Farmer Mathematics: Complex Numbers I

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June 30, 2022

Hi boys, today we will be going over complex numbers.
Complex numbers take the form $a+bi$, where i is the imaginary number.
Let $z = a + bi, z \in \mathbb{C} \Rightarrow \Re(z) = a, \Im(z) = b$
For instance, $(1 + 5i)(5 + i) = 5 + i + 25i - 5 = 26i$.
Each complex number z has a complex conjugate $z*$, such that
$\Re(z*) = \Re(z)$ and $\Im(z*) = -1 \cdot \Im(z)$.
$z + z*$
$= (a - bi) + (a - bi)$
$= a + bi + a - bi$
$= a + a + bi - bi$
$= (a + a) + (bi - bi)$
$= 2a + 0$
$= 2a$
$z \cdot z*$
$= (a + bi) \cdot (a - bi)$
$= a^2 - abi + abi + b^2$
$= a^2 + b^2$
Complex conjugates are particularly useful when doing complex division.
$\frac{a+bi}{c+di}$
$= \frac{(a+bi)(c-di)}{(c+di)(c-di)}$
$= \frac{ac-adi+bci+bd}{c^2+d^2}$
Which eliminates the imaginary part of the denominator.

An argand diagram of $z \in \mathbb{C}$ would have $\Re(z)$ in the horizontal, and $\Im(z)$ in the vertical axis.

Farmer's Easy Problems Wahoo!

1) A certain rabbit wants to multiply $23 + 3i$ by $3 + 23i$. Help him!

2) For some reason, a botanist has $78.75 - 26.25i$ plants but only $\frac{1}{16} - \frac{i}{8}$ plant pots. How many plants per pot would he plant on average? (Problem, dimensional analysts?)

3) i) The certain rabbit now wants to solve $6x^2 + 9x + 4 = 0$. Help him!

ii) Plot the solutions on an Argand diagram.

iii) Draw a line between the two points and find the equation of the line.

iv) What do you notice about the line from part (ii)?