MATH 244 Homework 5

Due Thursday, October 4 at beginning of class

1. Consider the sequence 6, 11, 16, 21, 26, ...

   a) What is the next term in the sequence?
   b) Find a formula for the n^{th} term of this sequence, assuming that $a_1 = 6$.

2. Evaluate the sums:

   a) $5 + 8 + 11 + \cdots + 131$
   b) $5 + 15 + 45 + \cdots + 5 \cdot 3^{20}$
   c) $\frac{2}{5} - \frac{4}{25} + \frac{8}{125} - \frac{16}{625} + \cdots + \frac{2^{40}}{5^{40}}$

3. Rewrite each of the following using either summation (Σ) or product (Π) notation:

   a) $1 + 5 + 9 + \cdots + 425$
   b) $\left(\frac{3}{4}\right)\left(\frac{4}{5}\right)\left(\frac{5}{6}\right)\cdots\left(\frac{90}{91}\right)$
   c) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{121}$

For each of #4-8, prove the statements using Mathematical Induction:

4. For any $n \geq 1$, $\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$.

5. Let f_n be the nth Fibonacci number. Recall that $f_n = f_{n-1} + f_{n-2}$; $f_0 = 0$, $f_1 = 1$.

   a. Prove that for any $n \geq 0$, $f_0 + f_1 + f_2 + \cdots + f_n = f_{n+2} - 1$.

   b. Prove that for any $n \geq 1$, $f_1 + f_3 + f_5 + \cdots + f_{2n-1} = f_{2n}$.

6. For any $n \geq 1$, $\sum_{k=1}^{n} k \cdot 2^k = 2 + (n-1) \cdot 2^{n+1}$.

7. For any $n \geq 1$, $\sum_{k=1}^{n} (2k-1)^3 = n^2(2n^2 - 1)$.

8. $7^n - 2^n$ is divisible by 5 for all integers $n \geq 0$.