan2(r, k);
         PointLD t1 = p + rotate(vec, alpha) * k, t2 = p + rotate(vec, -
      alpha) * k;
        return {t1, t2};
    }
};
template<typename T>
istream &operator>>(istream &is, circle_t<T> &c) {
    is >> c.c >> c.r;
    return is;
```

2.37 Стрессы (TheEvilBird)

```
# files: generators.py gen.py stress.py
# -----
import random
import heapq
def gen_num(L: int, R: int):
   Generates a number between L and R.
   return random.randint(L, R)
# -----
\# abcdefghijklmnopqrstuvwxyz
def gen_string_abc(LEN: int, ALPH_LEN: int = 26):
   Generates a string of length LEN using the first ALPH_LEN lowercase
   letters of the alphabet.
   abc = "abcdefghijklmnopqrstuvwxyz"
   s = abc[:ALPH_LEN]
   res = "
   for i in range(LEN):
      res += random.choice(s)
   return res
def gen_string_any_aplh(LEN: int, ALPH: str):
   Generates a string of length LEN using ALPH as the alphabet.
   # ALPH_LEN = len(ALPH)
   for i in range(LEN):
      kek = 1
      res += random.choice(ALPH)
   return res
# -----
def gen_tree(N: int):
   Generates a tree with N vertices.
   edges = []
   for i in range(2, N + 1):
      v = gen_num(1, i - 1)
       edges.append((v, i))
   return edges
def gen_DAG(N: int, M: int):
   Generates a directed acyclic graph with N vertices and M edges.
   edges = []
   # for i in range(1, N):
        if len(edges) == M:
           break
         v = gen\_num(i + 1, N)
        edges.append((i, v))
   while len(edges) < M:
```

```
v = gen_num(1, N - 1)
        u = gen_num(v + 1, N)
        edges.append((v, u))
    return edges
def gen_graph(N: int, M: int):
    Generates a graph with N vertices and M edges.
    edges_set = set()
    for i in range(M):
          u = 0, 0
         while (v, u) in edges_set or v == u:
            v, u = gen_num(1, N), gen_num(1, N)
        v, u = min(v, u), max(v, u)
edges_set.add((v, u))
    return list(edges_set)
def gen_multigraph(N: int, M: int):
    Generates a multigraph with N vertices and M edges.
    edges = []
    for i in range(M):
        v, u = -1, 0
        while v == -1:
             v, u = gen_num(1, N), gen_num(1, N)
v, u = min(v, u), max(v, u)
        edges.append((v, u))
    return edges
def gen_directed_graph(N: int, M: int):
    Generates a directed graph with N vertices and M edges.
    edges_set = set()
    for i in range(M):
        v, u = 0, 0
        while (v, u) in edges_set or v == u:
    v, u = gen_num(1, N), gen_num(1, N)
edges_set.add((v, u))
    return list(edges_set)
def gen_connected_directed_graph(N: int, M: int):
    Generates a directed connected graph with N vertices and M edges.
    edges_set = set(gen_tree(N))
    for i in range(M - (N - 1)):
        \mathbf{v}, \mathbf{u} = 0, 0
        while (v, u) in edges_set or v == u:
    v, u = gen_num(1, N), gen_num(1, N)
edges_set.add((v, u))
    return list(edges_set)
def gen_connected_graph(N: int, M: int):
    Generates a connected graph with N vertices and M edges.
    edges_set = set(gen_tree(N))
    for i in range(M - (N - 1)):
        \mathbf{v}, \mathbf{u} = 0, 0
         while (v, u) in edges_set or v == u:
        v, u = gen_num(1, N), gen_num(1, N)
v, u = min(v, u), max(v, u)
edges_set.add((v, u))
    return list(edges_set)
def gen_connected_multigraph(N: int, M: int):
    Generates a connected multigraph with N vertices and M edges.
    edges = gen_tree(N)
    for i in range(M - (N - 1)):
        v, u = 0, 0
        while v == u:
            v, u = gen_num(1, N), gen_num(1, N)
        edges.append((v, u))
    return edges
# -----
def gen_perm(N: int, FIR: int = 1):
    Generates a permutation of length N with \min element FIR.
    arr = [FIR + i for i in range(N)]
    # arr = arr[1:]
    random.shuffle(arr)
    return arr
def gen_array(N: int, L: int, R: int):
    Generates an array of length N with elements between L and R.
    arr = [gen_num(L, R) for i in range(N)]
```

```
def gen_array_pairs(N: int, L: int, R: int):
    Generates an array of pairs of length N with elements between L and R
    arr = [(gen_num(L, R), gen_num(L, R)) for i in range(N)]
    return arr
def gen_array_pairs(N: int, L1: int, R1: int, L2: int, R2: int):
    Generates an array of pairs of length N with the first elements of
     each pair between L1 and R1 and between L2 and R2 for the second
    element.
    arr = [(gen_num(L1, R1), gen_num(L2, R2)) for i in range(N)]
   return arr
# -----
def gen_tree_ivanq(N: int):
    Generates a tree with N vertices. Code by IvanQ.
    code = [random.randint(1, N) for _ in range(N - 2)]
     histogram = [0] * (N + 1) 
    unused = list(set(range(1, N + 1)) - set(code))
   heapq.heapify(unused) for u in code:
       histogram[u] += 1
    res = []
   for \mathbf{u} in \mathsf{code}:
       v = heapq.heappop(unused)
       res.append((v, u))
histogram[u] -= 1
        if histogram[u] == 0:
   heapq.heappush(unused, u)
return res + [tuple(unused)]
# -----
import sys
from generators import *
SEED = 228
if len(sys.argv) > 1:
    SEED = int(sys.argv[1])
random.seed(SEED)
# print(gen_num(1, 666))
# -----
import os
import sys
os.system("g++ -std=c++17 smart.cpp -o smart")
# os.system("g++ -std=c++17 -g -fsanitize=undefined -fsanitize=bounds -
     fsanitize=address -D_GLIBCXX_DEBUG smart.cpp -o smart")
os.system("g++ -std=c++17 stupid.cpp -o stupid")
def print_testcase():
    test = open("test.txt").read().strip()
   print(test)
   print("=====
    ans1 = open("smart.out").read().strip()
    ans2 = open("stupid.out").read().strip()
   print(ans1)
   print("==
   print(ans2)
   print("====="")
while True:
   os.system(f"python3 gen.py {i} > test.txt")
   r1 = os.system("./smart < test.txt > smart.out")
   r2 = os.system("./stupid < test.txt > stupid.out")
    if r1 + r2 != 0:
        print(f"Runtime! {i}")
        print_testcase()
        exit(0)
   ans1 = open("smart.out").read()
   ans2 = open("stupid.out").read()
    if ans1 != ans2:
       print(f"POPALCYA! {i}\n")
        print_testcase()
        exit(0)
    print(f"OK: {i}")
    i += 1
```

Задача	Саша	Дима	Федя	О чём
A				
В				
С				
D				
Б				
E				
F				
G				
Н				
I				
1				
J				
K				
L				
M				
N				
IN				
О				
P				