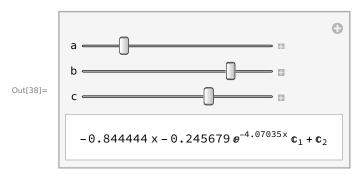
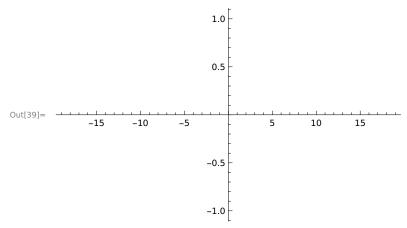
```
 \begin{aligned} & \text{In}[13] \coloneqq & \text{DSolve}[y \,'[x] + y[x] * \text{Tan}[x] == Sin[2 * x], \, y[x], \, x] \\ & \text{Out}[13] = & \left\{ \left\{ y[x] \to c_1 \, \text{Cos}[x] - 2 \, \text{Cos}[x]^2 \right\} \right\} \\ & \text{In}[14] \coloneqq & \text{DSolve}[\left\{ y \,'[x] + y[x] * \text{Tan}[x] == Sin[2 * x], \, y[0] == 0 \right\}, \, y, \, x] \\ & \text{Out}[14] = & \left\{ \left\{ y \to \text{Function}[\left\{ x \right\}, \, -2 \left( -\text{Cos}[x] + \text{Cos}[x]^2 \right) \right] \right\} \right\} \\ & \text{In}[21] \coloneqq & \text{Sol} = & \text{DSolve}[\left\{ y \,'[x] + y[x] * \text{Tan}[x] == \text{Sin}[2 * x], \, y[0] == 0 \right\}, \, y, \, x] \\ & y[x] \, / \cdot \, \text{sol}[[1]] \\ & \text{Plot}[y[x] \, / \cdot \, \text{sol}[[1]], \, \left\{ x, \, -6 \, \pi, \, 6 \, \pi \right\} \right] \\ & \text{Out}[21] \coloneqq & \left\{ \left\{ y \to \text{Function}[\left\{ x \right\}, \, -2 \left( -\text{Cos}[x] + \text{Cos}[x]^2 \right) \right] \right\} \end{aligned}
```

-2





 $\begin{aligned} &\mathsf{ndsoll} = \mathsf{NDSolve}[\{\theta''[\mathsf{t}] + \theta'[\mathsf{t}] + \theta + \theta^{3} == 10 \, \mathsf{Cos}[1.5 \, \star \, \mathsf{t}], \, \theta[\mathsf{0}] == 0, \, \theta'[\mathsf{0}] == 3\}, \, \theta, \, \{\mathsf{t}, \, \mathsf{0}, \, \mathsf{10}\}] \\ &\mathsf{NDSolve} : \, \mathsf{Equation} \, \mathsf{or} \, \mathsf{list} \, \mathsf{of} \, \mathsf{equations} \, \mathsf{expected} \, \mathsf{instead} \, \mathsf{of} \, \mathsf{True} \, \mathsf{in} \, \mathsf{the} \, \mathsf{first} \, \mathsf{argument} \\ &\{\theta + \theta^{3} + \theta'[\mathsf{t}] + \theta''[\mathsf{t}] == 10 \, \mathsf{Cos}[1.5 \, \mathsf{t}], \, \mathsf{True}, \, \mathsf{True}\}. \end{aligned}$ $&\mathsf{Out}_{[7]} = \, \mathsf{NDSolve}[\{\theta + \theta^{3} + \theta'[\mathsf{t}] + \theta''[\mathsf{t}] == 10 \, \mathsf{Cos}[1.5 \, \mathsf{t}], \, \mathsf{True}, \, \mathsf{True}\}, \, \theta[\mathsf{t}], \, \{\mathsf{t}, \, \mathsf{0}, \, \mathsf{10}\}]$

In[207]:= eq = x ' '[t] + (ω^2) * x[t] + ϵ b x[t]^3 x[t_] = x0[t] + ϵ x1[t] Expand[eq] Collect[Expand[eq], ϵ] eq0 = Coefficient[Expand[eq], ϵ , 0] eq1 = Coefficient[Expand[eq], ϵ , 1] s0 = DSolve[{eq0 == 0, x0[0] == 0, x0 '[0] == 0}, x0, t] eq1 /. s0[1]] s1 = DSolve[{(eq1 /. s0[1]) == 0, x1[0] == a, x1 '[0] == 0}, x1, t] x[t] xa[t_] = x[t] /. s0[1] /. s1[1] a = b = ω = 1; Plot[xa[t] /. ϵ \rightarrow 0, {t, 0, 6 π }]

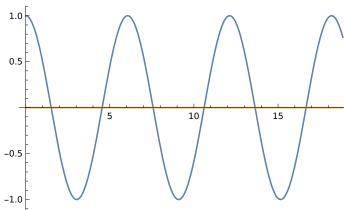
```
Out[207]=
           x0[t] + \epsilon x1[t] + \epsilon (x0[t] + \epsilon x1[t])^3 + x0''[t] + \epsilon x1''[t]
Out[208]=
           x0[t] + \epsilon x1[t]
Out[209]=
           \times 0[t] + \epsilon \times 0[t]^3 + \epsilon \times 1[t] + 3 \ \epsilon^2 \times 0[t]^2 \times 1[t] + 3 \ \epsilon^3 \times 0[t] \times 1[t]^2 + \epsilon^4 \times 1[t]^3 + \times 0''[t] + \epsilon \times 1''[t]
Out[210]=
           Out[211]=
           x0[t] + x0''[t]
Out[212]=
           x0[t]^3 + x1[t] + x1''[t]
Out[213]=
           \{\{x0 \rightarrow Function[\{t\}, 0]\}\}\
Out[214]=
           x1[t] + x1''[t]
Out[215]=
           \{\{x1 \rightarrow \mathsf{Function}[\{\mathsf{t}\},\,\mathsf{Cos}[\mathsf{t}]]\}\}
Out[216]=
           x0[t] + \epsilon x1[t]
Out[217]=
           \epsilon Cos[t]
Out[219]=
            1.0
            0.5
                                                         10
           -0.5
           -1.0
```

 $\text{In}[250] := \ \, \text{nds} = \text{NDSolve}[\{X \,' \,' \,[t] + \omega^2 \, X[t] + 0.1 \, b \, X[t]^3 == 0 \,, \, X[0] == a \,, \, X \,' \,[0] == 0 \}, \, X \,, \, \{t \,, \, 0 \,, \, 6 \, \pi \}]$ $\text{Plot}[\{X[t] \, / \, . \, \text{nds}[1] \,, \, xa[t] \, / \, . \, \epsilon \rightarrow 0.1 \}, \, \{t \,, \, 0 \,, \, 6 \, \pi \}]$

Out[250]=

 $\left\{ \left\{ X \to InterpolatingFunction \middle[\begin{array}{c} \blacksquare \\ \hline \\ Output: scalar \end{array} \right] \right\} \right\}$





```
ln[44] := eq = x''[t] + (\omega^2) * x[t] - \epsilon b (x'[t]^2) x[t]
                                    x[t_{}] = x0[t] + \epsilon x1[t]
                                  Collect[Expand[eq], \epsilon]
                                    eq0 = Coefficient[Expand[eq], \epsilon, 0]
                                    eq1 = Coefficient[Expand[eq], \epsilon, 1]
                                    s0 = DSolve[{eq0 == 0, x0[0] == 0, x0 '[0] == 1}, x0, t]
                                    s1 = DSolve[{(eq1 /. s0[1]) == 0, x1[0] == 0, x1'[0] == 0}, x1, t]
                                   xa[t] = x[t] /. s0[1] /. s1[1]
                                    a = b = \omega = 1;
                                    nds =
                                         NDSolve[{X''[t] + (\omega^2) * X[t] - 0.1 b (X'[t]^2) X[t] == 0, X[0] == 0, X'[0] == 1}, X, \{t, 0, 6\pi\}]
                                   Plot[{X[t] /. nds[1], xa[t] /. \epsilon \to 0.1}, {t, 0, 6 \pi}]
 Out[44]= \times 0[t] + \epsilon \times 1[t] - \epsilon (\times 0[t] + \epsilon \times 1[t]) (\times 0'[t] + \epsilon \times 1'[t])^2 + \times 0''[t] + \epsilon \times 1''[t]
 Out[45]= \times 0[t] + \epsilon \times 1[t]
 \text{Out}[46] = \quad \times 0[t] - \epsilon^4 \times 1[t] \times 1'[t]^2 + \epsilon^2 \left( - \times 1[t] \times 0'[t]^2 - 2 \times 0[t] \times 0'[t] \times 1'[t] \right) + \epsilon^2 \left( - \times 1[t] \times 0'[t]^2 - 2 \times 0[t] \times 0'[t] \times 1'[t] \right) + \epsilon^2 \left( - \times 1[t] \times 0'[t] \times 1'[t] \right) + \epsilon^2 \left( - \times 1[t] \times 0'[t] \times 1'[t] \right) + \epsilon^2 \left( - \times 1[t] \times 0'[t] \times 1'[t] \times 1'[t] \right) + \epsilon^2 \left( - \times 1[t] \times 0'[t] \times 1'[t] \times 1'[t] \right) + \epsilon^2 \left( - \times 1[t] \times 0'[t] \times 1'[t] \times 1'[t] \right) + \epsilon^2 \left( - \times 1[t] \times 0'[t] \times 1'[t] \times 1'[t] \times 1'[t] \right) + \epsilon^2 \left( - \times 1[t] \times 0'[t] \times 1'[t] \times 1
                                          \epsilon^{3} (-2 x1[t] x0'[t] x1'[t] - x0[t] x1'[t]<sup>2</sup>) + x0''[t] + \epsilon (x1[t] - x0[t] x0'[t]<sup>2</sup> + x1''[t])
 Out[47]= x0[t] + x0''[t]
 Out[48]= x1[t] - x0[t] x0'[t]^2 + x1''[t]
 Out[49]= \{\{x0 \rightarrow Function[\{t\}, Sin[t]]\}\}
Out[50]= \left\{\left\{x1 \rightarrow \operatorname{Function}\left[\left\{t\right\}, \frac{1}{32}\left(-4 \operatorname{t} \operatorname{Cos}[t] + 8 \operatorname{Sin}[t] - 8 \operatorname{Cos}[t]^{4} \operatorname{Sin}[t] + \operatorname{Cos}[t] \times \operatorname{Sin}[4 \operatorname{t}]\right)\right]\right\}\right\}
Out[51]= Sin[t] + \frac{1}{32} \epsilon \left(-4 \text{ t Cos[t]} + 8 \text{ Sin[t]} - 8 \text{ Cos[t]}^4 \text{ Sin[t]} + \text{Cos[t]} \times \text{Sin[4 t]}\right)
Out[54]=
                                                                                                                                                                                                                                               15
                                    -1.0
```

Out[8]= $\omega^2 x[t] + \epsilon bx[y] + x''[t]$