Farmer Mathematics: Matrix Transformation

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A certain rabbit told me he hates this topic!

However, he is wrong, and this topic is very useful!
2D transformations can be written as 2×2 matrices, and likewise, 3D transformations can be written as 3×3 matrices, with which points in the form of column vectors can be multiplied to undergo the transformation.

Enlargement For enlargement with centre $(0, 0)$, all you need is a scalar multiple of the identity matrix by the scale factor.

For specific axes, simply have the scalar multiple of the value in the corresponding leading diagonal for the axis. Usually from top left to bottom right these go x, y, z in the first 3 dimensions, above which I doubt you'll encounter.

$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \text{ and } \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

Shears Another transformation you now need to know about is called a shear, which can be best described as sliding of a point with the magnitude of the displacement being in proportion to its distance to another point, usually the origin.

In 2-dimensional spaces, shearing comes in 2 varieties; x-invariant and y-invariant.

x-invariant shears are in the form $\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$, x-invariance meaning points move parallel to the x-axis, and hence the x-axis is not affected by the shear.

Likewise, y-invariant shears are in the form $\begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$, y-invariance meaning points move parallel to the y-axis, and hence the y-axis is not affected by the shear.

Rotation in 2D Rotations are easy to learn, hard to master. In 2D, anticlockwise rotations around the origin are in the form $\begin{bmatrix} cos\phi & -sin\phi \\ sin\phi & cos\phi \end{bmatrix}$. If you want a clockwise rotation, simply use negative values for ϕ, and if you want a point other than the origin, use relative position.

Rotation in 3D 3D rotation differs based on which axis you will be rotating around.

If you are rotating about the x-axis: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\phi & -sin\phi \\ 0 & sin\phi & cos\phi \end{bmatrix}$

If you are rotating about the y-axis: $\begin{bmatrix} cos\phi & 0 & sin\phi \\ 0 & 1 & 0 \\ -sin\phi & 0 & cos\phi \end{bmatrix}$

If you are rotating about the z-axis: $\begin{bmatrix} cos\phi & -sin\phi & 0 \\ sin\phi & cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

is the matrix you need.

To do any sort of graph work, just multiply the individual points by the transformation matrix.

For multiple transformations at once, the multiple of the individual transformation matrices will give you the resultant transformation matrix.

To find invariant points/lines, simply equate the matrix multiplied by the point (x, y) to (x, y) or variations of (x, y) fitting a given linear equation.

gl hf in your next exam lol.

1 The Farmer's Easy Problems

1) A certain rabbit is snooking some stars. A snooker ball at position $(44, 72)$ is reflected in the x-axis and then undergoes an x-invariant shear of 3 units. Calculate the final position of the ball.
2) A rotten egg was using my pull-up bar so I rotated him about the z-axis clockwise by $\frac{\pi}{3}$ relative to the origin. What is the matrix that this transformation can be conveyed by?
3) I went to G's fitness club and my arms grew relative to the origin by scale factor 5 in the x-axis, 7 in the y-axis and 11 in the z-axis. What matrix reflects this change?
4) This time I decided to snook some stars. I was trying to play the game normally, but due to my extreme buffness I accidentally rotated the table by 45 degrees anticlockwise around the y-axis and then 90 degrees clockwise around the x-axis, all relative to the origin. Find a matrix that describes this.
5) i) An inclined slope has points $O(0, 0)$, $A(7, 5)$ and $B(7, 0)$. Find a, b where the slope is invariant to the transformation $\begin{bmatrix} 8 & b \\ a & 7 \end{bmatrix}$.
ii) Find the angle θ by which this rough shlope of coefficient of friction $\frac{1}{2}$ is inclined. Particle P of mass mg is on the slope and connected to particle Q of mass $3kg$ by a string passed through a smooth frictionless pulley. Given that the system is in limiting equilibrium, find the Tension T.

2 The Farmer's Easy Problems: Solutions

1) $(260, 72)$

2) $\begin{bmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

4) $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix}$

5) i) $a = \frac{-30}{7}, b = \frac{-49}{5}$
ii) $29.4N$