Farmer Mathematics: Matrix Algebra I

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A certain rabbit told me this topic can challenge some people quite a bit. Luckily, since you're reading this, I doubt you're one of them.

Matrices can be simply put as a 2D array of numbers.

The order of a matrix is $m \times n$, where m is the number of rows and n is the number of columns.

Matrices can be transposed such that:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \Leftrightarrow A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

Matrix Operations Matrix operations work a bit differently to individual values.

Matrix addition just adds each value to its correspondent.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 4x & 3x \\ 2x & x \end{bmatrix} = \begin{bmatrix} 1+4x & 2+3x \\ 3+2x & 4+x \end{bmatrix}$$

Matrix subtraction works similarly.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 4x & 3x \\ 2x & x \end{bmatrix} = \begin{bmatrix} 1-4x & 2-3x \\ 3-2x & 4-x \end{bmatrix}$$

When multiplying a matrix by a scalar, just multiply each individual element by the scalar.

$$\lambda \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} \lambda & 2\lambda \\ 3\lambda & 4\lambda \end{bmatrix}$$

To multiply one matrix by another, follow this logic:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} a+2b & c+2d \\ 3a+4b & 3c+4d \end{bmatrix}$$

For matrix multiplication to work, the first matrix needs to have as many columns as the second matrix has rows. Otherwise, the result is non-conformable.

The Identity Matrix The Identity Matrix is a matrix that when multiplied (on either side) by another matrix, the result is unchanged. It has the same relationship to square matrices of the same order as the number 1 has to real (and complex) numbers.

The placement of the 1's in the Identity Matrix is always on what is known as the leading diagonal, so that's how you can find identity matrix of different (square) orders.

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \cdots$$

$$I \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Another thing to note is by multiplying scalar multiples of the identity matrix with another matrix, you get the scalar multiple of the other matrix.

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \lambda I \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \lambda a & \lambda b \\ \lambda c & \lambda d \end{bmatrix}$$

1 The Farmer's Easy Problems

1) A certain rabbit wants to find $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} 9 & 10 \\ 11 & 12 \end{bmatrix}$. Help him!

2) i) Carry out the following multiplication: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

ii) Now find: $(\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}^T)^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

iii) What do you notice about the answer to part (ii)?

3) i) Find M where $M \begin{bmatrix} 5 & -7 \\ 11 & -23 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

ii) What is M called in relation to $\begin{bmatrix} 5 & -7 \\ 11 & -23 \end{bmatrix}$?

4) Solve $\begin{bmatrix} 1 & 3 \\ 7 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ 23 \end{bmatrix}$

2 The Farmer's Easy Problems: Solutions

1) $\begin{bmatrix} 413 & 454 \\ 937 & 1030 \end{bmatrix}$

2) i) $\begin{bmatrix} 30 & 36 & 42 \\ 66 & 81 & 96 \\ 102 & 126 & 150 \end{bmatrix}$ ii) $\begin{bmatrix} 30 & 66 & 102 \\ 36 & 81 & 126 \\ 42 & 96 & 150 \end{bmatrix}$

iii)The result is the answer to part (ii) transposed.

3) i) $\begin{bmatrix} \frac{23}{38} & \frac{-7}{38} \\ \frac{11}{38} & \frac{-5}{38} \end{bmatrix}$

ii) The inverse of $\begin{bmatrix} 5 & -7 \\ 11 & -23 \end{bmatrix}$

4) $(\frac{70}{22}, \frac{16}{22})$