Planck-Bound Unified Framework (PBUF) — Mathematical Supplement (v9.0)

Compiled: 2025-10-20
Principal Investigator: Fabian Olesen
Repository: github.com/TheExiledMonk/PBUF

1 · Field Equations and Elastic Extension

The starting point is the standard Einstein–Hilbert action, augmented by an elastic term:

$$S = \int d^{4x} \sqrt{-g} \left(\frac{R}{16\, pi\, G} + L_{elastic} + L_m \right)$$

Variation with respect to the metric yields the **elastic Einstein equation**:

$$G_{mu\,nu} + sigma_{mu\,nu} = 8\, pi\, G\, T_{mu\,nu}$$

The **elastic tensor** derives from the variation of the elastic Lagrangian:

$$sigma_{mu\,nu} = \frac{-2}{\sqrt{-g}}\, delta\, \frac{\left(\sqrt{-g}\, L_{elastic}\right)}{delta}\, g^{mu\,nu}$$

Energy–momentum conservation is automatically preserved:

$$\nabla \cdot (G + sigma) = 0$$

2 · Choice of Elastic Lagrangian

A curvature-bounded realization uses a tanh-type deformation of the Ricci scalar:

$$L_{elastic} = \frac{1}{16\, pi\, G} \left(f(R) - R \right)$$

with

$$f(R) = R_{star} \tanh\left(\frac{R}{R_{star}} \right) + lambda\, R$$

The parameter `R_star` sets the **saturation curvature scale**, while `lambda` allows small low-curvature renormalization.
The elastic modification ensures

$$|R| \leq R_{star}$$

thus removing the classical singularity.

3 · Cosmological Background

For an FRW metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2\, dOmega^2\right)$$

the **modified Friedmann equations** become

$$H^2(a) = H_0^2\left(Omega_m a^{-3} + Omega_r a^{-4} + Omega_k a^{-2} + Omega_{sigma}(a)\right)$$

and

$$\frac{\ddot{a}}{a} = \frac{-H_0^2}{2}\left[Omega_m a^{-3} + 2\, Omega_r a^{-4} - 2\, Omega_{sigma}(a)(1+3w_{sigma})\right]$$

The **elastic energy density term** is parameterized as

$$Omega_{sigma}(a) = alpha\left(1-e^{-a/R_{max}}\right)$$

with effective equation-of-state

$$w_{sigma} = \frac{p_{sigma}}{rho_{sigma}}$$

and conservation law

$$\frac{d\, rho_{sigma}}{dt} + 3H\left(rho_{sigma} + p_{sigma}\right) = 0$$

4 · Perturbative Stability

Linearizing about a background metric gmunu=bargmunu+hmunu:

$$delta\, G_{mu\,nu} + delta\, sigma_{mu\,nu} = 8\, pi\, G\, delta\, T_{mu\,nu}$$

For scalar perturbations in Newtonian gauge, the rigidity term modifies the potential equation:

$$\left(\nabla^2 - k_{rig}^2\right) Phi = 4\, pi\, G\, a^2\, delta\, rho$$

where krig=(1−ksat)1/2H0 defines an effective rigidity scale controlling growth damping.

5 · Curvature-Bound Derivation

Expanding the tanh term for |R| ≪ R★:

$$f(R) \approx R + \frac{1}{3}\left(\frac{R^3}{R_{star}^2}\right) - \ldots$$

At |R| ≫ R★:

$$f(R) \rightarrow R_{star}\, sign(R) + lambda\, R$$

This guarantees smooth transition and bounded curvature:

$$\lim_{R \rightarrow \infty} f(R)/R = lambda$$

and therefore

$$|R| \leq R_{star}$$

6 · Energy–Momentum Conservation

The total stress–energy tensor satisfies

$$\nabla^{mu}\left(T_{mu\,nu} + sigma_{mu\,nu}/8\, pi\, G\right) = 0$$

ensuring full diffeomorphism invariance and covariant conservation.

7 · Limiting Behavior

Low-curvature limit (| R | <<Rstar):

$$sigma_{mu\,nu} \rightarrow 0,\, G_{mu\,nu} = 8\, pi\, G\, T_{mu\,nu}$$

→ General Relativity recovered.

High-curvature limit (| R | →Rstar):

$$sigma_{mu\,nu} \rightarrow -G_{mu\,nu}$$

→ curvature saturates, preventing divergence and producing a cosmological bounce.

8 · Interpretation

- ksat: dimensionless rigidity parameter — fraction of GR stiffness retained.
- Rmax: saturation curvature scale (~Planck curvature).
- alpha: coupling amplitude linking elasticity to expansion.

These parameters connect macroscopic cosmic behavior to microphysical rigidity of spacetime.

9 · Summary

The **Planck-Bound Unified Framework (PBUF)** introduces a Lorentz-covariant elasticity term that bounds curvature, preserves conservation laws, and reproduces cosmological observables with one additional physical constant.
Analytically, it avoids singularities, predicts late-time acceleration, and allows a cyclic bounce within a single unified set of equations.