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Homework 3

9. $\vec{PQ} = \langle 2, 2 \rangle$, $x = y$ $\quad \|u\| = \sqrt{2x^2} = 1$

$2x^2 = 1$

$x^2 = 1/2$

$x = \frac{1}{\sqrt{2}}$, so $\boxed{\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle}$

a. $\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

b. $\boxed{\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}}$

15. $\vec{a} = 2i + j$; $\vec{b} = i + 3j$

a. $\underset{\vec{a}}{(2i+j)} + \underset{\vec{b}}{(i+3j)} = \boxed{3\hat{i} + 4\hat{j} = \langle 3, 4 \rangle}$

b. $\underset{\vec{a}}{(2i+j)} - \underset{\vec{b}}{(i+3j)} = \boxed{\hat{i} - 2\hat{j} = \langle 1, -2 \rangle}$

c. $\|\vec{a} + \vec{b}\| \le \|\vec{a}\| + \|\vec{b}\|$

$\sqrt{3^2+4^2} \le \sqrt{2^2+1^2} + \sqrt{1^2+3^2}$

$\sqrt{9+16} \quad \sqrt{5} + \sqrt{10}$

$\boxed{5 \le 2.236 + 3.162}$ ✓

$\|\vec{a} - \vec{b}\| \ge \|\vec{a}\| - \|\vec{b}\|$

$\boxed{\sqrt{5} \ge \sqrt{5} - \sqrt{10}}$ ✓

d. $2(2\hat{i}+\hat{j}) = 4\hat{i}+2\hat{j} = \boxed{\langle 4, 2 \rangle}$

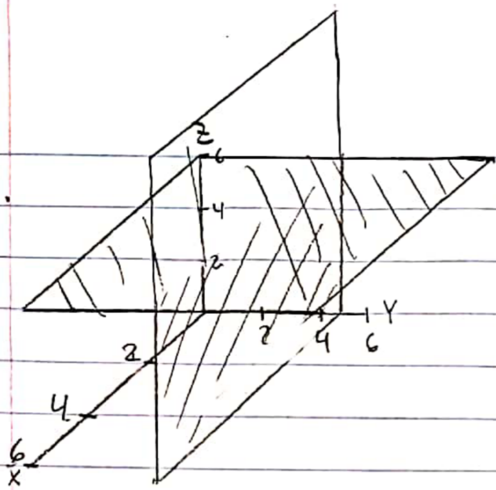
$-(\hat{i}+3\hat{j}) = -\hat{i} - 3\hat{j} = \boxed{\langle -1, -3 \rangle}$

$(4\hat{i}+2\hat{j}) + (-\hat{i}-3\hat{j}) = \boxed{3\hat{i} - \hat{j} = \langle 3, -1 \rangle}$

27. $\vec{V} = 7\left\langle \frac{U_1}{\|U\|}, \frac{U_2}{\|U\|} \right\rangle$, $\|U\| = \sqrt{34}$, $\vec{V} = \left\langle \frac{7(3)}{\sqrt{34}}, \frac{7(-5)}{\sqrt{34}} \right\rangle$

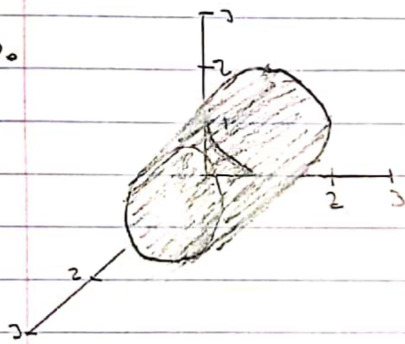
$\boxed{\vec{V} = \left\langle \frac{21}{\sqrt{34}}, -\frac{35}{\sqrt{34}} \right\rangle}$

63.



two Planes meeting
Plane $Y=5$ (Parallel to XZ)
Plane $Z=6$ (Parallel to XY)

65.



Cylinder, radius 1 on line $Y=1$, $Z=1$

75. $x^2+y^2+(z^2-4z+4)=1$

$x^2+y^2+(z-2)^2=1$

Center $= (0,0,2)$ Radius $= 1$

107. $P(x,Y,z)$

$$P(x,y,z) = \left(\frac{a_1+b_1}{2}, \frac{a_2+b_2}{2}, \frac{a_3+b_3}{2}\right)$$

$P = (0, \frac{1}{2}, \frac{1}{2})$ $\quad \frac{0}{2}, \frac{1}{2}, \frac{1}{2}$

$-2x+3y+z=2$

$-2(0)+3(\frac{1}{2})+\frac{1}{2}=2$

$2=2$ ✓

127. $a = \langle 2,0,-3\rangle$ $b = \langle -4,-7,1\rangle$ $c = \langle 1,1,-1\rangle$

$(a\cdot b)c = \langle 2(-4)+0(-7)+(-3)(1)\rangle\langle 1,1,-1\rangle$

$-8+0+(-3)$

$-11\langle 1,1,-1\rangle$

$(a\cdot b)c = \langle -11,-11,11\rangle$

$(a\cdot c)b = \langle 2(1)+0(1)+(1)-1\rangle\langle -4,-7,1\rangle$

$2+-1$

$1\langle -4,-7,1\rangle$

$(a\cdot c)b = \langle -4,-7,1\rangle$

129 $a = \langle 1,1,0\rangle$ $b = \langle 1,0,-1\rangle$ $c = \langle 1,0,-2\rangle$

$(a\cdot b)c = (1+0+0)\langle 1,0,-2\rangle = \langle 1,0,-2\rangle$

$(a\cdot c)b = (1+0+0)\langle 1,0,-1\rangle = \langle 1,0,-1\rangle$

137. $\vec{a} = \langle 1,1,0\rangle$ $\vec{b} = \langle 0,1,-1\rangle$

$$\cos\theta = \frac{a\cdot b}{\|a\|\|b\|} = \frac{(0+1+0)}{\sqrt{1+1}\sqrt{1+1}} = \frac{1}{2}$$

$$\boxed{\cos^{-1}\left(\tfrac{1}{2}\right) = 0}$$

145. $b = \langle 3,4\rangle$

$\vec{b}\cdot\vec{v}_1 = (3(-4)+4(3)) = 0$

$\vec{b}\cdot\vec{v}_2 = (3(4)+4(-3)) = 0$

149. $a = \langle 2,3\rangle$ $b = \langle 9,\alpha\rangle$

$a\cdot b = 2(9)+3(\alpha) = 0$

$3(\alpha) = -18$

$\boxed{\alpha = -6}$

153. $A(1,1,8)$ $B(4,-3,-4)$ $C(-3,1,5)$

$\overrightarrow{AB} = \langle 3,-4,-12\rangle$
$\overrightarrow{AC} = \langle -4,0,-3\rangle$

$AB \cdot AC = \|AB\|\,\|AC\| \cos\theta$
$-12+0+36$
$24 = \sqrt{3^2+-4^2+-12^2}\left(\sqrt{-4^2+-3^2}\right)\cos\theta$
$\sqrt{9+16+144}\,(5)$
$\sqrt{25+144}$
$13(5)$
$65\cos\theta = 24$
$\cos\theta = \frac{24}{65}$
$\cos^{-1}\left(\frac{24}{65}\right) = 1.19 \text{ rad} \approx \boxed{68.33^\circ}$

161. $u = \langle 2,2,1\rangle$

a. $\cos\alpha = \frac{2}{\sqrt{2^2+2^2+1^2}} = \frac{2}{3}$
$\cos\beta = \frac{2}{\sqrt{9}} = \frac{2}{3}$
$\cos\gamma = \frac{1}{\sqrt{9}} = \frac{1}{3}$

b. $\cos^{-1}\left(\frac{2}{3}\right) = \alpha \approx 48^\circ$
$\cos^{-1}\left(\frac{2}{3}\right) = \beta \approx 48^\circ$
$\cos^{-1}\left(\frac{1}{3}\right) = \gamma \approx 71^\circ$

165. $u = \langle a,b,c\rangle$

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \left(\frac{a}{\sqrt{a^2+b^2+c^2}}\right)^2 + \left(\frac{b}{\sqrt{a^2+b^2+c^2}}\right)^2 + \left(\frac{c}{\sqrt{a^2+b^2+c^2}}\right)^2 = \left(\frac{a^2+b^2+c^2}{a^2+b^2+c^2}\right) = 1}$$

167. $W = Proj_u V = \frac{u \cdot v}{\|u\|^2} u = \frac{\langle 5,2\rangle \cdot \langle 2,3\rangle}{5^2+2^2} \langle 5,2\rangle = \frac{16}{29} \langle 5,2\rangle =$

a. $\left\langle \frac{80}{29}, \frac{32}{29} \right\rangle$

b. $Comp_u V = \frac{|u \cdot v|}{\|u\|} = \boxed{\frac{16}{\sqrt{29}}}$

169 $u = \langle 3,0,2\rangle \quad v = \langle 0,2,4\rangle$

$Proj_u V = \frac{u \cdot v}{\|u\|^2} u = \frac{\langle 3,0,2\rangle \cdot \langle 0,2,4\rangle}{3^2+0^2+2^2} \langle 3,0,2\rangle =$

a. $\frac{8}{13} \langle 3,0,2\rangle = \boxed{\left\langle \frac{24}{13}, 0, \frac{16}{13} \right\rangle}$

b. $\frac{|u \cdot v|}{\|u\|} = \frac{8}{\sqrt{13}}$

171. $u = \langle 4,-3\rangle \quad V = \langle 3,2\rangle$

a $Proj_u V = \frac{u \cdot v}{\|u\|^2} u = \frac{\langle 4,-3\rangle \cdot \langle 3,2\rangle}{4^2+(-3)^2} \langle 4,-3\rangle = \frac{6}{25} \langle 4,-3\rangle =$

$\boxed{\left\langle \frac{24}{25}, -\frac{18}{25} \right\rangle = W}$

b $\langle 3,2\rangle = \left\langle \frac{24}{25}, -\frac{18}{25} \right\rangle + \langle 3,4\rangle$

173. a $\sqrt{(-1-1)^2+(1-1)^2(1+1)^2}$

$\sqrt{4+4} = \sqrt{8} = \sqrt{4 \cdot 2} = \boxed{2\sqrt{2}}$

b. $\overrightarrow{OS} = \langle -1,-1,-1\rangle \quad \overrightarrow{OR} = \langle -1,1,1\rangle$

$\cos\theta = \frac{\overrightarrow{OS} \cdot \overrightarrow{OR}}{\|OS\| \, \|OR\|} = \frac{(1+(-1)+(-1))}{\sqrt{3}\sqrt{3}} = -\frac{1}{3}$

$\boxed{\cos^{-1}\left(\frac{1}{3}\right) = 109.47°}$

Scanned with CamScanner

175. $F = \langle 5, 6, -2 \rangle$, $W = F \cdot \vec{PQ} = \|F\| \|\vec{PQ}\| \cos\theta$

$\vec{PQ} = \langle 2-3, 3+1, 1-0 \rangle = \langle -1, 4, 1 \rangle$

$-5 + 24 - 2 = \boxed{17 \ N \cdot m}$