

Numerical Exercise #1

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1 Question #1

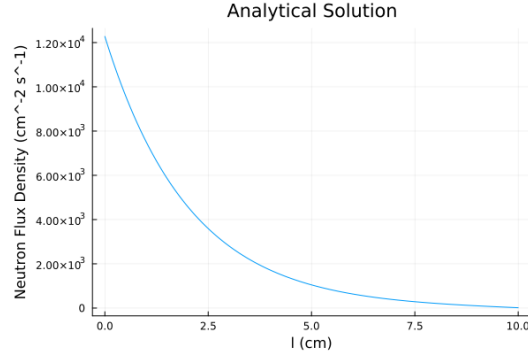


Figure 1: Analytical Solution for the neutron flux as a function of depth.

Flux values at the following locations (4 significant digits):
 Flux value at x_0 : $13.5896 \text{ cm}^{-2} \text{ s}^{-1}$
 Flux value at 0^+ : $12276.897875372839 \text{ cm}^{-2} \text{ s}^{-1}$

2 Question #2

The differential Equation is given as

$$D \frac{d^2}{dx^2} \Phi - \Sigma_a \Phi = 0 \quad (1)$$

Right hand side

$$\frac{1}{dx^2} (D\phi_{n-1} - (D + dxBC_n + \Sigma_a dx^2)\phi_n) = 0$$

Left hand side:

$$\frac{1}{dx^2} (-(D + \Sigma_a dx^2)\phi_1 + D\phi_2) = -\frac{S}{2dx}$$

Middle:

$$\frac{D}{dx^2} (\Phi_{i-1} - 2\Phi_i + \Phi_{i+1}) - \Sigma_a \Phi_i = 0$$

Relationship between Φ_i , Φ_{i+1} , Φ_{i-1} at any point within the material:

$$D \frac{d^2}{dx^2} \Phi - \Sigma_a \Phi = 0 \quad (2)$$

Coefficients of the matrix A (4 significant digits), for a mesh size of 0.1cm:
 Coef $A_{i,i} = -200.2411 \text{ cm}^{-2}$:
 Coef $A_{i-1,i} = 99.9999 \text{ cm}^{-2}$:

Coef $A_{i+1,i} = 99.9999cm^{-2}$:

Relationship between Φ_i , $\Phi_{i-1/2}$, $\Phi_{i+1/2}$,
at the source:

$$-- \quad (3)$$

at the RHS of the problem:

$$J_x^-(x_{n+\frac{1}{2}}) = 0 = \frac{1}{4}\Phi(x_{n+\frac{1}{2}}) + \frac{D}{2}\frac{d}{dx}\Phi \quad (4)$$

$$= \frac{\Phi_{n+\frac{1}{2}}}{4} + \frac{D}{2}\frac{\Phi_{n+\frac{1}{2}} - \Phi_n}{\frac{dx}{2}} \quad (5)$$

$$(6)$$

Associated coefficients of the matrix A (4 significant digits), for a mesh size of 0.1cm: at the source:

$$A_{1,1} = -100.2412cm^{-2} \quad (7)$$

$$A_{1,2} = 100.0000cm^{-2} \quad (8)$$

$$A_{2,1} = 100.0000cm^{-2} \quad (9)$$

at the right hand side:

$$A_{n,n} = -146.5724cm^{-2} \quad (10)$$

$$A_{n-1,n} = 100.0000cm^{-2} \quad (11)$$

$$A_{n,n-1} = 100.0000cm^{-2} \quad (12)$$

3 Question #3

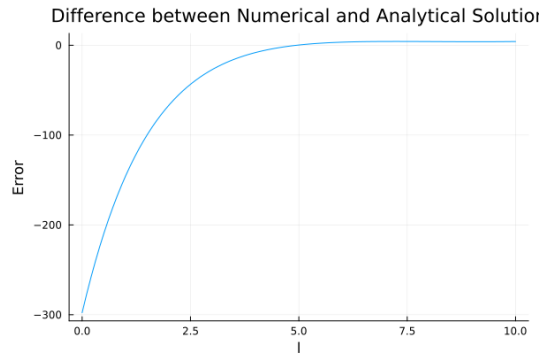


Figure 2: Distance between the solutions at each mesh point for a mesh size of 0.1 cm.

Flux values from the numerical solver at the following locations (4 significant digits):
 Flux value at $0^+ = 11979.0980 \text{ cm}^{-2} \text{ s}^{-1}$:
 Flux value at $x_0 = 17.732 \text{ cm}^{-2} \text{ s}^{-1}$:

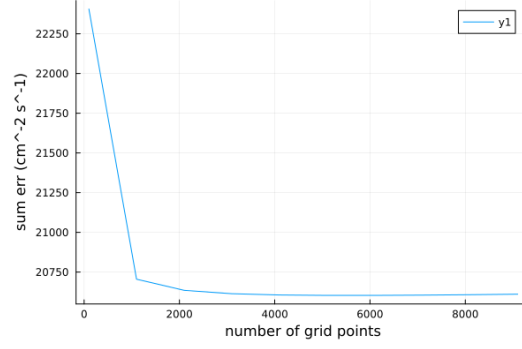


Figure 3: Evolution with mesh size of the absolute error of $\Phi(x_0)$.

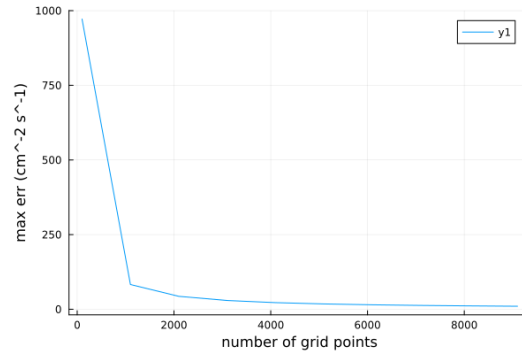


Figure 4: Evolution with mesh size of the absolute error of $\Phi(x_0)$.

In figure 3 you can see the sum of the errors and in 4 you can see the maximum deviation from the reference solution.