

Numerical Exercise #1

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1 Question #1

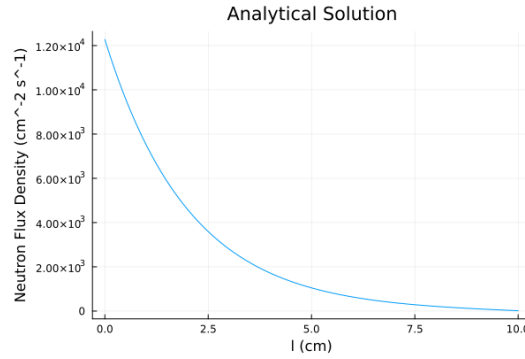


Figure 1: Analytical Solution for the neutron flux as a function of depth.

Flux values at the following locations (4 significant digits):

Flux value at x_0 : $13.589586060473522 \text{ cm}^{-2}\text{s}^{-1}$

Flux value at 0^+ : $12276.897875372839 \text{ cm}^{-2}\text{s}^{-1}$

2 Question #2

Relationship between Φ_i , Φ_{i+1} , Φ_{i-1} at any point within the material:

$$\frac{1}{dx^2}(\Phi_{i-1} - 2\Phi_i + \Phi_{i+1}) - \frac{1}{L}\Phi_i = 0 \quad (1)$$

Coefficients of the matrix A (4 significant digits), for a mesh size of 0.1cm:

Coef $A_{i,i} = -200.2411 \text{ cm}^{-2}$:

Coef $A_{i-1,i} = 99.9999 \text{ cm}^{-2}$:

Coef $A_{i+1,i} = 99.9999 \text{ cm}^{-2}$:

Relationship between Φ_i , $\Phi_{i-1/2}$, $\Phi_{i+1/2}$,
at the source:

$$J_x^+ \left(x_{n+\frac{1}{2}} \right) = \frac{S}{2} = \frac{1}{4}\Phi \left(x_{n+\frac{1}{2}} \right) - \frac{D}{2} \frac{d}{dx} \Phi \quad (2)$$

$$= \frac{\Phi_{n+\frac{1}{2}}}{4} - \frac{D}{2} \frac{\Phi_{n+\frac{1}{2}} - \Phi_n}{\frac{dx}{2}} \quad (3)$$

$$(4)$$

at the RHS of the problem:

$$J_x^-(x_{n+\frac{1}{2}}) = 0 = \frac{1}{4}\Phi(x_{n+\frac{1}{2}}) + \frac{D}{2}\frac{d}{dx}\Phi \quad (5)$$

$$= \frac{\Phi_{n+\frac{1}{2}}}{4} + \frac{D}{2}\frac{\Phi_{n+\frac{1}{2}} - \Phi_n}{\frac{dx}{2}} \quad (6)$$

$$(7)$$

Associated coefficients of the matrix A (4 significant digits), for a mesh size of 0.1cm: at the source:

$$A_{1,1} = -100.2412cm^{-2} \quad (8)$$

$$A_{1,2} = 100.0000cm^{-2} \quad (9)$$

$$A_{2,1} = 100.0000cm^{-2} \quad (10)$$

at the right hand side:

$$A_{n,n} = -146.5724cm^{-2} \quad (11)$$

$$A_{n-1,n} = 100.0000cm^{-2} \quad (12)$$

$$A_{n,n-1} = 100.0000cm^{-2} \quad (13)$$

3 Question #3

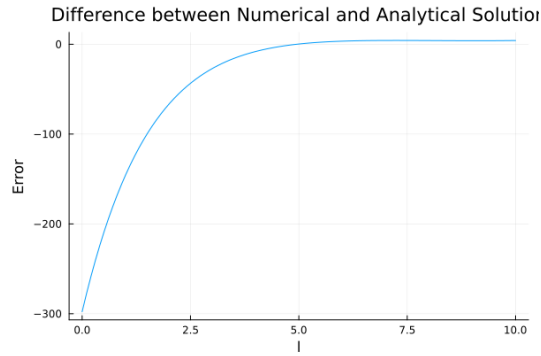


Figure 2: Distance between the solutions at each mesh point for a mesh size of 0.1 cm.

Flux values from the numerical solver at the following locations (4 significant digits):

Flux value at $0^+ = 11979.0980cm^{-2}s^{-1}$:

Flux value at $x_0 = 17.732cm^{-2}s^{-1}$:

In figure 3 you can see the sum of the errors and in 4 you can see the maximum deviation from the reference solution.

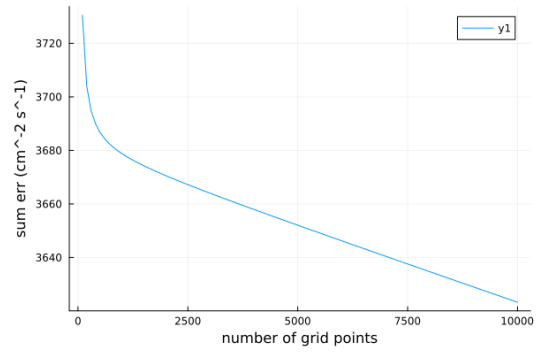


Figure 3: Evolution with mesh size of the absolute error of $\Phi(x_0)$.

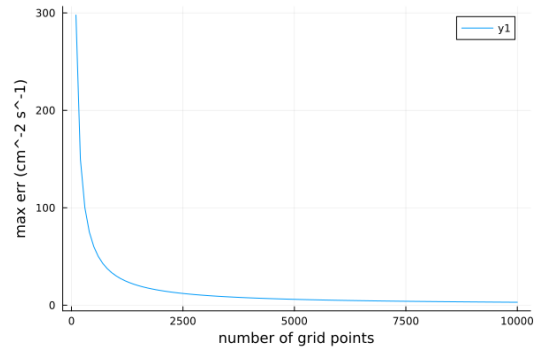


Figure 4: Evolution with mesh size of the absolute error of $\Phi(x_0)$.