Numerical Exercise #3

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1 Question #1 Numerical formulation

Using the laplace operator, that was implemented in the previous exercises

$$\triangle\phi_i = \frac{1}{dx^2}(\phi_{i+1} - 2\phi_i + \phi_{i-1})$$

the diffusion equation with downscattering and fission can be expressed at any point inside of the material as

$$D_1\triangle\phi_1 - \Sigma_{af}\phi_1 - \Sigma_{1\to 2}\phi_1 = -\frac{1}{k}(\nu\Sigma_{f_1}\phi_1 + \nu\Sigma_{f_2}\phi_2) \quad (1)$$

$$D_2\triangle\phi_2 - \Sigma_{as}\phi_2 + \Sigma_{1\to_2}\phi_2 = 0 \quad (2)$$

where ϕ_1 and ϕ_2 are understood to be at point i. The two equations can be organized in this matrix.

$$\begin{pmatrix} D_f\triangle - \Sigma_{af} - \Sigma_{1\to 2} & 0 \\ \Sigma_{1\to 2} & D_s\triangle - \Sigma_{as} \end{pmatrix} \begin{pmatrix} \phi_f \\ \phi_s \end{pmatrix} = -\frac{1}{k} \begin{pmatrix} \nu\Sigma_{ff} & \nu\Sigma_{fs} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \phi_f \\ \phi_s \end{pmatrix}$$

2 Question #2 Analytical solution for the bare system

We make a seperation of variables, seperating $\phi_g(x)$ into an energy independant $\phi(x)$ and an energy group g dependant amplitude φ_g:

$$\phi_g(x) = \varphi_g\phi(x)$$

then using that

$$\triangle\phi(x) = B\phi(x)$$

we can rewrite the equation

$$\begin{pmatrix} D_sB - \Sigma_{af} - \Sigma_{1\to 2} & 0 \\ \Sigma_{1\to 2} & D_fB - \Sigma_{as} \end{pmatrix} \begin{pmatrix} \varphi_f \\ \varphi_s \end{pmatrix} = -\frac{1}{k} \begin{pmatrix} \nu\Sigma_{ff} & \nu\Sigma_{fs} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \varphi_f \\ \varphi_s \end{pmatrix}$$

which is $A\varphi = \frac{F}{k}\varphi$ since A is a 2x2 matrix it can be easily inverted:

$$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} (D_fB - \Sigma_{as}) & 0 \\ -\Sigma_{1\to 2} & D_sB - \Sigma_{af} - \Sigma_{1\to 2} \end{pmatrix}$$

The determinant of A is given by:

$$\det(A) = (D_sB - \Sigma_a - \Sigma_{1\to 2})(D_fB - \Sigma_a)$$

$$M = A^{-1}F = \frac{1}{\det(A)} \begin{pmatrix} (D_fB - \Sigma_{as})\,\nu\Sigma_{ff} & (D_fB - \Sigma_{as})\,\nu\Sigma_{fs} \\ -\Sigma_{1\to 2}\nu\Sigma_{ff} & -\Sigma_{1\to 2}\nu\Sigma_{fs} \end{pmatrix} \quad (3)$$

The eigenvector of this matrix M can be found using numerical comutations.

$$k_{eff} = 1.0843329711143195 \quad (4)$$

keff (scientific format with 5 significant digits):

3 Question #3 Numerical solution of the bare homogeneous reactor

keff (scientific format with 5 significant digits): 0.0314159

fast buckling (scientific format with 5 significant digits):0.03135

thermal buckling (scientific format with 5 significant digits): 0.03135

analytical buckling : 0.03141

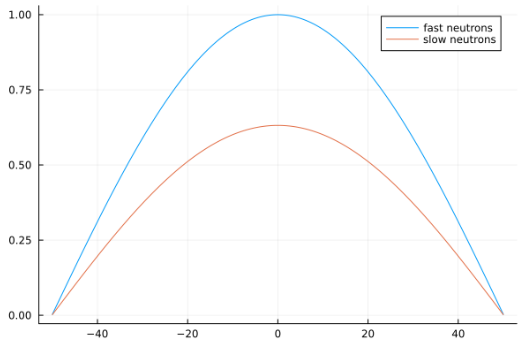

Figure 1: Fast and thermal fluxes in the bare reactor for a mesh size of 0.1 cm.

4 Question #4

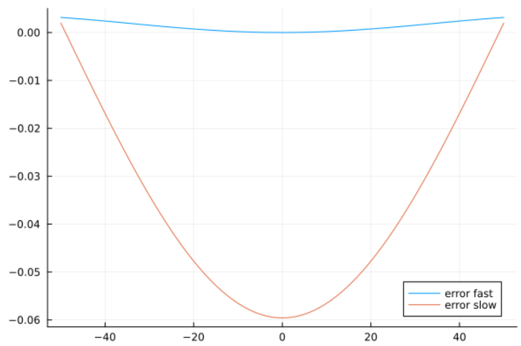

Figure 2: Difference between numerical and analytical solutions for the fast and thermal fluxes in the bare reactor for a mesh size of 0.1 cm.

5 Question #5 Numerical solution of the reflected reactor

keff (scientific format with 5 significant digits):

fast net current (scientific format with 5 significant digits) at the core/reflector interface:

thermal net current (scientific format with 5 significant digits) at the core/reflector interface:

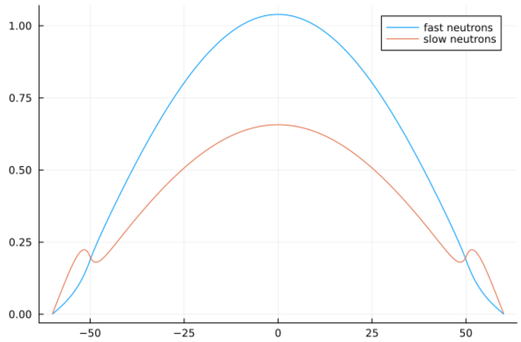

Figure 3: Fast and thermal fundamental fluxes in the reflected reactor for a mesh size of 0.1 cm.

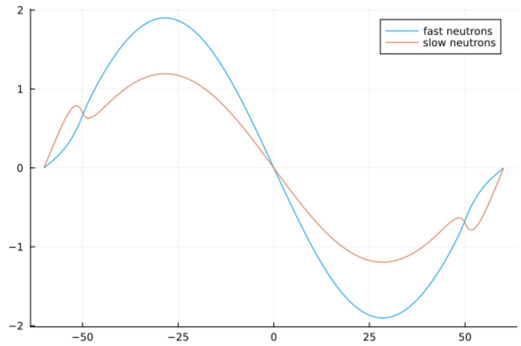

Figure 4: Fast and thermal first harmonic of the fluxes in the reflected reactor for a mesh size of 0.1 cm.

keff (scientific format with 5 significant digits):

first harmonic eigenvalue (scientific format with 5 significant digits):