

Numerical Exercise #1

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1 Question #1

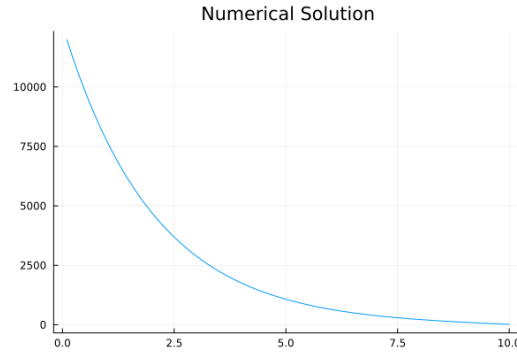


Figure 1: Analytical Solution for the neutron flux as a function of depth.

Flux values at the following locations (4 significant digits):
 Flux value at x_0 : $13.59 \text{ cm}^{-2}\text{s}^{-1}$
 Flux value at 0^+ : $12277 \text{ cm}^{-2}\text{s}^{-1}$

2 Question #2

The differential Equation is given as

$$D \frac{d^2}{dx^2} \Phi - \Sigma_a \Phi = 0 \quad (1)$$

Right hand side

$$\frac{1}{dx^2} (D\phi_{n-1} - (D + dxBC_n + \Sigma_a dx^2)\phi_n) = 0$$

Left hand side:

$$\frac{1}{dx^2} (-(D + \Sigma_a dx^2)\phi_1 + D\phi_2) = -\frac{S}{2dx}$$

Middle:

$$\frac{D}{dx^2} (\Phi_{i-1} - 2\Phi_i + \Phi_{i+1}) - \Sigma_a \Phi_i = 0$$

Associated coefficients of the matrix A (4 significant digits), for a mesh size of 0.1cm:
 at the source:

$$A_{1,1} = -8.31187 \quad (2)$$

$$A_{1,2} = 8.29187 \quad (3)$$

$$A_{2,1} = -8.29187 \quad (4)$$

at the right hand side:

$$A_{n,n} = 12.1536 \quad (5)$$

$$A_{n-1,n} = -8.29187 \quad (6)$$

$$A_{n,n-1} = -8.29187 \quad (7)$$

3 Question #3

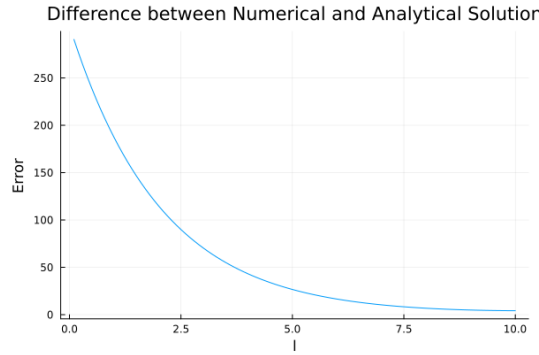


Figure 2: Distance between the solutions at each mesh point for a mesh size of 0.1 cm.

Flux values from the numerical solver at the following locations (4 significant digits):

Flux value at $0^+ = 11979 \text{ cm}^{-2} \text{ s}^{-1}$:

Flux value at $x_0 = 17.732 \text{ cm}^{-2} \text{ s}^{-1}$:

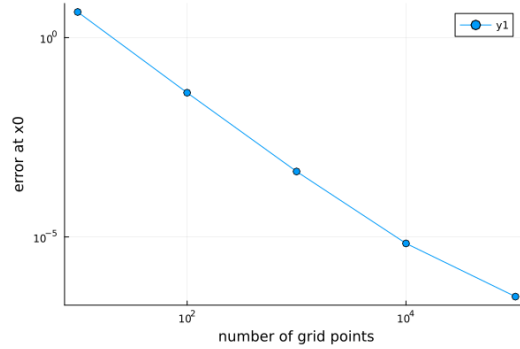


Figure 3: Evolution with mesh size of the absolute error of $\Phi(x_0)$.

In figure 3 you can see the sum of the errors and in 4 you can see the maximum deviation from the reference solution.

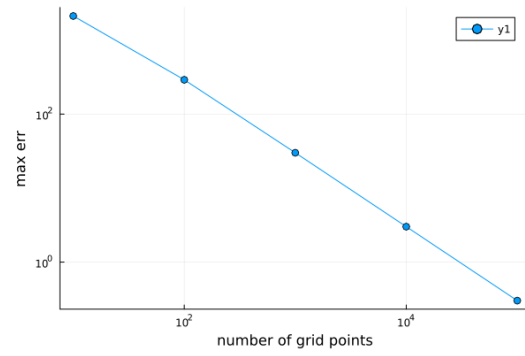


Figure 4: Evolution with mesh size of the absolute error of $\Phi(x_0)$.