

# Farmer Olympiad Round 2 Paper 1

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1. Which one of the following shapes is the odd one out?  
A. Coffee Mug   B. Circle   C. Pyramid   D. Doughnut   E. Triangle
2. A botanist is trying to understand the equation  $x^4 + 24x^2 + y + 16 = y^3 + 8x^3 + y^2 + 32x$ .
  - (a) He first draws the curve given by the equation. Determine whether the curve goes through the origin, and hence or otherwise, find the points at which the curve intersects the x and y-axes.
  - (b) Find points such that  $x \geq 1.4$  at which the curve intersects the curve given by the quintic equation  $x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32 = 0$
3. An  $n$ -th degree polynomial function  $f(x)$  is such that  $f(4) = f(8)$ .
  - (a) What can be deduced about the shape of  $f(x)$ ?
  - (b) Let  $g(x)$  be another polynomial function given by the equation  $g(x) = ax^2 + bx + c$ .
4. Let every 49th second of a minute be a grand second. Let every 49th minute of an hour be a grand minute.
  - (a) What is the smallest angle between the second and minute hands of a clock at the grand second of a grand minute?
  - (b) Let a grand day be a date in the form  $YYYY - MM - DD$  where the sum of each digit is 49. What is the earliest date for a grand day after the start of the year 2000?
  - (c) Let a grand time be a date and a time in the form  $YYYY - MM - DD - hh - mm - ss$  where the sum of each digit is 49. What is the earliest date and time for a grand time after the start of the year 2000?
5. Prove or disprove whether the following expressions converge.
  - (a)  $\sum_{n=1}^{\infty} \frac{kx^n}{n!}$
  - (b)
    - i.  $\int_0^{\infty} e^{-\omega t} (\sin^2(t) - 3\cos^3(t) + 7x^3)$
    - ii. Evaluate the expression in part (i).
  - (c)
    - i.  $\sum_{x=1}^{\infty} \frac{2(1+5x)^5 - 6250(1+x)^5 + 25000(1+x)^4 - 39998(1+x)^3 + 31994x^2 + 51194x + 21246}{x^5}$
    - ii. Evaluate the expression in part (i).