Numerical Exercise #1

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1 Question #1

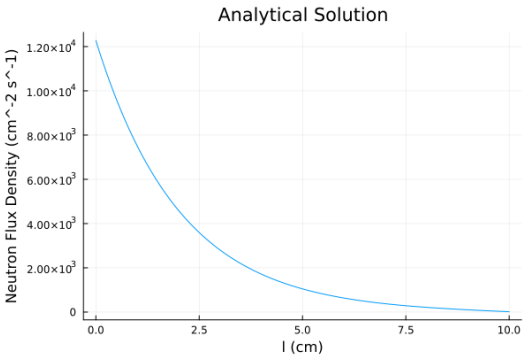

Figure 1: Analytical Solution for the neutron flux as a function of depth.

Flux values at the following locations (4 significant digits):
Flux value at x_0: 13.5896 $cm^{-2}s^{-}1$
Flux value at 0^{+}:12276.897875372839 $cm^{-2}s^{-}1$

2 Question #2

The differential Equation is given as

$$D\frac{d^2}{dx^2}\Phi - \Sigma_a\Phi = 0 \tag{1}$$

Right hand side

$$\frac{1}{dx^2}(D\phi_{n-1} - (D + dxBC_n + \Sigma_a dx^2)\phi_n) = 0$$

Left hand side:

$$\frac{1}{dx^2}(-(D + \Sigma_a dx^2)\phi_1 + D\phi_2) = -\frac{S}{2dx}$$

Middle:

$$\frac{D}{dx^2}(\Phi_{i-1} - 2\Phi_i + \Phi_{i+1}) - \Sigma_a\Phi_i = 0$$

Relationship between Φ_i, Φ_{i+1}, Φ_{i-1} at any point within the material:

$$D\frac{d^2}{dx^2}\Phi - \Sigma_a\Phi = 0 \tag{2}$$

Coefficients of the matrix A (4 significant digits), for a mesh size of 0.1cm:
Coef $A_{i,i} = -200.2411cm^{-2}$:
Coef $A_{i-1,i} = 99.9999cm^{-2}$:

Coef $A_{i+1,i} = 99.9999cm^{-2}$:

Relationship between $\Phi_i,\ \Phi_{i-1/2},\ \Phi_{i+1/2}$,
at the source:

$$--\tag{3}$$

at the RHS of the problem:

$$J_x^-\left(x_{n+\frac{1}{2}}\right) = 0 = \frac{1}{4}\Phi\left(x_{n+\frac{1}{2}}\right) + \frac{D}{2}\frac{d}{dx}\Phi \tag{4}$$

$$= \frac{\Phi_{n+\frac{1}{2}}}{4} + \frac{D}{2}\frac{\Phi_{n+\frac{1}{2}} - \Phi_n}{\frac{dx}{2}} \tag{5}$$

$$\tag{6}$$

Associated coefficients of the matrix A (4 significant digits), for a mesh size of 0.1cm: at the source:

$$A_{1,1} = -100.2412cm^{-2} \tag{7}$$

$$A_{1,2} = 100.0000cm^{-2} \tag{8}$$

$$A_{2,1} = 100.0000cm^{-2} \tag{9}$$

at teh right hand side:

$$A_{n,n} = -146.5724cm^{-2} \tag{10}$$

$$A_{n-1,n} = 100.0000cm^{-2} \tag{11}$$

$$A_{n,n-1} = 100.0000cm^{-2} \tag{12}$$

3 Question #3

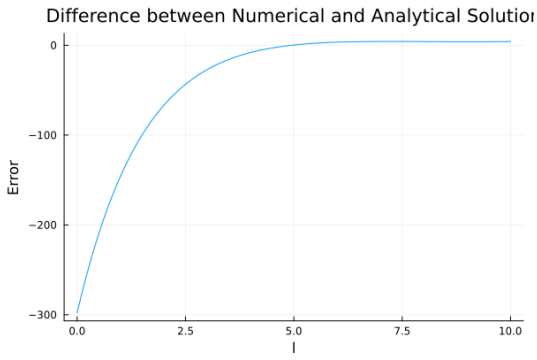

Figure 2: Distance between the solutions at each mesh point for a mesh size of 0.1 cm.

Flux values from the numerical solver at the following locations (4 significant digits):
Flux value at $0^+ = 11979.0980 cm^{-2}s^{-1}$:
Flux value at $x_0 = 17.732 cm^{-2}s^{-1}$:

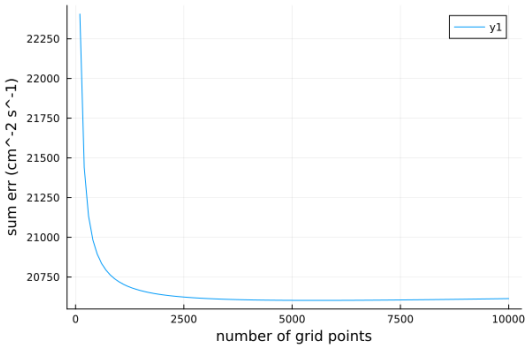

Figure 3: Evolution with mesh size of the absolute error of $\Phi(x_0)$.

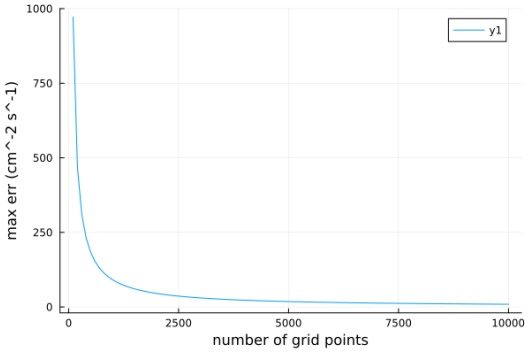

Figure 4: Evolution with mesh size of the absolute error of $\Phi(x_0)$.

In figure 3 you can see the sum of the errors and in 4 you can see the maximum deviation from the reference solution.