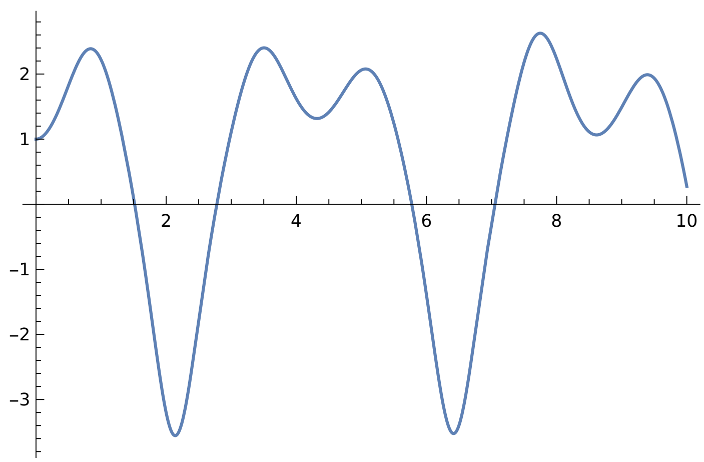
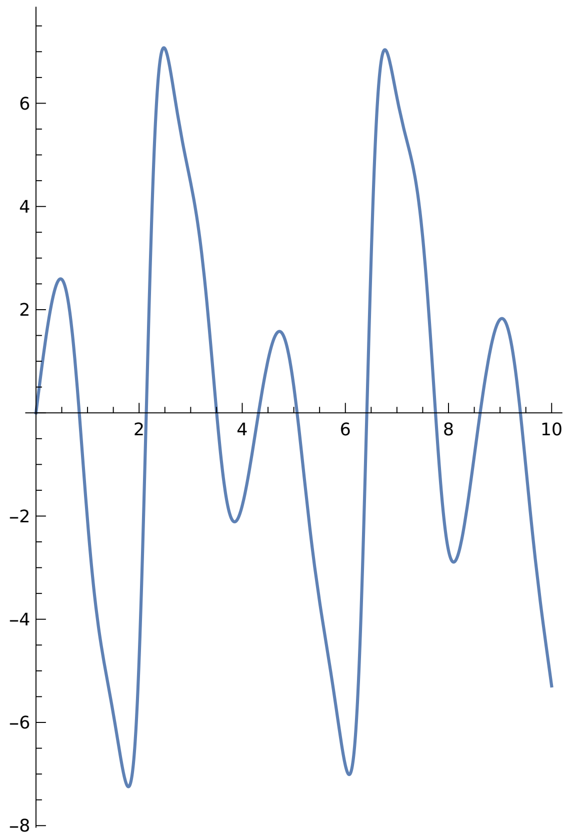
```
In[48]:= γ = 0.1; α = -1; f = 10; ω = 1.5
         ndsol = NDSolve[{x''[t] + γ * x'[t] + x[t] - α * (x[t])^3 == f * Cos[ω * t],
             x'[0] == 0, x[0] == 1}, x, {t, 0, 10}]

Out[48]= 1.5

Out[49]= {{x → InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ]}}

In[53]:= x[t] /. ndsol[[1]]

Out[53]= InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ][t]

Out[52]= InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ][t]

In[61]:= Plot[Evaluate[{x[t]} /. ndsol[[1]]], {t, 0, 10}, AspectRatio → Automatic]
         Plot[Evaluate[{x'[t]} /. ndsol[[1]]], {t, 0, 10}, AspectRatio → Automatic]
         Plot[Evaluate[{x[t], x'[t]} /. ndsol[[1]]], {t, 0, 10}, AspectRatio → Automatic]
```

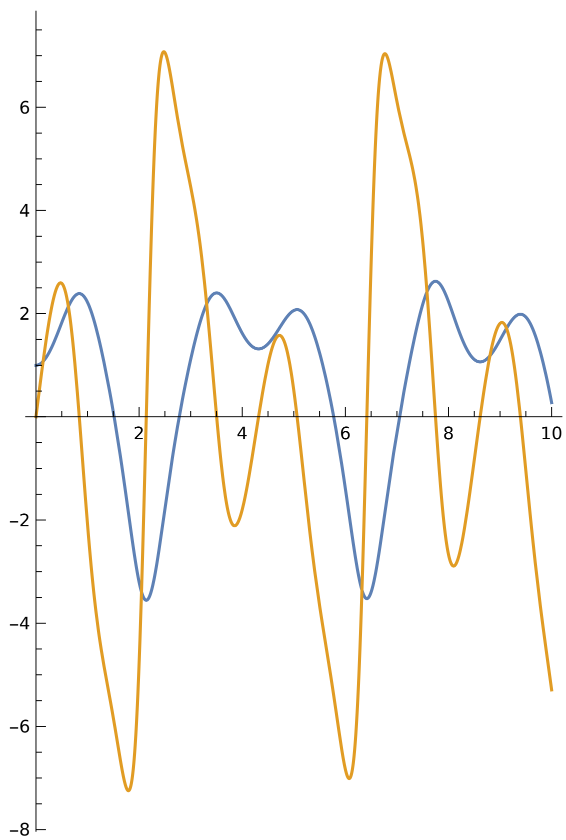
Out[61]=



Out[62]=



Out[63]=



In[64]:=
```
eq = x''[t] + ω^2 x[t] + ϵ * b * (x[t])^3
```
Out[64]= $2.25\, x[t] + b\, \epsilon\, x[t]^3 + x''[t]$

In[65]:=
```
x[t_] = x0[t] + ϵ * x1[t]
```
Out[65]= $x0[t] + \epsilon\, x1[t]$

In[66]:=
```
Expand[eq]
```
Out[66]= $2.25\, x0[t] + b\, \epsilon\, x0[t]^3 + 2.25\, \epsilon\, x1[t] + 3\, b\, \epsilon^2\, x0[t]^2\, x1[t] + 3\, b\, \epsilon^3\, x0[t]\, x1[t]^2 + b\, \epsilon^4\, x1[t]^3 + x0''[t] + \epsilon\, x1''[t]$

In[67]:=
```
Collect[Expand[eq], ϵ]
```
Out[67]= $2.25\, x0[t] + 3\, b\, \epsilon^2\, x0[t]^2\, x1[t] + 3\, b\, \epsilon^3\, x0[t]\, x1[t]^2 + b\, \epsilon^4\, x1[t]^3 + x0''[t] + \epsilon\left(b\, x0[t]^3 + 2.25\, x1[t] + x1''[t]\right)$

In[69]:=
```
eq0 = Coefficient[Expand[eq], ϵ, 0]
```
Out[69]= $2.25\, x0[t] + x0''[t]$

Out[68]= $2.25\, x0[t] + x0''[t]$

In[70]:=
```
eq1 = Coefficient[Expand[eq], ϵ, 1]
```
Out[70]= $b\, x0[t]^3 + 2.25\, x1[t] + x1''[t]$

In[72]:=
```
s0 = DSolve[{eq0 == 0, x0[0] == a, x0'[0] == 0}, x0, t]
```
Out[72]= $\{\{x0 \to \text{Function}[\{t\}, 1.\, a \cos[1.5\, t]]\}\}$

In[73]:=
```
eq1 /. s0[[1]]
```
Out[73]= $1.\, a^3\, b \cos[1.5\, t]^3 + 2.25\, x1[t] + x1''[t]$

In[78]:=
```
s1 = DSolve[{(eq1 /. s0[[1]]) == 0, x1[0] == 0, x1'[0] == 0}, x1, t]
```
Out[78]= $\{\{x1 \to \text{Function}[\{t\}, -0.111111\left(-1.\, a^3\, b \cos[1.5\, t] + 1.\, a^3\, b \cos[1.5\, t]^5 + 2.25\, a^3\, b\, t \sin[1.5\, t] + 1.\, a^3\, b \sin[1.5\, t] \times \sin[3.\, t] + 0.125\, a^3\, b \sin[1.5\, t] \times \sin[6.\, t]\right)]\}\}$

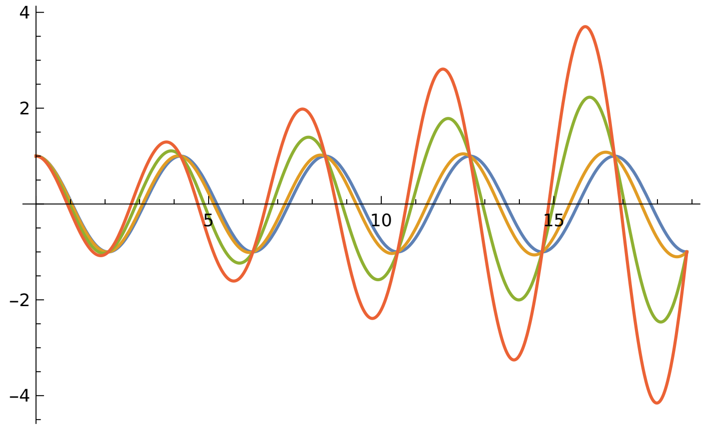
In[81]:=
```
x[t]
xa[t_] = x[t] /. s0[[1]] /. s1[[1]]
```
Out[81]= $x0[t] + \epsilon\, x1[t]$

Out[82]= $1.\, a \cos[1.5\, t] - 0.111111\, \epsilon\left(-1.\, a^3\, b \cos[1.5\, t] + 1.\, a^3\, b \cos[1.5\, t]^5 + 2.25\, a^3\, b\, t \sin[1.5\, t] + 1.\, a^3\, b \sin[1.5\, t] \times \sin[3.\, t] + 0.125\, a^3\, b \sin[1.5\, t] \times \sin[6.\, t]\right)$

In[83]:=
```
a = b = ω = 1;
```

In[85]:=
```
Plot[ Evaluate[xa[t] /. ϵ → {0, 0.1, 0.5, 0.9}], {t, 0, 6 π}]
```

Out[85]=

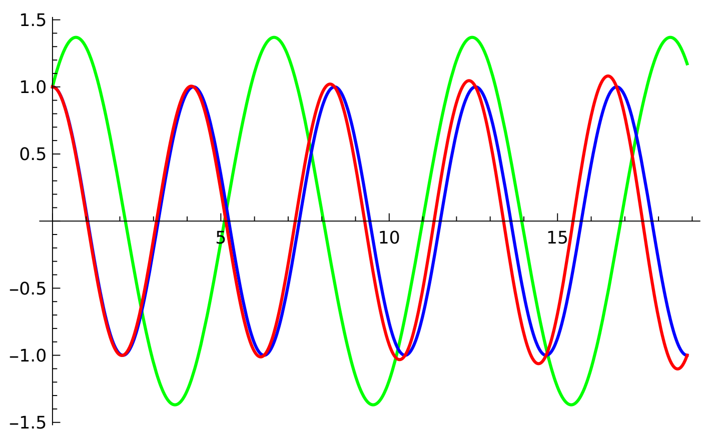


In[87]:=
```
nds =
 NDSolve[{X''[t] + ω^2 * X[t] + 0.1 b * X[t]^3 == 0, X[0] == a, X'[0] == b}, X, {t, 0, 6 π}]
```

Out[87]=
```
{{X → InterpolatingFunction[ Domain: {{0., 18.8}} Output: scalar ]}}
```

In[89]:=
```
Plot[{X[t] /. nds[[1]], x0[t] /. s0[[1]], xa[t] /. ϵ → 0.1},
 {t, 0, 6 π}, PlotStyle → {Green, Blue, Red}]
```

Out[89]=



In[91]:=
```
Plot[{X[t] /. nds1[[1]], x0[t] /. s0[[1]], xa[t] /. ϵ → 0.1},
 {t, 0, 6 π}, PlotStyle → {Blue, Red, Green}]
```

ReplaceAll: {nds1[[1]]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

ReplaceAll: {nds1[[1]]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

ReplaceAll: {nds1[[1]]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General: Further output of ReplaceAll::reps will be suppressed during this calculation.

Out[91]=