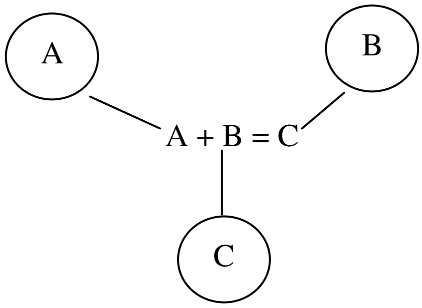
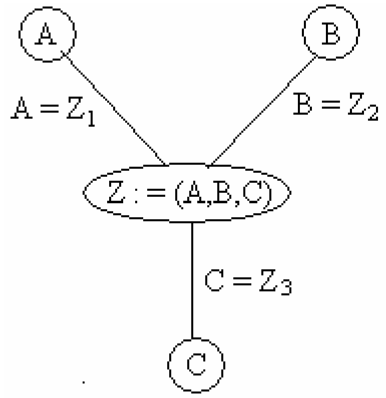
Sheet V, exercise 1.b - Show how a single ternary constraint such as “A+B=C” can be turned into three binary constraints by using an auxiliary variable. You may assume finite domains (Hint: consider a new variable that takes on values which are pairs of other values, and consider constraints such as “X is the first element of the pair Y”.)

Solution:
n-ary constraints always can be reduced to **binary constraints** by introducing additional **auxiliary variables** with the *cartesian product* of the original domains as new domain and the original **n-ary** constraint as unary constraint on the **auxiliary variable**.



Constraint Hyperhraph



Binarized Constraint Graph

Example:

After the introduction of the additional variable *Z* for the ternary constraint “A+B=C”, we have:

Variables: *A, B, C, Z*

Now consider the following finite domains for each variable:

domA : {0,1}
domB : {0,1}
domC : {0,1,2}
domZ : The constraint $C_{\{A,B,C\}}$ has a corresponding auxiliary variable, *Z*, whose domain can be the set {1,2,3,4} (a unique identifier for each of the four tuples in the constraint). We can define a correspondence between the values of *Z* and the tuples in $C_{\{A,B,C\}}$ as follows:

$$1 \mapsto (0,0,0), 2 \mapsto (0,1,1), 3 \mapsto (1,0,1), 4 \mapsto (1,1,2)$$

Notice that the tuple (0,0,1), for example, does not appear in the tuples above since the assignment $A \leftarrow 0, B \leftarrow 0, C \leftarrow 1$ violates the constraint “A+B=C”.

We then impose a constraint between the pairs of variables {A,Z},{B,Z},{C,Z}, giving the binary constraints:

$$C_{\{A,Z\}} = \{(0,1),(0,2),(1,3),(1,4)\}$$
$$C_{\{B,Z\}} = \{(0,1),(0,3),(1,2),(1,4)\}$$
$$C_{\{C,Z\}} = \{(0,1),(1,2),(1,3),(2,4)\}$$

For example, for $C_{\{A,Z\}}$, the value 2 for *Z* corresponds to the tuple (0,1,1) in which *A*=0 . Therefore, *Z*=2 is only compatible with *A*=0. More generally, the binary constraint between *A* and Z consists of a unique value for *A* for every value of *Z*.