Farmer Olympiad Round 3

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You have 45 minutes to complete all 3 questions. Give answers on a separate sheet.
You may use a scientific calculator.

1. A certain (rather grand) computer scientist wants to sort his mp4 files. While he tries sorting them in groups of 7866 he ends up with 49 files left over. When he tries sorting them in groups of 49 he ends up with 4 files left over. What is the smallest possible number of mp4 files that fits this description? [19 marks]

2. Being given that $f(x) = \sin(x) + \cos(x)$, simplify $f(f(f(x)))$ into a form such that no function is given more than one term as an input.

   [15 marks]

3. A certain farmer is arranging his twenty crops. He can split his crops into groups of size $\mathbb{Z}^+ \cap [1, N]$ where N is the total number of crops (while obviously still retaining the total number of crops).

   i. Find the number of possible groupings the farmer can arrange these crops in. [6 marks]

   ii. Find the number of possible groupings such that each group has an odd number of crops. [4 marks]

   iii. Find the number of possible groupings such that no two groups have the same number of crops. [4 marks]

   iv. What do you realise? [1 mark]